Free Surface Flow
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Lecture 13

Welcome, students, to this lecture number 3 of our problems on critical flow, where we are solving problems related to the computation of critical depth, specific energy, and other parameters. So, with that being said, we are going to continue again. So, we will start with So, a trapezoidal channel with a bed width of 4 meters and side slopes of 1.5 horizontal to 1 vertical.

So, the channel is trapezoidal, B is 4, and the side slope is 1.5:1. It carries a certain discharge; that is the problem. So, the first part is based on observations, if the critical depth of the flow is estimated as 1.7 meters, calculate the discharge in the channel.

If this discharge is observed to be flowing at a depth of 2.5 meters in a reach, estimate the Froude number of the flow in that reach. So, as I tell you all the time, we have to look for the information in the question. So, the first information: the channel is trapezoidal, the bed has a width of 4 meters, and the slopes are 1.5:1.

In the first part, It is asking, based on the observation, if the critical depth. So, we have all; they have also given critical depth. So, we have to calculate discharge, right? So, this problem looks not that difficult, the first part especially.

So, how to tackle that? We know that for trapezoidal channel. First is to write the values: B is 4, m is 1.5, and yc is 1.7. So, for a trapezoidal channel at critical depth, A_c is $\left(B+my_c\right)\times y_c$. We know these values: $\left(4+1.5\times1.7\right)\times1.7$.

That gives us 11.135 m^2. So, this is the critical area. And what about the top width? Top width T_c is equal to $B + 2my_c$. This is the top width or $4+2\times1.5\times1.7$.

That gives us 9.10 meters. So, we have determined A_c and T_c, right? So, okay. So, let us start on a new page. At critical flow, We know that the critical flow condition is $\frac{Q^2}{g} = \frac{A_c{}^3}{T_c}$, and A_c we have already calculated as $\frac{(11.135)^3}{T_c}$, also we know it is 9.10. So, and so basically that means $\frac{Q^2}{9.81}$ is equal to $\frac{11.135^3}{9.10}$. The discharge we will get is Q equal to 38.58 m^3/s. So, that is the first part of the problem. Now, in the second part, it says the depth of the flow has changed.

It has changed. So, the question is to what value, right? That is given actually that y has become 2.5 meters, right, and we have to estimate the Froude number here. So, y is given as 2.50 meters. So, the area will be $(B+my)\times y$, that is $(4+1.5\times 2.5)\times 2.5$, which gives us 19.375 m^2. The top width T is given as $B+2my$ or $4+2\times 1.5\times 2.5$, which gives us 11.5 m.

And A we have found, we know $\frac{A}{T}$, which is $\frac{19.375}{11.50}$. You know, because we will use this $\frac{A}{T}$ for the calculation of the Froude number 1.685. So, we know $\frac{A}{T}$. So, first we calculate V as $\frac{Q}{A}$, where Q is what we already calculated before. We calculated from the previous step the discharge remains the same from part A. We calculate Q as 38.579, and the new area we have come up with is 19.375 or 1.99 meters per second. And therefore,

The Froude number formula F is given as $\frac{V}{\sqrt{g\frac{A}{T}}}$. The V value we write is 1.99, under the root of 9.81, and $\frac{A}{T}$ we have already calculated as 1.685. So, the Froude number is 0.490. So, in the new reach, the Froude number is 0.490. So, we will proceed to a new question.

The question is to calculate the bottom width of a channel required to carry a discharge of 15 m^3/s. as a critical flow at a depth of 1.2 meters. So, calculate the bottom width of a channel. So, basically B required to carry a discharge Q is given as a critical flow.

So, in a critical flow condition, y is also given as 1.2 meters, right? If the channel section is a) rectangular, So, b) it could be trapezoidal with a side slope s of 1.5:1. So, let us start with the rectangular section, which is mostly very easy to solve. So, here it is straight.

Forward solution. How y_c is $\left(\frac{q^2}{g}\right)^{1/3}$, that is q is nothing but $\sqrt{g y_c^{\,3}}$. q is under root 9.81, and this is given already. 4.117 $m^3/s/m$ implies bottom width is capital $\frac{Q}{q}$. Capital Q is given as 15/4.117. So, discharge Q was already given, and this value comes out to be 3.643 meters. So, B for the rectangular case.

Now, the second case. Trapezoidal channel. So, here it is important to know that for a trapezoidal channel, mostly the solution is using trial and error. So, first calculating $(B+1.5\times1.2)\times1.2$ will give us $(B+1.8)\times1.2$. Similarly, T_c will be $B+2\times1.5\times1.2$.

Ac is the formula $(B+my)\times y$, and T_c is $B+2my$. This is the formula that we have used, and this comes out to be $B+3.6$. The solution for the critical flow condiition is $\left(\frac{Q^2}{g}\right)$, which is the critical flow condition. This is the critical flow condition. So, we put A_c, T_c, and Q everything in this equation and try to come up with $(B+1.8)^3\times\frac{(1.2^3)}{(B+3.6)}=\frac{(15^2)}{g}$

, which is $9.81.\frac{(B+1.8)^3}{B+3.6}=13.273$. So, if you solve this, B can come out to be 2.54 meters. It is simple, but using trial and error, this is a little complicated to solve.

So, another problem is, See, this chapter is more about finding critical depth, right? So, another problem is for a different set of configurations; find the critical depth for a specific energy head of 1.5 meters in the following channels. It is a similar type, but here a specific energy head is fixed.

The first problem is a rectangular channel. B is equal to 2.0 meters. It is important to note that the energy head is given as 1.5, and we have to find the critical depth, which is y_c.

The second is a triangular channel where m = 1.5, okay. The third is a trapezoidal channel where B is equal to 2.0 meters and m is given as 1.0.

Lastly, there is a circular channel. D is 1.50 meters. So, this is the problem statement. So, we will, for a rectangular channel Very simple.

We know that specific energy is related to critical depth by 3/2 y_c, and this is fixed at 1.5 meters. E_c is fixed. y_c can be written as 2/3 or 2 * 1.5 / 3. Therefore, y_c is 1 meter for a rectangular channel. Triangular channel. We know that here also E_c is nothing but 1.25 y_c, and that is also equal to 1.50 meters.

So, y_c is nothing but $\frac{E_c}{1.25}$. So, $\frac{1.50}{1.25}$ gives 1.20. So, y_c for a rectangular channel is 1.20 meters, straight and simple. Now, trapezoidal channel. So, for a trapezoidal channel, E is given as $y_c + \frac{V_c^2}{2g}$, or in terms of Q because our data is generally given in terms of Q, $\frac{Q^2}{2gA_c^2}$.

And what is the critical flow condition? It is nothing but $\frac{Q^2}{g} = \frac{A_c^3}{T}$. Therefore, when we write E_c, we use this in this particular equation. So, using that in the previous equation, E_c can be written as $y_c + \frac{A}{2T_c}$, that is, E_c is given as 1.5. $y_c + \frac{(2.0 + y_c)y_c}{\left(2\times\left(2 + 2y_c\right)\right)}$. And again, this has to be solved then we get the value of y_c as 1.095 meters. Not too complicated, but yeah. Now, the d part for a circular channel. E_c can be written as $y_c + \frac{A}{2T_c}$. So, what we plan to do, or what we generally do, I have told in this lecture as well. The theoretical slides are by non-dimensionalizing with respect to the diameter D. After doing that, it comes out to be

$\frac{A_c / D^2}{2T_c / D}$ is equal to $\frac{E_c}{D}$ or $\frac{1.5}{1.5}$ is equal to 1.0, okay. So, either we use stable or use trial and error. You know tables are used, right? In that table, we have $\frac{E_c}{D}$ equal to 1, and we find out the corresponding value of $\frac{y_c}{D}$ as 0.69; therefore, y_c is equal to 0.69×1.50, which is 1.035 meters for the circular. So, the question is: Water is flowing at critical depth at a section in a triangular

shaped channel with the side slope of 0.5*H: 1V*. If the critical depth is 1.6, estimate the discharge in the channel and the specific energy at critical depth. So, first, draw a figure of a triangular section.

And this water is filled; this is 3 meters, 0.5, and this is the slope, and this water depth is 1.6 meters. So, we see here what is m equal to 0.5. So, what is the T_c? So, the slope is actually -0.5. T_c, the top width, is $3 - 2 \times 0.5 \times 1.6$, which is 1.40 meters, and the area is $\frac{(3+1.4)}{2}$.

That is multiplied by depth is 1.6. That is 3.52 square meters. So, now critical flow condition $\frac{Q^2}{g} = \frac{A_c^3}{T}$ and A_c is $\frac{3.52^3}{1.40}$, and that comes out to be 31.153. And therefore, discharge comes out to be $\sqrt{31.153 \times 9.81}$, and Q comes out to be 17.48 m^3/s, right? OK. So, second, I mean this is the discharge that we have calculated.

The next part is calculating the specific energy, OK. So, for specific energy, we calculate V_c is equal to $\frac{Q}{A_c}$; Q we have already calculated, A_c we have already calculated. So, we get 4.97 meters per second. Then, $\frac{V_c^2}{2g}$ is $\frac{4.97^2}{2 \times 9.81}$, and this comes out to be 21.257 meters.

Therefore, critical energy is $y_c + \frac{V_c^2}{2g}$; V_c came out to it was 1.6 + 1.257, or E_c is 2.857 meters. This was the final thing that we needed to calculate in our problem. All right. So that being said, I would like to thank you all, and I will see you in lecture 4 of module 3. Thank you so much.