Structural Analysis 1
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Lecture 7
Analysis of Statically Determinate Structures: Truss

Hello, welcome to the next step of our journey. You see the first week which was essentially the warm up week we introduced the idea of structural analysis. What is structural analysis? And also we reviewed some of the basic concept of mechanics such as free body diagram, degrees of freedom, then equilibrium equation and then concept of determinate and indeterminate structures, right?

Now this week and next few weeks we will be discussing various methods to analyse statically determinate structures. We will start with truss. Now if you remember there are two kind of responses we are interested in. One is internal forces and then deflections. This week we will see various methods to determine internal forces in truss, statically determinate truss and the next week we will see how to determine displacement in truss, okay.

So let us first understand what is truss? Truss is a structure composed of slender members connected together at their end points through pins.

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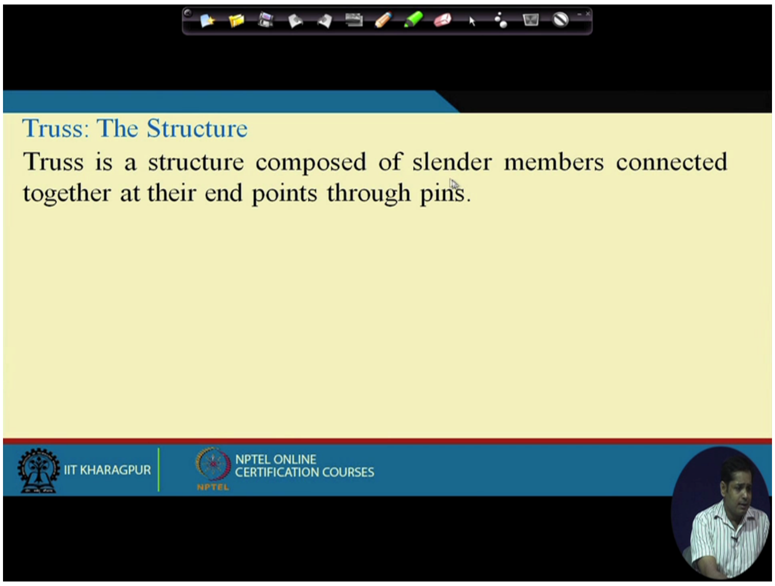

Slender means the cross section dimension is very small as compared to the length of these members. Now for instance suppose this is a truss, right?

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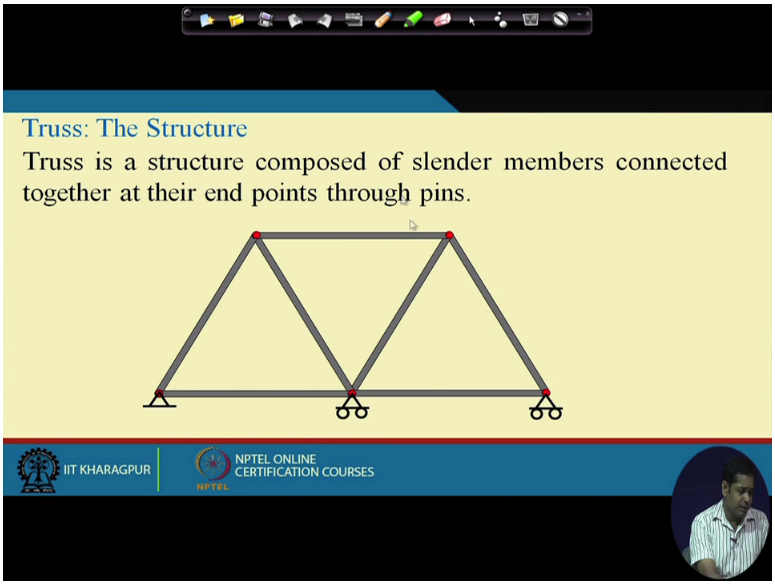

Now what it says? The structure is composed of slender members. Now we have several slender members. These are the members and these members are connected together at their end points.

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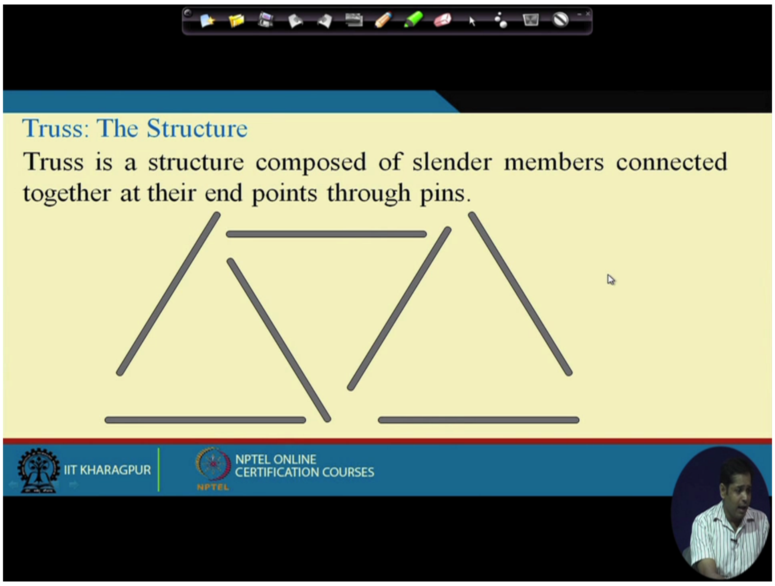

These members are connected together at the end points and how they are connected? All this connection should be through pins. So these are pins. And of course (supp) the truss is supported.

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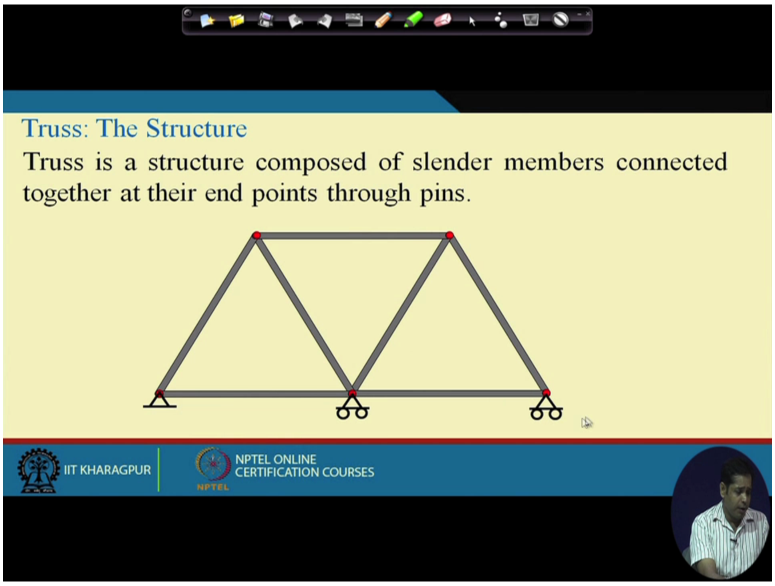

Now in this case we have three major structural components. One is a member itself which forms a body of the truss and their connection through pins and the supports. Now for better understanding of the structural arrangement let me take you to our model lab where I will show you one prototype of truss so that you can understand these members and their connection in a better way. This is the prototype of your Warren truss.

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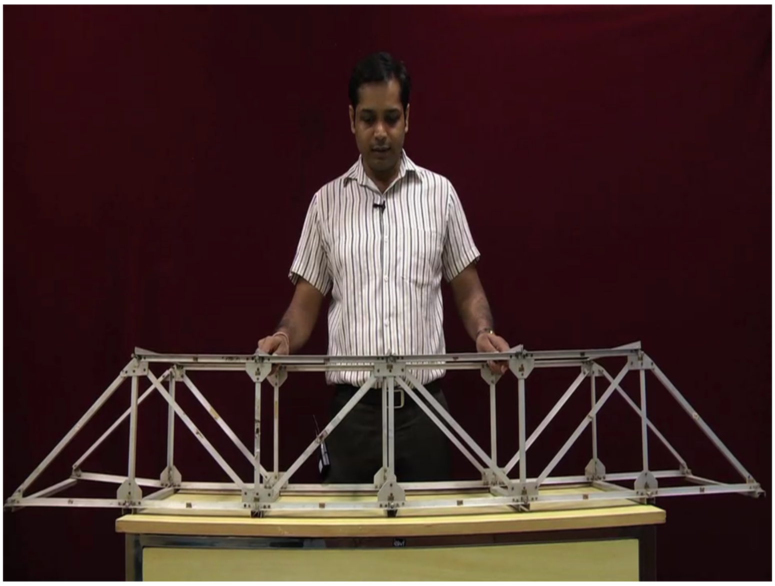

You look at the arrangements of different members in this truss. All of these are the members, right? All these are the members and these members are connected at their end. For instance this is the connection of this member, this member and this member.

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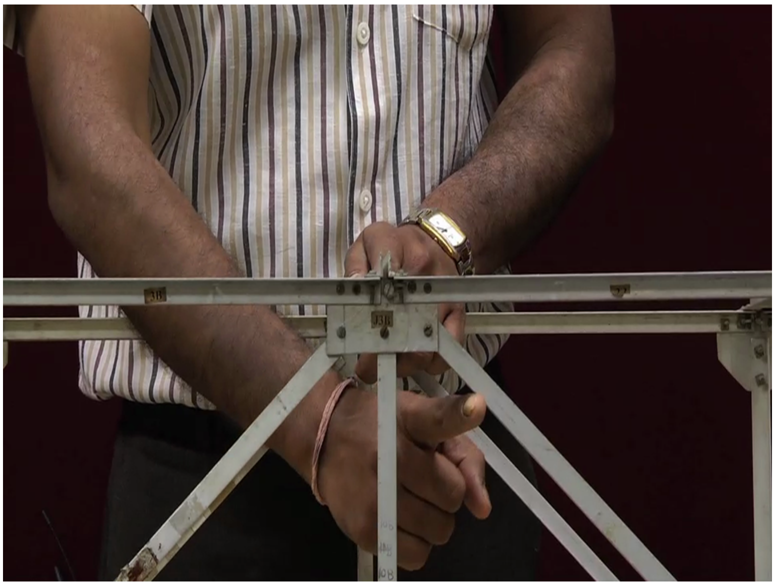

Now first thing is if you see the cross section of this members are very small as compared to the length. That is why these members are very slender.

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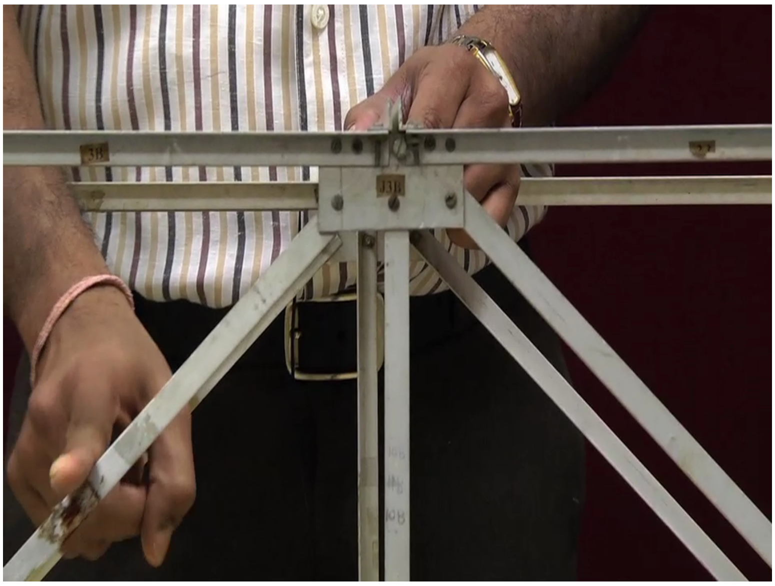

Now all these members are separate. This is called top cord and this is bottom cord. Even the top cord if you see, this is separate, this is separate and this is separate.

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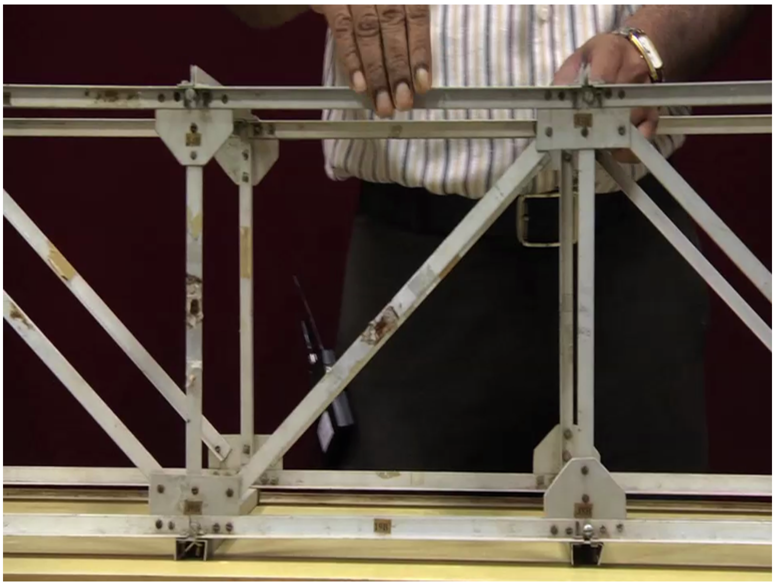

Now you look at the joint. You see this plate is called gusset plate.

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Now what happens is all the members in general they are not directly connected to the other members. They are connected to the gusset plate. So it is through the gusset plate, the force and everything gets transferred one point to another point. Now you see another important thing just now when we defined truss we said that every joints are pin joints. But in actual real life trusses these joints are not pin joints.

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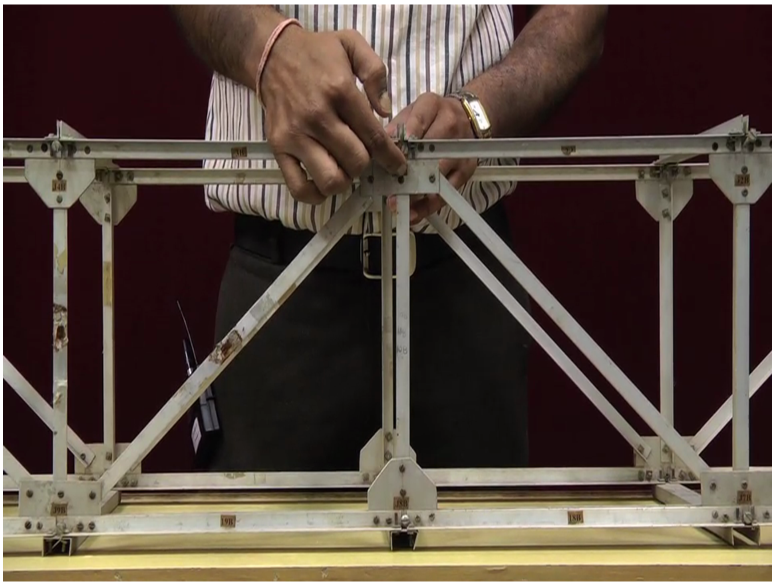

Even in this case you can see these joints are not pin joints because you see that the two screws are used here.

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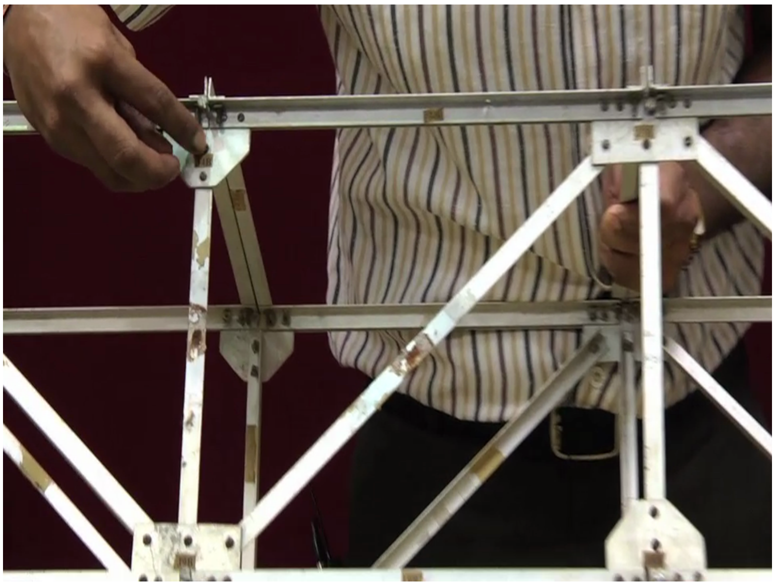

So if had it been one screw this would have behaved like pin joints. But in this case these are two screws. So this is not actually pin joint.

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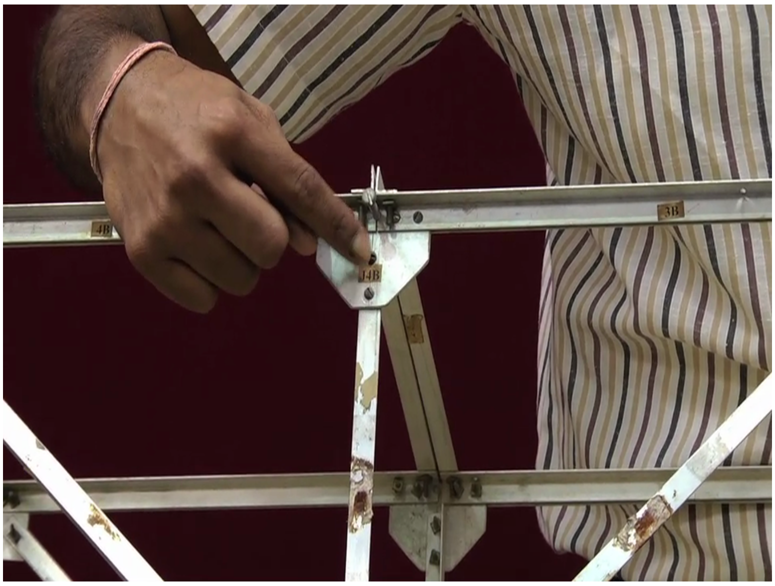

Now even if you see the real truss those joints are also not pin joints. But it is an idealization that we use. We assume those joints as pin joints. Now their effect to be very small but the rigidity that provided by those joints, the effect of that rigidity will be very small. And that idealization we can go with. Now you see if I just turn it like this, so you have two such things.

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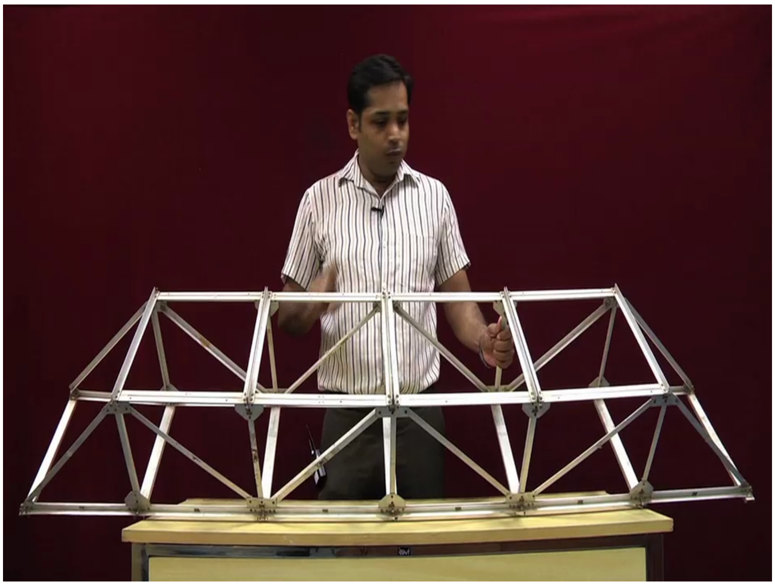

One is this and another in this and these two are connected to this blessing.

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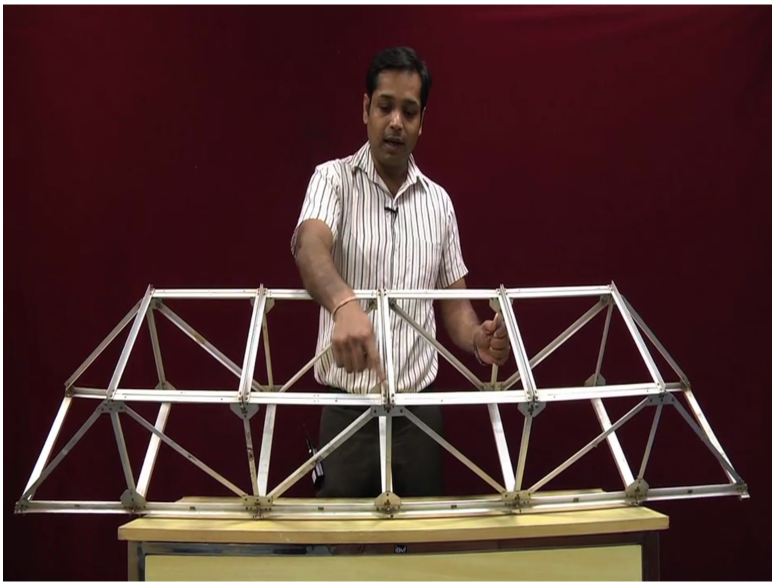

Now this is a space truss but what we do is we will analyse only one part. Only this plain part we will analyse, okay.

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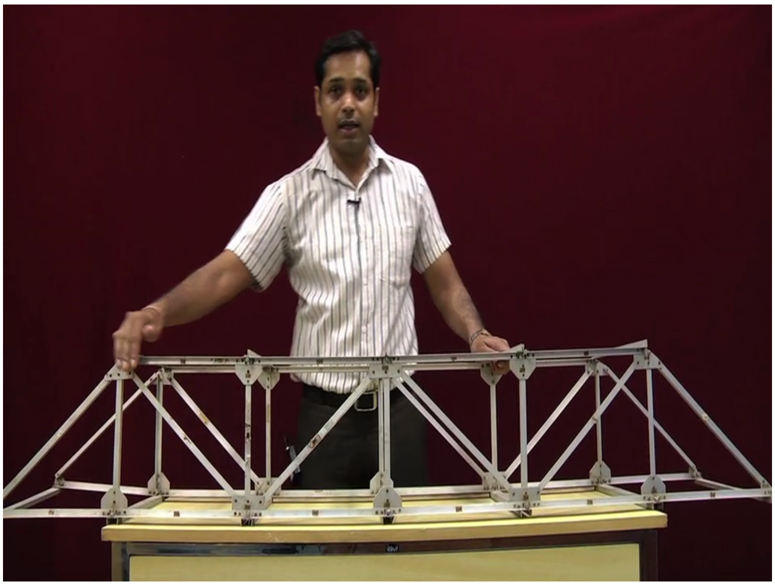

So let us again go back to the classroom and then continue with the discussion on structural analysis of truss. Okay that is what we could demonstrate in a classroom setup. But next time when you cross a bridge and see a truss then please look at the truss carefully.

See their arrangements, especially the joints or next time when you park your bicycle in a cycle stand which is made of truss then look at the truss or when you are walking through a field and see the transmission tower, you look at those towers and try to understand yourself their structural arrangements, okay. Now the analysis of the truss is based on certain assumptions and these assumptions are, the first assumption is all the joints interconnecting the members are pin joints.

Just now we defined truss that the members are pin joints but now you remember I told you in the module lab as well. Originally if you look at the different truss structure, those members are not pin connected. Their connection provides some rotational rigidity to the structure but this is the idealization, this is assumption that in our analysis we assume those connections are pin joints.

And that rigidity that provided by the joints is very small as compared to the other factor. That is why that rigidity can be neglected. Now the second is the loads are all (cons) concentrated and act exactly on the joints.

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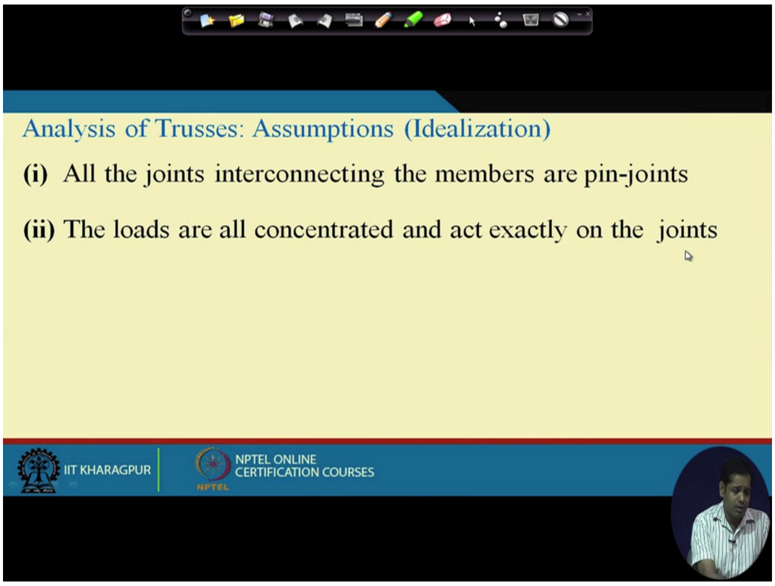

What does it mean? You see if this is a truss then the second idealization is, whatever load you have that load has to be always on the joints, okay.

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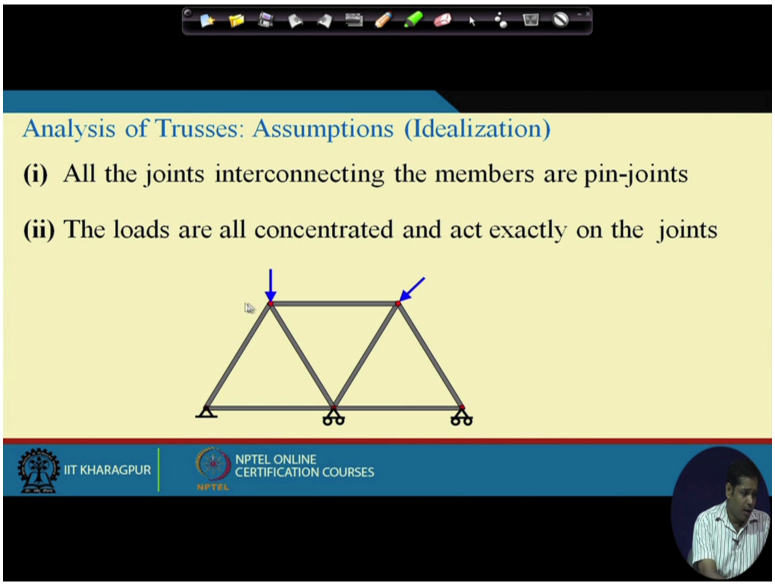

So these are correct load. You cannot have a load like this which is acting on the member. This load is not possible because under such kind of load the truss member will undergo bending which is not desirable in this case.

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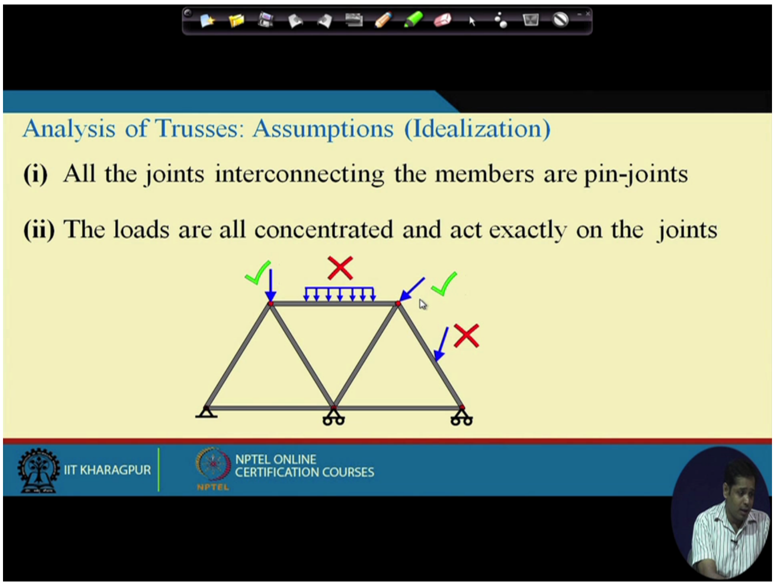

So the first assumption is our first idealization is all the members are connected through pins and the second idealization is our second assumption is the forces that is acting on the truss it is always on the joints, okay. Now you see let us introduce the concept of two force members. Now the name itself suggests a member which has two forces. Now what does it mean? You see this is a truss.

Now if I draw the free body diagram of this truss then this is the free body diagram, this is hinge support, this is roller support. So hinge support is represented by two reactions and roller support is one reaction here.

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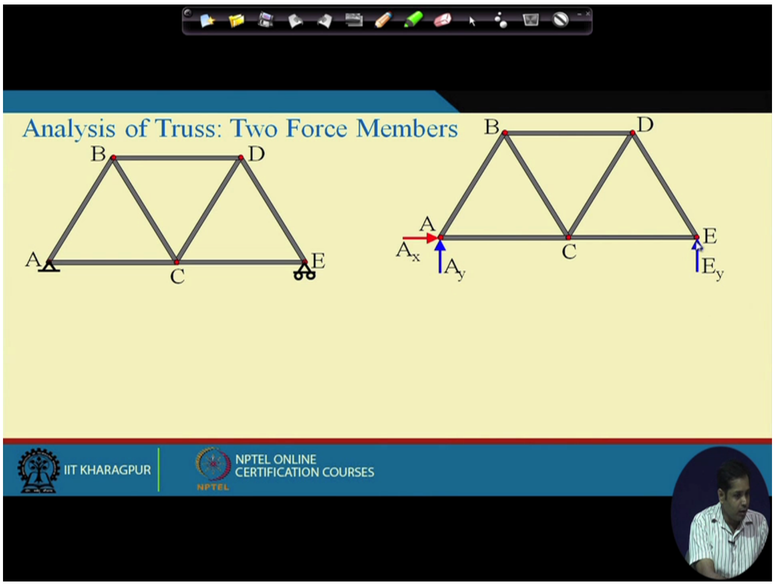

Now you take any member, suppose we take a member BC, okay. Now let us draw the free body diagram of the member BC. Now what are the end condition of this member BC? The BC end condition is a pin joints, okay. So naturally at point B and point C the characteristic of pin join is, it does not (rot) provide any constraint against rotation. So therefore the hinge forces we have two forces, one is horizontal and one is vertical direction. So at B we have this force and C we have this.

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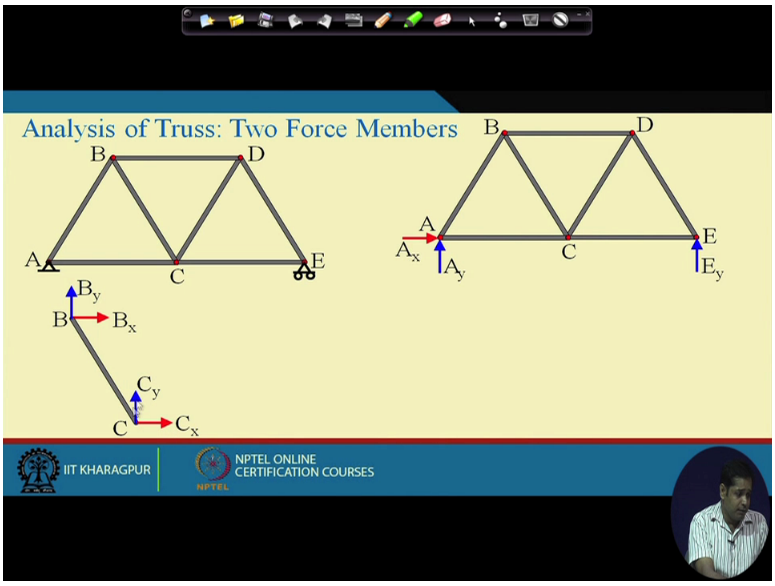

So this is a free body diagram of member BC, right? Now suppose RB is the resultant of Bx and By and similarly RC is the resultant of Cx and Cy. The value of RC and RB could be anything and in any direction. But suppose these are the resultant.

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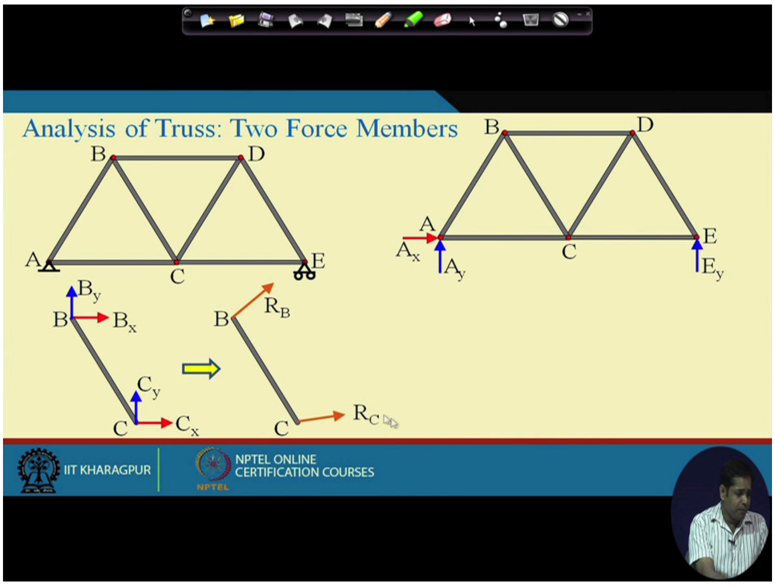

So this free body diagram now can be represented like this, right? These are equivalent. Now you see when we say a structure is in equilibrium then every point of the structure is in equilibrium, even the member of the structure is equilibrium, right? So in this case member BC is an equilibrium. Now what is the equilibrium condition? Equilibrium condition is that the net force acting on the body or net (more) moment acting on the body is zero, right?

There is no unbalanced forces or unbalanced moment. Now you see if it has to satisfy equilibrium condition the member BC then the first condition should be RB is equal to minus RC. So RB and RC they should be equal and opposite direction so that they can cancel each other.

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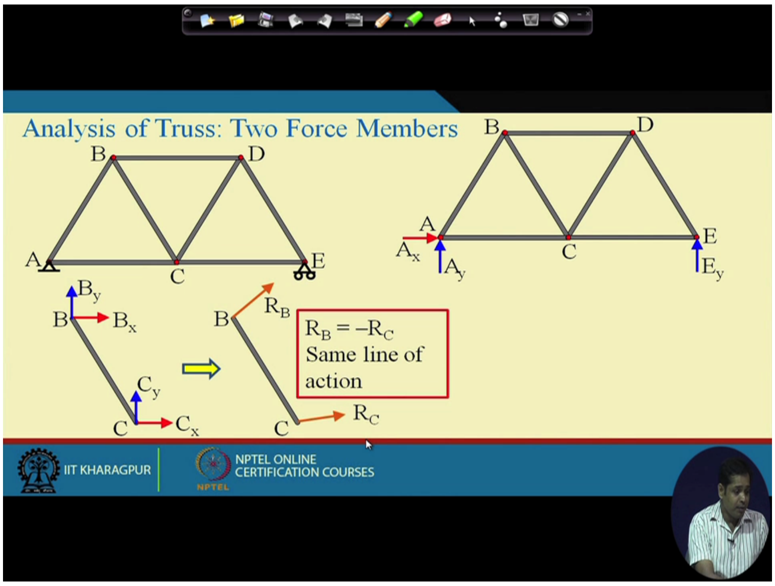

And so this will ensure that summation of forces is equal to zero. Now whatever force system you have they should not provide any unbalance couple in the system, right? Now this can be only insured when RB and RC their line of action is same and that line of action is along the member BC, right? So this RB and RC should be like this then only the equilibrium of BC will be satisfied, right?

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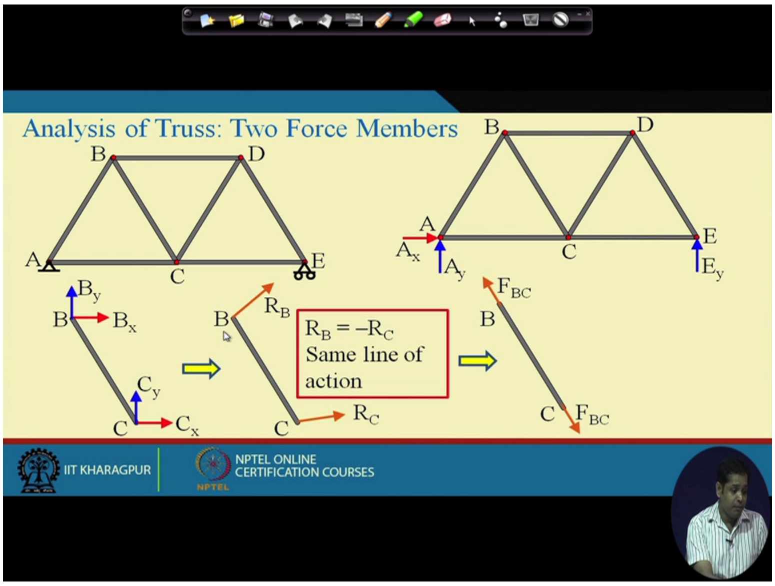

So in this case you see there is no net couple. The couple produced by this two forces is zero and they are equal and opposite so net force is also zero. So this is the free body diagram of member BC. Now what is says? Another possibility would be in this case, okay. Means in this case the member BC is under tension and in this case member BC is under compression.

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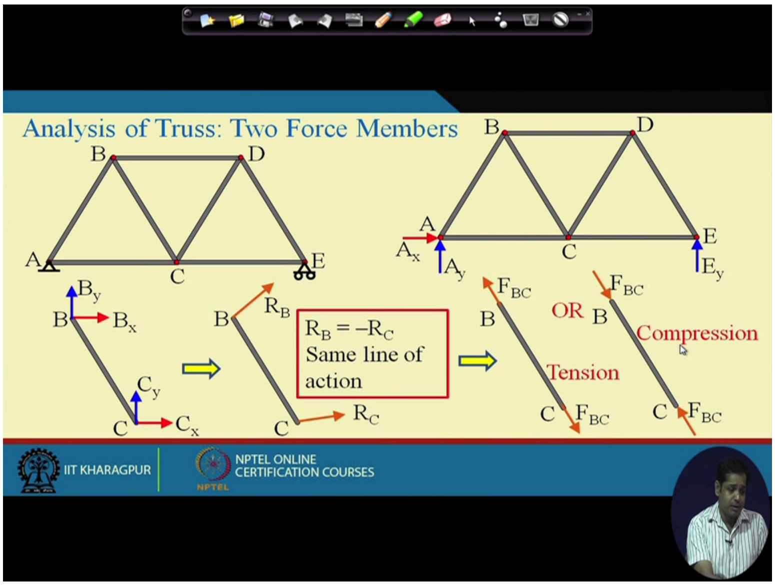

Now what it says? It says that if we take and this is true for any member in the truss, right? So it says that if we take any member from the truss then their free body diagram will be this. Those members are subjected to an actual force. It could be tension or it could be compression. So this kind of member is called two force members. Because in this member two forces are acting at the two end of this body, right?

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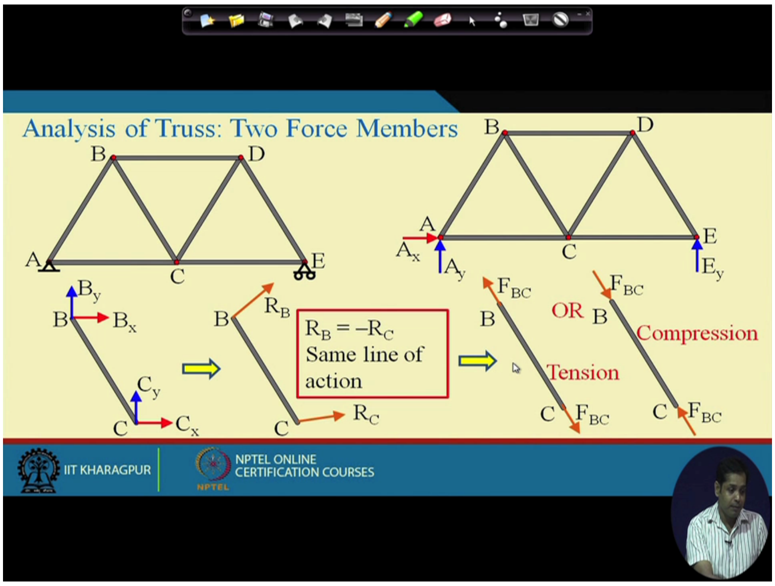

So in truss, all the members are two force members. and all the (mem) two force member and that is why in truss all the members are subjected to actual load, either compression or tension. Now if you recall when we discuss internal forces in beam then the internal forces in beams are bending moment and shear force. We did not consider actual force in beam. Now in truss there is no bending moment, there is no shear force.

Only force is that each member in the truss is the actual force, right? Now another important assumption is because of this actual force this member still does not undergo any deformation. There is no actual deformation in the member, right?

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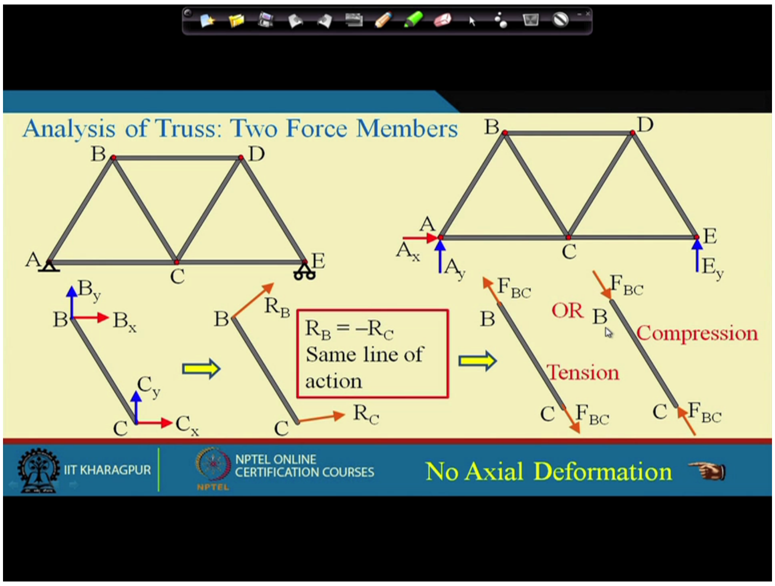

Now the free body diagram of BC we have just now discussed this is a free body diagram of BC. Similarly if I have to draw the free body diagram of other members, let us say member AB. So free body diagram of the member will be this. Now this is the standard representation of force in BC, FBC. FBC means this is the force in member BC, okay. Similarly when we write FBA it means it is the force in member BA. You can write FAB or FBA but whatever notation we use be consistent with that.

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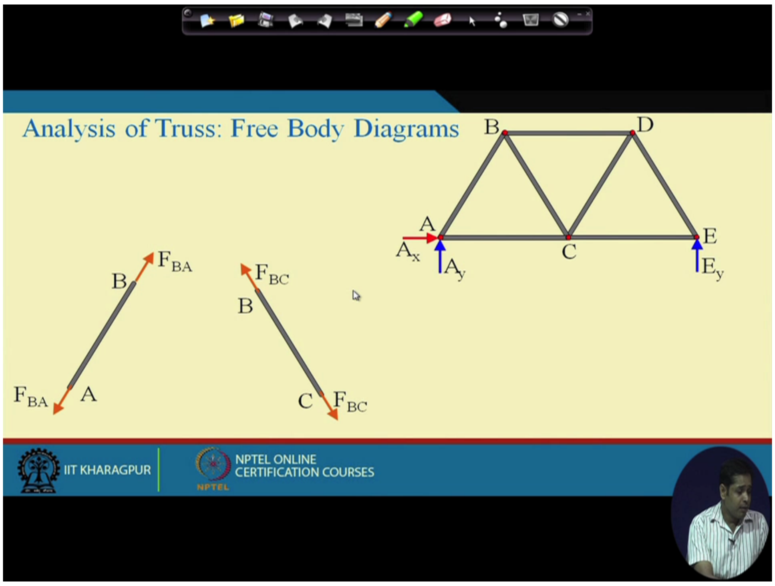

Now similarly we can have the free body diagram of member BD. Now you see member BC, BA and BD these three members are connected at B, okay. Now if I have to draw the free body diagram of this joint then what would be the free body diagram? This is the joint B.

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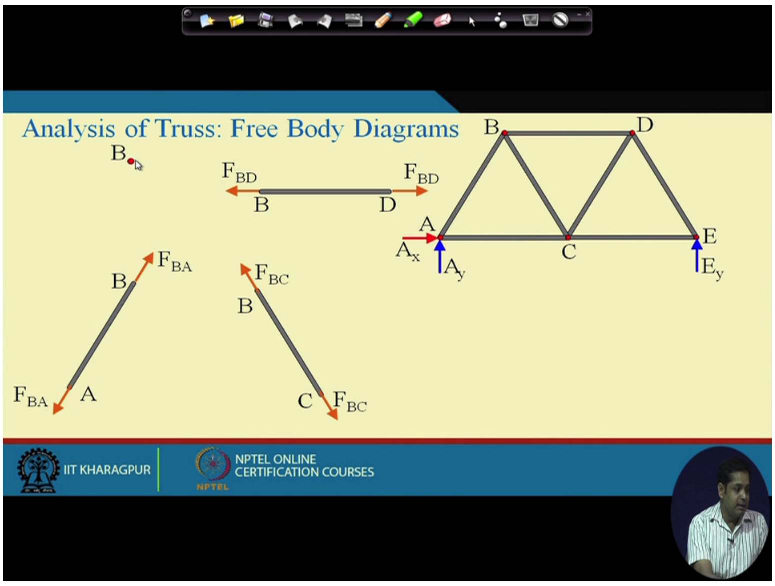

Now this point will get force from BD which is FBD and what will be the direction of this force? In this case it is in this direction. So when we draw the free body diagram of B the force has to be shown in this direction.

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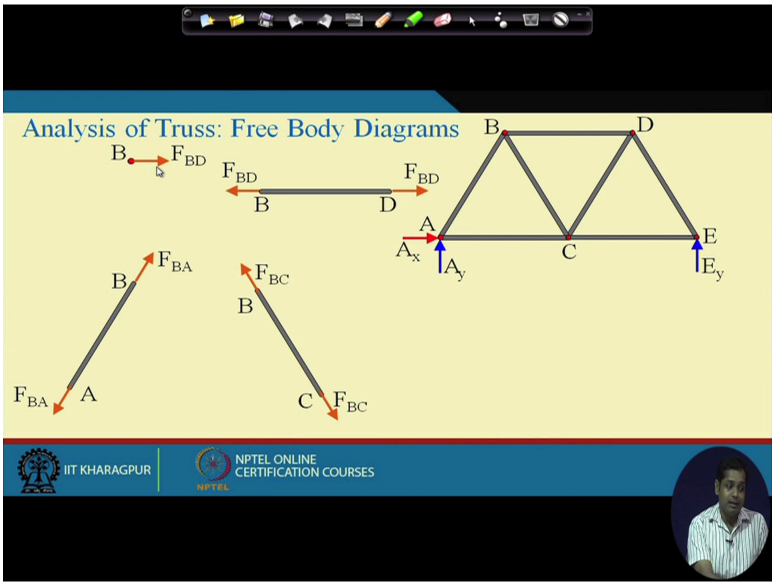

Similarly from BC there will be a force from BC and this BC will be the opposite direction that we have shown opposite to the direction in BC.

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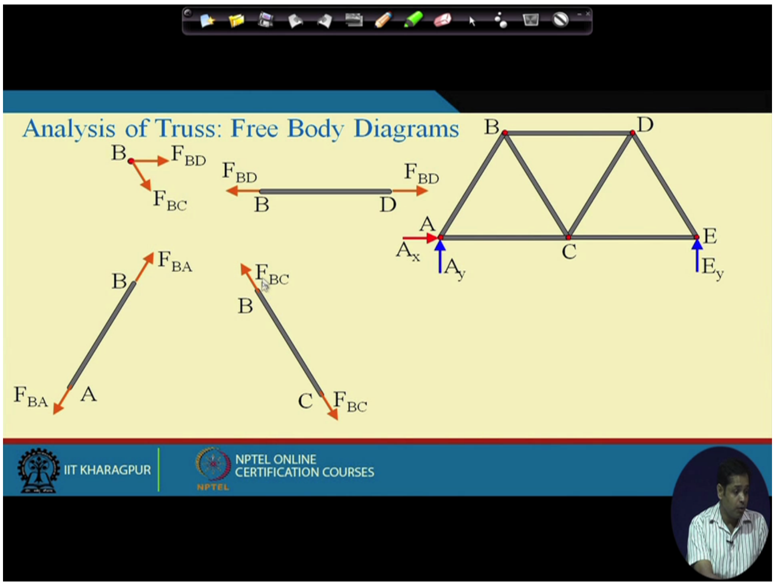

And similarly the force from member BA to this joint will be FBA. Now when these three members are connected with this joint then this force will balance this, this force will balance this and this force will balance this and the entire joint will be in equilibrium, right?

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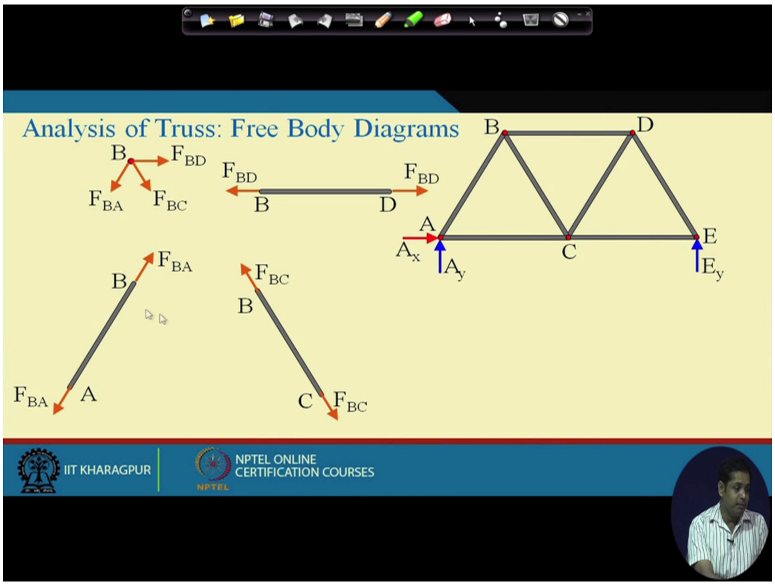

Now this is the free body diagram of the whole structure. And then these are the free body diagrams of member and this is the free body diagram of joint B, okay.

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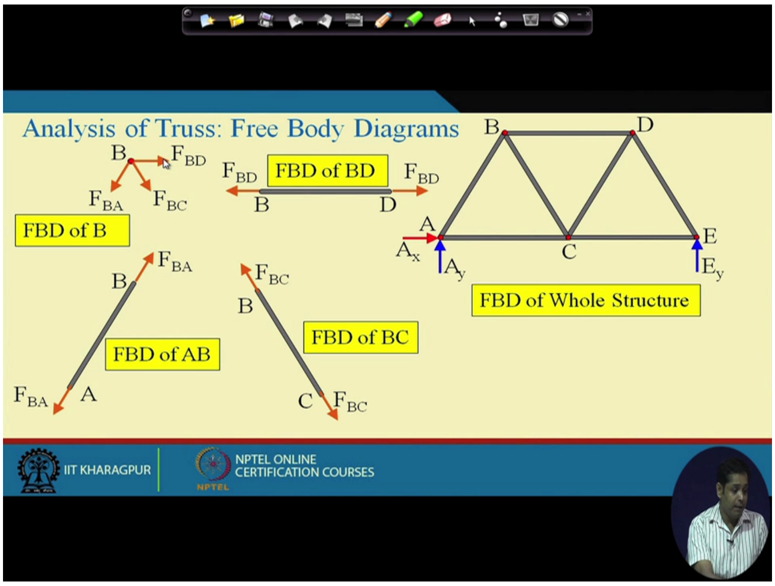

Now another important point here to note is our sign convention. We assume that the member if it is subjected to tension that it is positive and compression is negative. That is the reason why in all the members the forces as shown as if the members are in tension, okay. Now this is how we can draw the free body diagram of each member separately and free body diagram of joints. Now in this truss we have (sev) several joints.

Similarly free body diagram of all the joints can be obtained like this. This is a free body diagram of joint B. Similarly free body diagram of joint A if I have to draw then what are the forces at joint A? Reaction force Ax and Ay, then member force from AB and member force from AC. So this is the free body diagram of A. Ax, Ay and members force from AC and member force from AB.

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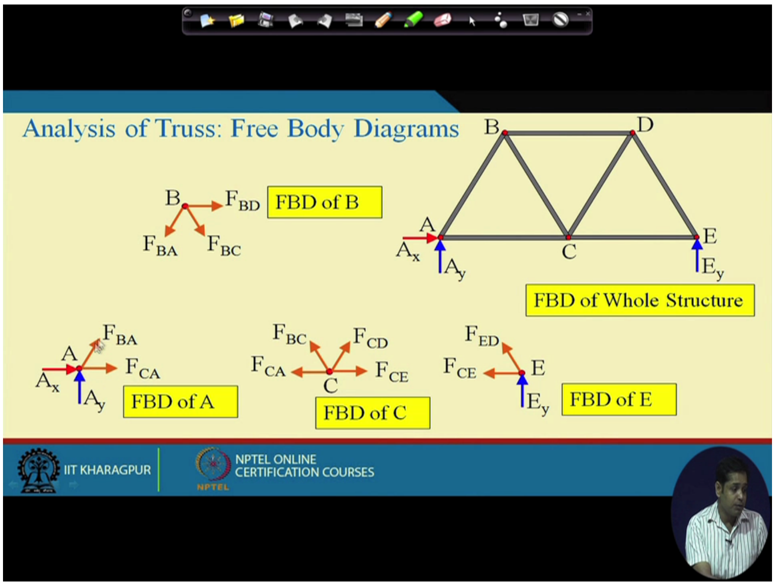

If we draw the free body diagram of joint C then we have member force from AC, member force from CE and member force from BC and member force from CD. So this will be the free body diagram of joint C.

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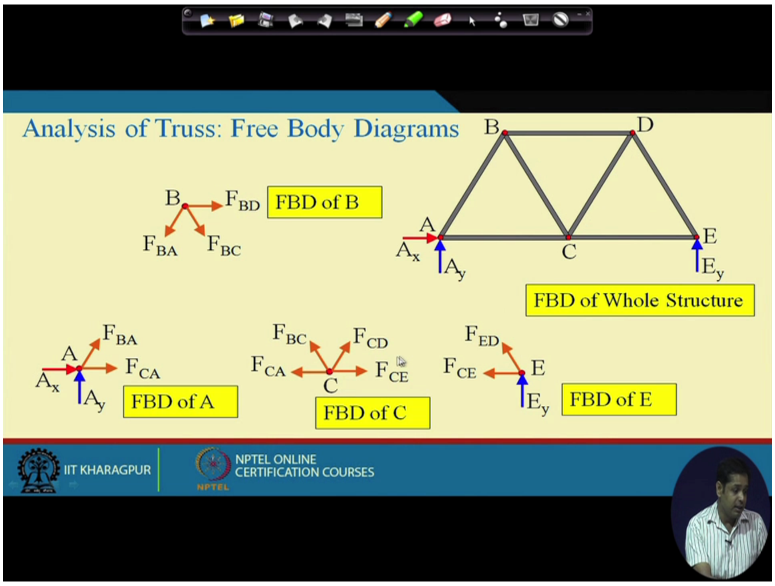

Similarly the free body diagram of joint E is reactions Ey and two member forces, member force ED and member force CE. This is the member force. So these are the free body diagram of different joints, right?

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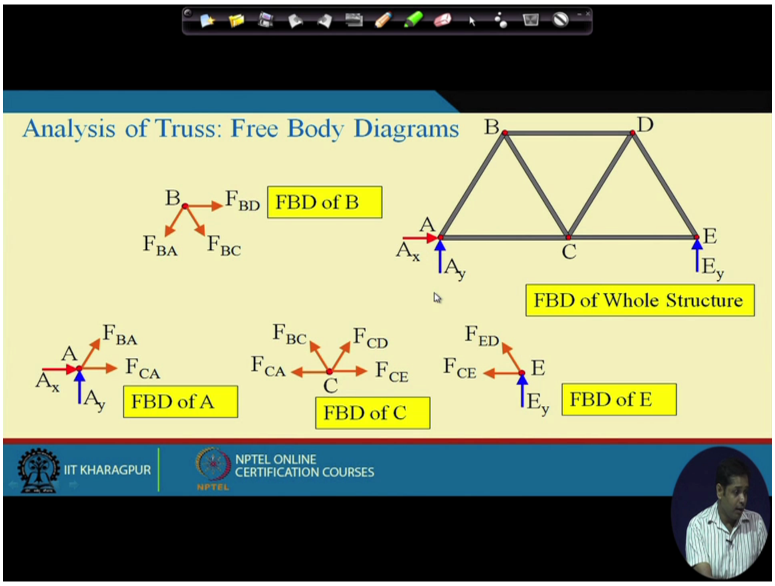

Now you see this FB and this when they are connected through members, this joint and this joint, member AB then BA and this force and this force will cancel each other and this will be in equilibrium, okay.

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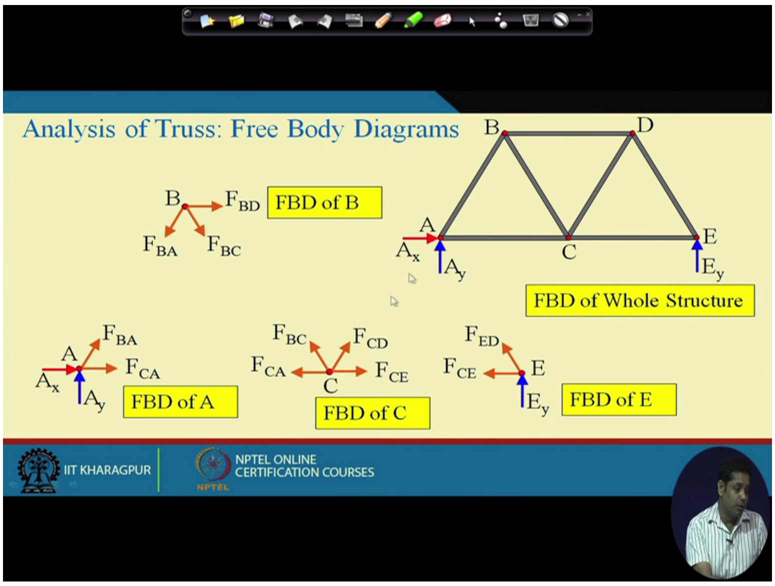

Now this is how we can draw the free body diagram of different joints, okay. Now once we know the free body diagram let us write the equilibrium equation on those free body diagrams, okay. Now what is the (equili) equilibrium equations we can have here? One equilibrium equation is this is the free body diagram of joint B. Now joint B is in equilibrium so what is the condition that joint B should satisfy?

The forces system must satisfy, one is summation of forces in x direction is equal to zero and summation of forces in y direction is equal to zero.

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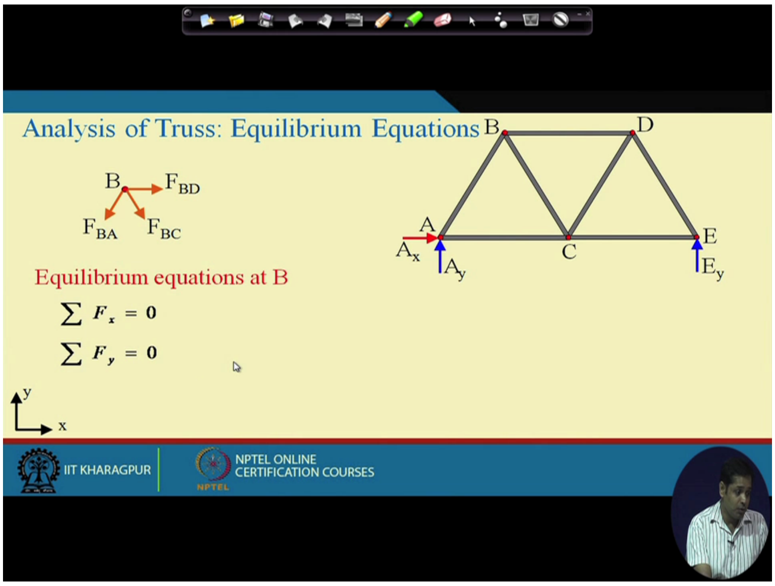

You see summation of moment at B is anyways zero because all the forces are passing through point B. So irrespective of the value of these forces, summation of moment is always zero. So if you take summation of moment is equal to zero it will not give you any additional information, right? So independent information we can get only from these two equations. Summation of Fx is equal to zero, summation of Fy is equal to zero.

Now therefore every joints will give us two equations, okay. Now if there are j number of joints in a truss then number of equilibrium equations we can have 2 into j, two equations per joint, okay.

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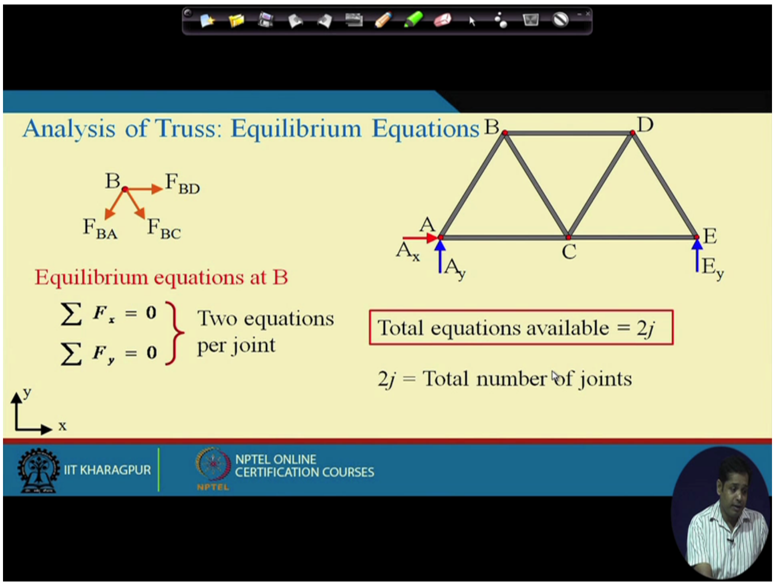

Now the number of equilibrium equations available is 2 into j. Now in order to be able to determine all the unknown the member of unknown should be less than number of equations and as number of equations are 2j so total number of unknown in the truss must be less than 2j, right?

Number of unknown must be equal to 2j, right? Okay before we discuss more on this aspect, the member of their relation, number of unknown, number of equations let us quickly see some concept of stability and static determinacy in this truss. Suppose this is a truss, okay. Now it has two members. Now this is not a stable configuration. You see now this is hinge support and this is roller support so number of support reactions are three.

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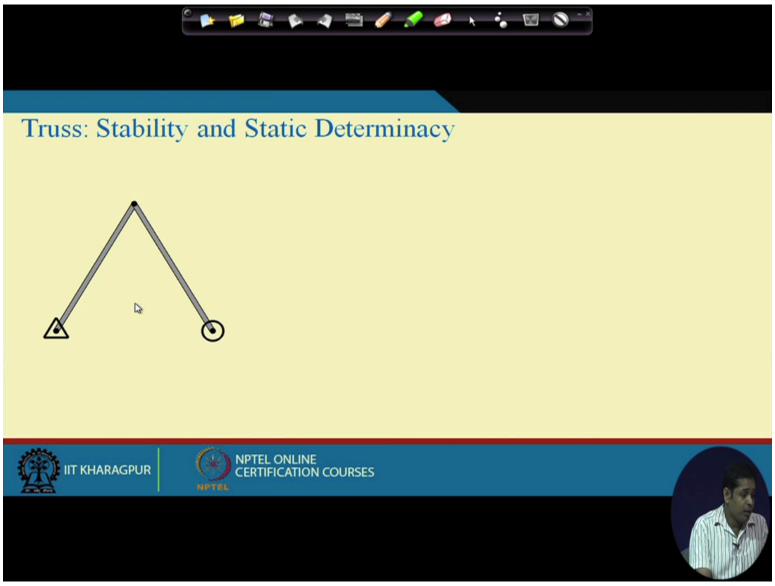

But in spite of that the truss is not stable because when it is subjected to a vertical load at this point then it experiences a mechanism like this, okay.

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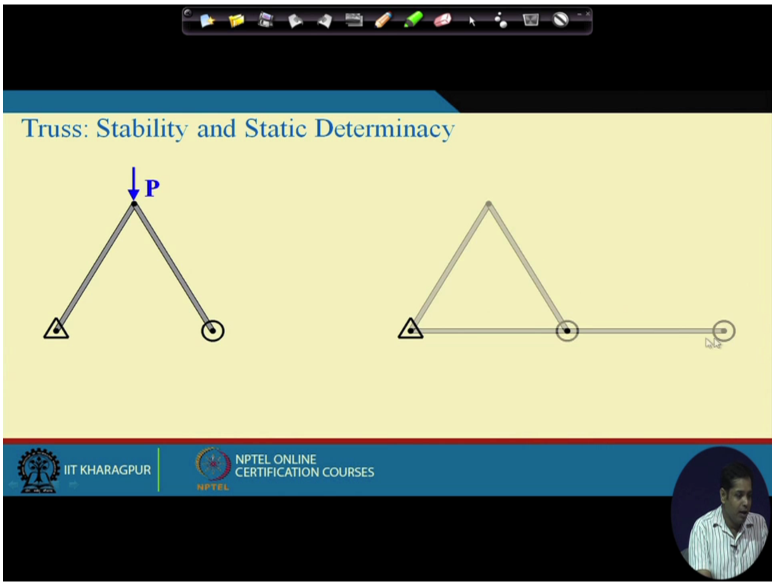

So this is not a stable configuration. Now let us see how I can make this stable. One, by intuition we can say that if we replace this roller support by a hinge support then the structure may become stable because it is a roller support which is allowing this point to move in this direction.

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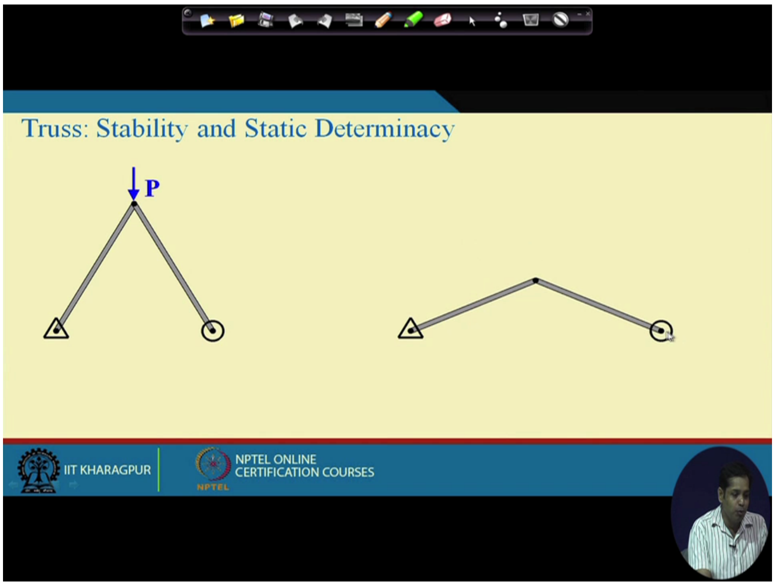

But if I provide a constraint in this direction then this point will not move and the configuration will be stable. So this is an unstable configuration, right? Now one thing we can do is we can replace this roller support by a hinge support and this becomes stable, okay. Now how many constraints we have given here? Constraints are here 2 constraints and here 2 constraints, okay. Four constraints are given.

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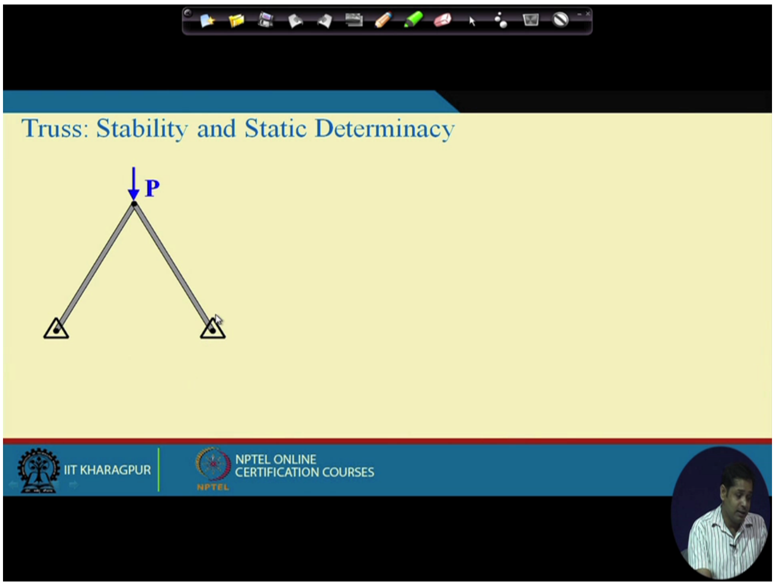

Now is there any other way through which we can make this truss stable? Yes, there is, still we can use roller support here but then if we joint this point and this point through a member then this becomes stable. Because when in absence of this member this point was free to move in this direction. But now even if it is roller support this will not move in this direction because this member will oppose that motion. So this is a stable configuration, right?

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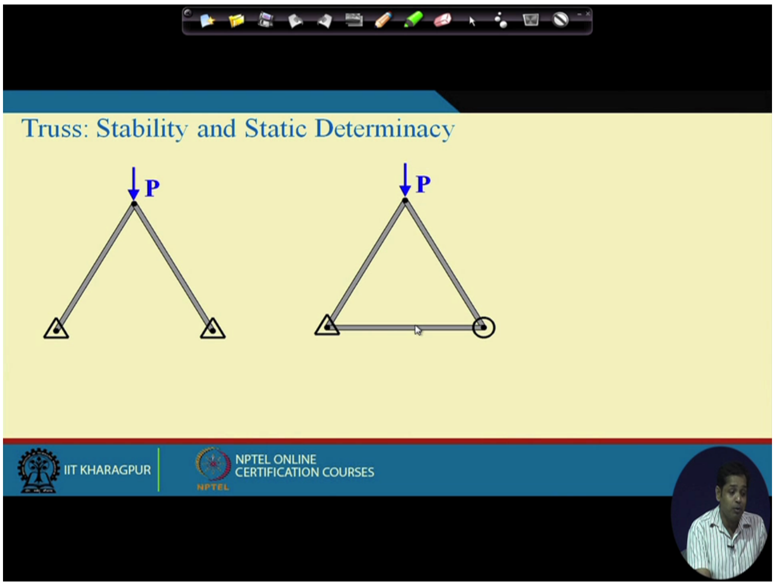

Now another thing we can do is instead of roller support again we provide both, we provide member between these two point and also the roller support is replaced by a hinge support.

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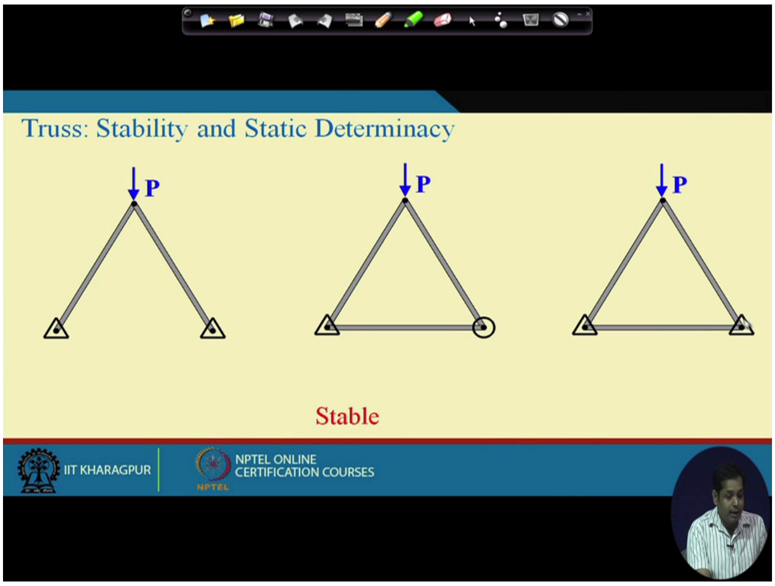

So now what happens in both this case, this case and this case? Both these two configurations is determinate configuration and this configuration is indeterminate configuration. Now why I am saying that this configuration is determinate and this configuration is indeterminate configuration, okay.

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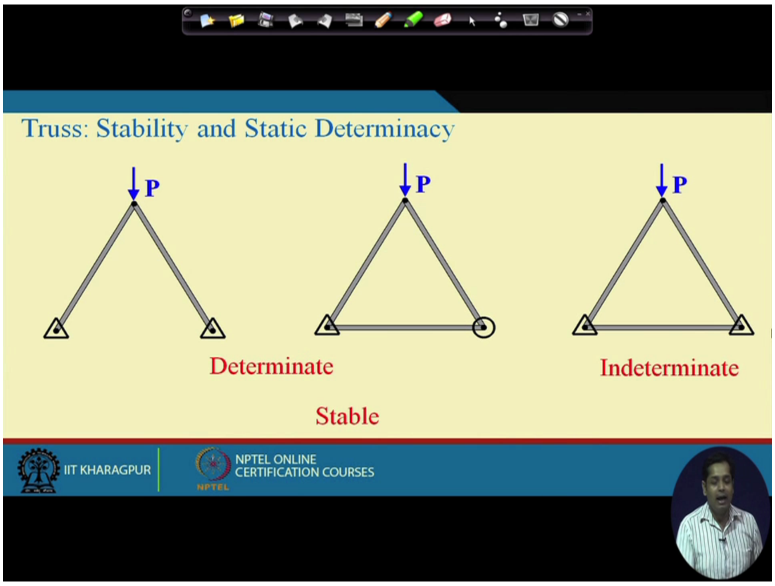

Now what is determinate? Go back to the definition of determinate structure. The number of equations available or the number of information available that is equal to the number of unknown to be determined. Then the structure is determinate. And indeterminate means when the number of unknowns are more than the number of equations available.

Now let us see what are the number of unknowns we have in this structure. Now in this case we need to determine support reactions. So here two support reactions, here two support reactions so total four support reaction.

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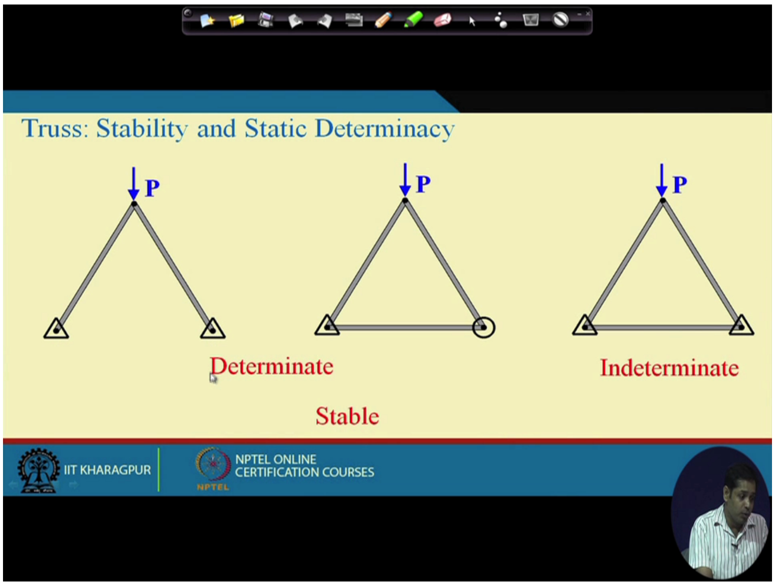

Is there any (oth) other unknown? Yes we need to determine the forces in this members also, right? Now we need to determine force in this member, we need to determine force in this member. So these are also unknown. So total number of unknown is two plus four, total number of unknown is six unknowns. Now let us see here how many unknown we have?

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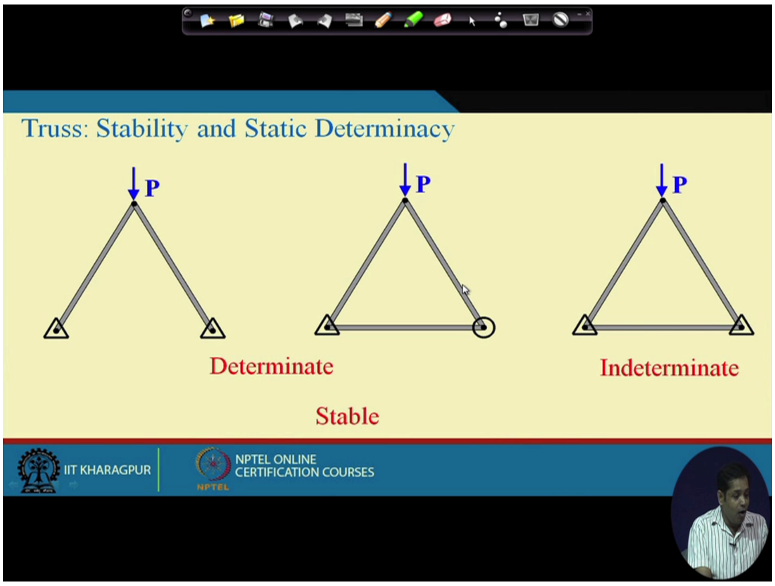

We have two reactions and one reaction here, total three reactions and then three members, means total number of unknowns are six here. Now how many equations we have? Just now we discussed that every joint will give us two equations and the total number of equations will be two into number of joints. Here number of joints are three so total number of equations are six. In this case also number of joints are three, total number of equations are six.

So therefore here in both these configurations your number of unknowns are six and the number of equations available are six. So these two configurations are determinate configurations.

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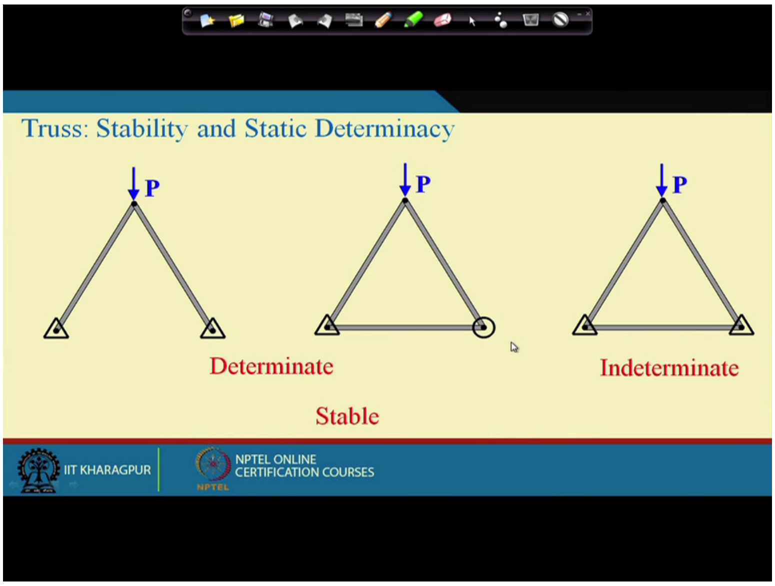

Now let us see the third one. Third one is we have two reactions here and then again two reactions here, total four reactions. And then three member forces, total seven unknown. But number of equations available is 2 into j therefore the structure is indeterminate structure, right?

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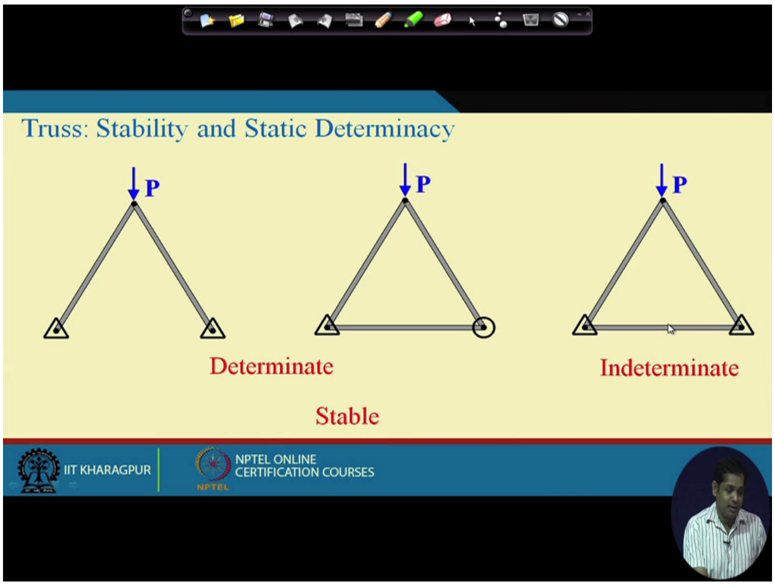

Now suppose the number of unknown support reaction is r and then number of unknown member forces are m. Means in a structure you have m members and then total member of unknown becomes m plus r, r is the support reactions and m is the member forces. And how many equations we have? Total number of equilibrium equations are 2j. J is the number of joints. Then depending on their value m plus r is a total number of unknown and the equation 2j we can have three situations, okay.

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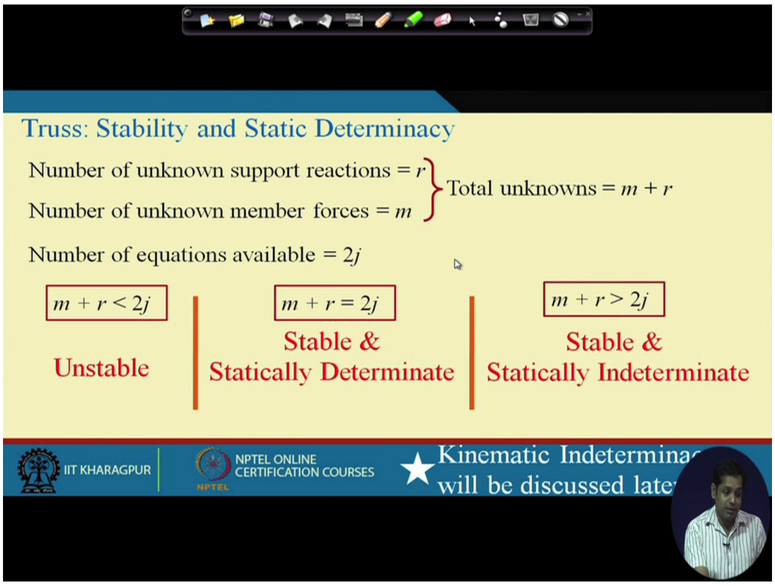

One is m plus r is less than 2j and then (pus) m plus r is equal to 2j and m plus r is greater than 2j. In the first case is unstable because you do not have sufficient constraint to make the structure stable. And in the second case the structure is stable and also statically determinate structure, okay. Now in the third case where the number of unknowns are more than number of equations then the structure is stable but statically indeterminate structure.

You see the (deter) indeterminacy that we are discussing here is static indeterminacy. There is another kind another kind of indeterminacy called kinematic indeterminacy that will be discussed later, okay. But you see when we say this is statically determinate and this is statically indeterminate, just based on this condition there are counter examples which show that though this conditions are satisfied but this structure behaves in a different way.

So these conditions are necessary condition but (con) these conditions are not sufficient to make any comment on the stability of the structure.

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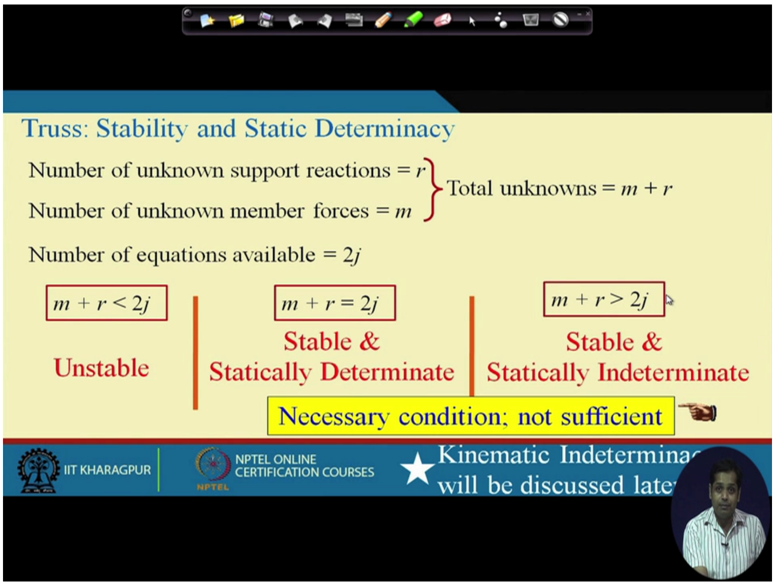

Just to give you some example. You see take one example, this. Now in this example number of member is nine. Now support reactions here two, hear one, so total support reactions are three. So m plus r becomes twelve and number of joints are six.

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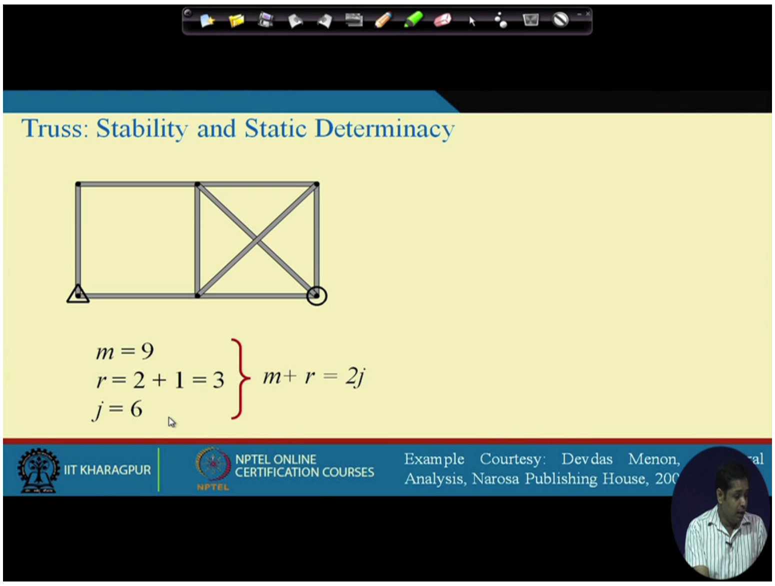

This is not a joint. This is one member and this is another member. These two members are not connected here.

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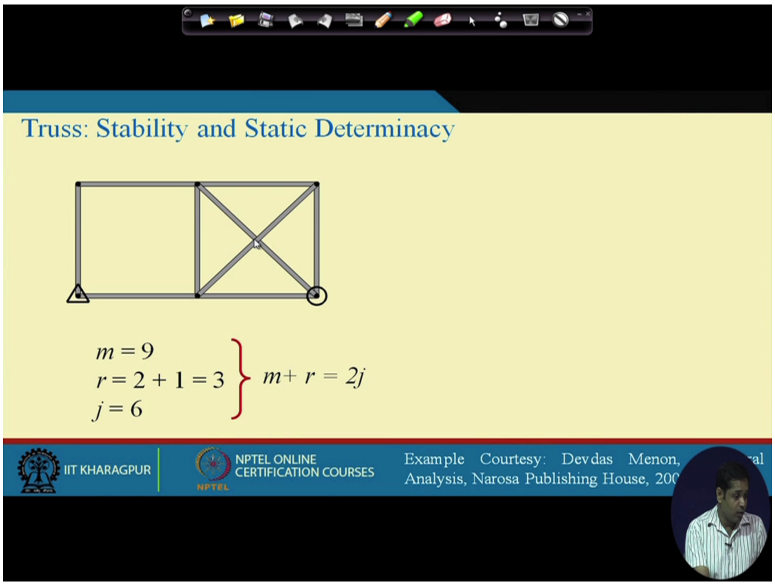

So number of joint is six. So m plus r is equal to 2j. So this structure should be as per our previous statement that m plus r is equal to 2j. The structure should be determinate and also stable. But you see if I apply vertical load like this then this may undergo mechanism like this, okay.

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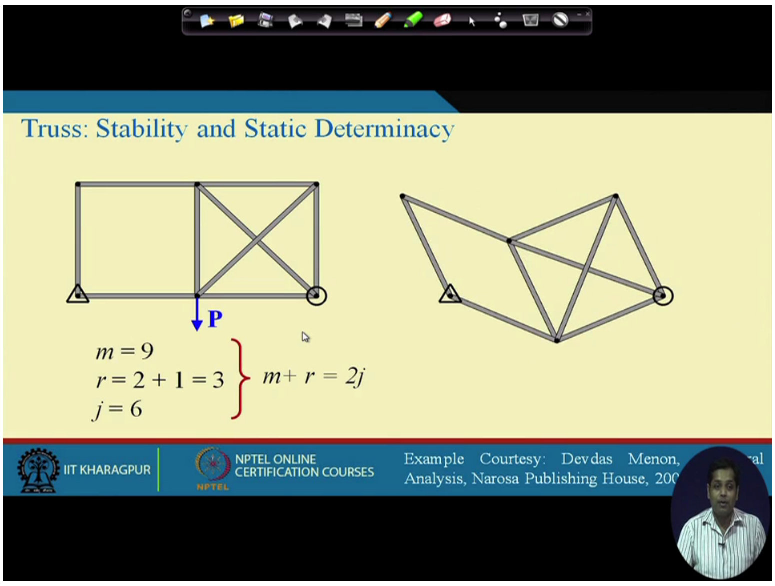

So this is not a stable configuration even though it satisfy the condition that m plus r is equal to 2j. So in this case m plus r is equal to 2j but still the structure is unstable.

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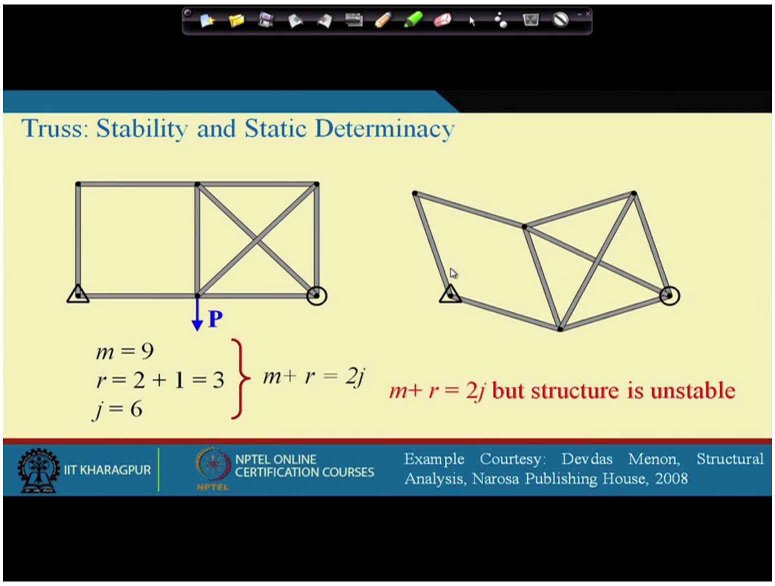

Let us see one more example, this example. Here number of members are fourteen and number of reactions are three, so total fourteen plus three, number of unknowns are seventeen and number of joints are eight. When we say number of joints that also include the joints where you have supports, okay. So number of joints are eight.

So number of equations available is sixteen and number of unknowns are seventeen. So m plus r greater than 2j. The structure is indeterminate but the structure has to be stable as per this relation, right?

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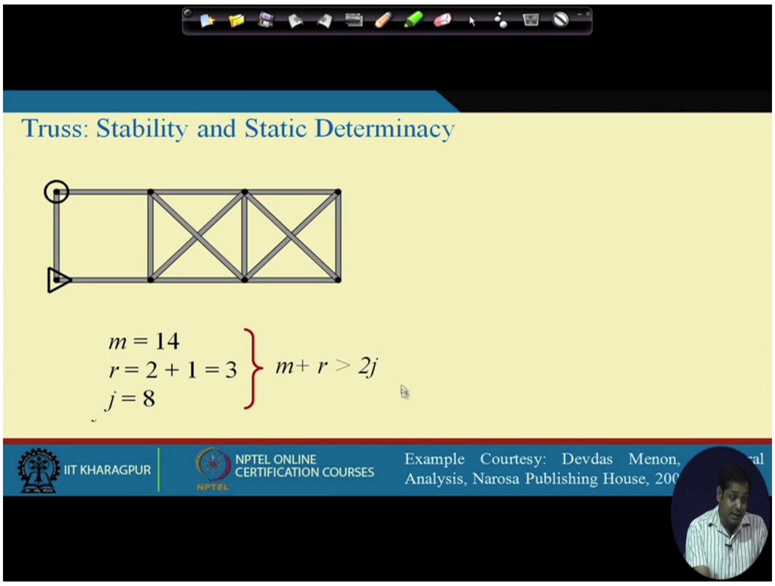

But if it is subjected to a vertical load like this, you see it undergoes mechanism like this. So this is again an example where the condition m plus r greater than 2j satisfy but structure is not stable, it is unstable.

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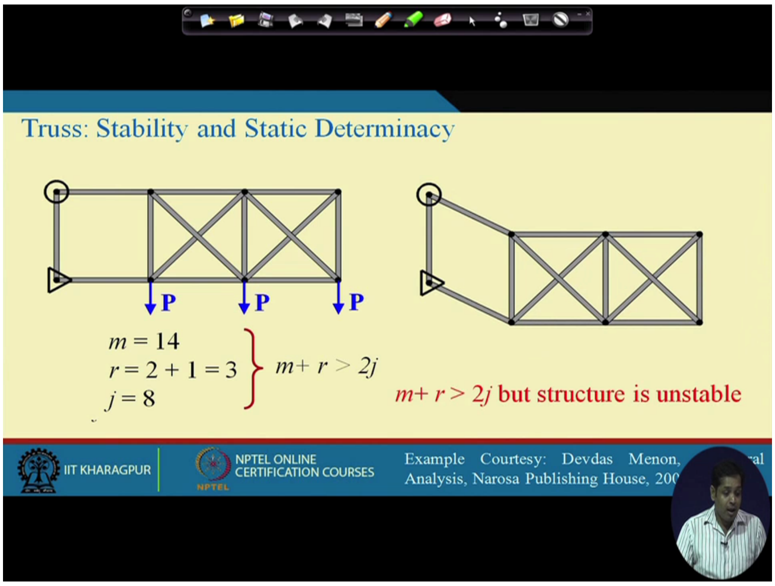

So the point is here I mentioned that in earlier classes as well, do not blindly use any equation or do not blindly use any condition. Apply your engineering sense and by intuition we need to judge whether any configuration of the truss can be stable or not, okay. So now these are some common types of trusses, some commonly used but there are many trusses. If you see any books you will get many more different kinds, okay.

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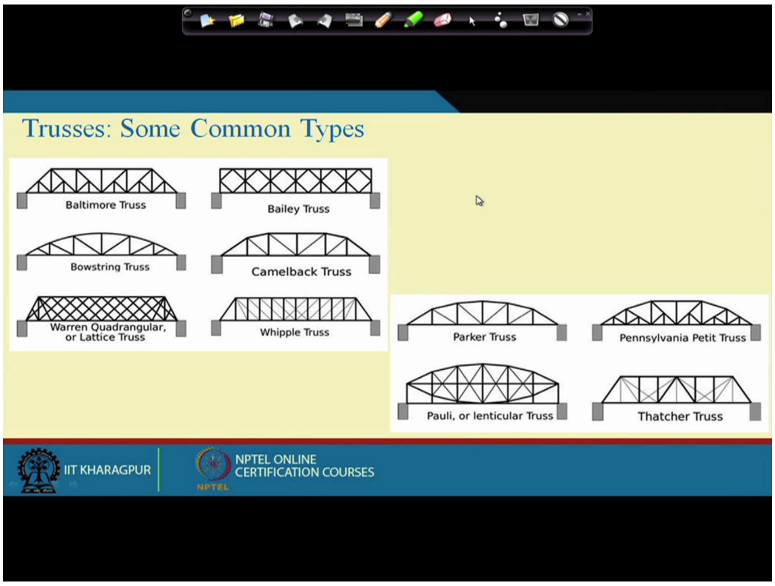

Now just to summarises this you see this week and next week we will (dis) be discussing analysis of statically determinate truss. This week we will see how to determine the member forces. And then the next week we will see week 3, how to determine joint displacements, okay. So there are two methods we will be studying to determine member forces, method of joints and method of sections, okay.

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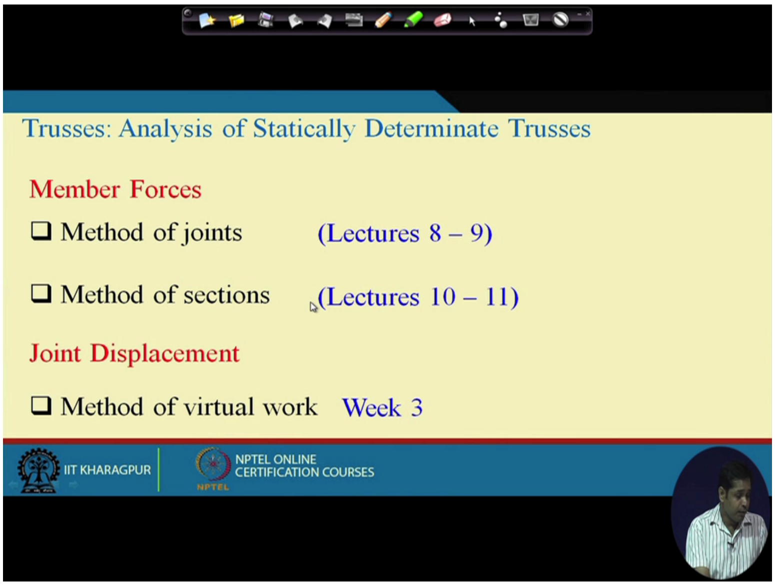

Now we will stop here. Next class we will see how to determine the member forces in a statically determinate truss using method of joints. Thank you.