

**Traffic Engineering**  
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**Lecture 09**  
**Single Regime Traffic Stream Models**

Welcome to Module B Lecture 4. In this lecture, we shall discuss about Single Regime Traffic Stream Models.

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### Recap of Lecture B.3

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- Macroscopic density parameter: Density, Occupancy
- Relationship between **speed, density** and **flow**
- Relationship between speed-flow (**v-q**), speed-density (**v-k**) and density-flow (**k-q**)
- Importance of **field location** and **comparison with theoretical curves**
- **Speed-flow** characteristics of a **modern freeway**



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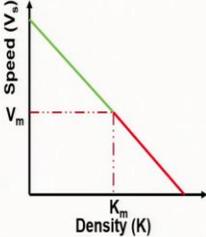
In Lecture 3, I mentioned to you about the macroscopic density parameter that is how to quantify, what are the difficulties, and if we want to estimate using occupancy then how we can estimate it. Then explained you the relationships between the speed, density and the flow among all these parameters, also the separate relationships between speed and flow, speed and density and flow and density and then highlighted the importance of field location where you are collecting the data and under which condition at what time.

That will determine that what data you will get, what curve we will fit and how that will match closely with the or how that will match either closely or remotely with the theoretical curves. Then also indicated how the speed flow curves look like for a modern freeway. With this background, today, we shall entirely focus on single regime models.

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### Single Regime Traffic Stream Models

- Several analysts attempted to fit relationships to observed data of speed and density considering a single regime phenomenon over the complete range of flow conditions
  - ✓ Green portion: Free-flow regime, Red portion: Congested flow regime
- Three major models: Greenshields' model, Greenberg's model, Underwood's model



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Why we call it a single regime? Several analysts attempted to fit relationship to observe data, as I said, to collect the field data and try to fit the relationship, the speed density relationship and also other relationships, considering single regime phenomenon over the complete range of flow condition. One relationship explaining the behavior say how density varies with flow or how density varies with speed or how speed varies with density not flow. How speed varies with density that relationship is explained, entire range is explained using a single expression, a single equation that is what I said that a single regime phenomenon over the complete range of flow condition. Obviously, this will include both stable flow and force flow condition.

Here, for example, you can see these green segments, it indicate the stable flow condition, capacity is somewhere here. And this red line indicates the congested or forced flow condition, but a single equation, a single regime phenomenon over the complete range of flow condition that is what we say that this is a model which is a single regime model. There are three major models that we are going to discuss in today's class. One is Greenshields' model, then Greenberg's model and Underwood's model. These are the three models we shall discuss now.

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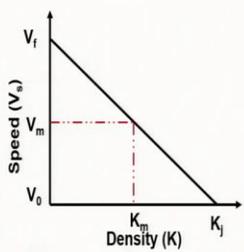
### Single Regime Traffic Stream Models

#### Greenshields' Model (1935)

- **Linear** relationship between speed and density

$$V_s = V_f - \left(\frac{V_f}{K_j}\right) \times K \quad (1)$$

Where,  $V_s$  = Speed of traffic stream  $V_f$  = Free-flow speed of traffic stream,  $K_j$  = Jam density  $V_m$  = Speed at maximum flow (optimum speed),  $K_m$  = Density at maximum flow (optimum density)



- **Boundary Conditions**

- $V_s \Rightarrow V_f$  When  $K \Rightarrow 0$ ; and  $V_s \Rightarrow 0$  when  $K \Rightarrow K_j$
- At  $Q_{max}$ ,  $V_s \Rightarrow V_m$  and  $K \Rightarrow K_m$



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First coming to the Greenshields' model. Greenshields' model assume linear relationship between speed and density, the simplest one, but I will also say a very practical one, simple one but a very practical and very meaningful one as well. So, the relationship can be explained using this equation as shown here.  $V_s = V_f - \left(\frac{V_f}{K_j}\right) \times K$ ;  $K_j$  is the jam density,  $V_f$  is the free flow speed, multiplied by  $K$ . So, this shows how the  $K$  changes and correspondingly  $V_s$ , so speed of traffic stream how it changes.

Now, there are certain boundary conditions which had to be satisfied. If  $V_s$  equal to  $V_f$  that means I am talking about the free flow condition, then what you will get, you will get  $K$  equal to 0. That is what you get here. And therefore, if you substitute put  $V_s$  equal to 0,  $K$  equal to then  $K_j$  that will be there. So, this is the free flow condition. When  $V_s$  equal to  $V_f$  free flow condition then density is 0. And when  $V_s$  equal to 0 standstill condition that means the density is equal to the jam density. At  $Q_{max}$ , let us consider. When the capacity flow occurs, then let us consider  $V_s$  is equal to  $V_m$  and at capacity flow  $K$  is equal to  $K_m$ .

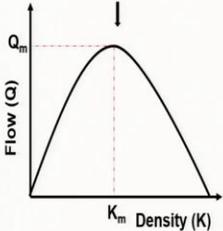
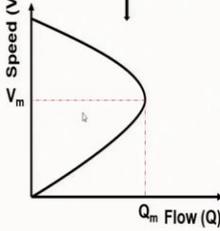
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### Single Regime Traffic Stream Models

- Flow-density and speed-flow relationship is **parabolic** in nature if we assume **linear speed-density** relationship
- Putting  $V_s = \frac{Q}{K}$  in equation 1 and  $K = \frac{Q}{V_s}$  equation 1

$$Q = V_f K - \frac{V_f}{K_j} K^2 \quad \dots (2)$$

$$Q = V_s K_j - \frac{K_j}{V_f} V_s^2 \quad \dots (3)$$



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### Single Regime Traffic Stream Models

#### Greenshields' Model (1935)

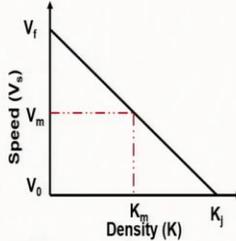
- Linear** relationship between speed and density

$$V_s = V_f - \left(\frac{V_f}{K_j}\right) \times K \quad \dots (1)$$

Where,  $V_s$  = Speed of traffic stream  $V_f$  = Free-flow speed of traffic stream,  $K_j$  = Jam density  $V_m$  = Speed at maximum flow (optimum speed),  $K_m$  = Density at maximum flow (optimum density)

**Boundary Conditions**

- $V_s \Rightarrow V_f$  When  $K \Rightarrow 0$ ; and  $V_s \Rightarrow 0$  when  $K \Rightarrow K_j$
- At  $Q_{max}$ ,  $V_s \Rightarrow V_m$  and  $K \Rightarrow K_m$



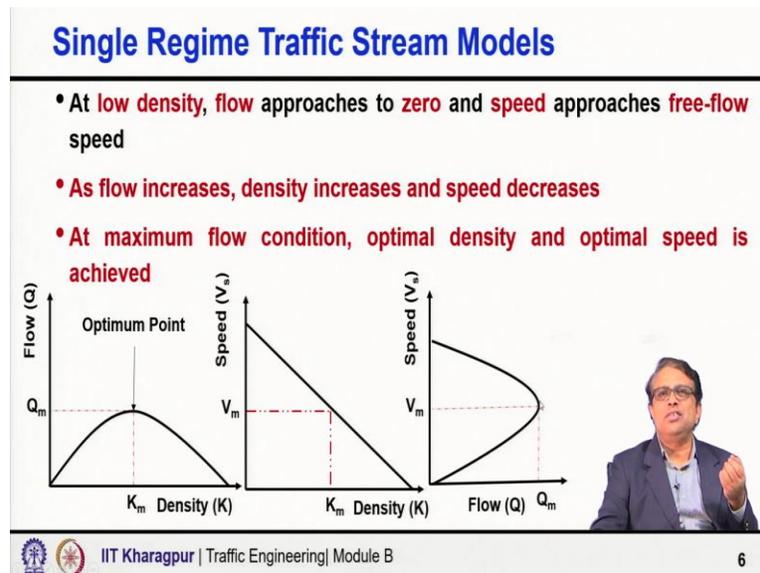

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Now, how if this is the linear speed density relationship then what will be the corresponding flow density and speed flow relationship. These two relationships will look like parabola. How we can get that. We can get it putting  $V_s$  equal to  $Q$  by  $K$  in equation 1 that means whatever equation we have shown here  $V_s$  equal to  $V_f$  minus  $V_f$  by  $K_j$  into  $K$ . In this equation we are substituting  $V_s$  using  $Q$  and  $K$ . So, this whole equation now becomes relationship between  $Q$  and  $K$ . You can see this equation and you can see that it is going to take a shape of a parabola.

Similarly, if you replace  $K$  with  $Q$  by  $V_s$  that substitution if we do in equation 1 instead of this substitution, that means I am replacing  $V_s$  here, but I am bringing  $K$ . So, now the relationship will

be, I am sorry, if we replace  $V_s$  equal to  $Q$  by  $K$  in equation 1 and then in another case, if we replace  $K$  equal to  $Q$  by  $V_s$  in equation 1, then we shall get these two equations both are actually parabolic equation. This shows the relation between  $Q$  and  $V$  and this shows the relation between  $Q$  and  $K$ . So, both cases you will get equations which are parabola.

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At low density flow appears to be 0 and speed approaches free flow speed as I have explained. So, if you take a point when the density is very low then the flow also becomes 0 and speed approaches to the free flow speed here. And as flow increases, density also increases. We are increasing the flow, so density is also increasing. And as the flow is increasing, the speed is decreasing. And at maximum flow condition, optimal density and optimal speed is achieved.

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## Single Regime Traffic Stream Models

- Estimation of **optimal density, optimal speed** and **maximum flow**

✓ Differentiating equation (2) w.r.t  $K$ ;  $\frac{dQ}{dK} = V_f - 2V_f \frac{K}{K_j} = 0 \Rightarrow K_m = \frac{K_j}{2}$

✓ Differentiating equation (3) w.r.t  $V_s$ ;  $\frac{dQ}{dV} = K_j - 2K_j \frac{V}{V_f} = 0 \Rightarrow V_m = \frac{V_f}{2}$

✓ Maximum flow,  $Q_m = V_m K_m = V_f K_j / 4$

Where,  $V_m$  = Speed at maximum flow (Optimal speed)

$K_m$  = Density at maximum flow (Optimal density)

- Although Greenshields' model used **extensively** in **transportation analysis because of its simple form** but **field observations** do not fully support the **shape** of the model



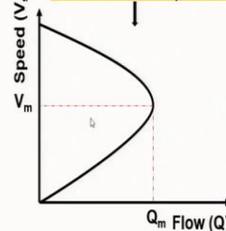
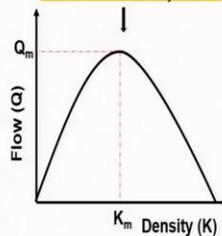
## Single Regime Traffic Stream Models

- **Flow-density** and **speed-flow** relationship is **parabolic** in nature if we **assume linear speed-density** relationship

✓ Putting  $V_s = \frac{Q}{K}$  in equation 1 and  $K = \frac{Q}{V_s}$  equation 1

$$Q = V_f K - \frac{V_f}{K_j} K^2 \quad \text{--- (2)}$$

$$Q = V_s K_j - \frac{K_j}{V_f} V_s^2 \quad \text{--- (3)}$$



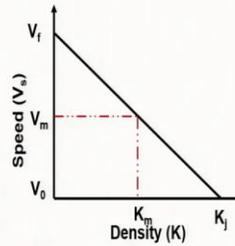
## Single Regime Traffic Stream Models

### Greenshields' Model (1935)

- **Linear** relationship between speed and density

$$V_s = V_f - \left(\frac{V_f}{K_j}\right) \times K \quad (1)$$

Where,  $V_s$  = Speed of traffic stream  $V_f$  = Free-flow speed of traffic stream,  $K_j$  = Jam density  $V_m$  = Speed at maximum flow (optimum speed),  $K_m$  = Density at maximum flow (optimum density)



#### Boundary Conditions

- $V_s \Rightarrow V_f$  When  $K \Rightarrow 0$ ; and  $V_s \Rightarrow 0$  when  $K \Rightarrow K_j$
- At  $Q_{max}$ ,  $V_s \Rightarrow V_m$  and  $K \Rightarrow K_m$



How to get that? We can differentiate equation 2 with respect to  $K$ . So, here we can calculate  $(dQ/dK)$  make it equal to 0. And another derivative we can take. We can, equation 3 we can take and differentiate this with respect to  $V_s$ . That is what this equation. So, if you do that, what we will get? You will get here,  $\frac{dQ}{dK} = V_f - \frac{2V_f}{K_j} \times k$ . And that indicates that when  $\frac{dQ}{dK} = 0$ ; if, when it will be 0, it will be at speed, when the capacity flow has reached. So, at capacity the density  $K_m = \frac{K_j}{2}$

Similarly, if you take this,  $\frac{dQ}{dK} = K_j - \frac{2K_j}{V_f} \times V = 0$ ; And we get speed at capacity is equal to  $V_f/2$ .

So, if these two are fine, that means, if the speed density relationship follows the Greenfield model as shown in equation 1, then at capacity as per that the density will be half of the jam density and speed will be half of the free flow speed. So, then what is actually the  $Q_m$ .  $Q_m$  capacity value would be  $V_m \times K_m$ . So,  $V_m = V_f/2$  and  $K_m = K_j/2$  so you get  $(V_f \times K_j)/4$ .

Now, again, this is valid if my assumption about the linear relationship of speed and density is valid as shown in equation 1. So, this is very simple but very useful. But although Greenshields' model used extensively in transportation analysis, because of its simple form, you can see anything linear is always very simple to use, but field observations do not fully support the shape of the model. Some cases people got little better match, but some cases it did not give that much good match. So, further development happens.

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## Single Regime Traffic Stream Models

**Example1:** Speed and density are related as,  $V=100-0.5K$ ;  $V=\text{kmph}$ ,  $K=\text{vehicles/km}$ . Calculate-maximum flow on that stretch?

**Solution-** For maximum flow,  $\frac{dQ}{dK} = 0$ ; We know,  $Q = KV$

$$Q=100k - 0.5k^2 \quad \frac{dQ}{dK}=100-kj=0; \Rightarrow k_j = 100 \text{ veh/km}$$

$$V_f = 100 - 0.5 \times 100 = 50 \text{ km/hr.}$$

$$Q_m = 100 \times 50 = 5000 \text{ veh/hr.}$$



But before we go that or before we discuss that, let us take an example problem. Speed and density are related as  $V$  equal to 100 minus 0.5K. Clearly this is a linear equation as proposed by Greenshield. So, it is a Greenshield model.  $V$  equal to in kmph,  $K$  in vehicles per kilometer. Calculate the maximum flow on the stretch, capacity flow. How you do? We can, for maximum flow we know that  $dQ/dK=0$  and we know  $Q= kV$ , so you take this  $Q= K \times V$ , so  $100K-0.5 K^2$ . Now, we take  $dQ/dK$  and we find  $K_j=100$  vehicle per hour that is the jam density.

Now,  $V_f$  will be how much then  $100-(0.5 \times 100)$ , so 50 kilometer per hour. Use this equation directly. The  $K$  is known so  $V$  is known. And once I know that, and this is not  $V_f$  rather it is  $V$  at optimal value and this is  $K$  also is at optimal flow. It is not  $K_j$ , it will be  $K_m$ . This is also not  $V_f$ , this would be  $V_m$ . So, density at capacity is 100 vehicle per kilometer and speed at capacity is 50 kilometer per hour. So, what do we get? The capacity value will be 50 into 100 so it is 5000 vehicle per hour .

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### Single Regime Traffic Stream Models

**Example2:** Space mean speed and density are linearly related;  $V_f=80\text{kmph}$ ,  $K_j=100\text{ veh/km}$ . Determine the capacity?

**Solution-** According to Greenshields' model,  $V_s = V_f - \left(\frac{K}{K_j}\right)V_f$

We know,  $Q = KV \Rightarrow Q = KV_f - V_f\left(\frac{K^2}{K_j}\right)$   
 $= 80K - K^2\left(\frac{80}{100}\right)$

For maximum flow,  $\frac{dQ}{dK} = 0 \Rightarrow \frac{dQ}{dK} = 80 - \left(\frac{8}{5}\right)K$

$K_m = 50 \text{ \& } V_m = 40$

Then,  $Q_m = 2000\text{ veh/hr}$ .



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Let us take another example. Space mean speed and density are linearly related. The moment we say speed and density are linearly related that means I am saying it is the Greenshield model.  $V_f$  equal to 80 kilometer per hour,  $K_j$  equal to 100 vehicle per kilometer that is the jam density, and we want to determine the capacity.

So, according to Greenshields' model, we know  $V_s = V_f - (V_f/K_j) \times k$  also, we know  $Q = KV$ . So, take the  $Q, K$  multiplied by this equation, so  $Vfk - \frac{Vf}{kj} k^2$ . So, you put the value here. We know that  $K_j$  is 100 vehicle per kilometer and  $V_f$  equal to 80. So, you put these values and we get this equation. Now, for maximum flow because we say at capacity, so  $dQ/dK$  will be 0. So, put that. We get what is the value of  $K$  at capacity so that is the  $K_m$ . And similarly, we get what is the speed at capacity that also you can get and then you can find out then  $K_m$  into  $V_m$  equal to the  $Q_m$ .

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### Single Regime Traffic Stream Models

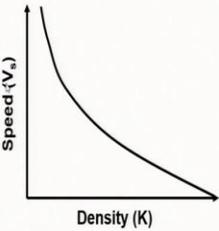
#### Greenberg model (1959)

- Observing the demerit of Greenshields model, nonlinear (logarithmic) relationship between speed and density was proposed

$$V_s = V_m \ln(K_j/K) \quad (4)$$

Where,  $V_m$  is speed at maximum flow (Optimal speed)

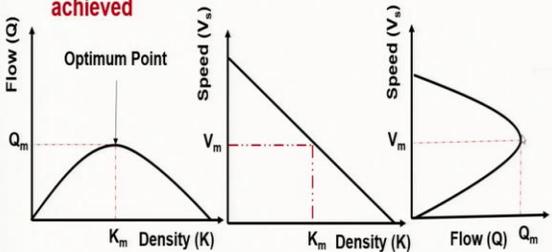
- Model breakdowns at low concentrations i.e.  $K=0$ , hence model can be used only in congested regime



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### Single Regime Traffic Stream Models

- At low density, flow approaches to zero and speed approaches free-flow speed
- As flow increases, density increases and speed decreases
- At maximum flow condition, optimal density and optimal speed is achieved



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Now, as I said that although Greenshields' model is very simple and quite a popular model among transport professionals or traffic engineers, and very simple one very practical one, but not that always it will give you a good fit. So, people tried, researchers tried different alternatives. And now I am going to show you a model which was suggested by Greenberg and it is also another very well known model called Greenberg's model.

And observing the demerit of Greenshields' model, Greenberg suggested non-linear rather a logarithmic relationship between speed and density. What was suggested is like this shown as equation 4 in the slide  $V_s$  equal to  $V_m$ , remember that it is not  $V_f$ , not the free flow speed, it is the

speed at maximum flow or capacity flow, you can also call it as optimal speed, into  $\ln K_j$  by  $K$ . So, this is a non-linear relationship, but what you find the model breaks down at low concentration. If you put  $K$  equal to 0 speed is infinity. Any value it can take.

So, therefore, this model normally is not used for low density region. This model is normally not used for the low-density region, but it is used predominantly for a range where the density is moderate to high, because here otherwise it will give you a free flow speed which will be unrealistic. But does it really, is there any basis for that, why it is considering infinity or so much highest value of speed. I will discuss it further. Let me complete the other model also. And then I will make a comparison.

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### Single Regime Traffic Stream Models

- **Derivation of Greenberg's model**

General form,  $V_s = a \ln \left( \frac{b}{K} \right)$

$Q = V_s \times K = aK \ln \left( \frac{b}{K} \right) = aK \ln \left( \frac{K_j}{K} \right)$

For maximum flow,  $\frac{dQ}{dK} = 0, \Rightarrow \frac{dQ}{dK} = -a + a \ln \left( \frac{K_j}{K_m} \right) = 0$

$K_m = \frac{K_j}{e}$ , Hence,  $V_m = a \ln e \Rightarrow a = V_m$

Therefore,  $V_s = V_m \ln \left( \frac{K_j}{K} \right)$

**Boundary Condition:**  
When,  $K = K_j, V_s = 0$   
Hence,  $b = K_j$





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We can derive this Greenberg model from a generalized equation. Generalized form could be  $V_s$  equal to  $a \ln b$  by  $K$ . This is the logarithmic relationship. But we know that there is a boundary condition when  $K$  equal to  $K_j$ , density is the jam density, then the speed is 0,  $V_s$  equal to 0. If you put that you will get here  $b$  equal to actually  $K_j$ . So, using this boundary condition  $K$  equal to  $K_j$  then  $V_s$  equal to 0. So, you can find out then  $b$  equal to  $K_j$ .

Now, what is then the  $Q$ ?  $Q = V_s \times K$ .  $Q$  equal to speed  $\times$  density. So,  $aK \ln(b/K)$ . Now, for maximum flow, we can take the derivative  $dQ/dK$  make it 0 and you get them  $dQ/dK$  as like this, as shown here, minus  $-a + a \ln(K_j/K_m)$  that is equal to 0. Now,  $K_m$  equal to  $K_j$  by  $e$ . And therefore,  $V_m$  you can write it as  $a \ln e$  which is equal to  $a$ . So, the  $V_m$  actually is the value of  $a$ .

So, what is the value of a? It is actually  $V_m$ . So, now, if you put it back to this equation, this will then become, a becomes  $V_m$  and b becomes  $K_j$ . So, the equation becomes  $V_s$  equal to  $V_m \ln K_j$  by  $K$ .

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## Single Regime Traffic Stream Models

**Example3:** The flow-speed relationship is given as  $Q=265 \times V - 79V \ln(V)$ , calculate (i) maximum flow, (ii) speed and density at maximum flow

**Solution:**  $Q = 265V - 79V \ln(V) \Rightarrow$  Differentiating this with respect to  $V$

$$\frac{dQ}{dV} = 265 - 79(1 + \ln V) = 0 \Rightarrow V_m = 10.53 \text{ km/hr.}$$

(i)  $Q_{max} = 832 \text{ veh/hr.}$

(ii)  $K_m = \left(\frac{832}{10.53}\right) = 79 \text{ veh/km and } V_m = 10.53 \text{ km/hr.}$



## Single Regime Traffic Stream Models

• Derivation of Greenberg's model

General form,  $V_s = a \ln\left(\frac{b}{K}\right)$

**Boundary Condition:**  
When,  $K = K_j, V_s = 0$   
Hence,  $b = K_j$

$$Q = V_s \times K = aK \ln\left(\frac{b}{K}\right) = aK \ln\left(\frac{K_j}{K}\right)$$

For maximum flow,  $\frac{dQ}{dK} = 0, \Rightarrow \frac{dQ}{dK} = -a + a \ln\left(\frac{K_j}{K_m}\right) = 0$

$K_m = \frac{K_j}{e}$ , Hence,  $V_m = a \ln e \Rightarrow a = V_m$

Therefore,  $V_s = V_m \ln\left(\frac{K_j}{K}\right)$



## Single Regime Traffic Stream Models

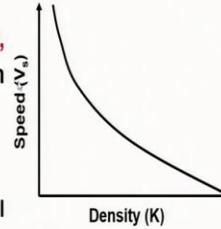
### Greenberg model (1959)

- Observing the demerit of Greenshields model, nonlinear (logarithmic) relationship between speed and density was proposed

$$V_s = V_m \ln(K_j/K) \quad (4)$$

Where,  $V_m$  is speed at maximum flow (Optimal speed)

- Model breakdowns at low concentrations i.e.  $K=0$ , hence model can be used only in congested regime



Let us take a small example. The flow speed relationship is given by  $Q$  equal to  $265 V$  minus  $79 V$  into  $\ln V$ . Now, this is a logarithmic relationship and we can understand that it has actually come from the Greenberg's model, Greenberg's logarithmic model. Now, this includes this term. Now, you can see here, now the speed flow relationship includes the term and you can see here that the speed flow relationship includes the term  $\ln V$ . So, you can understand it has come from the logarithmic relationship as suggested by Greenberg for or between speed and density. But here we are giving it flow equal to this expression. So, now we want to calculate maximum flow.

How we can calculate maximum flow? Take this and differentiate with respect to  $V$ . So, you can get  $dQ/dV$  and from here you can find out what is the  $V_m$ . Once you know the  $V_m$  or  $V$  value at capacity you can get the  $Q_{max}$  value. Put it simply in this equation. And once you know the  $Q_{max}$  capacity value and speed at capacity, you can also get the value of  $K_m$ . Simply it is  $Q$  by  $K$ ,  $Q$  by  $V$ . So, that way you can calculate that  $K_m$  equal to 79 vehicle per kilometer, speed at capacity  $V_m$  equal to 10.53 kilometer per hour and  $Q_{max}$  will give you 832 vehicle per hour.

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### Single Regime Traffic Stream Models

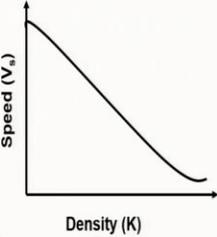
#### Underwood model (1961)

- As Greenberg's model fails at low concentration, an exponential relationship between speed and density was proposed

$$V_s = V_f \times e^{-\frac{K}{K_m}}$$

Where,  $K_m$  is density at maximum flow

- Does not represent zero speed at high concentrations



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Now, Underwood also suggested another non-linear model, because Greenshield suggested linear relationship. Contrary to that, Underwood also suggested non-linear relationship which looks like this  $V_s$  equal to  $V_f$  into  $e$  to the power  $K$  by  $K_m$ . Remember that note it or note it carefully that here we are not using  $K_j$ , but we are using  $K_m$ , but  $V$  is  $V_f$ . So, speed is we are using free flow speed, but the density, it is not the jam density, it is the density at optimal speed or capacity.

Why this was suggested, because primarily the Greenberg's model fails at low concentration. So, this is a model which does not fail at low concentrations. So, could be utilized effectively. But then at the other end it fails, because it does not represent 0 speed at high concentration. You can see here. You can see from equation also. Whatever may be the  $K$  value the speed is unlikely to be exactly 0. It will still have a very low value not 0. So, it does not represent 0 speed at high concentration.

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## Single Regime Traffic Stream Models

**Example 4:** The relationship between speed (km/h) and density (veh/km) for a traffic stream on road is given as  $V = 60e^{-0.01K}$ , Calculate (i) Density at maximum flow? (ii) Capacity of road? (iii) Free-flow speed of the stream?

**Solution-**  $V = 60e^{-0.01K}$  and  $Q=VK \Rightarrow Q = 60Ke^{-0.01K}$

At maximum flow condition,  $\frac{dQ}{dK} = 0 \Rightarrow \frac{dQ}{dK} = 60e^{-0.01K}[K \times (-0.01) + 1] = 0$

(i) Density at maximum flow,  $K_m = 100 \text{ veh/km}$

(ii)  $Q_{\max} = 100 \times 60e^{-0.01 \times 100} = 2207 \text{ veh/hr}$

We know,  $V_s = V_f \times e^{\frac{-K}{K_m}}$   
 $= V_f e^{-0.01K}$

(iii) Free-flow speed,  $V_f = 60 \text{ km/hr.}$



Let us take an example problem. The relationship between speed and density for a traffic stream on a road is given by this. You can clearly see that relationship between speed and density is given by this equation. Calculate density at maximum flow, capacity of road and free flow speed of the stream. Three parameters we need to calculate.

So, take the equation  $V$  equal to  $60e^{-0.01K}$  and you can take then  $Q$  equal to  $V$  into  $K$ , so  $60K$  into  $e^{-0.01K}$ . And at maximum flow condition  $dQ/dK$  equal to 0. So, you can take the derivative. Make it equal to 0. And you can get here what is the value of  $K$  at maximum flow condition. So, that is  $K$  is actually  $K_m$ .

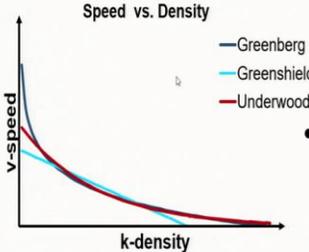
Once the density at maximum flow or capacity is known, you can actually calculate this  $Q_{\max}$  because we know  $60K$ . So,  $60$  into  $K$  is  $100$  into  $e^{-0.01 \times 100}$  and what is the  $K$ ,  $K$  value is  $100$ . So, you can get the  $Q_{\max}$ . So, now you know  $V_s$  equal to  $V_f$  into  $e^{-K/K_m}$  by  $K_m$ . So, you put it here and it clearly tells you that what is the value of  $V_f$ .  $V_f$  is actually  $60$  kilometer per hour.

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## Single Regime Traffic Stream Models

### Applications

#### Comparison between Single Regime Models



- Greenshields' model touches both the axis but Greenberg model did not touch Y-axis and Underwood model did not touch X-axis
- Each model has its advantages and disadvantages: Applicability depends on site conditions



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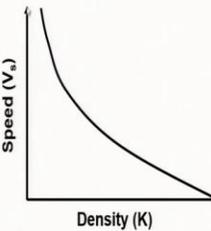
## Single Regime Traffic Stream Models

### Greenberg model (1959)

- Observing the demerit of Greenshields model, nonlinear (logarithmic) relationship between speed and density was proposed

$$V_s = V_m \ln(K_j/K) \quad (4)$$

Where,  $V_m$  is speed at maximum flow (Optimal speed)



- Model breakdowns at low concentrations i.e.  $K=0$ , hence model can be used only in congested regime



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Now, let us come to the applications part of it. There are three models, Greenshields', very simple linear relationship, then Greenberg's and then Underwood's model. Let us go one by one. As I said already that Greenshield model is very simple, but very practical. Why I said that? If you look at the Greenberg's model, let me go back there also, you can say it actually breaks down at low concentration.

So, if I try to plot all the three-model using a single plot then this Greenshields' the blue line is touching both x and y axis that is both K and V it is touching clearly. But then the Greenberg's model is touching this end, but not touching the speed axis. The speed axis it is not touching.

Density axis it is touching. As you have seen earlier that it is not touching the speed axis, Greenberg's model. So, here also Greenberg model is shown that it is not touching. What is the actual implications?

Now, theoretically, this may be all right as well. I considered this could be alright. Why? If there is no vehicle around, hardly any vehicle around, then why the density should influence your speed, what we are trying to capture here, trying to state here, how density influenced the speed. If there is no vehicle around then there is practically, why your, there should be loss in the freedom of movement. There is no loss.

So, theoretically, a vehicle can travel at any speed as far as the density is concerned. Density is not a factor. Any speed could be possible theoretically. So, I feel that Greenberg's this discontinuity here, what it say, it breaks down is theoretically not wrong. What happens? Real data will show you like this, no. But that reason is different.

When you design a road, you will have a design speed. You will have maximum speed limit as well. So, a vehicle can no way exceed those speed limits. So, the constraint whatever is coming it is because of the road design. The constraint will also come because of the vehicle technology. A vehicle is not an airbus.

So, it cannot travel at any speed it wants. There are limitations with the technology, other considerations of safety, road design, all these factors will actually make even the free flow speed a particular value only for a given condition or a given context. But understand that that restriction is not because of a density value.

So, theoretically, it might be alright to say that, that there is a discontinuity at the low-density level. But because of the vehicle technology limitations, safety consideration, road design considerations and any other considerations other than the density there will be a defined or a definite value of the free flow speed.

Let us consider the other end now, Underwood model. So, the red line is shown as Underwood model. It is touching the speed axis, but it is not touching the density axis. It is again same that density will never be, the speed will never be 0. Is this also very unrealistic theoretically? Again, I feel no. It is not unrealistic. Why, because yes, the freedom of movement will come down as

there are more and more vehicles around. But why the vehicle should come to a standstill with 0 speed? Why the 0 speed should come theoretically.

If there are more vehicles, just consider you were driving in a very lean period, no vehicle is around and then there are a few vehicles and the number of vehicle is more and more it will affect your speed. But why because of more vehicle the speed will come down to 0 specifically. It should not come. Still there should be. Vehicle should move. Why the speed should come to 0. Even if you consider the congested operation, 0 speed average how that is possible.

So, I also feel that theoretically it might be that it will not touch this x-axis. It is coming down. It is not telling exactly 0. In reality, it will come so low that you can consider that it is touching. Beyond certain density value which you can consider as jam density, you can consider the speed is practically 0. That could be very realistic assumption as well that to assume that density could be up to a maximum value. That approximation is fine.

So, altogether, each of these models, three models, Greenshield, Greenberg, and Underwood's model, each of this model has its own merits as well and considerations. The fact remains true what is common in all the models that as density starts building up there will be loss in the freedom of movement and therefore the speed will come down.

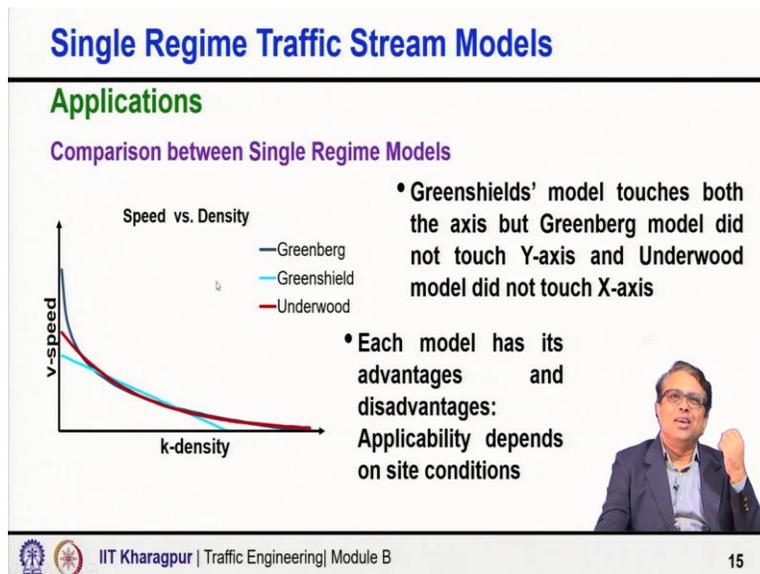
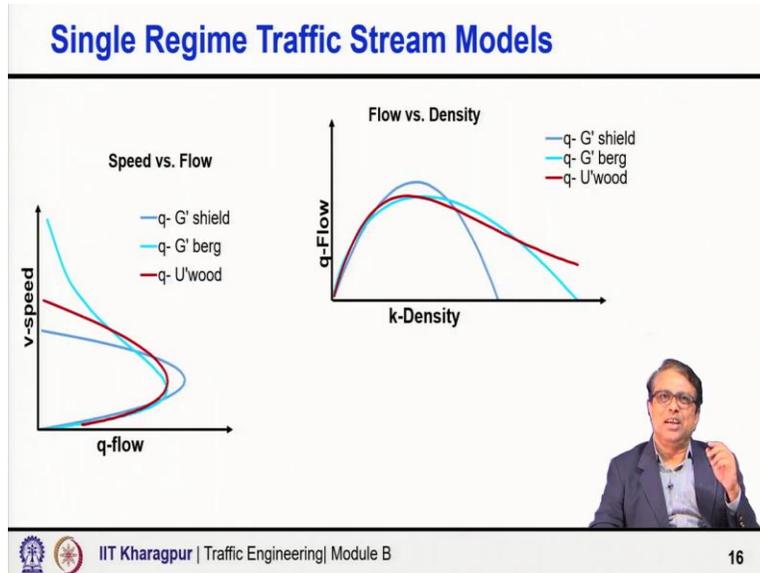
How exactly speed will come down, whether it will follow a relationship as given by Greenshield, exactly linear may not follow. It may not follow even Greenberg's model, it may not follow an Underwood's model as well, because the way the density will influence the speed will also depend on the many other conditions.

Remember my previous lecture, where you were collecting the data, on which road, what you can get in an arterial, you will not get for a freeway. At what location you are calculating or recording the data. So many things, so many considerations will be there. What nature, how it will get reduced following Greenshields's curve, following Greenberg's curve or Underwood's curve or any other curve that is a different issue.

It will vary depending on prevailing roadway, traffic and control conditions and may be many other factors where you are doing what, when you are measuring what is the observed thing. Accordingly, we will get the data and try to fit a curve. And this discontinuity at very low density

and also discontinuity at very high density, they also, the theoretical part and the practical considerations that connection is very interesting.

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Now, as I have shown here the speed density curve for this three models Greenshield, Greenberg and Underwood, I have shown here also the corresponding speed flow curve and flow density curve. As you can see here, here the Greenberg's model given infinity. It does not touch the free flow point on speed axis. So, here also Greenberg it does not touch. It looks like this. And how the flow density will look like? Here also the red one, Underwood model will not touch the density axis. So that is what is shown here as the red line.

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## Single Regime Traffic Stream Models

### Calibration of Single Regime Models

- Traffic models were used to determine stream characteristics: **Jam density; speed and density at maximum flow**
- Usually involves collecting **appropriate data** and **fitting** the data points to a suitable model
- **Regression analysis** is often used to find coefficients **a** and **b** from equation,  $y = a + bx$  where, **y=dependent variable x= independent variable a & b are constants (Method of least square)**
- **Co-efficient of determination ( $R^2$ ), signs and values of a and b should be checked while performing this analysis**



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Now, when you want to calibrate single regime model, that is very important, we need to calibrate with the field data. So, traffic data were used to determine stream characteristics. We try to find out what is the jam density, what is the speed and density at maximum flow and such meaningful measurements. Usually involves collecting appropriate data and fitting the data points to suitable model. And in almost all the cases we use linear regression model.

Generalized form is something like  $y = a + bx$ , where  $y$  is the dependent variable and  $x$  is the independent variable and  $a$  and  $b$  are the coefficient estimates which we obtain for a given data how best these coefficients are estimated. Using the list square techniques you know some of the squares of errors that we are trying to minimize through the fit. So, we are accepting a model generally what are the properties that we should check carefully.

We should look at obviously the  $R$  square value, the coefficient of determination, higher  $R$  square is the better, obviously better fit, but we also look at the values of  $a$  and  $b$  what value we are getting actually, because there is a physical meaning of that. So, that has to be realistic. Even the sign everything we should check before accepting this and also, we should check the statistical significance of the coefficient estimate. You get some value of  $a$  and  $b$ . But are these estimates statistically significantly different from 0 that also you need to check. It is significant but statistically significant or not that also, that we should check.

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## Single Regime Traffic Stream Models

**Example 5** Calibration using Greenshields model by the use of following

data

K	10	20	30	40	50	60	70	80	90	100	100	110
U	75	75	65	55	40	50	30	35	20	30	10	10

**Solution-**

$$b = \frac{\sum_{i=1}^n x_i y_i - \frac{1}{n} (\sum_{i=1}^n x_i \times \sum_{i=1}^n y_i)}{\sum_{i=1}^n x_i^2 - \frac{1}{n} (\sum_{i=1}^n x_i)^2} = \frac{23400 - \frac{1}{12} (780 \times 495)}{65000 - \frac{(780)^2}{12}} = -0.614$$

$$a = \frac{\sum_{i=1}^n y_i}{n} - \frac{b \sum_{i=1}^n x_i}{n} = \frac{495}{12} + \frac{(0.614 \times 780)}{12} = 81.16$$

Here,  $x=K$  and  $y=V$

$$\sum K_i U_i = 23400, \sum K_i = 780, \sum U_i = 495, \sum K_i^2 = 65000$$

$$U = 81.16 - 0.614K$$



## Single Regime Traffic Stream Models

$K_i$	$K_i^2$	$U_i$	$K_i U_i$
10	$= (10)^2 = 100$	75	$= (75 \times 10) = 750$
20	400	75	1500
30	900	65	1950
40	1600	55	2200
50	2500	40	2000
60	3600	50	3000
70	4900	30	2100
80	6400	35	2800
90	8100	20	1800
100	10000	30	3000
110	12100	10	1100
120	14400	10	1200
$\sum = 780$	$\sum = 65000$	$\sum = 495$	$\sum = 23400$

← Calculation



Now, there is an example problem here. The K and U values are given. Simple one. None of you will probably actually calculate it using your calculator. You will use Microsoft Excel or any other platform to readily fit the data with model, fit a model for the given data, readily fit a model for the given data and calculate the value of a and b. But here I have shown the equations and the calculations also. So, if you want to do it on your own for better understanding you can do it. You can match also, the calculation how we can calculate a and b. And then each calculation is shown here.

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### Single Regime Traffic Stream Models

**Example 6** Determine the best fitted model using the following speed-density data

Speed	Density
53.2	20
48.1	27
44.8	35
40.1	44
37.3	52
35.2	58
34.1	60
27.2	64
20.4	70
17.5	75
14.6	82
13.1	90
11.2	100
8	115

- The **best regression fit** for each model is estimated
- **Greenshields** model is the best fitted model ( $R^2 = 0.9468$ )
- **Jam density ( $K_j$ )** =  $\frac{62.556}{0.528} = 118$  veh/km
- **Max flow ( $Q$ )** =  $\frac{118 \times 62.556}{4} = 1845$  veh/hr.

(A)  $y = -0.528x + 62.556$   $R^2 = 0.9468$   
(B)  $y = -28.59 \ln(x) + 144.76$   $R^2 = 0.9216$   
(C)  $y = 97.771e^{-0.021x}$   $R^2 = 0.9509$

Legend: — Greenshields model (A), — Greenberg model (B), — Underwood's model (C)

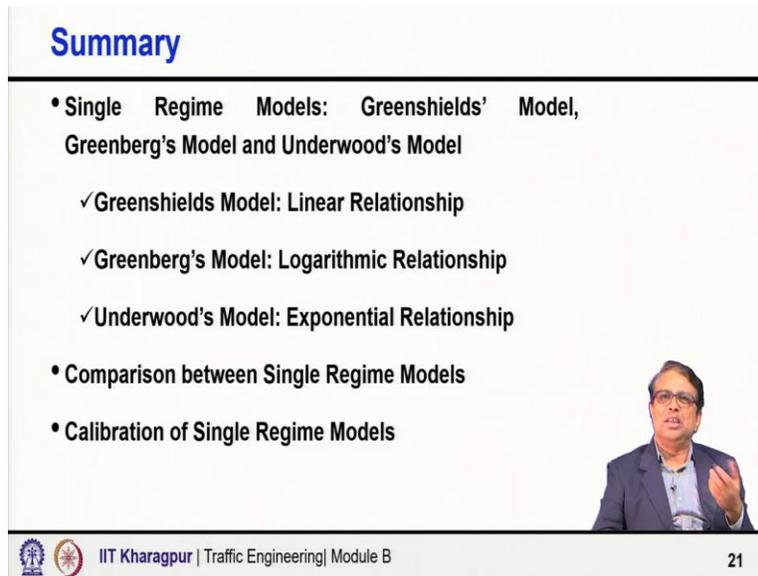
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Now, another example I have given. Determine the best fitted model using the following speed density data. So, I have given the speed data, I have also given the density data, and I am trying to fit Greenshields' model, Greenberg model and Underwood's model, all the three model to this given data again using regression equation only.

I have not shown here each and every step. You can simply try to generate the, what is the best model that you get out of the given data. If you consider Greenshields' model, if you consider Greenberg's model and if you consider the Underwood's model, you can use any computer software maybe Microsoft Excel to do that. Please do it and check it.

I have given you the equation here Greenshields' model A, Greenberg's model B, Underwood's model C for the given data, how they are fitting, the equations are given here. And you can see here for this case, for the given data the best fit is actually the Greenshield model in terms of the highest R square value. So, we take that because of this. And you can accordingly calculate what is the jam density and what is the maximum flow. Jam density you can calculate and also you can get the maximum flow that is 1845 vehicle per hour.

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**Summary**

- **Single Regime Models: Greenshields' Model, Greenberg's Model and Underwood's Model**
  - ✓ **Greenshields Model: Linear Relationship**
  - ✓ **Greenberg's Model: Logarithmic Relationship**
  - ✓ **Underwood's Model: Exponential Relationship**
- **Comparison between Single Regime Models**
- **Calibration of Single Regime Models**

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So, altogether what I discussed here is the single regime models, three particular models, Greenshields', Greenberg's and Underwood's model. Greenshields' assume linear relationship, Greenberg's assume logarithmic relationship and Underwood's speed density model assumes exponential relationships.

I have tried to explain you how we can compare these models and how we can calibrate these single regime models from a given data using least square model or using linear regression technique to estimate the coefficients that best fit the given data, best fit the model for the given data and how to select the best model. With this, I close. Thank you so much.