Free Surface Flow
Dr. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture 11

Welcome, students, to our module 3 and the first lecture, where we are predominantly going to solve problems for free surface flow and especially problems on critical flow. So, let us get started with the first problem. You see, we have, and we will be using several tables, right? You remember, we derived some equations for calculating the critical depths for different shapes, you know, rectangular, trapezoidal, and circular. So, this is for the circular channel.

We will use these tables to, you know, take the values out and then use them to solve our calculations, you know. Yeah. So, this is just to show you. So, we will start with. So, we will start with.

So, first, the question: A 2.5-meter rectangular channel has a specific energy of 1.5 meters. So, there are several things, but let me first write down the question. When carrying a discharge of 6.48 m^3/s. Simple problem. Calculate the alternate depths and the corresponding Froude.

Numbers. So, this is the question; see, many things are already given here. For example, so this is a 2.5-meter wide rectangular. So, the width is given. Solution.

So, the width is given. We are also given it is a rectangular channel. We have also been given a specific energy of 1.5 meters. So, what are we going to do? We are going to write energy in specific energy $y + \frac{V^2}{2g}$ and that in terms of Q can be written as $y+\frac{Q^2}{2gB^2y^2}$.

So, what are the things that we know? We know the specific energy here, we know Q here, we know B here, we know the value of g here. What do we not know? What we do not know is the value of y. So, we can write $1.5 = y + \frac{6.48^2}{2\times9.81\left(2.5^2\right)\left(y^2\right)}$.

1.5 is equal to, so from going from this step to this step, 1.5 is equal to y will remain y, and if we solve this, we get $\frac{0.342}{y^2}$ right. So, this will give a cubic equation in y. What are the different ways using which we can solve? The easiest is using trial and error. So, it is a cubic equation, 1 will come negative, and we will get 2

positive roots. If you solve this, I mean, I have already solved this on my calculator, are obtained as y_1 is equal to 1.296 and y_2 is 0.625. So, this is meters. So, if we solve it using the trial and error method, this is what we get.

So, we have y_1 and y_2. So, Froude $\frac{V}{\sqrt{gy}}$. See, one way is we know Q, right. So, we are able to calculate V as well, but anyways, we will write $\frac{6.48}{2.5y\sqrt{9.81y}}$

Or, in other terms, $\frac{0.82756}{y^{3/2}}$. So, the Froude number equation comes out to be this one. So, now what is going to happen is we are going to put y_1 and y_2 and obtain the corresponding critical. So, the alternating depth we have already obtained y_1 and y_2. So, continued at y_1 is equal to 1.296.

So, let me write down the Froude number final equation again here: $\frac{0.82756}{y^{3/2}}$. So, at y_1 we apply this, F_1 is going to be 0.561 and at y_2. is equal to 0.625 meter. Units are very important; implies if you put this value here, similarly this value here, we get F_1 and F_2 comes out to be 1.675. Now, this is what we were asked, F_1 and F_2, just the conclusion is.

The depth y_1 is equal to 1.296 meter is in the critical flow region, and the depth y = 0.625 m is in the super critical flow regime, and this completes our first problem. Yeah. So, I think you have understood that. So, now we will solve another problem.

Most of these will be different types of problems with some repetitions in between. In any case, let me write down the, so while writing down and when reading the problems as I have already mentioned, try to note down what parameter values are already given, and then when you find out what those parameter values are, a formula or how to approach that will automatically come to your mind. So, a flow. of 5 m^3/s is passing at a depth of 1.5 meters through a rectangular channel of 2.5 meters width.

See, so discharge is given, depth is given, width is given. We are also, the question also states that the kinetic energy correction factor alpha is found to be 1.20. We have seen that alpha and beta other than uniform conditions are always greater than 1. Now, the question is what is the

First question is what is the specific energy of the flow? Second part states that what is the value of the alternate depth existing depth if alpha is equal to 1.0 is assumed for the alternate flow. So, the first part was very simple, alpha was given and asking what is the specific energy. Secondly, it is asking if alpha is changed to 1.0, what is the value of the alternate depth?

So, we will say a solution here. So, the solution is The first thing is discharge is given, so we will find out the velocity at upstream. Well, discharge is given $\frac{Q}{A_1}$. So, let us write down Q is given as 5 m^3/s.

We also know that width is given, that is 2.5 meter, and y_1 is also given that is y_1 or y is given as 1.5 meter. So, $\frac{Q}{By_1}$ or $\frac{5}{2.5\times1.5}$. That gives us 1.33 meters per second. This is the velocity at section 1, right. Now, what is the, I mean, because of the kinetic energy friction, I mean, due to the kinetic energy correction factor, we calculate the value of alpha $\frac{V_1^2}{2g}$, okay.

Calculate So, alpha is given 1.20, alpha is also given 1.20 here into 1.33, which we have come from here. Right, and divided by 2g that is 9.81. If you solve this, you are going to get 0.1087 meter. Of course, you can write it 0.11 because of the significant digits, but I am just writing it like that. Now we have calculated V_1, we have calculated $\frac{V_1^2}{2g}$. Now we need to calculate specific energy. And that is the first part of our problem E_1. Now, how is it written?

$y_1 + \alpha_1 \frac{V_1^2}{2g}$ or we can simply write $\alpha \frac{V^2}{2g}$. In this case, what is y_1? y_1 is 1.5 plus alpha is or we have already calculated $\alpha \frac{V_1^2}{2g}$ as so simply we plug it in here that is 0.1087. And, the value we get 1.6087 meters or approximately 1.61 meters yeah. Now, this was the first part of the problem right.

The second part is if we make alpha as 1 what is going to happen? We have to find the alternate depth. So, for alternate depth y_c, $y_2 + \frac{5^2}{2 \times 9.81 \times 2.5 y_2}$. Let us say that the, sorry, y_c generally is given for the critical.

So, I will take y_2 and therefore, here also y_2. This whole square, see this will, the specific energy will remain same, right? So, since is equal to 1.6087 right. Then we get y_2 + if we solve this right we get its alpha here.

Here, alpha is equal to 1. So, $y_2 + \frac{0.2039}{{y_2}^2} = 1.6087$. This is a cubic equation again. Which will have different roots. To solve this, we will use trial and error, and using this.

On equation, we get y_2 is equal to 0.413 meters. So, if alpha is assumed to be 1, this is our alternating depth. Earlier, it was 1.5 meters. And this was the second part of the solution that was required. So, this gives us the solution to another problem.

So, now another question. Obtain the value of the first hydraulic exponent M for first, for a rectangular channel. And then a channel which is an exponential channel where the area A. Is given as $A = K_1 y$. So, we have to calculate the first hydraulic exponent M for first, a very simple case, a rectangular channel.

And second is an exponential channel where the area A is given by $A = K_1 y^n$. So, for a rectangular channel, A is equal to the area, which is nothing but By, and the top width is given as *B*. And we know that Z^2 is equal to $\frac{A^2}{T}$, which is equal to $B^2 y^3$ or $C_1 y^M$. These equations are directly given, all right, for the calculation of the hydraulic exponent. So, you see, if you equate the component This is what?

This C_1, it is a general equation to represent the hydraulic exponent *M*, where *Z* was, if you remember from our last slides, where *Z* was given as a function of the exponent of *y*. So, see Z^2 is also given as $C_1 y^M$, Z^2 is also written as $\frac{A^2}{T}$, and then we substitute *A* as *By*, so $B^2 y^2$. So, Z^2 is Z^2 is actually *A* raised to the power, yes. So, it is *A* is *By*. So, $B^2 y^2$.

So, basically it becomes $By.\frac{B^2 y^2}{y}$. So, it becomes something like (B^2y^3). And when you compute this, so *C* becomes B^2 and y^M. is equal to y^3, or directly looking, $M = 3.\frac{Z^2}{T}$. So, therefore, Z^2. So, *M* here is 3 if you look at the exponent in the first part, right.

However, this equation, the above value of *M* can also be obtained directly by using any empirical equation that we have covered in the slide. So, now the B part. Now, our area is given as $K_1 y^n$. How do we find *T*? *T* is nothing but $\frac{dA}{dy}$, or if you differentiate it, it will become $K_1 n y^{n-1}$.

So, if we use the equation from the slides, *M* is given by the formula, *M* is $\frac{y}{3}\left(3T - \frac{A}{T}\frac{dT}{dy}\right)$. So, actually, this equation can be used. So, I am very sorry, this is not *y* by 3, it is *y* by *A*. So, *M* here is just trying to substitute $\frac{y}{K_1 y^n}\left(3K_1 n y^{n-1} - \frac{K_1 y^n}{K_1 n y^{n-1}}\left(K_1 n\left(n-1\right) y^{n-2}\right)\right)$. Right. And then after, you know, there will be many cancellations K_1, K_1, and then common $K_1 y^n$. Then, *M* can be actually simplified as $(3n - n + 1)$ or $2n + 1$. So, *M* finally comes to $2n + 1$.

So, for rectangular, it also comes out to be 3 using this particular thing, all right. So, I think with this, we will finish this particular lecture, lecture 1 of module 3, and I will see you in another lecture where we are going to solve even more sets of problems. Until then, bye-bye.