Free Surface Flow
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Lecture 15

Welcome back, students, to the last lecture of problem-solving sessions on critical flow. And we will proceed. We are going to solve around 2 to 3 problems in this particular lecture, mostly indicating the importance of channel transitions. So, first, let me write down the question. Today's first question is a 2.5-meter-wide rectangular channel. It carries 6 m^3/s of flow, basically discharge, at a depth of 0.50 meters.

Now, calculate the minimum height of a streamlined, flat-topped hump required to be placed at a section to cause the critical flow over the hump. The energy loss over the hump can be taken as 10 percent of the upstream velocity head.

So, this is the question. So, the first thing that we are going to calculate here is discharge intensity. And that is q is equal to capital $\frac{Q}{B}$ or 6/2.5. It is a 2.5-meter-wide rectangular channel, and that will come out to be 2.40 m^3/s/m and, correspondingly, the velocity 1 will be $\frac{q}{y}$. Or V_1 is going to be 2.4/0.5, that is 4.8 m/s.

As we have seen before, it is very, I mean, very convenient to solve this value beforehand because this value is used for the calculation of specific energy. So, $4.8^2/2*9.81$, and that will come out to be 1.174 meters. Now, the energy loss will be h_L, what is given is 10% of the upstream velocity head. So, we get 0.1 of $\frac{V_1^2}{2g}$, that means 0.1 of 1.174 meters or 0.1174 meters, that is the energy loss.

Now, we will also calculate the Froude number F_1, that is $\frac{V_1}{\sqrt{gy_1}}$, or equal to $\frac{4.8}{\sqrt{9.8\times 0.5}}$.

This will come out to be 2.167. Right. As we have seen, we have derived many such, I mean, equations before.

So, I am using one of the equations if we use. So, in the derivation, we have seen many equations, right, in terms of F_1, F_2, from the theory, use the equation, which equation? $\frac{\Delta Z_m}{y_1}+\frac{h_L}{y_1}$ is equal to $\left[1+\frac{F_1^2}{2}-\frac{3}{2}F_1^{2/3}\right]$. If you remember, this equation we have seen in our theoretical, I mean, lectures, module 2. If we try putting in the value, this is what we need to find: $\frac{\Delta Z_m}{0.5}+h_L$, we know, so 0.1174, $1+\frac{(2.167^2)}{2}-\frac{3}{2}\times 2.167^{(2/3)}$, or on solving, we get $\frac{\Delta Z_m}{0.5}$ is equal to 0.6017, or ΔZ_m as 0.301, and this was what was needed to find. There is another way as well. ΔZ_m is nothing but $E_1-E_c-h_L$, and we need to find E_c, right? E_c is $\frac{3}{2}y_c$ or 1.5 times $\left(\frac{q^2}{g}\right)^{1/3}$. You can write 3/2 itself or $\frac{3}{2}\times\left(\frac{2.4^2}{9.81}\right)^{1/3}$

So, our E_c is going to be 1.256 meters. And we have already seen that h_L was 0.1174, and E_1 was nothing but y_1 plus $\frac{V_1^2}{2g}$. That is 0.5 + 1.174, which is 1.674. Therefore, using this equation. ΔZ_m is 1.674 - 1.256E_c - 0.1174, and using this also, ΔZ_m comes out to be 0.301 meters.

Both methods are correct. If you do not remember this particular formula, no problem. This is the first approach, and this is another approach. So, a rectangular channel is 3.5 meters wide and conveys a discharge of 15 m^3/s at a depth of 2 meters. It is proposed to reduce the width.

Of the channel at a hydraulic structure, assuming the transition to be horizontal and the flow to be frictionless, and determine the water flow. Surface elevations upstream and downstream of the when the constricted width is 2.5 meters and 2.20 meters. So, this is the question set.

So, how are we going to solve this particular problem? So, we say let suffixes 1 and 2 denote sections upstream and downstream of the transition Discharge Q can be written as B_1, y_1, *and* V_1. So, we will calculate V_1 as equal to $\frac{15}{3.5\times 2}$, 2.143 m/s, and similarly, the

Froude number or Froude number at 1 will be equal F_1 will be equal to $\frac{V_1}{\sqrt{gy_1}}$, that is $\frac{2.143}{\sqrt{9.81\times 2}}$, and that will come out to be 0.484. What do we conclude? The upstream flow is subcritical, which is important, right? What will the transition therefore,

The transition will cause a drop in water surface. So, how do we calculate E_1? It is nothing but $y_1+\frac{V_1^2}{2g}$. This is $2.0+\frac{2.143^2}{2\times 9.81}$ or 2.234 meters. So, this is the specific energy at 1.

We say let B_{2m} be equal to the minimum width at section 2, which does not cause choking. Then E_{cm} is equal to E_1, which is equal to 2.234 meters. And therefore, y_{cm} will be 2/3 E_{cm} or 2/3 * 2.234, which is 1.489 meters. And we also know, since y_{cm}^3 is equal to $\frac{Q^2}{gB_{2m}^{\ 2}}$, B_{2m} is going to be equal to $\left(\frac{Q^2}{gy_{c2}^{\ 3}}\right)^{(1/2)}$. Or $\left(\frac{15^2}{9.81\times 1.489^3}\right)^{0.5}$ =2.636 m .

So, this is the initial thing. Now we will go when it says for a part when the constructed width is 2.5 meters, and in the second case, it is 2.20 meters. So, we start with our first case. Now these are the basic essential steps. When B_1 is equal to 2.5 meters. So, here what is happening is B_2 is less than B_{2m}. So, B_{2m} here is greater than this one or B_1.

So, breadth is greater than B_{2m} minimum, and hence the choking condition prevails. Therefore, the depth at section 2, y_2, is equal to y_{c2}. The upstream depth, y_1, will increase to y'. So, actually, $q2$ will become $\frac{12}{2.5}$, which is 6.0 m^3/s/m, and y_{c2} will be $\left(\frac{q_2^{\ 2}}{g}\right)^{1/3}$.

1.542, and E_{c2} is coming out to be $1.5y_{c2}$ or 1.5 * 1.542, which is 2.3136 meters at the upstream section. So, we are going to solve at the upstream section because the choking condition is prevailing. At the upstream section 1, E_1' is equal to E_{c2}, which is equal to 2.3136, with a new upstream depth. Such that q_1 is equal to y_1V_1, which is equal to 15/3.5 or 4.2857 $m^3/s/m$. Hence, y_1' is equal to

$\frac{V_1^2}{2g}$ is equal to 2, sorry, this is plus. 2.3136 or $y_1 + \frac{4.2857^2}{2 \times 9.81 \times y_1^2}$ is equal to *y₁* plus $\frac{0.9362}{y_1'^2}$ is equal to 2.3136. We need to solve it using trial and error. Solving by trial and error and selecting a root that shall give subcritical flow.

We will get y_1' equal to 2.102 meters. This is the part a solution. Part b is when B is 2.20 meters. So, we see here that B is again less than B_{2m}. This means the choking condition prevails, okay.

So, almost repeating the same procedure, depth section 2 is equal to y_2 is equal to y_{c2} and q_2 is equal to $\frac{15.0}{2.20}$, that is 6.8182 meter square per second per meter. y_{c2} will be equal to $\left(\frac{q^2}{g}\right)^{1/3}, \left(\frac{6.8182^2}{9.81}\right)^{1/3}$ is equal to 1.6797 meters and therefore, E_{c2} will be $1.5y_{c2}$, that is 2.5195 meters at upstream section 1 new upstream depth is equal to y_1' and E_1' is equal to E_{c2} is equal or q_1 is equal to $V_1'y_1'$ is equal to same $\frac{15}{3.5}$, that is 4.2857 $m^3/s/m$. $y_1' + \frac{V_1'^2}{2gy_1'}$ is equal to 2.5195 or $y_1' + \left(\frac{4.2857^2}{2 \times 9.81 \times y_1^2}\right)$ is equal to 2.5195 or $y_1' + \frac{0.9362}{y_1'}$ is equal to 2.5195. So, this is again to be solved using trial and error.

And choose depth such that the flow remains subcritical, and we get y_1' is equal to 2.350 meters. So, it is important to note that for the same discharge, when B is less than B_{2m}, that is under choking conditions, the depth at the critical section will be different from y_{cm}, and it depends on the value of B. So, you see this value has changed here; it is 2.35, and here it was 2.10. So, this is basically a clear-cut demonstration of our choking conditions and other parameters. So, I think with this, we will close this lecture and this particular module, and I will see you in our next lecture and the next module on uniform flow. Thank you so much.