Free Surface Flow

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Lecture 14

Welcome, students, to yet another lecture of our third module, which is problems on critical flow. In the second module, we studied the theory about critical flow, and among this set of lectures, this is the fourth lecture where we are going to solve the problems again. So, without losing much time, I think we will get started. So, first, we write down the question: a rectangular channel has a width of 2 meters. And it carries a discharge of 4.80 m^3/s with a depth of 1.60 meters at a section. So, at a certain section, a small, smooth hump with a flat top and a height of 0.10 meters is proposed to be built. So, the hump size is also given. Now, we have to calculate the likely change in the water surface. Here, we shall neglect any energy loss. So, we will see the solution. So, there is an upstream section and there is a downstream section, right? So, let the suffixes 1 and 2 refer to the upstream.

And downstream sections respectively. So, what are we going to do? We know small q is nothing but capital Q divided by B, or this is 4.80 discharge, breadth is 2. With that is the breadth or width. So, we get small q as 2.40 meter cube per second per meter.

And therefore, you know velocity V_1 will be q divided by y_1. So, we have 2.40 divided by 1.6 is equal to 1.50 meters per second. It will be easier if we also find out this value V_1 squared divided by $2g$. So, V_1 is 1.5^2 divided by (2 * 9.81).

This will come for us to be So, we are finding the Froude number first at the upstream section, that is V_1 divided by (gy_1) to the power of 0.5, and that is V is V_1 is 1.5 divided by (gy_1) to the power of 0.5. 9.81 and then y_1 we have 1.6, and this will come out to be 0.379 Froude number, which will imply hence the upstream flow is subcritical, and the reason is the Froude number is less than 1. And therefore, the hump, what will it do?

It will cause a drop in the water surface. So, energy will be E_1, which is going to be $y_1+\frac{V_1^2}{2g}$, and y_1 *is* $1.6+\frac{V_1^2}{2g}$, as we have already calculated. That is 0.115, which is

1.715 meters. This is the energy at section 0.1. So, here we have calibrated almost everything at section 1.

Now, what will happen at section 2? At section 2, we know that energy, because of the hump, will decrease by ΔZ, and therefore, it will be 1.715 - 0.10. So, the energy at section 2 will be 1.615 meters. And it is very simple to calculate y_c. What should y_c be? y_c is nothing but $\left(\frac{q^2}{g}\right)^{1/3}$.

q was $\left(\frac{2.4^2}{9.81}\right)^{1/3}$ and that comes to be 0.837 meters. And critical energy, therefore, is going to be 3/2 y_c, or 3/2 * 0.837, which will give us 1.256 meters. So, the minimum specific energy at section 2, that is E_{c2}, is less than E_2.

The available Specific energy at that section. Hence, y_2 is greater than y_c, and the upstream depth y_1 will, therefore, remain unchanged. This is what we studied in quite detail in our theoretical lectures.

Now, how do we solve for depth y_2? The depth y_2 is calculated by solving the specific energy relation. That is, $y_2 + \frac{V_2^2}{2g}$ is equal to E_2, that is $y_2 + \frac{2.4^2}{2 \times 9.81 \times y_2^2}$ is equal to 1.615. So, how is this going to be solved?

By trial and error. And if we solve this using trial and error, we are going to get y_2 equal to 1.481 meters, 1.481 meters. So, y_2 has become 1.481. So, this concludes our first problem. We are going to go to another problem, this more like a derivation sort of thing.

The critical depth y_c is related to the alternate depths y_1 and y_2 in a channel by the equation below. And what is this equation that we have to prove? We have to derive this equation $\left(\frac{2y_1^2 y_2^2}{y_1 + y_2}\right)^{1/3}$. How are we going? So, we know that at alternate depths y_1 and y_2, the specific energy is going to be the same.

It is the same; that is, E_1 is equal to E_2. Therefore, y_1 is equal to $y_2+\frac{V_2^{\ 2}}{2g}$. This is the main equation. This is the specific energy equation.

Therefore, how are we going to write y_1 + instead of V_1? We are going to write $\left(\frac{Q^2}{B^2}\right)\frac{1}{y_1^{\ 2}}$, right? Because V can be written as $\frac{Q}{By}$, $\frac{Q}{A}$, and A is nothing but B. So, V is $\frac{Q}{A}$ or in general right. This is the thing that we have used and utilized here, right? It is equal to $y_2+\frac{1}{2g}\left(\frac{Q^2}{B^2}\right)\frac{1}{y_2^{\ 2}}$, or we can write $y_1+y_1+\left(\frac{q^2}{g}\right)\left(\frac{1}{2y_1^{\ 2}}\right)$ is nothing but $\frac{q^2}{g}$, right? Just rearranging $\frac{Q^2}{B^2}$ is small q, $2y_1^{\ 2}$ to $y_2+\frac{q^2}{g}$ square by $g\times\frac{1}{y_1^{\ 2}}$ and the reason is small q is capital $\frac{Q}{B}$, ok.

And now, what is $\frac{q^2}{g}$? What is $\frac{q^2}{g}$? Is y_c nothing but one-third of $\left(\frac{q^2}{g}\right)^{1/3}$? So, we are going to write $y_1+\frac{y_c^{\ 3}}{2y_1^{\ 2}}$ is equal to $y_2+\frac{y_c^{\ 3}}{2y_2^{\ 2}}$. So, the next step is if we multiply the above equation by $\frac{2y_1^{\ 2}}{y_2^{\ 2}}$, then we are going to get. So, which equation are we talking about?

This particular equation. If you multiply by $\frac{2y_1^{\ 2}}{y_2^{\ 2}}$, then we are going to get $\frac{2y_1^{\ 3}}{y_2^{\ 2}}+\left(y_2^{\ 2}\right)\left(y_c^{\ 3}\right)$ is equal to. $2y_1^{\ 2}y_2^{\ 3}+y_1^{\ 2}y_c^{\ 3}$ or $y_2^{\ 2}y_c^{\ 3}-y_1^{\ 2}y_c^{\ 3}$. We keep this term and this term here on the left-hand side. This term and this term we keep on the left-hand side.

And this term and this term we are keeping on the right-hand side. Then we are writing $2y_1^2y_2^3 - 2y_1^3y_2^2$ or just take y_c^3 common it becomes $y_2^2 - y_1^2$ is equal to $2y_1^2y_2^2(y_2 - y_1)$ or y_c^3 will be written as. $\frac{2y_1^2y_2^2(y_2 - y_1)}{(y_2 + y_1)(y_2 - y_1)}$, right? Or y_2. In the next step, this term and this term will get canceled.

So, we get $2y_1^2y_2^2(y_1 + y_2)$. Yes, or yc will be written as $\left[2y_1^2y_2^2(y_1 + y_2)\right]^{(1/3)}$, and this was something that we needed to derive or prove. So, this is just one example of how a derivation is done, I mean, in terms of, you know, without any numerical values, all right. So, we are going to head to our next problem; I mean, maybe we will quickly see a very small problem which we have done before. So, let me just take it up as well.

A rectangular channel, which is 3 meters wide, very simple, carries a discharge of 10 m^3/s. It has its specific energy of 2 meters of water. The question is to calculate alternate depths and number. So, a simple problem, therefore, a simple solution.

So, what is specific energy? It is $y + \frac{V^2}{2g}$ or $y + \frac{Q^2}{2gA^2}$ or $y + \frac{Q^2}{2gB^2y^2}$. Or we can simply substitute in the values. What is the specific energy given as? 2 is equal to y plus what is Q? That is 10, $2 \times 9.81 \times 3^2 \times y^2$, or this can be written as 2 is equal to $y + \frac{100}{176.58y^2}$. So, what is the solution? The solution is to solve using trial and error; it is a cubic equation.

So, what we get, we solve by trial and error. So, we get two roots, three roots; one is positive, one is negative. So, sorry, two are positive, one is negative. y_1 is 1.84 and y_2 is 0.64 meter. Now, we are going to calculate the Froude number.

So, these are one part of the solution. Alternate depths. So, calculating the Froude numbers, let us calculate Froude number 2 as $\frac{V}{\sqrt{gy_2}}$ *or v* is $\frac{10}{3 \times 0.64}$. Froude number 2 is 0.64 and $\sqrt{9.81 \times 0.64}$. So, this comes out to be 2.08.

This is greater than 1. Froude number 1 will be $\frac{V}{\sqrt{gy_1}}$, a very simple problem. $\frac{10}{3\times1.84\times\sqrt{9.81\times1.84}}$ that is 0.4264, this is less than 1. This is supercritical, corresponding to this is subcritical. So, we will have one more problem.

So, we saw this problem, right? This one: a rectangular channel has a width of 2 meters and carries a discharge of 4.8 with a depth of 1.6 meters at a certain section. A small smooth hump with a flat top and a height of 0.1 meters is proposed to be built. Calculate the likely change in the water surface here. We shall neglect energy losses. So, the next question is, in that particular, problem of the hump, that is the first question of this lecture. If the height of the hump is 0.5 meters, estimate the water surface elevation on the hump and at a section upstream.

That is the first part. Second part, estimate the minimum size of the hump to cause critical flow over the hump. So, from our first problem, we found out that F_1 was 0.379, E_1 was 1.715 meters, and y_c is equal to y_{c2}, which is equal to 0.837 meters.

This data we are taking from our previous findings, and then we saw that the available specific energy. at section 2 is equal to E_2, which is equal to $E_1 - \Delta Z$, and E_2 was 1.715 - 0.5, that is 1.215, because now the height of the hump has been increased to 0.5 meters. So, the available specific energy is less. We also know that E_2 is nothing but 3/2 y_{c2}, and that is equal to 1.256. So, 3/2 y_{c2} was 0.837, right?

So, 3/2 y_{c2} was 0.837. So, this came out to be 1.256 meters, okay, right? So, what we see is that the minimum specific energy at section 2 is greater than E_2, correct? That is the available specific energy at that section. Hence, what is going to happen?

The depth at section 2 will be at the critical depth. This is different from our first problem in the lecture. which means that y_2 will be equal to y_{c2}, that is equal to 1.256 meters. Now, because it will remain at critical depth, the upstream depth will increase. to *a* depth y_1' such that the new specific energy at the upstream section 1 is given

E_1' is equal to $E_{c2} + \Delta Z$. *or* E_1' is equal to $y_1' + \frac{V_1^2}{2g}$ is equal to $E_{c2} + \Delta Z$ or $y_1' + \frac{Q^2}{2g{y_1'}^2}$ is equal to 1.256 + 0.50, that is 1.756. *or* $y_1' + \frac{2.4^2}{2 \times 9.81 \times {y_1'}^2}$ is equal to 1.756 sorry, this is not y_1 square $y_1' + \frac{0.2936}{{y_1'}^2}$ is equal to 1.756. Now, what is how are we going to solve this?

By trial and error, and if we use that, we will select the positive root, right? And which positive root gives. y_1' is greater than y_2. So, out of this, we get y_1' as 1.648 meters, and this is the solution to our problem. So, with this, I will end today's lecture, and I will continue in the next one. Thank you so much.