Hydraulic Engineering
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Lecture – 34
Non-Uniform Flow and Hydraulic Jump

Welcome students. This is the 7th lecture for this broad topic, that is, open channel flow and as we were going to start this gradually varied flow as promised in the last lecture. Until now, we have studied that in open channel flow with the classification based on space, the dimensions, I mean, there are 3 type of flows; one is uniform flow and other is non-uniform flow. So, non uniform have 2 different categories; the first is gradually varied flow and the second is rapidly varied flow.

So, the next 4 lectures would be dedicated on these 2 topics gradually varied flow and rapidly varied flow, in the open channel flows.

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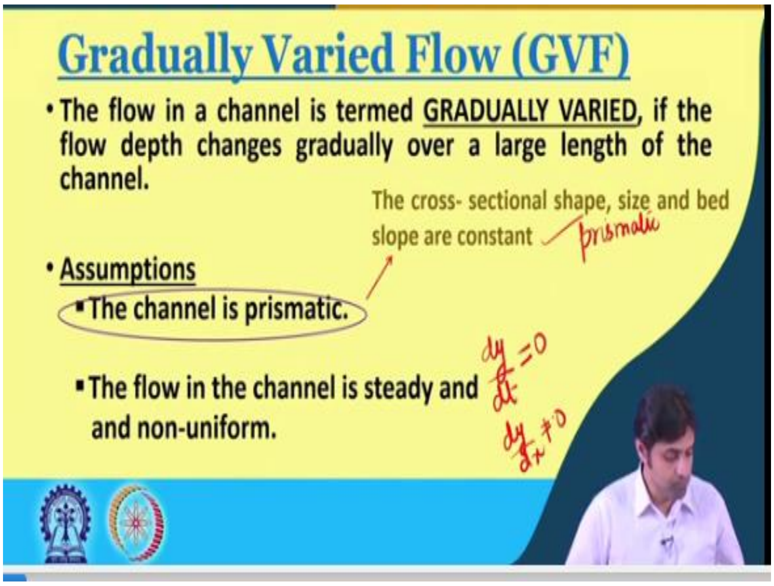

So, to get started, we should understand what exactly a gradually varied flow is. We have already derived an equation before, but to make it more clear we will see it in a little different way, a derivation of a different sort. So, what is gradually varied flow? The flow in a channel is termed as gradually varied, if the flow depth changes gradually over a large length of the channel.

We said that, that dy by dx is very much less than 1. So, this is the definition. So, I will just erase this. So, what are the assumptions behind gradually varied flow? First, that the channel is prismatic, this is the first assumption, this means. What does it mean by the channel is prismatic? That the cross sectional shape, size and the bed slope are constant, so this is what it means, being prismatic.

Second assumption is that the flow in the channel is steady and non-uniform. Non-uniform means that dy by dx, a steady means, dy by dt is 0 but dy by dx is not equal to 0. So these are the some assumptions when we deal with the gradually varied flow.

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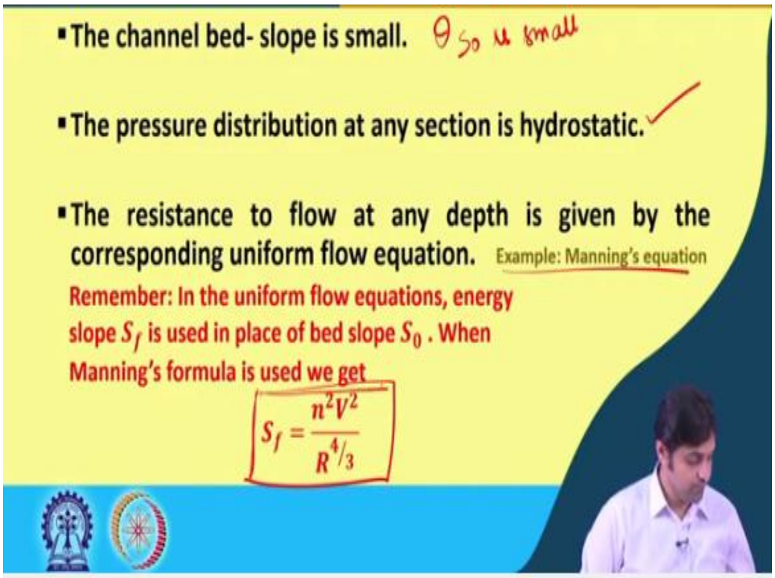

The third one is the channel bed slope is small. So, theta or S 0 is small, S0 as we have been seen in the open channel flow. The another approximation assumption is that the pressure distribution at any section is hydrostatic. This is an important one. Apart from that, the resistance to the flow at any depth is given by the corresponding uniform flow equation. This is another assumption when we are going to derive the different parameters and the properties of gradually varied flow.

So, this is another assumption that the resistance to flow at any depth is given by the corresponding uniform flow equations. For example, uniform flow equations like the Manning's equation, for example or Chezy equation. So, example is Manning's equation. You have to remember that in uniform flow equations energy slope Sf is used in place of bed slope S0. When Manning's formula is used, we get Sf is equal to n square V by R to the power 4 by 3.

So, these we have to, you know, keep into account. So, instead of the bed slope S0, the energy slope Sf is used.

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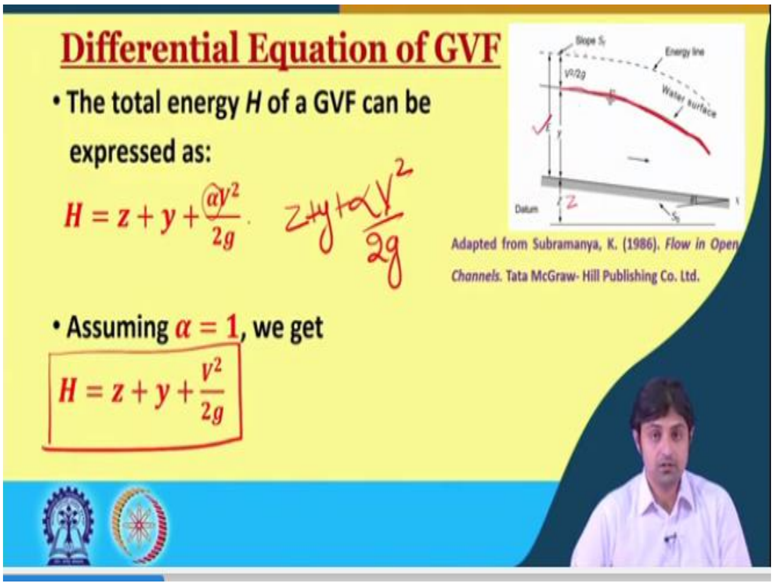

Now, what the going to the differential equation of the gradually varied flow. So, first we have to draw, you know, figure the; I have shown you a figure here, where there is a small bed slope, there is a channel with a small bed slope S 0, so basically, this theta is the angle, the datum is given by this, you know, this is the datum and the height at any in the height of the bed, at any point is at a distance z from the datum.

The water depth is given by y, a specific energy E is given here and the energy line is given dotted like this and this is the water surface. So, first I will remove this. So, the total energy H of a gradually varied flow can be expressed as; so now, there is a little change from the equation that we have seen. In the uniform flow we have seen, that it was,

$$H = z + y + \frac{\alpha V^2}{2g}$$

, here a parameter is introduced, alpha, it is a parameter depending upon different conditions.

If we assume, alpha is equal to 1 that we have already assumed previously, so when we wrote the equation of the total head H in any, you know, in any type of flow, we got, z + y + V square by 2g, so that was actually under the assumption that alpha is equal to 1. In principle, we must have this equation; z + y + alpha V square by 2g.

So, if we assume, alpha is equal to 1, we get this equation as first. H is equal to z + y + V square by 2g, so the velocity head, plus the pressure head, plus the potential head.

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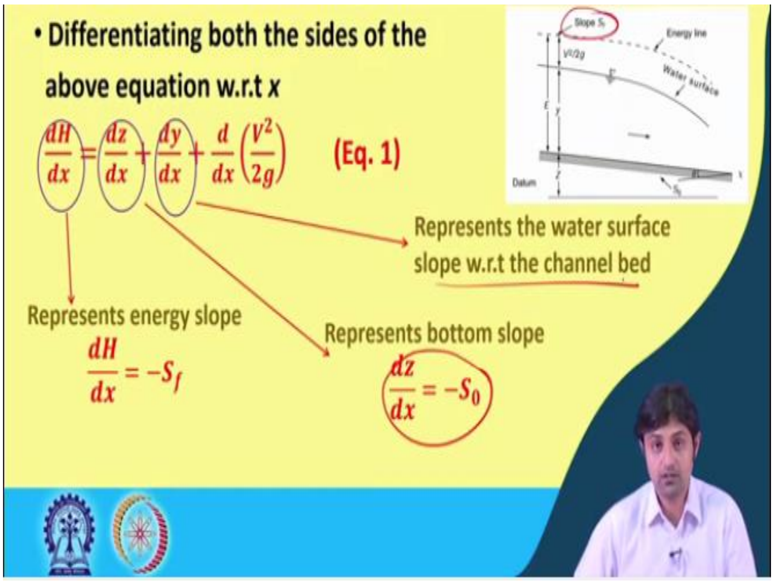

So, now if we differentiate both sides of the equation with respect to x, if we do that, we are proceeding in the same way as we did in the beginning, so dH by dx is will be

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

, that we call equation number 1. So, now this actually represents the energy slope, that is, dH by dx is equal to minus Sf and this dz by dx represents the bottom slope, minus S_0.

This dy by dx represent the water surface slope with respect to the channel bed alright. So, the 3 terms; dh dx represents the energy slope, dz dx represents the bottom slope and the dy by dx represents the water surface slope, with respect to the channel bed.

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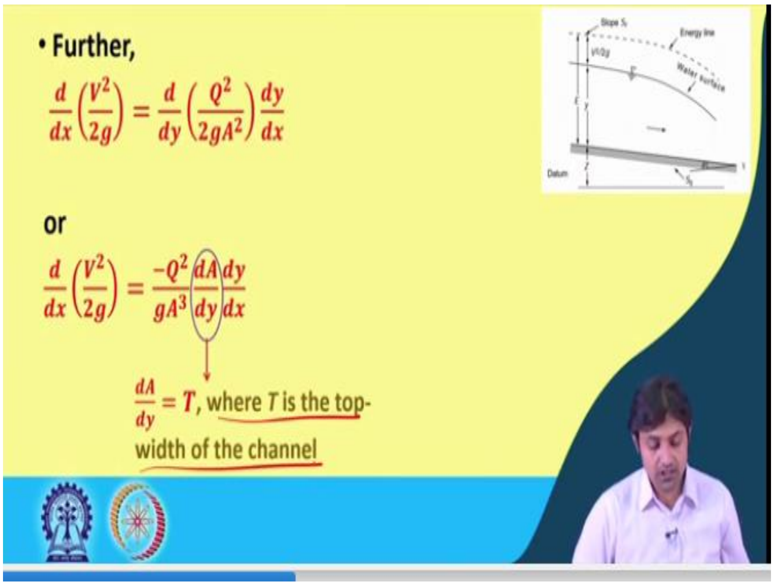

Now, if you see, I mean, for your convenience I am just keeping this figure here all the time so, you can actually follow, if something comes I will just point it out. So, further, V square we say d, I mean, just going into the sub part of that equation, for $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$, so $\frac{V^2}{2g}$ can also be written as, V is Q by A, so it becomes Q square by 2g A square and this is what has been put here, this you should be clear.

Now, what we do, what we say is, so instead of writing $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$, so let us say this is A, for example, capital A, so what we write is $\frac{dA}{dy}$ into dy/dx, this is a simple differentiation rule that you already know. So, dA/dy can be written as d sorry, sorry, dA, so dA by dx can be written as dA by dy into dy by dx. So, same principle we have used here, or we simply write ddx of V square, left hand side remains the same.

Now, looking at the right hand side, what we say is, so Q square is constant, 2g is constant, so it comes out, but we differentiate A with respect to the y, so it becomes, so let us differentiate this, I mean, let us see this part, so it will become Q square by 2g will be there and when 1 by A square will be differentiated, will become minus 2 by A cube into dA by dy, this particular part.

And then, so this is what exactly is written, 2 and 2 gets cancelled, this becomes Q square by gA cube, you see, Q square divided by gA cube into dA by dy and this dy by dx comes as it is. So, this dA by dy, rate of change of area, with respect to the depth is called T, where T is the top width of the channel. So, we are trying to obtain an equation in a slightly different form than what we have seen before.

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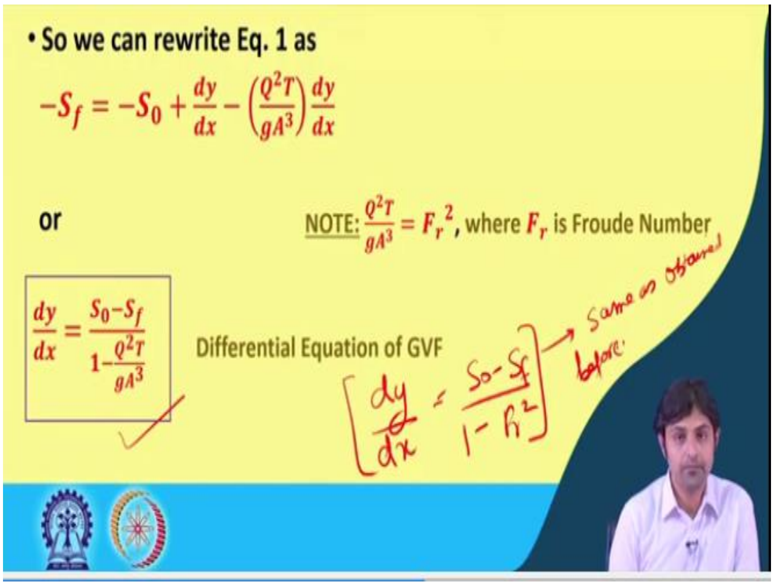

So, here T is the top width of the channel, so this equation can be rewritten as, so if we rewrite equation 1, the dH by dx was - S_f, dz by dx was minus S_0, this dy by dx is the same and this becomes Q square T by gA cube, from the previous slide and this dy by dx. Each term you have understood now, so we rearrange and we take this S_0 on this side and dy by dx we take common, so let me do it for you here and then you will follow.

So, dy/dx, so basically, so I am taking S0 this side, so this becomes, and Sf will be the same is equal to dy /dx common, 1 - $Q^2 T/gA^3$, so dy/dx is going to be S_0 - S_f divided by 1Q square T by gA cube,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}$$

this is exactly same as this. So, it is written in a more, you know, so just directly we have written, but now we have shown also. So, this equation is a differential equation of gradually varied flow.

We have actually obtained the same equation just that we did not have T and it was not in form of T and A, if you remember, I can, you know, take you to that particular slide, so I will show you, where we had done that.

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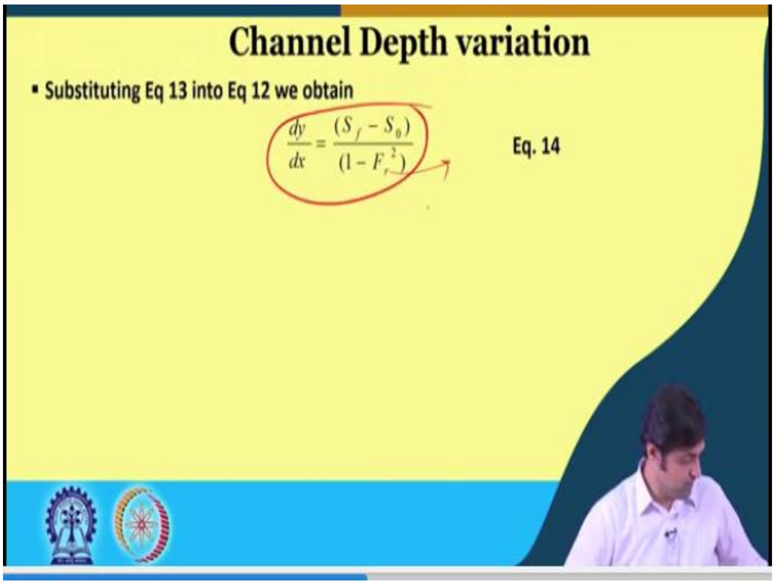

See, we have obtained this similar equation here, you remember, this one, so it is of a same form just that we will later see in the gradually varied flow, this is actually Froude number whole square, the way of derivation was different. So we go back, we were here, so actually this value, Q square T by gA cube is Froude number whole square, where Fr is the Froude number.

So, you can try, Froude number whole square is equal to V square by gy. So, it will be Q square by A square into gy, so if you multiply A, this one, this one, so it will be Q square A divided by A cube g into A, sorry, into y. So, A by y is can be written as T, top width. So, Froude number whole square is Q square T by gA cube because Froude number is V by under root gy.

So, you have also seen that yg Q square T into gA cube is Froude number whole square. So, this equation is the same as we obtained, we obtained was S0 – Sf 1 minus same.

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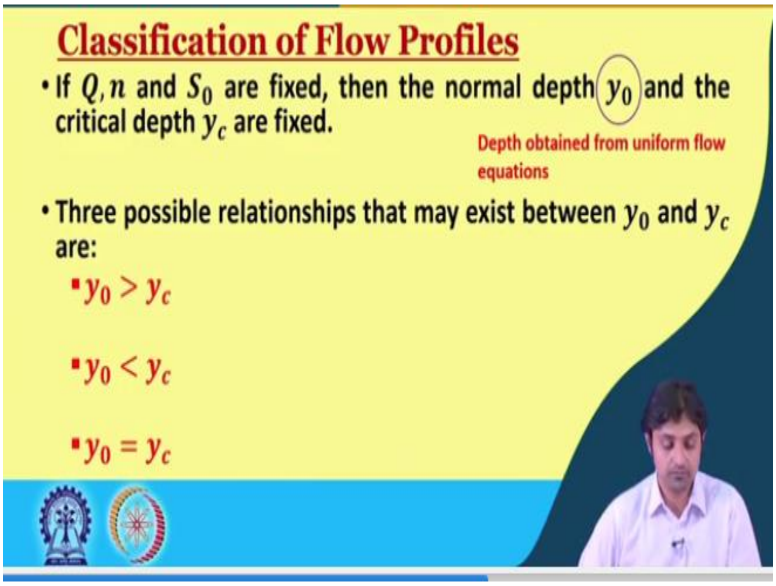

So, now the classification of flow profiles. What are the flow profiles in gradually varied flow? So, if the flow rate Q, Manning's number n and S0 are fixed, then the normal depth y0 and the critical depth yc is also fixed. You understand this? See, Q is equal to 1 by n A, sorry, I will just, sorry, the Manning's equation is V by n S to the power 1/2 Rh to the power 2 by 3 alright.

So, S0 is fixed, n is fixed, V is there, then Q, you know, depending upon the cross section, Q by A that is also going to be, you know, fixed. So, this normal depth y0 and the critical depth yc is also fixed, if we know all these 3 values. So, this is the depth, y0 is the depth which is obtained from the uniform flow equations. So, there could be 3 possible relationships that may exist between the normal depth y0 and the critical depth yc.

One, that the normal depth can be greater than the critical depth. Second is that the normal depth is less than the critical depth. And the third one is that the normal depth is the same as the critical depth.

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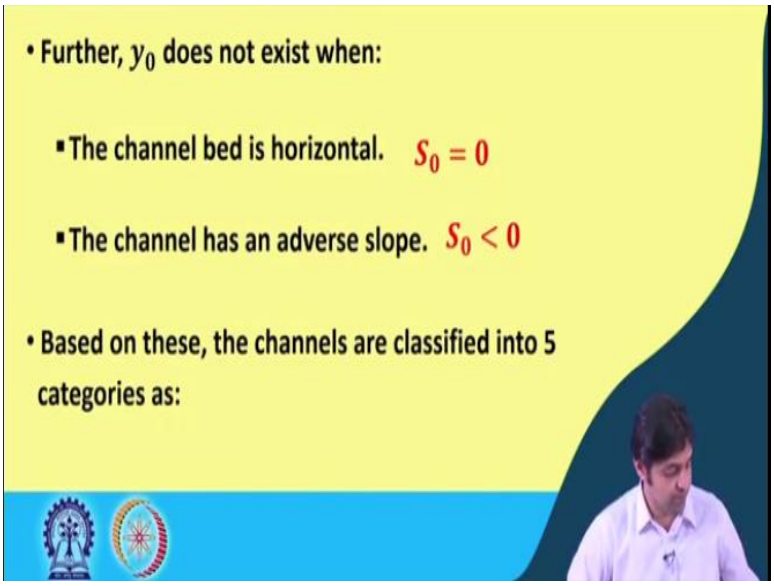

Further, y0 does not exist when, I mean, there will be no y0, if the channel bed is horizontal, that means, S0 is equal to 0. The channel if it has an adverse slope then, also there will not be any normal depth, that means, S0 is less than 0 and based on these, the channels, so based on these, you know, conditions the channels are classified into 5 categories, the channels itself.

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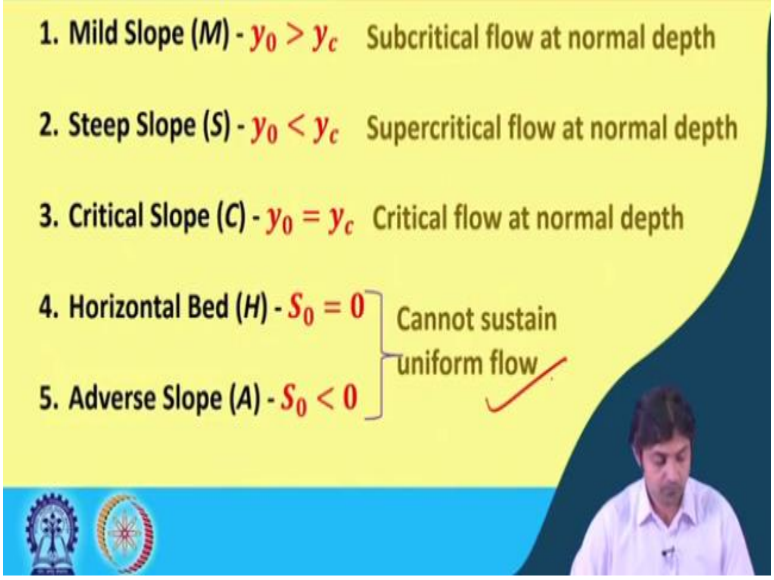

One is mild slope, that is called M, where the normal depth y0 is greater than the critical depth. So, this implies that the flow is subcritical at normal depth. See, Fr is equal to V under root gy0, if y0 is greater than critical depth, then Froude number is going to be less than 1. So, if the normal depth is greater than the critical depth, that is, subcritical flow at normal depth, then this is called mild slope.

A steep slope is when y0 is less than yc, y0 means normal depth is less than the critical depth. Here, there will be supercritical flow at normal depth. How? Fr is V by under root gy0, if y0 is less than yc, that means, the numerator is decreasing, sorry, denominator is decreasing therefore, this will increase, so this is going to be greater than 1, so that is it is supercritical. At critical slope, this is called the critical slope C, so mild slope is denoted by M, a steep slope is denoted by S and critical slope is denoted by C.

y0 is equal to yc, indicates that at normal depth, there is a critical flow alright. The horizontal bed means S0 is equal to 0 that is very simple to understand, horizontal bed means S0 is equal to 0. And then in the adverse slope S0 is equal to 0. So, using all the conditions before, there could be 5 different type of channels; one is mild sloped channel, steep slope and critical slope.

And when the normal depth does not exist, in that case it is simply defined either as the horizontal bed because there is going to be no normal depth in horizontal bed. And also in the case of adverse slope, when the slope is less than 0, so in that case also there is not going to be any normal depth, so it is just defined based on S0 itself called horizontal bed and adverse slope.

Mild slope, steep slope, critical slope, horizontal bed and adverse slope, M, S, C, H and A, so this is important to remember. The mild is where y0 that the normal depth is greater than the critical depth, steep is when the normal depth is less than the critical depth, horizontal bed is very clear, adverse slope is also very clear and critical depth is the easiest one to remember.

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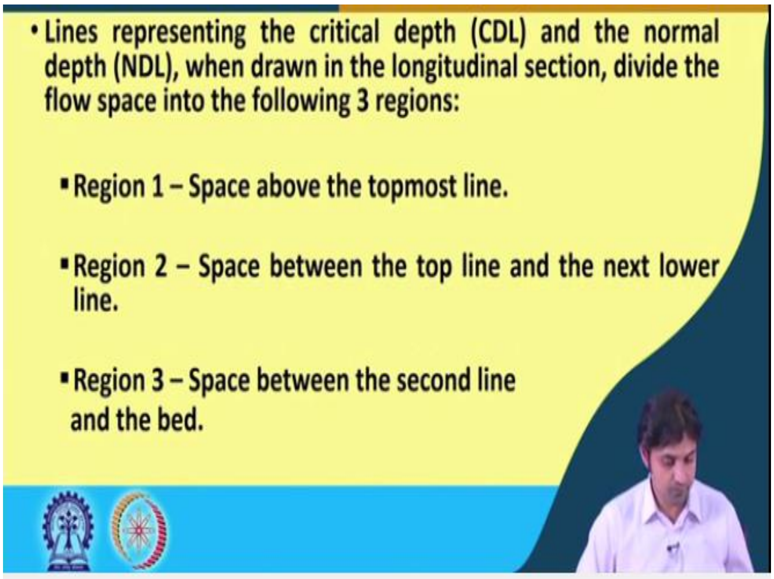

So, this 2; the horizontal bed and adverse slope cannot sustain uniform flow. So, the lines which represents a critical depth called CDL and the normal depth is called NDL, when drawn in longitudinal section, divide the flow space into following 3 regions, we are going to see. Region 1 is, so at the critical depth if we draw that line in the longitudinal section at like this, this, you know, then that line which represents the critical depth is called CDL and the one that represents the normal depth is called NDL.

So, for example, just I will draw this for example, this is there, this is a channel, for example, I mean, does not even necessarily, so suppose this is, this could be anything, so this represents line 1, this is line 2, so actually it is going to divide into 3 regions, region 1, region 2, region 3, so that is what we are actually talking.

We will show you the exact, you know, these NDL, HDL, in upcoming slides, but just to tell you why there are, you know, the several regions. So, region 1 is the space above the topmost line, very normal as I showed you, region 2 is the space between the top line and the next lower line and region 3 is going to be the space between the second line and the bed or the lowermost region. So, these are the 3 regions depending upon where the CDL and NDL is.

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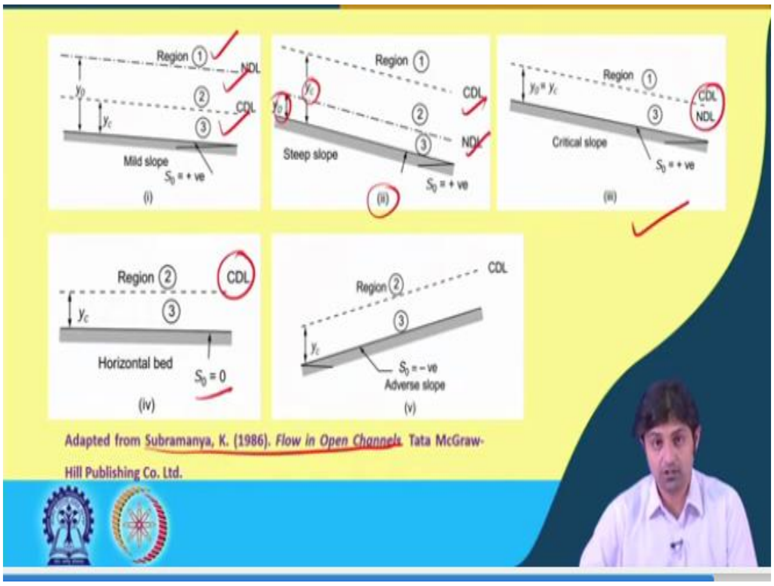

So, this is an example we have just, I mean, this diagram number 1, I mean, indicates in case of a mild slope, it shows 2 regions. What was the criteria? The normal depth, so this is normal depth and this is critical depth, so this is the line that shows, NDL corresponds to normal depth, CDL corresponds to critical depth. So, this represents a mild slope.

So, as you see, this is region 1, this is region 2 and this is region 3. So here, this figure represents a steep slope. Why? Because the critical length yc, critical depth yc is greater than y0, so CDL is above NDL. The third region is critical slope, so CDL and NDL are same. So, here, there will not be any region 2, so region between the CDL and NDL is called region 2.

So, basically there is going to be 2 regions; region 1 and region 3. The fourth is horizontal bed, there will be no NDL here, so there is only be a CDL because the bed is horizontal. So, there is not going to be, there will be only region 2 and region 3. Similarly, here, in case of adverse slope also, there is not going to be existence of NDL, only CDL will be there, so that existence of only region 2 and region 3.

So, this has been, this figures have been adopted from Subramanya, Flow in Open Channels, this book. I think it is a very nice book for open channels, so of course, very detailed, might not be fully, you know, this is only one 6th of the course this open channel flow. So, I mean, you can read these topics from there that will give you a good idea.

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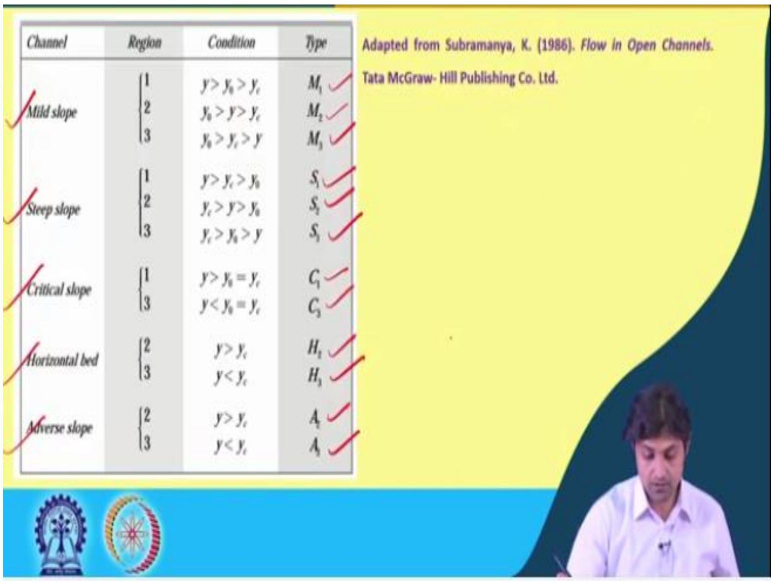

So, there can be 3, you know, as we have already seen that the profiles, there is mild slope, there is steep slope, there is critical slope, there is horizontal bed and the adverse slope. So, depending upon the 3 regions, you know, there could be 3 different types of profiles. If, y is greater than, so region 1 is called M1, region 2 is M2, you know, region 3 is M3, same S1, S2, S3, C1 and C3 and this one is H2 and H3 and this adverse slope is A2 and A3.

This is very clear, because if the y is located both above the normal depth and the critical depth M1, if it is located in between, it is M2. So just by region, region wise, you see, these regions here, and so on. Here, H2, H3, A2, A3 and this is C1 and C3, C1 and C3.

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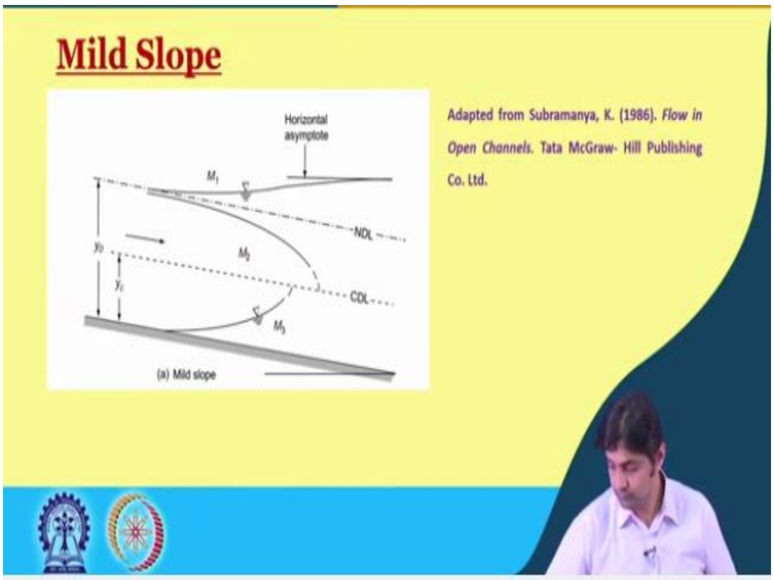

So, mild slope, this is a figure showing mild slope, we have already drawn this.

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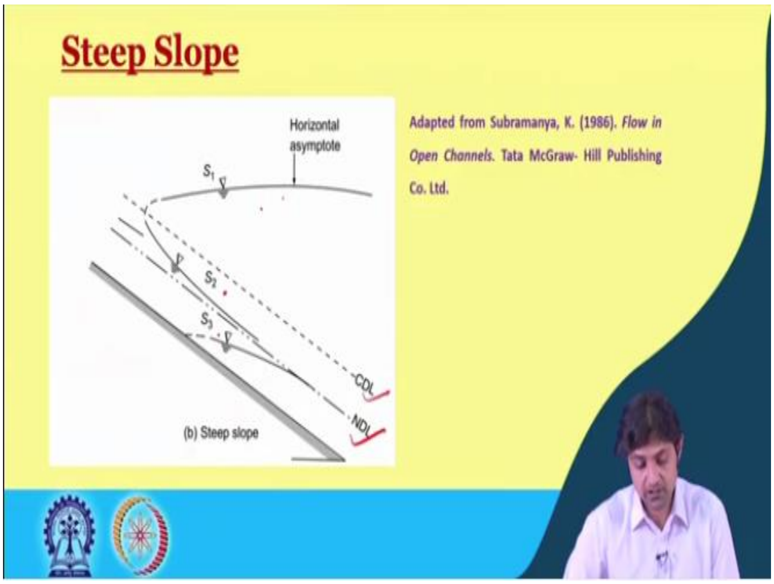

The steep slope, which presents like this, you see, the CDL, the NDL, you see, S1, S2, S3, this is horizontal asymptote.

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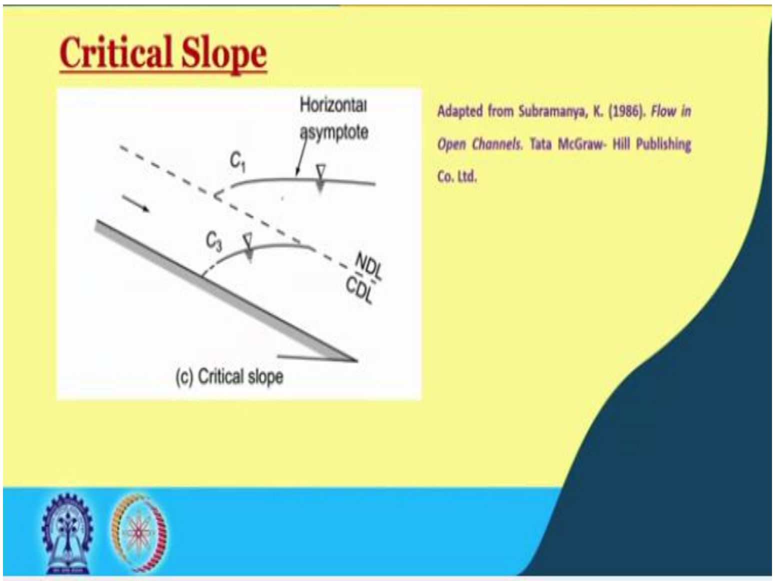

So, critical slope, so the horizontal asymptote C1 and C3, so I mean, there is much more detail to it but at undergraduate level only what we had, I mean, what I am teaching is only required, the identification of the regions, what is mild slope, what is adverse slope, what is horizontal bed.

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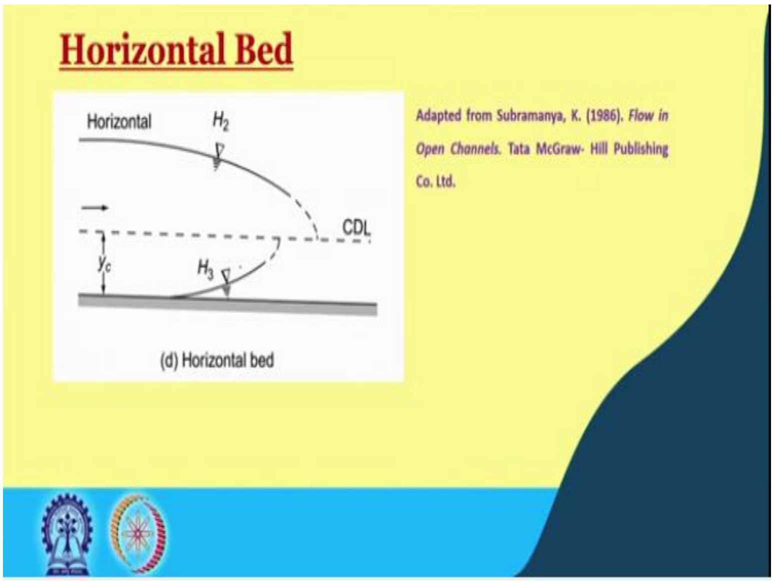

So, the horizontal bed is like this, you know, this area is H2 and this area is H3. So, if the profile is like this and if the profile is like this, which is H2, so the profile is like H2, this is H3, which is just to show you.

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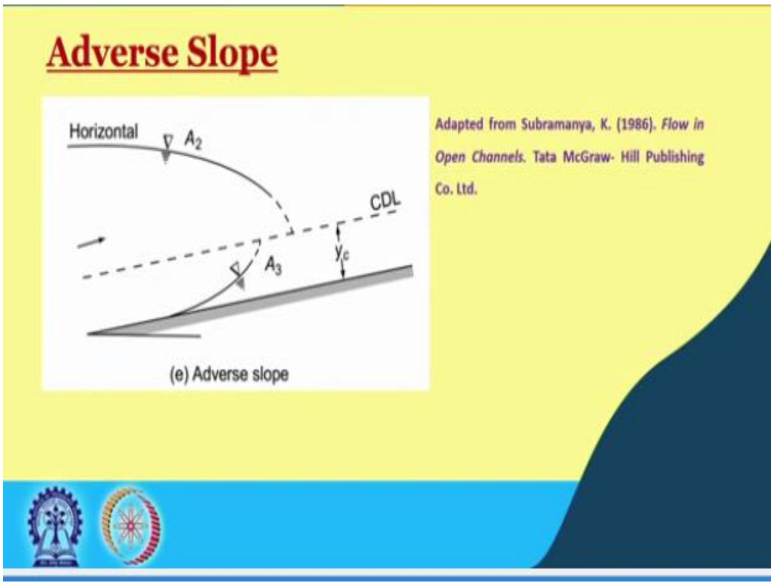

Adverse slope looks something like this, which you already know. If, the water level is like this, it falls in A2, if this is in this one, below CDL, it is A3. So, this all figures have been taken from Subramanya. So, I think this is nice and appropriate point to stop. So, we will solve couple of problems in our next lecture, for related to the gradually varied flow and then we will continue to the other topic called the rapidly varied flow. So, thank you so much for listening. I will see you in the next lecture.