Structural Analysis 1
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Lecture 8
Analysis of Truss: Method of Joints

Hello welcome back. Today what we will see is we will introduce method of joints for determining member forces in statically determinate truss, okay. Now let us understand the method of joints through some examples, okay. In the last class we have seen how to draw free body diagrams of different truss members.

We have also seen how to draw free body diagram of different joints. The method of joints essentially where the free body diagrams of different joints are used, okay. Every joints we can have two equilibrium equations. So total number of equilibrium equations will be 2 into J. J is the number of joints. And then with all these equilibrium equations we have to determine the number of unknown.

Number of unknown includes support reactions and also member forces, okay. Let us demonstrate that through one example first, okay. Now take a very simple truss. This is triangular shaped truss.

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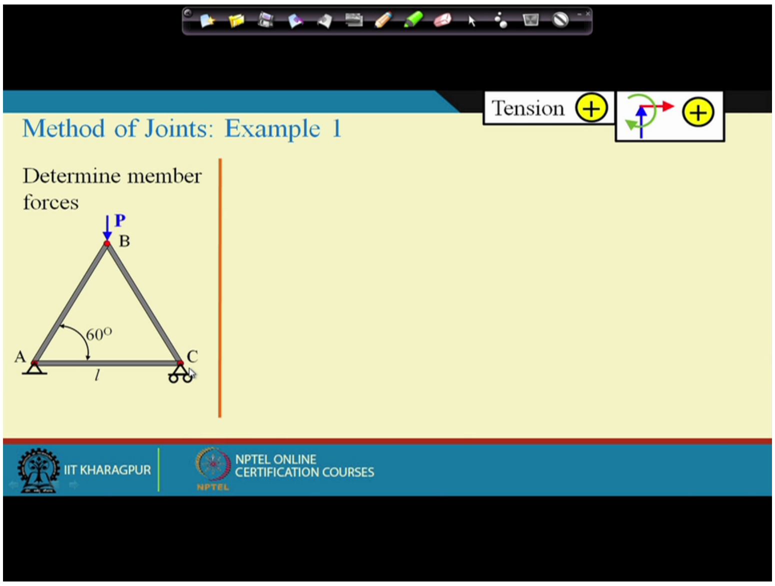

Now first let us see whether it is statically determinate or not. Now number of reactions we have two reactions here and here you have one reaction. So therefore r is equal to 3, right?

Now number of members. Members we have three members so m is equal to again 3. Now how many joints we have? We have three joints and every joint will give us two equilibrium equations.

So, total six equilibrium equations are available. So six number of unknown are 3 plus 3, 6. So m number of unknown is equal to number of equilibrium equation. So this is a determinate truss, okay.

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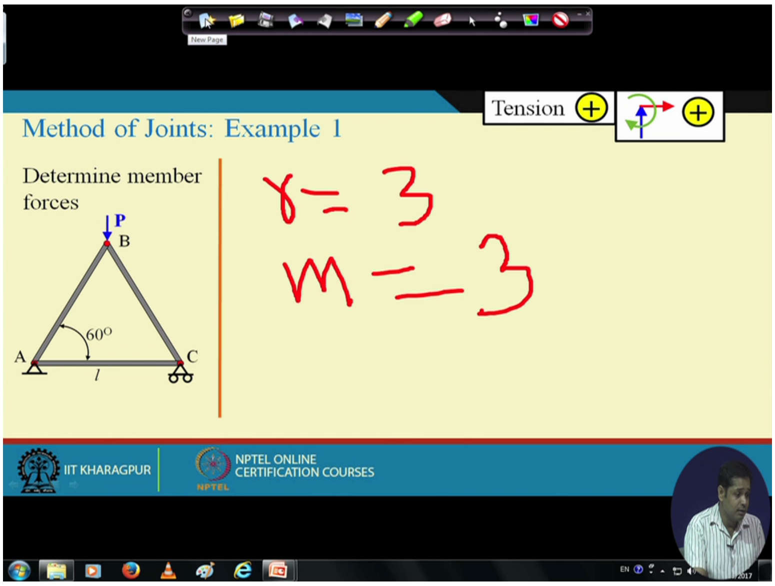

Okay now let us first determine the support reaction of this truss. Draw the free body diagram of the entire structure. Suppose this is the free body diagram of the entire structure, okay.

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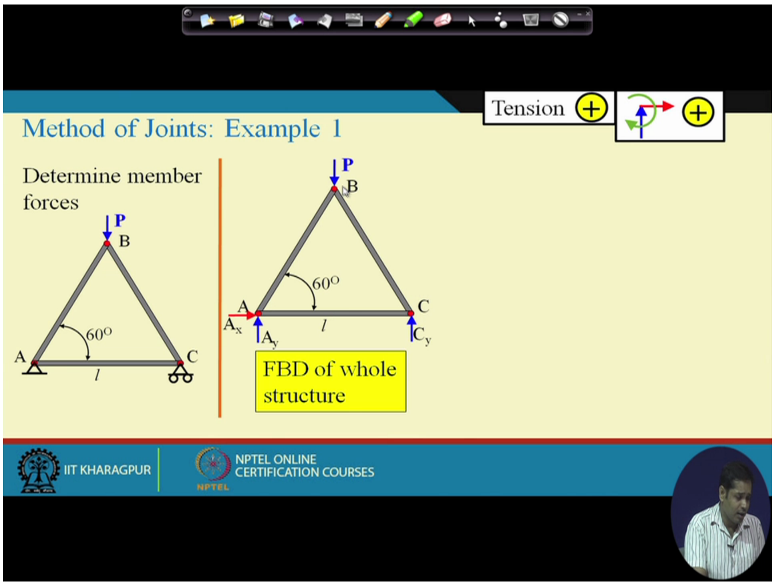

Now we apply the equilibrium conditions on this free body. But before we do that let us clarify our sign convention. Now tension we assume as positive when we draw forces in members. And for all algebraic operations we use this kind of forces are positive, okay. You can use any sign convention whatever you want but whatever you use please be consistent with sign convention.

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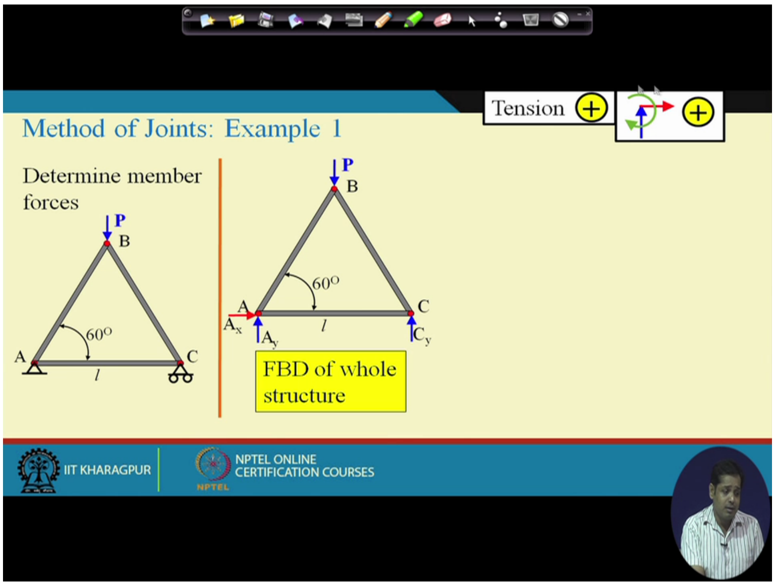

Now this is the free body diagram of the whole structure. Now apply the equilibrium equation. The first equilibrium equation is equal to summation of force in x direction is zero which directly gives you the value of Ax is equal to zero or the reaction is zero.

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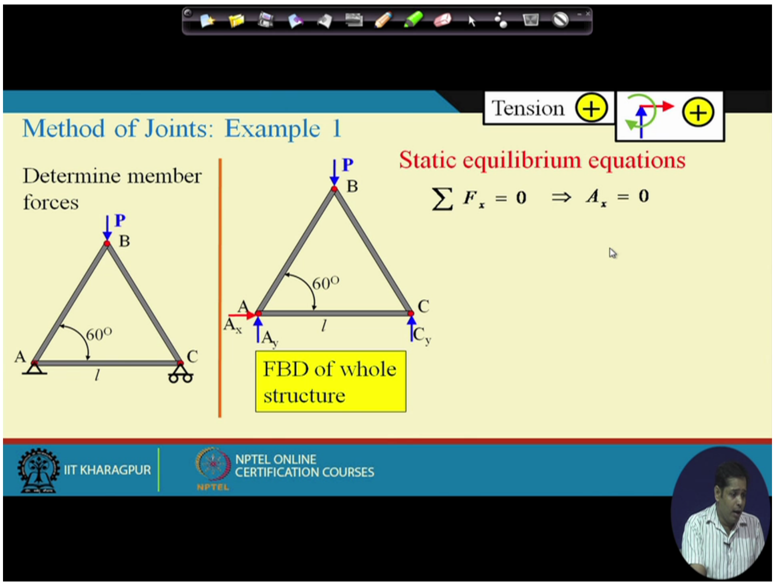

Now in the class I am writing this but every time you may not really need to write summation of Fx is equal to zero and Ax is equal to zero because this is obvious from this truss. There is no horizontal component of the force. So naturally this Ax will be zero, okay.

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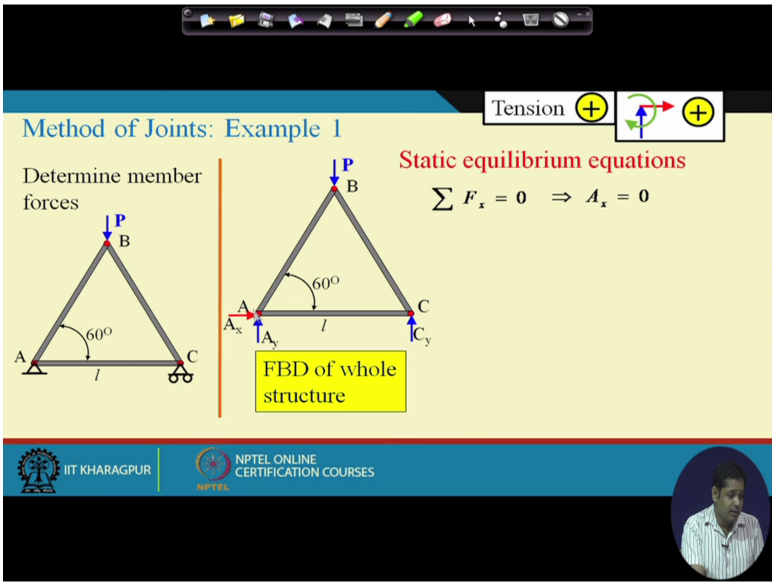

So by just looking at the truss we should be able to say that Ax is equal to zero. Now Ma is equal to zero, moment at A is equal to zero. Then it gives you the movement contribution to this equation will be the force P and then support reaction Cy and this immediately gives us Cy is equal to P by 2. And the another equation is summation of Fy is equal to zero and this gives you Ay plus Cy is equal to zero. Total support reaction should balance the external load and immediately Ay is equal to P by 2.

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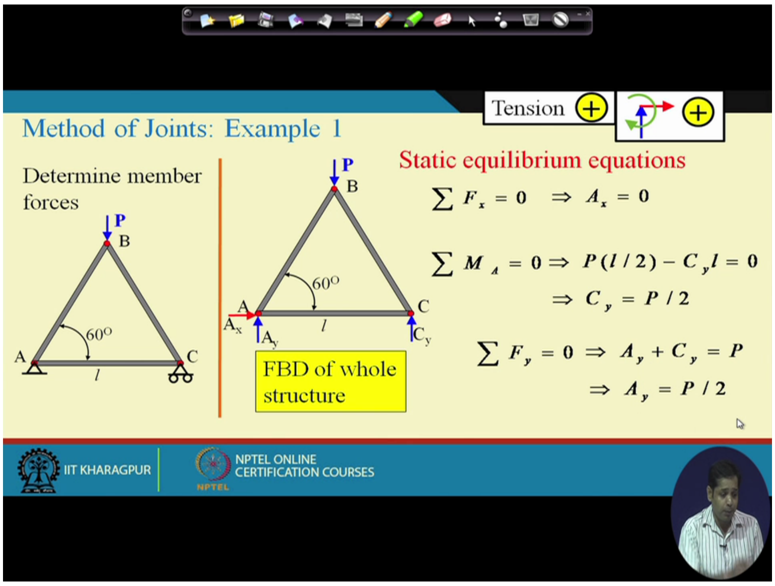

Now here one point please note, when we use the free body diagram of joints then we had only two equilibrium equations, right? Summation of force in x direction is zero, summation of force in y direction is zero. The reason we did not have summation of moment is equal to zero because moment is anyway zero because all the forces are passing through the same joint. So moment is equal to zero will not give you any additional information.

But in this case since it is entire structure we can use summation of moment at any point is equal to zero which gives us some additional information, right? So generally first step we do is we (deter) determine the support reactions. So these are the support reactions, okay.

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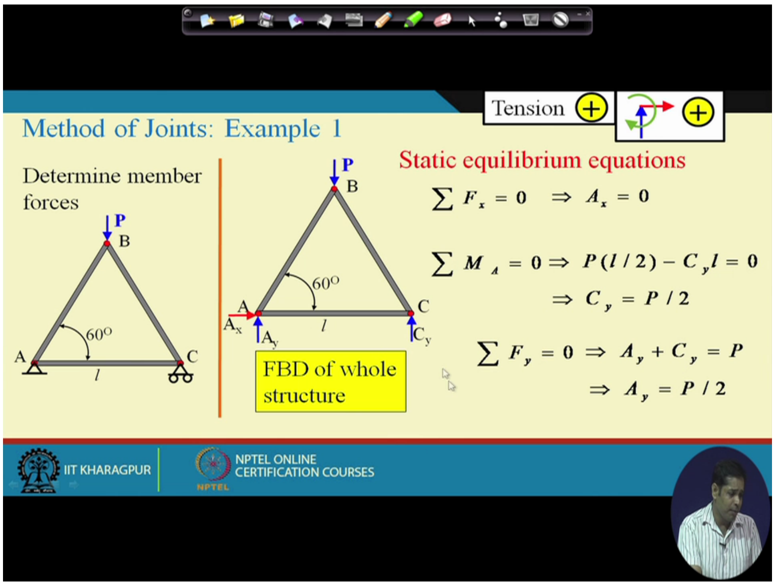

Now next method of joints is we need to (sep) draw the free body diagram of different joints and apply the equilibrium equation on that free body diagram. Now these are the reactions. Let us first take joint A. Now what are the forces we have at joint A? Support reaction Ay. Ax is equal to zero that is why it is not shown here explicitly.

So Ay and then member force AC, FCA and then force in member AB and when we show like this it means that our tension is positive, okay, because when a member is in tension, the forces in the corresponding joint will be away from the joint. We have seen that in last class.

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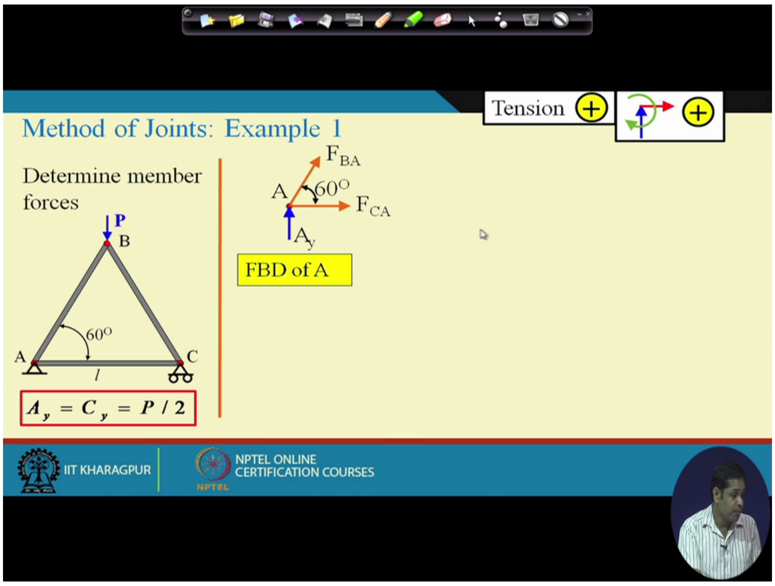

Now apply equilibrium condition on this free body diagram. Summation of Fy is equal to zero, the component of Ay and the component of By. So Ay plus By this is equal to zero and it gives us finally FBA is equal to minus P by root 3. So force in member AB is P by root 3.

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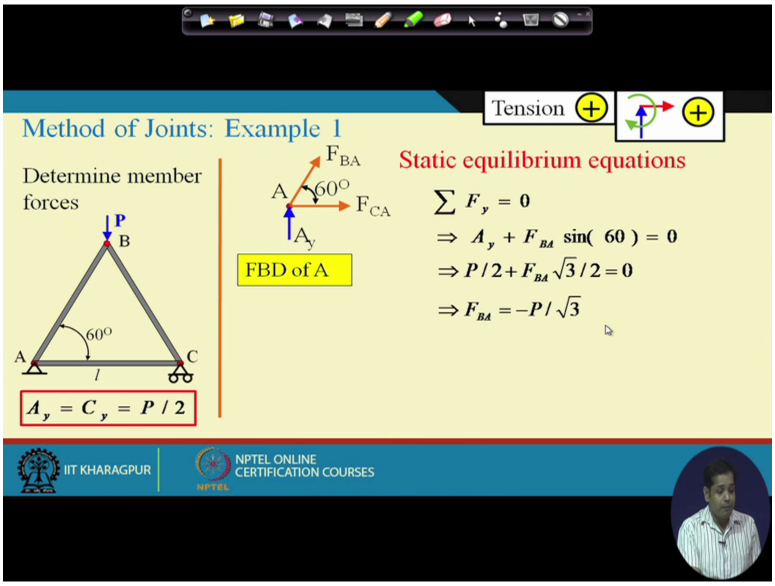

You see what is the advantage of being consistent with the sign convention? All purposes for all member forces we assume that our sign convention was tension is positive. Now if you are getting the positive value after calculation means the member is in tension. If you are getting the negative value then the member is in compression. So just by looking at the sign you can say the member is in compression or tension.

But be consistent with the sign convention. Some free body diagram we show compression is positive, in some free body diagram we show tension is positive and if you mix up the sign convention then the sign associated with any value will not give you any information whether the member is in tension or the member is in compression, right? Now similarly this is member force BA.

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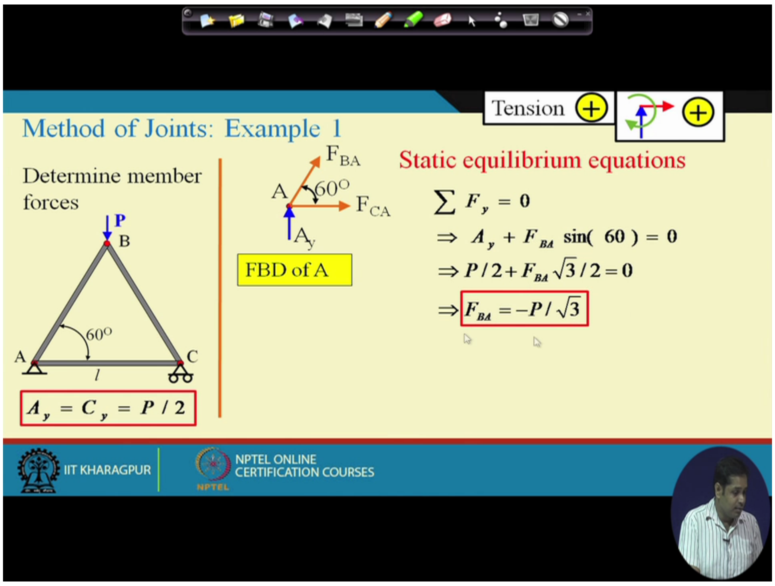

Now let us take another equilibrium condition Fx is equal to zero and the components are FCA and component of FBA and if we substitute that, FBA already determined here and finally we get FCA is equal to this, okay.

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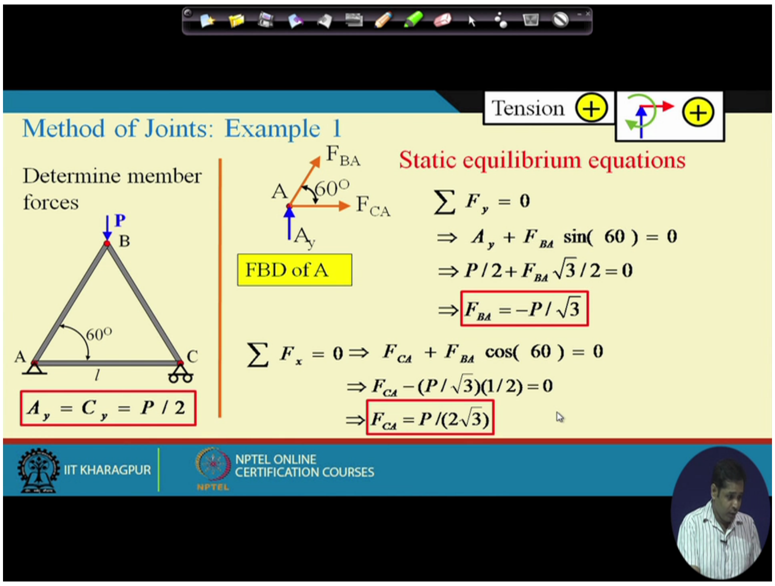

So this is the member force in AB and AC. Now next only member left is BC. Now we can take either free body diagram of joint B or free body diagram of joint C to determine the force in this member. So take free body diagram of joint C. Cy is the support reaction, FCA is the force in this member and FCB is the force in this member. Apply again equilibrium condition. Summation of Fy is equal to zero and this gives you finally CB is equal to minus P by root 3.

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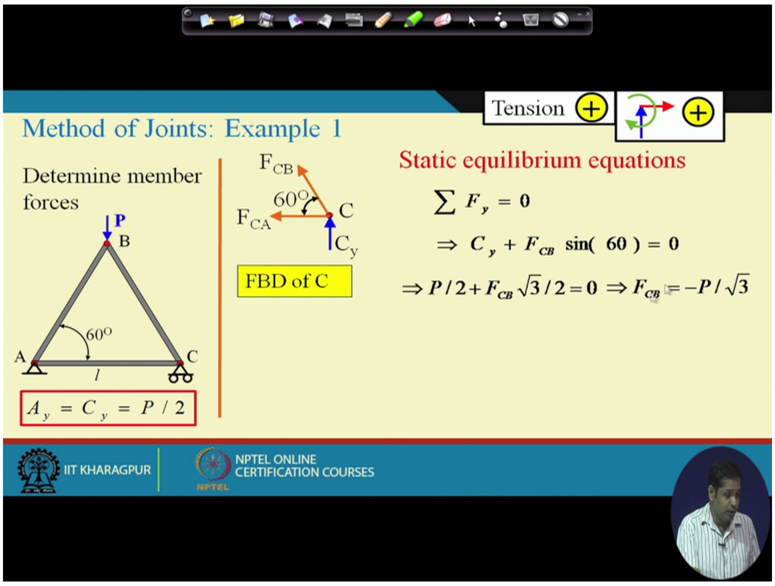

So this is a good check because you see this is a symmetric truss, okay. And the loading is also symmetric and therefore it is expected that the force in member AB and force in member BC they should be same. So in the last slide we obtain the member force in AB is minus P by root 3 and member CB is minus P by root 3. Now again by intuition itself we can say that if a member is subjected to force like this, so naturally this member and this member will be subjected to compression, right?

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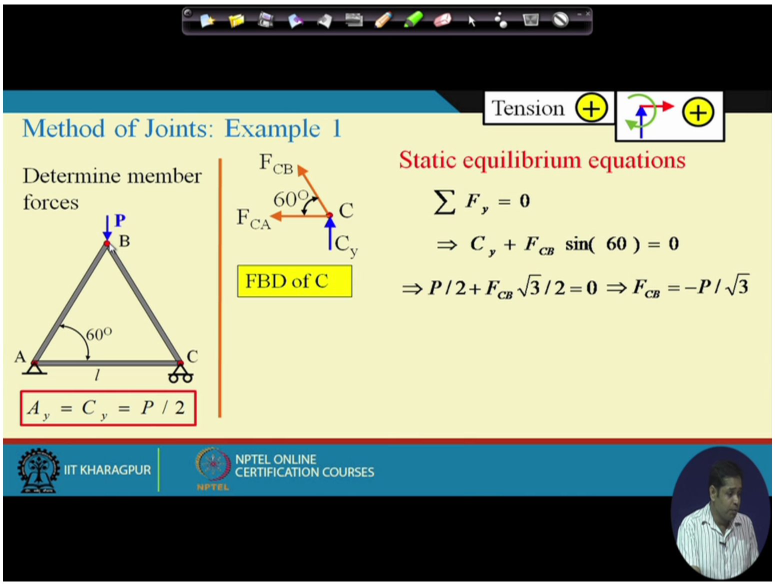

And we are getting the compression (bec) which is evident from this negative value. Now member force in AC was tension and these two forces are compression. Even without analysis we could say that member AC will be in tension because you see when you apply a force like this, what happens? This roller support since it is roller support a compression will transfer in this direction. The horizontal component of this force will try to move this roller support here in this direction.

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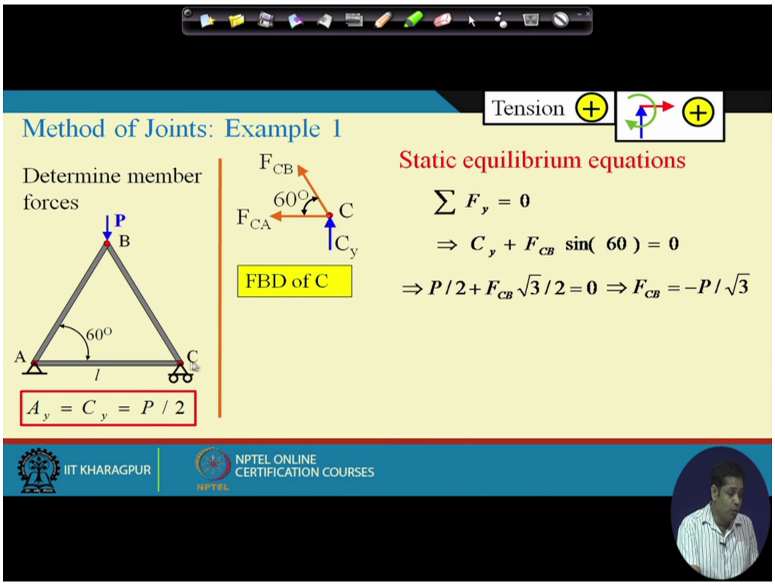

The mechanism we saw in the last class. But now we will restrict that motion and when this member C tries to move in this direction and this moment is restricted by member AC so naturally member AC will be in tension. So why I am saying that because you do analysis, get some solution but we should cross check our solution without intuition, right?

Because many problems by the end of this course we should be able to develop a sense so that just by looking at the structures, by looking at the geometry, looking at the loading pattern we can say that what could be the probable tension member, compression member or in the case of beam frame bending member or shear force diagram.

And then we cross check, then we need all this computation to get their values, okay. So it is always a good practice to first try to understand the truss and their member forces based on our intuition and then check whether our intuitions are correct or not, okay. Now these are the forces we have computed.

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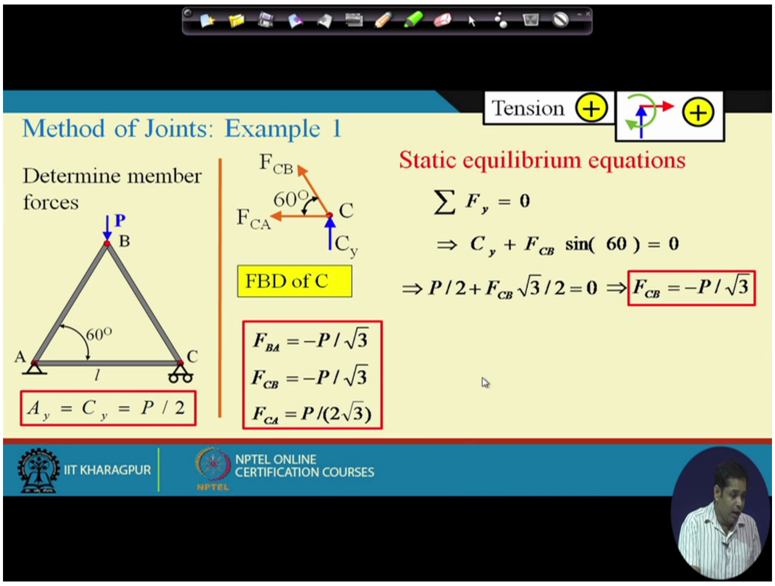

Now you can represent these forces in two ways. One is negative with sign and since we have already mentioned here the tension is positive so naturally it is evident it is very clear all the forces which has negative sign are compression and positive sign are tension. Otherwise you can write these forces like this. Write the magnitude in bracket whether it is tension or compression.

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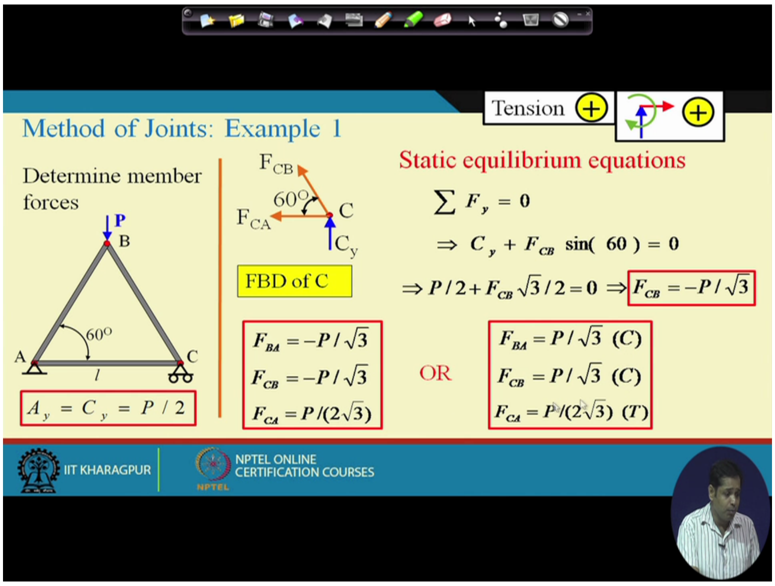

So both the (repre) representations are fine, okay. Now so these are the forces in members. Let us draw like shear force and bending moment diagram here. Here we do not have shear forces and bending moment. What forces we have is actual force, right? So let us draw the actual force diagram or member force diagram.

The member force diagram will be, member AB is a compression. So the value is P by root 3. Member BC is in compression and then again member AC is in tension, the value is this, okay.

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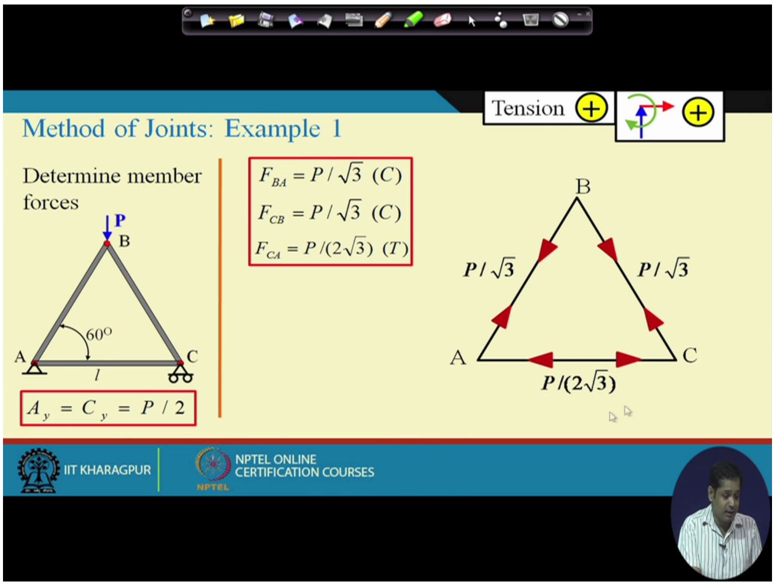

This is called member force diagram. You see these arrows are on the members, not at the joints, okay. So it says that the member (com) AB is compressed. This represents that member AC is elongated, okay. Now this is the member force diagram.

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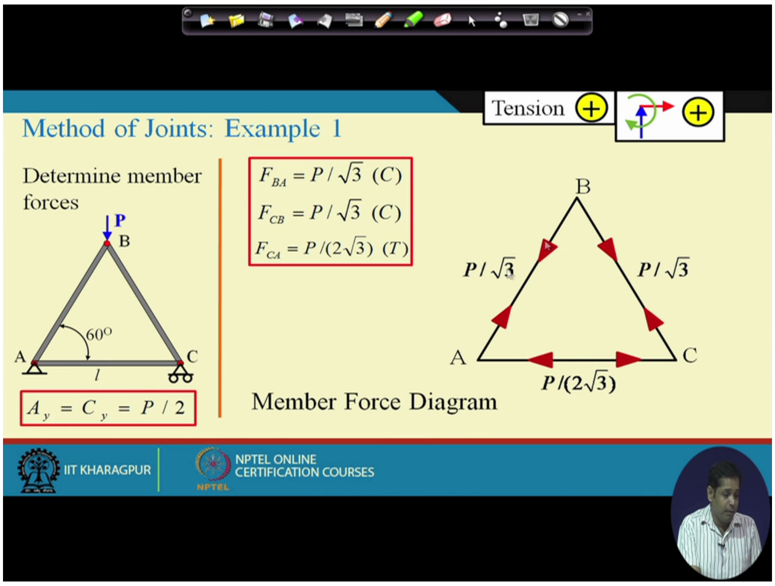

Now so this was method of joints, okay. So method of joint is essentially what? We take several different joints, draw the free body diagram of these joints, apply the equilibrium conditions on the free body diagram, get the equations and then solve those equations to find out the unknown member forces and support reactions. Now before I give you some more examples let us see one very important concept is zero force member, okay.

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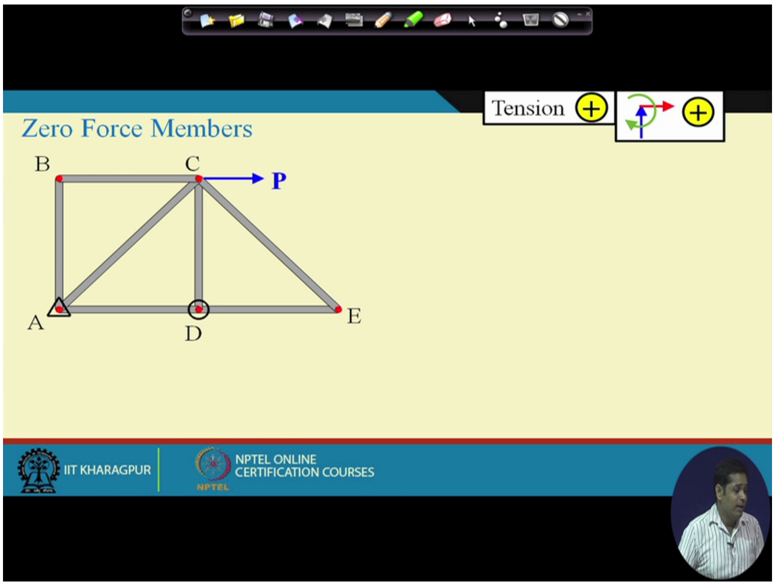

Now what is zero force member? As the name suggests that the members which are zero force, right? Now why is understanding of zero force member important? Okay. Now for instance take these examples, okay. Now let us first see whether it is statically determinate or indeterminate truss. We have three reactions and then number of members are 1, 2, 3, 4, 5, 6, 7, 7 members.

So total 10 unknowns and the number of joints 1, 2, 3, 4, 5, 5 into 2, 10 equations. So this is a statically determinate truss, right? Now the method of joint just now we demonstrated. Using that method of joints we can determine the forces in this member. All the members we can determine the forces.

Now, but there are some members in this truss. In order to say the force in those members, really we do not need any calculation, okay. Just by looking at those members and their connection with other members we can say what could be the force in this member or in a most severe case at least some of the members which have zero forces can be identified.

Let us see how it can be identified? Now suppose if I have to analyse this structure as it is. Then we have to take several joints first, determine the support reactions then several joints and do the (compe) calculation for all member forces, okay.

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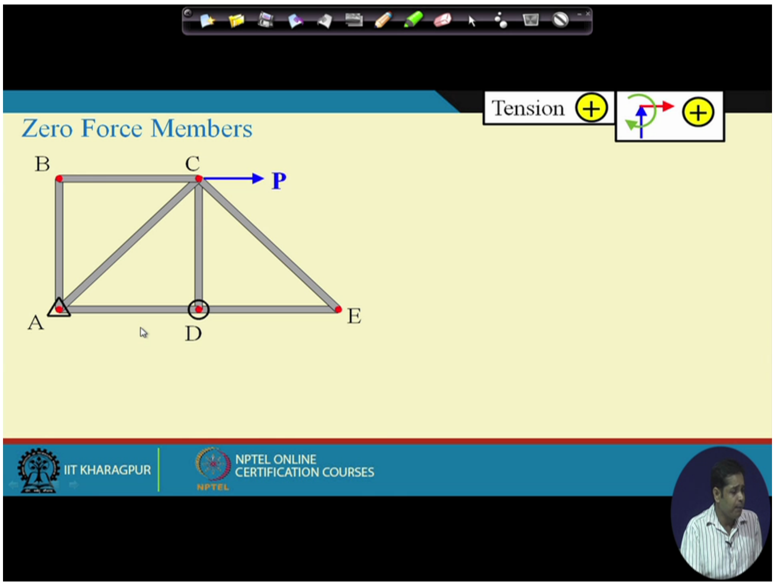

Now look at joint B, okay. Now if I draw the free body diagram of joint B so this is the force in FBC and this force is FBA.

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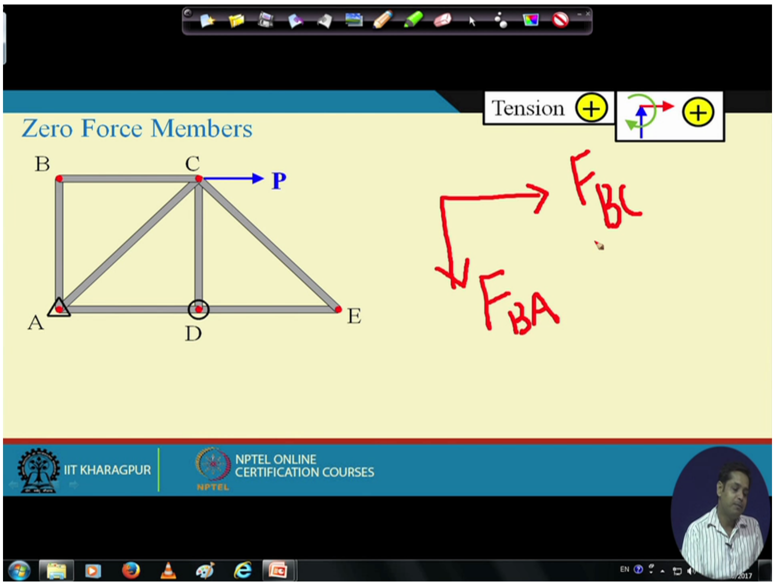

You see, now apply equilibrium condition. There is no other horizontal force here, no other vertical force here. So naturally this will be zero and this will be zero, okay.

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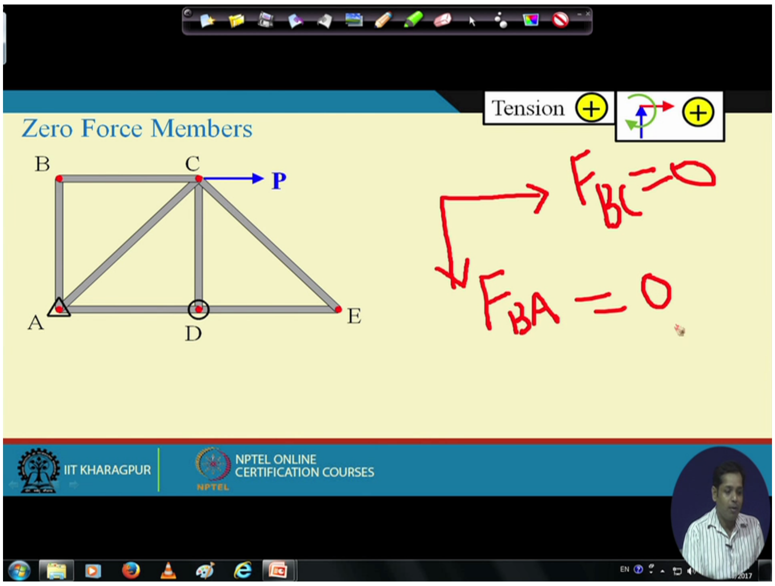

Now to say that FBC is equal to zero and FBA is equal to zero we really do not need this free body diagram and the equilibrium equation. Just look at this joint B. This joint B has only 2 members and no other external forces, okay. Now if the force in member BC is non zero there is no other force to balance this non zero force. Similarly if AB has non zero member force there is no other member force which can balance the force in AB which is evident from this joint.

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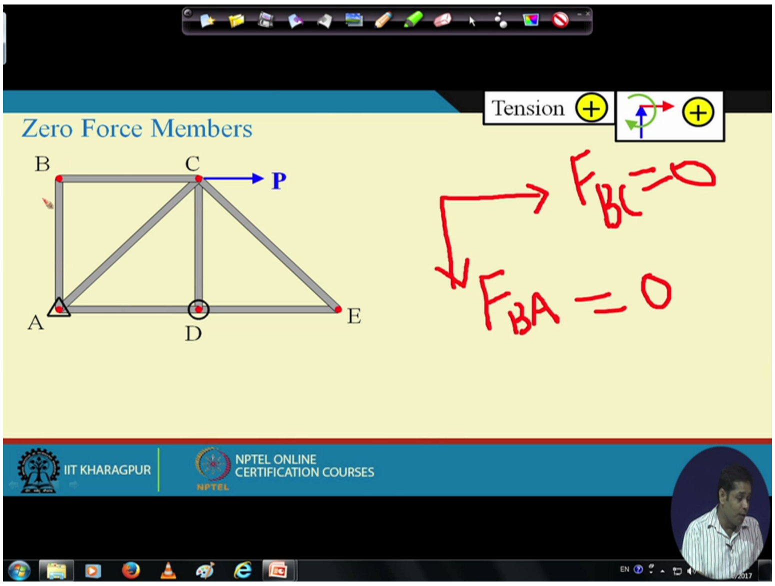

So straight away we are doing all this calculation we can say that this is a zero force member, the force in this member is zero and the force in this member is zero. So this member BC and member BA they are zero force member.

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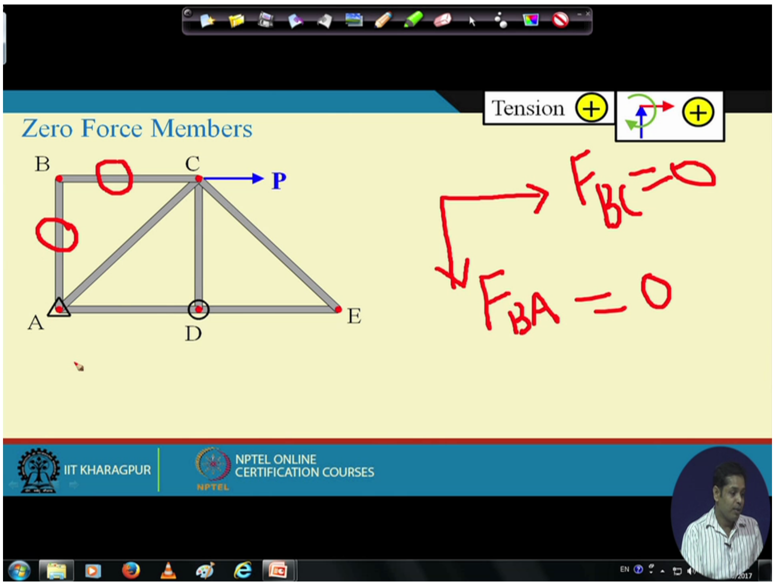

Now similarly look at joint E. Now you see if I draw the free body diagram of joint E then this will be FED . And similarly we have another FCE .

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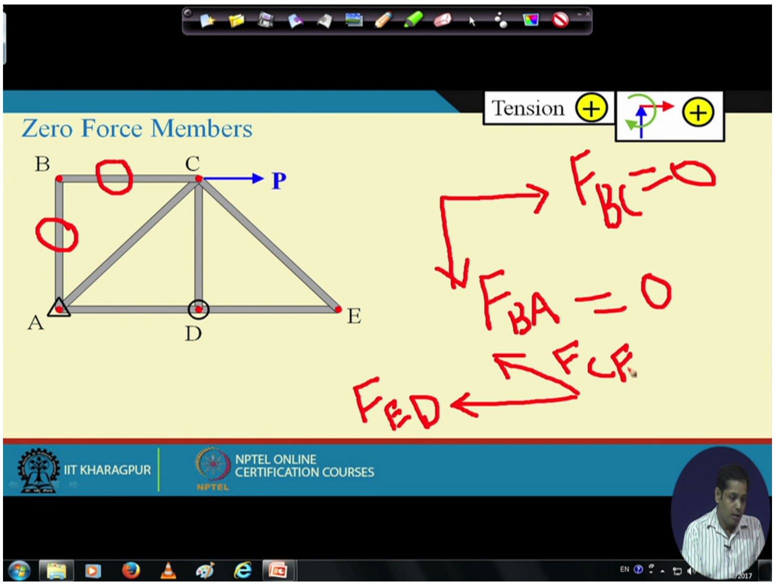

Now this is horizontal and this is inclined. It has some horizontal component, it has some vertical component. Now there is no other vertical force which can balance the vertical component of FCE, okay. So therefore FCE has to be zero. Now if FCE is equal to zero there is another horizontal component which can balance FED. So FED has to be zero. So this is zero and this is zero, okay.

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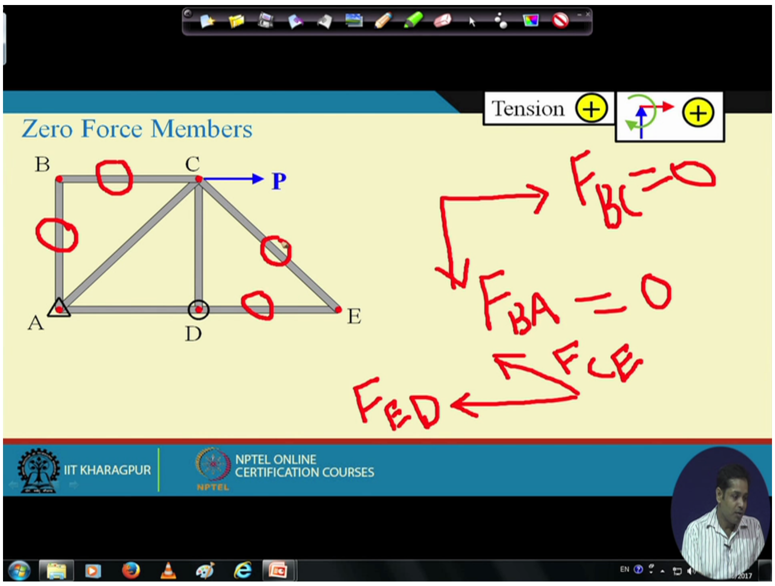

So without drawing any free body diagram we could say that these are the four members where the forces are zero. Then rest how many member forces we need to determine? We need to determine for this member, for this member and for this member.

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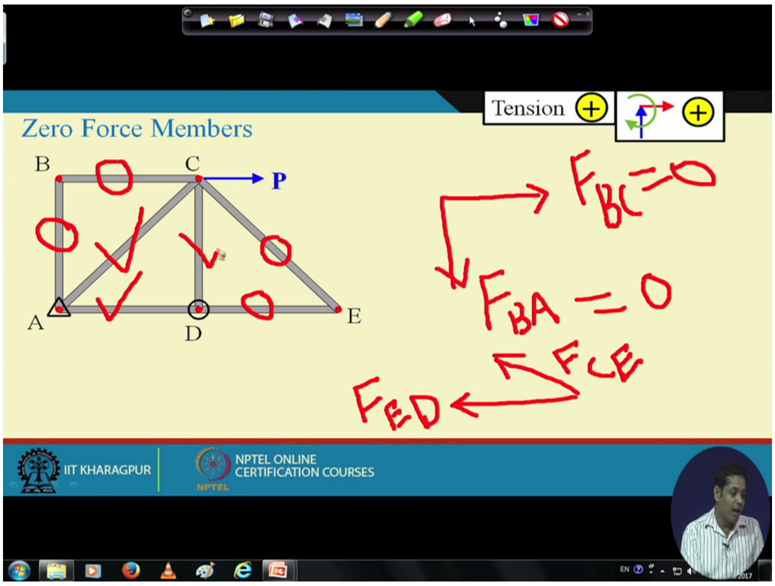

Now instead of 7 member forces now we need to determine only 3 member forces, right? So that is why identifying zero force member is very important or it will reduce your computation, okay. Now let us see with another example. Now this is another example.

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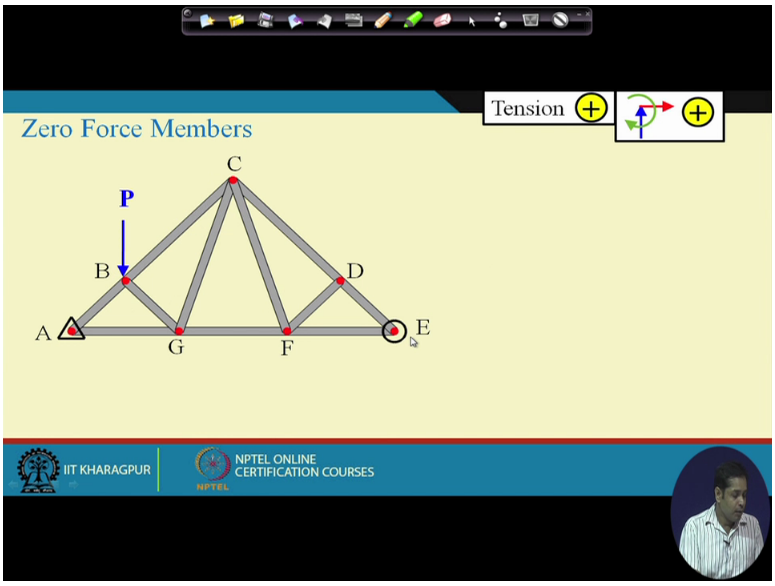

Now look at joint D. Joint D is if I take the forces in this member is this direction and the forces in this member are perpendicular to this direction, right? This is perpendicular direction. So this is 90 degree, okay.

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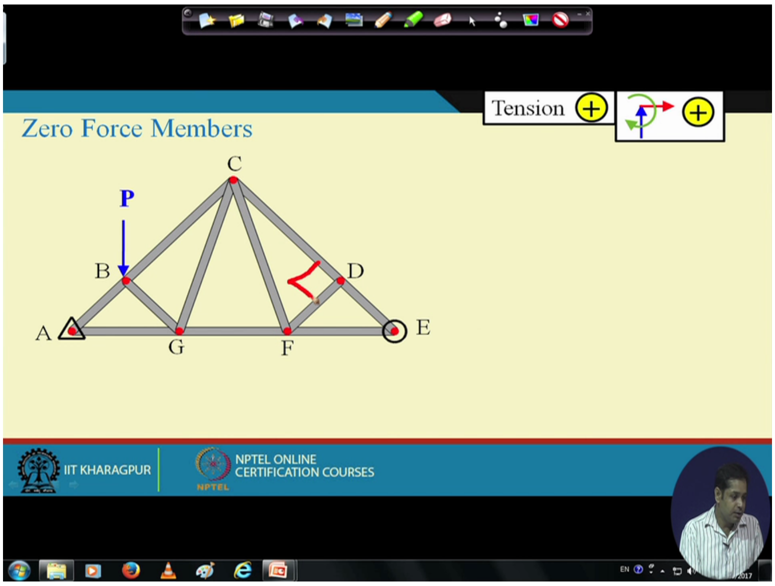

Now so in order to balance this force let us draw the free body diagram of joint D. So this is D, right? This is a free body diagram of joint D. This is CD, this is DE and this DF. Now this and this can balance each other because they are in opposite direction but there is no other force in this direction which can balance this force, right?

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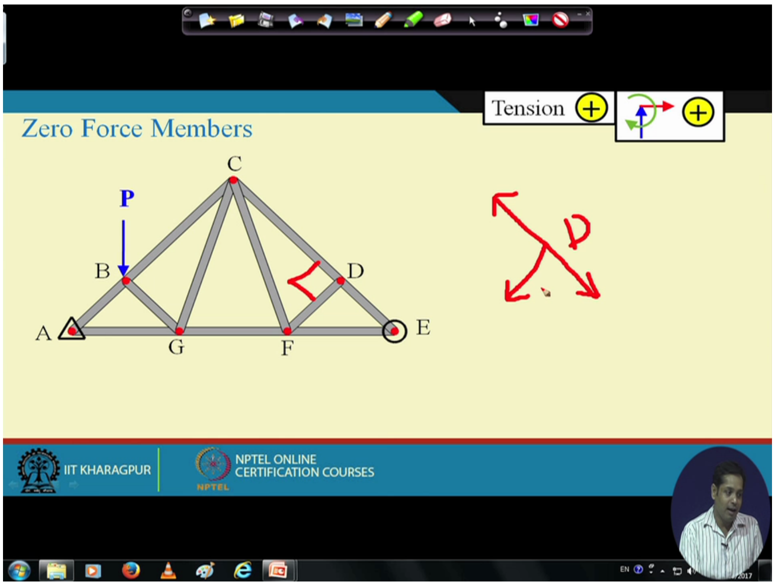

So therefore this member is a zero force member, okay. Now if this member is zero force member let us see joint F. If this member is zero force member then this is the free body diagram of joint F. This is F. I am not showing the member force in FD because this is zero, okay. So this is FCE, this is CE and this is FG and this is FE .

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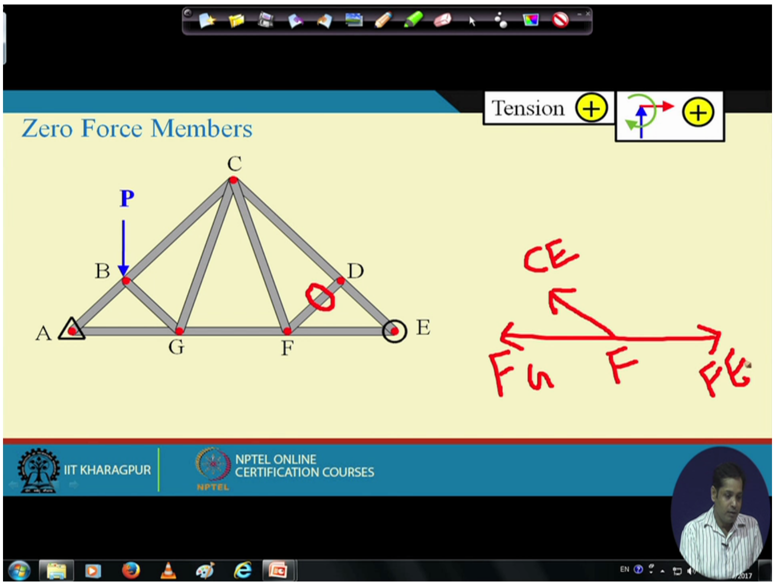

Now force CE has some vertical component. But there is no other vertical force which can balance this. So therefore CE has to be zero. So member CF has to be zero . So this force is again a zero force member, okay.

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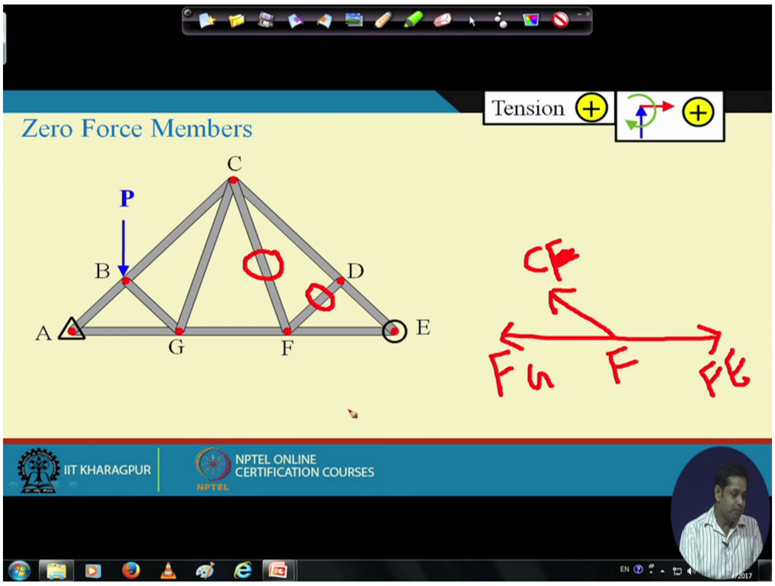

Now similarly can we say the same thing for member BG? We cannot say because we have a vertical external load here. The component of P in this direction will balance this force.

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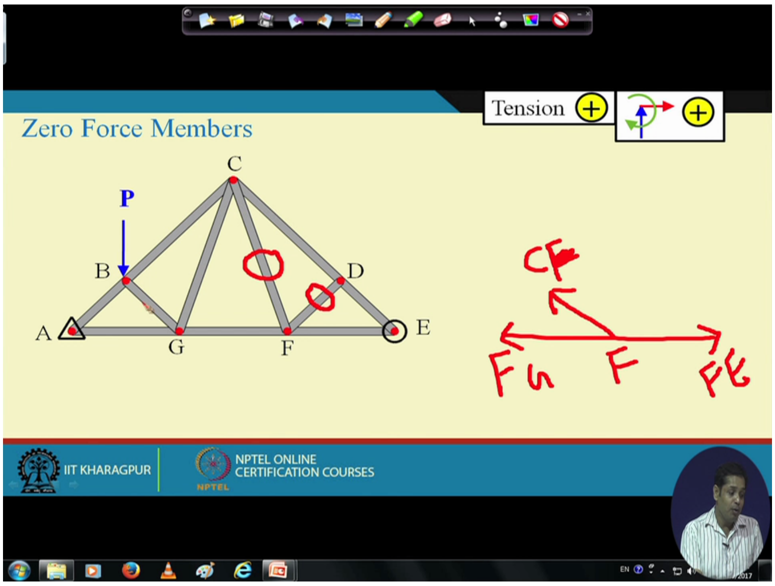

So we cannot say whether the force in this member is zero or not, okay. So we can easily identify that these two members are zero force member. The advantage is now instead of solving for many members you need to solve for fewer members. And another advantage is you see at every joint you have at most two equations, right?

So when you select a joint we need to select in such a way that the joint has only two unknown, because if the joint have more unknown then again only two equation cannot give you all the unknown. So when we select a joint we select in such a way that it has only two unknown, okay, or one unknown. Now suppose if we do not identify this member and this member as zero force member and let us draw the free body diagram of F.

Then if we draw the free body diagram of F then we have the four member forces 1, 2, 3, 4, 4 member forces. But at this point we have only two equations, right? So we cannot start with joint F.

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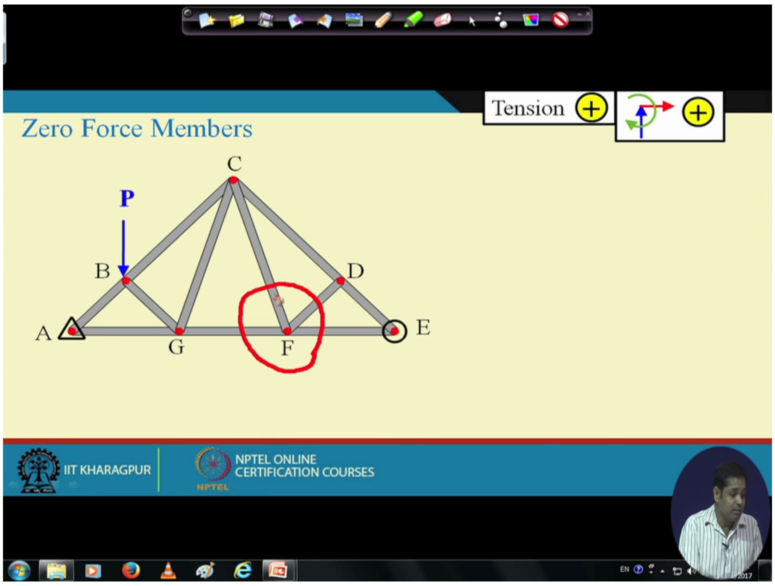

Again if you take joint D then also we have three unknowns. So now once we identify that these two members are zero force member then in the free body diagram of joint F only we have non zero forces this and this. So this is (ho) why (under) identifying zero force member is very important, okay. Now quickly let us see one more example, okay. Now this is a truss.

Now what you need to determine is determine the forces in members ED and the member EF and then member GF and member BH. These are the four members we need to determine the member forces.

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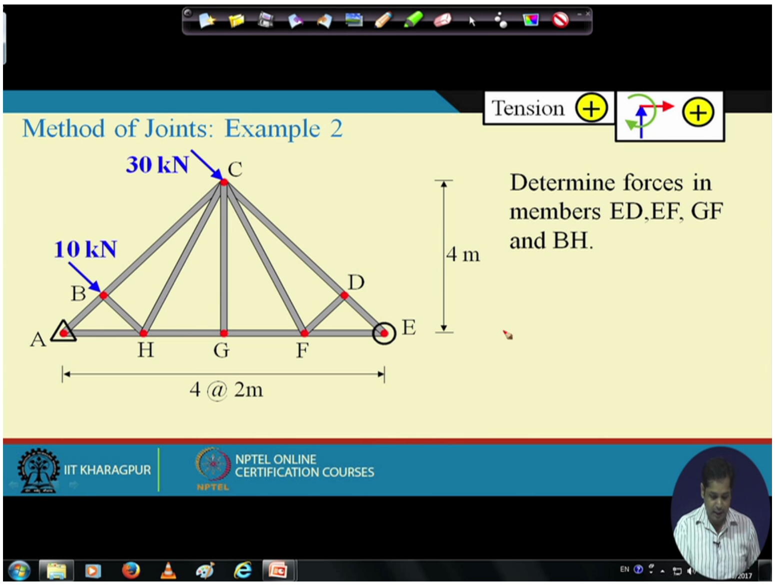

Let us see this, okay. Okay now first we need to calculate the support reactions, okay. Now let us draw the free body diagram on this itself, okay.

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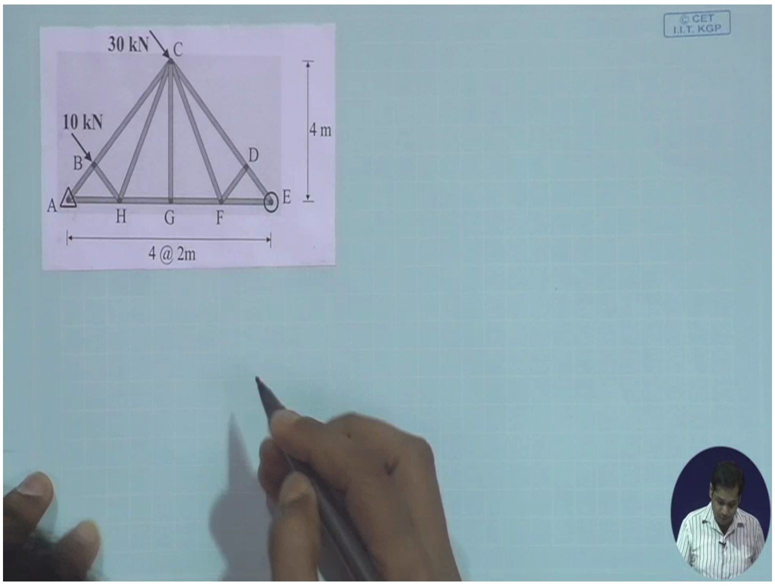

This will be Ey and then this will be Ay and this will be Ax, right? But remember one thing here just I have to draw another figure that is why I am showing the support reaction here itself. But when you draw free body diagram the first thing is the body has to be free from support, right? So when you are replacing a support by forces you cannot show the support. So this I am drawing because I can save some time, okay.

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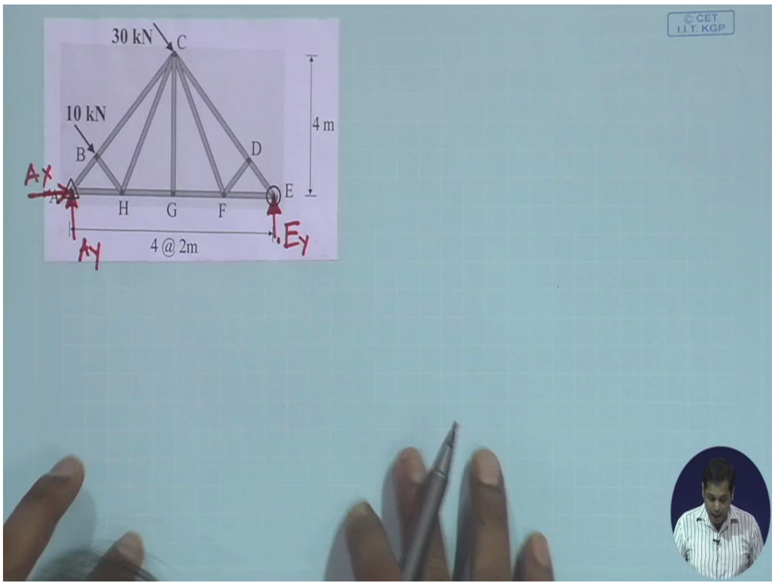

Now we can apply the equilibrium equations on this free body diagram. Summation of Fx is equal to zero, summation of Fy and summation of moment about A or about E is equal to zero and if you do that then we have support reaction this. We have Ey is equal to and I am not doing it once again here because we have discussed this step many times in this class. Ay is equal to 5 point 3 kilo Newton.

Do not forget to write unit because without unit any data is meaningless. Without unit any data does not gives any information, okay. These are all support reactions that you can obtain by free body diagram of this, okay.

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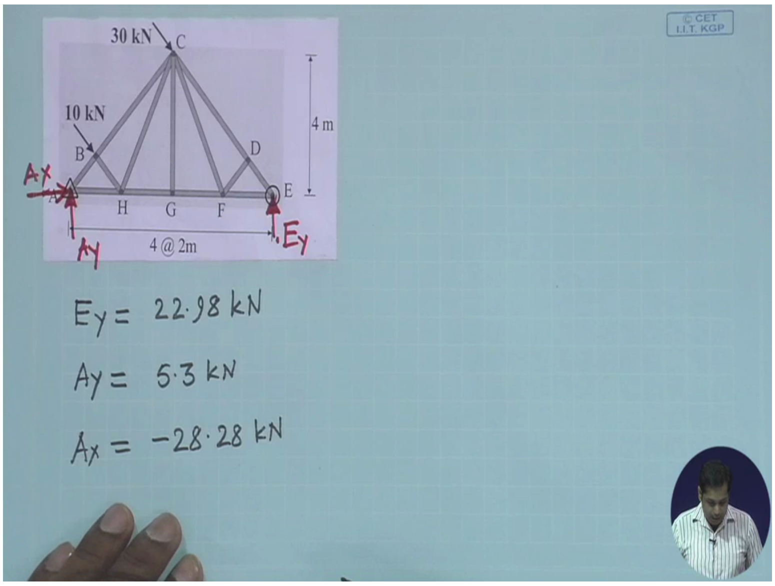

Now let us draw free body diagram of joint FBD of E. FBD of E will be what? We have reaction force which is Ey, then another (reac) member force which is FEF and then this member force as FED, okay. And these angles are 45 degree, okay.

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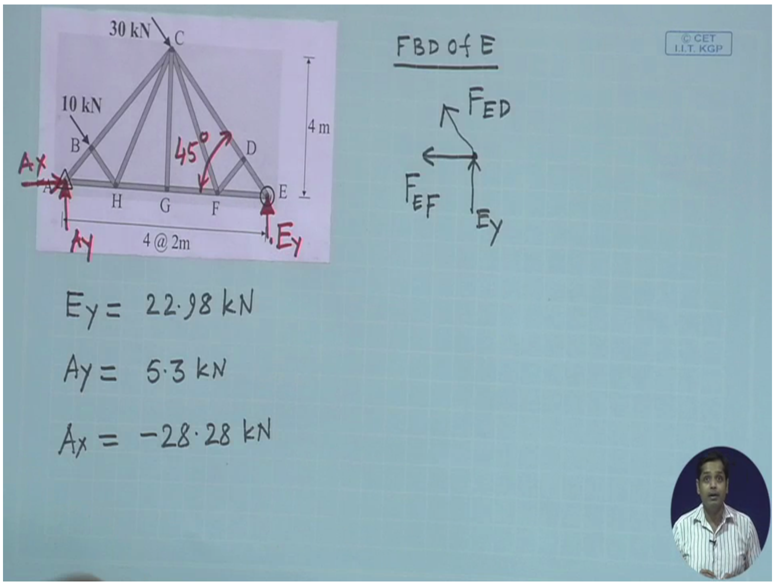

Now so this is 45 degree . Now apply summation of Fy is equal to zero. If I take summation of Fy is equal to zero the component of Fy is Ey and the component of this gives us Ey which is upward positive plus FED. And we know already Ey is equal to this and this gives us FED is equal to minus 32 point 5 kilo Newton, okay. Now this is just solving this equation, okay.

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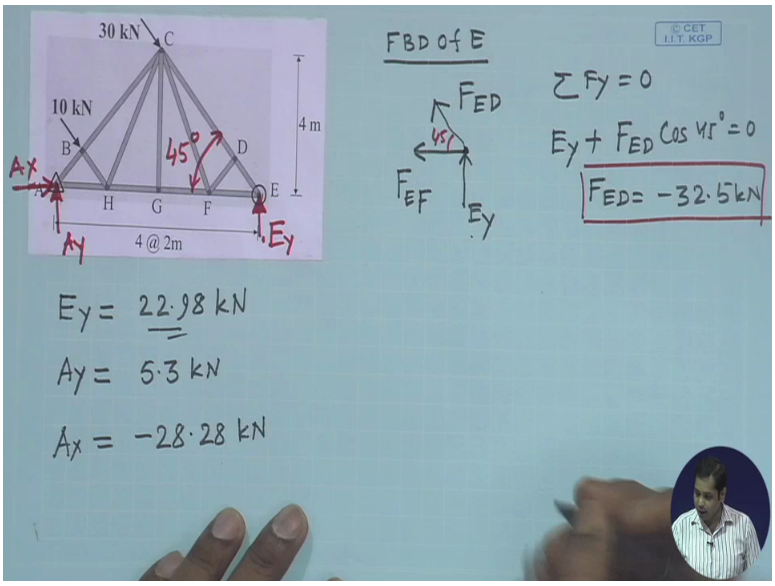

Now similarly next we take summation of Fx is equal to zero and summation of Fx is equal to zero will give us minus FEF minus FED cos 45 degree is equal to zero. Minus because they are in this direction, okay.

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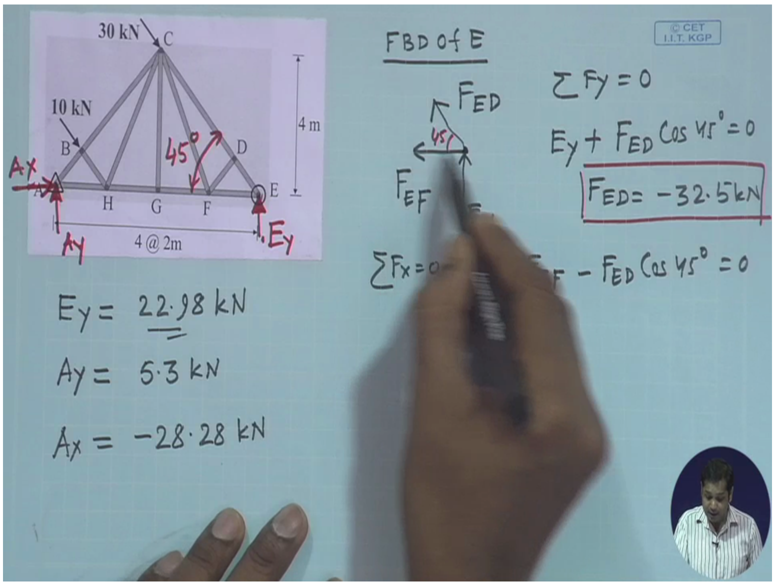

Our sign convention was if you remember this is positive, right?

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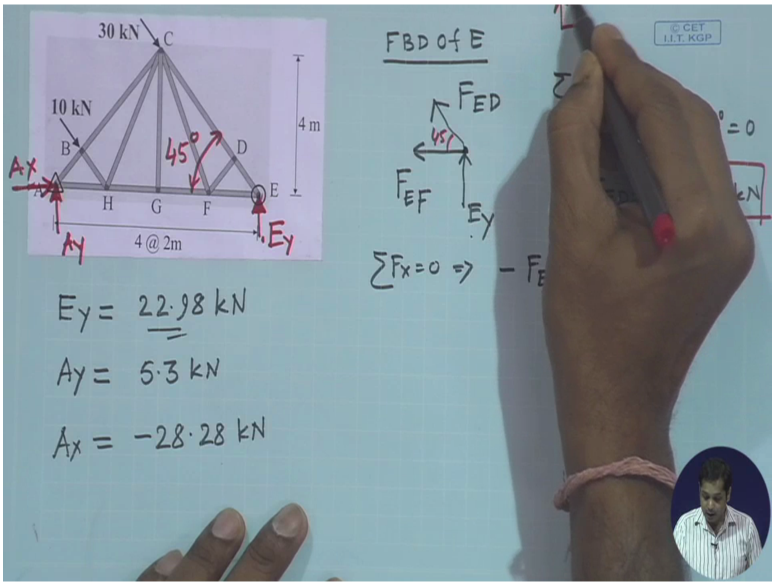

Okay now if we substitute ED here then we get FEF is equal to 22 point 98 kilo Newton , okay.

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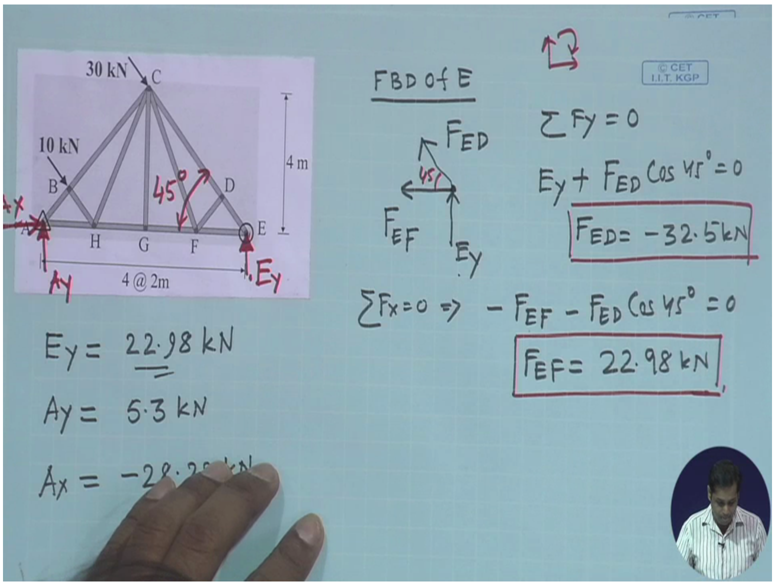

Now let us identify is there any zero force member here? Yes this is a zero force member. This member is F this is a zero force member and this is also a zero force member.

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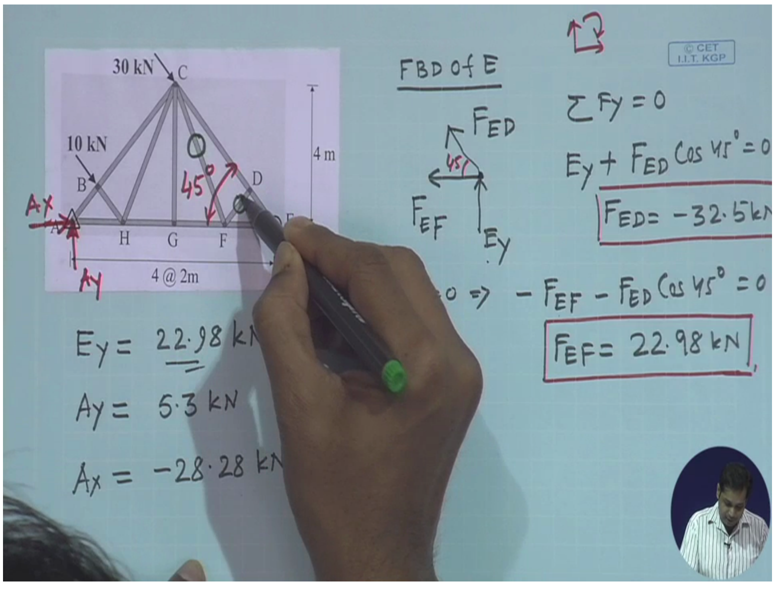

Just now we discussed. This is a zero force member, this is zero force member. Now once when we say that these are zero force members let us now draw the free body diagram of joint F. Now FBD of F that gives me this is FEF and this is FGF. These two forces we do not have to show because these are zero.

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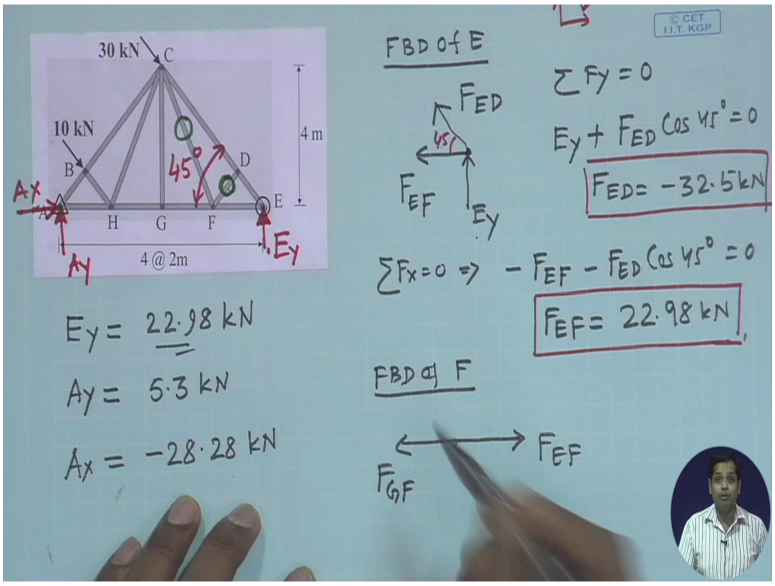

And our FEF just now we determined here. So naturally from this free body diagram we can say that if FEF is equal to FGF. So FGF is equal to same 22 point 98 kilo Newton .

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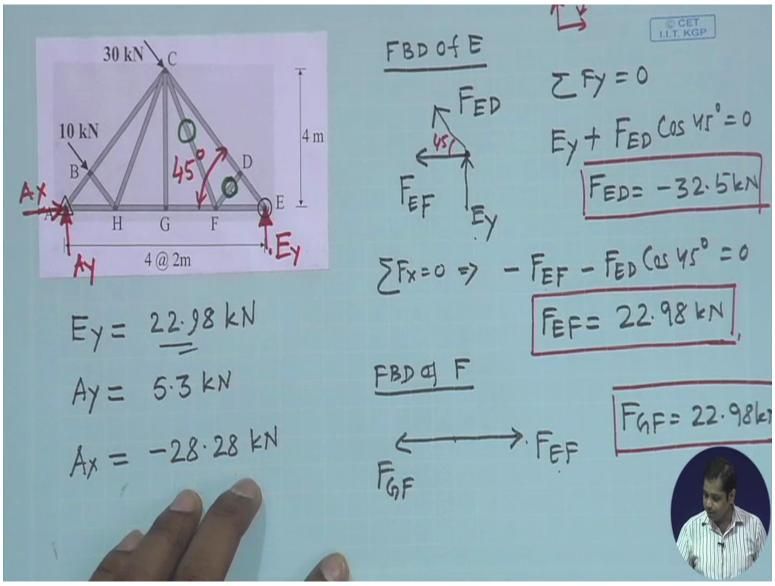

Now let us last draw the free body diagram of joint B . Free body diagram of joint B will be. Let us do it here. Now joint B is this. This is the member force in FBC, this is member force in FAB then 10 kilo Newton and then this is FBH .

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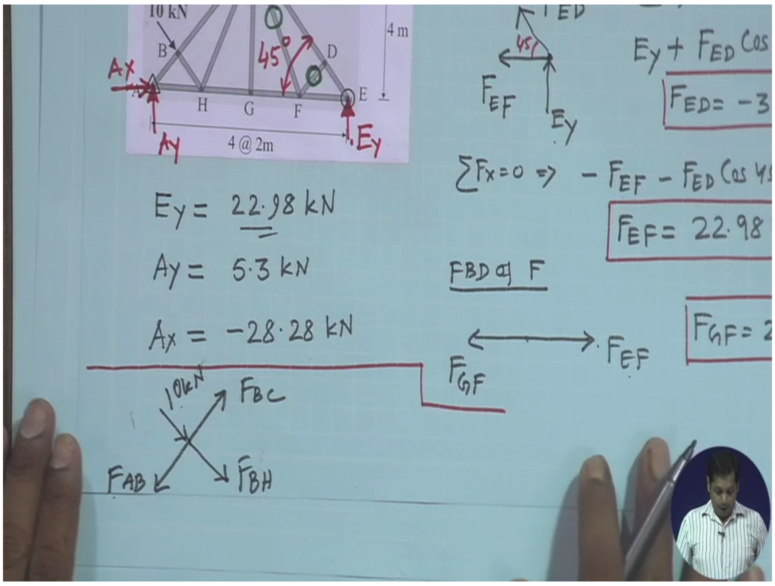

Now the equilibrium equation does not say that summation of force in x direction is equal to zero or summation of force in y direction is equal to zero. The equilibrium equation says that if we take any two independent coordinates system then summation of forces in each coordinates are zero. It could be X and Y or any arbitrary oriented coordinate system. Now so why I am saying that? Now look at this free body diagram .

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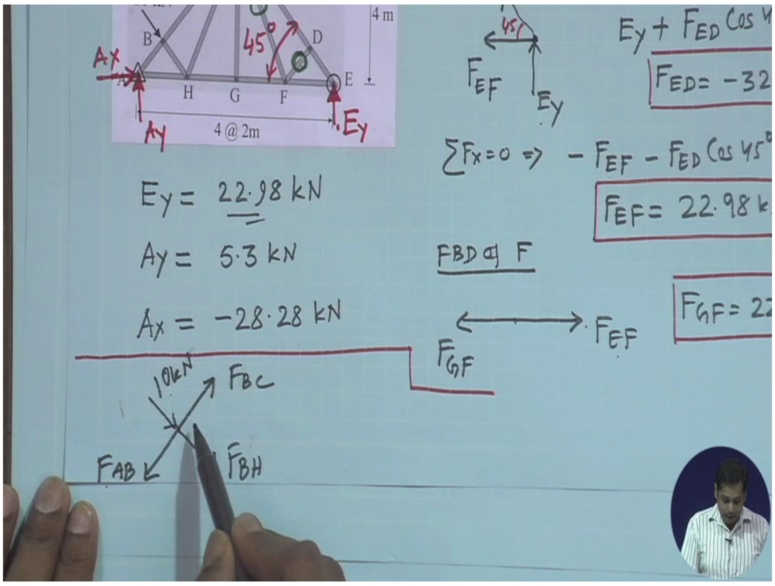

Now instead of saying the summation of Fx is equal to zero, summation of Fy is equal to zero, what we can say that summation of forces in this direction is equal to zero and summation of forces in this direction is equal to zero.

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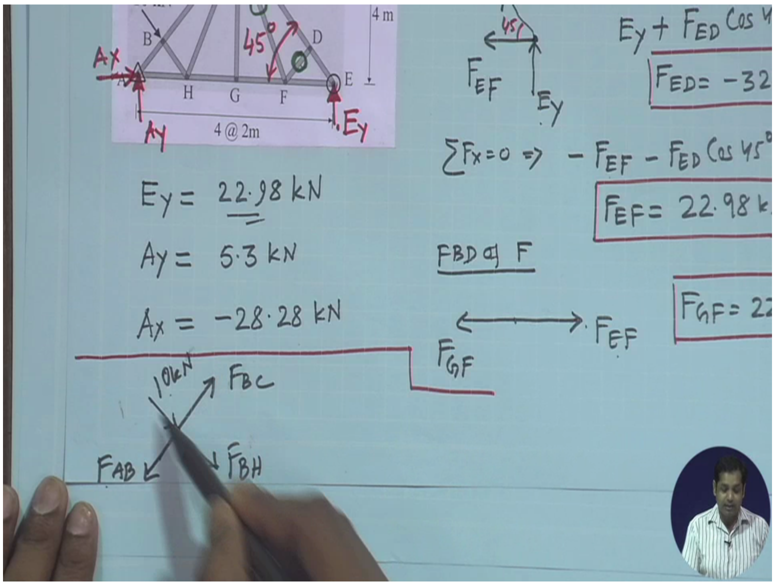

Now if I say summation of forces in this direction is equal to zero then directly we can get that FBH plus 10 is equal to zero and FBH is equal to minus 10 kilo Newton, okay.

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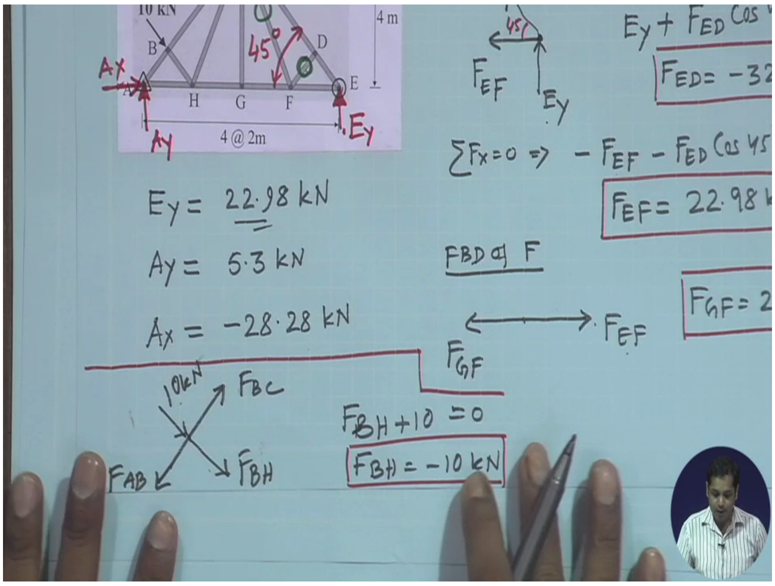

So now other member forces also you can determine following the similar approach, okay. Now what we do is next class will have some more example on determination of statically determinate truss using method of joints.

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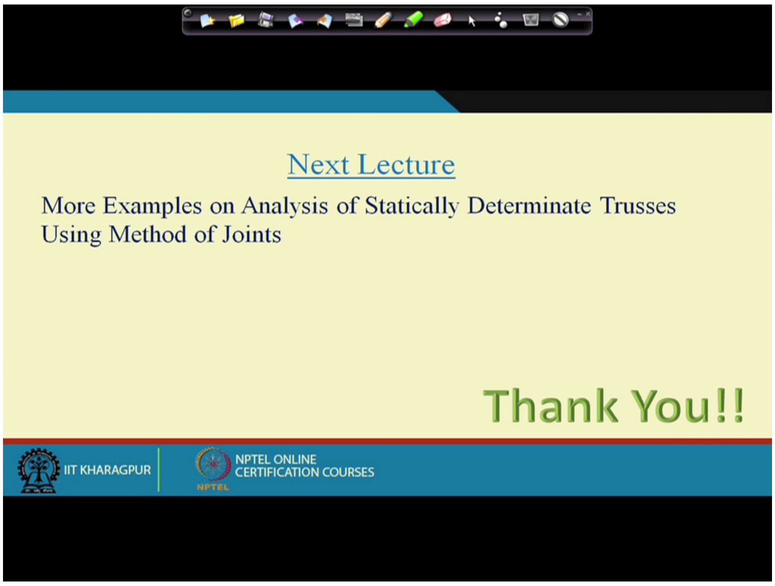

Thank you.