Traffic Engineering
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Lecture 46
Car Following Theory-3

Welcome to module F lecture 3. In this lecture, we shall continue our discussion about car following theory.

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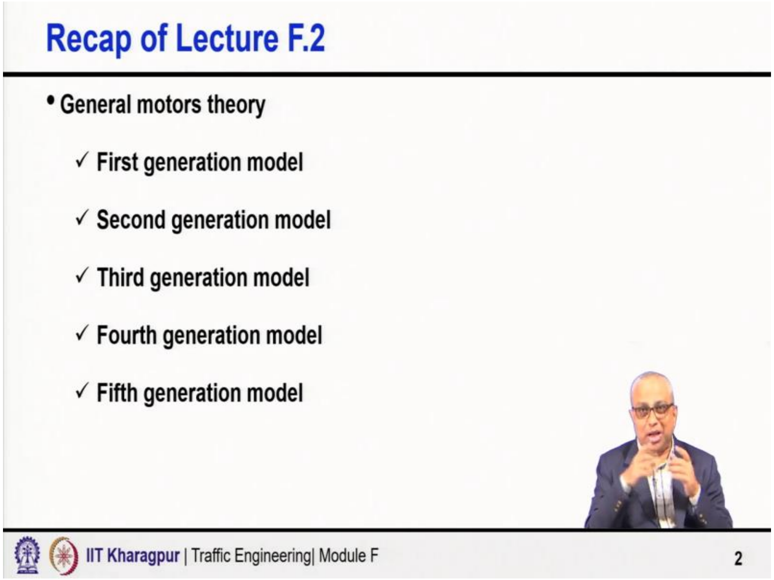

In lecture 2, I explained to you various models first generation, second generation, third generation, fourth generation, and fifth generation models all these models which were developed by General Motors, I explained to you fifth generation model is the most generalized model and all the other four models are special cases of the generalized fifth generation model. While I explained to you the basic idea or the concept and the model form, I said that we will continue our discussion with an example problem. So, that is what we are going to do now.

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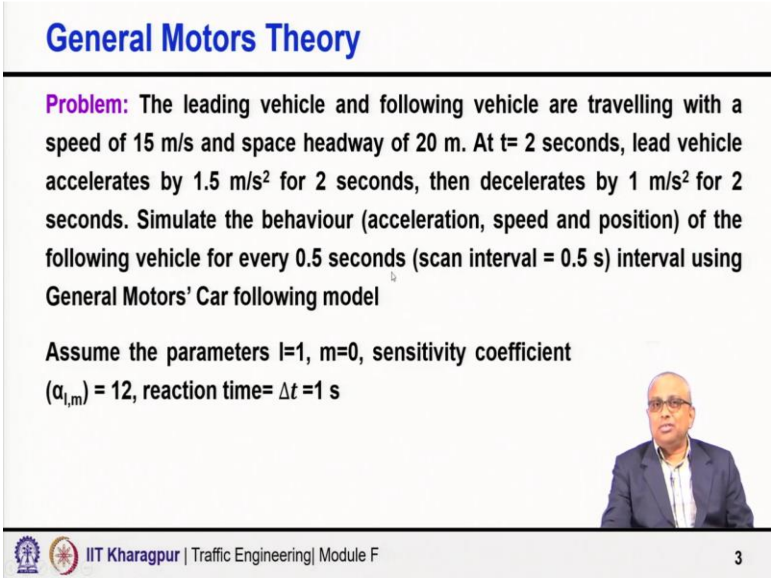

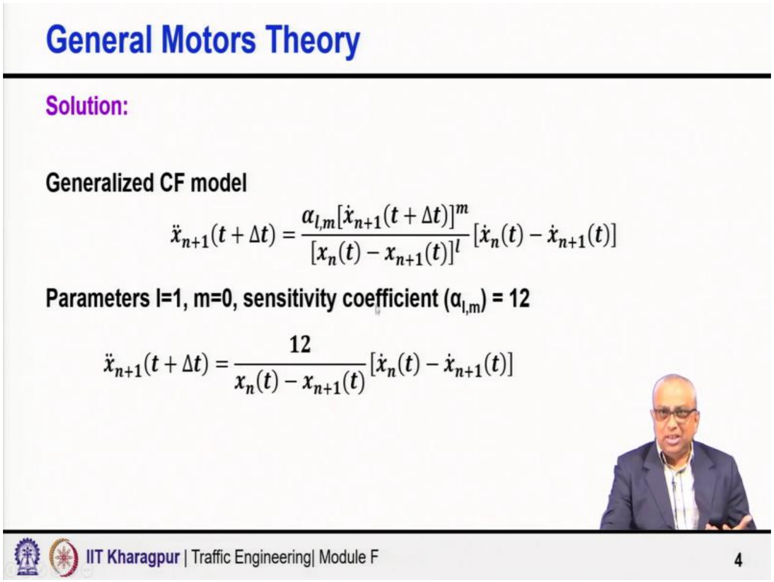

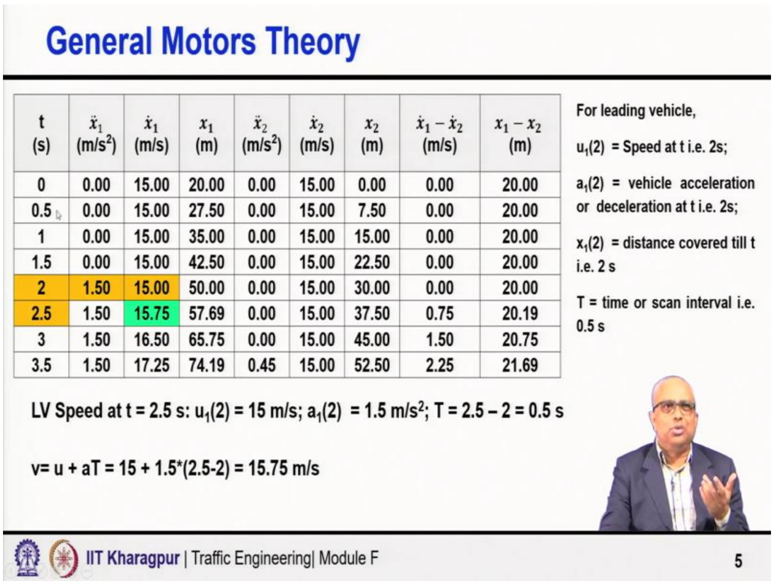

So, let us take an example problem, the leading vehicle and following vehicles are traveling with the speed of 15 meters per second. So, both are traveling at the same speed and with a space headway of 20 meter. So, initial speeds both vehicles are same spacing or space headway is 20 meter then at time t equal to 2 seconds the lead vehicle accelerates by 1.5 meters per second square for 2 seconds.

And then further decelerates by 1 meter per second squared for the next 2 seconds. With this background, what we want we want to simulate the behavior, behavior means the response in terms of acceleration speed also the position of the following vehicle for every 0.5 second interval. So, every 0.5 second interval we want to update, so, you can call it also a scan interval.

Using General Motors car following model where l is 1, m is 0, you know already what is l what is m and the sensitivity coefficients alpha lm equal to 12 the reaction time is 1 second. So, this is reaction time that is 1 second and we want the update at 0.5 seconds interval. That is the scan interval.

So, how to calculate it before we show you the calculation before I explain you the steps, let us look at the generalized model car following model as proposed by General Motors is the fifth-generation model. So, this model form is known to you the response in terms of acceleration deceleration at time t plus delta t depends on the alpha a constant which is already given.

This is the speed of the following vehicle at time t plus delta t but since m equal to 0 this term will get omitted then divided by xn t minus xn plus 1 t. So, if the distance headway between space headway between vehicle n and n plus 1 to the power l, l is taken as 1. So, m is 0, l is 1. So, it actually indicates that this is the third-generation model as proposed by General Motors multiplied by relative speed of two vehicles.

So, x dot n at time t minus x dot n plus 1 at time t. Now, remember that time t what is happening that the response we are trying to model after time delta t which is the reaction time this is very important. So, this simplified form of this generalized model as per the third-generation model given coefficient indicate giving values indicate that it is actually a third-generation model. So, I have written it.

Now, using this model we shall calculate all the values which are asked in the example problems or question that what will be the acceleration what will be the speed what will be the position of the following vehicle at scan interval of 0.5 seconds. Here I have shown the calculation for the first 3 and half seconds 3.5 seconds simply because otherwise it will be too lengthy.

So, I wanted to show it only for 3.5 seconds and explaining the calculation, then I will show you the entire calculation. Let us go column wise we want the scan interval of 0.5 seconds. So, start with 0 and 0, 0.5, 1, 1.5, 2, 2.5, 3, and 3.5 like that it will go for 15 minutes. Because that is what we eventually want or for even longer time. Now so, that is understood x double dot one leading vehicle acceleration.

So, we know at time t equal to 0, it is 0 for the first four interval time t 0.5 1 1.5 at 2 seconds it starts doing the acceleration. So, at 2 seconds the value will be 1 for the next 2 seconds. So, that means 1.2, 2.5, 3 and 3.5 All will be actually 1.5 seconds look at this problem carefully at t equal to 2 seconds and lead vehicle starts acceleration accelerating at 1.5 meters per second squared for 2 seconds.

So, when it is exactly t equal to 2 acceleration starts. So, at that time, so, at t equal to 2 we have taken the acceleration. So, these are basically inputs. So, similarly, in subsequent time also when I show you the full table you will see this is given input. So, first column is given input second column is again given input third column is the speed.

Now, this can be calculated initially we said that it is traveling at 15 meter per second both the leading vehicle and the following vehicle. So, now we are doing the leading vehicle calculation and leading vehicle we know it starts accelerating from t equal to 2 seconds onwards.

So, at t equal to 2 seconds also it will be 15 meter per second speed, because that is the time when it starts accelerating. So, how to calculate the remaining values I am showing you one calculation suppose at t equal to 2.5 how we can get the x1 dot leading vehicle speed. So, the cell which we are going to calculate I have indicated it in green.

We are going to use a simple formula which you all know v equal u plus aT. So, here we are talking t equal to 2.5 seconds what is going to happen? So, when you are talking about trying to calculate the value at t equal to 2.5 seconds at 0.5 seconds scan interval as I have indicated here that as capital T so, that means our u is at 2 seconds.

So, what is the initial speed at t equal to 2 seconds 15 meters per second is given what is the acceleration at t equal to 2 seconds 1.5 meters per second square and what is my this interval T capital T it is nothing but the scan interval always it will be 0.5 but this is the time we have here between 2.5 seconds and 2 seconds that 0.5 seconds. That is why I have shown the calculation here. The remaining is simple.

So, v equal to u plus aT, you put the u value as 15 meter per second, a as 1.5 meters per second squared and capital T equal 0.5 seconds or 2.5 minus 2. So, you get 15.75. So, this cell you get and now using the same approach you can calculate all the remaining cells in the same column. So, column 1, input column 2 is also direct input, column 3, we know how to calculate.

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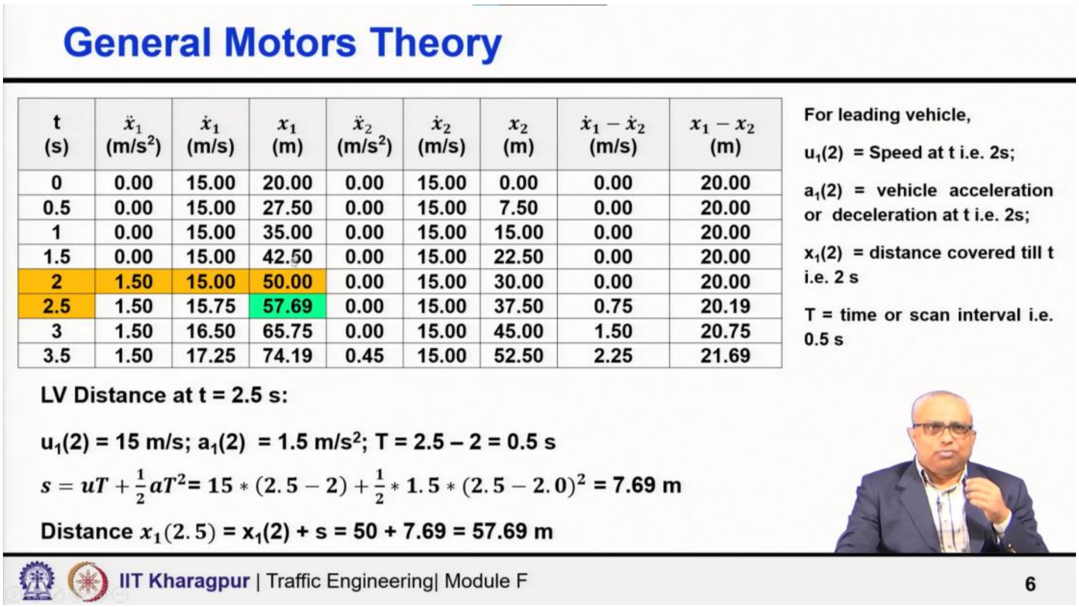

Now next go to column four. So, in this slide I am going to explain you how we can calculate a cell in Column four. Here, I am going to again explain you that at t equal to 2.5 seconds, how we are going to calculate the value of x1 all leading vehicle till now. So, leading vehicle acceleration is known we calculated leading vehicle then the speed and we want to calculate the x1 the distance from the reference point.

So, how we can calculate the distance. So, this was like initially as it was, it is said that the initially it is said that the space headways 20 meter. So, we are considering the x1 at t equal to 0 it is 20 meter, then we are going to use again a simple formula s equal to uT plus half of aT squared a is the acceleration.

Now, in all these calculation initial calculation, the a is 0 because you can see the x dot values are 0 and therefore, wherever a will be other than 0 this half aT square also will come, let us take the generalized s equal to uT plus aT squared and try to see how we can calculate the x1 value at t equal to 2.5 seconds.

So, when we are talking at trying to calculate the distance as t equal to 2.5 seconds, we are trying to calculate how much distance it will move in between 2 and 2.5 seconds. So, within that 0.5 seconds, how much distance it will move. So, what is the u, u will be taking a speed at time t equal to 2 that is 15 meter per seconds.

a is again at time T equal to 2 what is the acceleration 1.5 meters per second square and what is the T value is the difference between 2.5 and 2 scan interval so, that is 0.5. So, s equal to uT plus half aT square a value is known, u is known capital T is known. So, you get the s as 7.69, then what will be the distance x1 it will be at 2 seconds it was 50.

So, in between 2 and 2.5 seconds at 2.5 meters traveled another 7.69 meter. So, the total is 50 plus 7.69 57.69. So, you know how to calculate the cell. So, once you know this, you can calculate all the cells in column x1. So, we are through with cell one input, cell two as input cell three calculation, cell four calculation.

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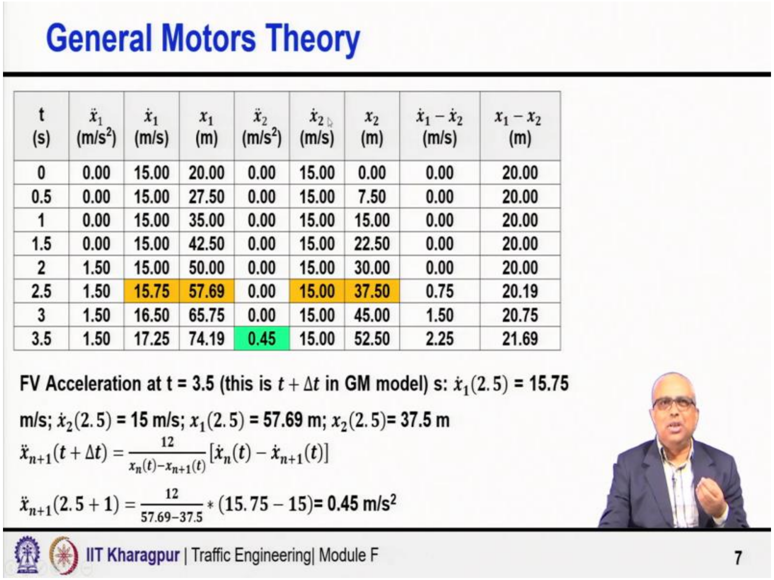

Now we go to cell five here we are actually trying to calculate the response acceleration deceleration whatever it is. Obviously, you know you put to this formula you know when both vehicles are traveling at the same speed obviously first the first 0 at 0.5 at 1 and at 1.5 even at 2 also the you know the deceleration whatever will happen let us that we can also calculate but the initially it will be all 0.

The actual the response will acceleration deceleration that response will be there. Once the vehicle in front of the leading vehicle start accelerating or decelerating. So, let us try to see how we can calculate a cell let us take the cell corresponding to time t equal to 3.5 seconds. Now, this is the cell the green one that we are going to calculate.

So, what will be the acceleration or deceleration of the following vehicle at time t equal to 3.5 seconds, but as you know if we are trying to calculate the cell it is actually in this equation t plus delta t that is the t here because we are taking the time clock. So, at 3.5 seconds what is going to happen? That is the response is happening after 1 second because that is what the reaction time.

So, delta t is 1 second. So, we will take the values like the speed at 2.5 seconds, because the reaction time is 1 second. So, x1 dot at 2.5 we will take we know that it is 15.75 x2 dot at 2.5 it is we know that it is 0 sorry it is 15 meter per second then x1 2.5 this is 57.69 x2 2.5 the following vehicle it is 37.50 then we go to this equation third generation general motor model we know it is 12 by xn t minus xn plus 1 t.

So, this will be simply x1 and x2 you can consider. So, the x1 values are known also the x dot n and x dot n plus 1 all we are taking a time t 2.5 second what is the value that is the basis for calculating the response after delta t time the reaction time. So, t plus delta t is the time clock. So, you calculate it as 0.45 meters per second square. So, exactly in the same way now, you can calculate all the cells in this column x2 dot that means, the response of the following vehicle. So, next it will be x2 dot and x2.

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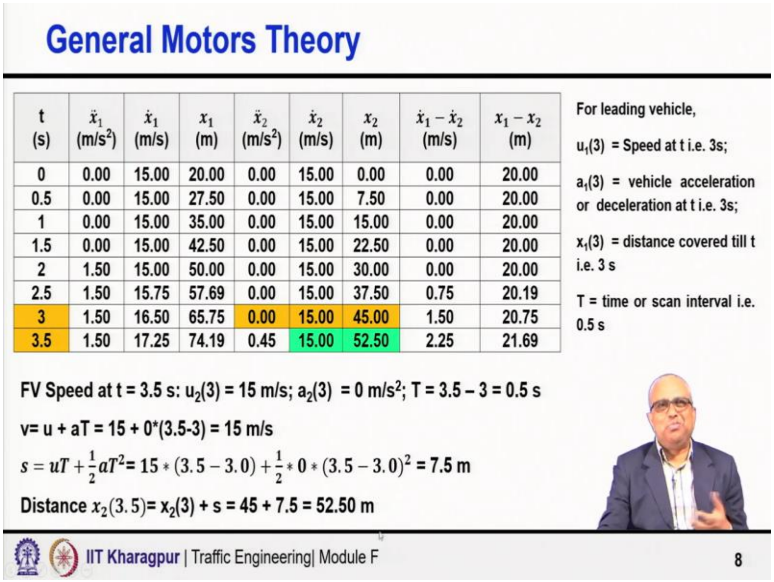

So, we are going to explain you now how you can calculate x2 dot and x2 let us take that both you know we are trying to calculate at 3.5. Again, the basic equation is v equal u plus aT. Simply you can get this value. So, at 3.5 means 0.5 second interval, that is the scan interval. So, we will take the u value and a value at t equal to 3 seconds. And obviously, delta t again capital T is again 0.5 seconds that is the scan interval.

So, you can get v you can get s, s you can get using this formula uT equal to half aT squared. So, once you get this s so, what will be then 45 plus s and once you get the speed that is going to be used for the calculation of you know the other values. So, that way the whole calculation is explained.

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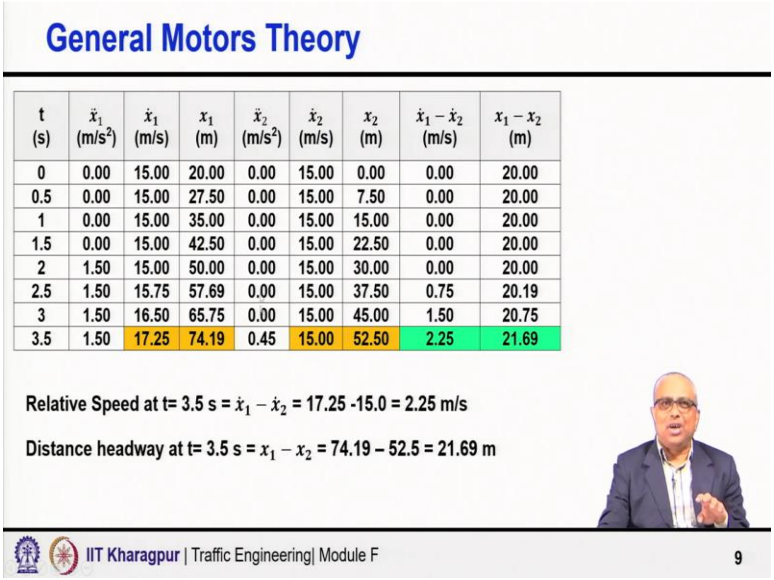

t (s)	$\ddot{x}_1$ (m/s²)	$\dot{x}_1$ (m/s)	x_1 (m)	$\ddot{x}_2$ (m/s²)	$\dot{x}_2$ (m/s)	x_2 (m)	$\dot{x}_1 - \dot{x}_2$ (m/s)	$x_1 - x_2$ (m)
0	0.00	15.00	20.00	0.00	15.00	0.00	0.00	20.00
0.5	0.00	15.00	27.50	0.00	15.00	7.50	0.00	20.00
1	0.00	15.00	35.00	0.00	15.00	15.00	0.00	20.00
1.5	0.00	15.00	42.50	0.00	15.00	22.50	0.00	20.00
2	1.50	15.00	50.00	0.00	15.00	30.00	0.00	20.00
2.5	1.50	15.75	57.69	0.00	15.00	37.50	0.75	20.19
3	1.50	16.50	65.75	0.00	15.00	45.00	1.50	20.75
3.5	1.50	17.25	74.19	0.45	15.00	52.50	2.25	21.69

Now, what is remaining is really x1 dot minus x2 dot and x1 minus x2 that is very simple because if we know x1 dot also you know x2 dot also you will know the speed of lead vehicle and following vehicle at any time t. So, we can find out what is the relative speed we also know x1 we also know x2.

So, we can calculate at any time interval t what is going to be the value of x1 minus x2 I have shown it here for time t equal to 3.5 seconds. So, you can see that, so, that way all the cells now, you are through with all the columns. So, 1 is input 2 is input 3 onwards 3, 4, 5, 6, 7, 8, 9 all these columns how we calculate I explained.

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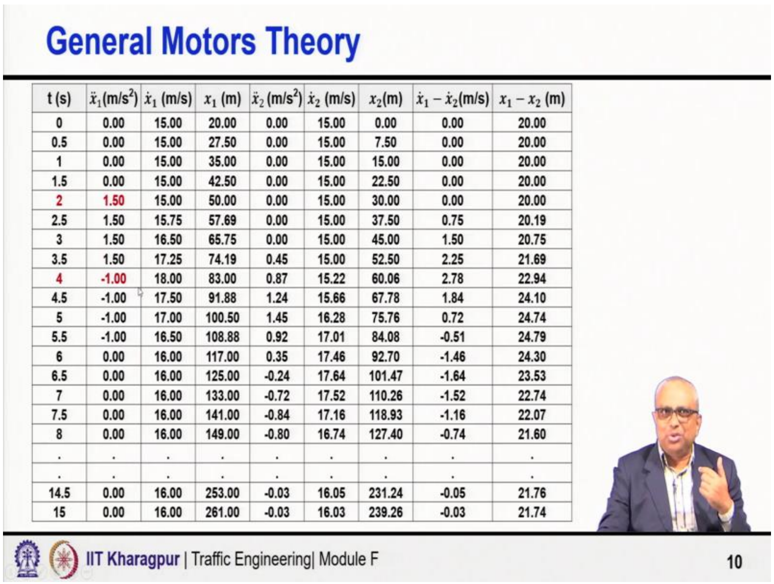

t (s)	$\ddot{x}_1$(m/s²)	$\dot{x}_1$ (m/s)	x_1 (m)	$\ddot{x}_2$ (m/s²)	$\dot{x}_2$ (m/s)	x_2(m)	$\dot{x}_1 - \dot{x}_2$(m/s)	$x_1 - x_2$ (m)
0	0.00	15.00	20.00	0.00	15.00	0.00	0.00	20.00
0.5	0.00	15.00	27.50	0.00	15.00	7.50	0.00	20.00
1	0.00	15.00	35.00	0.00	15.00	15.00	0.00	20.00
1.5	0.00	15.00	42.50	0.00	15.00	22.50	0.00	20.00
2	1.50	15.00	50.00	0.00	15.00	30.00	0.00	20.00
2.5	1.50	15.75	57.69	0.00	15.00	37.50	0.75	20.19
3	1.50	16.50	65.75	0.00	15.00	45.00	1.50	20.75
3.5	1.50	17.25	74.19	0.45	15.00	52.50	2.25	21.69
4	-1.00	18.00	83.00	0.87	15.22	60.06	2.78	22.94
4.5	-1.00	17.50	91.88	1.24	15.66	67.78	1.84	24.10
5	-1.00	17.00	100.50	1.45	16.28	75.76	0.72	24.74
5.5	-1.00	16.50	108.88	0.92	17.01	84.08	-0.51	24.79
6	0.00	16.00	117.00	0.35	17.46	92.70	-1.46	24.30
6.5	0.00	16.00	125.00	-0.24	17.64	101.47	-1.64	23.53
7	0.00	16.00	133.00	-0.72	17.52	110.26	-1.52	22.74
7.5	0.00	16.00	141.00	-0.84	17.16	118.93	-1.16	22.07
8	0.00	16.00	149.00	-0.80	16.74	127.40	-0.74	21.60
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
14.5	0.00	16.00	253.00	-0.03	16.05	231.24	-0.05	21.76
15	0.00	16.00	261.00	-0.03	16.03	239.26	-0.03	21.74

So, here I am showing the complete table for the first 15 seconds not shown the complete table really they initially I have shown here up to 8 seconds and then the last 14.5 and 15 seconds you can even calculate and try to match what is important for you to note that at 2 second it starts accelerating.

So, the x1 dot value at 2 seconds is taken as 1.5 and then it is said after 2 seconds it start decelerating. So, again at 4 second onwards, the value is taking us minus 1. So, note that at 2 second also it will be 0 because it is 2 seconds on it starts accelerating at 2 seconds. So, at 2 seconds it starts accelerating. So, the value is 1.5 not 0. So, similarly after 2 second means, at 4 seconds, we will start taking the minus 1 the remaining all calculation I have explained.

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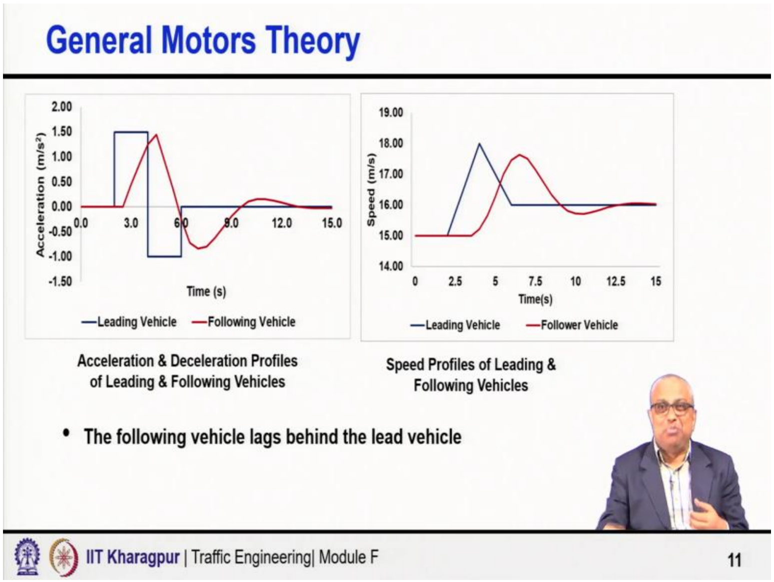

Now, here I am trying to show how the curve will look like x axis is the time y axis is the acceleration. So, the black line shows the leading vehicle for how the acceleration is always changing over time. And the red one shows for the following vehicle which we tried to model using the car following model actually we tried to capture using car following model.

So, you can see that it is showing interesting to see that we are actually able to model the response in terms of acceleration or deceleration, because here you know, in this segment it is acceleration and in this segment it is deceleration. So, how the response is happening and all is the following vehicle lags behind the lead vehicle.

So, it starts doing the acceleration then it is also start doing after a lag and it can decelerate, so, it also starts reducing and finally decelerate also some gap. So, it is trying to catch up here it is shown the speed profile of leading and following vehicle, you can also see the speed is increasing, here also after some time it is increasing, it is reducing this also is then the following vehicle is also reducing the speed.

But it is not just a shift in time but the overall pattern and values are also very different. So, the pattern here is the black line pattern and the red line pattern is different here also the pattern for the variation for the red black line and the pattern what is shown as the red line, they are not same not just lateral shifted, but the values are also different but suddenly it is going to respond. So, if the leading vehicle lead vehicle is accelerating the following vehicle is also trying to accelerate but not exactly in the same way. There is of course a gap and but also not exactly the same manner.

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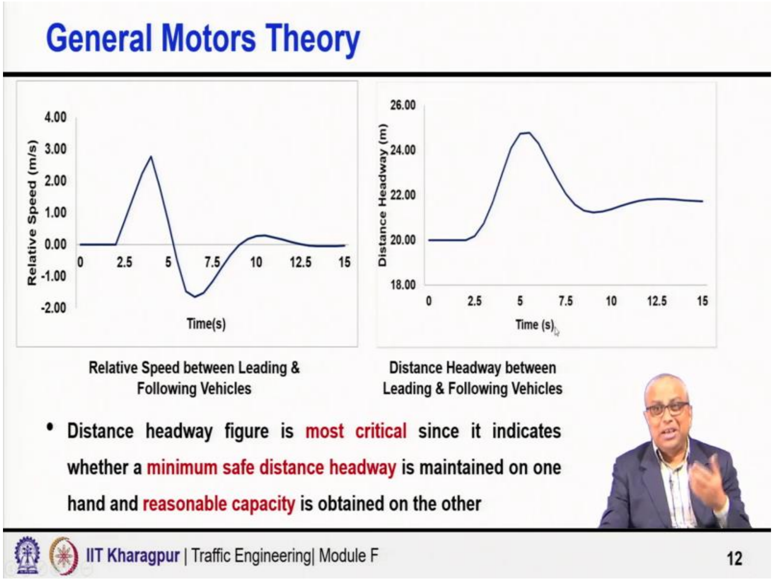

Here we are trying to show the relative speed and here it is the distance headway very interesting distance headway the relative speed also shows that it is always trying to catch up. So, relative speed is suddenly the front vehicle or lead vehicle accelerate. So, the speed relative speed becomes higher but then immediately in the response the following vehicle also start doing the acceleration. So, that the it can catch up.

So, it is always trying to catch up trying to catch up and you know the relative speed almost ending with some oscillation it is trying to keep the relative speed 0 trying to do that. But there is a pattern is different lag is different, there will be time lag also in the response, the nature of response is also not exactly the way the lead vehicle behaved the following vehicle will behave exactly in the same manner not like that, you get seeing that everywhere.

So, this is what it is and the distances headway figure is most critical. What I have shown here, since it indicates whether a minimum safe distance headway is maintained at one end the reasonable capacity in a larger context overall context a reasonable capacity is obtained on the other hand.

So, the distance headway that is why it is very critical because it influence the safety it will tell you that whether the response is going to be safe it is that that safety is very important because it depends on the space headway and also the finally the capacity also depends on the headway or inverse of headway is 1 by headway is actually the distance headway is actually the density. The density influences the throughput the capacity also will depend on that and all traffic parameters it is going to be influenced by this behavior. So, the space headway or the distance headway is very, very important. And we are able to get that we are able to model that.

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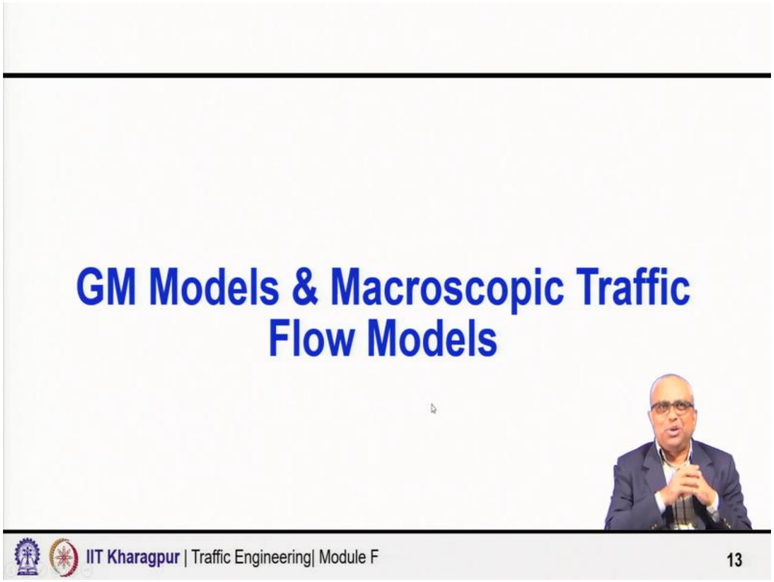

So, with this now, let us go to another topic. It is the General Motors model, which is the microscopic model and the macroscopic traffic model what you have studied earlier Greenfield’s model Greenberg's model and so, on. Is there any relation? Are they can they be linked? That is the part we are going to discuss now.

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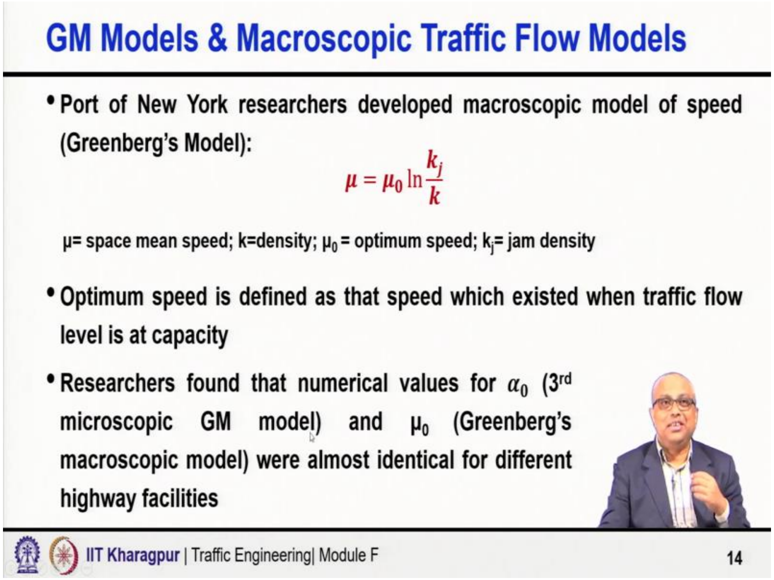

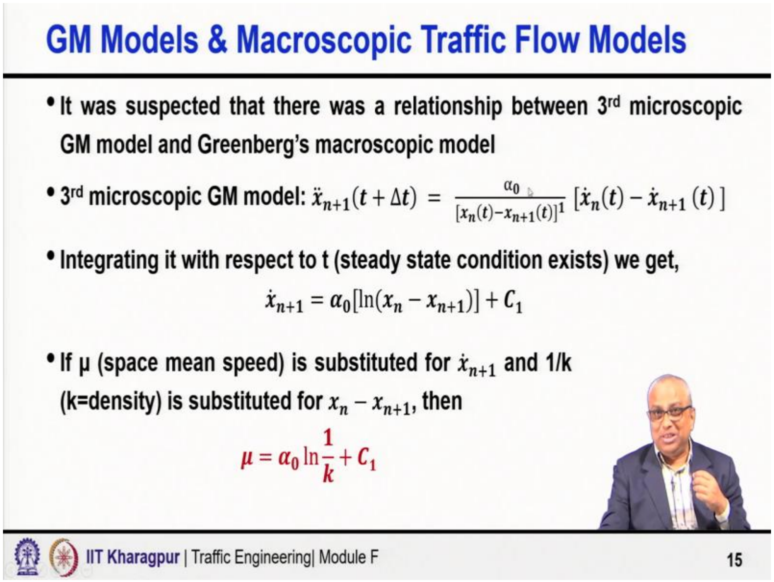

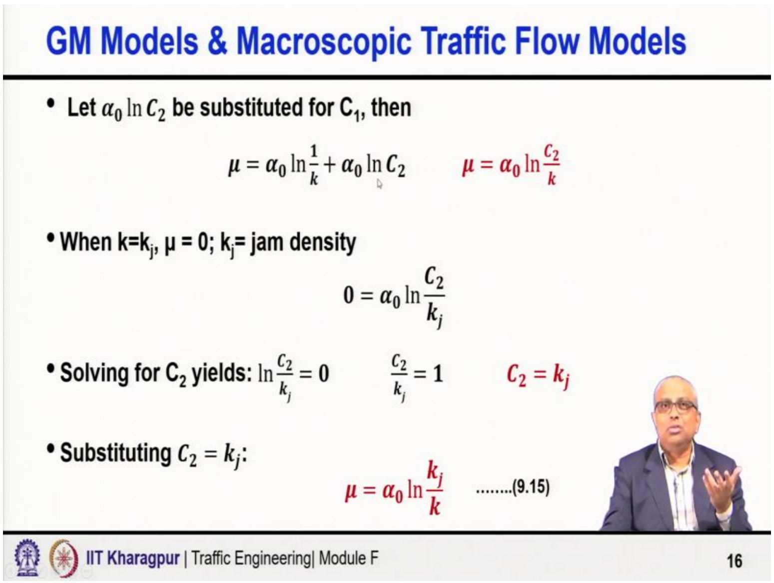

So, what happened port of New York researchers develop macroscopic model of speed Greenberg's model you can see mu equal to mu0 into ln kj by k mu is the space mean speed obviously k is the density mu is here the optimum speed that is the speed at capacity speed at capacity and kj is the jam density.

$$\boldsymbol{\mu = \mu_0 \ln \frac{kj}{k}}$$

Now, optimum speed as I have said it is the speed when existed when the traffic flow level is that capacity. Now, interestingly rather very interestingly researcher found that numerical value of alpha 0 in the third-generation model you know this term alpha 0 in the third-generation model and the value of mu0 here the Greenberg's model.

This is Greenberg's model mu0 they were almost identical for different highway facilities that data came and they found what is the value of mu0 in Greenberg's model and then in General Motors third generation model what is alpha 0 these values are almost identical for different highway facilities.

Then they started thinking is it just a sheer coincidence or is really there is they are same what is the mu0 here in Greenberg's model and what is alpha 0 in the third-generation microscopic model proposed by General Motors are the same thing because one is macroscopic one is microscopic.

So, they started doing the research or started working further investigations and eventually they found a connection that I am going to explain you how they got. So, let us take the third generation microscopic model remember it is microscopic model and whereas, Greenberg is macroscopic.

$$\boldsymbol{\ddot{x}_{n+1}(t + \Delta t) \; = \; \frac{\alpha_0}{[x_n(t) - x_{n+1}(t)]^1} \, [\dot{x}_n(t) - \dot{x}_{n+1}\,(t)\,]}$$

So, can we get from this model can we really get this third generation General Motors model can we get the Greenberg's model that was the attempt. So, what if you integrate this with respect to stay t under steady state condition then it is x double dot acceleration. So, you get speed x dot n plus 1 alpha 0 and this whole term obviously becomes log of ln x 1 xn minus x n plus 1.

$$\boldsymbol{\dot{x}_{n+1} = \alpha_0[\ln(x_n - x_{n+1})] + C1}$$

Because here in the denominator it was xn are they omitted theta term here because for simplicity. So, xn minus xn plus 1 so, it comes ln of log of this plus a constant term. Now, if mu is the space mean speed in Greenberg's model is substituted for this x dot n plus 1. What is x dot n plus 1 it is speed.

So, if we consider that the space mean speed that is the way the whole traffic speed is behaving under steady state conditions so, this x naught n plus 1 can be replaced by mu quite possible and also what is xn minus xn minus 1 it is the distance headway spacing or space headway.

So, 1 by that is what density macroscopic parameter density. So, we do that x dot n plus 1 replacing by mu and this xn minus xn minus 1 we are replacing by 1 by k because 1 by distance headway is the density plus c1 we have moved one step one small step we have proceeded. Still it does not look yes 1 by k has come but u plus 1. So, now let us see what we can do with the C1.

$$\boldsymbol{\mu = \alpha_0 \ln \frac{1}{k} + C1}$$

C1 also can be written it is a constant it can be written as alpha 0 into ln C2. So, what was C1 a constant. We can substitute that and we can write it like another constant alpha 0. Here also alpha 0 is there. So, we write alpha 0 and ln C2 otherwise we can write because after all it is a constant. So, what do you get here? mu equal to alpha 0 into ln C2 by k. The form has come already it has come it has already come it now looks like something like a greenback model. Of course, there is still we do not know what is C2 where alpha 0 is there. whether how can know that that is really mu 0. That we do not know. Because our this form is mu 0.

But this overall looks has come these look very similar. mu equal to alpha 0 into ln C2 by k. Now, our next step was to understand what is the C2 how we can get the value of C2. Because here it is actually kj by k is that C2 is equal to kj what they did let us take when k equal to kj a jam density, then obviously mu equal to 0 speed is 0.

So, you take speed is 0, k equal to kj then alpha 0 into ln C2 by k. Now this, what it tells you, this is 0, alpha 0 is again a constant. So, obviously it indicates that C2 is actually kj So, we got it mu equal to alpha 0 into ln kj by k mu equal to mu 0 into ln kj by k the only difference is here is mu 0 and here it is alpha 0 even by comparing these two also you can now say this must be mu 0 and alpha 0 was the same. You got the other form but those still let us see if we can even bring with a little bit of more analysis, if we can bring alpha 0 as mu 0.

$$\mu = \alpha_0 \ln \frac{kj}{k}$$

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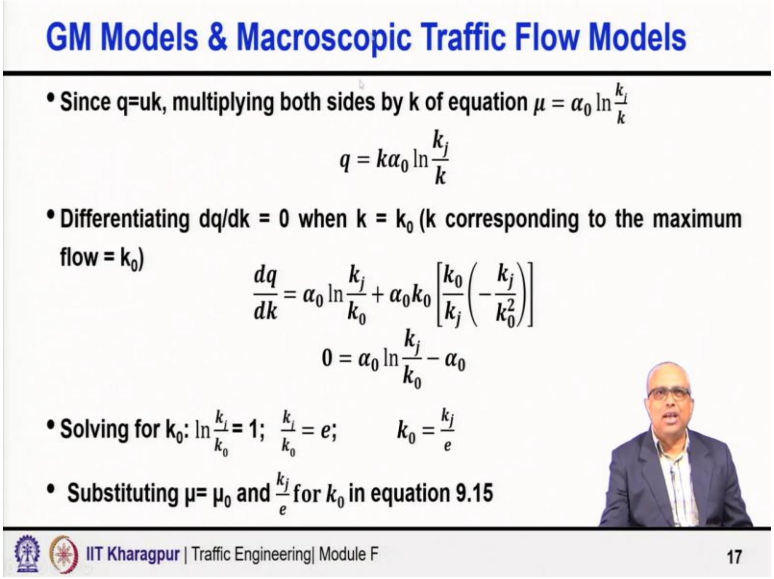

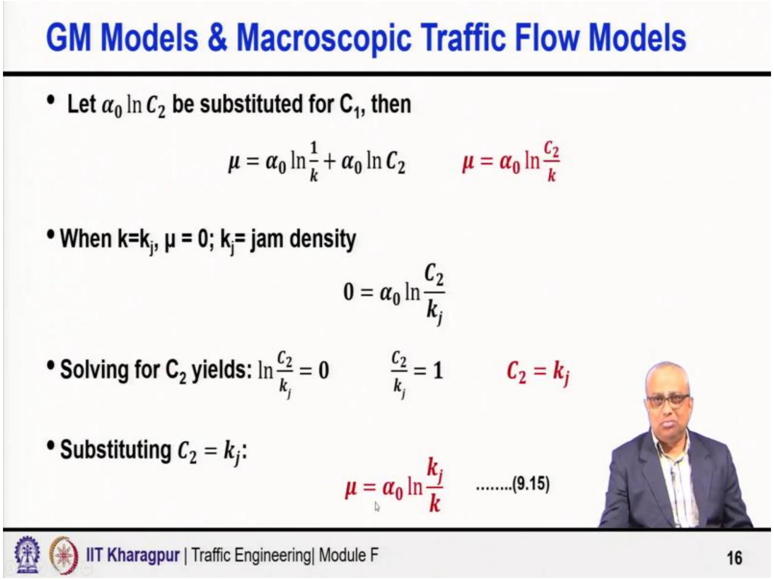

What was done you know q equal to uk flow equal to speed into density at macroscopic level, speed flow density are related. So, let us multiply both sides by k. So, k into mu speed flow and here k into alpha 0 ln k by k same alpha 0 only we have taken the k multiplied by k both sides.

Now, if we differentiate this flow with respect to q by that is what the capacity. So, at capacity we know k equal to k0 optimum density when the dq dk equal to 0 then whatever is the k value that is the optimum density at capacity. So, dq dk you take to the derivative make it 0 and when you make it 0 and already we have taken know that, that k equal to k 0 in that case.

So, if you solve you get actually k0 equal to kj by e fine. Now, go back to this equation 9.15 mu equal to alpha 0 into ln kj by k. Substitute mu equal to mu 0 and k 0 because that is the capacity point that we are trying to understand. What happens in this equation. So, mu equal to mu 0 and k 0 we are replacing by kj by e.

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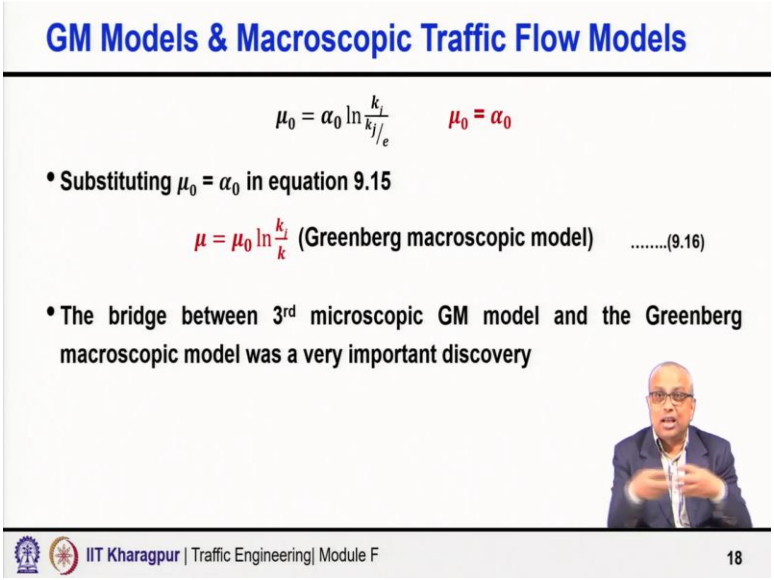

What we get here, we get here then mu0 equal to alpha 0 ln kj by kj by e. That means mu0 equal to alpha 0 actually. So, now you simply substitute in equation 9.15. What was their mu equal to alpha 0 ln kj by k, but since alpha 0 equal to mu0. See you substitute that make it mu0 what do you got eventually got Greenberg's macroscopic model.

$$\mu = \mu_0 \ln \frac{kj}{k}$$

So, we started with third generation microscopic model as proposed by General Motors and with a simple with some simple steps and substitutions we could derive this Greenberg's macroscopic model; strong tie and this is really amazing and very important discovery. Because, looking at the traffic step you look at the microscopic with help on microscopic parameter look at with the help of macroscopic parameters, there is a connection. So, that was established.

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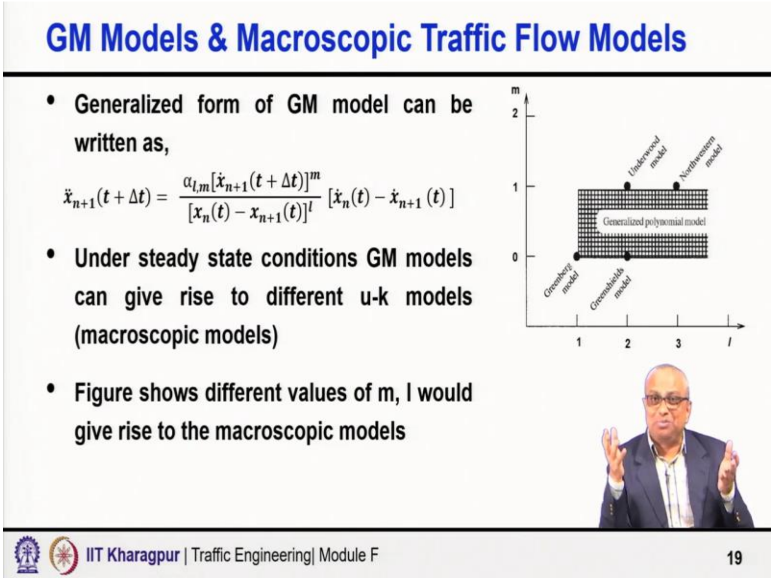

In fact, the generalized GM model which can be written as I have shown it in terms of l and m actually I will stop here with this closing kind of statement. Actually, you can see and you can get with what value of l and m you are taking. Then it is possible that the GM model generalized model with specific different values of l and m can actually give you several microscopic model as I have shown you for some combination third generation models specifically it gives you Greenberg's model when l equal to 1 m equal to 0.

Similarly, with some other combination of n l m you can also get with m as 0 and l as 2 you can get actually the Greenshield’s model, another combination can give you underwood’s model. So, that shows the strength of the generalized model fifth generation generalized model as developed by General Motors that with different l and m, you can actually get several models established models which have been developed independently by researchers and which are all macroscopic models. So, there is a strong connection.

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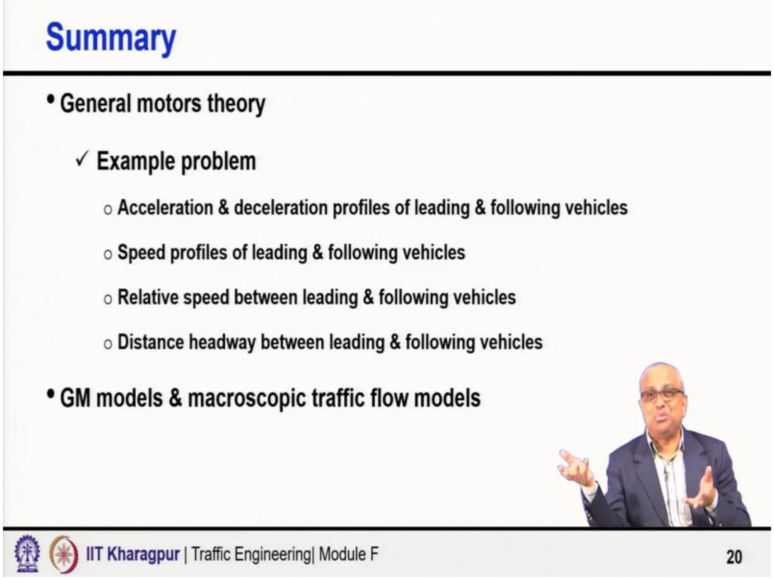

So, what we discussed? We discussed the General Motors theory in the previous lecture, we took an example problem to explain that and then also the other part the connection between generalized motors model the generalized fifth generation model GM model. Because all other models are specific cases of the generalized model fifth generation model first, second, third, fourth generation all at some specific cases of the fifth-generation model. So, generalized GM model General Motors model and the connection between that and the macroscopic traffic flow model. I explained only Greenberg, but you can get Greenshield you can also get other models from that. So, thank you so much.