Free Surface Flow

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Lecture 12

Welcome back, students, to the second lecture on problems of critical flow, where we are going to continue solving problems on critical flow. In the last lecture, we solved three problems. Now, we are going to start solving those problems again. So, let us get started. Firstly, I would like you to know that in this particular lecture, we are probably going to use this set of tables in some problems that we will encounter.

So, now we are starting with another problem, which is the fourth problem. The question is, it is required to have a channel in which the Froude number F remains constant. at all depths. So, this is a hypothetical situation where we have been asked to design a channel where the Froude number *F* will remain constant at all depths.

Now, it says if the specific energy E is kept constant, show that for such a channel, what we have to show is that for such a channel $\frac{T}{B}=\left(\frac{E}{E-y}\right)^{1+\frac{F^2}{2}}$. This is what has to be shown *T* and *B* are the top width and bottom width of the channel respectively. This is the complete set of questions. So, let us see how we are going to approach this problem. So, first, we start by writing down what we know. So, we have been seeing this term specific energy.

So, let us start writing down the specific energy *E* is equal to $y+\frac{V^2}{2g}$, and *V* can be written as $\frac{Q}{A}$. So, we write $y+\frac{Q^2}{2gA^2}$ or *y* + So, you know, for what we can write $\frac{Q^2}{gA^2}$, you know what it can be written as? It can be written as $\frac{F^2}{2}\frac{A}{T}$. So, $y+\frac{F^2}{2}\frac{A}{T}$. Froude number is $\frac{V}{\sqrt{gy}}$. I mean for this channel *gy* and then you see this *F*, you can write $\frac{V^2}{gy}$ is.

By and *T* is the width, it will come out to be Q^2. So, g^2, I mean you can just leave it actually *A* like this only. So, if you do some manipulation here, you are going to get $\frac{Q^2}{gA^2}$ is $F^2\frac{A}{T}$. So, or we can write so, if you bring y on this side, we can write $E-y=\frac{AF^2}{2T}$. So, this is one important equation.

Now, this important equation shows that *F* is constant. The Froude number is constant at all depths. So, what we are going to do next is differentiate with respect to *y* and note that *F* is a constant. So, what are we going to do?

We are going to differentiate *dE*, or you can write $\frac{dE}{dy}$ as equal to So, $\frac{dE}{dy}-1=\frac{F^2}{2}\left(T\frac{dA}{dy}-A\frac{dT}{dy}\right)$ because this is constant. by T^2. Since E is also constant, $\frac{dE}{dy}$ is equal to 0. Also, we know $\frac{dA}{dy}$ is equal to *T*. The above equation here can be written as $\frac{F^2}{2}\left(1-\frac{A}{T^2}\frac{dT}{dy}\right)=-1$.

$\frac{AF^2}{2T}\frac{1}{T}\frac{dT}{dy}=\left(1+\frac{F^2}{2}\right)$. Substituting for $\frac{AF^2}{2T}$ is $(E-y)\left(\frac{1}{T}\frac{dT}{dy}\right)=1+\frac{F^2}{2}$. Or $\frac{dT}{T}$ is equal to $\left(1+\frac{F^2}{2}\right)\left(\frac{dy}{E-y}\right)$. Now, this is integration. So, on integration, we get ln *T* is equal to $\left(1+\frac{F^2}{2}\right)\left(-ln\left(E-y\right)\right)+C$. So, at F^2

At $y = 0$, T is equal to B, and hence C is equal to 0. $\ln B + (1 + \frac{F^2}{2}) \ln E$ or $\ln \frac{T}{B}$ is equal to $\left(1 + \frac{F^2}{2}\right)$. It is more of a manipulation problem where you have to rearrange items ln $\frac{E}{E - y}$ or $\frac{T}{B}$ is E, or $\frac{T}{B} = \frac{E}{E - y} \left(1 + \frac{F^2}{2}\right)$. This was something

that was asked to prove. So, this was a lengthy question but simple to do in a sense that you had to just differentiate. We will go for another question now. This problem will employ the tables that we have. The question is to calculate the critical depth

and the corresponding specific energy for a discharge of 5 m^3/s in the following type of channel. A is a rectangular channel having a width equal to 2.0 meters; second is a triangular channel m is equal to 0.5. Third is a trapezoidal channel where B is equal to 2.0 meters and m is equal to 1.5. And the last type of channel

Circular channel having D as 2.0 meters, all right. So, this is a very classical and typical problem where we have to calculate the critical depth and the corresponding specific energy. The discharge is given; Q is fixed at 5 m^3/s for different types of cross-sectional areas, all right. So, we start with the most basic type, which is the rectangular channel. Rectangular channel. For the rectangular channel, first, we know what is given: Q is given as 5 m^3/s, which is common for all, and B is given as 2.0 meters.

The first thing for the rectangular channel is we need to calculate small q, which is nothing but capital Q divided by B. Capital Q is 5, and B is 2. So, we get 2.5 meters cubed per second per meter. And the formula for y_c is nothing but $\left(\frac{q^2}{g}\right)^{1/3}$. So, Q is $\left(\frac{2.5^2}{9.81}\right)^{1/3}$, and this value is going to come to 0.86 meters. The second part of this is we

have also been asked to calculate the specific energy. So, we know for the rectangular channel, What is E_c? $3/2\ y_c$. Therefore, E_c is 1.5 times 0.860.

That is, E_c will come out to be 1.29 meters. So, the first part y_c is known, and E_c is known. So, now the solution continues. The second part was that we have a triangular channel. Triangular channel.

So, for the triangular channel, we derived the formula, if you remember. Here also, first write down Q is known: 5 meters cubed per second, and we know m as 0.5. So, if you remember, for the triangular channel in terms of m, y_c, and Q, we derived the equation y_c was $\left(\frac{2Q^2}{gm^2}\right)^{1/5}$, if you remember. So, simply substituting $\left(\frac{2\times5^2}{9.81\times0.5^2}\right)^{1/5}$ will be equal to 1.828 meters. For the triangular channel, we found out that.

$\frac{E_c}{y_c}$ was 1.25, or E_c was $1.25y_c$, or 1.25×1.828, that is 2.284 meters. So, for the triangular channel, y_c came out to be 1.828 meters, and E_c came out to be 2.284 meters, right? Now, the next part. So, it starts to get a little complicated. That is part C, that is trapezoidal.

So, trapezoidal, if you remember, we needed to calculate some values. Here, we also know Q as 5 m^3/s, we know the value of B as 2 meters, and m was 1.5. So, we need to calculate one quantity, ψ, that is $\left(\frac{Qm^{3/2}}{\sqrt{g}B^{5/2}}\right)$, I think. So, Q was 5, m was $1.5^{3/2}$, divided by $\left(\sqrt{9.81}\times2^{5/2}\right)$, and on solution, using your calculator, you will get 0.0518. This is the value of ψ that we get.

So, if we know the value of ψ, we can usually find out the value of zeta. All right. So, let me try going back to the trapezoidal channel, right. This is circular; this is trapezoidal. And what is the value of ψ that we got? 0.51843, right, 0.51843.

Yeah, so using this table, it is 0.0, 0.1; we need to find 0.5, 0.51. So, point to the value of 0.518. So, somewhere here, it should be an interpolation of somewhere here. So, if we interpolate, because I have done this, you see it should be between, sorry, not this one, sorry. So, for this one, it should be somewhere between 0.535 and 0.440.

So, since I have already seen our value was 0.518, right, 43. So, between this and this, our value of zeta should come out to be 0.536, which is between these two values. So, let us go and use this value. So, using the table. The corresponding value of zeta is $\frac{my_c}{B}$ is equal to 0.536.

So, substituting the value. *m* and *B*, y_c can come out to be 0.715 meters. A_c, that is the area at critical, will be 2 + 1.5 or first let me write down the formula for your convenience $(B+my_c)\times y_c$. So, $(2+1.5\times 0.715)\times 0.715$ that comes to be 2.197 meters square, this is the area. Now, we need to find the critical velocity, that is $\frac{Q}{A_c}$.

Q is 5, A_c we have already calculated. $\frac{Q}{A_c}$ that comes $\frac{5}{2.197}$ is equal to 2.276 meters per second. And also $\frac{V_c^2}{2g}$ is, if you substitute the value, it will come to 0.265. Now, energy at critical depth is E_c is equal to $y_c+\frac{V_c^2}{2g}$, y_c came out to be 0.715 and using this V_c and $\frac{V_c^2}{2g}$, it comes to be 0.265. So, E_c comes out to be 0.979 meters.

y_c is the third part for trapezoidal, where we have used and made the use of the equations here. Now, again continuing part *D*, this is one of the most standard problems or the main problems of critical flow. So, the last part is for the circular channel. So, for the circular channel, Z_c is $\frac{Q}{\sqrt{g}}$, which is $\frac{5}{\sqrt{9.81}}$ or 1.5964. And therefore, Z_c, which is also calculatable, is equal to $\frac{1.5964}{2^{2.5}}$, and this value comes out to be 0.2822.

Now, we will use the table that is the relationship of $\frac{Z}{D^{2.5}}$ with $\frac{y}{D}$ and the value of $\frac{y_c}{D}$. Corresponding to $\frac{Z_c}{D^{2.5}}$, which is equal to 0.2822, is found by, like in the last one that we did, suitable linear interpolation. And what will be the value? So, we know that it is 0.2822.

So, the best is to go and look at the curve Z_c divided by, you see, Z divided by our value here is 0.2822. So, it should be So, let me see $\frac{y_c}{D}$ is $\frac{y}{D}$. So, it should be 0.2. So, somewhere between this and corresponding $\frac{y}{D}$ should be between this. Because our value is 0.2822.

So, we linearly interpolate, and we linearly interpolate this as well. I already have the answer, which means the value $\frac{y_c}{D}$ after linear interpolation results in 0.537. So, let me go back. So, the value that we get is $\frac{y_c}{D}$ is 0.537 implies y_c is nothing but 1.074 meters.

This is one answer. And if we have y_c, we can always calculate things. Now, just to make it more complete, we can also use determine y_c by empirical equations. One could be Swami's equation, which says for circular.

So, F_D is $\frac{Q}{D^2\sqrt{gD}}$. So, $\frac{5}{2^2 \times \sqrt{9.81 \times 2.0}}$ that comes to be 0.2822. So, $\frac{y_c}{D}$ is given by $\left[0.77\left(F_D^{-6}+1\right)\right]^{-0.085}$ or $\frac{y_c}{2}$ is given by $\left[0.77\times\left(0.2822^{-6}\right)+1\right]^{-0.085}$. This comes to be 0.5363 or y_c comes to be 1.072 meters using Swamy's equation.

Another method is using Straub's equation, which is $y_c = \left(\frac{1.01}{D^{0.265}}\right)\left(\frac{Q}{\sqrt{g}}\right)^{0.506}$. If we use this particular equation, y_c will come out to be $\frac{1.01}{2^{0.265}}$. I do not suggest using these empirical equations, but they can help sometimes to verify.

1.065 meters. So, you see all these values are very close: 1.074, 1.065, 1.062. So, I think with this, I will conclude this particular chapter, and we will continue solving some more problems in Lecture Number 3 of our Problems Module: Problems in Critical Flow. Thank you so much. See you. Thank you.