

Traffic Engineering
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Lecture 44
Car Following Theory-2

Welcome to Module F, lecture 2 in this lecture also we shall continue our discussion about car following theory.

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Recap of Lecture F.1

- Introduction
 - ✓ Space headway
 - Influence on road capacity & LOS
 - ✓ Notations & definitions
- Pipes theory
 - ✓ Min. space headway & min. time headway
- Forbes theory
 - ✓ Min. space headway & min. time headway



In lecture 1 I mentioned to you what is really car following, the context of car following, the importance of the space headway, time headway and then discussed about pipes model and Forbes model.

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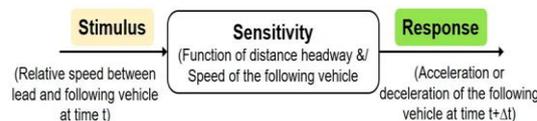
General Motors Theory



General Motors Theory

- The car following theories developed by GM are much **extensive** and of particular importance because of the accompanying comprehensive field experiments and **discovery** of the **mathematical bridge** between **microscopic and macroscopic theories** of traffic flow
- The fundamental concept:

Response = function (sensitivity, stimuli)



Now, with this background, let us continue the discussion and in this lecture we shall discuss about the car following models developed by General Motors. The car following theories developed by General Motors are much more extensive and of particular importance because of the accompanying comprehensive field experiments a lot of field experiments were carried out.

So, that is really the core strength of the General Motors model and discovery of the mathematical bridge between microscopic and macroscopic traffic flow. You have seen that in earlier two models what are the kind of assumptions that we made and what was the real basis for developing car following model.

In General Motors theory according to the theory proposed by General Motors, the response is a function of sensitivity and stimuli. What is the stimuli or stimulus, it is the relative speed between the lead and the following vehicle you know that we always have two vehicles one is the lead vehicle another is the following vehicle.

So, what is the relative speed between the lead and following vehicle that is the stimulus. Sensitivity is actually function of distance headway and speed up the following vehicles. And what is the response? Response is acceleration or deceleration of the following vehicle at time t plus delta t . So, whatever was the stimulus at t and what is the sensitivity accordingly the response at time t plus delta t in terms of acceleration and deceleration is actually model as a part of the car following model.

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General Motors Theory

Notations & Definitions

n = leading vehicle (LV); $n+1$ = following vehicle (FV)

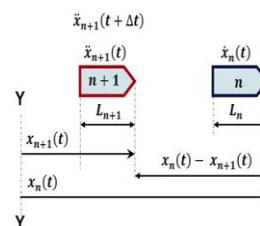
L_n = length of LV; L_{n+1} = length of FV

x_n = position of LV; x_{n+1} = position of FV

\dot{x}_n = speed of LV; \dot{x}_{n+1} = speed of FV

\ddot{x}_{n+1} = acceleration of the FV

Δt = Interval of time between the (i) time a unique CF situation occurs (t) and (ii) time the driver of the FV decides to apply a specified acceleration (deceleration); also known as reaction time



General Motors Theory

First Generation Model

- The first model assumed that the sensitivity term was a constant (α)

$$\ddot{x}_{n+1}(t + \Delta t) = \alpha[\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad \dots\dots(9.9)$$

- The stimuli term can be positive, negative, or zero, which could cause the response to be an acceleration, deceleration or constant speed. These three conditions are given as:

✓ $\ddot{x}_{n+1}(t + \Delta t)$ is positive, when $\dot{x}_n(t) > \dot{x}_{n+1}(t)$

✓ $\ddot{x}_{n+1}(t + \Delta t)$ is negative, when $\dot{x}_n(t) < \dot{x}_{n+1}(t)$

✓ $\ddot{x}_{n+1}(t + \Delta t) = 0$, when $\dot{x}_n(t) = \dot{x}_{n+1}(t)$



Now, although I mentioned about all these notations in my lecture 1, but still I have produced this slide once again to remind you that we are talking about two vehicles one is a leading vehicle and other is the following vehicle. So, leading vehicle is denoted as n and the following vehicle n plus 1.

L_n is the length of lead vehicle L_{n+1} length the following vehicle and particularly what are the important thing in this context is x_n is the position of lead vehicle x_{n+1} is the position of following vehicle \dot{x}_n speed of lead vehicle \dot{x}_{n+1} speed of the following \ddot{x}_{n+1} is the acceleration of the following vehicle and Δt is the response.

Now, the first general motors model we can write it like this the sensitivity term what we say stimulus and sensitivity stimulus is the relative speed. So, \dot{x}_n at time t minus \dot{x}_{n+1} at time t. So, at time t what is the relative speed between the following the lead vehicle and the following vehicle multiplied by in this case the sensitivity is taken as constant.

$$\ddot{x}_{n+1}(t + \Delta t) = \alpha[\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

And then let us try to they tried actually to calibrate this value based on the field experimentation. Now, obviously this stimulus can be positive negative or 0 which would cause the response to be either acceleration or deceleration or a constant speed. For example, if \ddot{x}_{n+1} rather if \dot{x}_n the speed of vehicle n is greater than the speed of the following vehicle that means, the lead vehicle speed is higher than the speed of the following vehicle.

$$\ddot{x}_{n+1}(t + \Delta t) \text{ is positive, when } \dot{x}_n(t) > \dot{x}_{n+1}(t)$$

Obviously, what would be the response? Response would be acceleration because the gap is increasing. So, \dot{x}_{n+1} at time t plus Δt would be actually positive indicating that it would be acceleration. Similarly, if \dot{x}_n at time t is less than \dot{x}_{n+1} at time t that means, the following vehicle is now becoming closer and closer to the lead vehicle in that case, what is the logical response is deceleration.

$$\ddot{x}_{n+1}(t + \Delta t) \text{ is negative, when } \dot{x}_n(t) < \dot{x}_{n+1}(t)$$

So, obviously using this formula, what is shown here as equation 9.9 that will mean that \ddot{x}_{n+1} at time t plus Δt will be negative indicating that this will be deceleration the responses was be deceleration because you are becoming closer to the lead vehicle.

And when both speed of the lead and following vehicle is same generally that means the distance is not changing. So, what is generally expected that it will be neither acceleration nor deceleration the same speed will be maintained. So, the acceleration deceleration will be 0 very fundamental, very simple and the basic formulation that is what is the first generation model.

$$\ddot{x}_{n+1}(t + \Delta t) = 0, \text{ when } \dot{x}_n(t) = \dot{x}_{n+1}(t)$$

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General Motors Theory

Parameter Values for First GM Model		
Measured Value	Reaction Time, Δt	Sensitivity, $\alpha(\text{sec}^{-1})$
Minimum	1.0	0.17
Average	1.55	0.37
Maximum	2.2	0.74

- Significant range in the sensitivity (0.17 to 0.74) alerted investigators that the **spacing between vehicles** should be **important** and this led to the development of second model



Now, when this model was actually calibrated based on the field data, then this table shows that how the reaction time was calculated how the reaction time was rather obtained and what was the value of alpha the sensitivity that was obtain you can see here the reaction time varies from 1 second to 2.2 seconds with an average of 1.55.

Similarly, the sensitivity average is 0.37 but minimum is 0.17 maximum is 0.74. What it indicates? The variation of the range is quite big 0.17 to 0.74. It is quite a wide range. So, they started thinking that why this range is so high. And it came out that the sensitivity is also a function of the speed spacing of vehicle what it came out the sensitivities also a function of the spacing between two vehicles. When two vehicles are closer and when these two vehicles are quite at a reasonable distance, the sensitivity is not the same. So, instead of one value they suggested let us have two values alpha 1 and alpha 2. That is what is basically this second model.

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General Motors Theory

Second Generation Model

- In second model, sensitivity have two different states
 - ✓ When vehicles are too close, a high sensitivity value (α_1) should be employed
 - ✓ When vehicles are far apart, a lower sensitivity value (α_2)

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_1}{\alpha_2} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \dots\dots(9.10)$$

- Difficulties in selecting the value of α_1 and α_2 in discontinuous state has led investigators to include **space headway in sensitivity term** leading to develop third GM model



So, two values of sensitivity were taken or suggested. That is the second-generation model. So, what was assumed that when the vehicles are too close obviously the sensitivity is high. Because irrespective of whatever will be the role of relative speed that remains, but the sensitivity will be higher or highly sensitive value.

High sensitivity value we will get when the vehicle set to close and when the vehicles are far apart a lower value of sensitivity is logical. So, they suggested a formulation like this the only difference between the first-generation model and the second-generation model is instead of having a constant value of alpha in the first simulation model here two values were suggested one when the vehicles are too close a high sensitivity value.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_1}{\alpha_2} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

And when vehicles are far apart in lower sensitivity value. Now, the difficulty is in selecting the values of alpha 1 alpha 2, where to select alpha 1 where to select alpha 2 in discontinuous state that led investigators to include that instead of saying higher or lower why do not we include this space headway in the sensitivity term itself. Because it may not be just too discrete value for two ranges, but it could be a term itself in the sensitivity value or the sensitivity term itself must may include space headway that was done that led to the development of third generation models.

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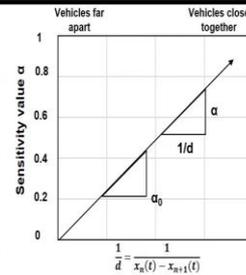
General Motors Theory

Third Generation Model

- Values for the sensitivity term (α) were measured as an inverse function of the space headway and data points closely followed a linear curve sloping upward to the right from the origin
- Slope of that line was designated as α_0 and sensitivity is given as

$$\alpha = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} \quad \dots\dots(9.11)$$

- Sensitivity term in third model is a function of a constant α_0 , and space headway



General Motors Theory

- As the vehicles come closer, sensitivity term becomes larger
- Dimension of α_0 constant in sensitivity term is (feet/s or m/s), or velocity
- Third model resulted as

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

.....(9.12)

Parameter Values for Third GM Models		
Location	Reaction Time, Δt	Sensitivity, α_0
GM Test Track	1.5	40.3
Holland Tunnel	1.4	26.8
Lincoln Tunnel	1.2	29.8



So, various values of sensitivity terms alpha were measured as an inverse function of the space headway and data points closely followed a linear curve sloping upward to the right of the origin that you can see here, that is what I have shown. So, this x axis is 1 by d, d is the distance or spacing when the distance is higher spacing is higher, then the 1 by d will be lower value.

And your sensitive value will be also lower and when they are close enough so, $x_n t$ minus $x_{n+1} t$ is a small value obviously, 1 by d is becoming higher. So, the sensitivity is also closely following two vehicles following closely so, the sensitivity is obviously higher and the sensitivity was expressed like this α equal to α_0 a constant value divided by x and t minus $x_{n+1} t$.

$$\alpha = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)}$$

So, what was a really alpha in the first-generation model in second generation model it was taken as alpha 1 and alpha 2 two values and in the third generation it was alpha 0 divided by the space headway or the distance headway or the space headway. And that is trying that we were trying to explain how the whole sensitivity is changing.

So, again here as the spacing between two vehicles is reducing sensitivity is going to be higher and higher. You can see here in this equation also the value is reducing $x_n(t) - x_{n+1}(t)$ lesser the value difference then higher will be the value of alpha sensitivity term in third model is a function of constant alpha 0 and space headway as I explained.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

And obviously, with this as I said as vehicle comes closer sensitivity term become larger dimension of alpha 0 constant in self is sensitivity term is maybe in meters per second or the velocity or it can be feet per second. So, the third model looks like this what was constant alpha in first generation model in second generation model we used alpha 1 and alpha 2 two values.

Now, the whole term is getting replaced by alpha 0 divided by $x_n(t) - x_{n+1}(t)$ the space headway or the distance headway multiplied by relative speed as usual $\dot{x}_n(t) - \dot{x}_{n+1}(t)$ at time t. So, the relative velocity of two vehicles at time t.

Then the parameter values for third generation General Motors model the values are shown here. In different tracks, this was tested the data were collected and calibrated. And the reaction time was found in a quite a close range I should say 1.2 to 1.5 with a and 1.4 here, two cases closer 26.8 value of alpha 0 29.8. But in GM test track first case it was 40.3.

So, something could be explained. But still it looks like there is something which is still influencing alpha 0 it is not really a constant. That is the indication what do we considered as alpha. Now, partly it is explained variation when we include this distance headway term. But then whatever still we are getting as alpha 0. It looks like it is still not a constant because it is varying. So, something else is actually again influencing the value of alpha 0.

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General Motors Theory

Fourth Generation Model

- Fourth Model was a further development towards improving the sensitivity term by introducing the **speed of following vehicle**
- **Concept:** As the speed of the traffic stream increased, the driver of the following vehicle would be more sensitive to the relative velocity between lead and following vehicle

- Fourth Model resulted as

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha' [\dot{x}_{n+1}(t + \Delta t)]}{[x_n(t) - x_{n+1}(t)]} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad \text{.....(9.13)}$$

- Sensitivity has three parts: a dimensionless constant 'α', speed of following vehicle, and distance headway



General Motors Theory

- As the vehicles come closer, sensitivity term becomes larger
- Dimension of α₀ constant in sensitivity term is (feet/s or m/s), or velocity

- Third model resulted as

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad \text{.....(9.12)}$$

Parameter Values for Third GM Models		
Location	Reaction Time, Δt	Sensitivity, α ₀
GM Test Track	1.5	40.3
Holland Tunnel	1.4	26.8
Lincoln Tunnel	1.2	29.8



So, further thinking further development. So, fourth model was a further developed towards improving the sensitivity term. By introducing speed of following vehicle this is okay that is the relative velocity sensitivity obviously the distance between the lead vehicle and the following vehicle if I am closer, my response sensitivity will be high.

If I am at a distance it is not so close, my sensitivity will be low. But it is not only the distance, but also the speed of the following vehicle. Let us consider the same distance between the lead vehicle and the following vehicle. But in one case, vehicle is following vehicle is traveling at a higher speed.

And in another case, the following vehicle is traveling or moving at a lower speed both case sensitivity cannot be same. So, another term was introduced. So, what they say they introduced

speed up the following vehicle that also was introduced in the sensitivity term. So, the concept here is as the speed of the traffic stream is increased or increases the driver of the following vehicle is expected to be more sensitive to the relative velocity between the lead and following vehicle.

For a given relative speed, for a given distance, higher the speed higher will be the sensitivity. So, it remains in this form as a multiplied in the numerator. So, what is the fourth model? What was here in earlier case as alpha 0 the remaining thing it is the relative velocity it is the distance between alpha 0 by $x_n(t) - x_{n+1}(t)$. Now, alpha 0 gets replaced by alpha dash another constant multiplied by speed of the following vehicle $\dot{x}_{n+1}(t)$ at time $t + \Delta t$.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha' [\dot{x}_{n+1}(t + \Delta t)]}{[x_n(t) - x_{n+1}(t)]} [x_n(t) - x_{n+1}(t)]$$

So, now the sensitivity has got three parts. Alpha dash a dimensionless constant then $\dot{x}_{n+1}(t + \Delta t)$ speed up the following vehicle and third $x_n(t) - x_{n+1}(t)$ the distance headway three terms are there. So, the sensitivity depends on a constant the speed of the following vehicle and the spacing between the lead vehicle and the following vehicle.

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General Motors Theory

Fifth Generation Model

- Fifth and final model was a continued effort to improve and generalize the sensitivity term

- This was implemented by introducing 'm' & 'l' components as seen equation:

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{l,m} [\dot{x}_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [x_n(t) - x_{n+1}(t)] \quad \dots\dots(9.14)$$

l = Distance headway exponent; m = Speed exponent

- This was the final car following model developed and all previous GM car following models were special cases of this generalized model as shown in figure (next slide)



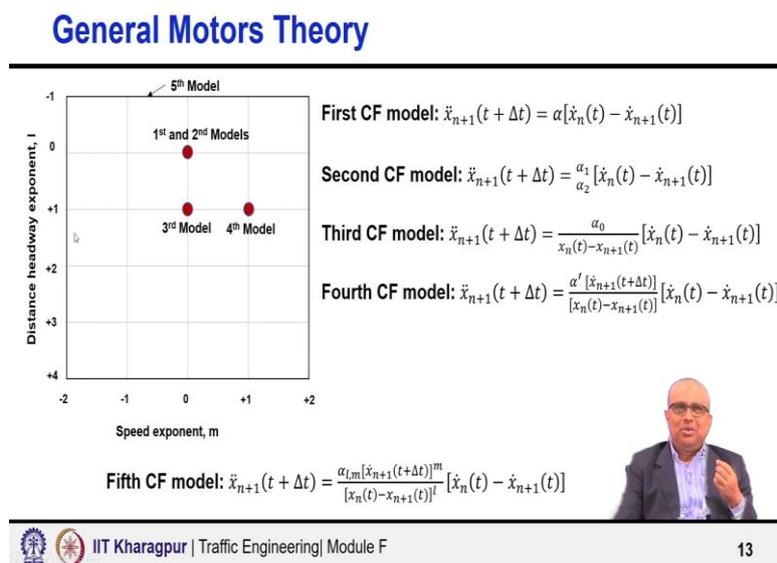
Now, fifth generation model was actually a continued effort to improve and generalize the sensitivity term. This was implemented by modifying the fourth-generation model, but trying to make it more generalized by introducing two components m and l. Then the equation looks like the response is not alpha only but alpha l,m it depends on l it depends on m.

The speed of the following vehicle to the power m what we considered earlier case in fourth generation model m equal to 1. So, now it makes it a generalized term to the power m divided by $x_n(t) - x_{n+1}(t)$ to the power l what was considered in the fourth-generation model l as 1 into the relative speed as it was there in first generation model second generation model, third generation model and fourth generation model same term continued also in the fifth generation model.

$$\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{l,m} [\dot{x}_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

So, this was the final car following model which was developed and it was eventually found or realized that all previous general motorcar following models were actually special cases of this generalized fifth generation model. What is l? l is the distance headway exponents that is what because this is the distance headway to the power l. So, it is the distance headway exponent and what is m it is the speed exponent because it is \dot{x}_{n+1} at time t plus delta t.

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Now, if you see if we try to plot this speed exponent m and distance headway exponent l and then try to see what values make what model out of all this first generation and second generation, third generation, fourth generation and fifth is obviously generalized one. Now, if you see when the l is 0 and also the m is 0 m is here m is 0, l is 0.

Then it is actually first- and second-generation model. That is what you found. That is what is our first- and second-generation model. Because, if you put in the fifth-generation model, I

think in this slide also I have written this is the generalized fifth generation model m is 0 means the whole term becomes 1.

l is 0 again the whole term becomes 1 then what remains is only α a constant that was nothing but the first-generation model car following model as developed by General Motors, then second is also that l is 0 m is 0, but we use two values of α rather than one constant value here we use two different values.

So, that is again l equal to 0, m equal to 0. Now, if you see the third-generation model then actually what is taken m is taken as 0 and l is taken as 1 that is the case. So, again in the fifth-generation model, if you consider m is 0 then this term in the denominator, the speed of the following vehicle will not be there it will be 1 and l is 1.

So, a constant divided by the relative speed relative spacing or the actually displacement with the relative position of two vehicles. So, which is the distance headway or the space headway. So, that gives you the third-generation model, fourth generation model you get when you take both l as well as m both as 1, m is 1, l is also 1 then a constant multiplied by the speed of the following vehicle divided by the spacing between the lead and the following vehicle. So, this l is 1, m is 1 that is nothing but the fourth-generation model.

So, fifth generation model is actually a generalized model and all the first, second, third and fourth generation model became the special cases of the fifth-generation model depending on what value of speed come exponent you are taking, and what value of distance headway exponent you are taking.

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Summary

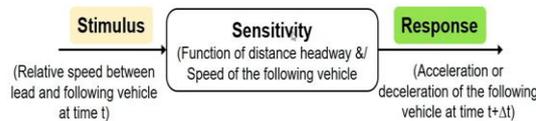
- General motors theory
 - ✓ First generation model
 - ✓ Second generation model
 - ✓ Third generation model
 - ✓ Fourth generation model
 - ✓ Fifth generation model



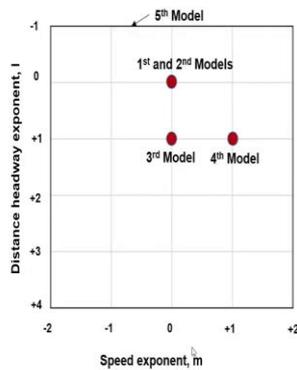
General Motors Theory

- The car following theories developed by GM are much **extensive** and of particular importance because of the accompanying comprehensive field experiments and **discovery** of the **mathematical bridge** between **microscopic and macroscopic theories** of traffic flow
- The fundamental concept:

Response = function (sensitivity, stimuli)



General Motors Theory



First CF model: $\ddot{x}_{n+1}(t + \Delta t) = \alpha [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$

Second CF model: $\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_1}{\alpha_2} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$

Third CF model: $\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_0}{x_n(t) - x_{n+1}(t)} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$

Fourth CF model: $\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha' [\dot{x}_{n+1}(t + \Delta t)]}{[x_n(t) - x_{n+1}(t)]^t} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$

Fifth CF model: $\ddot{x}_{n+1}(t + \Delta t) = \frac{\alpha_{l,m} [\dot{x}_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$



So, altogether what we discussed in the present class is about the General Motors theory response depends on two things going to the first slide response depends on stimulus and the sensitivity. Stimulus is relative speed what could be the sensitivity? Finally, the sensitivity depends on the distance headway and also the speed of the following vehicle.

And depending on how are what value you are considering for the speed exponent m and the distance exponent l you get all these models first generation model sometimes with certain assumption of $l=0, m=0$ you get first generation $l=0, m=0$ you get again second generation $l=1, m=0$ you get third generation $l=1, m=1$ you get fourth generation and any generalized value l and m any other value you get actually the generalized fifth generation model. So, with this I close this lecture and in the next lecture, we shall take a numerical example and show you the calculations based on the General Motors model. Thank you so much.