Traffic Engineering
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Shock Wave and Queueing
Lecture – 12
Analysis of Shock Waves - II

Welcome to module C lecture 2. In this lecture we shall continue our discussion about analysis of shock waves.

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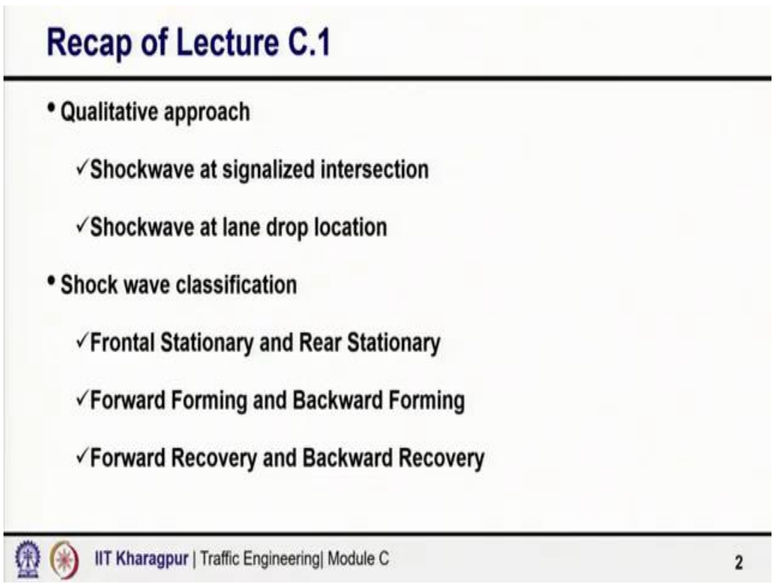

What we discussed in our previous lecture is the qualitative approach for analysis of shock waves and we took two examples, shock waves at signalized intersections and also so how the shock waves are generated due to lane drop and at different flow levels. Mentioned to you, I also mentioned to you about the classification of shock waves, stationary and moving; stationary again frontal, rear; frontal stationary, rear stationary.

And moving shock waves maybe again forward, backward, forming or recovery; so forward forming, backward forming, forward recovery, backward recovery. With this background today we shall focus on quantitative analysis of shock waves.

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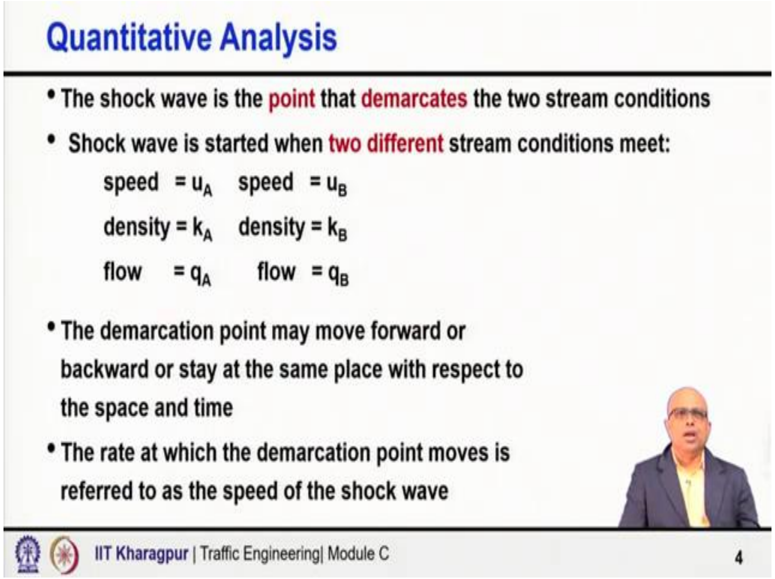

As you know the shock wave is the point that demarcates two stream conditions. Along the length of the road when we travel the stream conditions are not same. They vary. So, wherever the stream condition is changing a boundary is created. And this boundary which demarcates two stream conditions is the shock wave. Shock wave is started when to stream conditions, they meet. So, let us consider two stream conditions A and B with speed u, density k and flow queue. So, for A and B it would qA, uA kA and kB, qB uB.

Now the demarcation point when the two stream conditions are meeting, the demarcation points may move forward, forward with means in the line of the movement or along the movement of the vehicles in the direction of the movement of the vehicles, or move backward in the direction opposite to the direction of the movement of vehicles, or stay at the same point with respect to space and time. The rate at which the demarcation point moves is referred to as the speed of shock wave, how that point is moving. The rate at which the point moves is referred to as the speed of shock waves.

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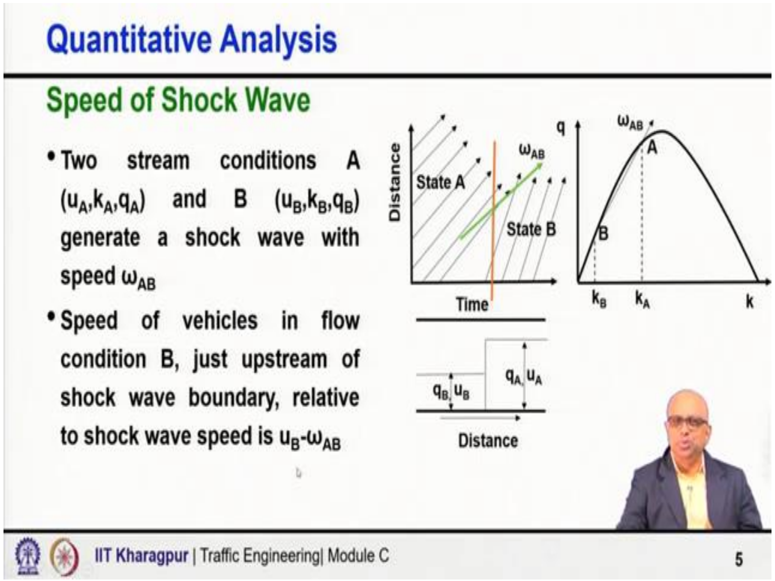

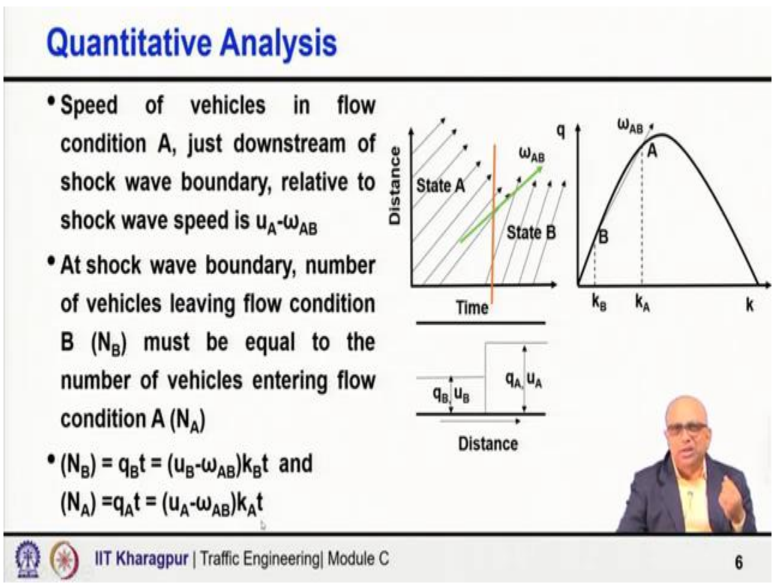

Now if we consider two stream conditions A and B as I have mentioned and they generate a shock wave with the speed of omega AB as shown here, this is stream condition A, this is Stream condition B in the time distance diagram and they are meeting. So, a shock wave is created. And the speed of this shock wave is $\boldsymbol{\omega_{AB}}$.

Now if you consider speed of vehicle in flow condition B relative to the speed of shock wave, relative to the speed of shock wave what is that speed? That is $\mathbf{u_B}$**-**$\boldsymbol{\omega_{AB}}$. uB is the speed of vehicles travelling in traffic state B and omega AB is the speed of shock wave. So, what is the speed of vehicles in flow condition B relative to shock wave speed? That is $\mathbf{u_B}$**-**$\boldsymbol{\omega_{AB}}$

Similarly, what is the speed of vehicles in flow condition A relative to the shock wave speed? In traffic state A the vehicles are moving with a speed uA. And the speed of the shock wave is $\boldsymbol{\omega_{AB}}$. So, what is then the relative speed of vehicles in flow condition A relative to shock wave speed? That is $\mathbf{u_A-\omega_{AB}}$.

Now at shock wave boundary if we consider two stream conditions and this is the shock wave boundary, the number of vehicles leaving flow condition B must be equal to the number of vehicles in entering flow condition A. These two numbers must be equal because shock wave is a boundary. So, the number of vehicles leaving flow condition B, let us call that as N_B number of vehicles and the number of vehicles entering the flow condition A on the other side, let us call that number as N_A. So, this N_B and N_A cannot be different. They must be same.

So, how we can calculate NB? How many number of vehicles are leaving flow condition A? It is the flow q_B multiplied by t. What is qB? How we can know flow rate? It is queue equal to u into k. But here relative to shock wave what is the speed? $\mathbf{u_B-\omega_{AB}}$. So, it is u be minus omega AB within bracket multiplied by density ($\mathbf{(u_B-\omega_{AB})k_B}$). That give me the flow into time t. You can call it t, you can call it delta t to indicate small time interval. We are talking about a small time interval.

And similarly, what is N_A? N_A is equal to q_A into t. And what is q_A? q_A-$\boldsymbol{\omega_{AB}}$, the whole in the bracket multiplied by k that is speed into density equal to flow multiplied by time t or delta t, whatever you take.

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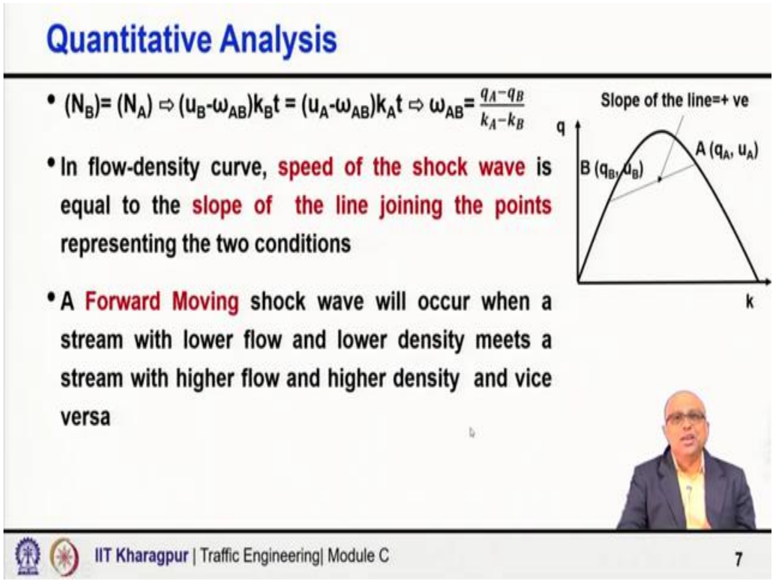

Now this N_A and N_B must be equal. So, you make them equal. And if you do that eventually you then find a very interesting thing that $\boldsymbol{\omega_{AB}} = \frac{q_A - q_B}{k_A - k_B}$. What is it? What does it mean? What is the physical interpretation? We are actually connecting two points on the flow density curve, connecting two points A and B and the slope of that line is actually qA minus qB divided by kA minus kB.

So, slope of the line joining two points on the flow density curve, that slope is actually the speed of the shock wave which will be generated at the meeting point of those two flow conditions A and B. So, that is what I have said in flow density curve. Speed of shock wave is then equal to the slope of the lining joining the points representing two conditions in the flow density curve.

Now given this background when then a forward moving shock wave will be created? Forward moving means with a positive slope. Obviously the line, the slope of this line will be positive only when a stream with lower flow and lower density, lower flow and lower density meets a stream with higher flow, higher density. Then only the slope will be positive. That is what you are saying here.

So, always a forward moving shock wave will be created when two stream conditions are meeting, one with low flow, low density and the other with high flow, high density. That is the context where a forward moving shock wave will be created.

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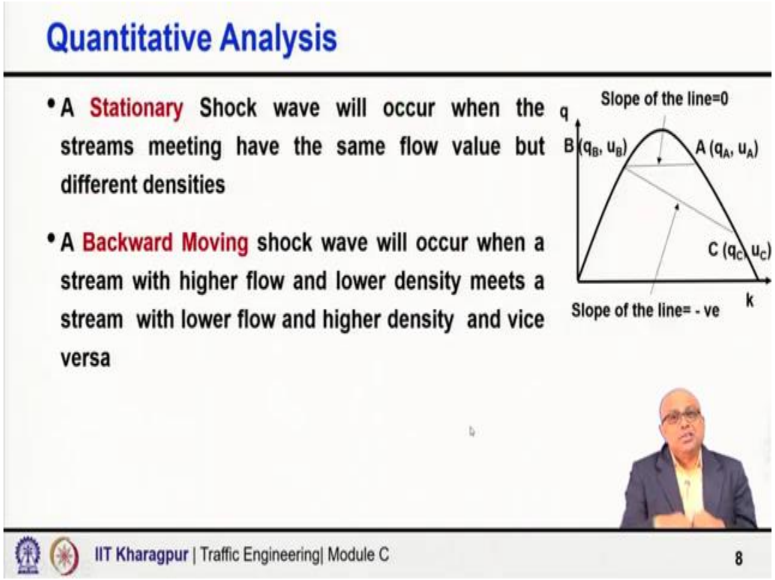

Similarly, when a stationary shock wave will be created? Slope 0 horizontal that means same flow but two different densities. So, a stationary shock wave will occur when the streams meeting have the same flow but different densities. You can see this line. Same flow, then only the slope will be 0 but two different densities. k values are different.

Now what is remaining is the backward moving. You can understand. When the slope will be negative? When the stream with higher flow, flow wise higher, lower density, flow wise higher, density wise lower meets a stream with lower flow and higher density. So, one side the traffic stream is having higher flow lower density. The other side lower flow higher density.

So now, we know based on the analysis, quantitative analysis and based on the understanding on the speed of shock wave we can tell now when the forward moving shock waves are likely to be created, when the backward moving shock waves likely to be created and when the stationary shock wave is likely to be created.

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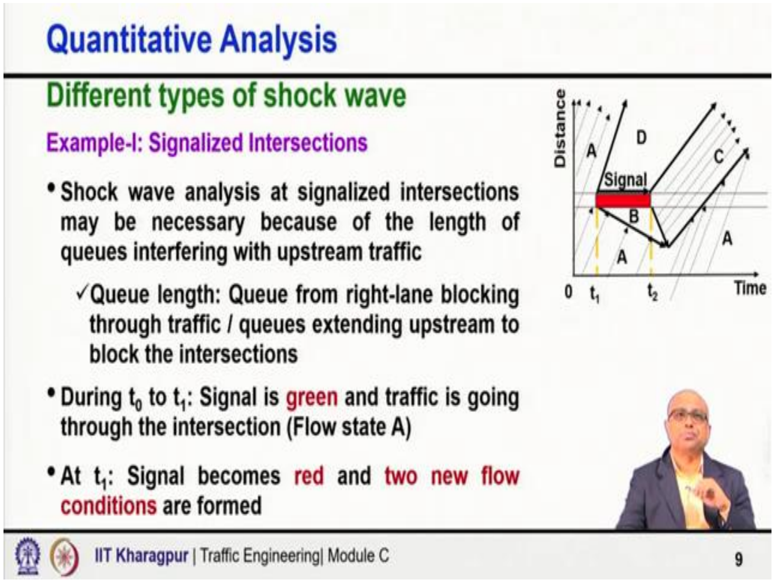

Now let us try to analyze different types of shock wave based on this quantitative analysis taking reference first with at signalized intersection. The context is known. The vehicles are arriving during the green time, passing the stop line or the signal without stopping, without slowing down. So, flow condition exists on both upstream and downstream of the signal or the stop line. Then the signal becomes red. And therefore, the new traffic state B is created.

Once the, also the traffic state B is created downstream of the signal during the red time because traffic stream condition in A and D, A and D these two steam conditions are different. Then once the signal turns red to green, again the vehicle get discharged with a traffic state, flow state C and then multiple shock waves will be created because of this whole phenomena.

Why we are interested in shock wave analysis at signalized intersection? For various reasons. Shock wave analysis at signalized intersection may be necessary because of the length of the queue interfering the upstream traffic. This is one possible reason. There are many other purposes the analysis may serve.

Say the queue length is really important because queue from the right lane might be blocking the through traffic. The right lane storage lane is exhausted and the spillover of vehicle is there which are actually right turning vehicle and that is altogether moving the through movement. Through vehicles are not able to pass because right turning vehicles are waiting in front, in front of those vehicles.

Or queue is extending upstream to block the intersection, that might be happening. So, you must do analysis what is really happening, when this traffic state B, how much it is forming, to what extent the formation is happening, when the traffic state is really getting dissipated, whether it is getting dissipated within a reasonable time, what is the maximum queue length, how much time it is taking to dissipate the queue, all such kind of questions can be answered.

So, as I said during time t0 to t1 you can say it is 0, or t0 you can call, to t1 signal is green, and traffic is going through the intersection with traffic state A upstream downstream of the signal or stop line when the flow condition is same, so no question of any shock wave. Now at time t1 signal becomes red. This is the point when the signal becomes red and therefore two new flow conditions are formed.

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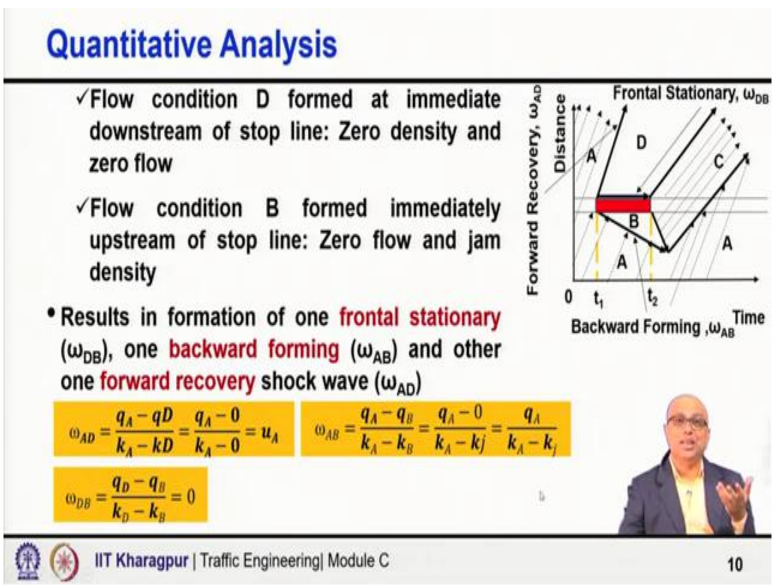

Which are those two flow conditions? One is at the downstream of the signal, and one is at the upstream of the signal. Now the flow condition is D is formed immediate downstream of the stop line. There is no vehicle there if you go further ahead then the traffic state is A. So a new traffic state D has been formed with zero density and zero flow. No vehicle is passing through the intersection during this red time.

And another flow condition is getting created which is B formed immediately upstream of the stop line. And here again the flow is zero. But density is the jam density because vehicles are coming and joining the queue, no movement is happening. So, the density is the jam density and the flow is 0. And altogether this phenomena that the traffic is turning into red, traffic light is turning into red from green, it results in formation of one frontal stationary shock wave.

What is the frontal stationary? Between D and B? And one backward forming shock wave between traffic state B and A, backward forming because it is moving towards upstream as more, more vehicles are joining the queue. And other one forward recovery shock wave w AD, forward recovery shock wave between traffic state A and traffic state D. So, that is the thing.

So, now we know how to calculate the speed of the shock wave. So, let us try to calculate the speed of these shock waves. Frontal stationary obviously will be 0. And you can check that $\omega DB = \frac{qD - qB}{kD - kB}$. So, here the qD is also 0. qB is also 0. kD is 0. But kB is not 0. So, altogether you get 0.

Similarly, you can calculate what will be then the shock wave of, the speed of the shock wave created by traffic state A and D. It is qA minus qD, sorry this D is not written properly here and divided by kA minus kD. So, qA minus qD is what? 0. Flow in the traffic state D is 0. And density in the traffic state D is also 0. So, it is actually qA by kA which is nothing but uA, speed of vehicles travelling in traffic state A.

And what will be the omega AB, the speed of shock wave AB which is the backward forming shock wave? It is qA minus qB divided by kA minus kB but qB equal to 0, all other are non zero, so it is qA divided by kA minus kj. So, you can calculate. If we know the speed flow density of different traffic state we can easily calculate then what will be the speed.

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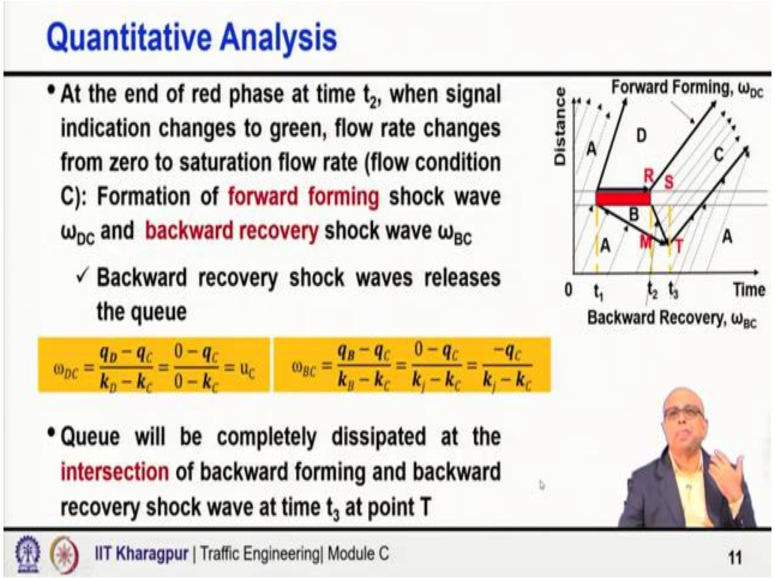

Now at the end of red phase at time t2 when signal indication changes to green flow rate changes from 0 to saturation flow rate and a new flow condition C is created. And there again one shock wave is created which is forward forming shock wave between D and C traffic states and a backward recovery shock wave between traffic state B and traffic state C. It is recovery because the shock wave created or the queue length what has been created due to the traffic state B and the shock wave between A and B traffic states, that will get dissipated by this recovery shock wave which will be created now due to the traffic states B and C.

So, again we can calculate what will be the speed of the shock wave created by traffic state D and C. That is $\omega DC = \frac{qD - qC}{kD - kC}$. Now qD is zero, kD is zero. So, it is qC by kC. It is nothing but uC, speed of the traffic state C. Similarly, you can calculate what will be the omega BC that is the speed of the shock wave created due to traffic state B and traffic state C. And you will get it. It is equal to minus qC divided by kj minus kC. And it is minus means moving backward. So, it is backward shock wave.

Now queue will be completely dissipate at the intersection of backward forming and forward recovery shock wave. Let us call it at time t3 where these two shock waves will meet. Both are backward. One is forming between A and B. Another is recovery shock wave between B and C. They will meet and the traffic state D will vanish. Let us consider that they will meet at time t3.

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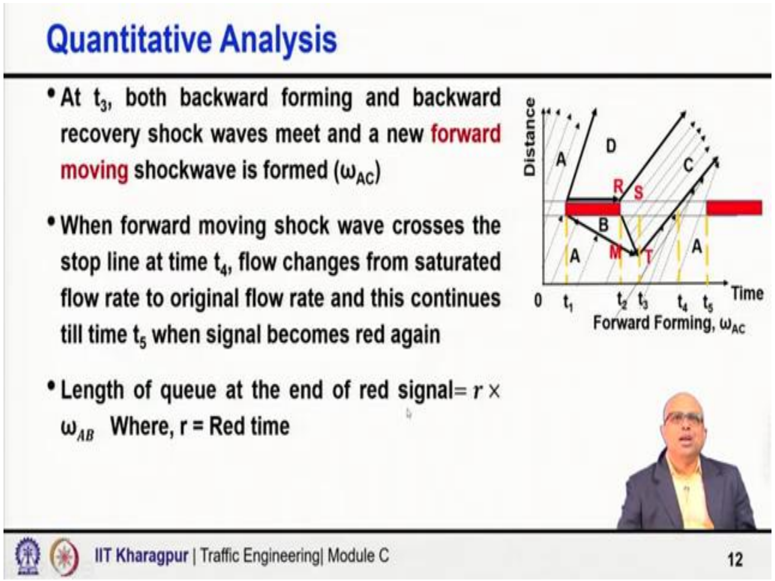

Now at t3 both backward forming and backward recovery shock waves will meet and a new forward moving shock wave will be formed between traffic state A and traffic state C, because when B vanishes, in between layer is gone. So, A and C directly are meeting. So, a new shock wave will be created which is forward moving shock wave **ω_{AC}**.

When forward moving shock wave crosses the stop line at t4, you can see that at time t4 during this green, the forward moving shock wave is crossing the stop line, then the flow changes from, if I am observing how the flow is happening from the stop line we will observe then the flow beyond t4 time is changing to a flow rate which was original, like the traffic state A. And this will continue till time t5 when the green time is over and again the signal becomes red.

So, what will then be the length of the queue at the end of red signal? What will be the length of the queue at the end of the red signal? It will be the speed of the shock wave AB multiplied by the time at speed, how much will be then the length, multiplied by the time. The time is here what? It is the red time. So, red time multiplied by the speed of the shock wave AB.

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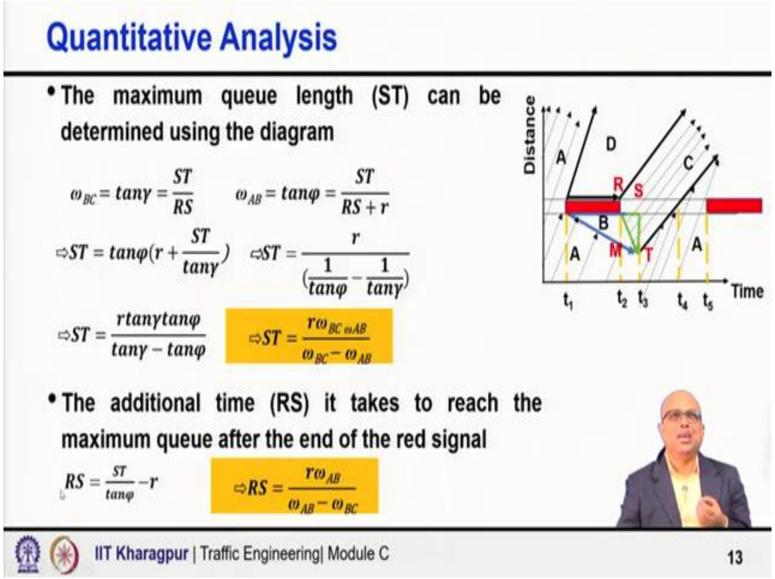

Now the maximum length queue length ST, that is the maximum queue length, can be determined using this diagram as I have shown here. $\boldsymbol{\omega_{BC} = tan\gamma = \frac{ST}{RS}}$ that is the $\boldsymbol{tan\gamma}$ that is the speed of the shock wave which will be created due to traffic state B and C.

Similarly, let us consider $\boldsymbol{\omega_{AB} = tan\varphi}$ equal to, what will be that? This will be this ST divided by this whole base. That means RS plus this is red portion that is nothing but the red time. So, from these two equations we can move forward. Then ST equal to, you can calculate like this, tan psi into r plus ST divided by tan gamma using this RS from this equation.

So, like that ST you can eventually calculate and you will see it is getting a form like this. So, I again this omega will be capital like omega BC. It has come all in the same line. So, you can calculate then the ST values. And also you can calculate the additional time RS, that means once this signal becomes green then how much time after that it will take to reach to the maximum queue after the end of the red signal.

So, that RS also can be calculated using this equation and knowing all the other values. So, you can calculate this RS also. Simple calculation you can follow. I am not telling each and every step. You can easily follow it. So, what we are trying to see, that from this diagram we can actually calculate the ST and RS. So, how much time, what will be the maximum queue length and what how much time it will take once the signal becomes green to clear that queue.

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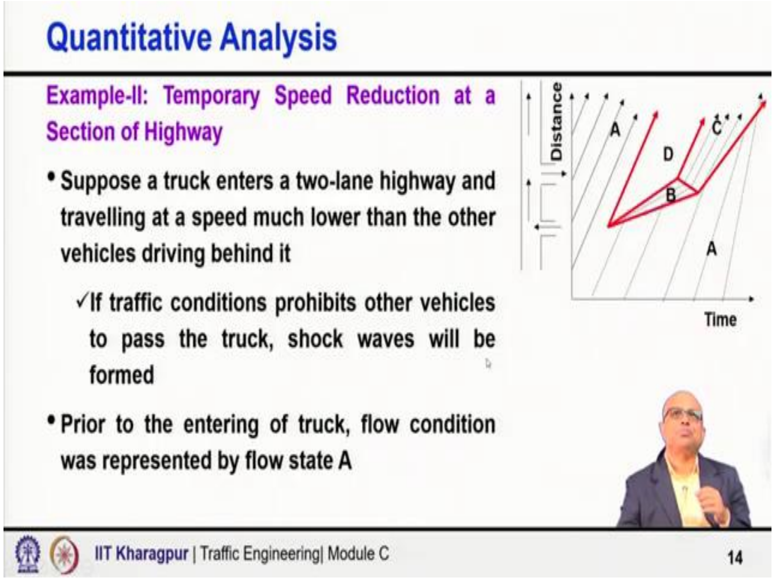

This is another example which is referring to the temporary speed reduction at a section of highway. So, suppose the traffic are moving like this. And suppose a truck enters at two lane highway and travelling at speed much lower than the other vehicles driving behind it. The truck is entering here with a lower speed.

Now if traffic condition prohibits other vehicles to pass the truck, then the shock waves will be formed. If vehicles are not able to, they will be forced to travel with the speed, same speed of the slow moving vehicle, following that slow moving vehicle. Now prior to entering the truck suppose the flow condition was represented by flow state A. Vehicles are moving like flow state A. Here you can say A. Here also it is A because till that time truck has not entered. Truck has entered at this point, at this location and at this time.

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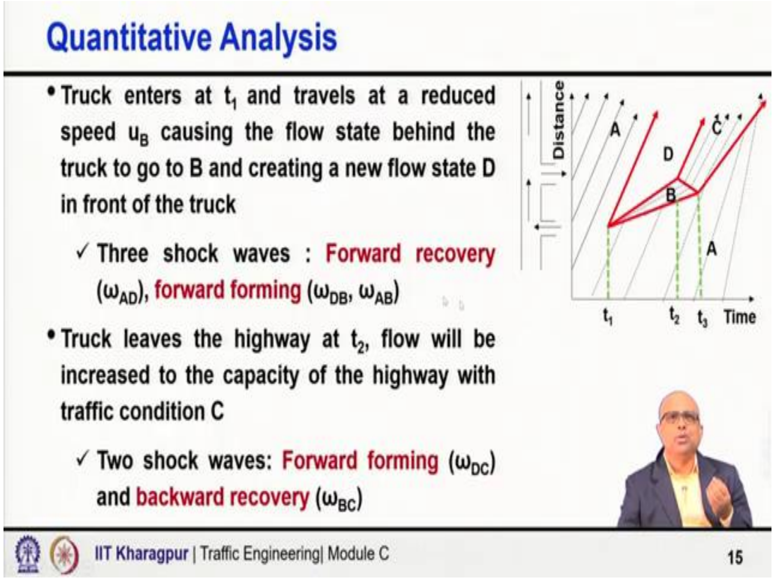

Now once the truck enters at time t1 and travels at a reduced speed uB causing the flow state behind the truck to go to B. So, now again a new traffic state will be created because of the queued vehicles. Queue is not completely stopping. The vehicles are moving with the speed of the truck. But it is a different flow condition now, or state that will be and also that will create a new flow state D in front of the truck, because immediately in front of the truck traffic state is not A. A traffic state has gone further ahead.

So, in between a new traffic state will be created again with zero flow, zero density. That is traffic state D. So, now this itself, this phenomena itself up to this will create three shock waves. One is a forward recovery shock wave ω_{AD} between traffic, ω_{AD} between traffic state A and traffic state D. And one or two forward forming shock waves. One is between D and B with speed of ω_{DB}. And another between traffic state A and B with a speed of ω_{AB}.

Now suppose at time t2 this is the time t2 the truck used the highway and the flow will be increased to the capacity of the highway and a new flow condition C will be created. Now as the truck leaves; when the truck entered three shock waves were created, now the truck left the road, gone back, here, it entered, left the main road, gone back the side road or minor road, again this will create two shock waves.

One is what, a forward forming shock wave between traffic state D and C with the speed omega DC; and a backward recovery shock wave between traffic state B and traffic state C with a speed of omega BC.

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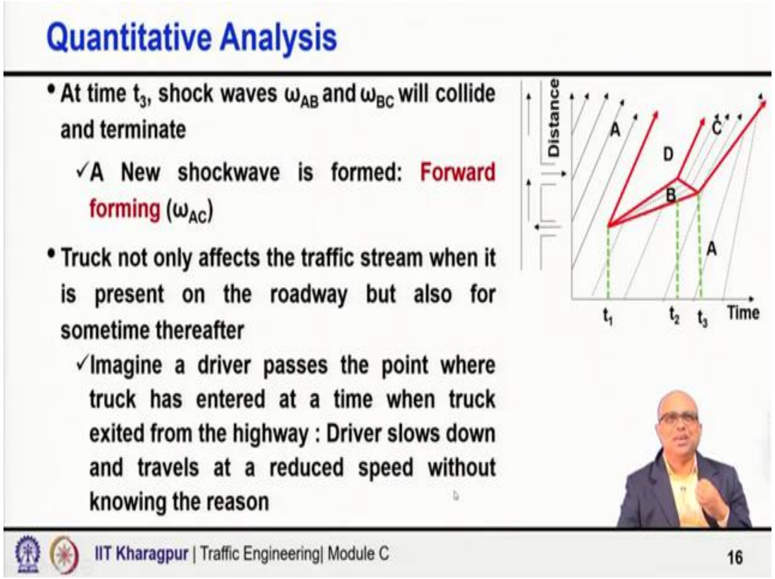

Now at time t3 these two shock waves, shock wave between A and B and the shock wave between D and C they will meet which are travelling with corresponding speed ω_{AB} and ω_{BC}. They will collide and then the whole traffic state D will vanish at time t3. But then a new shock wave will be created. What will be that shock wave?

That shock wave will be created between traffic state A and traffic state C and that will be a forward forming shock wave with a speed omega AC. Now truck not only affects the traffic stream when it is present on the roadway but also some time thereafter. The truck left at this point t2 but immediately after t2 the traffic doesn't come back to its original state. No way. The impact still remains.

Now imagine a driver passes the point where the truck has entered at a time when the truck exited the highway. The traffic state will be not the usual A as it was. Truck is not there. Truck has just exited. But then still the driver has to slow down after sometime and travel at reduced speed without knowing even what has happened. He does not know that the truck entered and left and, because he is experiencing still the impact after the truck has exited. The impact is still there in the traffic state.

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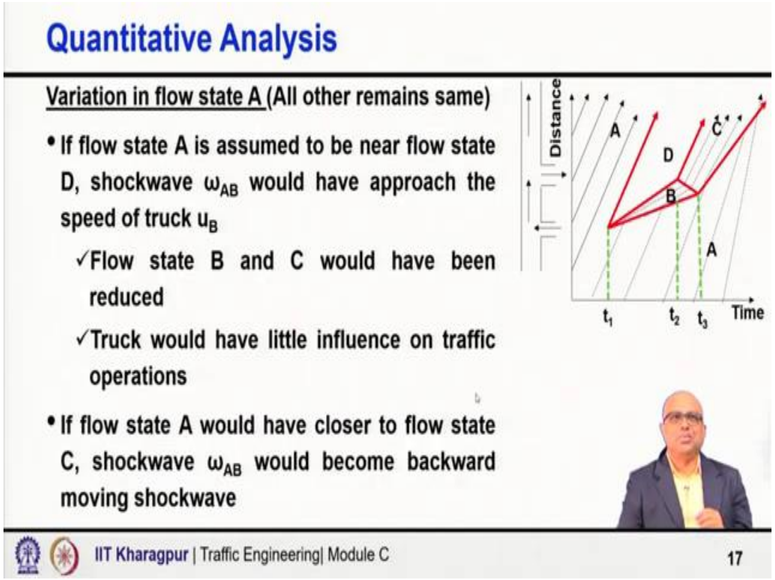

Now there could be several variations depending on different traffic state, how is the traffic state A, how is the traffic state B and so on. So, let us consider some of those. Variation in the flow state A, all others remain same. Now if the flow state A is assumed to be near the flow state D, free flow condition. Suppose the initial traffic state itself A is very similar to D. Then what will happen?

Shock wave with speed omega AB would have approached the speed of truck uB, because that is what you expect. Flow state B and C would have been reduced. The area whatever you are getting now and the overall the traffic state the presence of this traffic state would have reduced to some extent. And eventually truck would have little influence on traffic operation because traffic state A is equivalent to D. No vehicle. So what is the impact? A truck enters with a lower speed and leaves it, fine. There is not much overall impact on the traffic operation because traffic state itself is almost a free flow state, very little vehicle and no vehicle.

Now if the flow state A would have been closer to traffic state C operating at capacity and then this truck enters, slow moving truck, then the shock wave with speed omega AB would become backward moving shock wave. This line AB would not have been forward moving. It could have been probably like this, depending on, because the impact would be severe. So, the presence of traffic state B will be even further dominant. Earlier case here it got reduced. But here it will not get reduced. It will be even more prominent.

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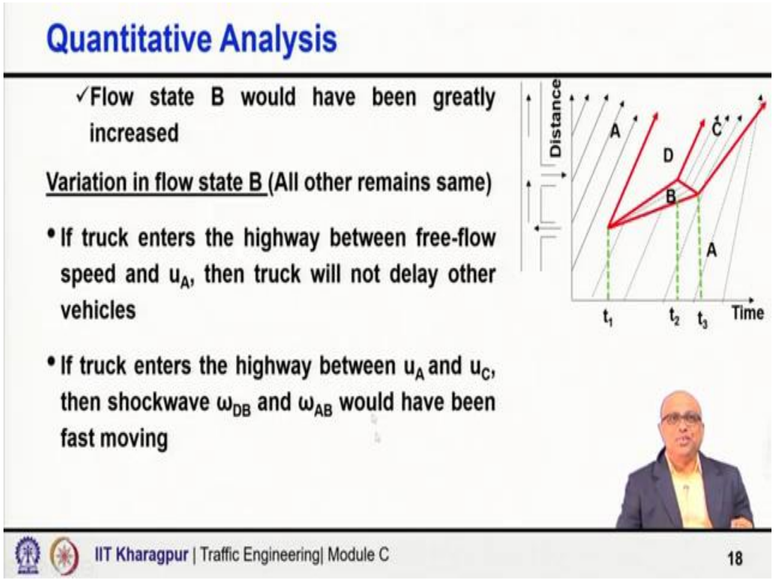

So, we said that flow state B would have been greatly increased. Impact would have been much higher. Now let us consider the variations in flow state B assuming that all others remain same. Just trying to develop some insights, sensitivity in a sense. If truck enters highway between free Flow speed and uA, speed of this traffic state A. So, truck enters at a speed which is either equal to the speed of the traffic stream which is moving or higher, maximum limit could be the free flow speed, then what will happen?

Then the truck will not delay any other vehicle. Traffic state A is travelling with, say 50 kilometer per hour. That is what is the speed of vehicles in traffic state. And truck enters at 50 kilometer per hour. No impact on the traffic state. Even enters, tries to enter with a higher speed, it will not be able to travel with that higher speed. It has to come down. But then the overall impact on the traffic speed is nothing. No impact practically.

Now if the truck enters the highway between speed uA and uC, lower than this A and uC, capacity discharge, capacity flow, then what would have happened? If the truck enters the highway between uA and uC, then the shock wave with speed omega DB and shock wave between A and B with omega AB would have been fast moving.

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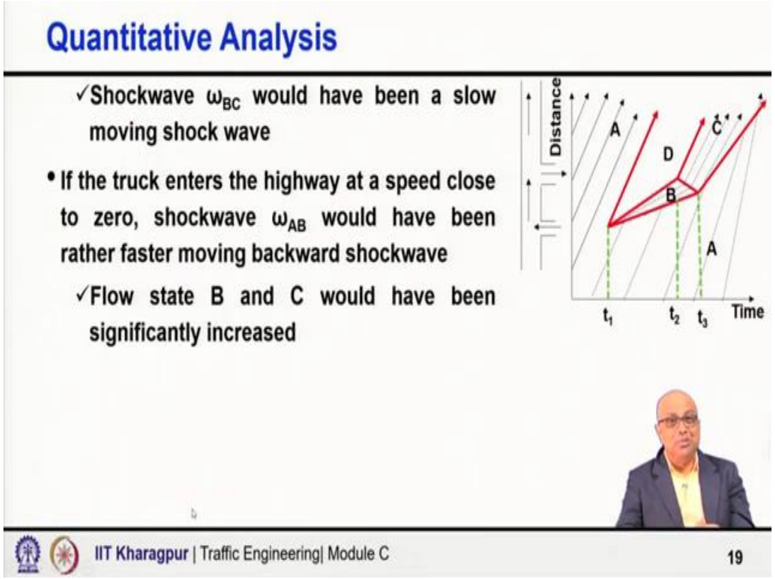

Now shockwave ω_{BC} would have been a slow moving shock wave, the speed, shock wave actually between traffic state B and C, that speed of that shockwave would have been lower. If the truck enters the highway at a speed close to 0, almost like a 0 speed it is entering, then the traffic state B again would have been significantly increased. Not only traffic state B, traffic state C also would have been increased very significantly.

Both would have got impacted heavily. You could have got a larger area of this traffic state A; I mean more dominance, more impact. So, if the truck enters the highway at a speed close to zero then shock wave ω_{AB} would have been rather faster moving backward shock wave and flow state B and C would have been significant increased.

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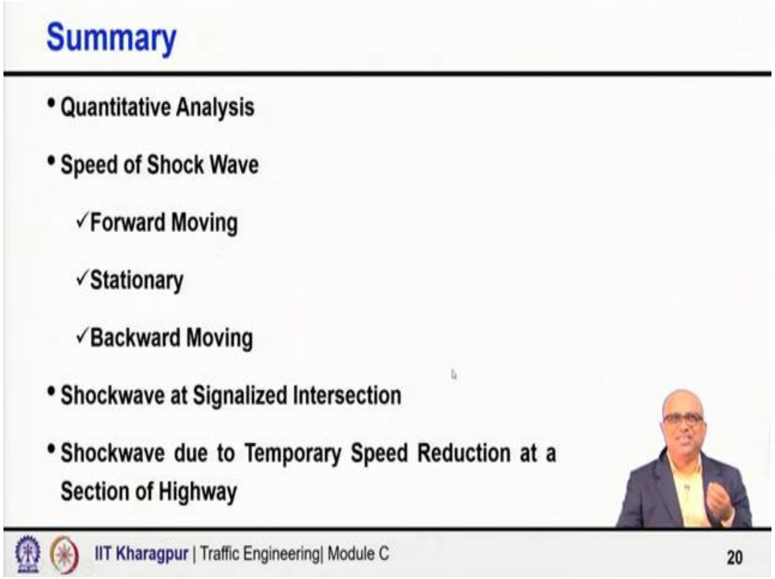

So, with this, what we discussed today is about the quantitative analysis, speed of shock waves, how we can calculate knowing the lower higher two traffic states, whether the density, how the density is compared, how the flow compares and the flow density curve itself comparing two traffic state we can say a whether the shock wave which is going to be created in-between two traffic states is going to be forward moving, stationary or backward moving.

Then we took an example of how the shock waves are going to get created, different traffic states are going to be created and dissipated eventually due to the operation of a signalized intersection. And also another example we took where the speed is temporarily getting reduced on a section of highway because the slow moving vehicle is entering and then travelling sometime and then leaving the highway again.

And then again most interestingly we have seen that how things would have been different depending on various possibilities of traffic state A and also various possibilities of actually variation in the traffic flow state B but primary depending on at what speed the truck is entering, how that would have impacted the traffic flow state B and also traffic flows state C. So, with this, let us close this lecture. Thank you so much.