Availability and Management of Groundwater Resources
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Lecture - 27
Law of Groundwater Movement, Darcy's Law and Application (Continued)

Welcome you all in the part 3 of the module 6 law of groundwater movement Darcy law and application. So, in the last 2 part; we have seen about the different stretch approaches; different formula related to the law of groundwater movement because, underneath the earth's surface the water flows from one aquifer to another aquifer and the movement of the water depends upon the different factors.

Generally, water needs energy for its movement and it flows from the higher hydraulic head to the lower hydraulic head. So, generally this thing usually happens from the pressure concept we have seen, pressure difference is very important for the movement of the your groundwater inside the earth's surface because different factors also we have seen which are very important for the permeability that is the ability to transmit water.

Now, in this module, in this part three module 6 we will discuss about the some of the important empirical formula.

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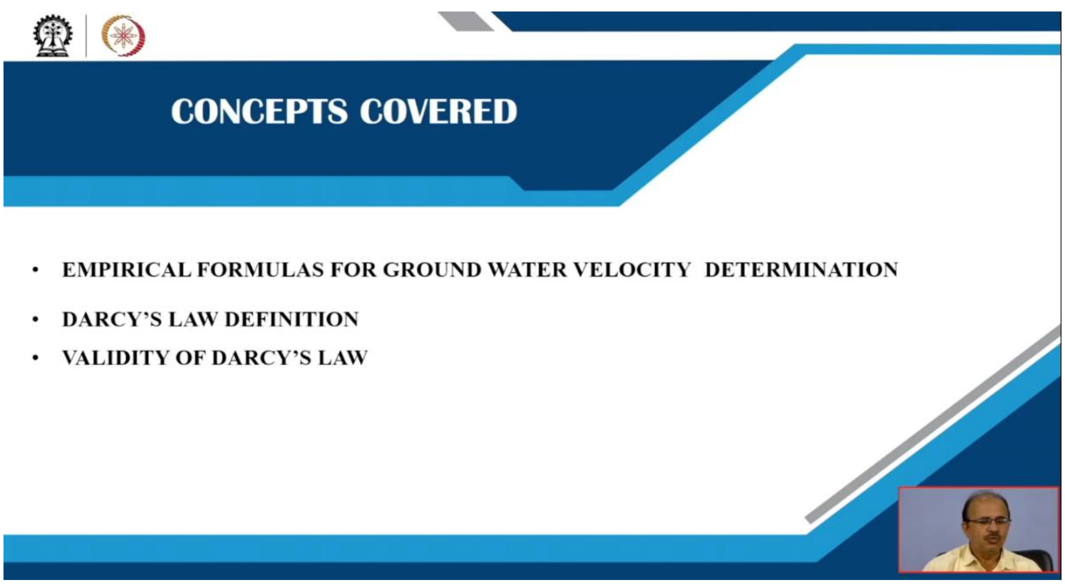

For groundwater velocity determinations the important Darcy law and the validity of Darcy law. So, what is prior to the Darcy law?

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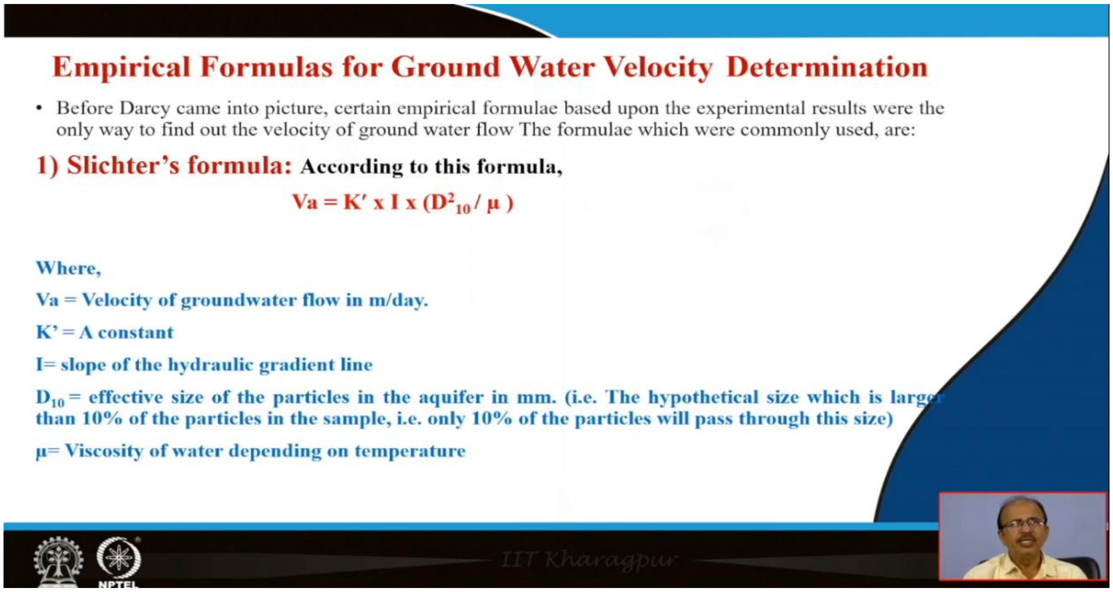

We have some empirical formulas for groundwater velocity determination and this generally came before Darcy came into picture and this formula where based on some of the concept related to the slope of the hydraulic gradient line. So, the two different formulae were very important before the Darcy law came into picture; the one is the Slichter formula. This is very important Slichter formula is based on certain factors, that is the slope the effective size of the particles viscosity of the water etcetera.

So, you can see that in the Slichter's formula

$$Va = K' \times I \times (D^2_{10} / \mu)$$

So, Where,

Va = Velocity of groundwater flow in m/day.

K' = A constant

I= slope of the hydraulic gradient line

D_{10} = effective size of the particles in the aquifer in mm. (i.e. The hypothetical size which is larger than 10% of the particles in the sample, i.e. only 10% of the particles will pass through this size)

μ= Viscosity of water depending on temperature, So, this is about the Slichter's formula.

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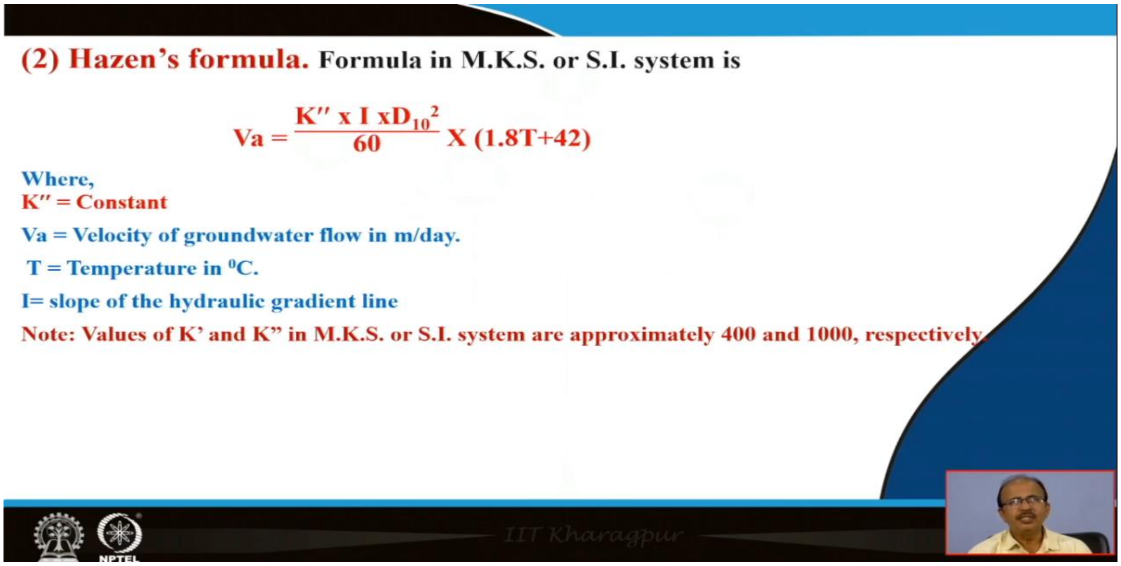

And Hazen's formula the second formula usually in M.K.S or S.I system is

$\mathbf{Va = \frac{K''\Box x\ I\ xD_{10}^2}{60}\ X\ (1.8T+42)}$ in which one more parameter has been added here that is the temperature. So, with the switches and hazard formula generally prior to the Darcy's law the calculation for the groundwater velocity were being done. In this Hazen's formula we have seen that

$$Va = \frac{K''x\ I\ xD_{10}^2}{60}\ X\ (1.8T+42)$$

Where,

Va = Velocity of groundwater flow in m/day.

T = Temperature in ^{0}C.

I= slope of the hydraulic gradient line

Note: Values of K' and K" in M.K.S. or S.I. system are approximately 400 and 1000, respectively

So, this value is generally being considered in the M.K.S or S.I system 400 and 1000.

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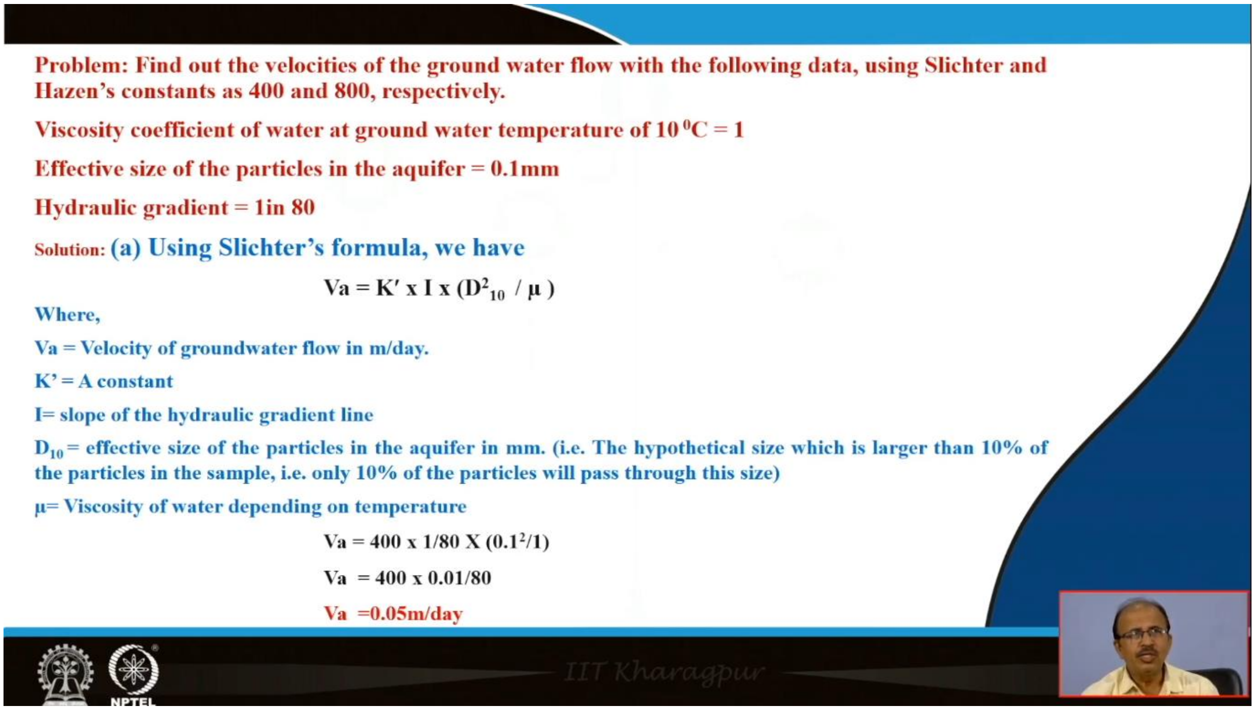

Now, based on the Slichter's and Hazen's formula one small problem is here; Find out the velocities of the ground water flow with the following data, using Slichter and Hazen's constants as 400 and 800, respectively.

Viscosity coefficient of water at ground water temperature of 10 ^{0}C = 1

Effective size of the particles in the aquifer = 0.1mm

Hydraulic gradient = 1in 80

Solution: (a) Using Slichter's formula, we have

$$Va = K' \times I \times (D^2_{10} / \mu)$$

Where,

Va = Velocity of groundwater flow in m/day.

K' = A constant

I= slope of the hydraulic gradient line

D_{10} = effective size of the particles in the aquifer in mm. (i.e. The hypothetical size which is larger than 10% of the particles in the sample, i.e. only 10% of the particles will pass through this size)

μ= Viscosity of water depending on temperature

$$Va = 400 \times 1/80 \times (0.1^2/1)$$

$$Va = 400 \times 0.01/80$$

$$Va = 0.05 m/day$$

So, this is coming as per your Slichter's formula.

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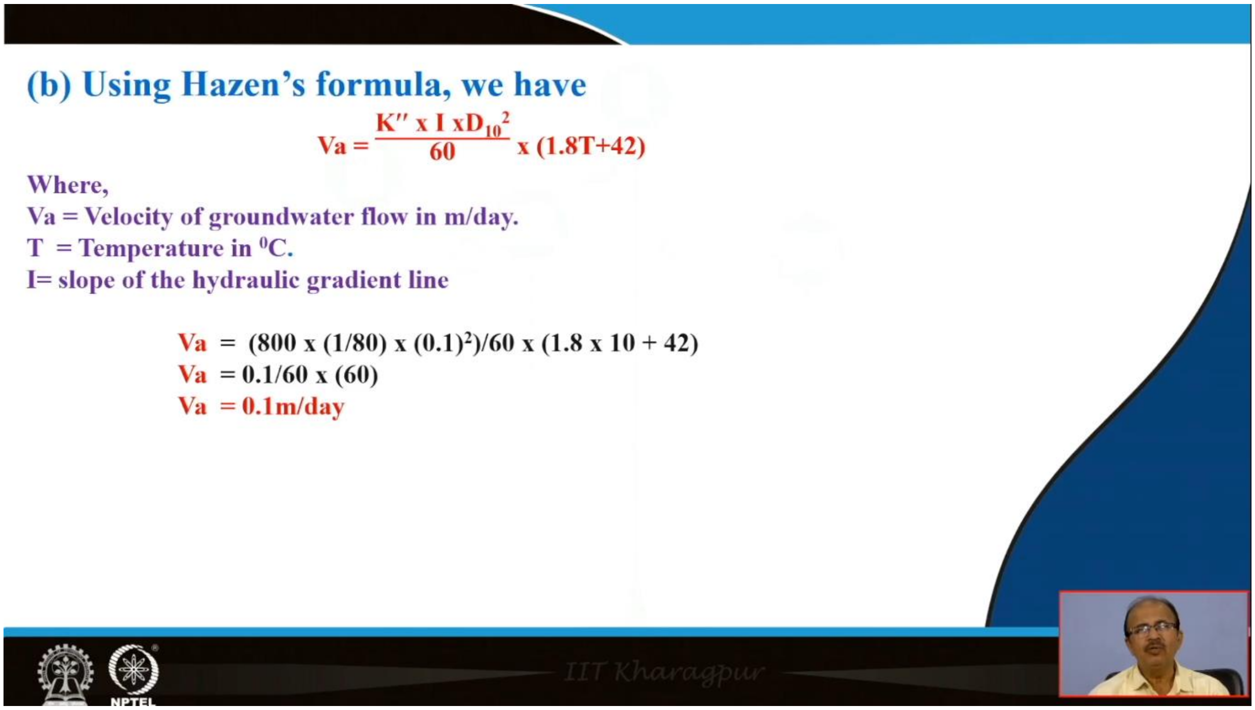

And now if we will use the Hazen's formula what will be the case? So, here the velocity of groundwater flow will be determined by

$$Va = \frac{K'' x\ I\ xD_{10}^2}{60} x\ (1.8T+42)$$

Where,

Va = Velocity of groundwater flow in m/day.

T = Temperature in ^{0}C.

I= slope of the hydraulic gradient line

$$Va = (800 \times (1/80) \times (0.1)^2)/60 \times (1.8 \times 10 + 42)$$

$$Va = 0.1/60 \times (60)$$

$$Va = 0.1 m/day$$

So, these two formulae were being used just prior to the onset of the Darcy's laws equations. So, generally these are being used for finding out the velocity of the groundwater flow inside the surface.

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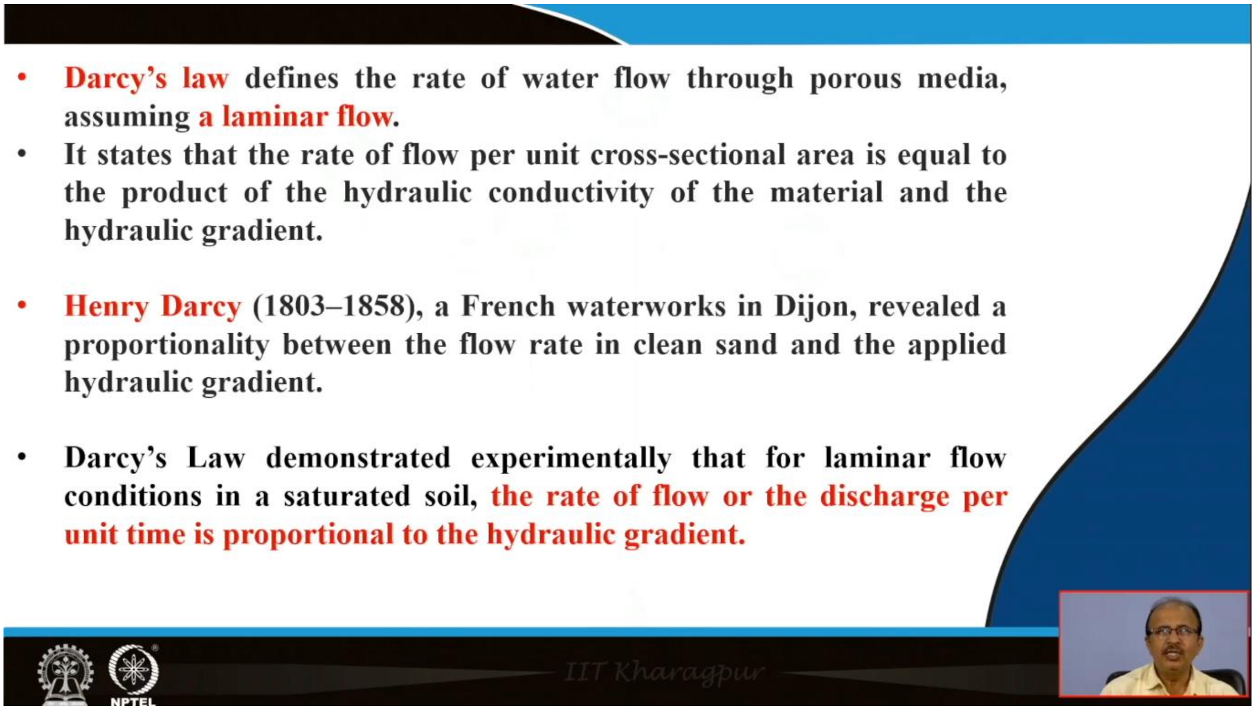

Now, regarding the Darcy's law it defines the rate of water flow through porous media and the condition is that it assumes a laminar flow only. So, Darcy law is defined the as the rate of water flow through porous media. So, it is very important and second assuming a laminar flow. It states that the rate of flow per unit cross sectional area is equal to the product of the hydraulic conductivity of the material and the hydraulic gradient.

So, hydraulic conductivity multiplied by hydraulic gradient, it will remain equal to the rate of flow per unit cross sectional area. This Henry Darcy, French waterworks in Dijon, revealed the proportionality between the flow rate in clean sand and the applied hydraulic gradient. So, he has just derived some relation that is the permeability behaviour between the flow rate in clean sand and the applied hydraulic gradient work.

So, Darcy law demonstrated experimentally that for laminar flow the condition is important this laminar flow is very important for Darcy law so and second is the porous media. So, these two are very well related term with the Darcy law, porous media and the assumption is the laminar flow. So, Henry Darcy has derived some of the personality relation and the relation between the flow rate and the hydraulic gradient.

Darcy law demonstrated experimentally that for laminar flow conditions in a saturated soil, so it is porous media is definitely in the media is saturated soil. The rate of flow or the discharge per unit time discharge per unit time is proportional to the hydraulic gradient. So, the rate of flow or the discharge per unit time is directly is proportional to the hydraulic gradient.

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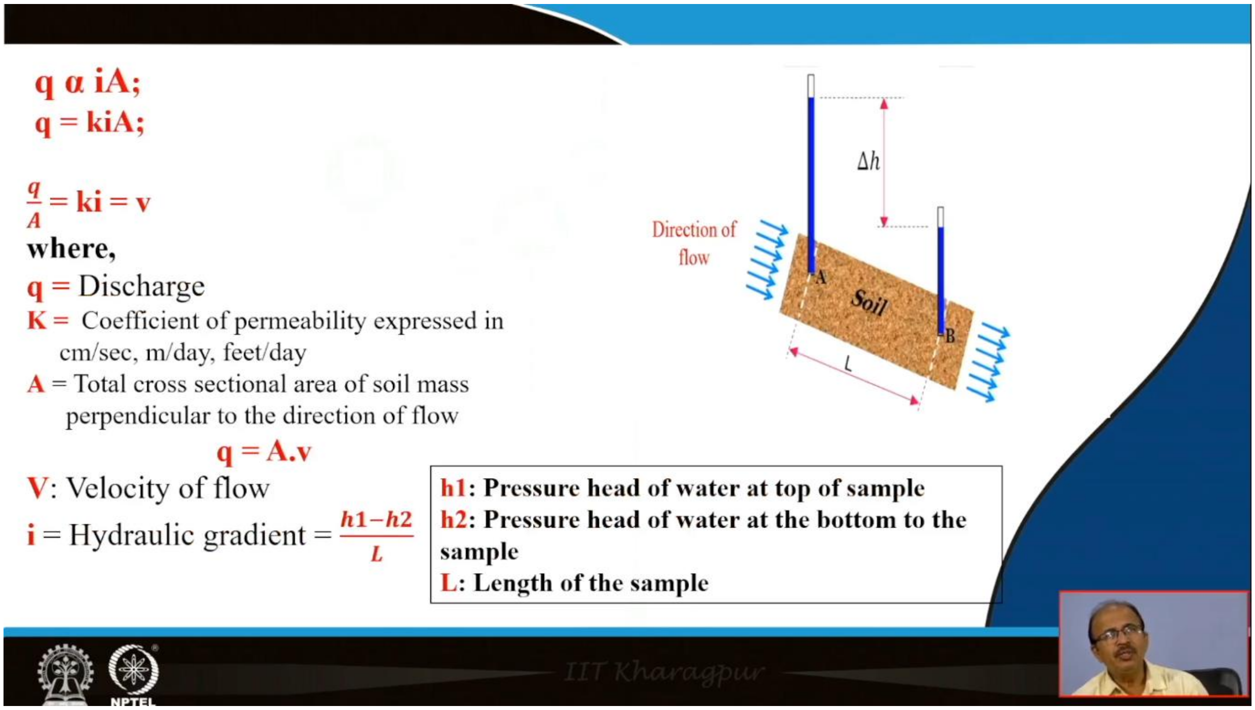

Now, means what means

q α iA;

q = kiA;

just if you will just break the proportionality some constant will come, this constant is the coefficient of permeability expressed in centimetre per second meter per day or feet per day i is the hydraulic gradient we are knowing h1 – h2 divided by L and A is the cross sectional area of the media of the soil mass through which the water is flowing.

So, we can write

$\frac{q}{A}$ = ki = v

where,

q = Discharge

K = Coefficient of permeability expressed in cm/sec, m/day, feet/day

A = Total cross sectional area of soil mass

perpendicular to the direction of flow

$$q = A.v$$

where v is the velocity of flow and i is the hydraulic gradient. So, h1 this we are knowing that

h1: Pressure head of water at top of sample

h2: Pressure head of water at the bottom to the sample

L: Length of the sample.

That is the difference length of the sample difference between the length of the sample. So, with this, we can find out that your hydraulic gradient = $\frac{h1-h2}{L}$

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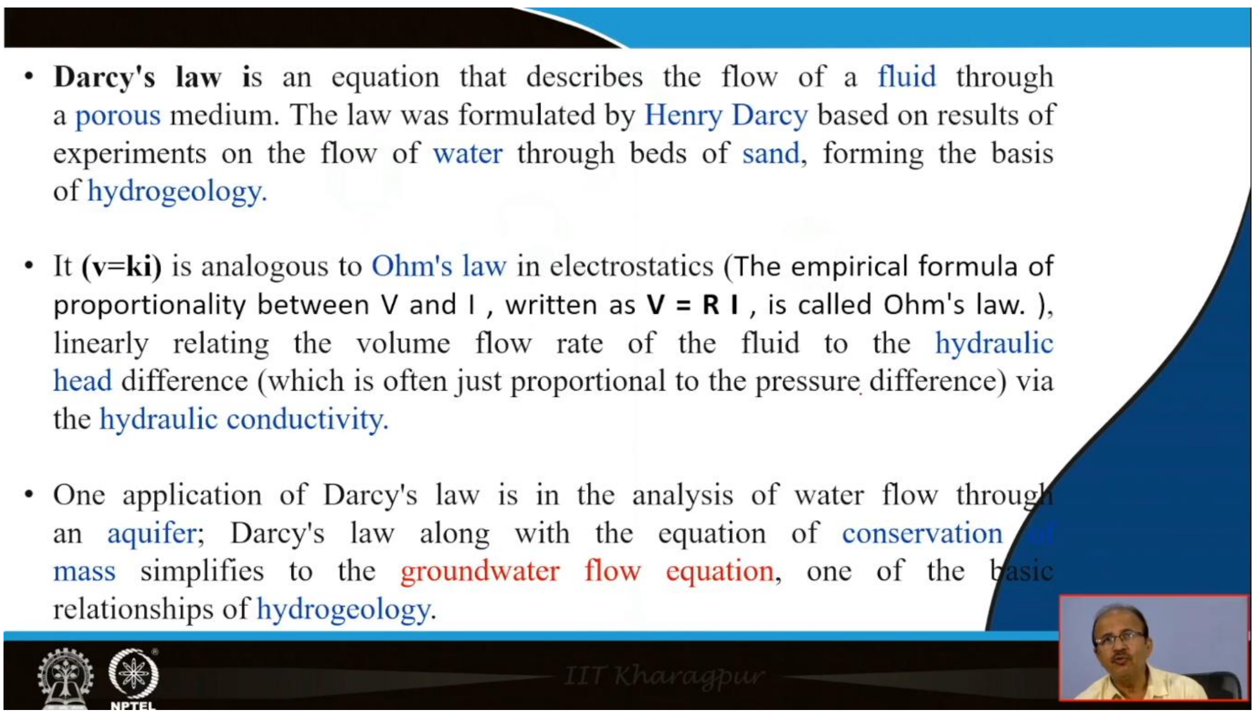

So, we can say in other way we can say that Darcy law is an equation that describes the flow of fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand, forming the basis of hydrogeology. It is forming one of the important basis of hydrogeology, hydrogeological equations are being derived from the Darcy law also.

Generally, this Darcy law v = k * i is very much analogous to ohm's law in electrostatics. If you will just recall they are also the empirical formula proportionality between V * I were written as V = R*I and this was the ohm's law. So, this Darcy law is linearly relating with the volume flow rate of the fluid to the hydraulic head difference; it is via the hydraulic conductivity.

One application of Darcy law is in the analysis of water flow through an aquifer. Aquifer is also a porous media and this is the; movement of water flow through the aquifer is also one of the application of Darcy law. Darcy law along with the equation of conservation of mass simplifies to the groundwater flow equation. One of the best relationship of hydrology one or we can say which makes the basics relationship of hydrogeological your chapters hydrogeological contributions.

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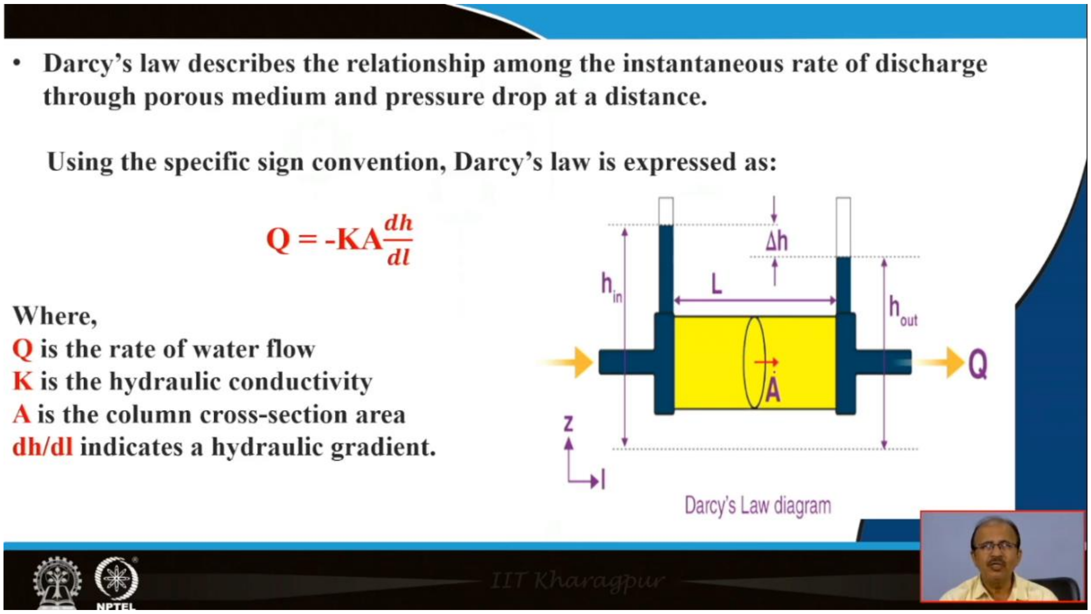

So, a bit more we can just learn the relation of the Darcy law, Darcy law describes the relationship among the instantaneous rate of discharge through porous medium and the pressure drop. In total if you will see this has happened and it is just showing the relationship, a relationship among the instantaneous rate of discharge to porous medium and the pressure drop at a distance.

And using the specific sign convention Darcy law is expressed as

$$Q = -KA\frac{dh}{dl}$$

Where,

Q is the rate of water flow

K is the hydraulic conductivity

A is the column cross-section area

dh/dl indicates a hydraulic gradient

So, through the diagram also we can understand it here, we are seeing here this is we have learnt in the previous slide also in flow the pressure rate is remains at the top whereas outflow it remains at the bit lower.

There will be pressure head difference will be here the distance between the 2 different wells or tubes are is l and this is the cross sectional area of the media. So, this is generally a very good diagram of Darcy law which shows the details about the queue is about the different pressure head and the distance between the different column.

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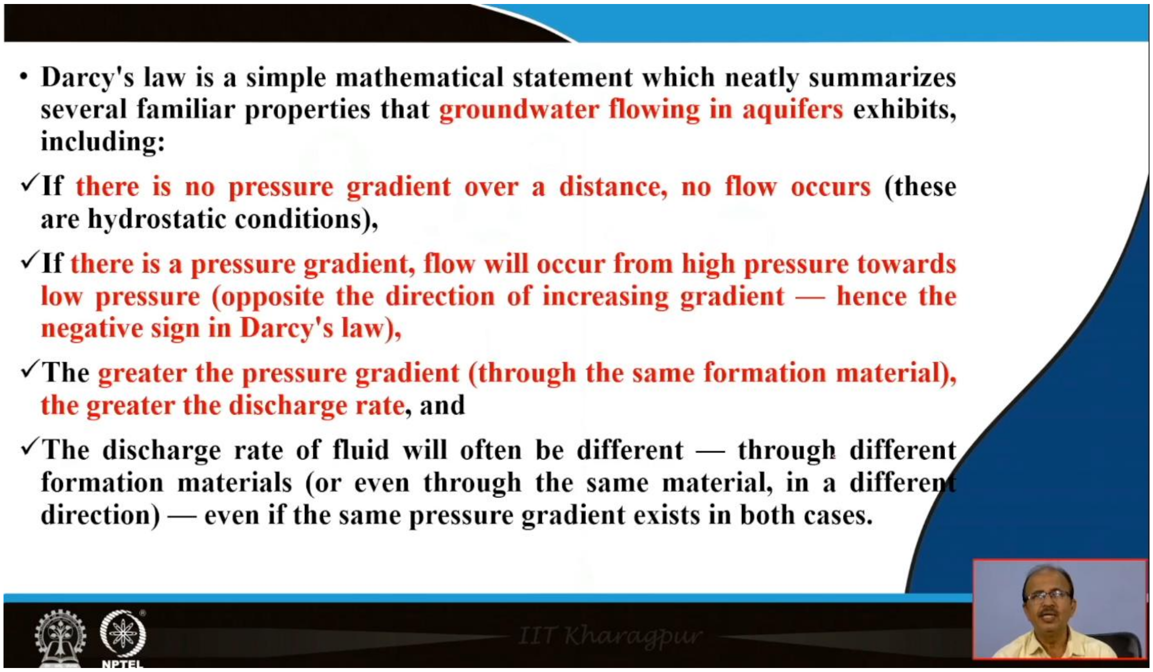

Now Darcy law is, if you will see it is a simple mathematical statement which nearly summarizes several familiar properties that groundwater flowing in aquifer exhibits. So, through this simple mathematical statement which is mentioned in the Darcy law, several your properties of regarding the groundwater of flow is being solved and this just it is mentioned here if there is no pressure gradient over distance no flow occurs.

So, if there will be no pressure gradient over a distance no flow occurs these are hydrostatic conditions. If there is a pressure gradient flow will occur from high pressure towards low pressure. So, opposite the direction of increasing radiance has the negative sign in Darcy law thatis negative sign is given minus you perhaps you have seen Q = -KA$\frac{dh}{dl}$. So, as because of the Darcy law the concept has come that if there is a pressure gradient flow pressure gradient their flow will occur from high pressure towards low pressure.

Towards low pressure, hence the negative sign is given mentioned in Darcy law. Another concept came the greater the pressure gradient the greater the discharge rate if the same formation material will remain. So, greater the pressure gradient the discharge rate will be greater. The discharge rate of fluid will often be different through different formation materials or even through the same material in a different direction and even if the same pressure gradient exists in both cases.

So, because of the Darcy law many more concept has come many more your consideration has come for because underground within the earth surface for finding out the ground of flow movement is the Darcy law is making it much more easier. Although, it is a bit difficult but does according to Darcy law we can find out the concept of the flow of groundwater movement inside the earth surface.

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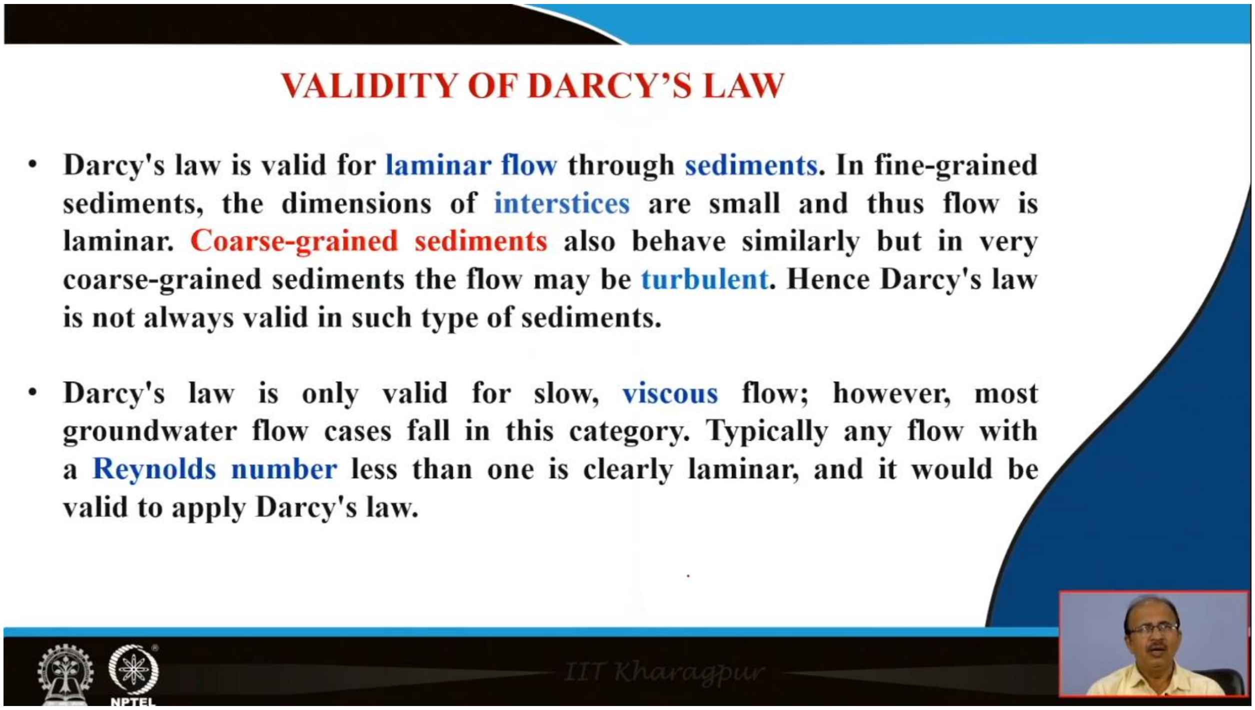

So, some more points for the validity of Darcy law. Darcy law are valid for laminar through sediments, laminar flow through sediments only. So, in fine grained sediment the dimension of interstitials are small thus flow is laminar. Coarse grained sediments also behave similarly but in very coarse-grained sediment the flow may be turbulent and Darcy law in such case is not valid in such type of sediments.

So, Darcy law is only valid for slow viscous flow however most groundwater flow cases fall in this category. Typically, any flow any flow with Reynolds number less than 1 is clearly laminar, if any flow is with a Reynolds number less than 1 then it indicates about it is laminar behaviour and it will be valid for valid to apply Darcy law in that condition also. So, Darcy law is only valid for slow movement.

Viscous flow most groundwater flow falls in this category only and typically the Reynold number if it is remaining less than 1 then, definitely it shows the laminar behaviour of the movement. So, this is all about the validity of Darcy's law.

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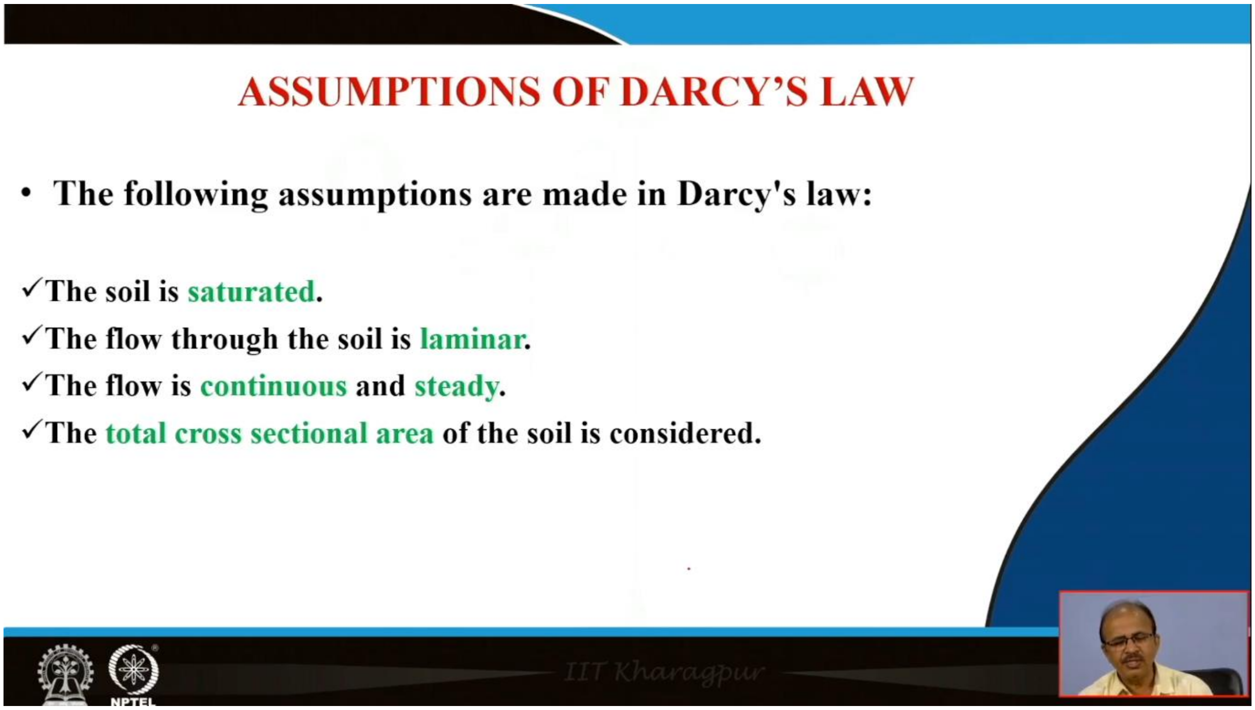

Next is the assumption of Darcy's law. So, here the Darcy's law has definitely given a law related to the grandeur movement but, what has happened thus some assumptions are also made by Darcy. Assumption was the soil is saturated the soil through which the water moves will be remain saturated, the flow through the soil is laminar the media through which it is flowing it is flow will be laminar.

The flow is continuous and steady the flow is continuous and steady and the total cross-sectional area of the soil is considered. So, all the four assumptions are very important any formation through which the water is moving if this is the formation and the water down water flow is taking place then this media should be saturated one filled up with water, totally filled up with water. So, the media or the soil will should be saturated.

Second, thus the flow through the soil in laminar it is it should not be turbulent it is laminar only. So, and third the flow is continuous and steady it is continuous and steady so no stop is there no discontinuity is there only the flow will remain continuous and steady then only the Darcy's law will be applied. The total cross sectional area of the soil is also considered here the total cross sectional area is also considered for the Darcy's law.

So, because you have seen that capital A was there which is the cross sectional area of the media through which the groundwater movement is flowing, groundwater movement is there or groundwater is flowing.

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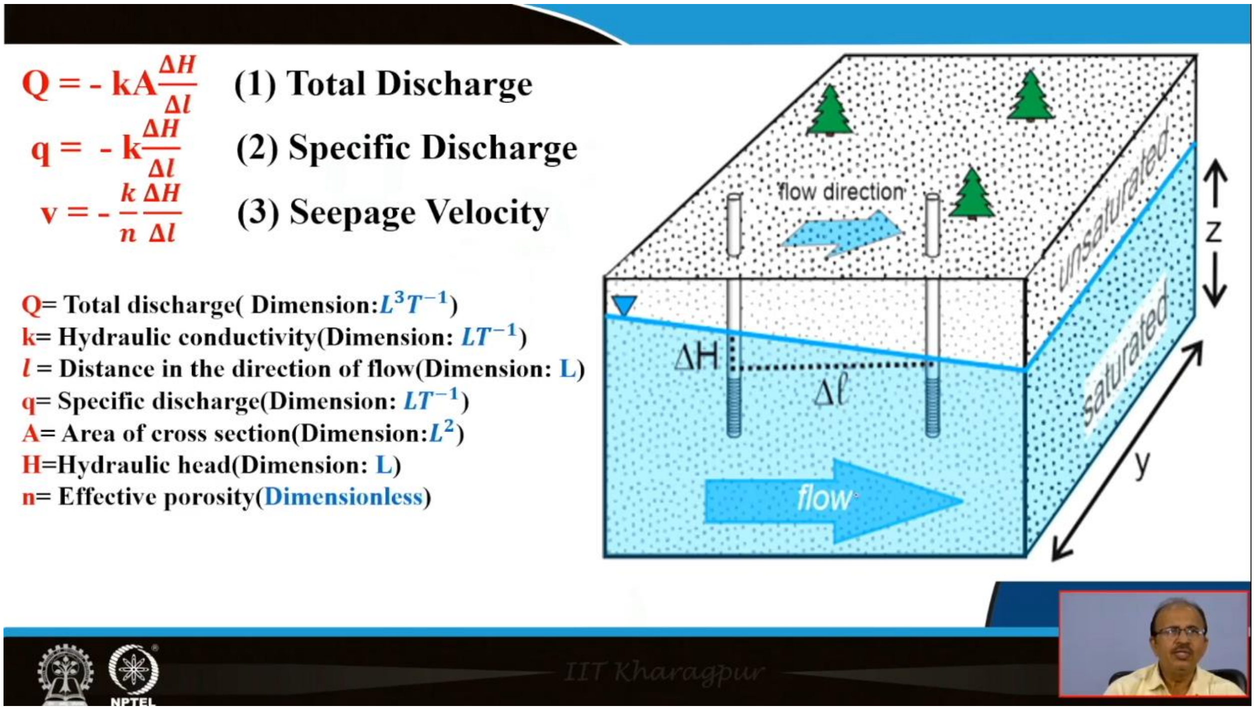

Now, next is the what we have just summarized with the whole concept related to the Darcy law that Q = - kA $\frac{\Delta H}{\Delta l}$ that is the total discharge it shows the total discharge q = - k$\frac{\Delta H}{\Delta l}$ by delta l it shows the specific discharge and **v = -** $\frac{k}{n}\frac{\Delta H}{\Delta l}$ is so the seepage velocity. So, you can see the flow is in the saturated formation only and the flow direction you can also see with the head of the difference in the head.

You can see the distance delta H and the difference here is delta H and the flow direction is from this to this direction. So, what is happening? Q is the total discharge and your k is the hydraulic conductivity and L is the distance in the direction of flow, q is the specific discharge, A is the area of cross section H is the hydraulic head and n is the effective porosity which is a ratio no dimension.

So, this whole concept tells us that, with the help of Darcy’s law we can find out the at least the flow direction of the groundwater movement underneath the earth's surface. Without it, it is very difficult to know the actual direction of the groundwater movement inside the formations inside the earth's surface. So, this is all about the concept of the Darcy’s law. So, what we have understood we have started from knowing about the different important basic terms related to the groundwater movement.

We have seen that the some of the energy is required for the movement of the water underneath the earth's surface also because, underneath the earth surface definitely the water will move through certain media and it will flow through certain rocks, rocks means the aquifer means any formation which is just having the water. So, water having the water means what it is remaining saturated.

So, Darcy law is very well related with the saturated media first second the flow will be laminar flow it will be laminar flow. So, and it is very well understood that because of the pressure head difference because of the hydraulic conductivities, hydraulic gradient the water flows inside also from one aquifer to another aquifer. And then only the availability of the groundwater resources also remains at one place in plenty and other places in viscosity. So, this is all about the details about the Darcy’s law. Thank you very much.