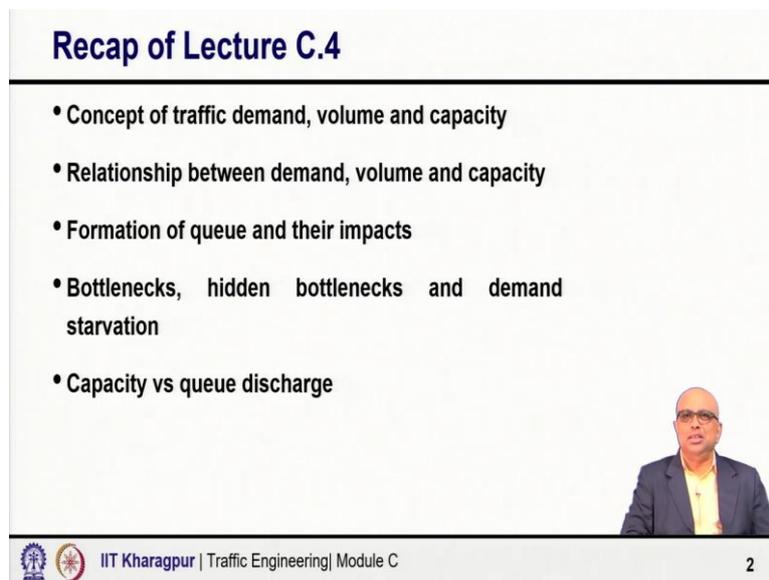


Traffic Engineering
Professor. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture 15
Queueing Analysis - II

Welcome to Module C, lecture 5, this is the last lecture of this module. And we shall continue our discussion in this lecture about the queueing analysis.

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The slide is titled "Recap of Lecture C.4" and lists five key topics. A small video inset of Professor Bhargab Maitra is visible in the bottom right corner of the slide content. The footer of the slide includes the IIT Kharagpur logo, the text "IIT Kharagpur | Traffic Engineering | Module C", and the page number "2".

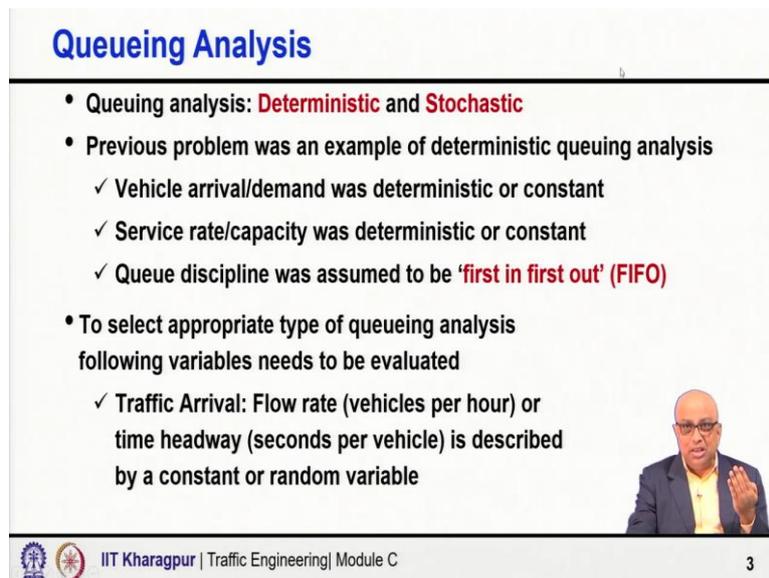
Recap of Lecture C.4

- Concept of traffic demand, volume and capacity
- Relationship between demand, volume and capacity
- Formation of queue and their impacts
- Bottlenecks, hidden bottlenecks and demand starvation
- Capacity vs queue discharge

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What we discussed in the previous lecture, the three major important variables or components which are extremely important in the context of queueing, the concept of demand, the observed volume, and the capacity. And then, how or how they relate, how these parameters or the variables relate in the context of queueing. Then formation of queue and their impacts, also the concepts of bottleneck, hidden bottleneck and demand starvation that I explained with an example. And also explained to you the difference between the capacity and the queue discharge, queue discharge is likely to be little lesser than the capacity.

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Queueing Analysis

- Queuing analysis: **Deterministic** and **Stochastic**
- Previous problem was an example of deterministic queueing analysis
 - ✓ Vehicle arrival/demand was deterministic or constant
 - ✓ Service rate/capacity was deterministic or constant
 - ✓ Queue discipline was assumed to be **'first in first out' (FIFO)**
- To select appropriate type of queueing analysis following variables needs to be evaluated
 - ✓ Traffic Arrival: Flow rate (vehicles per hour) or time headway (seconds per vehicle) is described by a constant or random variable

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3

Now, one important thing I should mention at this stage when I started our discussion in the last lecture about the queueing analysis, I just started talking about the key, but queueing analysis actually could be deterministic as well as stochastic.

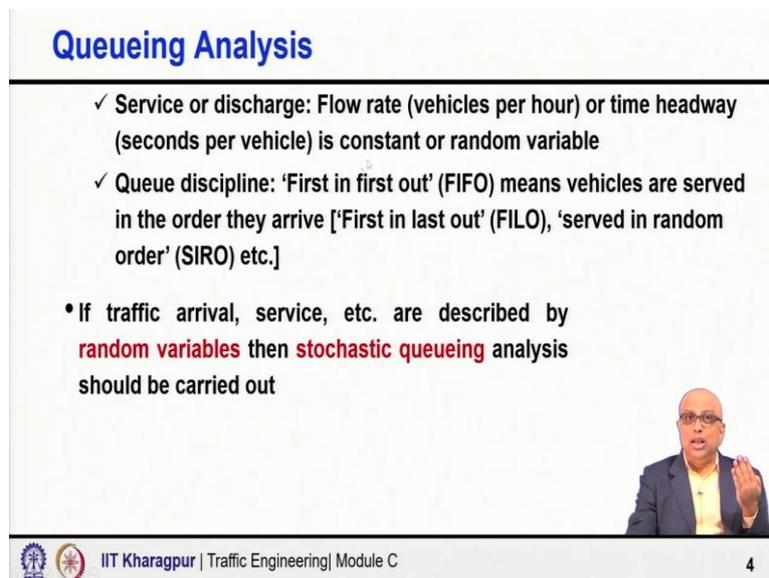
Now, the problem what we took the example problem what we took in the previous lecture was an example of deterministic queueing analysis why it was deterministic because the vehicle arrival a demand was deterministic, it was a fixed demand, we said that for a given period of time the demand is so many vehicles per hour, it is fixed. Of course, another period of time the demand may be different, but that comes, that may be called as the pattern of demand. But for a given period of time that demand is fixed, that means, there is no variation in the arrival rate during that time period.

So, it was constant or deterministic, the service rate or the capacity discharge was also deterministic at constant over a period of time, yes, when there are three lanes, when there are two lanes, the capacity values are different, but for a given time, but a given context the value is constant and deterministic. Queue discipline was also assumed in built assumptions that it was FIFO first in first out, so, the vehicle which is arriving fast is getting discharged fast. That is the simple thing.

Now, to select the appropriate type of giving analysis the following variables need to be evaluated. First type of traffic arrival say the flow rate how many vehicles per hour or time headway, how many seconds per vehicle, are these constant or they are described by a random variable, that means, every small interval if we see the rates is not constant.

If the rate is not constant, it follows a it is described by a random variable or described by some distribution, then it is not a constant value, then the analysis should be done following stochastic queuing analysis. If it is constant, I say that so many vehicles per hour, every or the so many seconds per vehicle means, every that when many seconds one vehicle is arriving and no variation from that, then we can carry out deterministic analysis. So, traffic arrival we need to check whether the flow rate or time headways described by a constant or by a random variable.

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Queueing Analysis

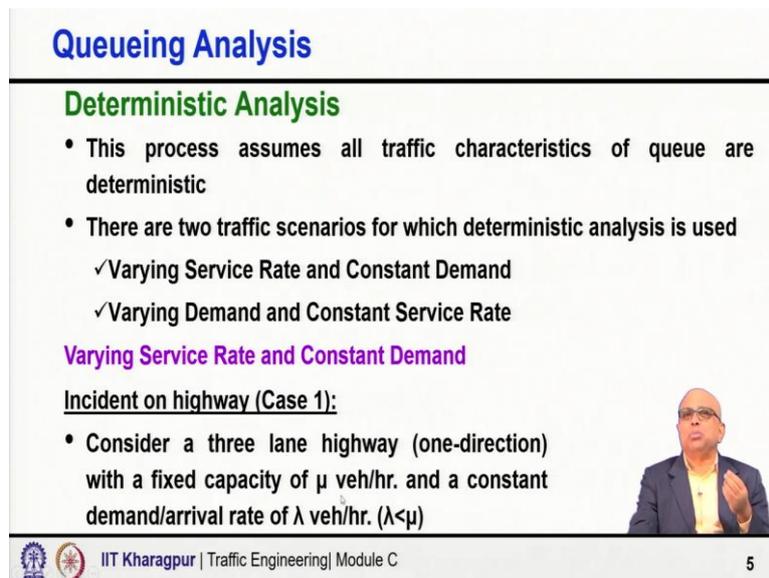
- ✓ Service or discharge: Flow rate (vehicles per hour) or time headway (seconds per vehicle) is constant or random variable
- ✓ Queue discipline: 'First in first out' (FIFO) means vehicles are served in the order they arrive ['First in last out' (FILO), 'served in random order' (SIRO) etc.]
- If traffic arrival, service, etc. are described by **random variables** then **stochastic queueing** analysis should be carried out

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Similarly, whether the service or the discharge in this context again flow rate or time headway is constant or random variable and what kind of queue discipline that is followed. First In First Out or First In Last Out, served in random order many things are possible. So, if the traffic arrival service etcetera are described by random variables, then stochastic queuing analysis should be carried out.

If not, the arrival rate is fixed a constant value, service rate is fixed a constant value, does not change during a specified duration and is not random in that sense, then we will carry out deterministic analysis.

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Queueing Analysis

Deterministic Analysis

- This process assumes all traffic characteristics of queue are deterministic
- There are two traffic scenarios for which deterministic analysis is used
 - ✓ Varying Service Rate and Constant Demand
 - ✓ Varying Demand and Constant Service Rate

Varying Service Rate and Constant Demand

Incident on highway (Case 1):

- Consider a three lane highway (one-direction) with a fixed capacity of μ veh/hr. and a constant demand/arrival rate of λ veh/hr. ($\lambda < \mu$)

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With this background let us relook at the deterministic analysis one second, although we took some examples, some basic concepts we discussed in the previous lecture, but we shall build further on deterministic analysis. So, in deterministic analysis, the process assumes that all traffic characteristics have QR deterministic, no randomness.

So, their outcome is not described by any distribution, it is a fixed value and such kind of deterministic analysis is used under two scenarios. First, varying service rate, service rate is not constant. It varies, but different periods, different service rate. Again, it is not random, but it is different rates during different time periods. An example will make it even more clear and constant demand.

So, demand is constant, but the service rate is not same throughout the period, maybe during some period it is one value, during another period it is another value. Simple example, maybe when three lanes are operational, then the service rate is something, when for half an hour, if that lane one lane is blocked due to an accident or due to any other incident or maintenance work or so, whatever, then the service rate will be different during that period. And once that is removed, again service rate is restored to its original value.

So, that way it is varying but not random, that means when the three lanes are available, then the service rate does not vary over time. The second scenario is varying demand, which was earlier constant, demand was constant in this case, varying demand and constant service rate. Demand is vary, sometimes the demand may be more than the capacity for some period, the demand may be less than the capacity. So, demand is varying over time. And the service rate is constant. All the three lanes are available, for example, during the entire period.

So, first, let us discuss about varying service rate and constant demand. Let us take a case of an incident on highway that is our first case, consider a three lane highway in one direction with the fixed capacity of new vehicles per hour and a constant demand or arrival rate of λ vehicle per hour. And here λ is less than μ . That means the demand rate is less than the fixed capacity of three lane.

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Queueing Analysis

- An incident resulted in closure of one lane and thereby reduced capacity to μ_R for a period of t hr. which is the time to clear the incident ($\lambda > \mu_R$)
- Before the incident, there was no queue but after the incident, queue is formed
- Maximum queue length is the excess demand rate multiplied by duration of incident, $q_{max} = (\lambda - \mu_R)t$
- Average queue length, $q_{avg} = \frac{(\lambda - \mu_R)t}{2}$
- Time duration of the queue, $t_q = \frac{(\lambda - \mu_R)t}{(\mu - \lambda)}$




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6

Normally, queue will not get formed, but then an incident occurred and it resulted in closure of one lane leaving only two lane operational and thereby reduce the capacity to not μ , but μ_R , μ_R reduced for a period of t hour, which is the time to clear the incident. So it took t hour to clear the incident, maybe a breakdown vehicle, because of a breakdown vehicle or because of an accident or crash. So, it took t hour to remove that, to clear it.

So for t period of time t hour, the capacity was not μ , but it was μ_R and the demand for λ is actually greater than μ_R . The λ was less than μ original three lane capacity, but it is more than the two lane capacity, if one lane is occupied. Before the incident, there was no queue as usual because demand was less than the capacity, but after the incident during this t hour, the queue is formed.

So, maximum queue length in excess, is the excess demand rate multiplied by duration of incident. So, what will be the Q_{max} ? Q_{max} will be, if λ is the demand and minus μ_R , μ_R is the capacity, reduced capacity during the incident time of t hour. So, $\lambda - \mu_R$ into t , what will be then the average queue? Length half of this, the time duration of the $q_{avg} = \frac{\lambda - \mu_R}{2} t$, that is the reduced capacity into t . So, that is the μ_R is the reduced capacity,

original capacity is μ , so, into t divided by μ minus λ . So, how much, what will be the time duration of the queue.

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Queueing Analysis

- **Total delay is the time duration of the queue multiplied by average queue length,**

$$dT = \frac{(\mu - \mu_R)t}{(\mu - \lambda)} \times \frac{(\lambda - \mu_R)t}{2} = \frac{(\mu - \mu_R) \times (\lambda - \mu_R)t^2}{2(\mu - \lambda)}$$

Example 1 A three lane expressway (one-direction) is carrying a total volume of 4050 veh/hr. when a incident occurs resulting in closure of two lanes. If it takes 90 minutes to clear the obstruction, determine the following (Capacity= 2000 veh/hr./lane)

- a) Maximum queue length formed?
- b) The total delay
- c) Number of vehicles affected by the incident
- d) Average individual delay



Similarly, total delay you can easily calculate is the time duration of the queue, multiplied by average queue length. So, you have calculated time duration, you have also calculated the average queue length. So, you simply multiply these two to get the total delay. So, we can easily calculate this values.

So, let us take an example to explain it further. A three lane expressway in one direction is carrying a total volume of 4050 vehicles per hour. When an incident occurs resulting in closer of two lane if it takes 90 minutes to clear the obstruction, determine the following, maximum queue length form, the total delay, the number of vehicles affected by the incident and average individual delay, assuming that capacity is 2000 vehicle per hour.

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Queueing Analysis

Solution- Total capacity (μ) = $3 \times 2000 = 6000$ veh/hr. and reduced capacity (μ_R) = $2000 \times (3-2) = 2000$ veh/hr.

a) Max queue length = $(\lambda - \mu_R)t = (4050 - 2000) \times 1.5 = 3075$ vehicles

b) Total delay = $\frac{(\lambda - \mu_R)(\mu - \mu_R)t^2}{2(\mu - \lambda)} = \frac{(4050 - 2000) \times (6000 - 2000) \times (1.5)^2}{2(6000 - 4500)} = 6150$ hr.

c) Number of vehicles affected by the incident = $4050 \times 1.5 = 6075$ vehicles

d) Average individual delay = $(6150 / 6075) = 1.010$ hr.

Signalized intersection: (Case 2)

- Arrival rate is λ veh/hr. and service rate (μ) has two states: Zero when signal is red and up to saturation flowrate (s) when it is green



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So, the first component what is then the total capacity? Capacity is 6000 vehicle per hour when the three lanes are operational that is the μ and μ reduced is one lane is dropped. So, 3 minus 2. So, not one lane but closer of two lanes in this case, so, two lanes are drop so 3 minus 2, so, 1 into 2000 so, original capacity was 6000 vehicle per hour and during the incident of t hour, the reduced capacity is 2000 vehicle per hour.

So, what will be the maximum queue length, directly used the formula, this is the demand 4050 but getting discharged during the incident is 2000 and that is happening for 1 and a half hour. So, you have total 3075 vehicles that is in the queue. And what will be the total delay, again you can calculate easily that is the total delay using this equation.

Then number of vehicles affected by the incident how many vehicles, because of that 4050 multiplied by 1.5. So, it takes 90 minutes to clear the obstruction. So, all those vehicles got affected by the incidents and what will be the average delay, you know the total delay is 6150 and you know that number of vehicles affected were this. So, average individual delay you can calculate. Do not worry about the values is just example numbers, mean the calculation or how such kind of things can be analyse that we are trying to show.

Taking another case of a signalized intersection, let us consider arrival rate λ vehicle per hour and service rate is μ , μ has now two states, service rate, two states, we say varying service rate, earlier case also varying service rate; one during closer of two lanes and another under normal condition. Here also to service rates we are considering, one is when the signal is red, then 0 and saturation flow rate when the signal is green. So, two distinct values, one during red, one during green.

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Queueing Analysis

- Queue assumed to follow first in, first out (FIFO) system
- During the red period, service rate is zero (horizontal line) and at the start of green period, service rate is equal to saturation flow rate
- At intersection point of arrival and departure curve: Queue is dissipated

Flowrate (veh/hr.)

Time

Green

Green

Green

μ

s

λ

Cumulative vehicles

Arrivals

Departure

Time

Cycle length

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9

Now, queue assumes to follow first in first out system; FIFO, during the red phase service rate is 0, you can see, when it is rate, service rate is 0, when it is green service rate is equal to the saturation flow μ , then again green is over, red comes it is 0, again green goes, μ saturation flow rate comes down to 0 and like this.

And how the cumulative vehicles and time may look like this is an example, some assumption is also there, we are assuming that when the signal is red, then vehicle are, this is the arrival curve, arrival is constant, you can see it is a constant, one line, no variation, rate is constant, that is our deterministic. And departure, again follows straight line, but when the red is their signal, then it is 0.

So, no discharge, no departure during green time discharge at saturation flow rate, it is catching up then these two lines are meeting together, additional green time whatever is the arrival is getting, all getting discharged. Then, the again red time, again during green time it is catching the arrival curve and then sometime as the vehicles are arriving, they are getting discharged without any slowdown or delay or even stop like this.

So, here one assumption is there that there is no signal failure that means, whatever, every individual cycle, whatever is the queue vehicles during the red time all are getting discharged during the immediate next green interval. So, at intersection point of arrival departure queue is dissipated.

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Queueing Analysis

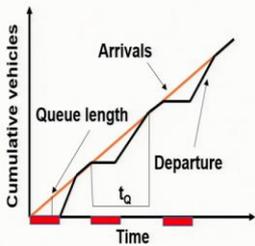
- Time duration of queue (t_Q): Horizontal projection of queueing triangle
- Starts at the beginning of red period and continues up to the point when queue is dissipated

$$\lambda t_Q = \mu(t_Q - r)$$

$$\Rightarrow t_Q(\mu - \lambda) = \mu r$$

$$\Rightarrow t_Q = \frac{\mu r}{(\mu - \lambda)}$$

- Percent time queue is present (Pt_Q):

$$Pt_Q = \frac{100t_Q}{C}$$



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Now, the time duration of the Q, t_Q . What is the time duration of the queue if you take one cycle, it starts at the beginning of red period here fine, beginning of the red period and continues up to the point when the queue is dissipated, it is going up to this point, when the two lines arrival and cumulative arrival and cumulative departure are catching up. So, that is the time duration of the queue.

So, we can calculate it easily say λt_Q , that is the discharge arrival rate equal to $\mu(t_Q - r)$, r time there is no discharge. So, this is the must be equal and you can get the value of t_Q . Then percentage time Q is present, also can be calculated, it is t_Q time in a cycle length of C, Q should be subscript, it is an, it is a problem sometimes the, this sorts of notations will get distorted. So, P t_Q proportion or percentage of time Q is present, Q is present at t_Q time over a cycle in C, so 100 into t_Q by C that much percentage $Pt_Q = \frac{100t_Q}{C}$.

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Queueing Analysis

- Number of vehicles queued (N_Q): $N_Q = \frac{\lambda t_0}{3600}$
- Number of vehicles per cycle (N): $N = \frac{\lambda C}{3600}$
- Maximum queue length (Q_m): $Q_m = \frac{\lambda r}{3600}$
- Average queue length while queue is present (\bar{Q}_Q): $\bar{Q}_Q = \frac{Q_m}{2}$
- Average queue length (\bar{Q}): $\bar{Q} = \frac{Q_m t_0}{2C}$
- Individual delay is represented by horizontal distance across the triangle and vehicle arriving at the beginning of red experiences maximum delay (d_M), $d_M = r$ and average delay = $r/2$ (Queue present)
- Average individual delay (d): $d = \frac{r t_0}{2C}$



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Similarly, you can calculate all other parameters, which are very simple, you can understand easily number of vehicles queued, number of vehicles per cycle all very simple calculation, maximum queue length, average queue length while the queue is present, average queue length all these you can calculate very easily, using this graph and using simple equation, logical equation. Also the individual delay, average individual delay, all this can be expressed.

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Queueing Analysis

- Total delay can be represented by the cross sectional area of the queueing diagram and expressed in veh-secs (TD):
$$TD = \frac{N_Q r}{2} \text{ or, } TD = \frac{Q_m t_0}{2} \text{ or, } TD = d \times N$$

Varying Demand/ Arrival Rate and Constant Service Rate

- The **same procedure** is valid in this case if it is **assumed that demand changes at specific times and not gradual**
- Suppose service rate (μ) is constant over the entire time period and arrival rate (λ) gradually increase in the early portion of peak period and then gradually decrease in the later portion of peak period
- Time period DT_0 : Constant arrival rate λ_0



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So I have, I am not going to discuss each and every calculation. It is not necessary because they are quite simple and you can understand them easily. Second case is the varying demand or arrival rate and constant service rate. The same procedure is valid in case if it is assumed that the demand changes at specific time and not gradual.

Suppose service rate is constant over the entire time period, like in previous case, lane was blocked, but now, we are not considering that kind of situation, the discharge is same, constant and arrival rate gradually increase in the early portion of the peak period and then gradually decrease in the later portion of the peak period. So, if we consider time period DT 0, the constant arrival rate is $\lambda = 0$.

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Queueing Analysis

- Time period DT₁: Arrival rate (λ_1) increases to λ_2 from λ_0
 - ✓ Sometime during this time period, arrival rate exceeds service rate
- Time period DT₂: Constant arrival rate λ_2
- Time period DT₃: Linear decrease from λ_2 to λ_4
 - ✓ Sometime during this time period, arrival rate became less than service rate
- Beyond DT₃: Arrival rate remains constant λ_4




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13

So, during time DT 0, the constant rate of flow is the demand is DT 0, then the demand goes during time DT 1 from lambda 0 to lambda 2, then over DT period it remains same lambda 2, then further reduces to lambda 3 to lambda 4 sorry, during the time DT 3. And in between somewhere it is, when it is increasing, is this crossing that capacity value or service rate, maximum service rate value and while coming down also at some point it is crossing that value.

So, time period DT 1 arrival rate lambda 1 increases to lambda 2 from lambda 0, sometime during this period the arrival rate exceeds the service rate here you can see that is the point, then time to the DT 2 constant arrival rate lambda 2, time period DT 3 linear decrease from lambda 2 to lambda 4 sometimes during this period the arrival rate becomes less than the service rate, say at this point and beyond DT 3, beyond this period the demand remains constant as lambda 4.

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Queueing Analysis

- Time when arrival rate exceeds service rate from the start time: $TM = DT_0 + DT_1 \left(\frac{\mu - \lambda_0}{\lambda_2 - \lambda_0} \right)$
- Time when arrival rate becomes less than service rate after the end of TM time period : $TI = DT_1 \left(\frac{\lambda_2 - \mu}{\lambda_2 - \lambda_0} \right) + DT_2 + DT_3 \left(\frac{\mu - \lambda_2}{\lambda_4 - \lambda_2} \right)$
- Duration of queuing process when queue is dissipated within DT_3 : $TQ_p = TI + \left(TI + DT_2 \right) \left(\frac{\lambda_2 - \mu}{\mu - \lambda_4} \right)$

14

If it is so, then we can again easily calculate what is the time when arrival rate exceeds service rate? We can assume a linear variation, simple, at time they sit was this, at a time this is alpha 2. So, a linear variation you can easily calculate it. So, time when they access the service rate from the start time assuming linear variation. Again time when arrival rate becomes less than the service rate after the end of this TM period. So, that again you can from the TM period, how much time it will take the TI that also can be calculated easily.

And then duration of queuing process when the queue is dissipated. Now, this queue may be dissipated within this period theoretically, when it is coming down and by the time it is reaching to demand level alpha 4. So, within this DT 3 time, DT 3 time itself the queue may get dissipated. If so, then the duration of queuing process can be calculated using this formula.

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Queueing Analysis

- Duration of queuing process when queue is dissipated after DT_3 : $TQ_N = \frac{TI}{2} \left(\frac{\lambda_2 - \mu}{\mu - \lambda_4} + 2 \right) + \frac{DT_2}{2} \left(\frac{\lambda_2 - \mu}{\mu - \lambda_4} \right) + \frac{DT_3}{2} \left(\frac{\lambda_4 - \mu}{\lambda_4 - \lambda_2} \right)$
- Number of vehicles adversely affected by the bottleneck: $N_Q = \mu(TQ)$
- Total delay in vehicle-hours: $TD = \int_0^{TQ} [\lambda(t) - \mu(t)] dT$
- The maximum number of vehicles in queue: $Q_M = \int_0^{TI} [\lambda(t) - \mu(t)] dT = \frac{\lambda_2 - \mu}{2} (TI + DT_2)$

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If not, like if the duration of queuing process when Q is dissipated after DT_3 period, it depends on what is the value and what is the difference in the rates and then how much time it will take. It may be getting dissipated completely within this DT_3 period itself or it may go even beyond that. If it is going beyond that, then how the calculation will look like.

Now, the number of vehicles adversely affected by the bottleneck you can again calculate, it is that TQ into μ total delay in vehicle per hours maximum number of vehicles in the queue all our simple calculation, you can calculate, no problem easily we can calculate and this is the deterministic case. So, the all calculations are very simple, we can take the area, we can assume linear variation, we can assume some simple triangle and to find out the area such kind of calculations are involved. So, everything you can understand very easily.

The maximum number of vehicles in the queue all, all you can do, when the maximum Q will be there, this will be there during 0 to TI period, this is the TI period, when it is crossing and when again coming down it is touching the capacity value. So, during that period is the TI .

(Refer Slide Time: 24:39)

Queueing Analysis

Example-2 Given the demand pattern shown in the figure, calculate the following:

- How long does it take the demand rate to increase to the capacity of 1,200 veh/hr. ?
- How long does it take the demand rate to decrease back to the capacity of 1,200 veh/hr. ?
- What is the size of queue developing during the period when demand > capacity?
- How much time it will take to dissipate the queue?

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16

So now, let us take an example to explain this further. Say given the demand pattern shown in figure calculate the following. Here as you say, during the first one hour demand is 1000, then increasing following a linear variation, going to 1300 at hour 1.5, in between somewhere it is crossing the capacity value of 1200, then it is there up to 2.5 hours, then coming down coming down up to this value 900 in 3.5 hours, in between it is again somewhere touching this value 1200. Then farther beyond 3.5 to 5 hours, it is further getting reduced at 5 hours at 600 vehicle per hour and then remain constant.

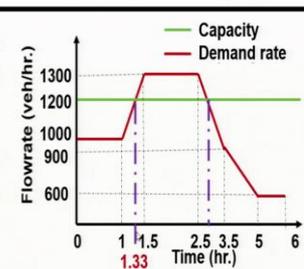
So, you can use some kind of formulas, similar kind of formulas as I have explained in the previous two or three slides. Also, we will try to show that how for this example problems to value can be calculated, I have not used the formulas directly, you can use it, the same, same result same thing. How long does it take the demand rate to increase to the capacity 1200 vehicle per hour?

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Queueing Analysis

Solution- (i) How long does it take the demand rate to increase to the capacity of 1,200 veh/hr. ?

- Demand rate is increasing at a rate of $(1300-1000)= 300$ veh/hr. in $(1.5-1)=0.5$ hours i.e. 600 veh/hr.
- Time taken to increase of demand flowrate to 1200 from 1000 is $(200/600)=0.33$ hr.
- Therefore, it will take 1.33 hr. for the demand rate to reach the capacity



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Let us take one by one, how long does it take, the demand rate to increase to the capacity of 1200 vehicle per hour? You know the demand is increasing at a rate of what 1300 minus 1000 so, 300 vehicle per hour that much increase is happening in 1 to 1 and a half hour. So, that means in 0.5 hour that rate is happening. So, the actual rate is 600 vehicles per hour that is the rate it is increasing.

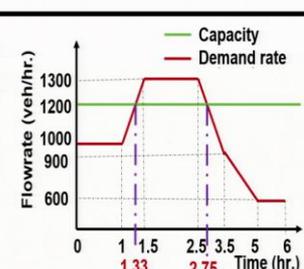
So, the time taken to increase the demand flow rate to 1200 would be how much? 1000 to 1200. So, 200 increase means, in 1 hour, 600 increases happening, that is the rate. So, 200 includes will happen in 0.33 hour. So, therefore, it will take 1 hour plus 0.33 hour, so, 1.33 hour for the demand rates to reach to the capacity.

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Queueing Analysis

Solution-(ii) How long does it take the demand rate to decrease back to the capacity of 1,200 veh/hr. ?

- Demand rate is decreasing at a rate of $(1300-900)= 400$ veh/hr. in $(3.5-2.5)=1$ hours i.e. 400 veh/hr.
- Time taken to decrease of demand flowrate to 1200 from 1300 is $(100/400)=0.25$ hr.
- Therefore, it will take 2.75 hr. for the demand rate to reach the capacity



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The second part, how long does it take the demand rate to decrease back to the capacity 1200 vehicles per hour at this point what is the time that is what we want to find out. Demand is just decreasing at a rate what, 1300 to 900 in 2.5 to 3.5; so, in 1 hour. So, 400 decrease in one hour. So, 1300 to 200 means how much decrease, it is only 100 decrease.

So, 1300 decrease if it takes 1200 to 1300 a so, demand is decreasing see how much 1300 to 900, so 400 decrease in 1 hour, so 100 decrease in how much 0.25 hour. So, how much it will take it was 1.1 hour and then 2.5 plus 0.5. So, 2.5 plus 0.5, 2.5 up to this and then 0.5 this portion. So, it takes 2.75 hour.

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Queueing Analysis

Solution-(iii) What is the size of queue developing during the period when demand > capacity?

- Area of demand and capacity curve when demand > capacity gives the queue length during that period = Area of trapezium
 $ABCD = 0.5 \times [(2.75 - 1.33) + (2.5 - 1.5)] \times (100) = 121$ vehicles

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19

Third what is the size of the queue developing during the period when demand was greater than capacity? During the period when the demand was greater than capacity. What you can do you can calculate the area of the demand and capacity curves when demand was greater than capacity. So basically, the area of this trapezoid, so 0.5 into this is 2.75 minus 1.33 and this one is how much 2.5 minus 1.5 that is AB. So, 0.5 of this plus this into this difference is 1300 to 1200, so it is only 100 is the difference, so 121 vehicles.

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Queueing Analysis

Solution-(iv) How much time it will take to dissipate the queue?

If queue is dissipated at t hr. from start

- Assuming time to dissipate queue is between 2.75 and 3.5 hours, then area of triangle CFE = 121 i.e., $0.5 \times (t - 2.75) \times (1200 - 900) = 121$, $t = 3.556$ hr.
- Thus, the assumption is wrong
- Number of vehicles dissipating between 2.75 and 3.5 hr. are = $0.5 \times 0.75 \times 300 = 112.5$

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Queueing Analysis

- Thus, $121 - 112.5 = 8.5$ vehicles are left to clear
- This many vehicles will clear after a short time of 3.5 hr.
- Area of rectangle FIHE + Area of triangle EGH = 8.5
- If queue is dissipated at t hr. from start, Then $300 \times (t - 3.5) + 0.5 \times (t - 3.5)^2 \times 200 = 8.5 \Rightarrow t = 3.528$ or 0.47 hr.
- Queue will be dissipated at 3.528 hr.

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Similarly, how much time it will take to dissipate the queue? The queue is dissipated at one hour from start. If the queue is dissipated, at t hour from the start, how much time it will take? Then assume that the time to dissipate Q is between 2.75 to 3.5. That means, within been this period, 2.5 point, 2.75 when it is just below 1200 or just touching 1200 to 900 during this period, so 2.5, 75 to 3.5 hours within that.

And in that case the area of the triangle is how much, how total Q 121? How we got 121? We have calculated that earlier, we know that 121 vehicles will be in the queue. So, how much time it will take 121, and again if you take this area, this area is again you can also call it equivalent to this area. So, that will be 0.5 into t minus 2.75 that is the base this or this one and into 1200 minus 900. So, that is the area of this triangle.

So, equal to 121. So, you can get t equal to 3.566 so, it is not within 3.5 hours, it is going beyond. So, the assumption is wrong and we have to now go for the other equation or other calculation assuming that it will extend beyond 3.5 hours. So, the number of vehicles that can be dissipated or discharged between 2.5 and 3.5 that you can calculate now, the whole area, so, 0.5 into 0.75 because 2.75 to 3.5 0.75 into 300 because 1200 to 900 so, this whole area you can see that when it is going.

And this here earlier, which considered that the time t will be before or within this 3.5. So, the 2... t minus 2.75 is this portion wherever, it will be equal that time was taken. In this case, the whole area is taken. So, we know 112.5 vehicles can be cleared, dissipated up to time 3.5 and then whatever will be remaining is 8.5 vehicles.

And this may be clearer obviously, within a short time after 3.5 hour and what will be this? Again you take that area and assume that time t and you can find out it is 0.47 hour. So, it is 3.528 or 0.47 hours. So, you can take that value and accordingly you can say that when that queue will be get dissipated, it will be 3.528 hour.

(Refer Slide Time: 32:44)

Queueing Analysis

Stochastic Analysis

- This **assumes** that variables are **random** and therefore, the queuing process is **not deterministic**
- A queue is formed when arrivals wait for a service or an opportunity
 - ✓ Arrival of an **accepted gap** in a main traffic stream
 - ✓ Collection of **tolls** at toll-booths
 - ✓ **Parking fees** at parking garage
- **Proper analysis** of the **effects of queue** needs the following characteristics to be known
 - ✓ **Nature of arrival distribution:** Uniform (heavy traffic), Poisson (random), etc.




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22

Now, coming to the part, next part is the stochastic analysis. Now, this assumes that the variables are random as I have said and therefore, queueing process is not deterministic. Now, when a queue is formed, it is formed because the arrivals wait for a service or an opportunity.

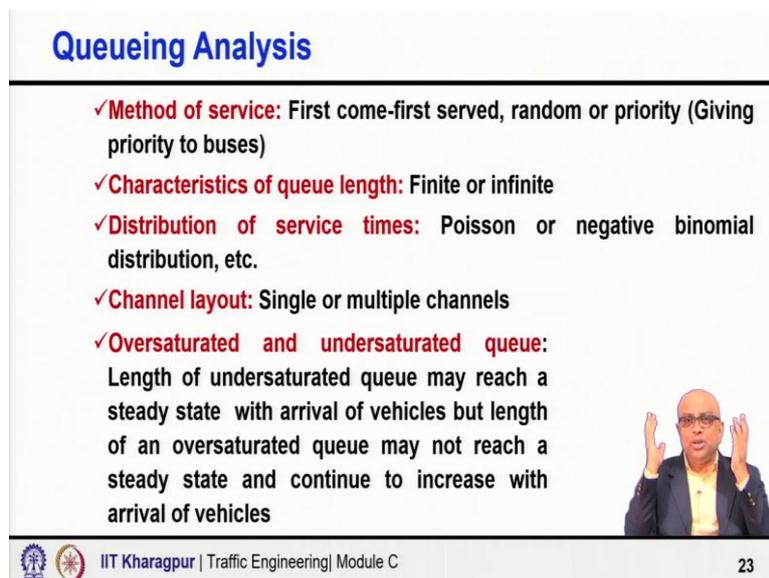
Example could be maybe arrival of an accepted gap in the main traffic stream and vehicle is trying to do main over so, looking for an acceptable gap not every gap is acceptable. So, looking

for a gap or collection of toll at toll-booth or maybe parking fee at the parking garage, vehicle is coming out of parking lot and trying to pay the fee.

Now, remember that in all these cases, this is realistic assumption would be that they follow or they are described by a random variable, why? Because not every vehicle will take the same time to pay the toll or pay the parking charge or that not every gap will be accepted. So, such kind to acceptable get may not come at every equal interval that is what is the process. So stochastic analysis is more useful and more practical under such scenarios.

So, proper analysis of the effect of queue needs the following characteristics to be known. Nature of arrival distribution, how the vehicle are arriving? You can think of a counting distribution or the corresponding headway distribution. So, whether it is uniform, whether it is Poisson, so, what kind of traffic state and what distribution is best suited under the given condition. So, whenever the outcome is described by a random variable, so, we need to understand also what distribution it follows. So, nature of arrival distribution.

(Refer Slide Time: 34:56)



Queueing Analysis

- ✓ **Method of service:** First come-first served, random or priority (Giving priority to buses)
- ✓ **Characteristics of queue length:** Finite or infinite
- ✓ **Distribution of service times:** Poisson or negative binomial distribution, etc.
- ✓ **Channel layout:** Single or multiple channels
- ✓ **Oversaturated and undersaturated queue:** Length of undersaturated queue may reach a steady state with arrival of vehicles but length of an oversaturated queue may not reach a steady state and continue to increase with arrival of vehicles

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Similarly, method of service whether it is FIFO, LIFO or random or some kind of priority, maybe giving priority to buses, bus priority is there, so you give additional priority to buses. Characteristics of queue length, it could be finite or infinite, sometimes this is very important because you are considering a right turn vehicles are accumulating.

And if that right turn storage lane is exhausted, then that overflow of vehicles, right turning vehicles may block the through traffic or it may be other ways, the through traffic queue may

block the right turning lane. So, vehicles actually want to go to the right turning lane, but not getting an entry because the queue is long.

So, what is the? It is the characteristics of queue length finite or infinite, then distribution of service time again it several distributional assumptions maybe their possible, say Poisson, a negative binomial distribution based on that what is the service time or actual headway distribution is happening.

So, the counting distribution and the corresponding what is the headway distribution? Then channel layout; single or multiple channels in a toll-booth, multiple channels, number of channels you have changed the whole processing time delay, queuing everything will be different. Also whether it is oversaturated or non-saturated queue, so it maybe as I have taken that signal, traffic signal example, all the queues which are getting formed during the red all are getting cleared during that immediate green.

So, it is not an oversaturated situation, but sometimes you will find the multiple signal failure is happening, not all vehicles will be cleared in the immediate green so, and it may continue to grow there. So, then it could be oversaturated or under saturated queue. Length of the under saturated to meet, may reach to a steady state with arrival of vehicles but length of an oversaturated queue may not reach to a steady state and continues to increase with arrival of vehicles. If you assume that always the vehicle same rate, high rate the vehicles are arriving. So, the queue length will continue to grow, then signal failure is also happening.

So, the distributional assumption is very important, we are stochastic analysis means we are assuming that these are not fixed, not like deterministic analysis a single value, but it follows a distribution. So, it is a random variable and which follows a distribution. So, the outcome anything we want to calculate as all the parameters we are calculating earlier, we can calculate here also the expected value what will be the key maximum queue length, what is the probability, all will be expressed in terms of probability, the outcome as you know that if it is a random variable following certain distribution the outcome will be expressed in terms of probability.

(Refer Slide Time: 38:24)

Queueing Analysis

Single Channel, Undersaturated, Infinite Queues

- Assuming Poisson arrival and negative exponential service time and undersaturated condition

- Probability of n units in the system,
$$p(n) = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$
- Expected number of units in the system,
$$E(n) = \frac{\lambda}{\mu - \lambda}$$
- Expected number of units waiting to be served in the system, (mean queue length)
$$E(m) = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Single Channel Queue

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24

So, we can calculate all the parameters for example, if you assume single channel under saturated and infinite queues, then if we assume that Poisson arrival and negative exponential service time and under saturated condition, then all these values can be calculated. Obviously, it will not be because it is a stochastic process, because it is random variables following certain distributions it will depend on what distribution you are assuming, but you can calculate things like probability of any units in a system, what will be the probability?

The outcome obviously, will be expressed in terms of probability because we are talking about random variable and which follows a particular distribution. Similarly, you can calculate the expected number of units in the system, what will be the expected number? Expected number of units waiting to be served, all this you can calculate. Obviously, you change the distributional assumption you may come out with a different equation.

(Refer Slide Time: 39:33)

Queueing Analysis

✓ Average waiting time in the queue, $E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$

✓ Average waiting time of an arrival (Including queue and service)
 $E(v) = \frac{1}{\mu-\lambda}$

Example 3 On a given day 425 veh/hr. arrival at a toll booth located at the end of an off-ramp of an expressway. If the vehicles can be serviced by single channel only at a rate of 625 veh/hr., determine i) the percentage of time operator of the toll booth will be free? ii) average number of vehicles in the system? iii) average waiting time for vehicles that wait?



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So, let us take an example I have just taken an example to show you on a given day 425 vehicle per hour arrive at a toll booth located at the end of an off-ramp of an expressway. If the vehicle can be serviced by a single channel, only at a rate of 625 vehicle per hour, determine the percentage of time operator of the toll-booth will be free, average number of vehicles, what will be the average number of vehicles in the system? What will be the average waiting time for the vehicles that with, all this can be calculated.

(Refer Slide Time: 40:08)

Queueing Analysis

Solution- For the operator to be free, the number of vehicles in the system must be zero, $p(n) = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$ $n=0$

$p(0) = \left(\frac{425}{625}\right)^0 \left(1 - \frac{425}{625}\right) = 0.32$

i) Operator will be free 32 % of the time

ii) Average number of vehicles in the system, $E(n) = \frac{\lambda}{\mu-\lambda} = \frac{425}{625-425} = 2.125 \sim 2$

iii) Average waiting time for vehicles, $E(v) = \frac{1}{\mu-\lambda} = \frac{1}{625-425} = 0.005$ hr. = 18 sec

• Queueing analysis can be extended to single Channel, Undersaturated, finite Queues and multi channel queues (Refer books on traffic flow theory)



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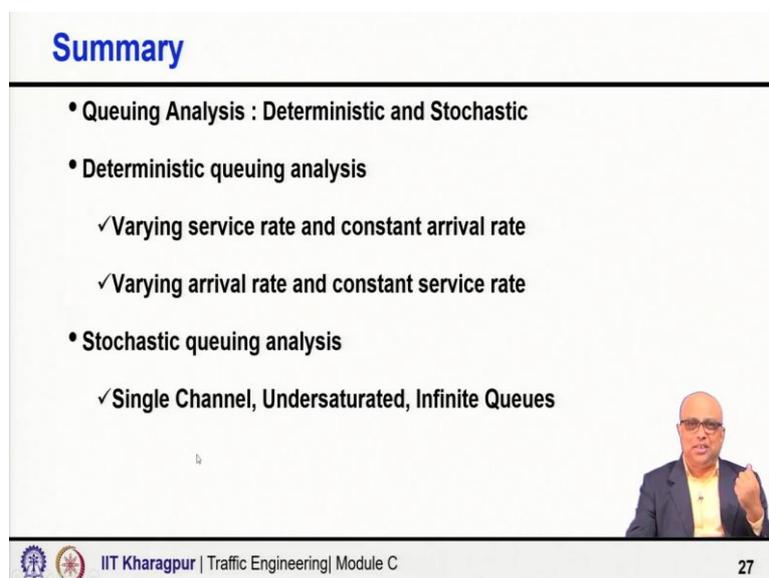
So, for the operators to be free, the number of vehicles in the system must be 0. So, what is the probability that there will be 0 vehicle arrival? So, that you can calculate, you can get that 0.32 is the value. So, you can assume that operators are expected to be free 32 percent of the time.

Similarly, average number of vehicles in the queue, what is the expected value that you can calculate? Average number of vehicles in the system, expected number of units in the system, what is the expected value, $\lambda / (\mu - \lambda)$. So, that you can calculate.

So, 2: average waiting time for the vehicle E_w , E_w you can calculate here, average waiting time for an, of an arrival including queue and service. So, all these calculations you can do. Now, remember that the hot distribution you are assuming the actual calculation of probability expected value all these are expected number all this will actually change based on that, but you can calculate and the outcome will be in terms of generally the probability.

As I have said here that the number of vehicles, what is the probability that there will be 0 vehicles during an interval? The probability you can calculate. So, once you calculate the probability you said that maybe 32 percent of the time the operator will be free. So, what we discussed here is can be extended further to single channel, under saturated finite queues and multi channels all so, many combinations so, many possibilities are again there, but I will not extend further in this class. If you are interested you can refer to any book on the traffic flow theory, you will see that so, many further developments that are there, you can access those material.

(Refer Slide Time: 42:24)



Summary

- **Queuing Analysis : Deterministic and Stochastic**
- **Deterministic queuing analysis**
 - ✓ Varying service rate and constant arrival rate
 - ✓ Varying arrival rate and constant service rate
- **Stochastic queuing analysis**
 - ✓ **Single Channel, Undersaturated, Infinite Queues**

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So, what we discussed here about the queueing analysis, mentioned to you that queueing analysis could be deterministic, could be stochastic, when it is deterministic? When it is stochastic that is explained clearly. Then how the deterministic queueing analysis can be done under two scenarios, varying service rate and constant arrival rate and varying arrival rate and constant service rate that I explained, took examples also to explain you that.

And also, gave you an introduction to stochastic queueing analysis and just explained you with certain distributional, single channel, certain other assumptions, how you can actually calculate the various parameters of interest. With this, I close this session and close this module also. Thank you so much.