Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 11
Macroscopic Balances (Contd.)

Welcome. Today we shall do a few problems based on the Macroscopic Balances.

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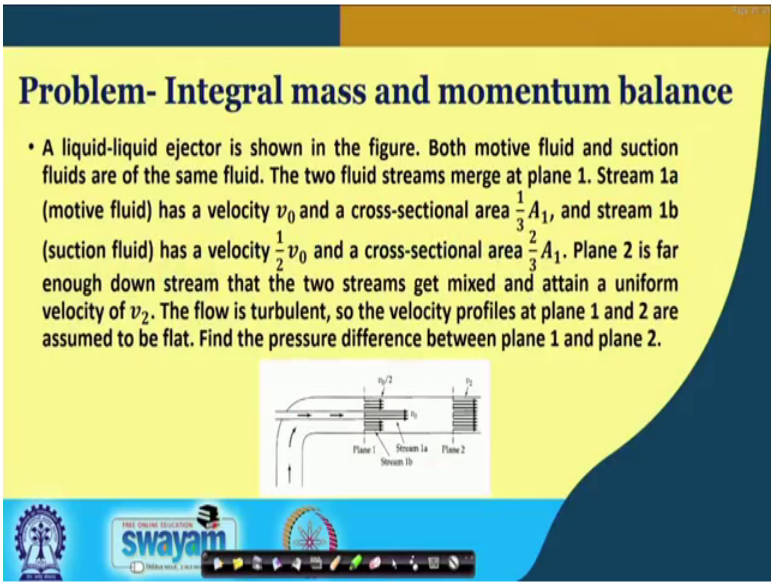

So, first let us look at this problem. In this problem we have a liquid-liquid ejector and here this is an application of pumping out a fluid. And in this perhaps you know that we use here what we call a motive fluid and a suction fluid. And this is the working of this particular ejector is based on the Bernoulli's theorem in the sense that inside this thin tube what you find some motive fluid comes. And here it comes as an through a nozzle and what happens that it velocity increases and that cross as a decrease in the pressure and this decrease in the pressure causes suction fluid from another channel.

So, this is the broad way the working of a liquid-liquid ejector. So, in this particular problem we have been told that the two fluid streams are of the same fluid and they merge at a plane one. So, here you see this is a plane 1, this plane one is situated right after the inner tube and so, one liquid is coming from here another is coming from here and then they mix together.

So, here it is said the stream 1 A that is the motive fluid as a velocity v naught and cross sectional area of 1 by 3 A1. So, here when it comes to this so, this is the total area is a 1. So, this small cross section is one-third of the total area and here the velocity is v naught and stream 1 B that is a suction fluid has a velocity of half v naught and a cross sectional area of two-thirds of the a 1; that means, the rest of the area that is that is if you would subtract the total area minus this area you get two-third. So, this rest of the area is two-third of A1.

Plane 2is far enough downstream that the two streams get mixed and attain a uniform velocity v2; that means, we have put a plane two over here and this is taken a sufficiently long distance from the inlet. So, that a it is ensured that both the fluids. I have got mixed thoroughly and also they attain a uniform a velocity ok.

The flow is turbulent so, the velocity profiles at plane 1 and 2 are assumed to be flat you know that turbulence causes a good mixing of the various fluids and this mixing causes a flat profile that is there is no distribution of the velocity along the radial direction ok. So, we have uniform velocities at this both the planes at plane 1 and plane 2. So, we have been asked to determine the pressure difference between plane 1 and plane 2. So, this is the problem.

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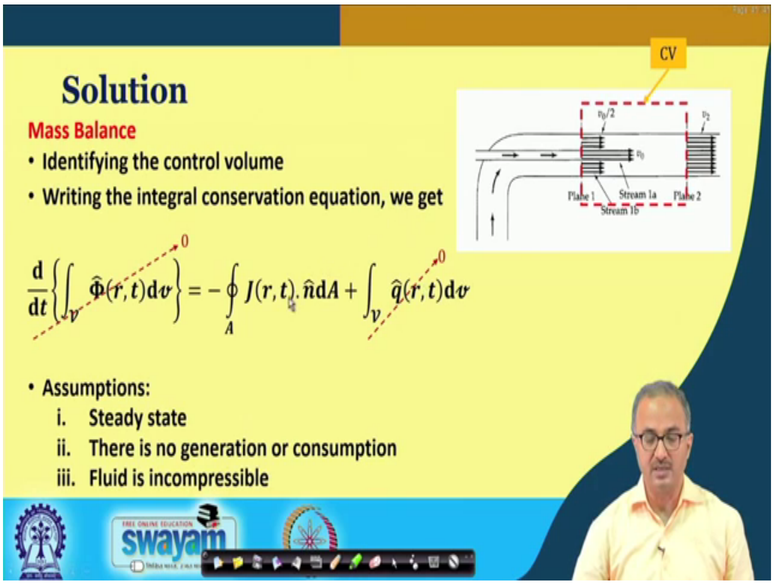

So, let us see how we do it. So, first let us look into the mass balance for that first we need to select the control volume. So, we select the control volume in a such a manner that it is in composing both plane 1 and plane 2 and we find that something is going inside this particular plane 1 and here is going out at the plane 2.

So, now we put cover integral conservation equation here and the assumption is that it has steady state. So, this first term goes to 0. Next assumption is there is no generation or consumption. So, that this term also goes to 0. So, what we are left with is this particular term and for this particular term. We assume that the fluid is incompressible that is the density is taken to be constant throughout the flow.

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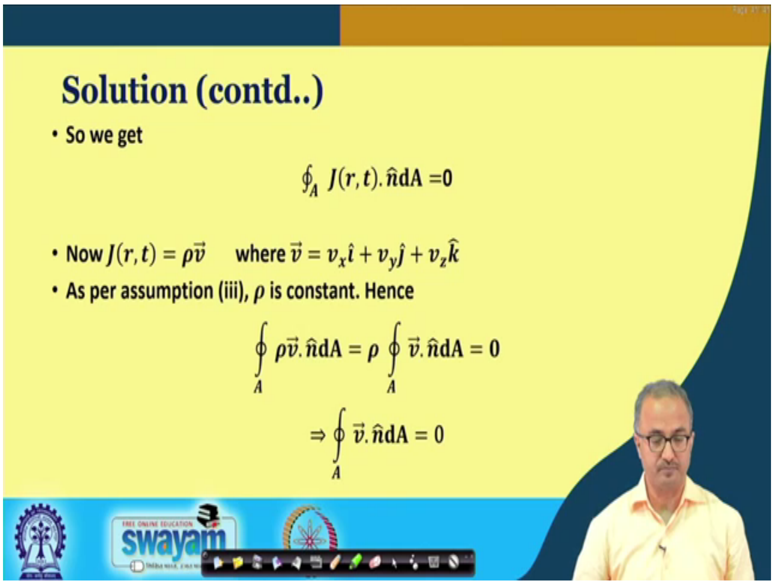

Now, once we write this particular expression from the conservation law and what we do for this j. We write rho into the velocity and velocity has 3 components and this density is constant. So, it may be taken out of the integral. So, we are left with this particular equation that is v dot n dA over the all the area.

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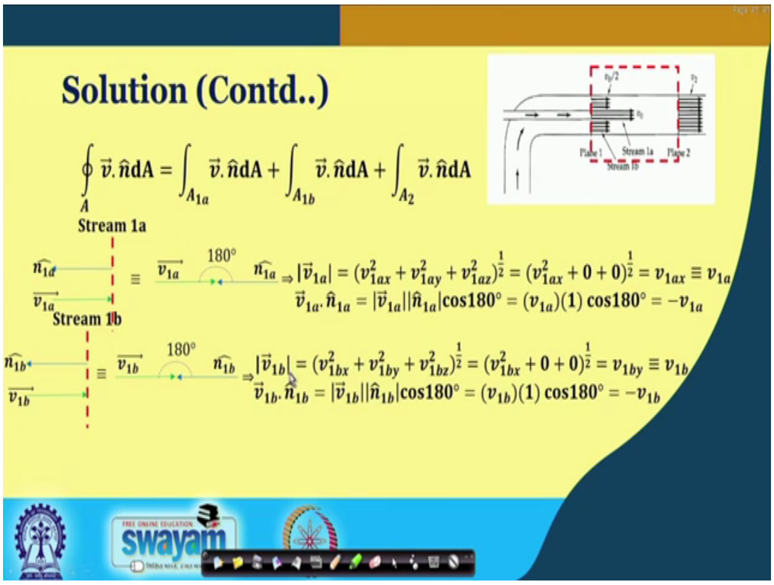

Now, we see if that the velocity is a having again a 3; it associated with 3 streams first stream is the inner tube that the motive fluid, second is the outer fluid that is the suction fluid and rest is the combined fluid. So, we have 3 terms one is for the 1 a for the short shorter tube 1 B for the outer it was section available to the suction fluid and A2 is available to the mixture of both this motive and the suction fluids. So, we break up this whole flux turn into these 3 components.

Now, first we come to this fluid which is coming out from 1A and here we see that the direction of the out word normal to this plane 1 and the direction of the velocity or opposite to each other. So, that the angle between them is one 180 degree. So, first we find out the magnitudes of the velocity, and what we find that here it is having only one component that is in x direction. So, this is 1 ax and we put it as 1 a and then we take the dot product. This is the magnitude of this velocity and this is the magnitude of this is unity.

So, putting this magnitudes and the cos of the 180 degrees angle between them we get minus v 1 a. And now we extend a similar logic to the another stream coming from the 1 b section. And again we put the similar kind of arguments do arrive at the particular velocity that is 1 b and we take the dot product and it is coming out to be minus v 1 b.

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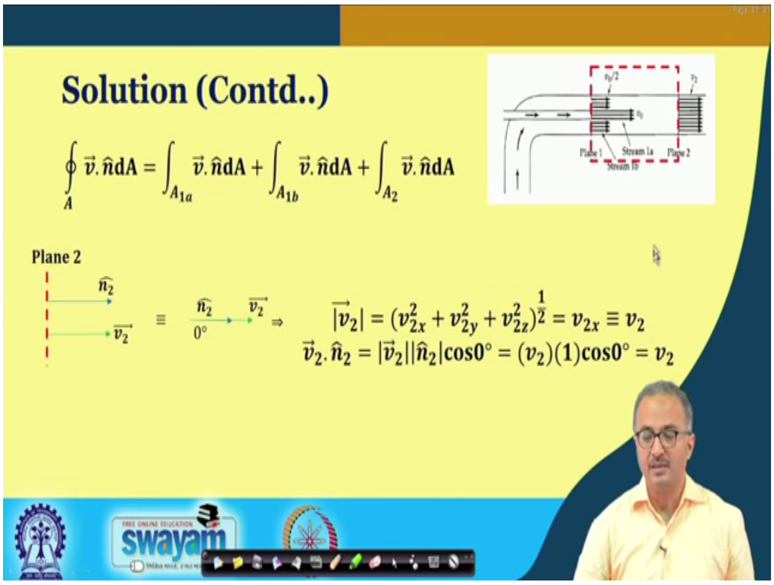

Next we go to the other plane 2. And here we find that the outward normal and the velocity have the same directions. So, that the angle between these two vectors is 0 degree. So, here we put the 0 degree and we find this is the value of the flux at this particular at a particular a plane.

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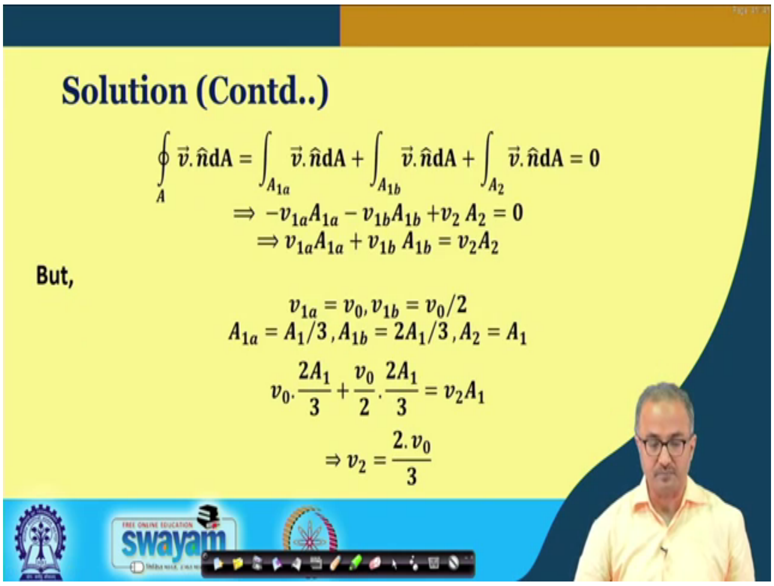

Now, putting in these values of these velocities over here we find that this is the expression we obtain ultimately. And you can see that velocity into this area is the volumetric flow rate.

Now, as per the given information in the problem we have v 1 a equal to v naught v 1 b equal to v naught by 2. So, and A1 a is given as a 1 by 3 A1 b is given as 2 A one by 3 and A 2 is equal to a 1. So now, what we do? This simply put all this values in this particular expression and we find the value of the v two in terms of v naught ok. So, this is the velocity that will be attained by the mixture of the motive fluid and the suction fluid.

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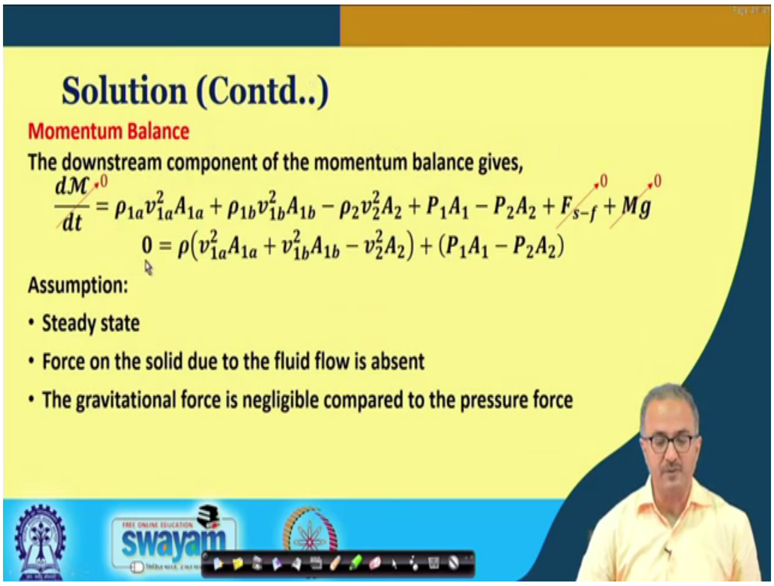

Now, to find out the pressure difference what we do we now go to the momentum balance. So, from the mass balance, we obtain and expression of the v two in terms of v naught. Now in the from the momentum balance, we shall obtain the pressure drop as asked in the problem.

So, first we again go back to the momentum balance equation and here you can see. These are the expressions we derived earlier in the earlier lecture. So, here we have the momentum going in momentum coming out and this is the pressure force difference and this is the force from the solid to the fluid and this is the body force term ok. So, I am not going into the detail of these expressions. So, what we find here is this that we put this because at steady state this is going to be 0 and then we say that the force on the solid due to the fluid flow is taken to be absent. So, this particular term goes to 0 and the gravitational force is taken negligible in comparison to the pressure force at this term is taken to be 0. So, what we are left with is this term.

So, here we put this and we take out the rho out form all this 3 a terms, because this is the in comfortable fluid and we just club this things together.

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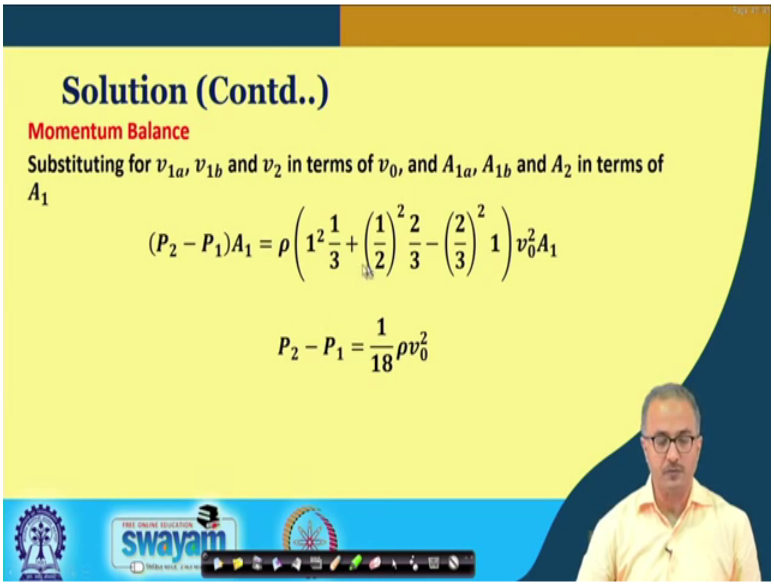

Now, what we do we substitute the values of v 1 a v 1 b and v 2 in terms of v naught and A 1 a, A 1 b and A 2 in terms of A 1.

So, now once you substitute all this values, what we get? We get an expression for the pressure difference between 0.2 and 0.1 and in terms of the v naught. So, this is our final solution.

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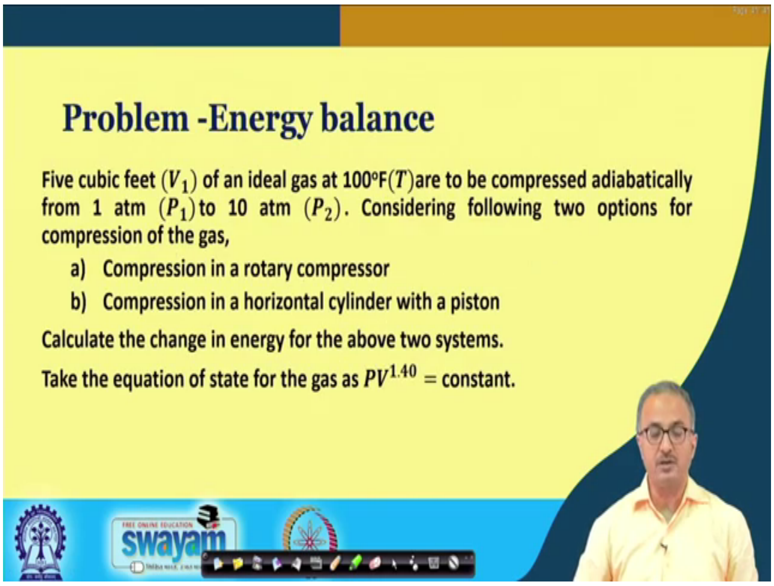

Next we come to another problem based on energy balance. In this we are told that 5 cubic feet of an ideal gas at this temperature 100 degree Fahrenheit are to be compressed adiabatically from 1 atmosphere to 10 atmosphere. So, initial volume is taken to be v 1 initial pressure has P1 equal to 1 atmosphere and initial temperature is taken has initial. So, final pressure is taken as 10 atmosphere P2.

Now, considering the two options; so, these are two options in which it is proposed to compress the gas. One is by a rotary compressor and another is by some horizontal cylinder with a piston, and we have to calculate the change in energy for the above two systems. And it has been told that equation of state for the gas is to be taken as PV 1 to the power 1 per equal to constant.

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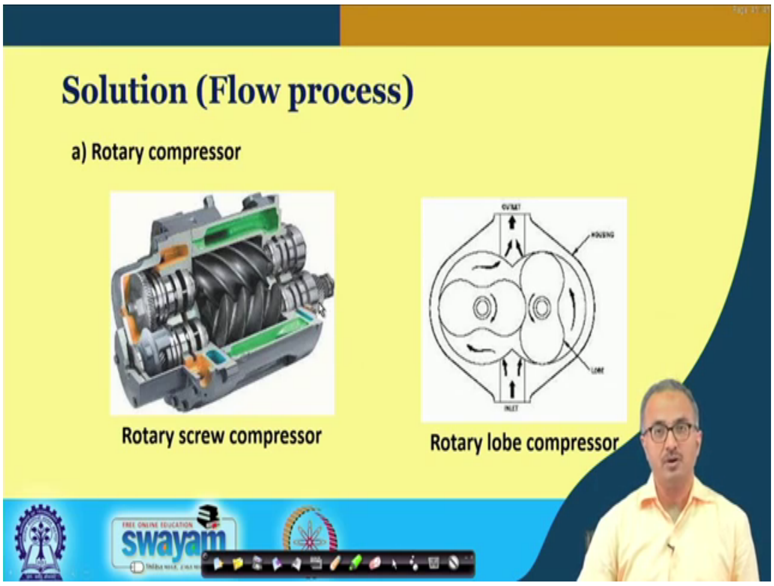

Now, first let us go to rotary compressor you know that there are various kinds of compressors rotary, centrifugal etcetera. So, this is rotary is one of this positive displacement compressors. And here I am showing some figure of two of types of the rotary compressor one is the screw compressor and another is the lobe compressor. I will not go into the detail of their working because these are generally dealt with in separate courses on the fluid moving machineries or pumps or compressor.

So, here just for your knowledge I am just showing you two types of rotary compressors.

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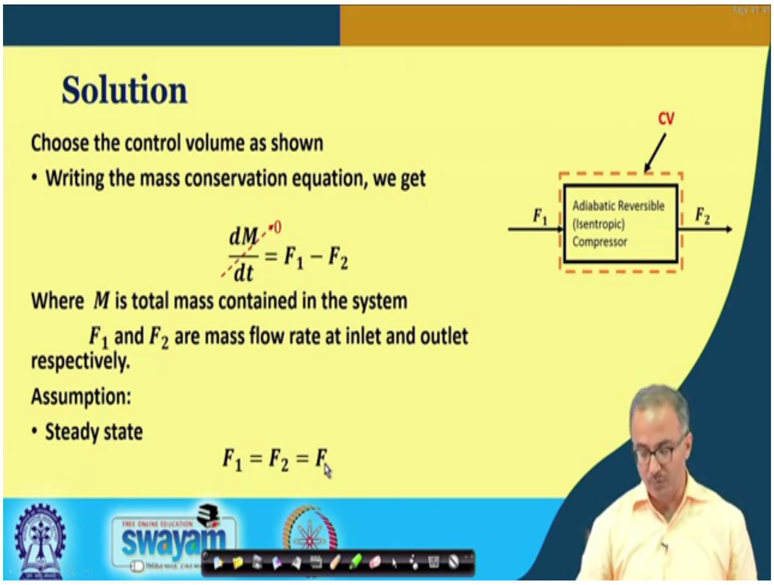

Now, let us go to our problem. Here we see that by this particular may block we represent the adiabatic reversible compressor. Adiabatic reversible perhaps you know this is called isentropic. So, in this compressor, we choose a whole compressor as a control volume in which their feed is going from one side and it is coming often other.

Now, naturally this particular stream F2, we have a greater pressure than the stream F1. So, first we are writing the mass conservation equation for this particular control volume. Here there is no generation or consumption of any of the species within the control volume. So, we are living out the generation term. Here we have the input term and the output term and this is the accumulation term.

Now, because it is a steady state so, this accumulation will go to 0 and so, we were left with F1 is equal to F2 equal to some kind of F putting a single value to this F1 and F2.

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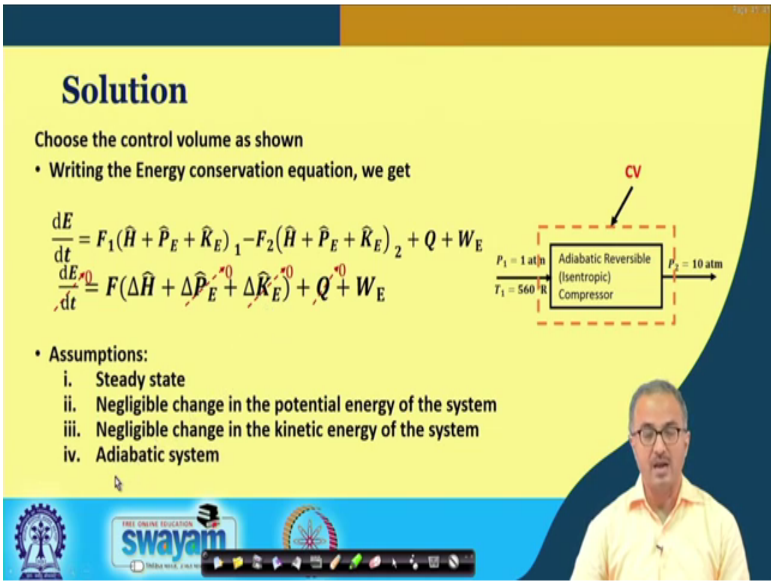

Now, we write the energy conservation equation. So, this also I am not explaining which I derived in or the earlier lecture. So, here you see that energy conservation is go to F1 into order energy is here this enthalpy, the potential energy, the kinetic energy and please mind it these are the specific values. And this is the heat interaction between the system and surroundings and this is the work interaction.

Now, in this case the work involves only the compression work because this work is a generally shaft work plus the compression work. Now in this case there is no shaft work because this is a rigid a body. So, here we have only the compression work, here we are writing in terms of expansion E and perhaps you recall that compression work is the negative of the expansion work.

Now, from the energy mass balance equation we find that F 1 and F 2 are the same and equal to F. So, we are rearranging this particular equation in terms of f. Now, again assuming steady state we have this first term as 0 and we assume that there is negligible change in the potential energy of the system from the inlet to the outlet. So, that this terms goes to 0, then we also neglect the change in the kinetic energy of the system. So, this term also goes to 0 and when we assume it to be adiabatic then this particular Q term also goes to 0.

So, what we are left with is this work term and the change in enthalpy term.

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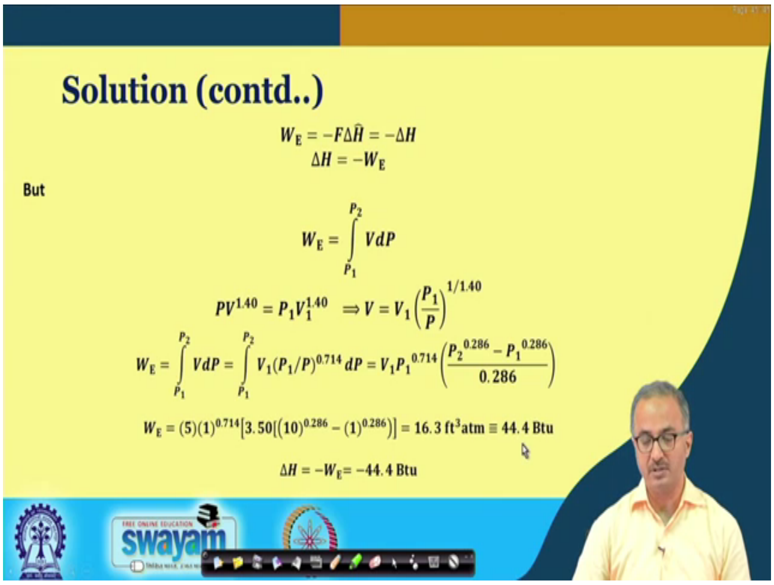

So, here we find the total enthalpy change equal to is delta H and this is equal to minus W E. So, now, the W E is to be found from this particular integral that is VdP and the pressure is going from P 1 to P 2. Now ins, here we what we do now, we replace this particular V in terms of the pressure from the given expression.

So, here we find that V equal to V1 into P 1 by P to the power 1 by 1.4. Now plugging in this particular term in this equation, we get this particular equation and now we see that during the integration what we ultimately find that this is the value of the work done and this is coming out to be this much of Btu. And this is this work done is nothing, but the change in the enthalpy. So, change in the enthalpy is a negative of this particular value.

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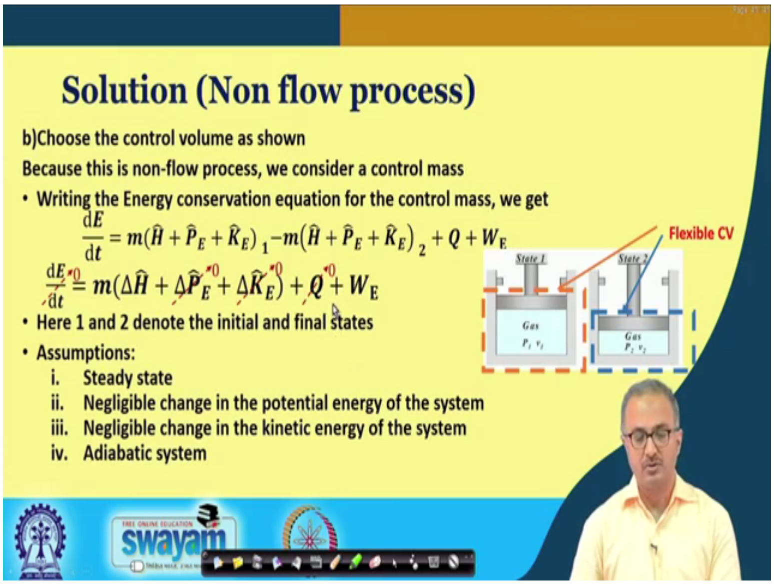

Now, let us look into the solutions of the second part of the problem. In this case we have this piston cylinder arrangement. So, in this figure you see that the piston will be moving up and down this in the cylinder and here we are assuming that the piston is frictionless that is we are neglecting the any work of due to overcome this particular frictional resistance by the cylinder well ok. And this movement of the piston is causing either a compression or an expansion as further direction of the movement, and because there is not any flow from the surroundings into the system or vice versa. So, we are not going to talk of any kind of flow, but we shall talk of a particular mass of the gas being expanded or compressed in a at a regular interval.

So, instead of taking about a control talking about a flow, we shall be talking about the mass of the system. And so we have the control mass that is a constant mass contained within the control volume. So, we shall be applying the mass balance and the energy balance on the control mass. Now, as the previous part showed that the mass flow rates where remaining constant at steady state and without any consumption and or generation. In similar fashion we can also show that the total mass of the system initially. And finally, are remaining constant under steady state and under no generation or consumption ok.

So, here we are putting in terms of the mass and the 1 and 2 are denoting the initial and final states respectively. So, again we make this assumptions of the steady state so, this particular term goes off then the negligible change in the potential energy between the two states and negligible change in the kinetic energy in a two states and taken to be adiabatic. So, what we are left with this only this enthalpy term and the work term.

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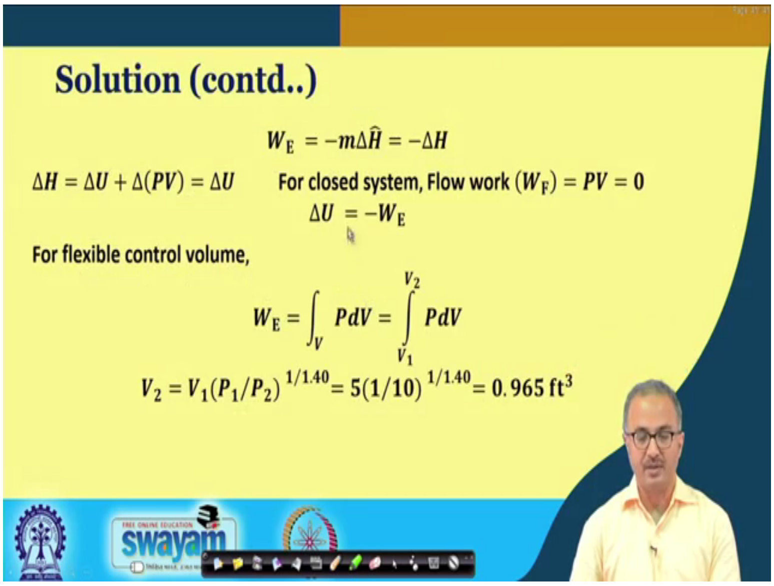

Now, here we find this is the particular expression for this and now we see because its a non flow system. So, the PV work that is associated with the flow work is taken to be 0. So, this delta H is essential reduced to a change in the internal energy of the system and as you know the internal energy is expressed in terms of the temperature of the system. So, what we expect is this that there will be a change in the temperature during this compression or the expansion process. The compression will lead to an increase in the temperature while expansion will lead to a decrease in the temperature.

So, now we go to this flexible control volume. So, we are writing in terms of PdV and what we are putting that we are putting this particular expression in terms of the V. Now we are now converting this P in terms of the V from our given expression. Now what we find here that the final V 2 the final volume is obtained from the given expression and this is the volume ok.

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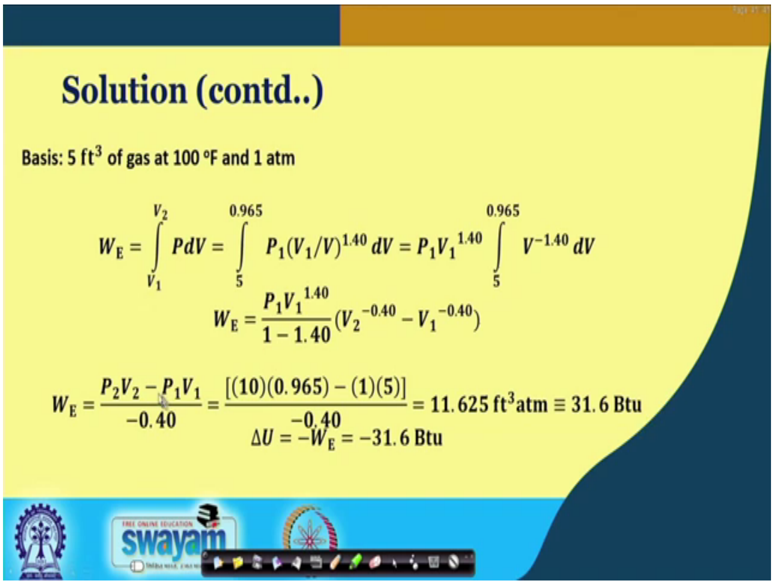

So, now doing this particular integral from the initial volume to the final volume, we find that this is the expression for the work done and this is the value of the work done. Now please note here that in this in converting this two this particular value, what we are doing? Basically is this we are again using the expression that P1 V1 to the power 1.4 is equal to P 2 V2 to the power 1.4.

So, when we are multiplying this term with this term, what we are doing? We are replacing this P1 V1 by P 2 V2. So, that we are getting this expression and for multiplying this by this term we are not changing anything here. So, we are getting P1 V1. So, with this knowledge we are able to find out the work done and this will give us the change in the internal energy of the system.

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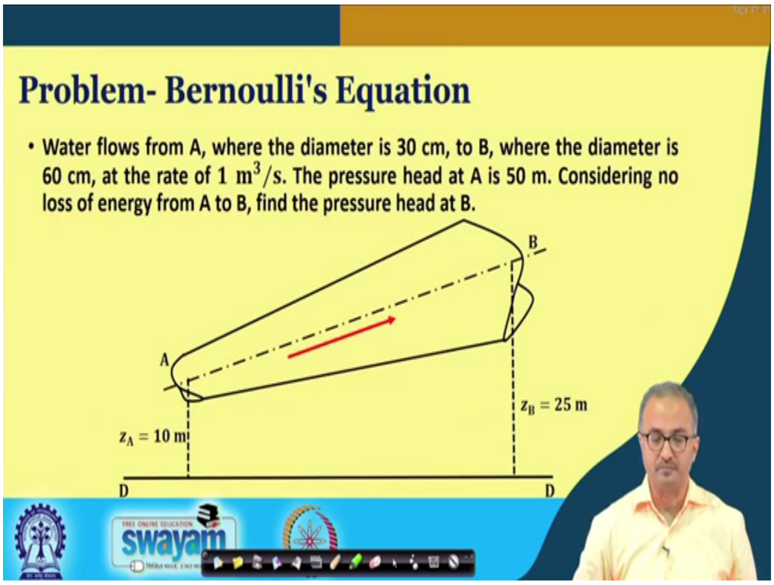

Next we come to another problem where we are going to apply the Bernoulli's equation. In this case, we find that water flows from a point A to a point B and the diameter of the pipe at the inlet is 30 centimeter, while at the discharge section it is 60 centimeter. The flow rate of water remains constant at this particular value of one cubic meter per second and the pressure head at point A is 50 meter and we consider that is no loss of energy from A to B. So, with this assumption we have to find out the pressure at point B. And here we see the elevations of this particular axis is given from the some datum at the inlet it is 10 meter, at the outlet it is 25 meter.

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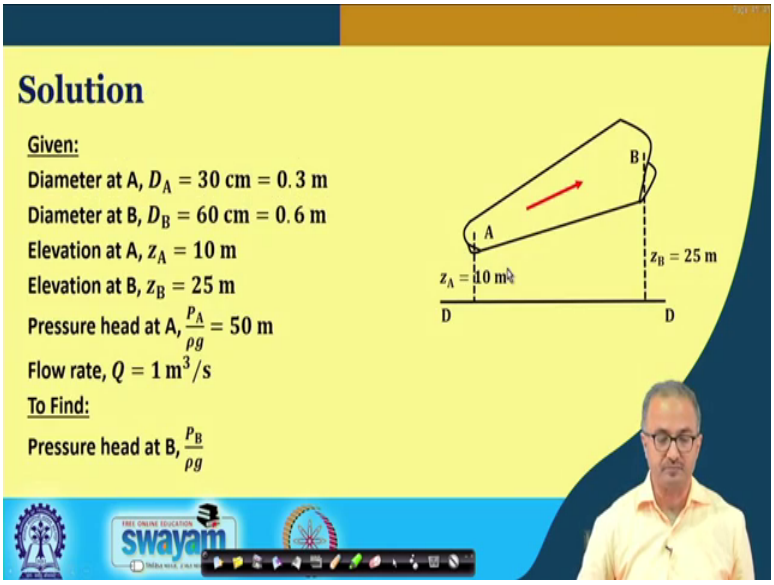

So, first what we do? We put all the data given in the problem. And we see the pressure head is nothing but the absolute pressure divided by the product of the density of the fluid and the acceleration due to gravity. So, this is given as 50 meters and we have to find the pressure head that is PB by rho g ok.

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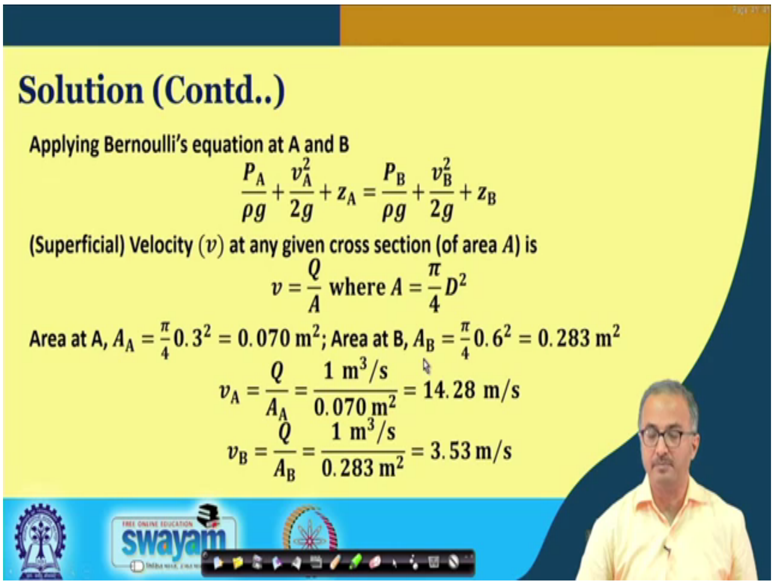

So, now let us write the Bernoulli's equation at point A and at point B and neglecting all kind of other energy losses. So now, you see that velocity at any point is given by this particular formula that is the ratio of the volumetric flow rate through the area of cross section. In fact, this particular velocity is called the superficial velocity which is based on hallow cross sectional area of the pipe. So, and the area of cross section is pi by 4 d square.

So, with this particular formula, what we find? We find out at point A, this is the cross sectional area and B this is the cross sectional area. Now with this we find out the superficial velocity of water at point A as this value and the same thing at point B at as this value. Here you see at both the cases, the volumetric flow rate is taken to be the constant.

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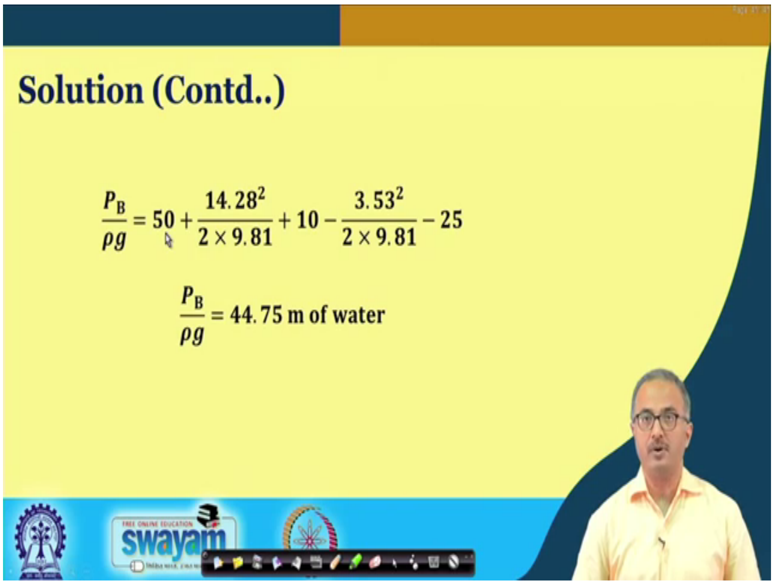

Now after plugging in the values of all the components so, these particular things are for the A side and these two terms are coming for the B side ok. So, this is the pressure rate, this is the velocity head, and this is the potential head, and this is the again velocity head and is the potential head at the point B and what we get? We get this particular head at point B in terms of water. Now what it means is this is a 44.75 meter of water and perhaps that 10 meter of water column corresponds to about 1 atmospheric pressure. That means, at the at the A point that we have about 5 atmospheric pressure, at the B point we have about 44.5; that means, there is a drop of about 5.25 atmosphere from the inlet to the outlet ok.

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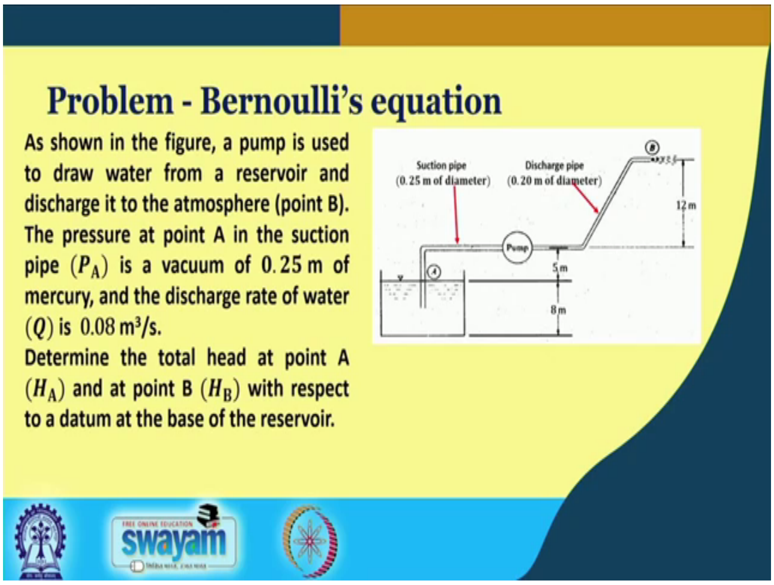

Next, we take up another problem also based on the Bernoulli's equation. In this problem, we have a pumping system. So, what we find this the pump over here which is drawing water from a reservoir. So, here reservoir and it is discharging it to the atmosphere at point B. So, here we see the pump is here. So, the reservoir water is taken out and it is being pumped at some height at point B. The pressure at point A is the suction pipe is a; these a suction pipe and the pressure is taken as the vacuum that is it is below the atmospheric pressure.

So, this is a negative pressure gauge. The rate of discharge is 0.08 cubic meter per second and it is that to determine the total head at point A and point B with respect to a datum at the base of the reservoir. So, here we have some datum with respect to this datum we are having the elevation at the various points; this is for the reservoir water table, this is for the pump and this is for the point B. So, here you can see point B is at an elevation of 12 plus 5 plus 8 that is 25 meter from the datum.

And in this case, the total head means the summation of the 3 heads. We have consider so, far that is the potential head, the kinetic head and the pressure head. So, we have to find out the summation of this three heads at the point A and point B.

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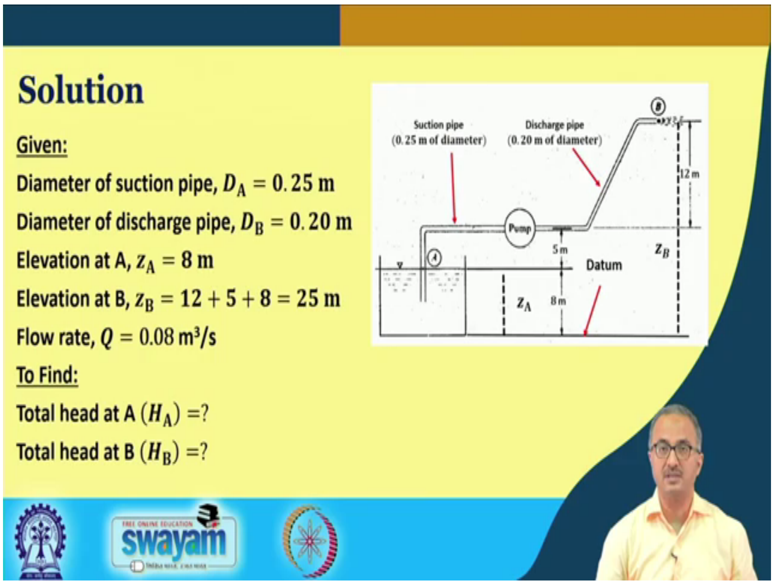

So, for this what we do is this we take all this data. So, we have been given the diameter of the suction pipe as DA to be 0.25 meter and the discharge pipe has a diameter of 0.2 meter the elevation at point A means at 8 meter and elevation at point B as a explained earlier is 25 meters and this is the flow rate of waters. And we have to find out the total head ha and HB at point A and point B.

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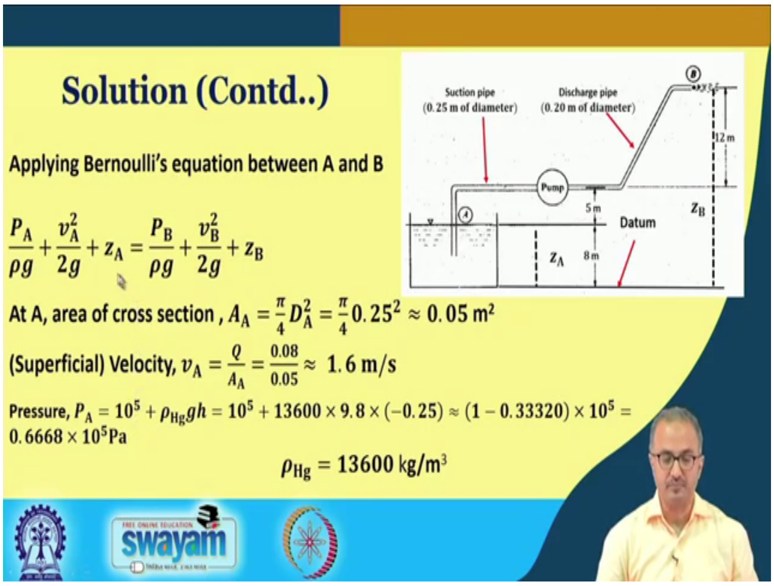

We write the Bernoulli's equation. So, here we find we have been given the pressures at the 2 points this elevations are also given, what we do not know is the velocity. So, to find out the velocity, we again go to the equation of this thing that the we divide the volumetric flow rate by the area of cross section of the pipeline and area of cross section of pipeline is given in terms of its diameter. So, for point A, we put the values of the diameter and this is the area of cross section of the pipeline and we get the velocity of water at point A as 1.6 meter per second.

Now, to find out the absolute pressure because we have been given the vacuum pressure that is that pressure difference between the absolute pressure, and the atmospheric pressure. So, it is a negative value. So, what we do? We add the atmospheric pressure with that pressure, only things it we consider this H given in terms of the mercury column that to be negative ok. So, that we get the pressure to be less than the atmospheric pressure. Here we use H rho g rho that is density of the mercury which is 13,600 kg per meter cube. So, we get this value of the pressure at point A.

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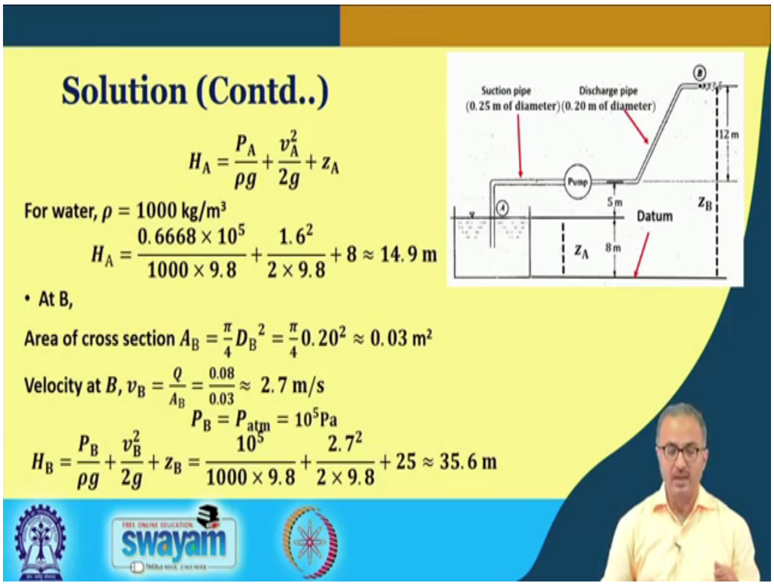

Similarly, we get the pressure at point B as the atmospheric pressure that is about 10 to the power 5 meter Pascal. Now, with these pressure head the pressure head is now obtained by dividing it by the density of the water and here we have the kinetic head has v square by 2 g plus the elevation and we get this is the head at point A. Similarly at point B we get this is the pressure head here, we put 10 to the power 5 atmospheric pressure. And we divide this by the density of water and g value is 9.8 and the velocity at point B 2.7 and again we put this 2 and into g plus 25, we get at the total head at point B.

So, this is how we are able to find out the total heads at point this two points. Only thing you must notice this that if you are given the gauge pressure where it is in positive or negative gauge pressure is called vacuum pressure. So, first you need to convert that gauge pressure in terms of the absolute pressure and then only you should put these figures to get the total head.

Now, when you are going to find out the head difference, then it does not matter to you to get the absolute pressure, because even if at the absolute pressure on both the sides of the Bernoulli's equation, the difference will come to be the same. So, in case you want to find out the difference in the pressure heads, then you can simply take the difference of the gauge pressures.

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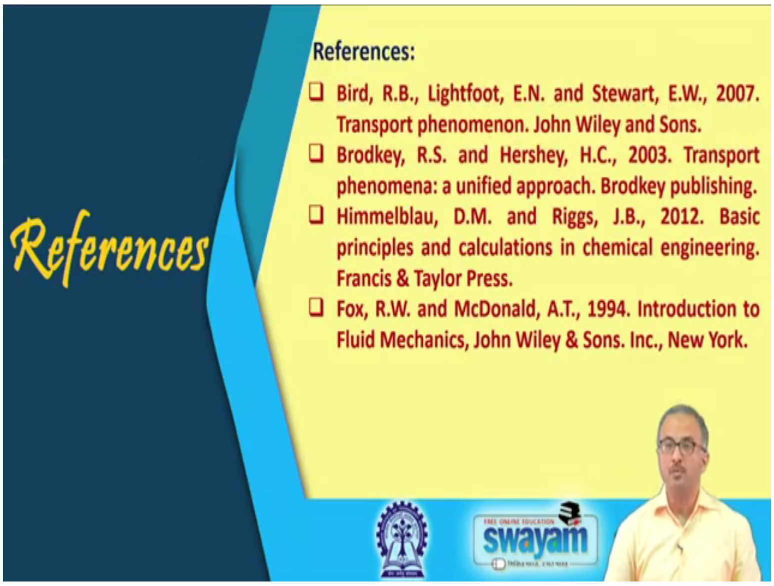

More can be found out from these particular references books.

Thank you.