Free Surface Flow
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Lecture 10

So welcome students to yet another lecture that is lecture number 4 of channel transitions continuation under the module of critical flow. So in the last lecture we discussed that we are going to start with transitions with change in width. What does that mean? So for example. What that means here is, see you see this has a width B_1 here and somehow the width has decreased, right?

It could be other way round as well. It could be that you have a smaller width in the beginning and it expands to a larger one. So, transition with the change in width means that the width is changing from one section to the other. So, there will be many cases, one of the cases subcritical flow if we assume that the. So, when we talk about flow that means the flow upstream, the condition of the flow upstream that is subcritical flow with a width constriction.

So when we talk flow conditions will always be at the upstream side. So, this is the upstream side. So, first case is subcritical flow in a width constriction. This is constriction. So, if we consider a frictionless horizontal channel of width B_1, right, let the width be B_1.

just to show you. So, this is width B_1 carrying a discharge Q at depth y_1. So, corresponding to this, this is the depth y_1. I mean of course, shown in figure. Now, at section 2, the channel has been constricted to width B_2 and therefore,

Of course, the water level will go to y_2 and this is a smooth transition. Smooth transition tentatively means trying to avoid loss due to the friction. Now, since there are no losses involved and since the bed elevations at section 1 and 2 are same, right? Bed elevations means it is a horizontal bed, right? The specific energy at section 1 is equal to the specific energy at section 2.

right, because the datum remains the same. This is quite important. The bed elevations are same and therefore, specific energy at section 1 is equal to the specific energy at section 2. Now, E_1 will be $y_1 + \frac{V_1^2}{2g}$ or Q is always V_1 by A_1. Sorry, it is opposite

V_1 is $\frac{Q}{A_1}$ or V_1 is $\frac{Q}{By_1}$. And then V_1 square will be $\frac{Q^2}{B^2 y_1^2}$ and this you substitute in this one and therefore, you get $y_1 + \frac{Q^2}{B^2 y_1^2}$ and divided by $2g$. So, it becomes like this, exactly this $2g(B_1^2 y_1^2)$. and E_2 correspondingly will be $y_2 + \frac{V_2^2}{2g}$ or in terms of V_2 and y_2,

$$y_2 + \frac{Q^2}{2g\left(By_2\right)^2}.$$

Now, when it comes to the matter of convenience, to analyze the flow in terms of discharge intensity, right? That is what we have been doing. So we normally do it discharge per unit width. So at section 1, what is going to be the

per unit with $\frac{Q}{B_1}$ and Q_2 will be $\frac{Q}{B_2}$. Since we know that B_1 is greater than B_2, see B_1 is greater than B_2, we know that q_1 will be smaller than q_2 or q_2 will be greater than q_1. So, in the specific energy diagram drawn with discharge intensity as the third parameter, point P on the curve q_1 corresponds to depth y_1 and the specific energy E_1. That is what we are going to see, this one here. Now, so since at section 2, you see the section 1 and 2 that we are talking about,

E_2 will be equal to E_1 and q is equal to q_2. And the point P, this point P will move downward to point R on the curve, q_2 to reach the depth y_2. Thus, In subcritical flow, the depth y_1 will be greater than y_2. This is very normal or y_2 will be smaller.

Thus, in the subcritical flow, y_2 will be smaller than y_1 and hence this particular figure. So, the water level will decrease and in which condition it is going to subcritical flow in a width constriction over the water level downstream will decrease. If B_2 is made smaller, then q_2 will increase and y_2 will still decrease. So, if you keep on decreasing the width, q_2 will keep on increasing and y_2 will keep on decreasing.

That is the important message for this particular part. So, the limit of the, so what is, the question comes what should be the limit of the width B_2, right? And this limit is obviously reached when corresponding to E_1, the discharge intensity q_2 goes to maximum. That is the maximum discharge intensity for a given specific energy or in other words, critical flow condition will prevail.

At this minimum width, y_2 will be equal to critical depth at section 2. So, y_2 here will be equal to critical so this is the maximum you know this is the maximum allowed or this is the maximum allowed waterfall the level being falling here okay Now, for a rectangular channel at critical flow, we all know that y_c is equal to $\frac{2}{3}E_c$ or E_c is 3/2. So, we know that we have derived that $\frac{3}{2}y_c$ and therefore, this will go here, this will go here, y_c will be $\frac{2E_c}{3}$.

Since E_1, we talked that E_1 is equal to E_{cm}. y_2 will be equal to y_{cm} or $\frac{2}{3}E_{cm}$ or $\frac{2}{3}E_1$. So, y_2 comes to be $\frac{2}{3}E_1$. And we know that y_c is what? $\left(\frac{Q^2}{B_{2m}{}^2 g}\right)^{1/3}$

Or if we take, you know, B this side, yc this side, take square root, then we end up with term $B_{2m} = \sqrt{\frac{Q^2}{g y_{cm}{}^3}}$. Or B_{2m} is in terms of E_1, if you put y_{cm}, you know, in terms of Qg and E_1, we can write $\sqrt{\frac{27Q^2}{8gE_1{}^3}}$. So, if we are writing this in terms of, you know, this. y_{cm} is $\frac{2}{3}E_1$, right? So, instead of y_{cm}, you put $\frac{2}{3}E_1$.

So, y_{cm} is $\frac{2}{3}E_1$, right? So, y_{cm}^3 will be $\frac{8}{27}E_1^3$. And this thing you put here and then you will end up in this one. This is an important thing to So, what happens if B_2 is less than B_{2m}, that is the width, the discharge intensity q_2 will be larger than q_m, that is the maximum discharge intensity consistent with E_1.

So, in this particular case for B_2, if B_2 is less than B_{2m}, the flow will not be possible with the given upstream conditions. And if that has to be taken care of somehow, the upstream depth will have to increase to y_1 so that new specific energy is formed. So, keeping the same depth, it is not possible, the upstream depth will have to increase to y_1, another depth, so that a new specific energy is formed and corresponding to that the calculations will be done. So, E_1 will be $y_1' + \left(\frac{Q^2}{2gB_1^2 y_1'^2}\right)$, which will just be sufficient to cause critical flow at section 2. It may be noted that the new critical depth at section 2 for a rectangular channel is this one.

See, at depth Q^2 divided by, it is a standard Q by, you know, $g^{1/3}$. So, instead of q, you put $\frac{Q}{B_2}$ here, ok. Then E_{c2} that is critical energy at C_2 will be $y_{c2} + \frac{V_c^2}{2g}$ and very simple equation that is $\frac{3}{2}y_{c2}$. This is all right. So, you see this is the variation of y_1 and y_2 in subcritical flow in a width constriction.

Now, we know that since B_{2m} is greater than B_2, y_{c2} will be larger than y_{cm}, very clear. Further, E_1' will be equal to $1.5y_{c2}$, which we have seen from the previous slide. Thus, even though critical flow prevails for all B_2 that is less than B_{2m}, the depth at section 2 is not constant as in the hump case, but increases as y_1 and hence E_1' rises, the variation of y_1. So, if we all plot the y_1, y_2 and E with B_2 by B_1, this can be seen from this particular figure. So, this is a detailed analysis of the subcritical flow that

That you can see from here. So, now the next case is obviously the supercritical flow in a width constriction. All right. So, what happens if the upstream depth y_1 is in the supercritical flow regime? A reduction of the flow width and hence an increase in the discharge intensity causes a rise in depth of.

So, you remember here the water level was decreasing, right? For supercritical flow, this in this water level will increase. So, in the above specific energy this one figure point I mean yeah the I mean this is just the representation I mean the specific energy curve that I have shown before. The point P' corresponded to y_1 and the point R' to y_2. As the width B_2 is decreased, R' moves up till the main important thing is in the supercritical flow, reverse is going to happen.

B_2 will be equal to $B_{2\text{m}}$. And if you further reduce B_2, it causes the upstream depth to decrease to y_1. So, very different from the subcritical flow. In subcritical flow, water level was falling, right? Now after keep on increasing the water level will go down downstream up to a certain level. But in the up when the flow is super critical, the water level will keep on rising. But if we reduce the constriction to even smaller than B_2, you know that the $B_{2\text{m}}$, what is going to happen? the upstream depth will decrease to y_1. And so that E_1 rises to E_1'.

Almost exactly similar scenarios as when we were talking about the hump. At section 2, the critical depth yc' corresponding to the new specific energy E_1' will prevail. And the variation of y_1, y_2 and E with $\frac{B_2}{B_1}$ in the supercritical flow regime is indicated here. So, what is going to happen if we keep on increasing the width from B_1 to B_2 for subcritical flow and supercritical flow are actually opposite. In one case, the water level keeps on decreasing up to a certain point and after that, that will remain constant and the water level will keep on rising for subcritical flow and will increase for supercritical flow.

Now probably one of the last topics is choking. What is choking of this particular module? We are what we are going to do after finishing this theoretical part of critical flow. The next module will be about problem solving in critical flow where we will solve a variety of problems and we will try to you know go into more details of how to actually tackle all these problems for subcritical, supercritical and mainly critical, but of course, subcritical and supercritical flow will come, the channel transitions or hump.

So now choking. So in the case of the channel with a hump and also in the case of width constriction, it is observed that the upstream water surface elevation is not affected by the conditions at section two. Till a critical stage is first achieved. Both place we saw that, right? Thus, in case of hump for all ΔZ greater than ΔZ_m, the upstream water depth is constant, okay?

And for all delta, okay, so this is not correct. So, for all hump ΔZ less than ΔZ_m, the upstream water depth is constant and for all ΔZ greater than ΔZ_m, the upstream depth is different from y_1. Then if you further keep on increasing the ΔZ, the water level upstream will change than the previous one. Similarly, in case of width constriction for B_2 greater than B_1, the upstream depth y_1 is constant. Yes, that is true.

While for all B_2 which is less than B_{2m}, the upstream depth undergoes a change. This onset of critical condition at section 2 is a prerequisite to choking. This onset of critical condition at section 2 is a prerequisite to choking. Thus all cases where ΔZ is greater than ΔZ_m or B_2 is less than B_{2m} are known as choked condition. So, those two condition where we have been discussing in a lot of detail in our previous slide where we said what happens if the hump size increases then the maximum allowed maximum allowed size

and the width decreases than the minimum allowed width, what is going to happen? Those two conditions are first prerequisite and these are called choking conditions. Choking condition is ΔZ greater than ΔZ_m or B_2 less than B_{2m}. Obviously, choked conditions are undesirable and need to be watched in design of culverts and other surface drainage features involving channel transitions. So, what we have been seeing as an exception after what happens after the limit is called the choking condition.

This is quite an important concept that you must understand. You know it from previous slides, but what exactly is the definition we have seen now. So, these are some of the references from where this material has taken. I think this particular slide, I mean this particular topic in terms of theory, we will conclude it today. However, we are going to start our module 3 from the next lecture and that is about mostly about the computations of the critical flow problems.

So thank you so much and I will see you in the next class. Thank you.