Traffic Engineering
Professor. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Shock Wave and Queueing
Lecture – 13
Analysis of Shock Waves - III

Welcome to module C lecture 3. In this lecture also we shall continue our discussion about analysis of shock waves.

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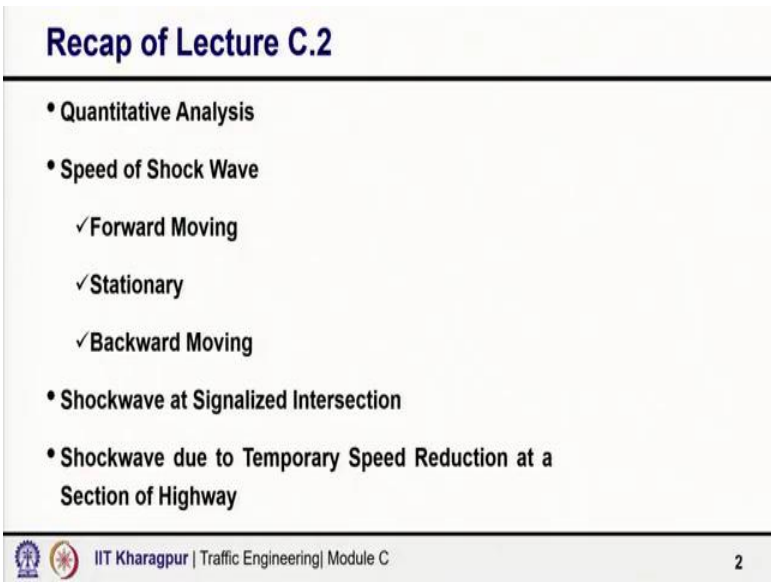

In lecture 2 I discussed about carrying about how to carry out quantitative analysis of shock waves, how to determine the speed of shock waves and how we can, say depending on two traffic states, their flows and densities, how we can say whether the shock wave that is likely to be generated is going to be forward moving or stationary or backward moving.

Then we took an example of signalized intersection and explained you how to calculate various components using the concept of shock waves and the speed of shock waves, then took another example of temporary speed reduction at a section of highway and then explain that under different conditions how the analysis can be carried out.

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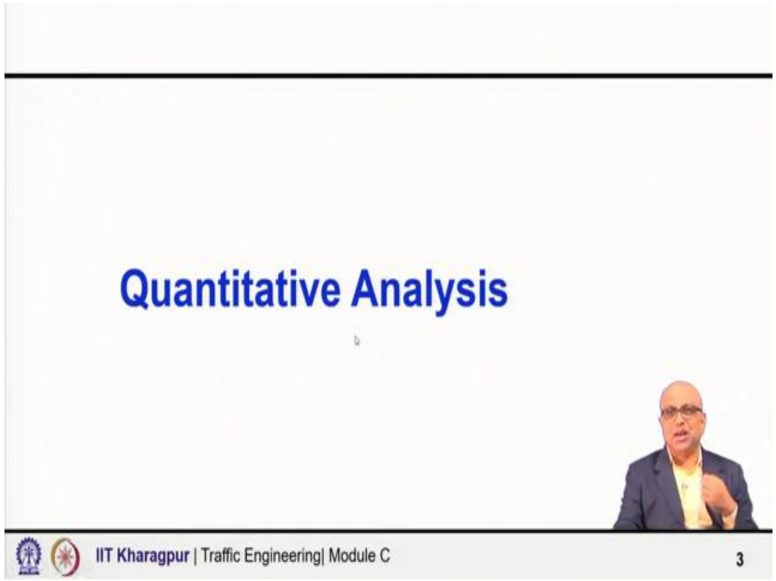

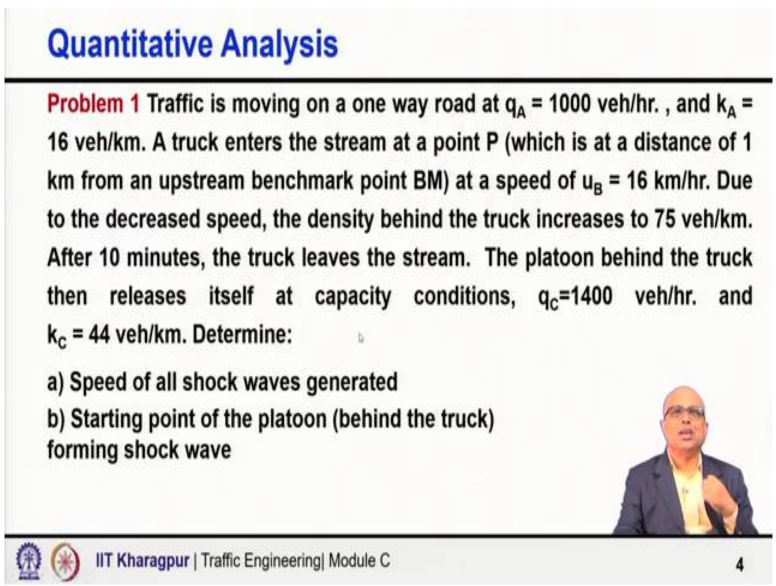

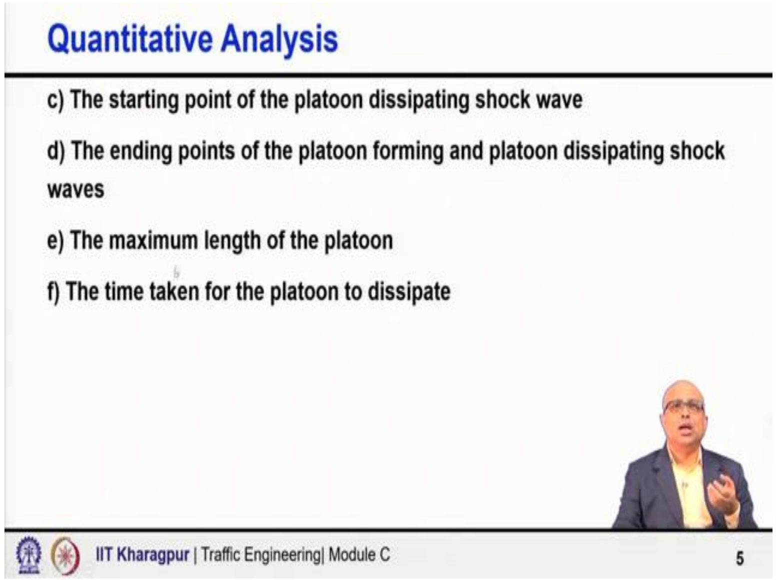

So with this background, we now continue our discussion about quantitative analysis. But in today's lecture we shall take up two example problems to explain you the application of quantitative analysis. Of course, there are so many different applications which are possible, but in this lecture we shall take up two examples.

First problem 1, let us consider that the traffic is moving on a one way road with a traffic state, the q flow and the density values are given. And then a truck enters the stream at a point, which you may consider at a distance of 1 kilometer from an upstream benchmark point, at a speed of 16 kilometer per hour.

In this case, we can compare then what was the speed of the traffic stream knowing this flow as 1000 vehicle per hour and density as 16 vehicle per hour. So, we know the speed. We can compare the speed of the traffic stream with the speed of the truck that is entering. And it will be a slow moving vehicle, speed lower than the speed of the traffic stream. And therefore, due to decreased speed the density behind the truck increases to 75 vehicle per kilometer.

Originally it was 16 vehicle per kilometer. That was the traffic state density value particularly. Now it is increased to 75 vehicle per hour. When the truck travels for 10 minutes, and then after 10 minutes the truck leaves the stream. So now, the platoon behind the truck is getting released itself at capacity condition with the discharge capacity or flow as 1400 vehicle per hour which is higher than the traffic state A and with a density of 44 vehicle per kilometer which is higher than again the value of k, original traffic state.

Now with all this data what we would like to do? we would like to calculate the speed of all shock waves which are expected to be generated, then the starting point of the platoon behind the truck when the shock wave is formed, then the starting point of the platoon dissipating shockwave, the ending points of the platoon forming and platoon dissipating shockwaves, the maximum length of the platoon and the time taken for the platoon to dissipate.

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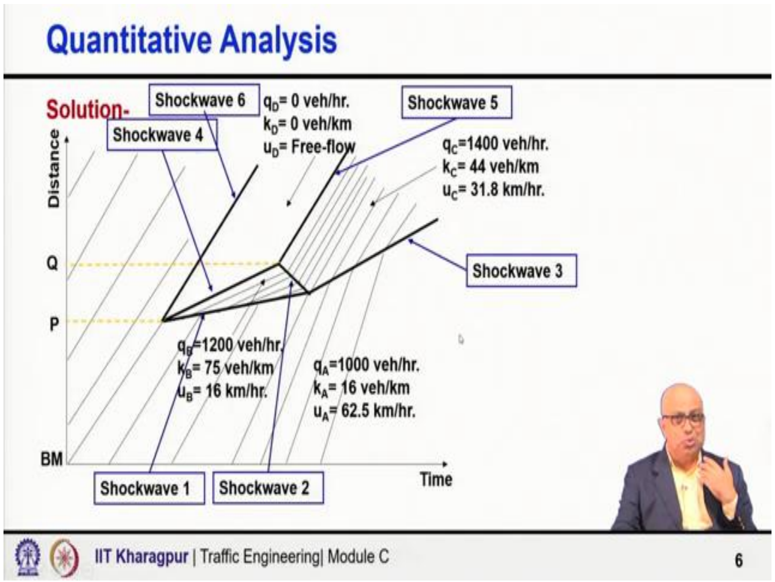

Let us go one by one. First we need to identify how many shock waves will be there and what will be the speed for the shock waves. As you know the original traffic state is A under normal condition till the truck enters at point B. So, that traffic state is shown here qA kA and we have calculated the corresponding speed value also; because the flow is known, density is known so we can calculate the speed.

Then a traffic state B is created behind the truck vehicle platoons, moving platoons of course and that traffic state is shown as traffic state B. So the qB, kB and uB values are there. Then another traffic state C is created, once the truck leaves the highway at point Q then the queued vehicles are released at its capacity flow. So, that is traffic state C. So, qC, kC and uC values are again known.

Then traffic state D will be created as long as the vehicle or the slow moving vehicle was moving from point P to point Q. And downstream of that there will not be any vehicle but that is the traffic state. So, with qD as 0, k as 0 and speed as the free flow speed. So, with these four traffic states now how many shock waves will be there? Altogether you know it; 6 shock waves are going to be created.

One is between traffic state A and traffic state B, another between traffic state B and traffic state C, another traffic state A and traffic state C, another shockwave traffic state A and traffic state D, shock wave 5 traffic state D and traffic state C or traffic state C and traffic state D, and shock wave 4 was traffic state B and traffic state D. And shock wave 6 will be actually traffic state A and traffic state D. So, all these 6 shock waves will be created.

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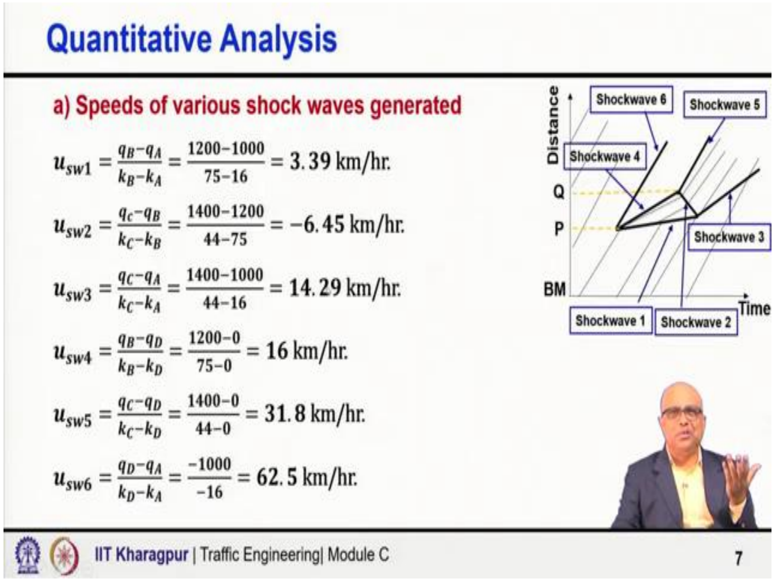

So now, based on our understanding how we can calculate the speed of the shockwave? It is the difference in flow by difference in density, so that slope of this line connecting two traffic state. So, we can calculate the speed of the shock wave. Say for example qB is1200, qA is 1000 and kB is 75, kA is 16. So, the first shock wave which will be created by this traffic state A and B, that shockwave 1 here it is shown as u_{sw1}, speed of the shock wave 1 you can calculate.

Similarly, you can calculate the speed of shock wave 2, 3, 4, 5 and 6. Some of the values are positive showing that they are forward moving shock waves. Some are negative. Say for example second shock wave usw is negative in this case. So, this is the second shockwave which is created between traffic state B and traffic state C. This is the recovery shock wave. So, it is negative, backward moving recovery shock wave. So, like that you can calculate all the speeds. I am not going to explain you the calculation of each and every shockwave. But all are given here. One I have explained.

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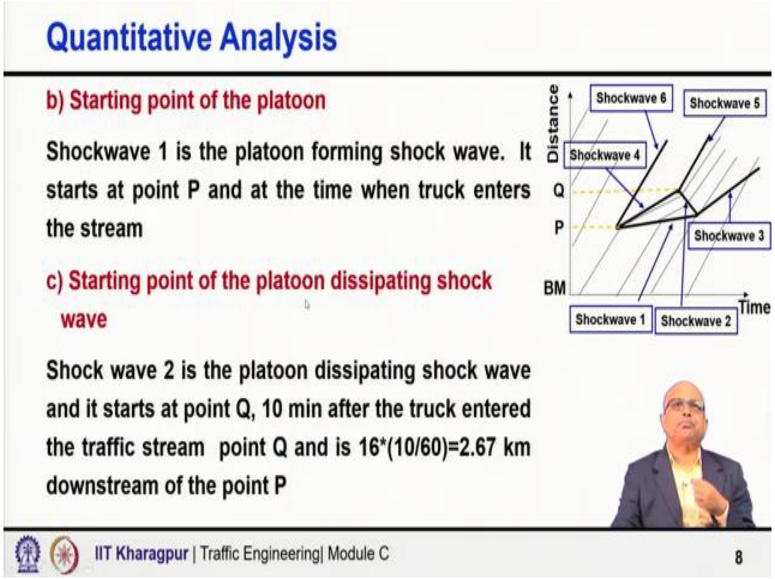

Then the starting point of the platoon, obviously the shock wave 1 is the platoon forming shock wave. As soon as the vehicle enters at point P the shock wave is generated here. It started here. Shock wave 1 is the platoon forming shock wave. And it starts at point P and at the time when the truck enter the stream. This is very easy.

Second what we want to calculate is the starting point of the platoon dissipating shockwave that means shockwave 2. When the shock wave 2 will start? Shock wave is the platoon dissipating shock wave and it starts at point Q when the truck leaves the traffic stream, slow moving truck, then the recovery shock wave or the platoon dissipating shock wave will start. And it starts obviously at point Q and after 10 minutes the truck entered the traffic stream at point P.

Now point Q is how much kilometer downstream of point P? It is 16 kilometer per hour. That is what is given. You can see here. The truck enters at a speed of u_B 16 kilometer per hour and leaves after 10 minutes. So, what will be the distance between this point P and point Q? 16 kilometer per hour, 10 minutes, so 10 by 60 hour, so 2.67 kilometer downstream of P. So, the entry point is P. Exit point is Q. And the distance between point P and point Q would be 2.67 kilometer.

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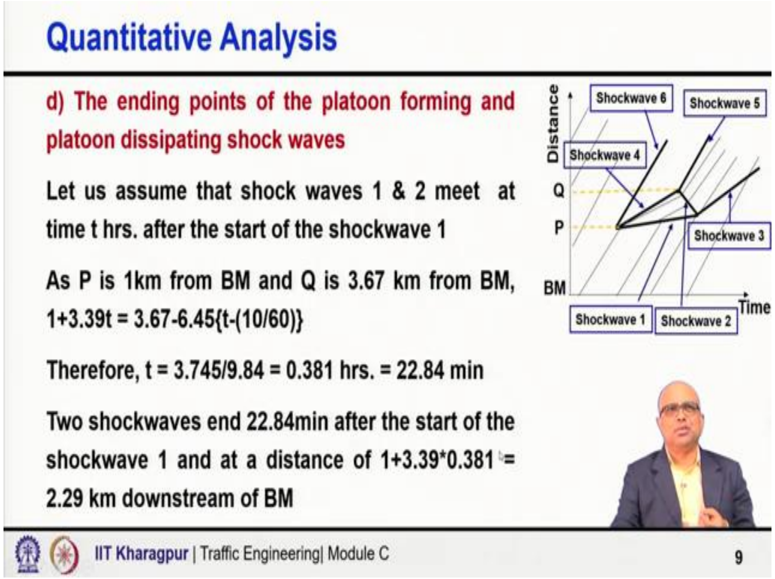

Now next what we want to calculate is the ending point of platoon forming and platoon dissipating shockwave. When this shock wave 1 and shockwave 2 will meet? That is the point. That we want to find out. Let us assume that the shock wave 1 and 2 meet at time t hours, this is the time axis, at time t hours after the start of shock wave 1. When the shockwave is getting started then shockwave 1, after the start of shock wave 1. So, when the shock wave 1 is getting started at point P, at that time. After that, after t hours this shockwave 1 and shockwave 2 will meet.

So, what will be then, as P is 1 kilometer from the benchmark point, we know it, it is given. You can check that once again, the problem. A truck enters at the stream at a point P which is at a distance of 1 kilometer from an upstream benchmark. The truck is entering at point P which is 1 kilometer from the benchmark and the Q will be how much? Q is 2.67 kilometer from point P.

That is what we have calculated here, 2.67 kilometer from the downstream point, 2.67 kilometer downstream of the point P. So, the distance between P and Q is 2.67 kilometer. So, what will then be the distance from this benchmark, point Q? It will be 2.67 kilometer plus 1 kilometer. The point P is 1 kilometer from the benchmark. So 3.67.

So, what we can do, 1 plus 3.39 t, what is 3.39? 3.39 is the speed of this shock wave. So at time t how much distance it will cover? 3.39 into t, that is equal to 3.67, the whole is 3.67 up to Q. 3.67 minus 6.45 is the speed of the shock wave 2 which is created between traffic state B and C. So, 6.45 into t minus 10 divided by 60. Why? It is after 10 minutes. The truck leaves after 10 minutes.

That is what is again given here. After 10 minutes the truck leaves the stream. So, the time wise the difference is after 10 minutes.

So, what we can get? At point P 1 kilometer from the benchmark, Q is actually 3.67 kilometer from the benchmark, so therefore as these two shock waves are meeting, 1 plus 3.39 t for the shock wave 2 and equal to 3.67 minus 6.45 into t minus 10 by 60. So, if you solve it you will get that t equal to 22.84 minutes. That means two shock waves end 22.84 minute after the start of shock wave 1 because everything, t is the time after the start of shock wave 1, so that way we got. So, two shock waves end at 22.84 minute after the start of shockwave 1.

And what will be the distance? The distance you can calculate here. 1 plus 3.39 t, put the value of t, so 2.29 kilometer downstream of the benchmark because all these distances are taken from the benchmark, with reference to the benchmark, 1 plus this and here 3.67 minus something, so all distances are taken from the benchmark reference. So, these two shock waves, the forming and recovery shock wave will meet, shockwave 1 and shockwave 2 will meet at a distance 2.29 kilometer downstream of the benchmark. That is what is the location.

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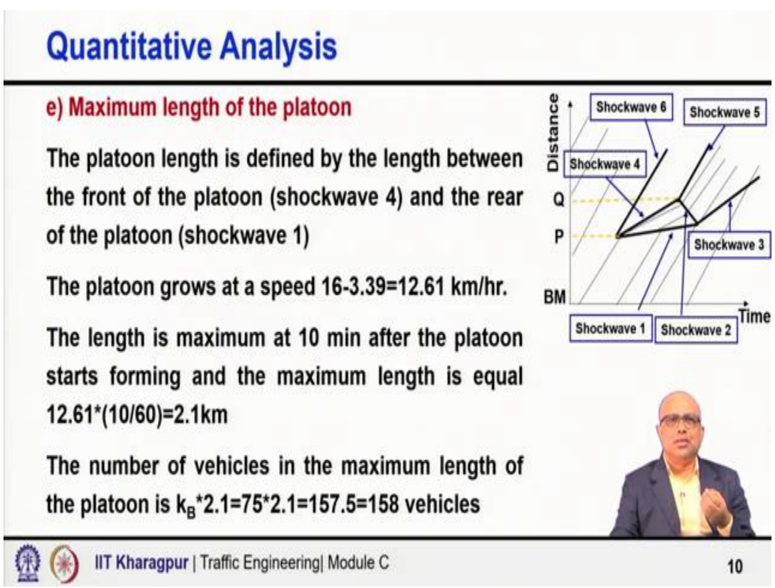

Now next is calculate the maximum length of the platoon. The platoon length is defined by the length between the front of the platoon that is shockwave 4 and the rear of the platoon that is shockwave 1. So, that is the length. Front is going with the platoon, shockwave 4. And the rear is going moving with shock wave 1. So, the length is actually the difference between the front of the platoon, that is shockwave 4 and the rear of the platoon that is shockwave 1. And the platoon grows.

How the platoon is growing? 16 minus 3.39 so 12.61 kilometer per hour. Why? Because truck is moving at 16 kilometer per hour, uB, so that is the front part. And what is the rear part? Rear part is the speed of the shock wave 1. What was the speed of the shock wave 1? It was 3.39 kilometer as you can see from the slide. So, the platoon grows at a speed of 16 minus 3.39 so that is 12.61 kilometer per hour. So, the length is maximum when the vehicle is leaving the highway at point P.

So, this is how much time? 10 minutes, because the truck was there for 10 minutes. So, the length is maximum at 10 minutes after the platoon starts forming. And then what will be the maximum length? 12.61 kilometer per hour multiplied by 10 minutes, so 10 by 60 hour, so you get 2.1 kilometer.

And how many vehicles will be there? Now you know the density of the traffic state. What is the density of the traffic state? B 75, it was also given, 75 vehicle per hour. So with that density 75 into 2.1 so there will be about 158 vehicles. That will be the maximum length of the platoon and that many vehicles will be there, so 2.1 kilometer length including about 158 vehicles.

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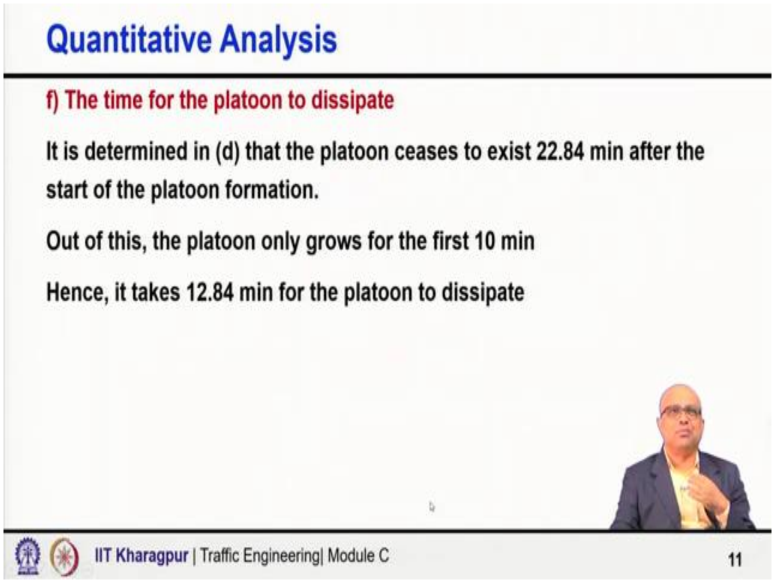

Then the last part, the time for the platoon to dissipate. It is determined in d that the platoon cease to exist 22.84 minutes after the start of the platoon formation. We have got this one already here, that 22.84 minutes after the start of shock wave 1. So, 22.84 minutes after the start of, and out of these the platoon is growing for the first 10 minutes, because as long as the slow moving vehicle is there in the traffic stream the platoon is growing. So, the growth is for the first 10 minutes. So, how much time it is actually taking to dissipate? The remaining time, 22.84 minus 10 minutes. So, 12.84 minutes for the platoon to dissipate.

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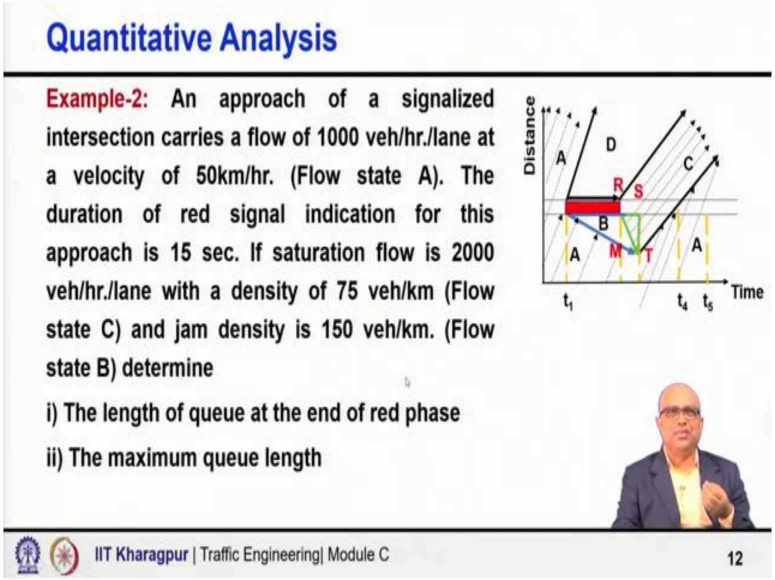

Now let us take another example, a small one but slightly different one. An approach of a signalized intersection, we have discussed a number of times about this signalized intersection, signal is turning from green to red and then how the traffic state is changing and again from red to green how the recovery shock wave is getting generated and the whole queue is getting eliminated eventually or the new traffic state will vanish in this case the traffic state B.

So, an approach of a signalized intersection carries a flow of 1000 vehicle per hour per lane at a velocity of 50 kilometer per hour. That is traffic state A. The duration of red signal indication for this approach is 15 seconds. So, this time is actually 15 seconds in x axis. If saturation flow is 2000 vehicle per hour that means when traffic state C is created the vehicle get discharged when the signal becomes green the vehicle get discharged with a saturation flow rate of or the capacity flow rate of 2000 vehicles per hour per lane with a density of 75 vehicle per hour. That is again for the traffic state C and a jam density of 150 vehicle per kilometer.

So, for the traffic state C we know the flow that is 2000 vehicle per hour. We know density as 75 vehicle per kilometer. So, we can easily calculate the speed from that. Flow is known, density is known, speed can be calculated for traffic state C. And the jam density is 150 vehicle per kilometer. That is for the traffic state B because vehicles are queued up in front of a signal which is red.

Now with all this data what we want to do? Simple two calculations, one is the length of the queue at the end of red phase, what will be the length? And what will be the maximum queue length? These are the two things we are calculating. Of course we can calculate many other things as well but only these two things we will calculate in this example problem.

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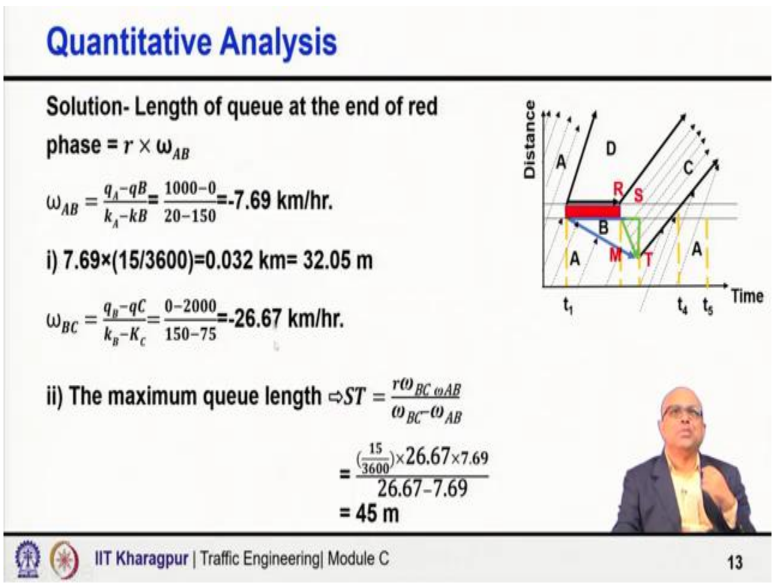

So, the length of the queue at the end of the red phase, remember our discussion in lecture 2, what will be that, r into omega AB. $\boldsymbol{\omega_{AB}}$ is the speed of the shockwave which will be formed at the meeting point of the traffic state A and traffic state B. So, what will be the speed of the shockwave AB? $\boldsymbol{\omega AB = \frac{qA - qB}{kA - kB}}$. So, qA you know 1000 vehicle per hour, that is what was the thing, 1000 vehicle per hour and with the velocity of 50 kilometer per hour. So, 50 kilometer per hour, so 1000 divided by 50, so kA is equal to 20.

So, we have put the value of $\boldsymbol{\frac{qA - qB}{kA - kB}}$, qB is 0 because it is the jam density state, so no movement, no vehicle is moving and the signal is red. So, this flow is 0. And kB is actually what? It is the jam density. So, jam density value is also given as 150 vehicle per kilometer. So, put that value.

We can calculate then what will be the speed of the shock wave which will get formed due to the traffic state A and B, at the meeting point of traffic state A and B. So, that speed is minus 7.69

kilometer per hour, minus because it is moving backward opposite to the direction of the traffic flow and it will be a forming shockwave. It is forming that congested condition. So, then what would be the length of the queue?

Length of the queue will be r into $\boldsymbol{\omega AB}$. $\boldsymbol{\omega AB}$ is 7.69 and r is 15 seconds, divided by 3600 to convert that second into hour, so you will get 0.032 kilometer or 32.05 meter, approximately. So, 32 meter is the length. Then what will be the speed of the shock wave which will get formed due to the traffic state B and C, that is omega BC also you can calculate. qB is 0. qC is 2000 vehicle per hour, saturation flow rate. kB is the jam density, and kC is 75 that is given.

That is said, with a density of 2000 vehicle per lane and with the density of 75 vehicle. So, you can calculate. Now we know then what will be the speed of the shock wave AB and what will be the speed of the shock wave BC. Both are backward moving. One is forming, another is recovery. And then what will be the maximum queue length. I have directly put this formula. This we explained, we discussed actually in the previous lecture.

So, $\frac{r\omega_{BC}\ \omega_{AB}}{\omega_{BC}-\omega_{AB}}$. So, you can get that. We know the red time is 15 seconds. Omega BC, speed of shockwave BC is 26.67 into omega AB is 7.69 divided by what is the value of omega BC? Omega BC is 26.67 actual value minus omega AB the speed is 7.69. So, we take that and we get maximum queue length as 45 meter.

So, we discussed two example problems to show you how the quantitative analysis can be applied, and to estimate some of the, to calculate some of the key values which are of interest to traffic engineers.

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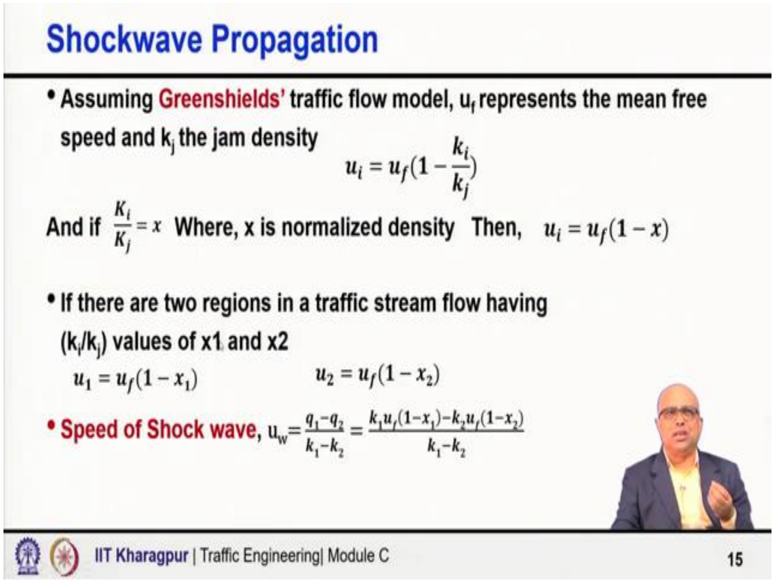

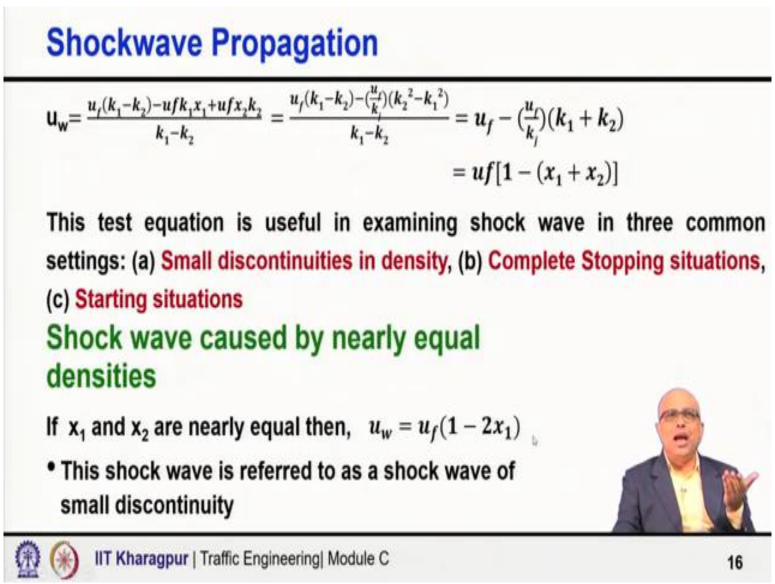

Now we switch over to another discussion. Let us discuss briefly about the shock wave propagation. Let us assume that the Greenshields traffic flow model is valid and uf is the free flow speed. kj is the jam density. Then if the Greenshields linear model is valid, linear speed density model is valid then what will be the equation? You are already familiar. You know this one. ui equal to uf into 1 minus ki by kj. So, for any traffic state where this I, where this speed is ui, density is ki then if uf the free flow speed, kj is the jam density then you can express that speed density using this linear equation.

Now if we take or if we consider that $\frac{K_i}{K_j} = \boldsymbol{x}$ which is the normalized density, instead of expressing the density ki I am expressing it is a normalized density normalized with respect to the jam density then I can write $\boldsymbol{u_1} = \boldsymbol{u_f}(\boldsymbol{1} - \boldsymbol{x1})$. If there are two regions in the traffic stream flow having ki by kj values as x1 and x2, because two different traffic states they only create the shock wave, so then I can individually write express these two traffic streams as $\boldsymbol{u_1} = \boldsymbol{u_f}(\boldsymbol{1} - \boldsymbol{x1})$, and $\boldsymbol{u_2} = \boldsymbol{u_f}(\boldsymbol{1} - \boldsymbol{x2})$. So, then these are the two streams which are meeting.

Then at the meeting point of these two stream conditions, whatever the shock wave will be created, what will then be the speed of this shock wave? It will be $\frac{\boldsymbol{q1}-\boldsymbol{q2}}{\boldsymbol{k1}-\boldsymbol{k2}}$. This is also quantitative analysis. We know it. So, now you replace q1 and q2, k1 to u1, so u1 is what, uf into 1 minus x1, minus q2 means k2 u2, k2 into u2, u2 replacing with uf into 1 minus x2. And eventually what we get here is it will be then uf into 1 minus x1 plus x2 in the bracket. This form you will get. So, that way also you can calculate the speed of the shock wave which will be formed by two different traffic states.

Now this equation or test equation is useful in examining shockwave in three common settings or scenarios. One, small discontinuity in density that means density is nearly equal. We are saying it x1 and x2 but they are almost like similar, so discontinuity there but we call it as small discontinuities in density, so where x1 and x2 nearly equal. Second, complete stopping situation, one of them is complete stopping. And then starting situation. These are the three things. Under these three settings we can examine this equation.

So first, shockwave caused by nearly equal densities, remember that we are talking about nearly equal densities. So, if x1 and x2 are nearly equal then we can simply write uw equal to uf into 1 minus 2 x1. So, not writing x1, x2 separately because x1, x2 are nearly equal. Simply we are taking x1 equal to x2 or x2 equal to x1 and we are expressing this speed in this form. Now this shockwave is referred to as shockwave of small discontinuity. Carefully observe this part, small discontinuity. That means x1 and x2 are nearly equal.

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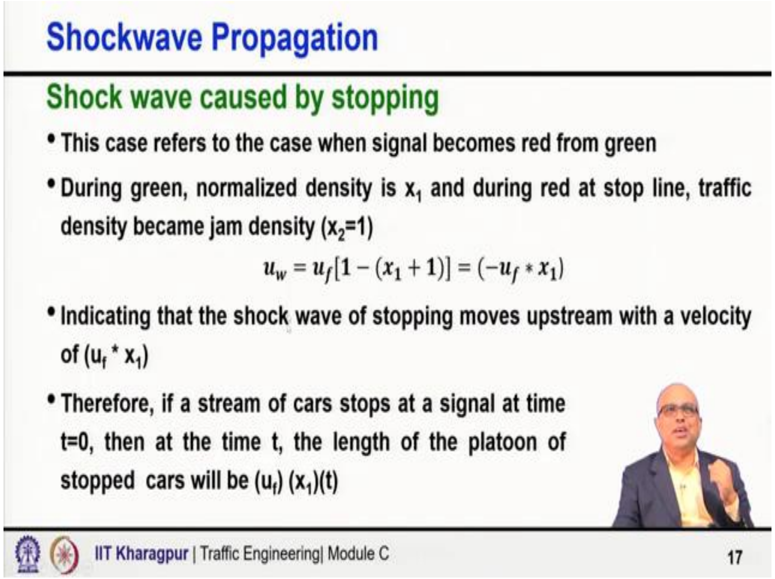

Now this case, second one, is shock wave caused by stopping. So, this case refers to the case where signal becomes red from green as I have shown earlier, number of times we have discussed. Signal was green and then turns it to red. So, the vehicle stops. So, during green normalized density is x1. That was the initial state. And during red at stop line traffic density becomes jam density. So, x2 becomes what? x2 is normalized x by xj, k by kj. So, the k becomes kj now. So, x2 equals to 1.

So then, in this case how this speed will look like? It will look like uf into 1 minus x1 plus 1. So, ultimately minus uf into x1. That is what. So, as I have said earlier examples traffic states between A and B the shock wave is created, and what is the speed. It is minus uf into x1 indicating that the shockwave of stopping moves upstream with a velocity of uf into x1.

Therefore, if a stream of car stops at signals at a time t equal to 0 and then at a time t other than, initially started with 0 so at time t what will be the length of the platoon? Simply it will be uw, so that is uf into x1 into time t, what is the time duration from the time the signal becomes red.

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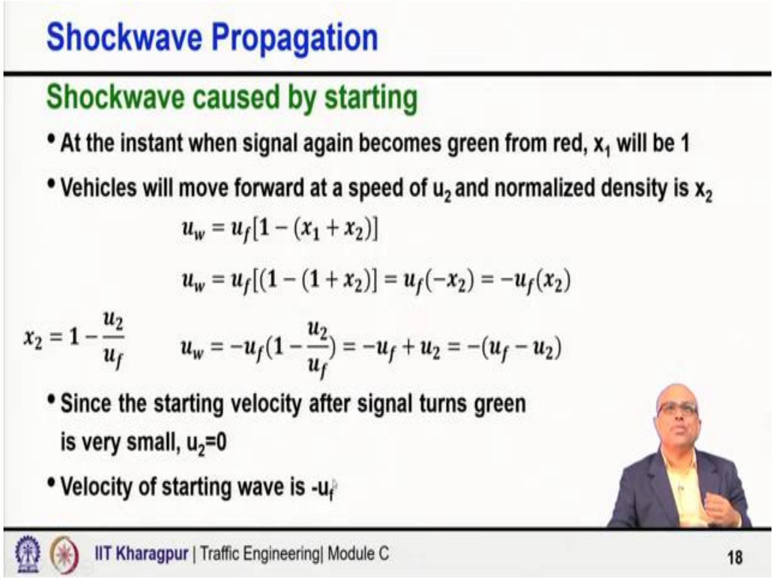

Now the other one, when the green turns, when the red turns into green. So, shock waves caused by starting. At the instant when signal again becomes green from red. In this case x1 is 1 because it was in the jam density state. So, the normalized density is 1, k by kj equal to 1, so x1 equal to 1. Now vehicles will move forward at a speed u2 and normalized density is x2. So, how it will look like? uw equal to uf into 1 minus x1 plus x2. So, you write it uf into 1 minus 1 plus x2. So, it is eventually minus uf into x2.

Now x2 is again 1 minus u2 by f. How we got it? You could write it from this original equation. u2 you can get it. Now if we do this simple substitution then we can get actually uw equal to minus uf minus u2. So, since the starting velocity after signal turns green is very small u2 may be considered as 0. So, velocity of starting wave is practically minus uf.

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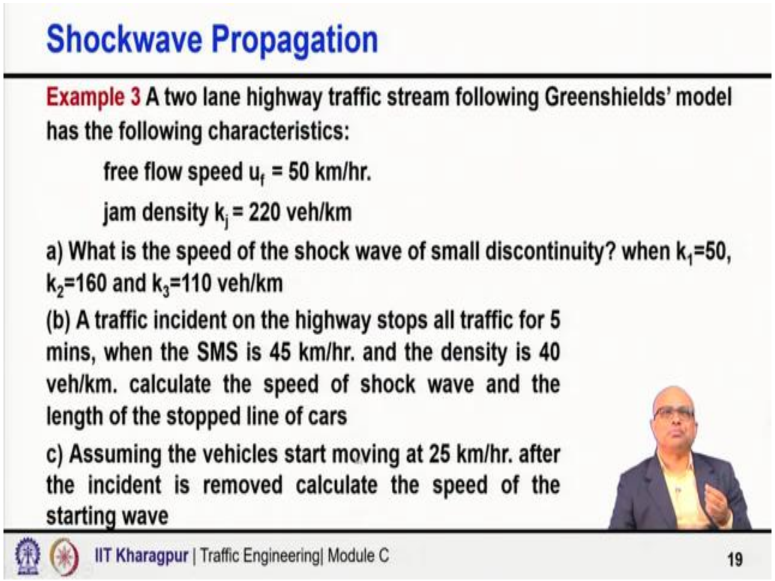

Let us taken an example. A two lane highway traffic stream following Greenshields model has the following characteristics. uf is given. kj is given. What we want you to calculate three components, first is what is the speed of the shock wave of small discontinuity. We are talking about small discontinuity when k1 equal to 50, k2 equal to 160, k3 equal to 110. These are different states, traffic states and in each case we want you to calculate the speed of shockwave of small discontinuity.

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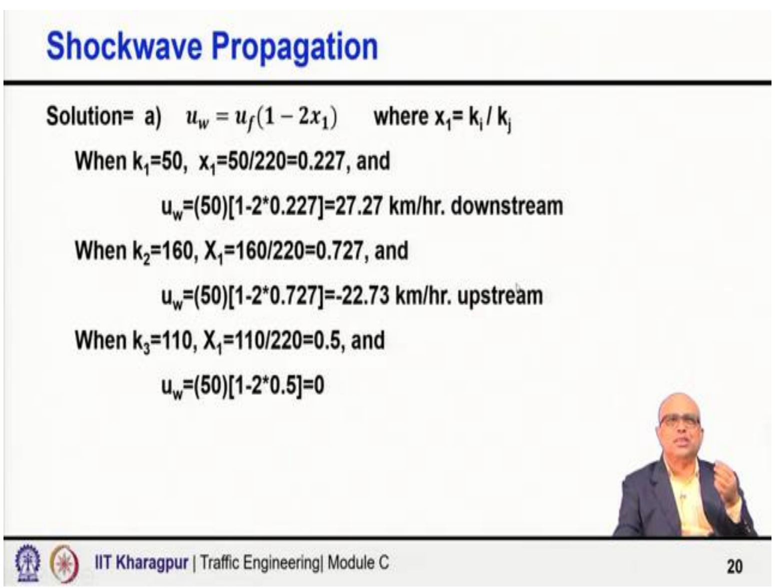

Let us take this part first. So, we know that uw is equal to uf into 1 minus 2 x1. In that case x1 and x2 is same. So, put the value of k1. And so the normalized value, what will be the value of x1 in this case? 50 by 220; in this case 160 by 220; in the third case 110 by 220. And you can straight away calculate then what will be the corresponding value of the speed of the shockwave. So, that you can get.

Part 2, a traffic incident on a highway, carefully observe, a traffic incident on the highway stops all traffic for 5 minutes. For 5 minutes all traffic are dropped means speed will be 0, density will be jam density. Then when the SMS is 45 kilometer per hour and the density is 40 vehicle per hour, that means earlier before stopping it was the case. Calculate the speed of the shock wave and the length of the stopped line of cars.

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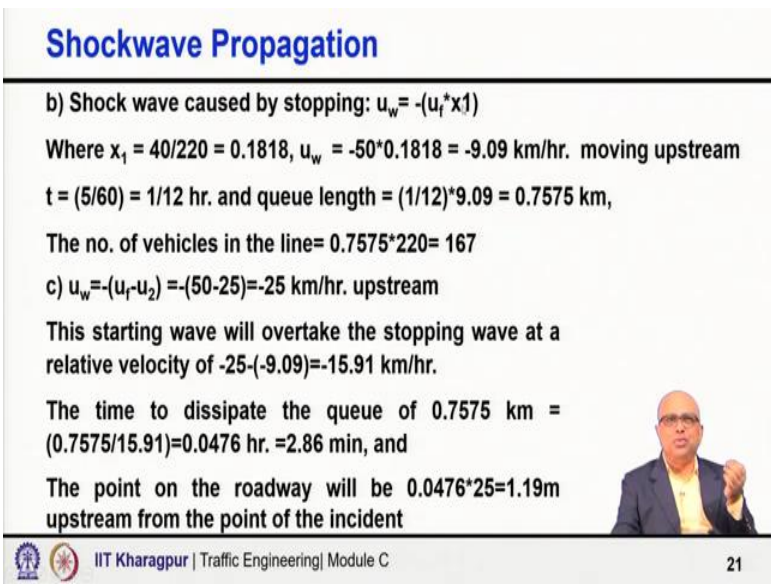

How we can calculate easily? Shockwave, it is caused by stopping so uw is equal to minus uf into 1. Here x1, x1 will be 40 because that was the state 40 vehicle per kilometer density, so 40 by 220 you can calculate. So, you can accordingly calculate the speed of this shock wave by stopping and this will move upstream. So, it stops only for 5 minutes. That is what is told. You can go and check it once again, that all the vehicle is stopping for 5 minutes.

So, this for 5 minutes, so if it is for 5 minutes, then t is equal to 5 minutes so therefore what would be the queue length? uw minus 9.09 kilometer per hour into 5 minute converted into hour, so 5 by 12, so 1 by 12. So, you can get. This is the going to be the length, queue length.

Third, assume that vehicle start moving at 25 kilometer per hour after the incident that means after 5 minute. Calculate the speed of the starting wave. This is the other case. Three cases I told you here, one is shock wave caused by nearly equal densities that is one, then we discussed also the application of shock wave caused by stopping, and now finally shock wave caused by starting.

So, here in this case uw equal to this uf minus u2, this starting wave will overtake the stopping wave at a relative velocity how much, this is 25 and other shock wave having 9.09. So, the relative velocity is 15.91. So the time to dissipate the queue of 0.7575 kilometer will be 0.7575 divided by this relative velocity, so 2.86 minutes. And the point will be how much? 0.0476 into 25, so 1.19 meter upstream of the point of the incident. So, you can make these calculations, not a complicated one.

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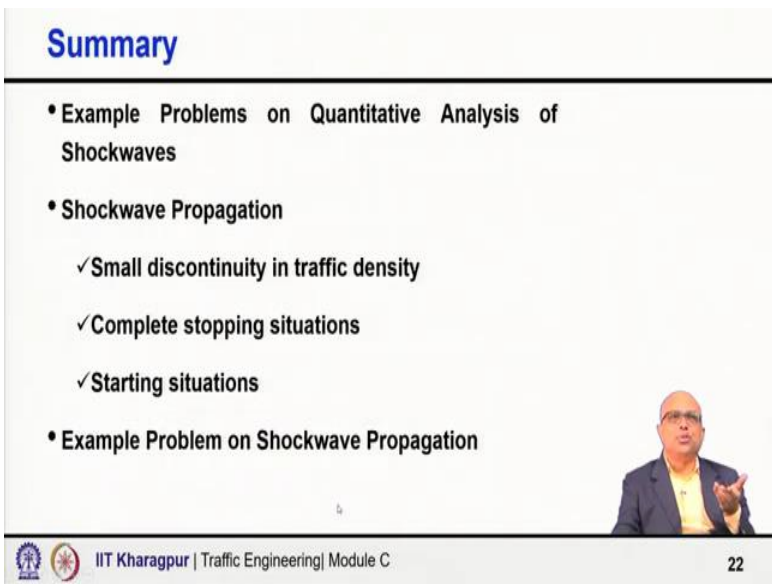

So, altogether what we discussed? We explained to you two example problems for carrying out quantitative analysis, discussed also about the shock wave propagation, three cases, three scenarios; small discontinuity in the traffic density, complete stopping situation and starting situation. And then took a small example to explain you the shock wave propagation, how these formulations, simple formulations can be used to solve real life problems. So, with this I close this lecture. Thank you so much.