Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture - 41
Illustration of Regression

Welcome back. Today we shall be looking into some Illustrations on the Regression. Now, as we have found out in our earlier lecture that, we try to estimate the values of some variables which are not directly obtained from any tabular chart by interpolation.

Now, many times as we have learnt earlier that we also carry out regression. Now, difference was that that in case of interpolation we are just concerned with the values of the variables; on the other hand in case of regression we are also concerned about the relationship between the various variables. Nonetheless, by getting the regressed equation we can also find out the values of the missing variables from the given equation and in that case, we do not really need to go for the charts. And as we have found out in our earlier lecture that the regression the regressed equation is obtained either from some experimental data or maybe from some tabulated data, ok.

So, what we shall do to illustrate this point? We shall be considering the same problem as we have done for the interpolation and only thing is this we shall be getting the value in terms from the regressed equation.

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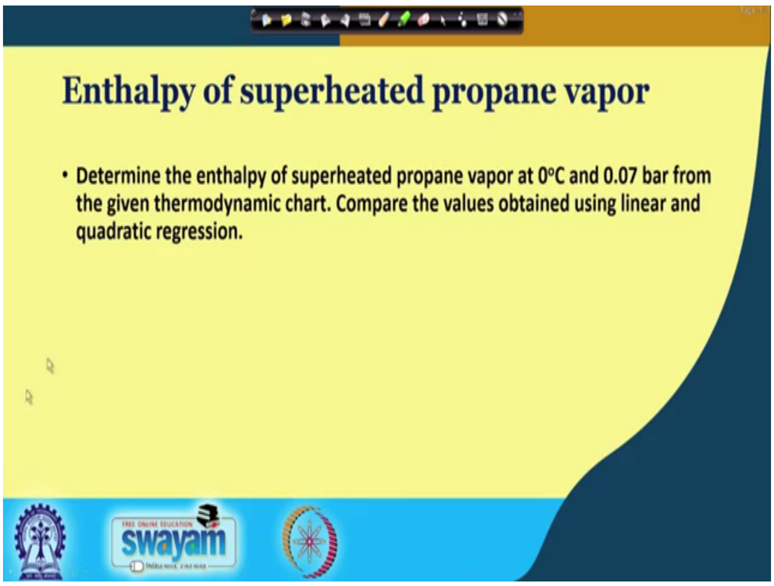

So, here we have the problem that determine the enthalpy of superheated propane vapour at 0 degree centigrade 0.07 bar from the given thermodynamic chart and compare the values obtained using linear and quadratic regression.

Now, you see that for here, we are taking the values for regression from the thermodynamic chart, in reality these values may come from some experimental data.

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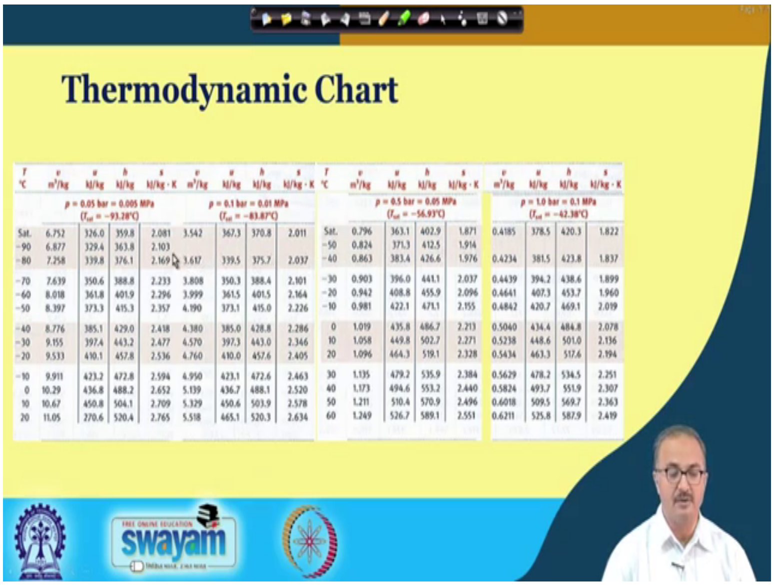

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 0.05 bar = 0.005 MPa (T_{sat} = −93.28°C)				p = 0.1 bar = 0.01 MPa (T_{sat} = −83.87°C)			
Sat.	6.752	326.0	359.8	2.081	3.542	367.3	370.8	2.011
−90	6.877	329.4	363.8	2.103				
−80	7.258	339.8	376.1	2.169	3.617	339.5	375.7	2.037
−70	7.639	350.6	388.8	2.233	3.808	350.3	388.4	2.101
−60	8.018	361.8	401.9	2.296	3.999	361.5	401.5	2.164
−50	8.397	373.3	415.3	2.357	4.190	373.1	415.0	2.226
−40	8.776	385.1	429.0	2.418	4.380	385.0	428.8	2.286
−30	9.155	397.4	443.2	2.477	4.570	397.3	443.0	2.346
−20	9.533	410.1	457.8	2.536	4.760	410.0	457.6	2.405
−10	9.911	423.2	472.8	2.594	4.950	423.1	472.6	2.463
0	10.29	436.8	488.2	2.652	5.139	436.7	488.1	2.520
10	10.67	450.8	504.1	2.709	5.329	450.6	503.9	2.578
20	11.05	270.6	520.4	2.765	5.518	465.1	520.3	2.634

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 0.5 bar = 0.05 MPa (T_{sat} = −56.93°C)				p = 1.0 bar = 0.1 MPa (T_{sat} = −42.38°C)			
Sat.	0.796	363.1	402.9	1.871	0.4185	378.5	420.3	1.822
−50	0.824	371.3	412.5	1.914				
−40	0.863	383.4	426.6	1.976	0.4234	381.5	423.8	1.837
−30	0.903	396.0	441.1	2.037	0.4439	394.2	438.6	1.899
−20	0.942	408.8	455.9	2.096	0.4641	407.3	453.7	1.960
−10	0.981	422.1	471.1	2.155	0.4842	420.7	469.1	2.019
0	1.019	435.8	486.7	2.213	0.5040	434.4	484.8	2.078
10	1.058	449.8	502.7	2.271	0.5238	448.6	501.0	2.136
20	1.096	464.3	519.1	2.328	0.5434	463.3	517.6	2.194
30	1.135	479.2	535.9	2.384	0.5629	478.2	534.5	2.251
40	1.173	494.6	553.2	2.440	0.5824	493.7	551.9	2.307
50	1.211	510.4	570.9	2.496	0.6018	509.5	569.7	2.363
60	1.249	526.7	589.1	2.551	0.6211	525.8	587.9	2.419

Now, for sake of understanding here I have shown the various values of these properties of superheated propane vapour for the different types of pressure. Here you can see that it is 0.05 bar, then 0.1 bar, 0.5 bar, then 1 bar and. Here you can also see that for each of these pressures the T set value is changing. Now, why because as you know that as the pressure increases.

So, does the T set that means, the boiling point is raised with the pressure. So, what we find that suppose you take this particular pressure 0.05 bar, so in that case the T set is minus 93.28 degree centigrade and we are getting these values of the enthalpies etcetera. So, this is a saturated vapor.

Now, in case of say 0.1 bar you find that the value will start from minus 83.87 degree centigrade for the saturated vapor. That means what? That here you are getting minus 90 for this part at this pressure which is giving superheated, but in this case, you have to go above this minus 83 that means, anything below 83 will be sub cooled liquid, ok. So, in we will or saturated liquid vapor mixture. So, we will now, so in this particular table for this superheated vapor we will not be getting the values of the enthalpies or any other thermodynamic properties which is below this temperature. So, you find that here we have the value of the enthalpy at this that saturated vapor and then it is directly going to minus 80 degree centigrade, ok.

Similarly, you can see that is 0.5 bar it is minus 56.93, and here you can see that the value starting from minus 50 onwards. And then for 1 bar you can see the saturated vapor in values and then the values starting from minus 40 which is more than minus 42. So, that means, you find that the range of the temperatures will keep on changing depending on the pressure value we are taking, ok.

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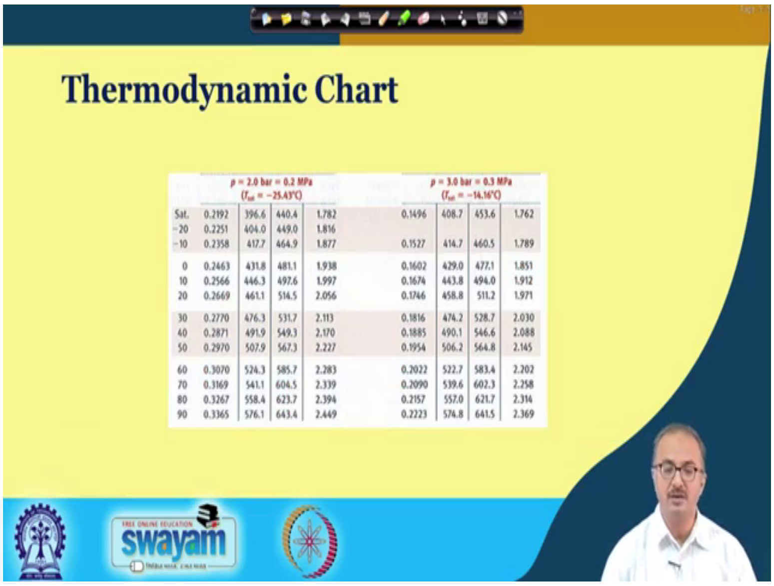

So, here we have also shown some more pressures at 2 bar and 3 bar. Why we are taking so many pressures? This is because that unlike the interpolation which we generally go for only 2 or 3 parts the neighbourhood of the actual independent variable value in the in regression we try to understand the correlation between the independent and dependent variable and for that the more the experimental values, we have the better will be our understanding about this kind of interrelationships between the independent and dependent variables. So, that is why in this particular problem we have taken the values of the enthalpies at so many different pressures and as per each pressure you are finding that the temperatures are also changing or rather the ranges of temperatures are also changing.

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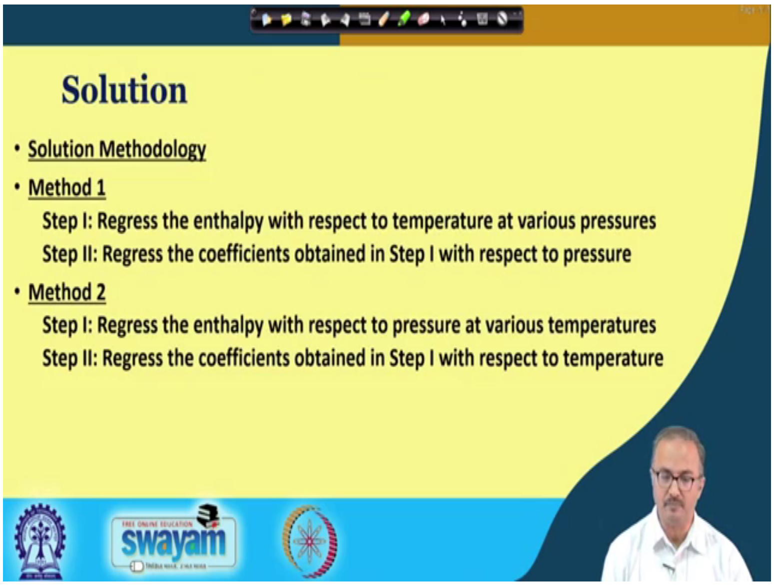

Now, in this case also we may adopt one of the two methodologies, one method is this first we can regress that enthalpy with respect to temperature at the various pressures and then the coefficients obtained in step one with respect to pressure we can. That means, here what we are doing that for various pressures we are regressing for the given temperature, ok. And then what we do?

From the in that equations we just put the pressure which have been given to us and we get the value of the enthalpy. In the second one, what we do? We regress first with respect to the pressures at the various temperatures and then we get the values of the enthalpy with respect to the given temperature and this I will illustrate with the some examples.

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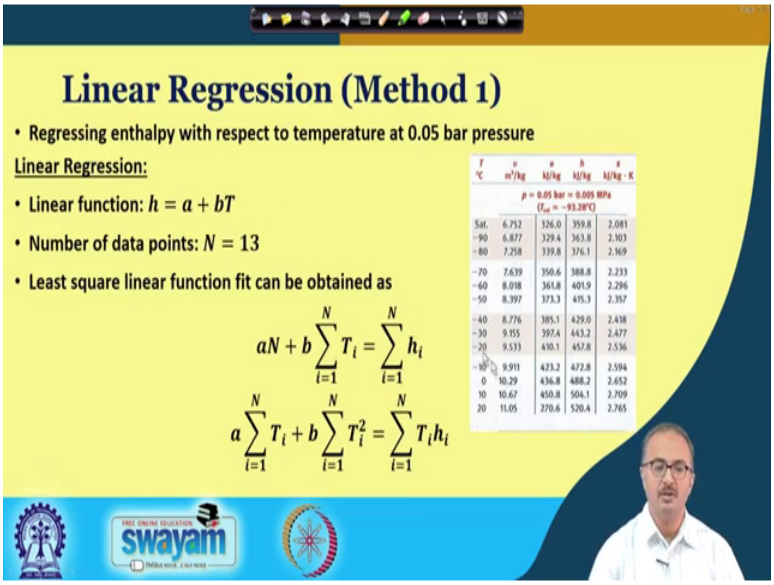

So, first let us drew with the method 1. So, here you see that for just arbitrarily, we go with the simplest regression that is the linear regression and first we are regressing the enthalpy with respect to temperature at 0.05 bar pressure, ok. Now, what we are doing? That here you have see that this is the table 0.05 bar and you can count here the number of data points here and this will come out to be 13, and this is the kind of equation with which we are going to regress the enthalpy as a function of temperature.

Now, please understand that because this enthalpy is a function of both temperature pressure the dependence of this a and b on the pressure should also be evident in our correlation. So, how that can be done we shall see later, ok. So, first we regress with respect to temperature knowing the total number of data points given to us. So, here these are the two equations which we obtain of obtain the least square fit, ok.

Now, you have two equations and two unknowns that is a and b and now we can put the values these summation of all the temperatures, this is the summation of the enthalpies. So, these all these things we are doing.

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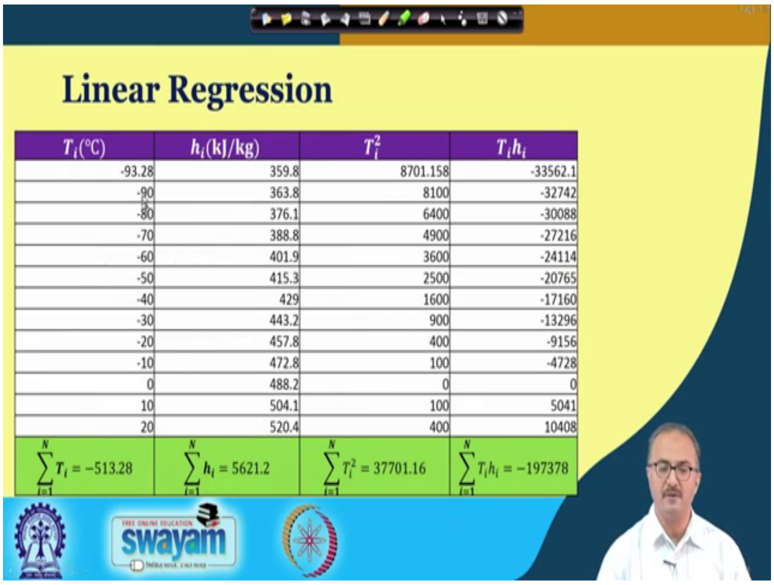

And here is the table where we are putting the values of temperatures, enthalpy, then the square of the temperature and the product of the temperature and the enthalpy, which are needed to the this least square fitting method.

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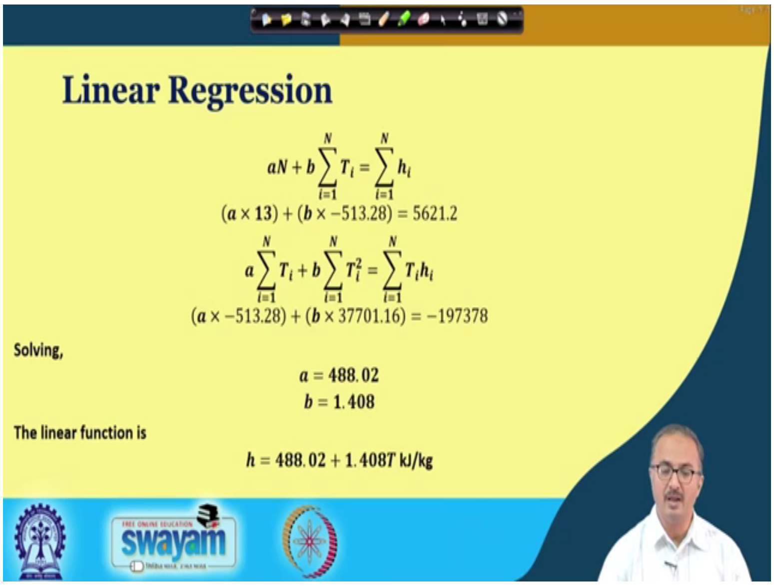

And after inputting those values that that the values here and what we get these are the values of the a and b, ok. Now, please understand that this we have done at pressure of 0.05 bar, ok. So, at 0 5 bar this is the enthalpy values. And you can see that, if we change the pressure this value of the a and b will also going to change, ok.

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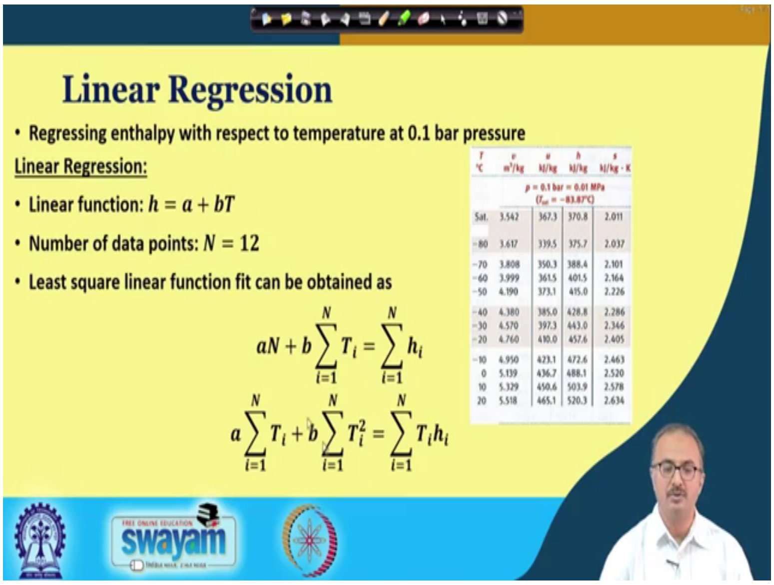

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
Sat.	3.542	367.3	370.8	2.011
−80	3.617	339.5	375.7	2.037
−70	3.808	350.3	388.4	2.101
−60	3.999	361.5	401.5	2.164
−50	4.190	373.1	415.0	2.226
−40	4.380	385.0	428.8	2.286
−30	4.570	397.3	443.0	2.346
−20	4.760	410.0	457.6	2.405
−10	4.950	423.1	472.6	2.463
0	5.139	436.7	488.1	2.520
10	5.329	450.6	503.9	2.578
20	5.518	465.1	520.3	2.634

So, next what we do? We go to 0.1 bar why because our pressure given pressure is 0.07 bar, ok. So, now, we go to and this next pressure 0.1 bar. Now, in this 1 bar again we count the number of data points given to us and it is coming out to be 12. So, N equal to 12. Again, we use the least square in your function fit and these are the equations.

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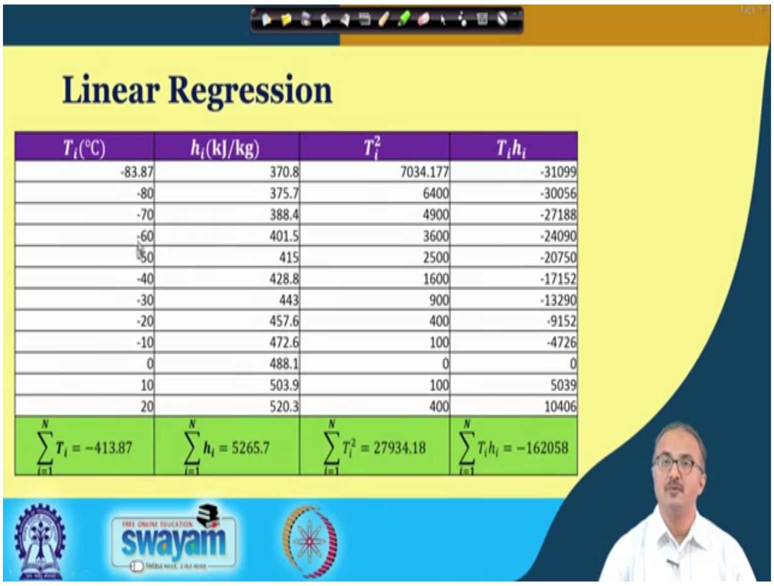

T_i(°C)	h_i(kJ/kg)	T_i^2	$T_i h_i$
-83.87	370.8	7034.177	-31099
-80	375.7	6400	-30056
-70	388.4	4900	-27188
-60	401.5	3600	-24090
-50	415	2500	-20750
-40	428.8	1600	-17152
-30	443	900	-13290
-20	457.6	400	-9152
-10	472.6	100	-4726
0	488.1	0	0
10	503.9	100	5039
20	520.3	400	10406
$\sum_{i=1}^{N} T_i = -413.87$	$\sum_{i=1}^{N} h_i = 5265.7$	$\sum_{i=1}^{N} T_i^2 = 27934.18$	$\sum_{i=1}^{N} T_i h_i = -162058$

So, these are the values of the temperature enthalpies the square of temperature the product of temperature enthalpy and these are the summations of all these things, which we are needed in this interpolation formulae and we put the values and we get the a and b value like this.

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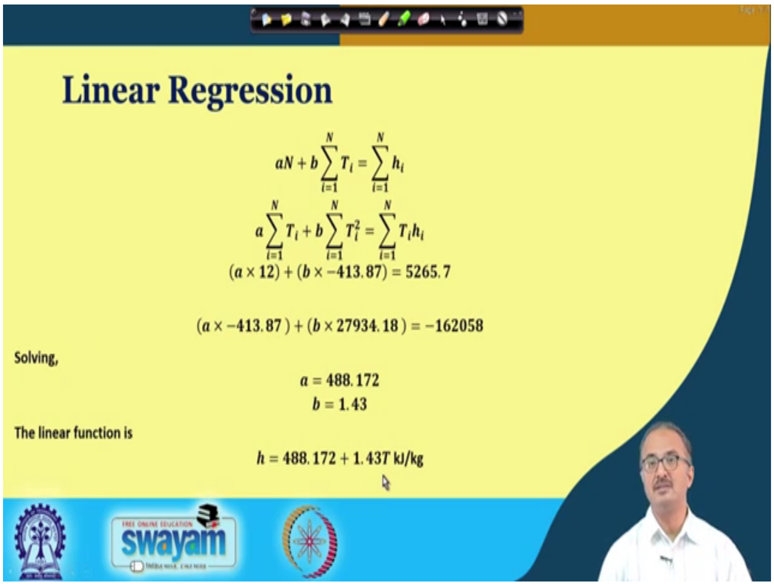

So, we find we get another set of a and b value for a different pressure, ok.

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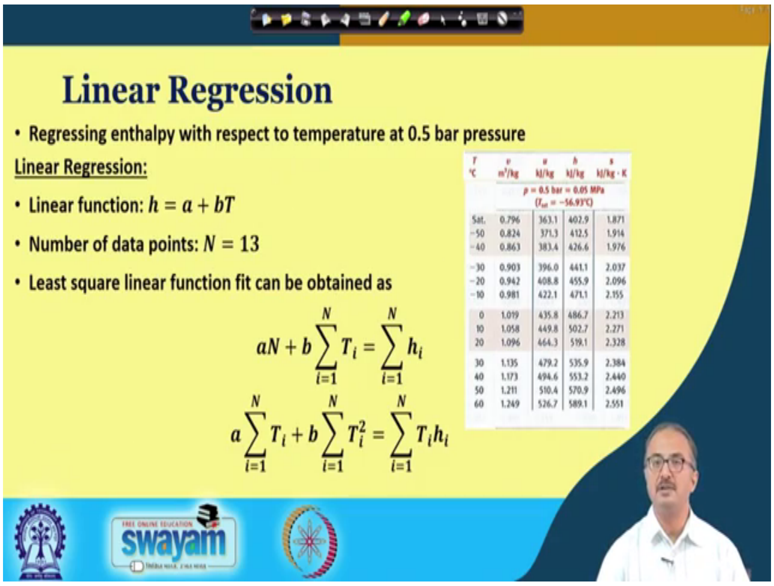

Now, this we can carry on, with this now we are going for 0.5 bar, again we are getting the values of a and b from the least square fit.

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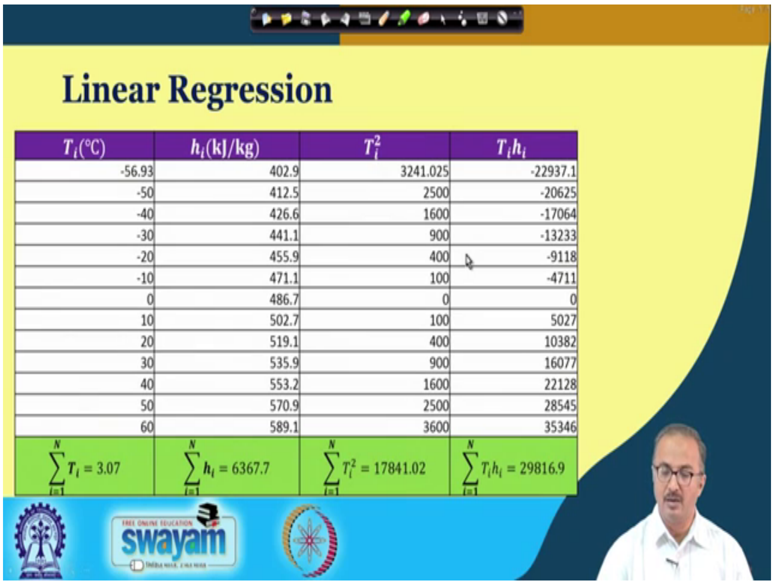

And these are the tables for this temperature enthalpies and other things and ultimately what we find that this is the regressed equation. And here you find that a and b values are like this and they have now changed, ok.

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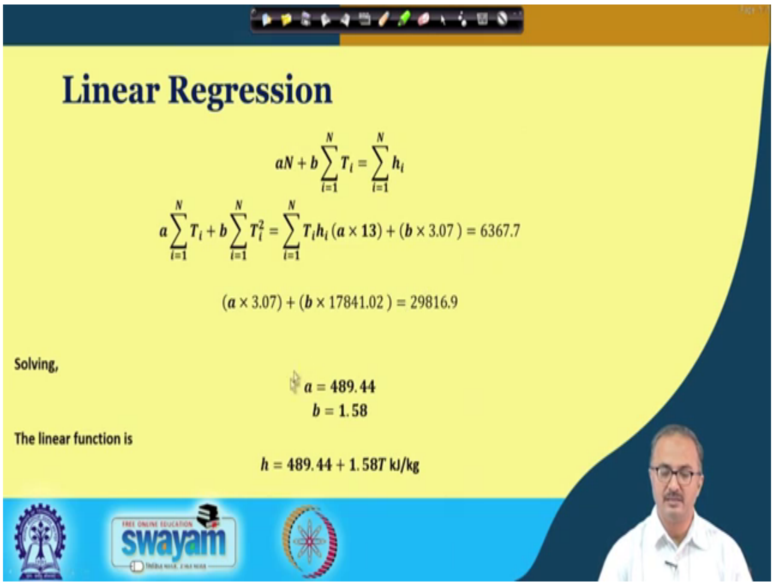

And this thing we can keep continuing for different pressures.

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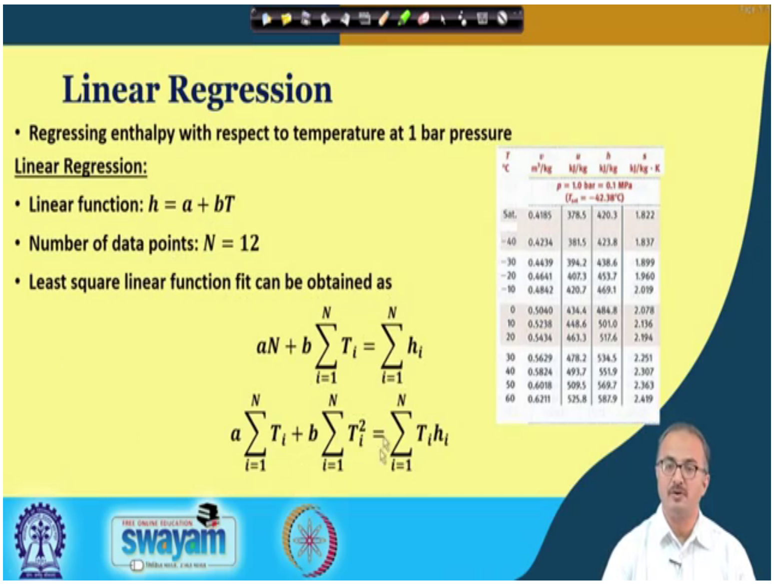

Now, how many pressures you will go up to, that is of course a personal choice. And as I told you the more data points you take and that will be better for us for understanding of the problem that is why I have chosen. However, please mind it again if you find that there is some kind of linear relationship between the dependent and the independent variable, in that case we need not take so many values or so many different pressures in this particular case, ok.

So, if we know a priori, the relationship, then it is fine if you do not know then perhaps it is always good to consider as many data points as possible, ok. So, here we are assuming that we do not know the nature of the variation of the enthalpy with pressure and temperature. So, it is get generalized case we are taking and accordingly we are considering many many pressures. So, here we have again for 1 bar pressure, again we count the number of data points given to us and in this particular equations we put these values.

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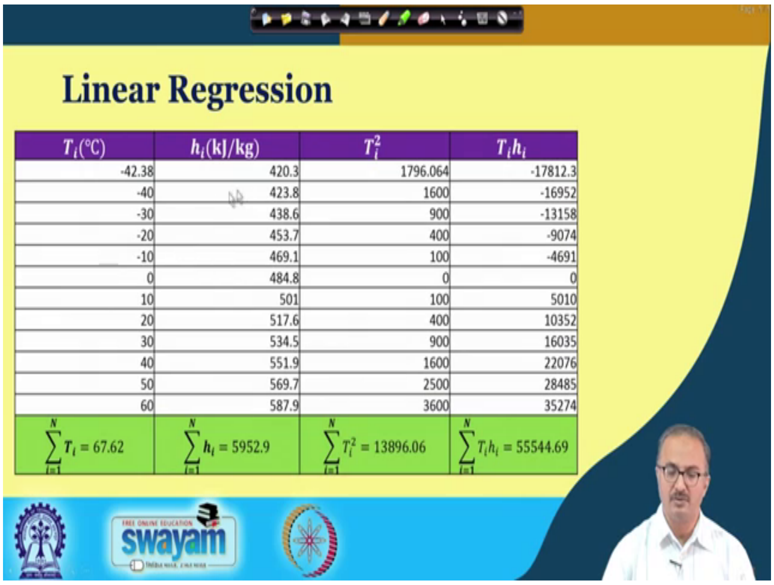

So, these are the tables for your understanding and this you can also check yourself, ok.

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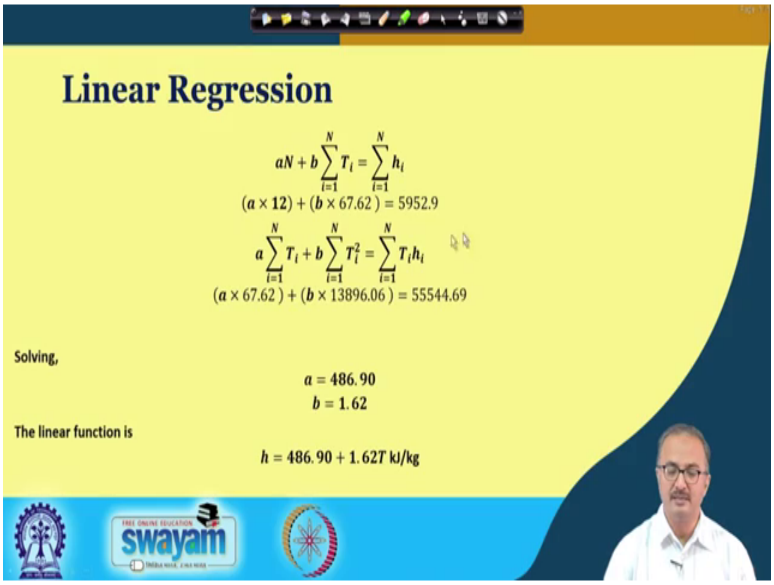

And then we find that these are the a and b values, ok.

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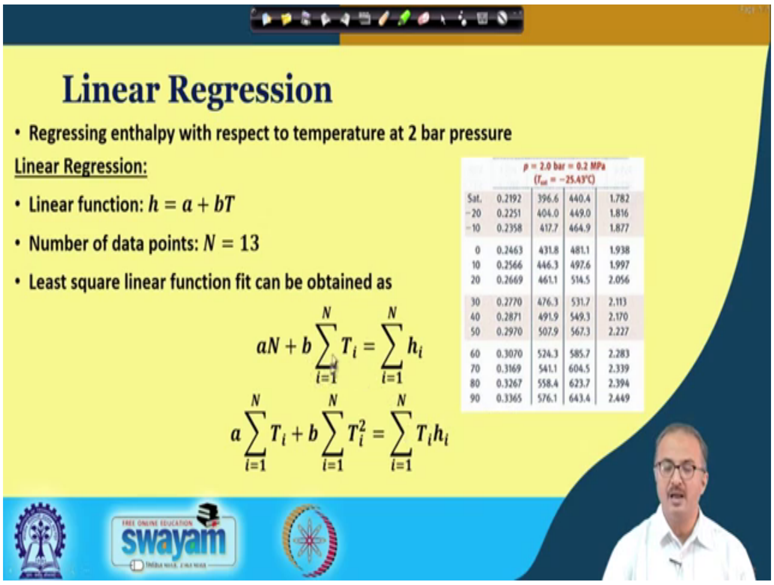

Next you go for 2 bar pressure. Again, we find how many data points are there here, and again we use these regression.

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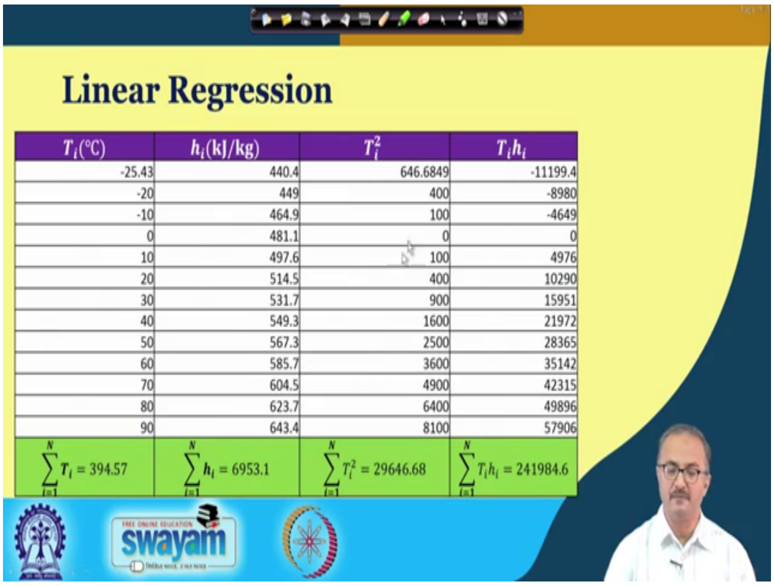

And we again that this table gives you temperature enthalpy etcetera.

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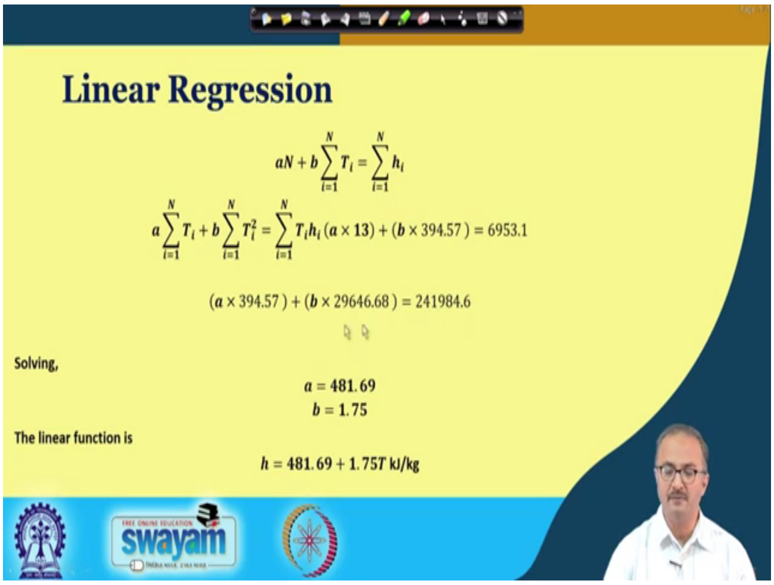

And we find these are the values of the a and b, ok.

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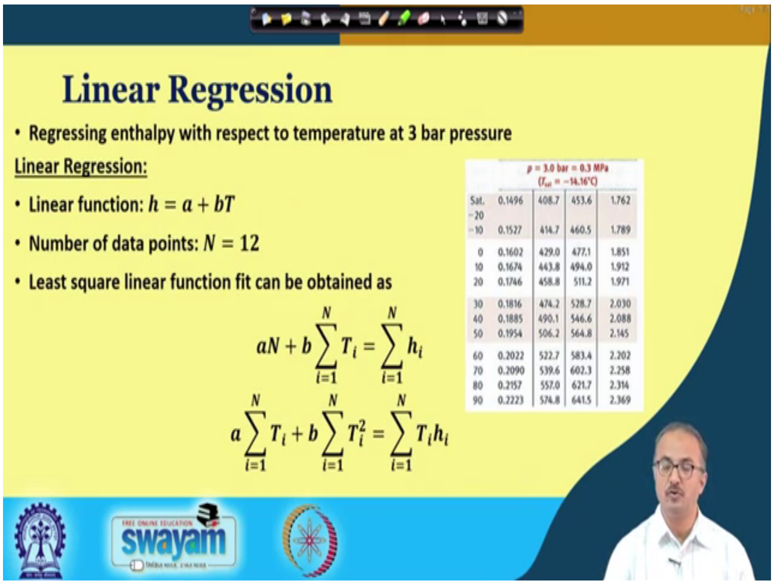

	$p = 3.0$ bar $= 0.3$ MPa ($T_{sat} = -14.16°C$)			
Sat.	0.1496	408.7	453.6	1.762
−20				
−10	0.1527	414.7	460.5	1.789
0	0.1602	429.0	477.1	1.851
10	0.1674	443.8	494.0	1.912
20	0.1746	458.8	511.2	1.971
30	0.1816	474.2	528.7	2.030
40	0.1885	490.1	546.6	2.088
50	0.1954	506.2	564.8	2.145
60	0.2022	522.7	583.4	2.202
70	0.2090	539.6	602.3	2.258
80	0.2157	557.0	621.7	2.314
90	0.2223	574.8	641.5	2.369

And then we go for 3 bar.

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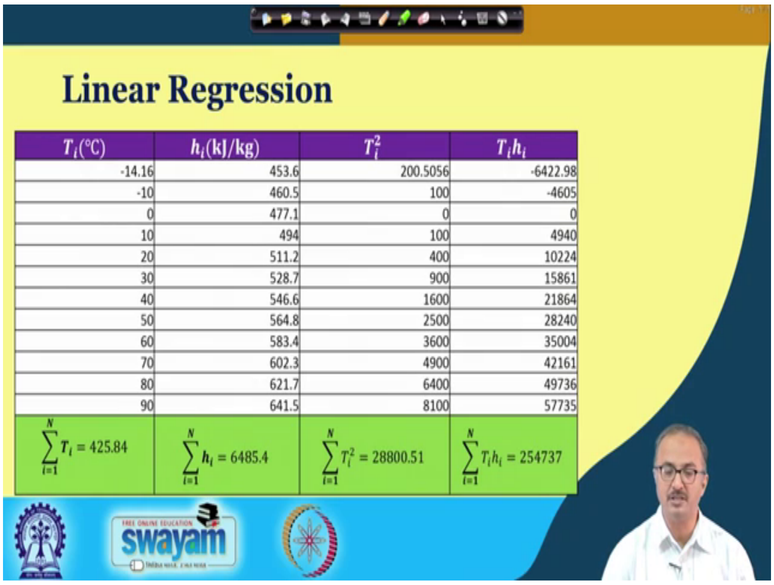

Again, we repeat the things, and we find that this doing all these calculations we find these are the values of the a and b.

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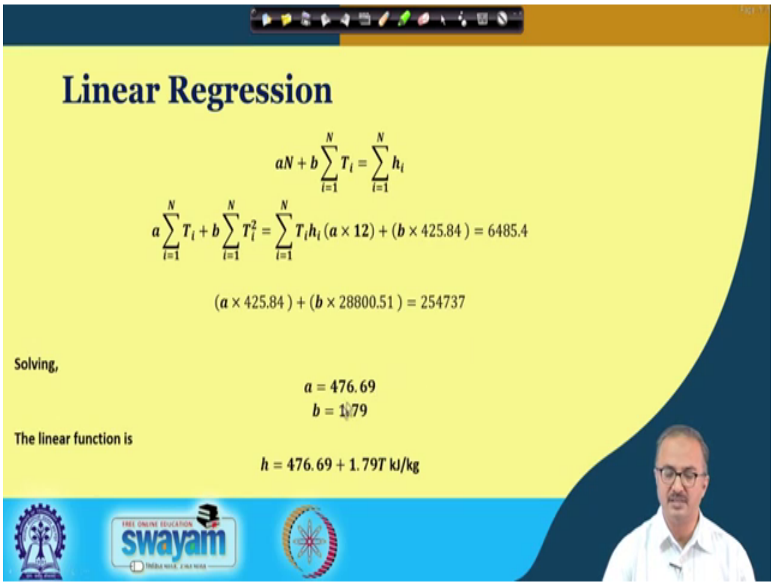

So, we find that as we take more and more pressures there is some kind of variation in the values of the a and b. So, here we have listed all the values of a and b with respect to various pressures.

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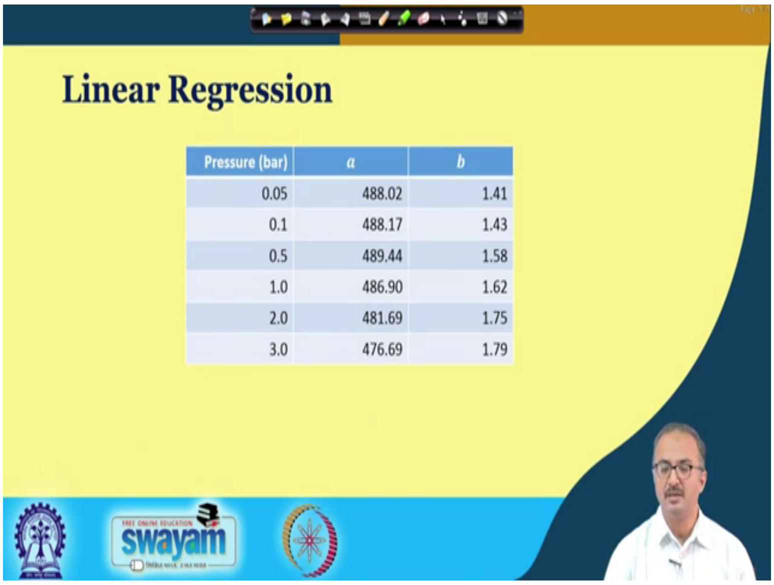

So, you can see that 0.05 bar we are going 0.1 bar and up to 3 bar. And how you are seeing this a values which is changing a changing a bit, it is kind of increasing initially and then decreasing. So, there is no clear trend on this and again we are finding here that it is also increasing throughout, ok.

So, but the whatever integer decrease if you find the percentage error it is not going to be much, ok. That anyhow as to illustrate the point of getting the regressed value with respect to both temperature pressure, what we do? Again we regress this a and b value with respect to pressure, that means, we put a as a function of pressure and b as a function of pressure.

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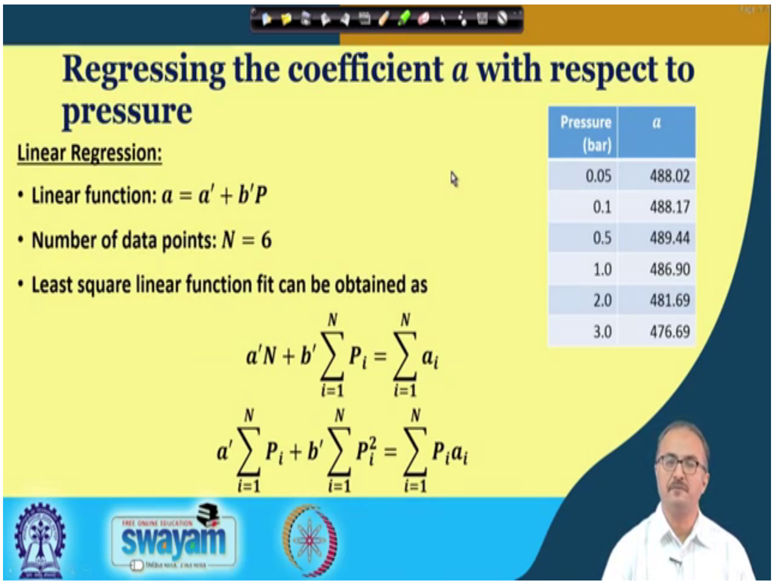

So, first we do the regression with respect to a. So, here we defining a equal to sum a prime plus b prime into P and we know that we have 6 data points with us. So, with this we again apply the least square methods.

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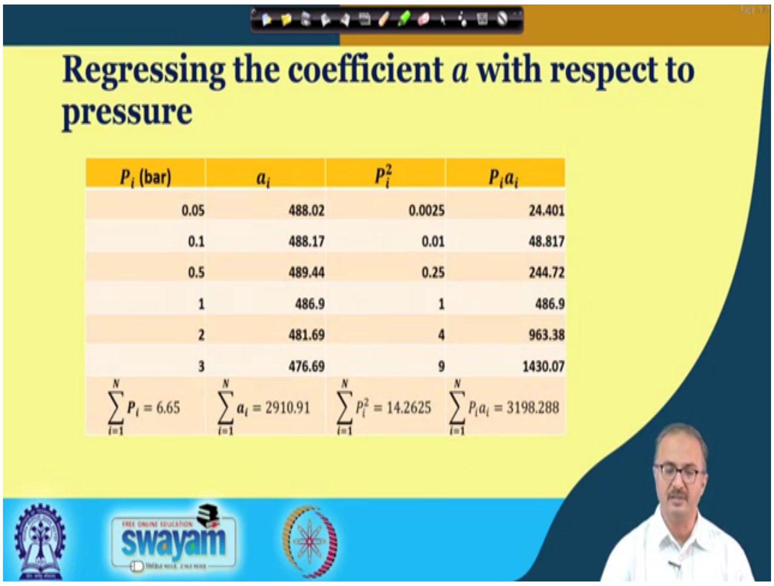

And this is the table where it is tabulating all the variable values which we need to do the regression.

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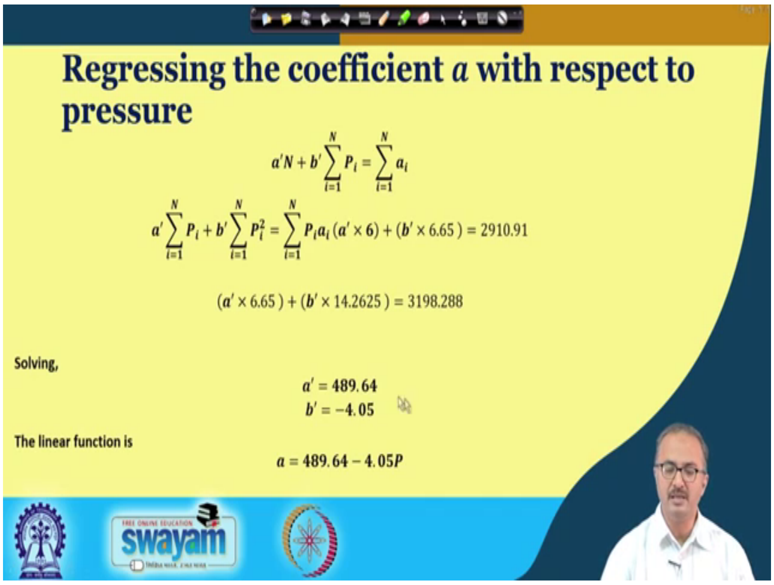

And finally, we get the values of the a prime and b prime and now we are getting that how a is changing with pressure, ok.

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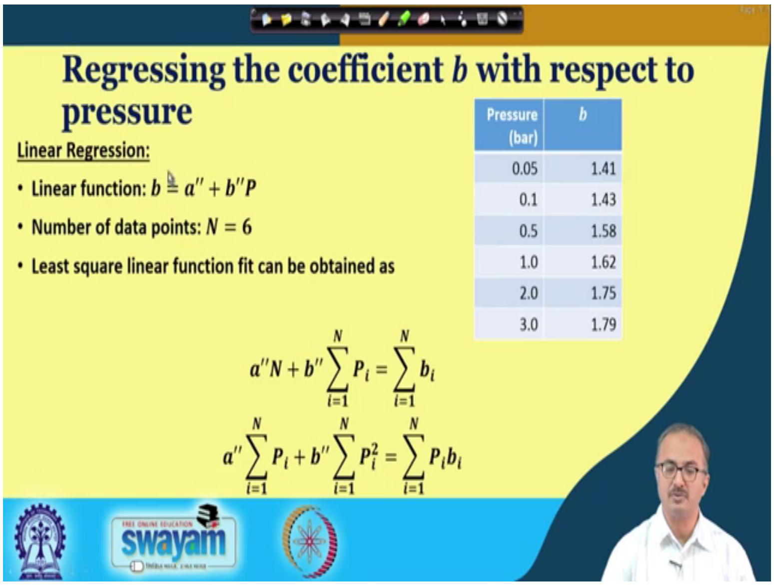

Pressure (bar)	b
0.05	1.41
0.1	1.43
0.5	1.58
1.0	1.62
2.0	1.75
3.0	1.79

In the similar fashion we go for b also. Now, we have defined we have taken two more variables that is a double prime and b double prime and now again we have 6 data points, and with these 6 data points using this linear square fir formula, and these values of the b and other variables here.

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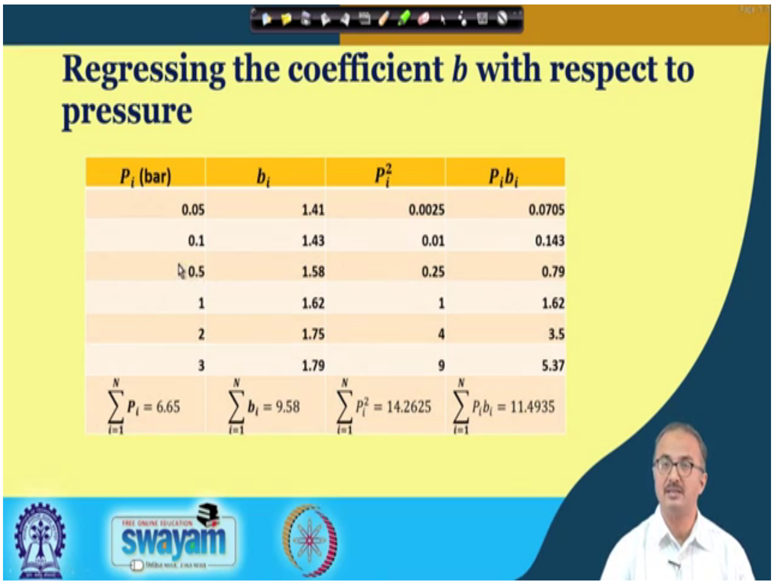

What we get here?

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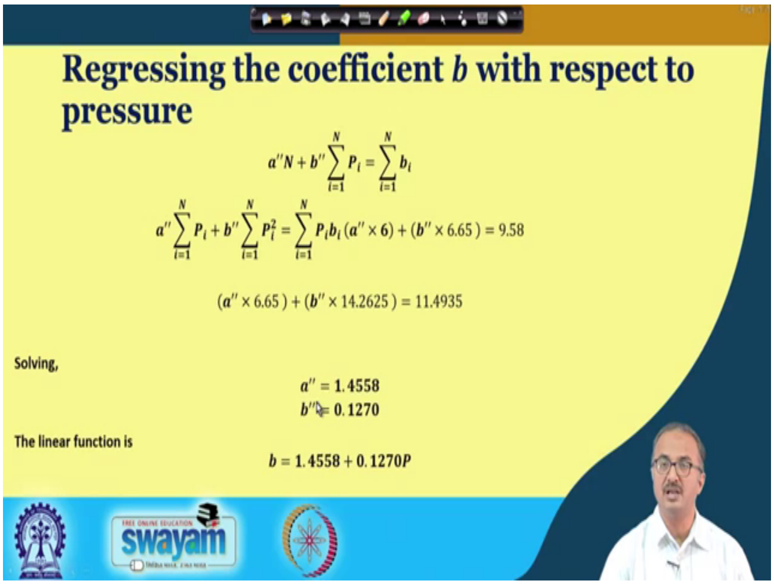

These values of the a double prime and b double prime and now, we find another expression for b as a function of the pressure.

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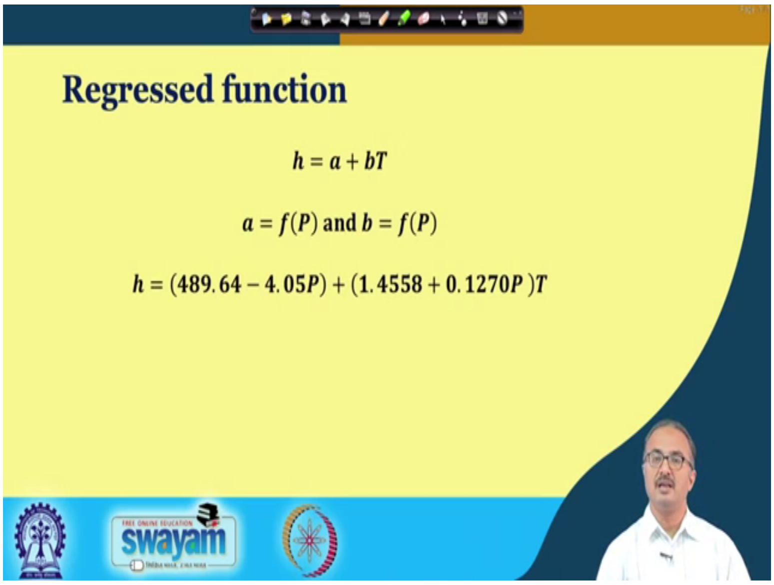

Now, after obtaining these a and b as function of pressure. Now, we can write the total expression and we will something like this that we put here for the P dependency of a and here we put the P dependency of b and the temperatures coming. So, now, you can see in this whole enthalpy we have both the temperature and pressure dependencies, ok.

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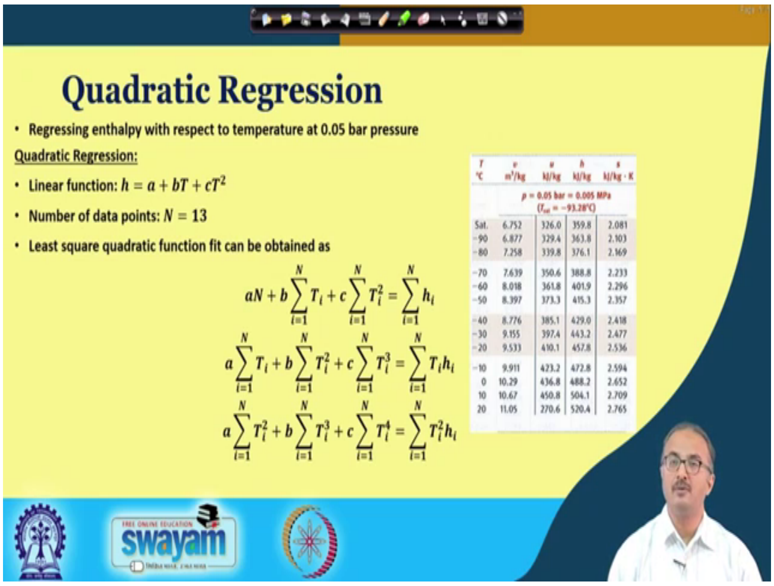

Now, just sake of testing what we do we go for quadratic regression. In quadratic as you know that we have this kind of formula that h equal to a plus bT plus cT square, and now, we apply the same formula as before. So, this is the, these that equations you will get after the linear quadratics least square feet, ok.

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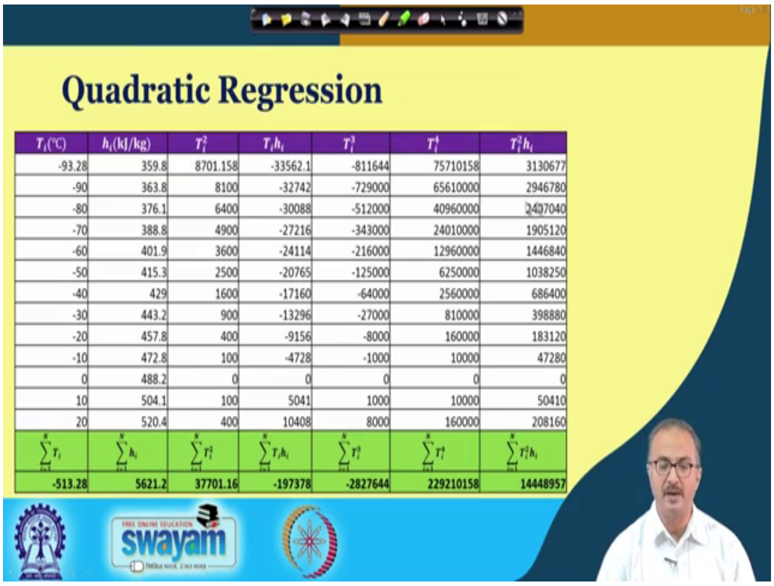

T_i(°C)	h_i(kJ/kg)	T_i^2	T_ih_i	T_i^3	T_i^4	$T_i^2h_i$
-93.28	359.8	8701.158	-33562.1	-811644	75710158	3130677
-90	363.8	8100	-32742	-729000	65610000	2946780
-80	376.1	6400	-30088	-512000	40960000	2407040
-70	388.8	4900	-27216	-343000	24010000	1905120
-60	401.9	3600	-24114	-216000	12960000	1446840
-50	415.3	2500	-20765	-125000	6250000	1038250
-40	429	1600	-17160	-64000	2560000	686400
-30	443.2	900	-13296	-27000	810000	398880
-20	457.8	400	-9156	-8000	160000	183120
-10	472.8	100	-4728	-1000	10000	47280
0	488.2	0	0	0	0	0
10	504.1	100	5041	1000	10000	50410
20	520.4	400	10408	8000	160000	208160
$\sum_{i=1}^{N} T_i$	$\sum_{i=1}^{N} h_i$	$\sum_{i=1}^{N} T_i^2$	$\sum_{i=1}^{N} T_ih_i$	$\sum_{i=1}^{N} T_i^3$	$\sum_{i=1}^{N} T_i^4$	$\sum_{i=1}^{N} T_i^2h_i$
-513.28	5621.2	37701.16	-197378	-2827644	229210158	14448957

And we are constraining this data which are given 0.05 bar and here you find that the if you go for higher degree regression then your mathematics also get more involved, you the time of computation also goes up.

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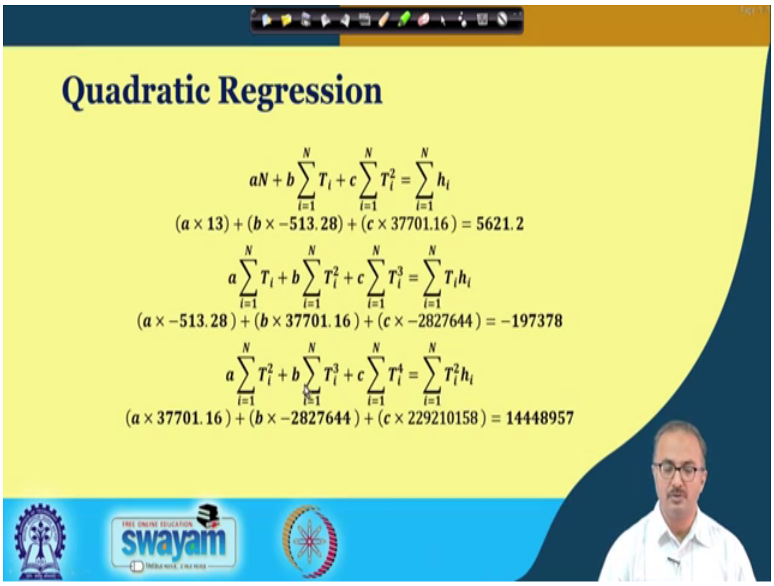

So, ultimately by putting all those values over here from the earlier chart we get the values of abc like this, ok.

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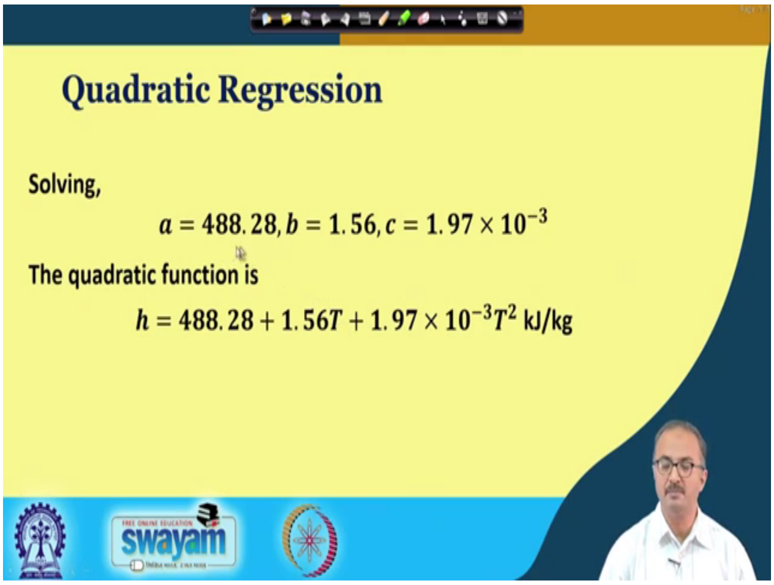

So, these are quadratic.

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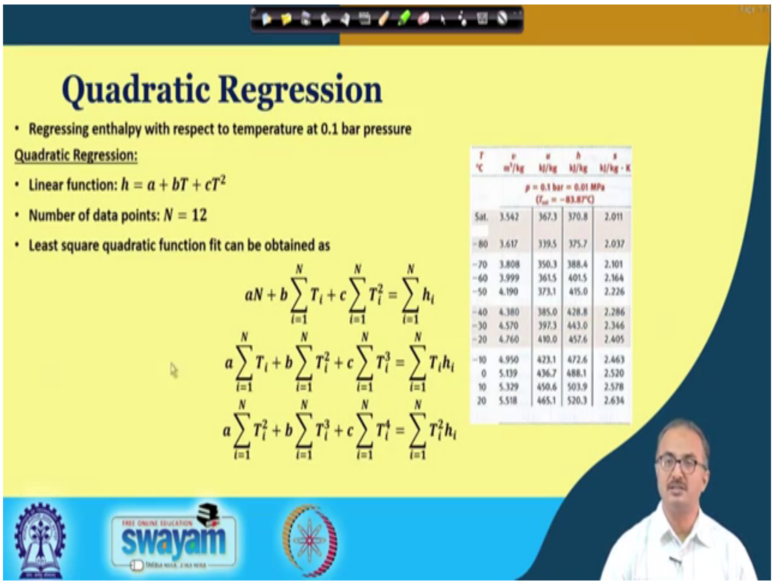

Now, similar way we also take some other pressures like 0.1 bar.

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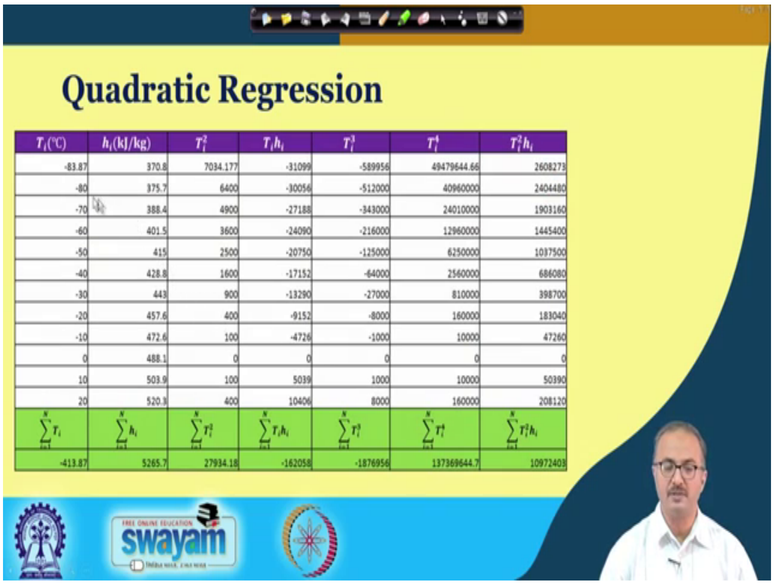

T_i(°C)	h_i(kJ/kg)	T_i^2	T_ih_i	T_i^3	T_i^4	$T_i^2h_i$
-83.87	370.8	7034.177	-31099	-589956	49479644.66	2608273
-80	375.7	6400	-30056	-512000	40960000	2404480
-70	388.4	4900	-27188	-343000	24010000	1903160
-60	401.5	3600	-24090	-216000	12960000	1445400
-50	415	2500	-20750	-125000	6250000	1037500
-40	428.8	1600	-17152	-64000	2560000	686080
-30	443	900	-13290	-27000	810000	398700
-20	457.6	400	-9152	-8000	160000	183040
-10	472.6	100	-4726	-1000	10000	47260
0	488.1	0	0	0	0	0
10	503.9	100	5039	1000	10000	50390
20	520.3	400	10406	8000	160000	208120
$\sum_{i=1}^{N} T_i$	$\sum_{i=1}^{N} h_i$	$\sum_{i=1}^{N} T_i^2$	$\sum_{i=1}^{N} T_ih_i$	$\sum_{i=1}^{N} T_i^3$	$\sum_{i=1}^{N} T_i^4$	$\sum_{i=1}^{N} T_i^2h_i$
-413.87	5265.7	27934.18	-162058	-1876956	137369644.7	10972403

And here we put these values of this various coefficients.

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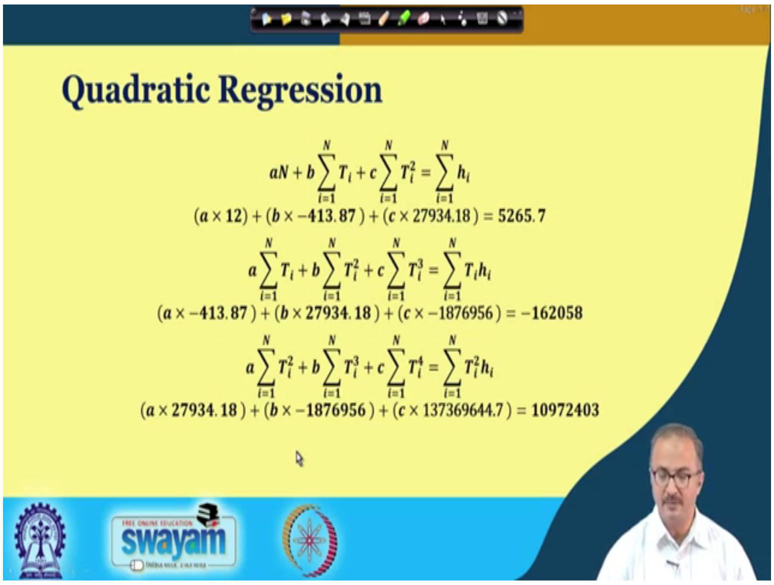

Then we find that we are getting the values of the abc like this.

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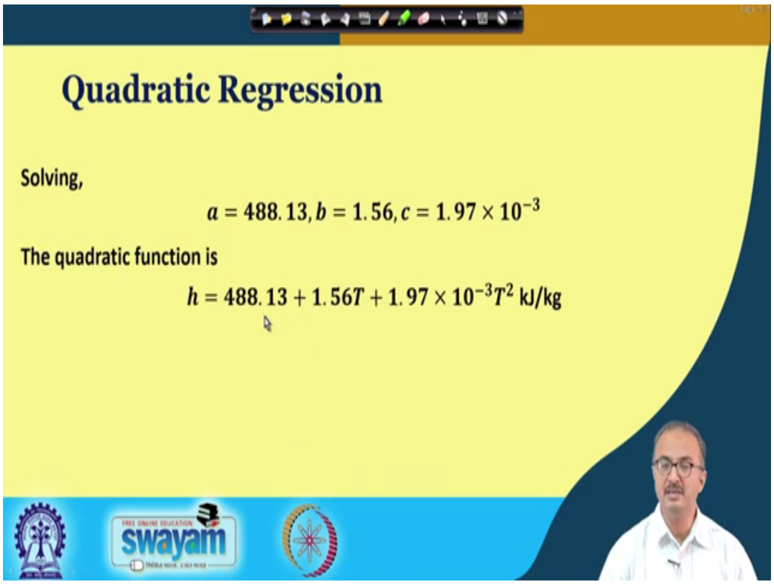

So, we find that there is slight change in the value of the abc.

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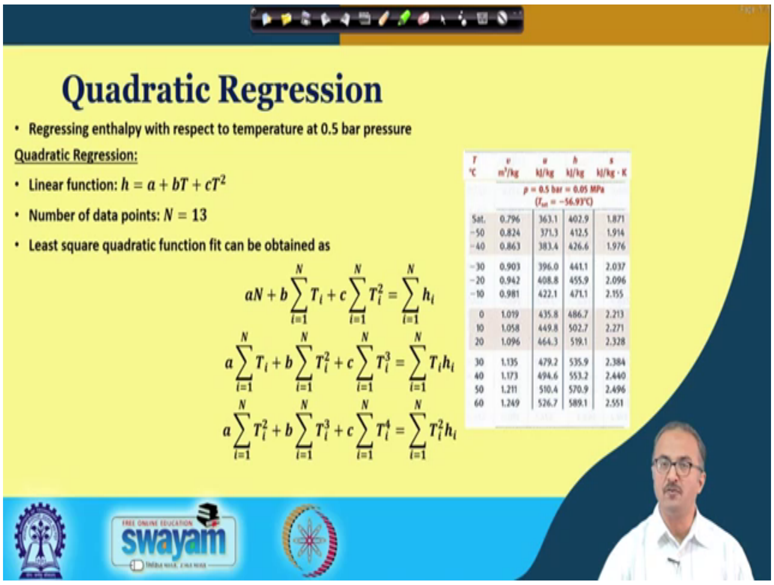

We go for another higher pressure 0.5 bar again we repeat the same things.

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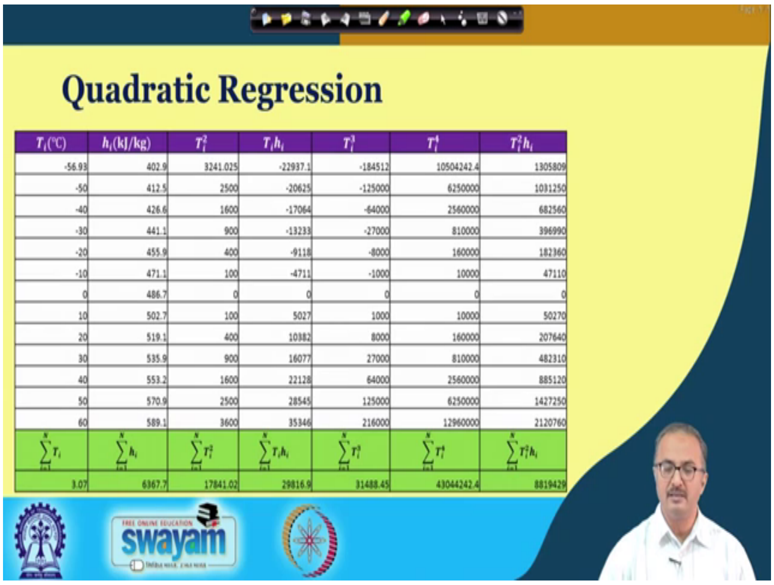

And we find that these are the values of the various variables.

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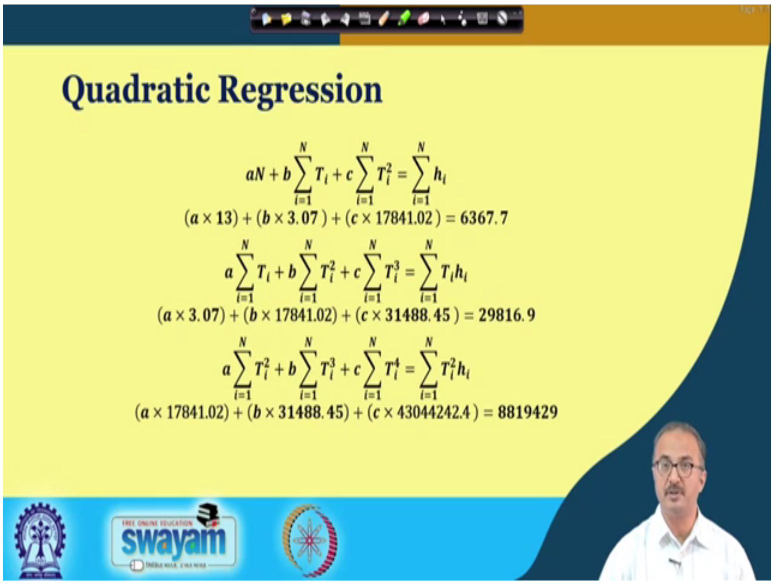

And these are the equations in which were plugging in the variable values.

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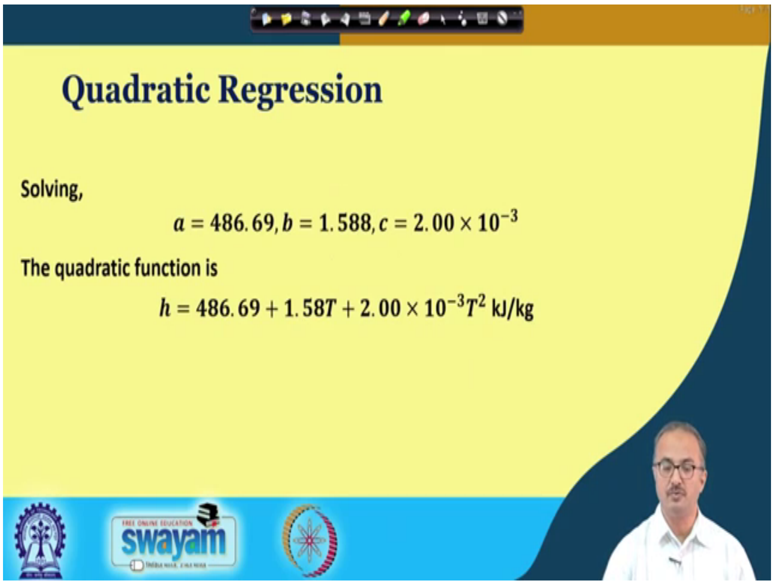

And here, here we have the another enthalpy function, with respect to temperature.

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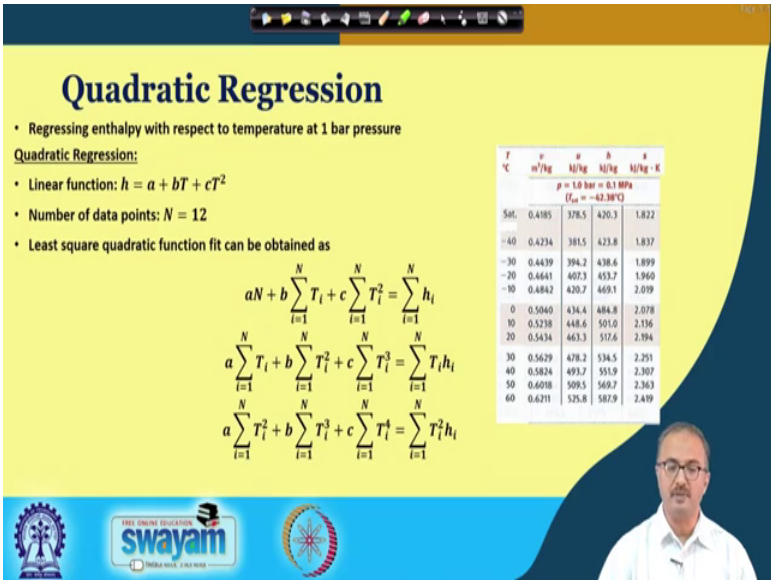

Then we go for 1 bar pressure here.

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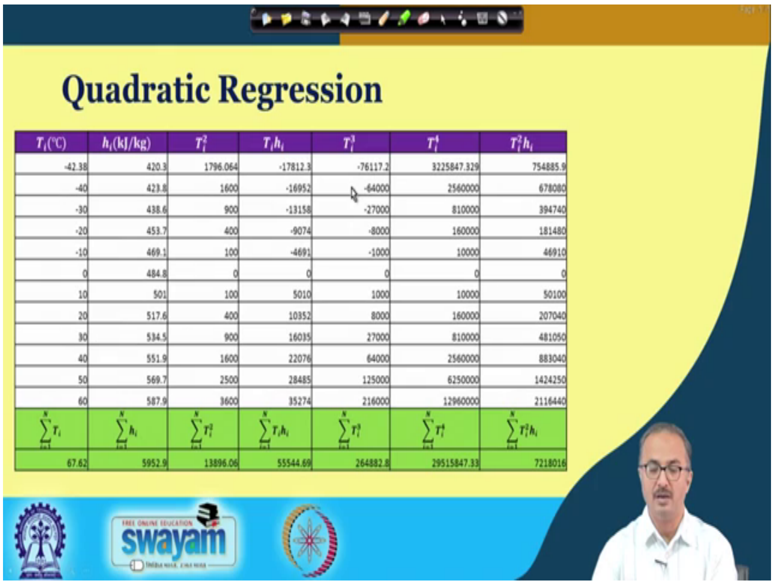

Similarly, repeating the same procedure we get the this quadratic function for enthalpy for another pressure.

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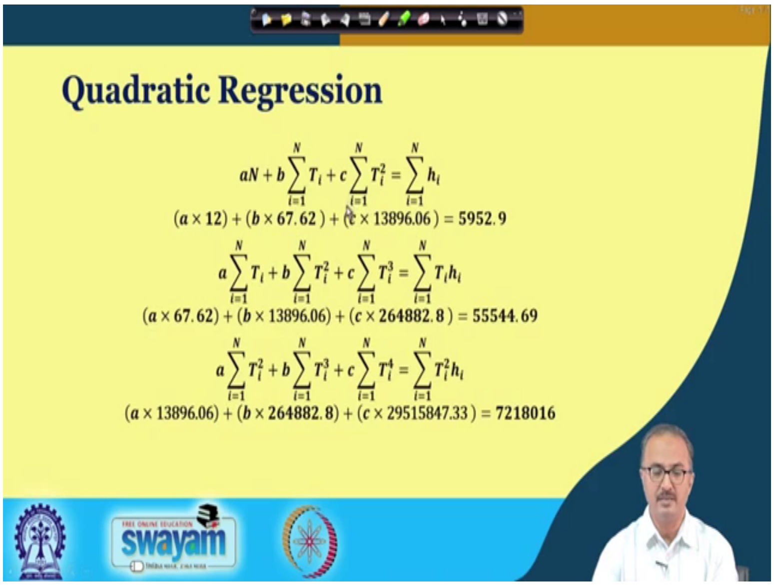

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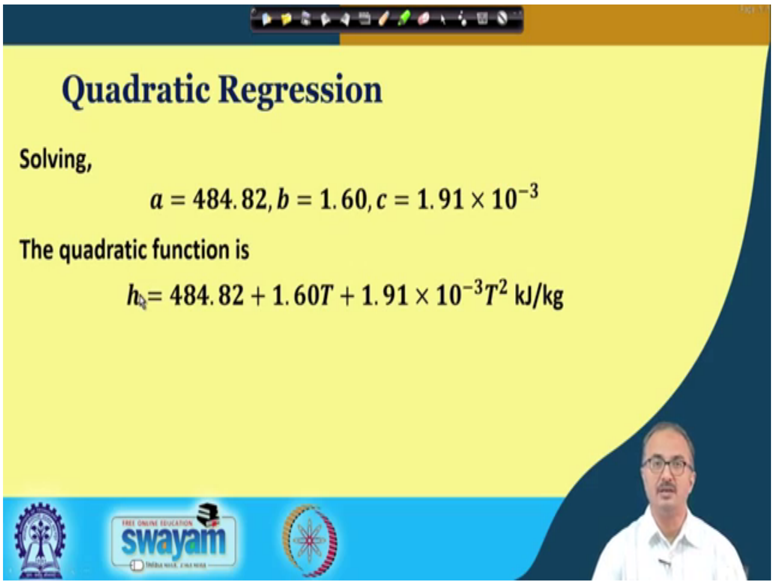

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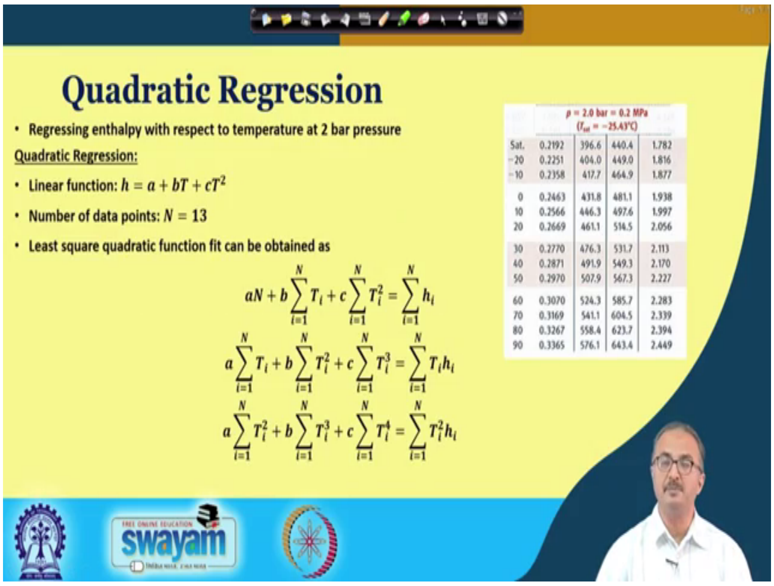

So, if we keep repeating this for different pressures you will find that these similar calculations are there and we are getting the values of abc, ok.

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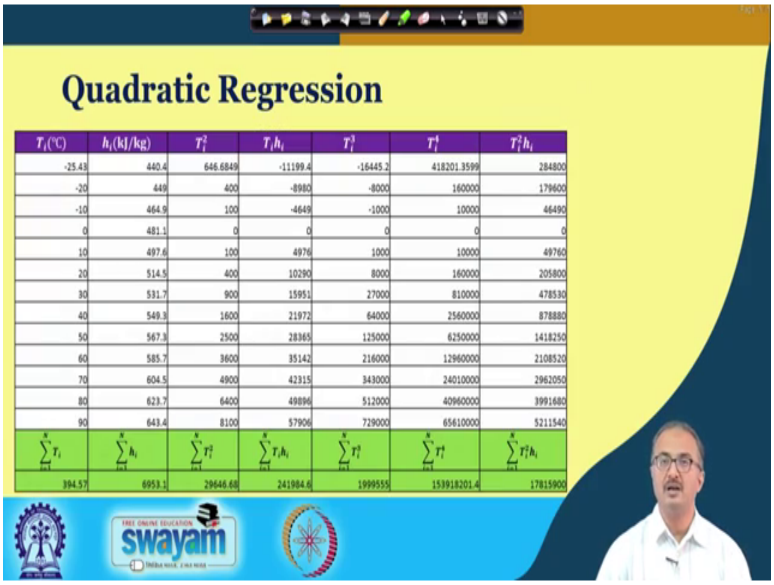

T_i(°C)	h_i(kJ/kg)	T_i^2	$T_i h_i$	T_i^3	T_i^4	$T_i^2 h_i$
-25.43	440.4	646.6849	-11199.4	-16445.2	418201.3599	284800
-20	449	400	-8980	-8000	160000	179600
-10	464.9	100	-4649	-1000	10000	46490
0	481.1	0	0	0	0	0
10	497.6	100	4976	1000	10000	49760
20	514.5	400	10290	8000	160000	205800
30	531.7	900	15951	27000	810000	478530
40	549.3	1600	21972	64000	2560000	878880
50	567.3	2500	28365	125000	6250000	1418250
60	585.7	3600	35142	216000	12960000	2108520
70	604.5	4900	42315	343000	24010000	2962050
80	623.7	6400	49896	512000	40960000	3991680
90	643.4	8100	57906	729000	65610000	5211540
$\sum_{i=1}^{N} T_i$	$\sum_{i=1}^{N} h_i$	$\sum_{i=1}^{N} T_i^2$	$\sum_{i=1}^{N} T_i h_i$	$\sum_{i=1}^{N} T_i^3$	$\sum_{i=1}^{N} T_i^4$	$\sum_{i=1}^{N} T_i^2 h_i$
394.57	6953.1	29646.68	241984.6	1999555	153918201.4	17815900

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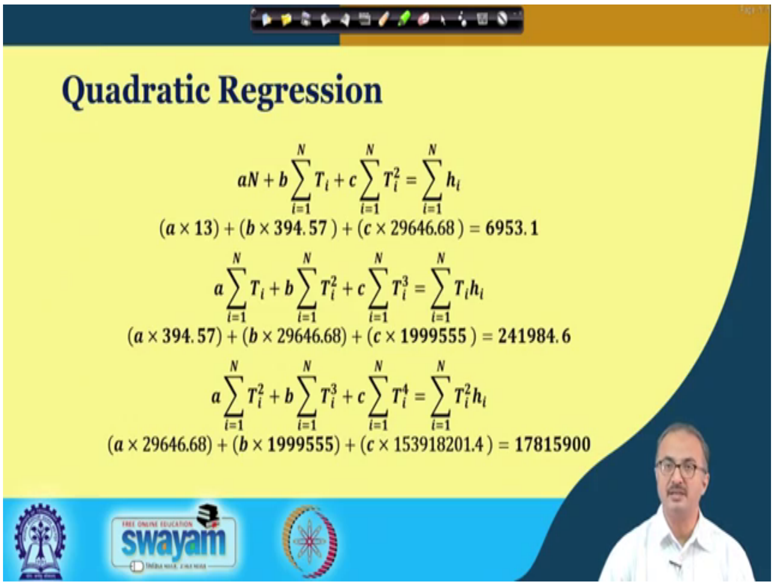

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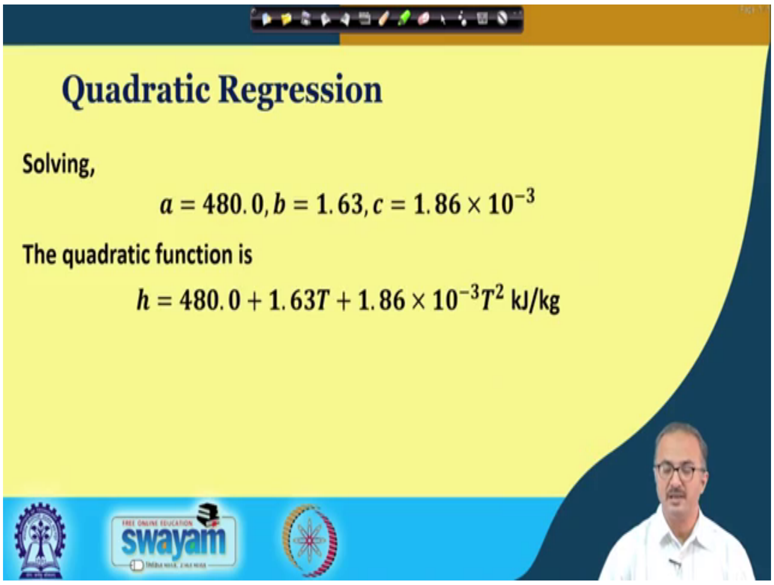

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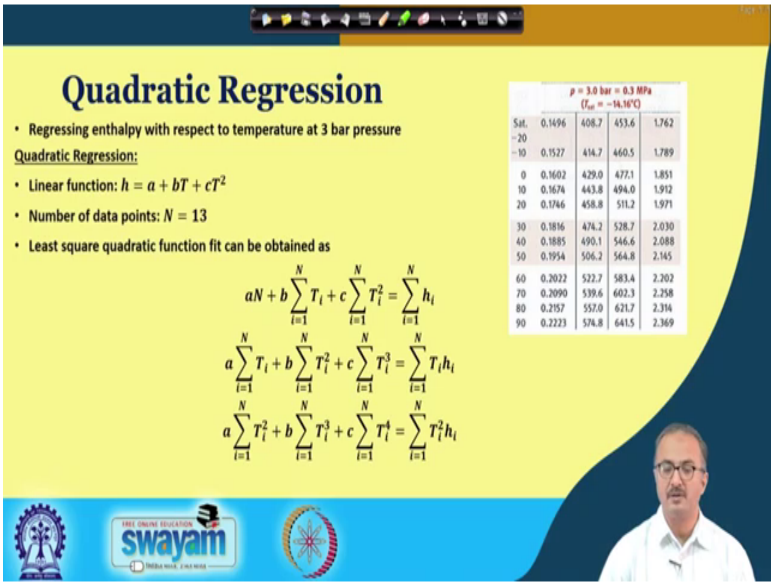

	p = 3.0 bar = 0.3 MPa (T_{sat} = −14.16°C)			
Sat.	0.1496	408.7	453.6	1.762
−20				
−10	0.1527	414.7	460.5	1.789
0	0.1602	429.0	477.1	1.851
10	0.1674	443.8	494.0	1.912
20	0.1746	458.8	511.2	1.971
30	0.1816	474.2	528.7	2.030
40	0.1885	490.1	546.6	2.088
50	0.1954	506.2	564.8	2.145
60	0.2022	522.7	583.4	2.202
70	0.2090	539.6	602.3	2.258
80	0.2157	557.0	621.7	2.314
90	0.2223	574.8	641.5	2.369

Now, next we go for 3 bar pressure.

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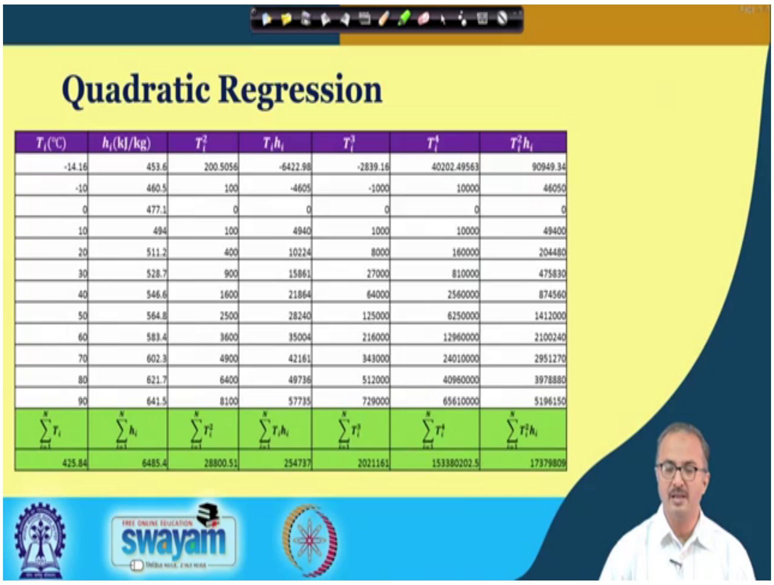

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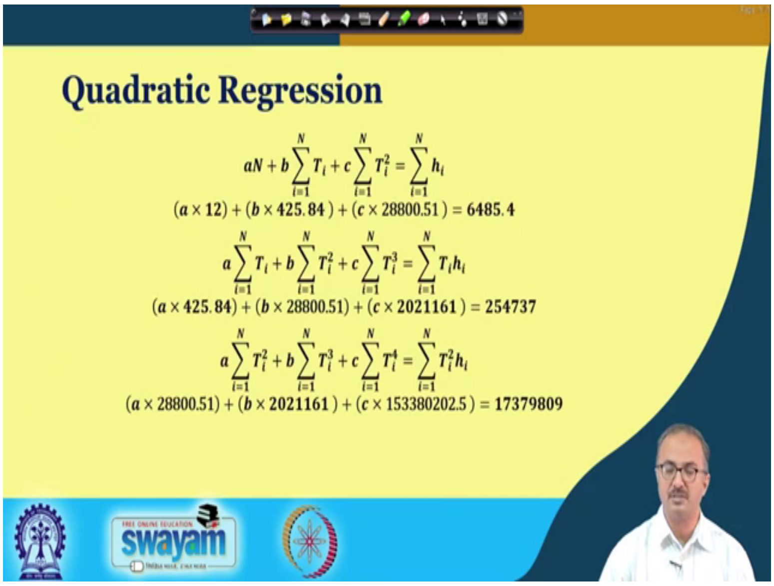

And again, we do all these calculations and we find this is the value of the, this is the regressed equation for enthalpy with respect to temperature and at 3 bar pressure.

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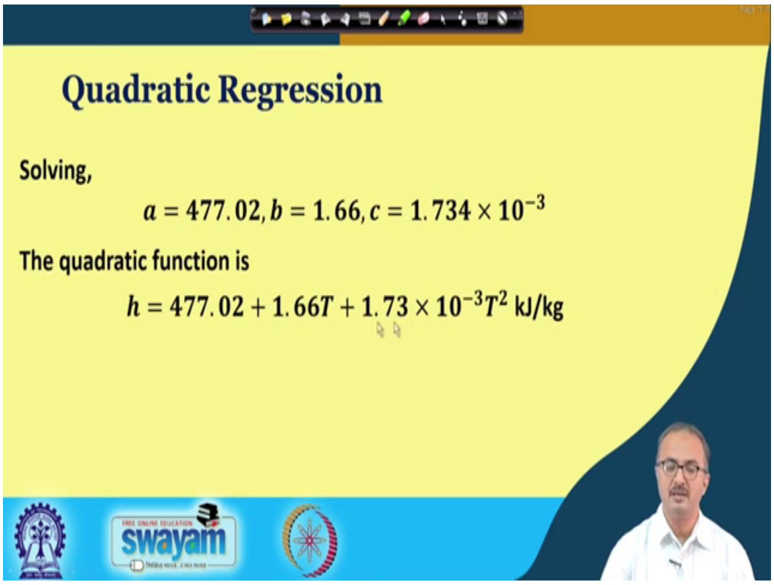

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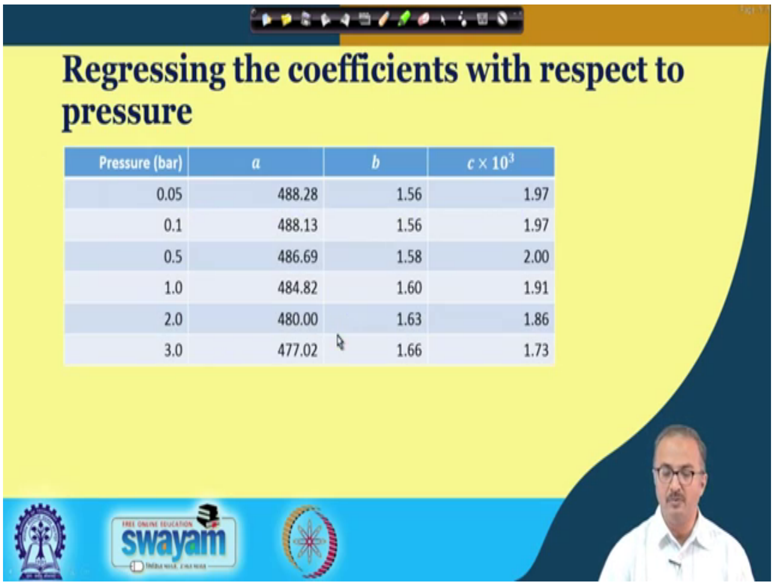

Now, summarizing what we are doing now, that will for the quadratic interpolation we put the values of abc over here and we find that for each of them there is slight change, ok. And anyway to for illustration purpose we again fit this abc with respect to pressure independently.

One thing you must notice that in going from the linear fit to the quadratic fit, we are hardly getting any change in the values of a and b, that means, for that c does not play much of a role that means, quadratic x term it does not play a significant role in this particular interpolation, but this is in this particular case it may not be so, far other cases.

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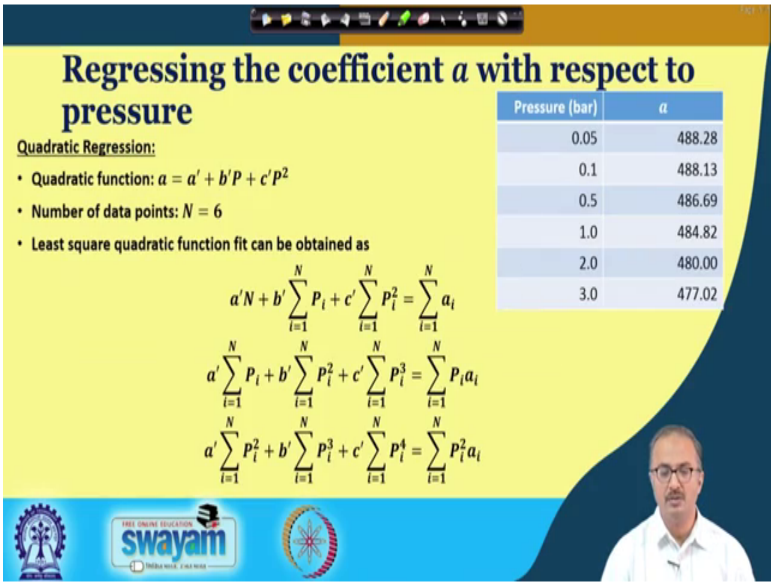

So, again we assume that a to be a quadratic function of P, ok. So, here we have this expression and again we know the number of data points with us. These are the equations we get after the least square fit.

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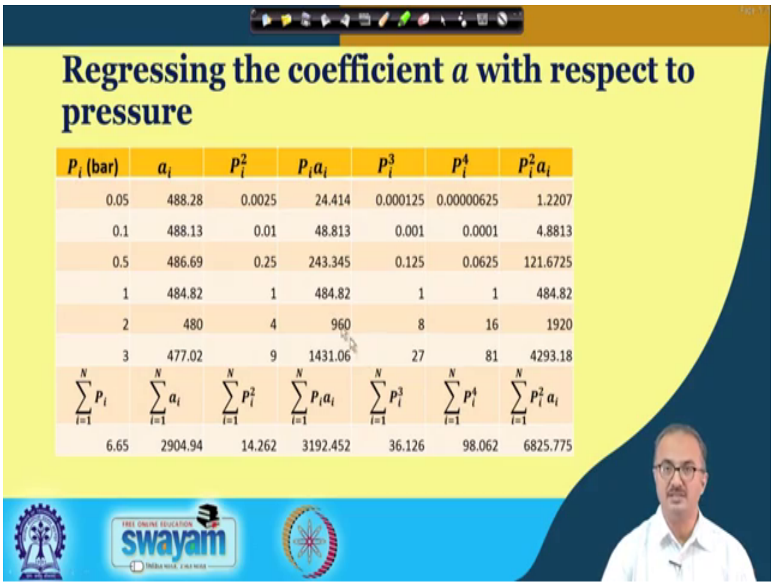

And this is the for table which is giving the values of various types of coefficients we need for the regression.

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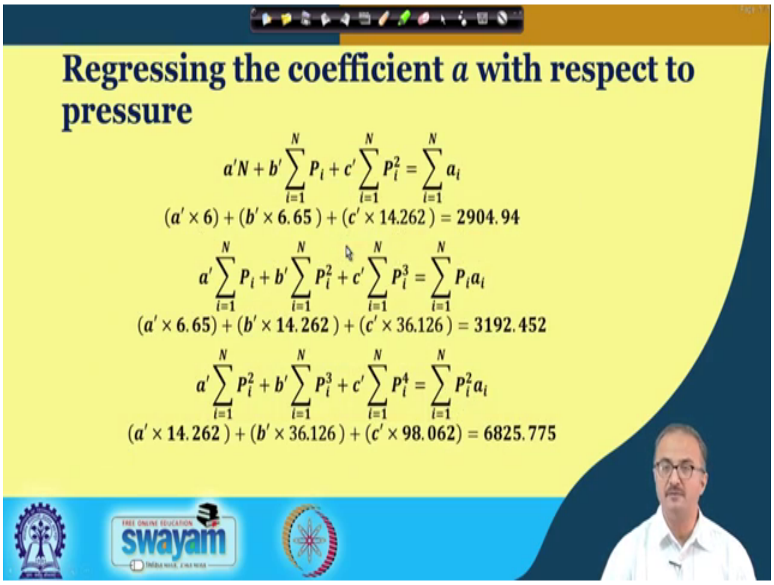

And here we have those equations where we have plugged in the values of the various coefficients.

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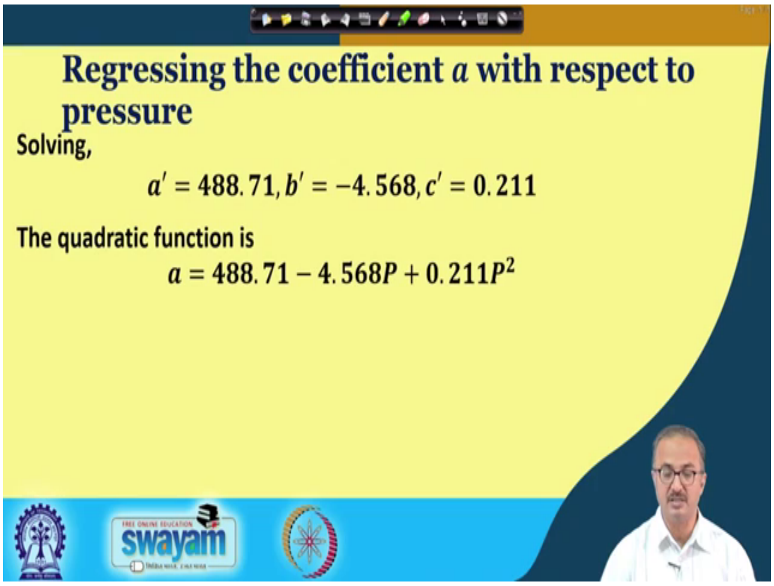

And here we have the a as a function of P and now, we see that we have put a quadratic functionality of a with respect to P.

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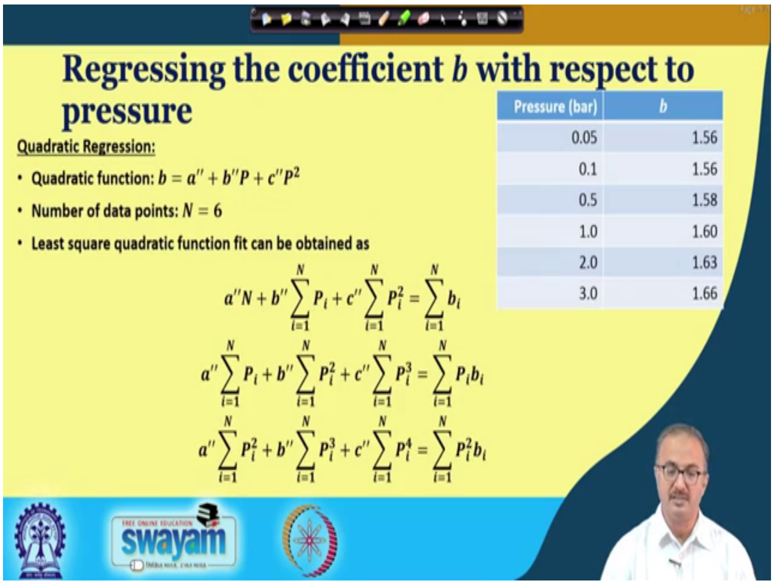

Similarly, we go for b. Again, we have this expression for b with respect to pressure.

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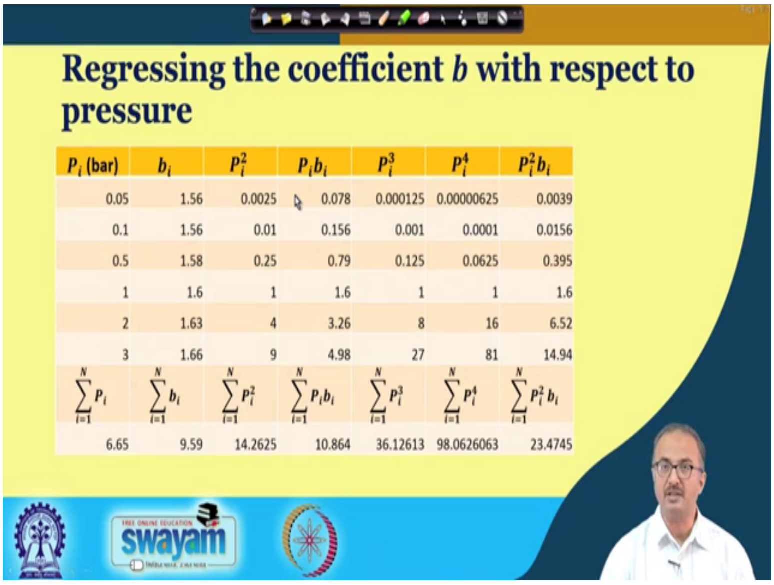

These are the various values, we need to regress.

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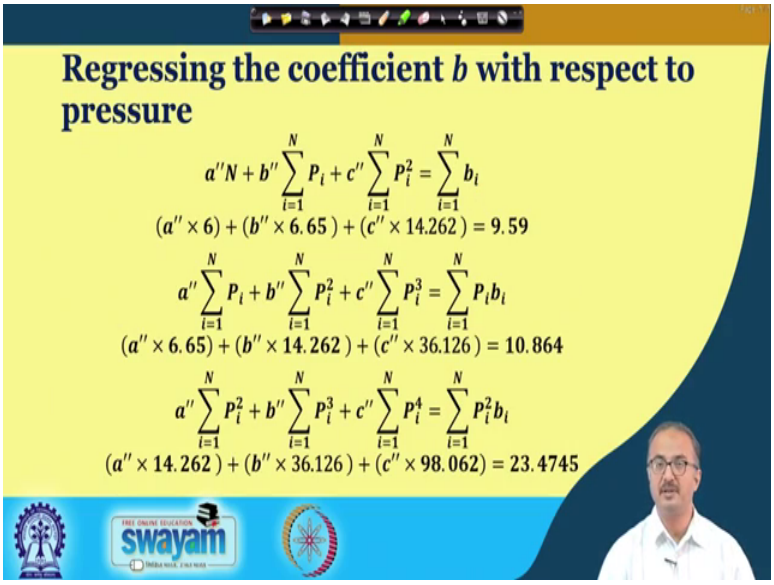

And here we plug in those values in the from the table and ultimately, we get a quadratic variation of b with respect to pressure, ok.

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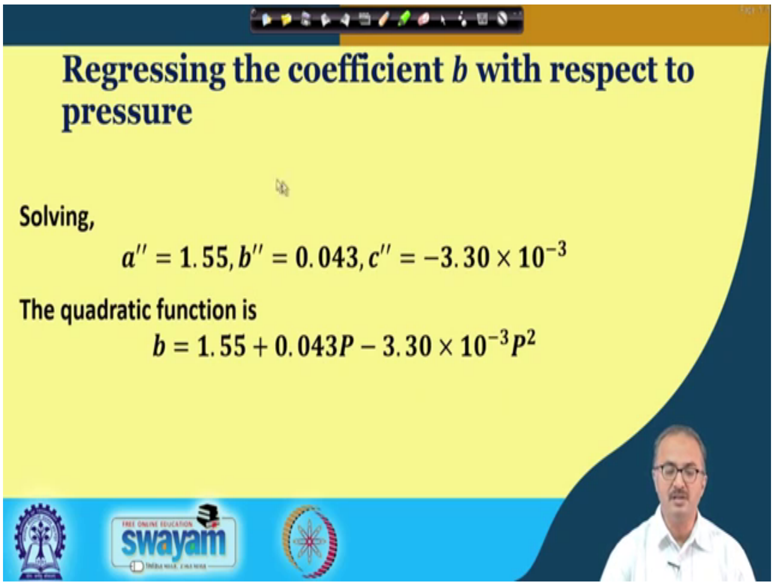

Now, with these two expressions of a and b with respect to pressure, what we do that we can figure out that how this will be changing.

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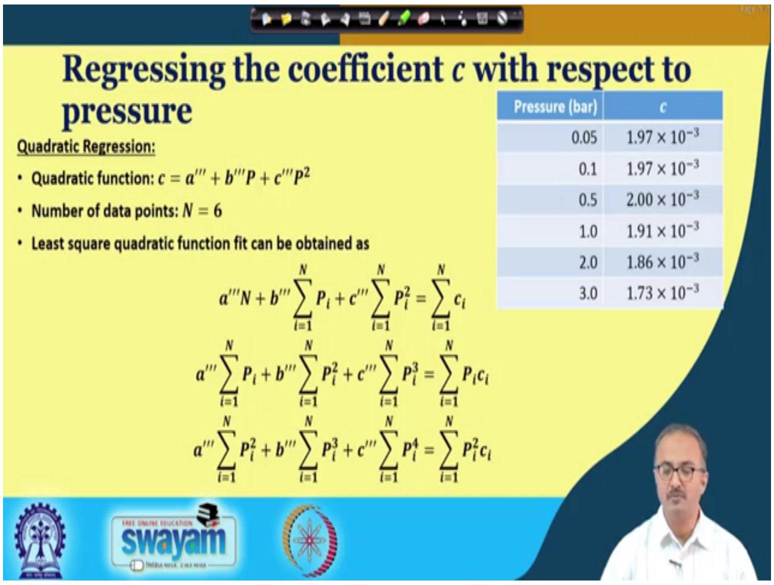

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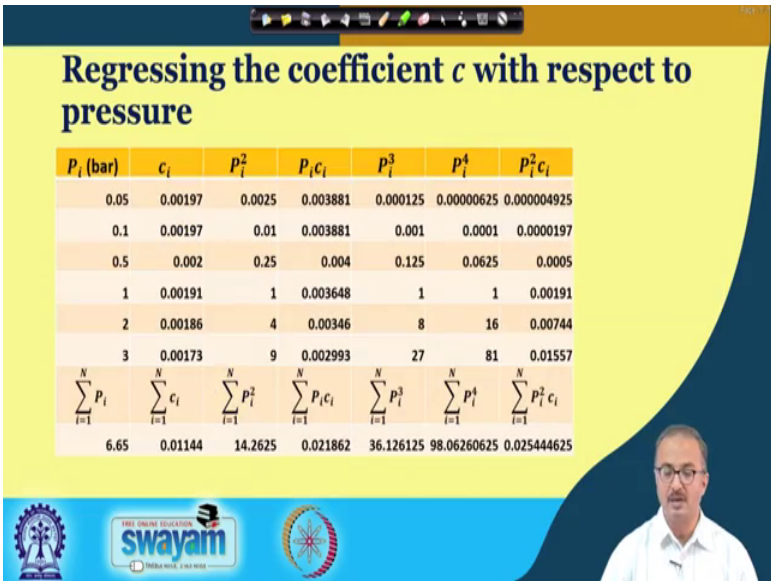

And we can figure out that how we are going to get the expression for the enthalpies, ok.

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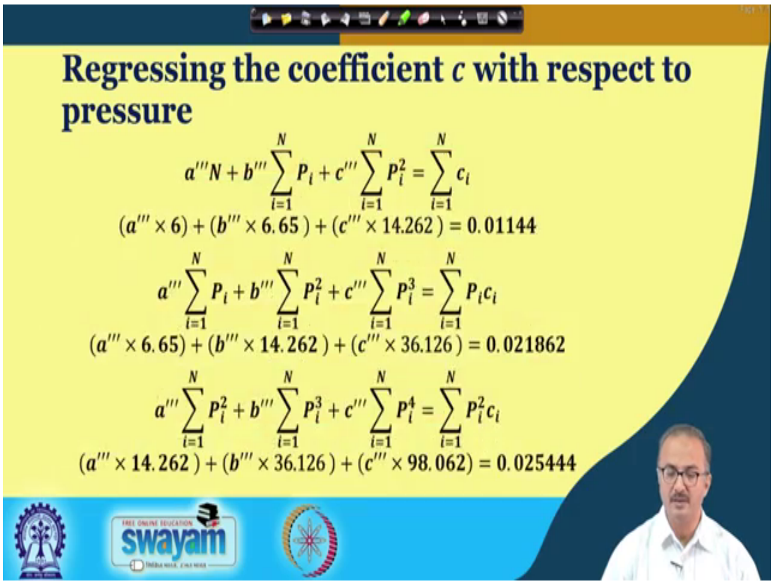

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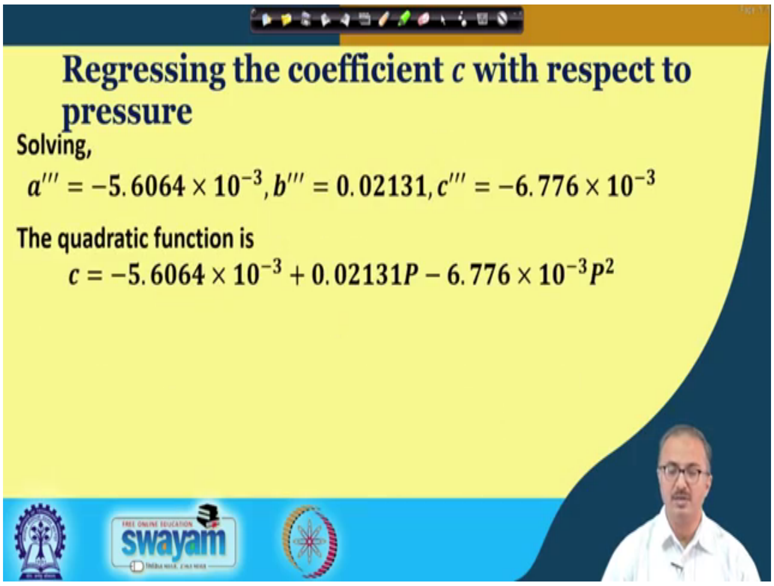

So, here the for the c.

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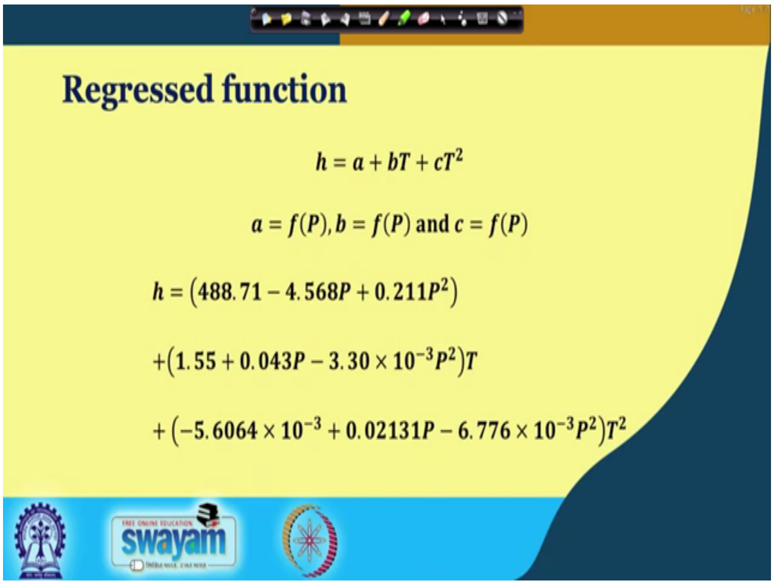

And here we have the final expression that h equal to a plus bT plus cT square that we assumed, ok. And then for a we put this expression, for b we put this expression, and c we put this expression and this way we are able to get the expression for enthalpy which depends on both pressure and temperature, ok.

So now, you see that unlike the interpolation we have got some kind of functionality of the enthalpy with the temperature and pressure, ok. Of course, in this case we have assumed the functionality and in real life that when this functionality will be assumed as per either the theory or the experimental data. So, here as I told you in my earlier lecture that either you can take some algebraic expression, you can also go for transcendental expression that means, that will be involving logarithm or some got hyperbolic functions or exponential function that also you can assume, ok.

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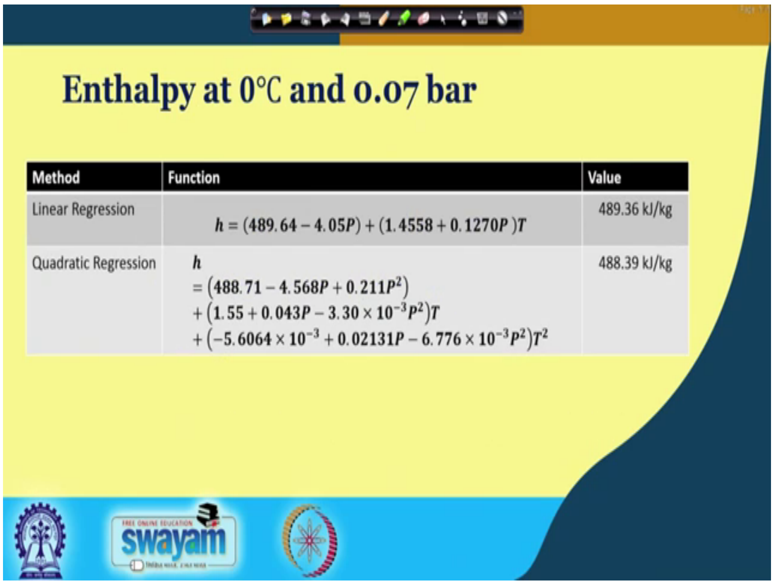

Now, here we have these two expressions of regressed enthalpy, one is h for the linear regression this is the thing we are getting and this is the one we have got for the quadratic regression. And here what we do now, we find the value of the enthalpy at this given temperature and pressure simply by plugging in the values over in this two equations. And what we find here is this that these two values are coming almost quite nearby each other.

Now, you see that how to decide that which kind of regression we should do. Suppose, we do not have the exact values available to us, in that case what we generally do is this we kind of apply this various regression formulae and we get this kind of values and check these values if they are coming, if their variations are not great then we choose the one which is simpler to adopt, ok. So, this is how we have performed the regression to get the values of the enthalpy at the given pressure and temperature.

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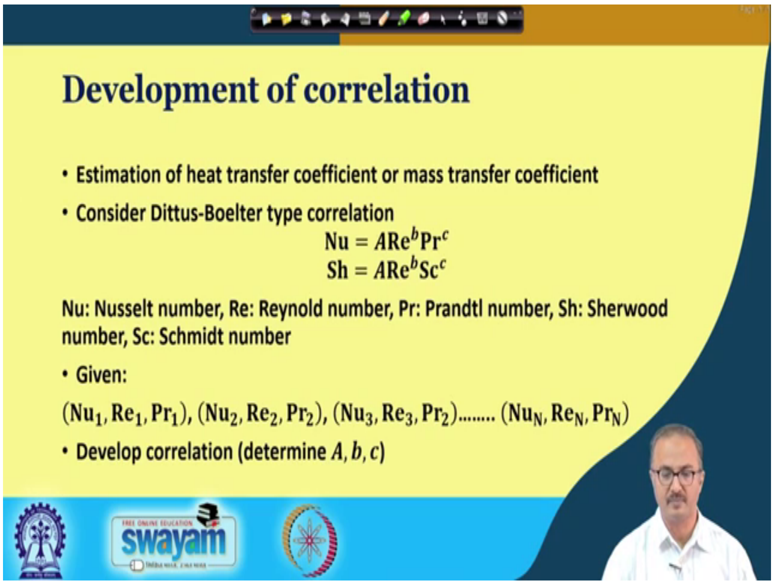

Now, this same thing can be adopted to develop the various correlations, ok. You come across many correlations and especially for example, in heat transfer mass transfer problem we obtain this heat transfer coefficient or mass transfer coefficient and many a times they are obtained from various correlations.

For your understanding I have just considered one of the very common ways of a estimating these heat transfer coefficient and the mass transfer coefficient. Here we have what we call Dittus Boelter type equation, and here we have put a generalized form that Nusselt number is equal to some constant a, then Reynolds number to the power b and Prandtl number to the power c. In similar fashion I we can also write Sherwood number equal to some constant a, Reynolds number to the power b, a Schmidt number to the power c.

Now, as you know that this Nusselt number involves the heat transfer coefficient and that is kl by d, k is the heat transfer coefficient, l is the some characteristic length and d is the say thermal diffusivity, ok. So, this way you can find this. In case Sherwood number it will be again similar expression, but in case in the heat transfer coefficient will be replaced by the mass transfer coefficient and the thermal diffusivity will be replaced by the mass diffusivity or simply diffusion coefficient, ok. And the Prandtl number as you learnt earlier its simply gives us the ratio of the thermal diffusivity to momentum diffusivity and the Schmidt number is giving us the momentum sorry the Prandtl number gives you the momentum diffusivity to thermal diffusivity as Schmidt number gives us a momentum diffusivity to the mass diffusivity, ok.

Now, when you want to develop such kind of correlations I will not go into specific examples, I will simply tell you the methodology. That suppose you are carrying out some experiment, and from those experiments you get the values of Nusselt number Reynolds number and Prandtl number, and you will find that you will be given a set of these values, ok. So, you see that you are given I have put in the parenthesis that for some Nusselt number Nu 1 we have with respect to Re 1 and Pr 1, ok. So, we you get such kind of sets of values, ok.

Now, in this case our problem is to find out the value of a, b and c. So, let us see how we do it.

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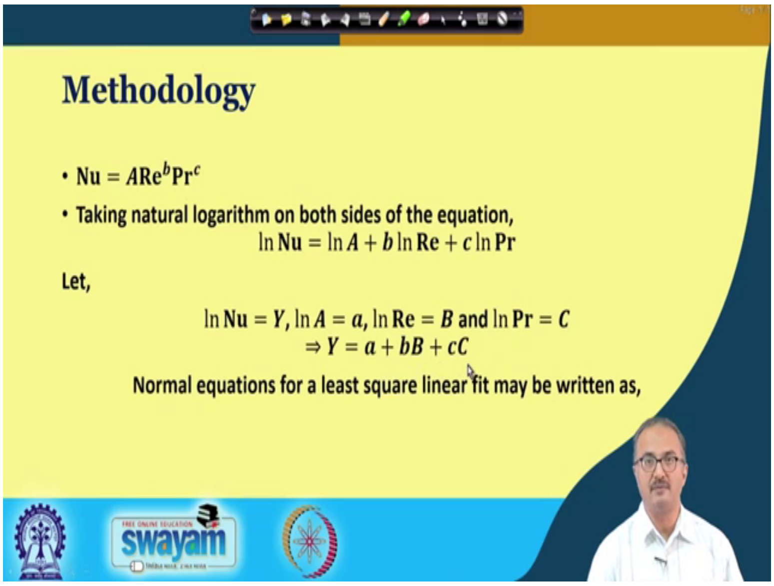

So, as earlier we saw that whenever we have a power law kind of equation, we take a logarithm. So, here we take a logarithm, and here we have to logarithm take a logarithm we get this expression. Now, what we do? We replace this log Nu by Y, log capital A by small a, log Re by capital B and log Pr by capital C. So, here we have those expressions.

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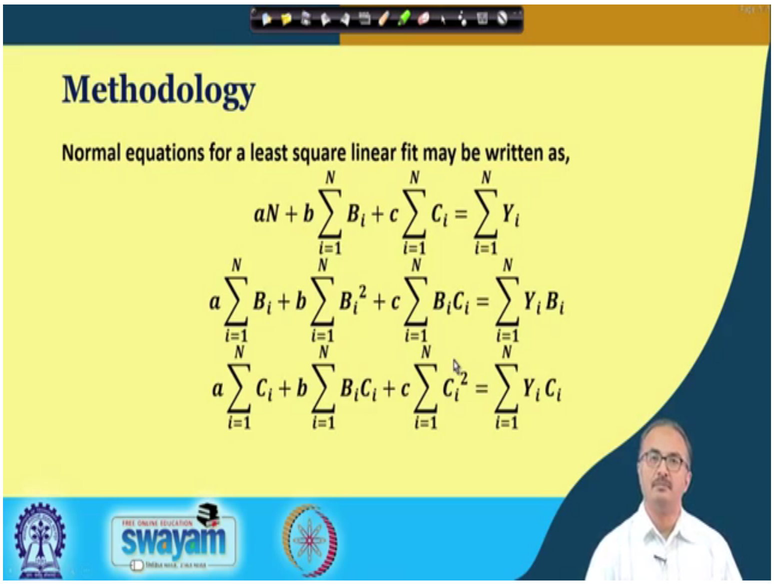

Now, you can now, apply the least square fit. So, here you have all these expressions which you get by the least square fit and then you can put these expression, these expressions what you obtain here in a matrix form.

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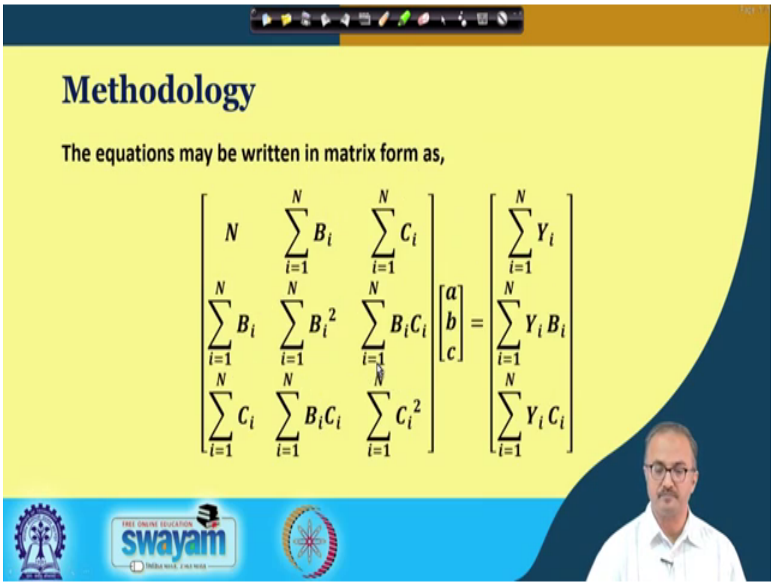

Now, when you put in a matrix form you see that this matrix is coming out to be symmetric, and this is generally the case in these kind of correlations. So, we get many a times this kind of symmetric matrix. Now, when we get symmetric matrix then we have to solve for this coefficients here abc we adopt some special methods matrix methods for this symmetric matrix. And one of these methods is the Cholesky method, ok.

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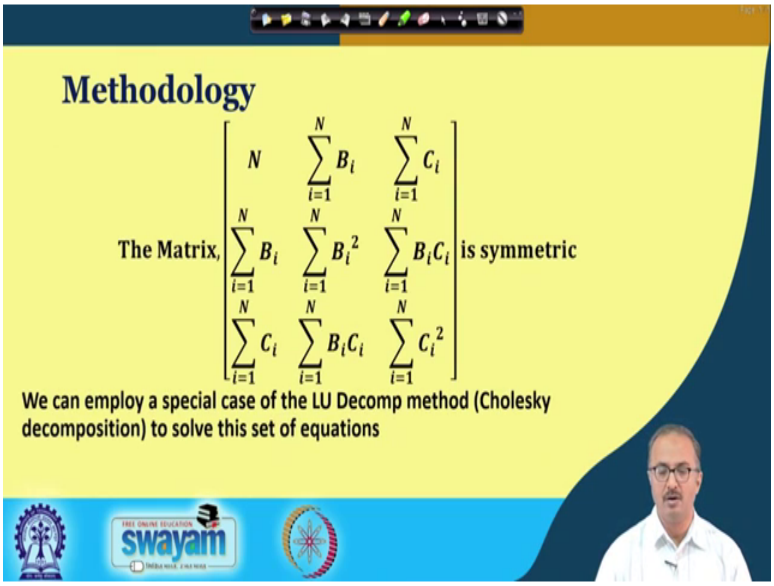

And this Cholesky method is based on the LU decomposition, that is lower triangle and upper triangle decomposition.

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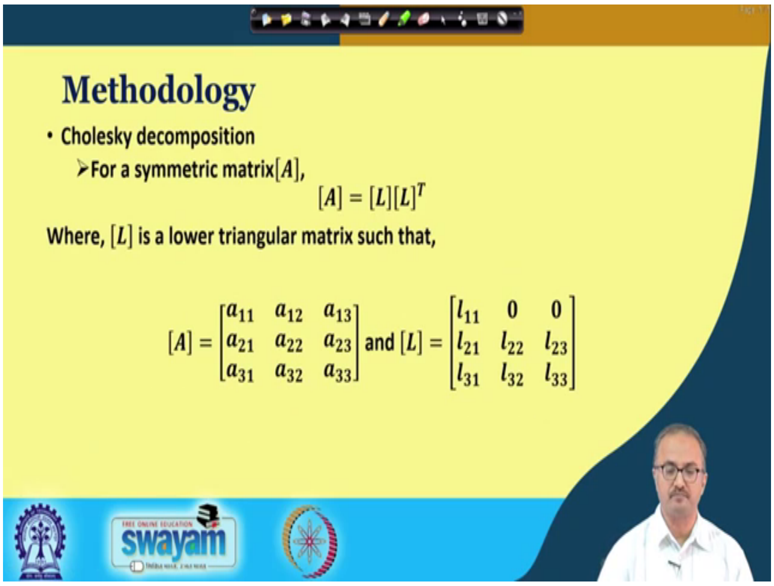

So, here we see that for symmetric matrix we find that the L and U matrices are transpose of each other that means, whenever you are writing L, L that means, a U matrix is transpose of L matrix, ok. So, here that means, what, that we need to only find the l matrix, ok. So, this is the kind of matrix we shall have. So, it will be L matrix it will be 0, ok. So, this kind of a matrix system we should get.

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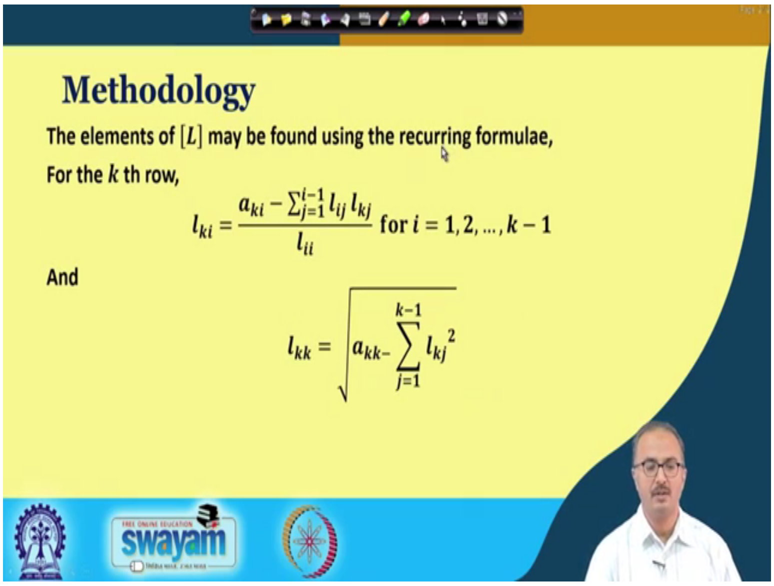

Now, to get this L matrix in the literature or in any standard numerical techniques book you will find that this is the recurring formula to obtain the values of the various coefficients in the lower triangular matrix, ok.

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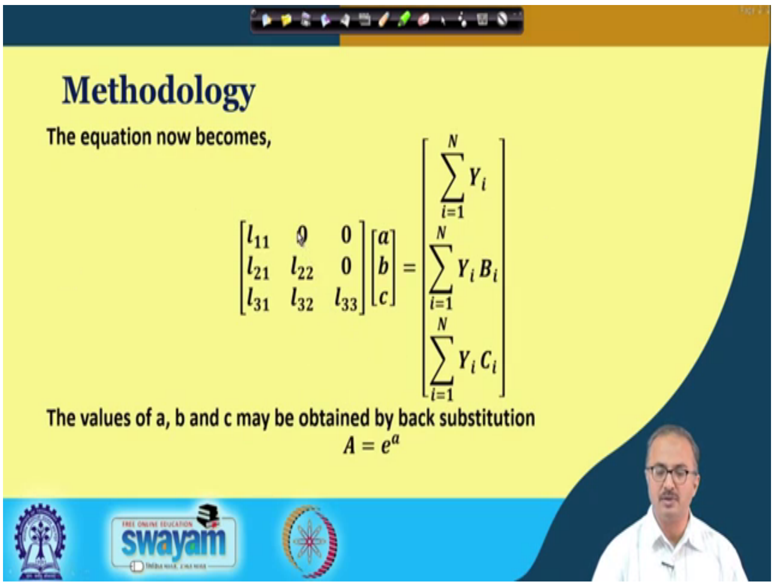

So, these this formula you use to get the L matrix and now, once you get these values of this elements then it is very easy for you. You can just do a forward swap that is you can start with the first row to get the value of a and then with this value of a you can get the value of b from the second row and then you get the third row to get the value of c from the values you obtained from a and b, ok. And then after that what you know that this a can give you the value of the capital A, which is there in the original equation and with this now you have the total correlation with you.

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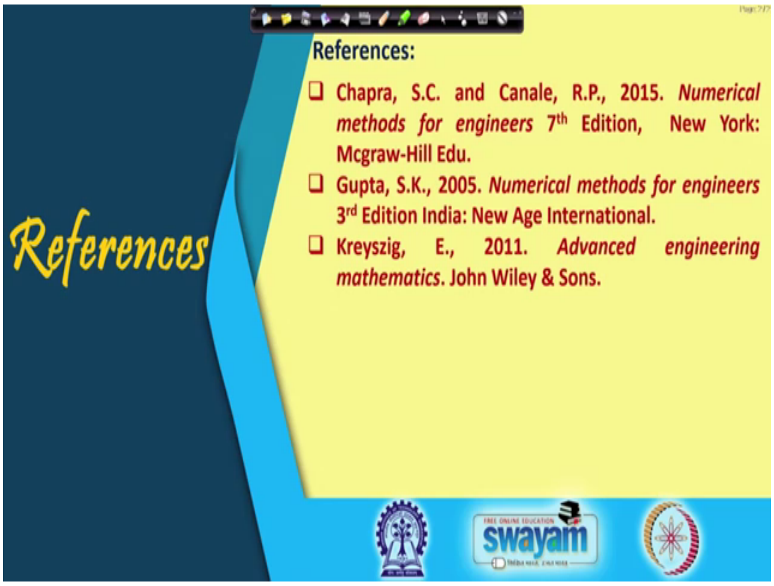

Now, you see that the same method, here we have shown one method, but this similar way you can also do it with other methods of regression. And you can obtain the similar expressions for this Nusselt number and maybe for other kind of correlations. So, these are the references you can refer to get some more idea about these methods.

Thank you.