Structural Analysis I
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Lecture 61
Direct Stiffness Method
Contd.

Hello everyone what we have been doing is we are just trying to understand what is direct thickness method and how it can be applied to different problems we have already done it for truss analyses, we have already done it for beam and today we will be we will be applying a stiffness method for frame. The main difference between the major difference between the frame and the beam and of course truss is you see truss we only have axial deformation.

Where as in beam there is no axial deformation, the deformations we had is rotation then transverse displacement, okay. Now frame just combine we have all the degrees of freedom we have axial deformation, transverse displacement as well as rotation and consequently the, so at every point our degrees of freedom a 3-2 (()) (1:20) and one rotation and consequently this stiffness matrix the size of the size of the members stiffness matrix will also increase, okay.

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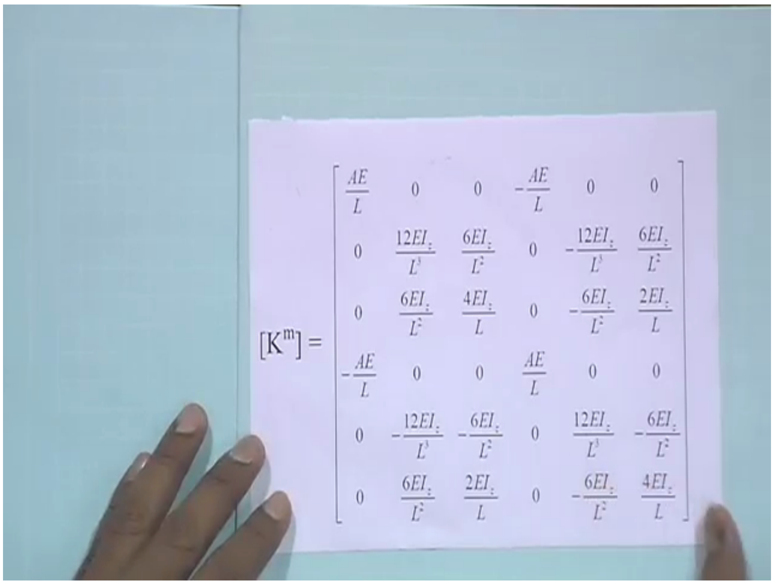

So the problem before we start any problem, just quickly recall that this is the element stiffness matrix, right for truss and if you remember this is for frame, for truss if you remember it was like this only we had this part, this part only we had, this part, this part, this part and this part, this is for the axial this, this, this is the contribution from the axial deformation.

And for this part, this part and this is, this is contribution from the transverse displacement and the rotation. So this part for truss axial deformation and this part we have already done for beam. So essentially when you are talking about frame, it is combination of 2, so you have component of axial deformation as same as truss and then the component of transverse displacement and bending and rotation same as beam and if we combine them element stiffness matrix for member stiffness matrix for frame.

And the Convention that is used here is like this, if we have any arbitrary member like this, see unlike beam where your members are always horizontal and therefore the sign convention, the Convention that we used while deriving stiffness matrix for member and that Convention remain same for all remember but whereas frame we have members oriented in a different direction.

We have horizontal member, vertical member inclined at certain angle, okay. So that orientation also we need to consider, okay. When we talk about this stiffness matrix here that is obtained by assuming that beam is like this, okay. Your member is like this horizontal, okay. Now the Convention is now, suppose we take any member we have discussed it just we are reviewing that.

Then this member is i, this joint is i and joint is j and then degrees of freedom are like this, this is first and this is second and then rotation, here it is 3 and similarly we have this is 4 and this is 5 and similarly rotation here is 6, total 6 degrees of freedom and here also it is 1, 2, 3, 4, 5 and it is 1, 2, 3, 4, 5, 6, okay. Now and then this is ith member and this is jth member means we are this is for any member m.

This is for any member m and this is node number i, this is node number j and we are moving in this way, okay. The element is represented as ij element, okay. Now then this is inclined at an angle Theta suppose this angle is Theta with horizontal this angle is theta. Now if this angle is Theta then this stiffness matrix as it is we cannot use.

Then this stiffness matrix will become the K will become , suppose this is K dash if we take then K will be is equal to T transverse T is the and then K dash into T, T is the transformation matrix which because of this orientation, change in orientation the T takes care of that orientation, okay.

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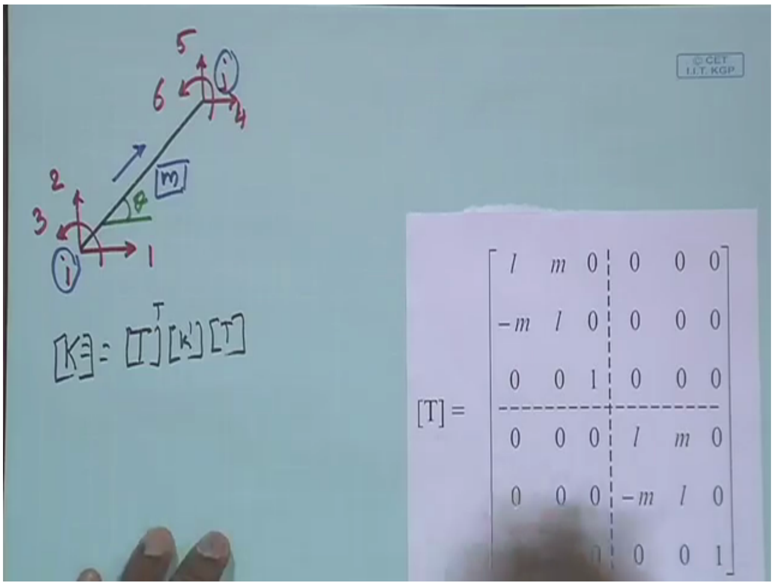

Now again if you recall that T is expressed as, the transformation matrix is expressed as this, this is the transformation matrix that we have seen where l and m components are l is equal to in this expression, l is equal to cos Theta and m is equal to sine Theta, okay.

Now then what we need to do? First is , once we first we first take member and then represent the member by their corresponding   corresponding node number and the degrees of freedom this is the Convention that we use degrees of freedom   if you are using this stiffness matrix in this form, this is a Convention you have to use.

But if you if you change the form means suppose if we take this is one, this is 2 and this is 3 then the same form you cannot use, elements will be what we have to do is then we have to just the corresponding row and column we have to change but the basic elements will be same but that row, column will now interchange, okay.

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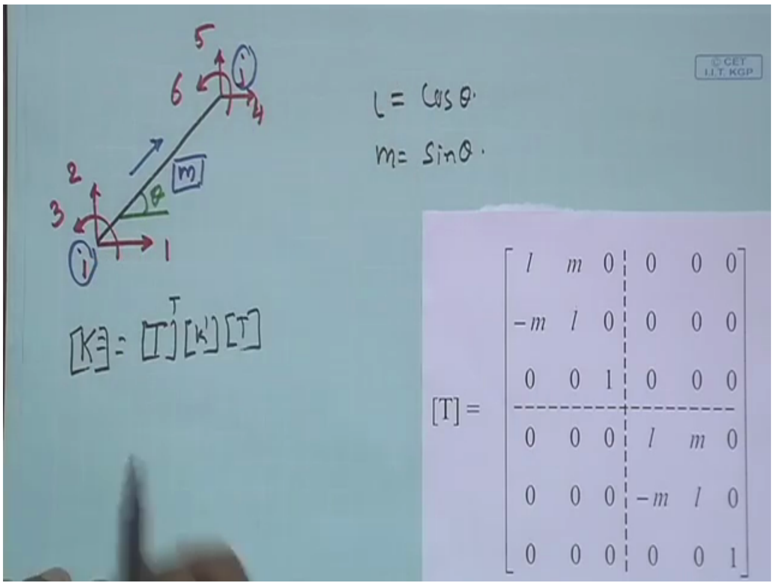

Now for this form this is a matrix, now then once we identify the element a member we have to get this matrix this is the K dash and to get the element stiffness matrix or the members stiffness matrix this would be the members stiffness matrix, we know orientation of these member and from this orientation calculate the transformation matrix and then holding the stiffness matrix as per this orientation of the beam you get the final stiffness matrix, okay.

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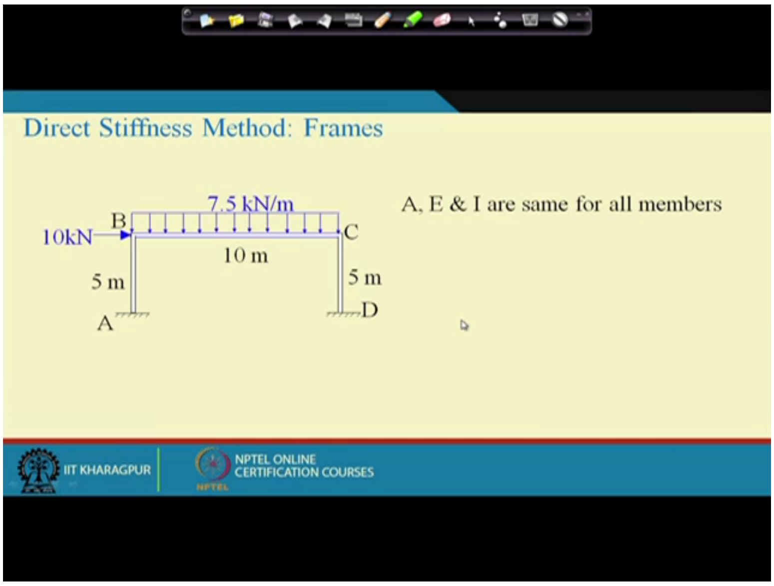

Now let us see suppose consider this example, this is example that we have we already solved using slope deflection method and then also we discussed how to use it using moment distribution method. Let us do it once again using direct stiffness method, now again E cross-section area Young modulus and second moment of area they are all constant for all the members, okay.

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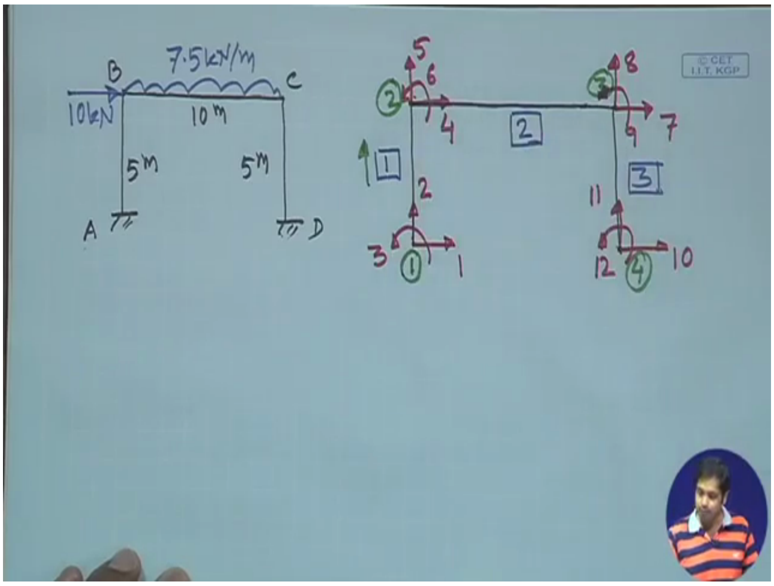

Now let us start it, so the problem is like this, okay. This is horizontal and then this is A, B, C, D and then we have uniformly distributed load here which is 7.5 kilo newtons per meter and in addition to that there is horizontal load 10 kilo newtons, okay. And then this is a 10 meter, this is 5 meter and this is 5 meter, the first step is bright the member number and the node number.

Corresponding node number and identify the degrees of freedom corresponding degrees of freedom, okay. So suppose this is node number 1 then node number 2 is this, node number 3 is this and then this is node number 4, okay. And the degrees of freedom are this is 1, this is 2 and the rotation here is 3.

And similarly at node number 2, this is 4, this is 5 and then rotation here is 6. Node number 3 then this becomes 7, this is 8 and rotation becomes 9 and then at this point this become 10, then it is 11 and then rotation here is 12. So we have total 12 degrees of freedom but out of these 12 degrees of freedom you see point 1 and point 4 are constrained.

So we know that these values are 0, these degrees of freedom are due, D1, D2, D3 and D11, D12 and D10 they are 0, okay. Now once this is done, now let us we for number 1 we write this is member 1, this is number 1, this is member 2 and this is member 3, okay. And suppose for remember 1 we go from this direction from this direction means member 1 is 1, 2.

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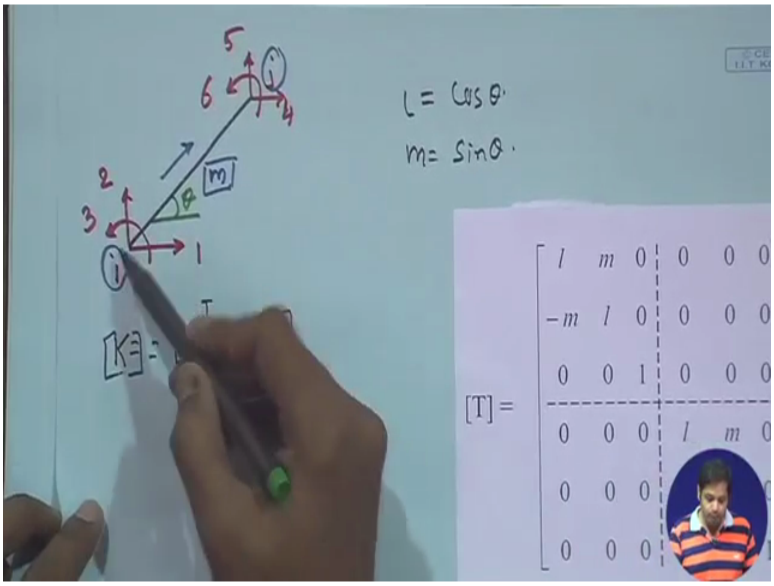

Member is connected between 1 and 2, so for member 1 point i becomes 1 and point to becomes, yes for member this is i starting point and this is the end point, so for member 1 the start point become one and end point is 2, for member 2 it is 2, 3 we go in this way and for member 3, suppose we go this way, so you start point becomes 3 and the end point becomes 4, okay.

This is very important because if we are writing stiffness matrix with summary presentation and then the problem here is not only this stiffness matrix, once you have that K dash you have to transform it as well. Now you write stiffness matrix in some orientation considering in some orientation and then transfer it in some orientation then that will be that will be correct, okay.

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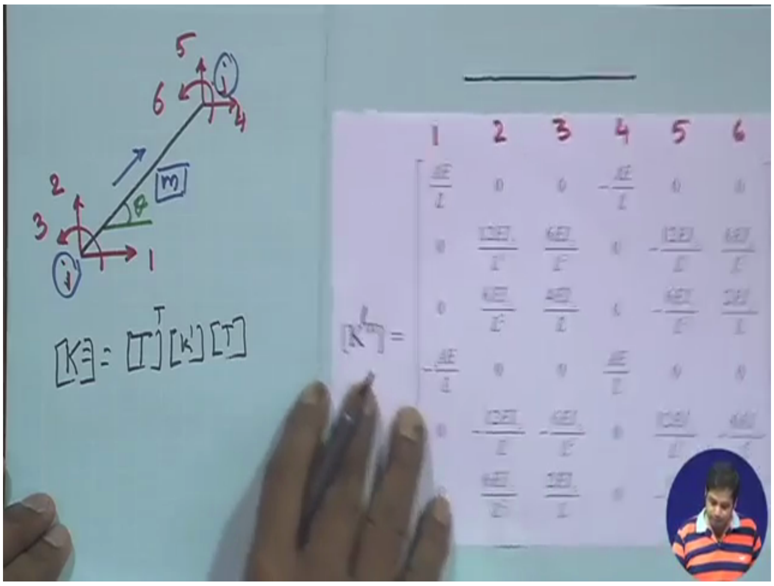

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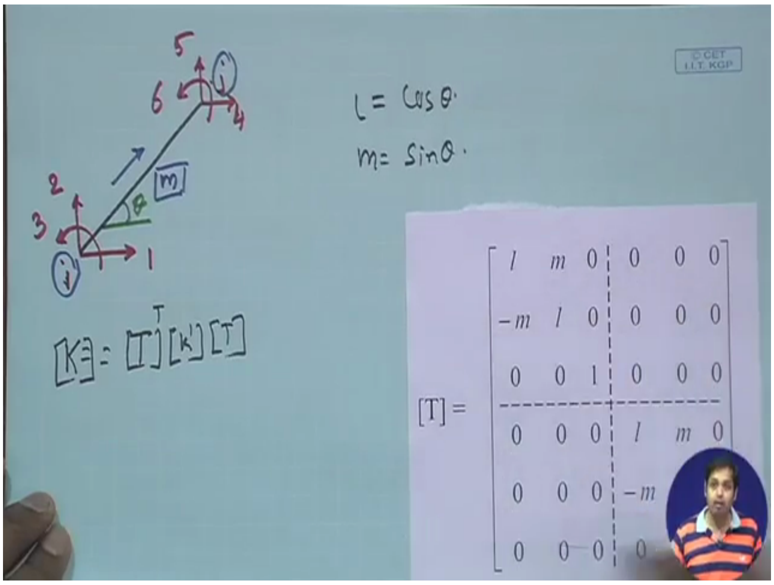

Now let us find out the stiffness matrices for different members, let member 1, now the member 1 you see , remember this when we write this stiffness matrix, when we write this transformation that this transformation needs to be obtained like this where Theta is l and m is given by this and Tata is the angle with horizontal, okay.

Now, so member 1 this angle is if you measure from this because this is the ith point measure from it this angle is Theta is 90 degree, if the dyes 90 degree then l is equal to which is cos Theta, l becomes 0 and m becomes 1, okay. Now we know what is l, we know what is the value of l and m.

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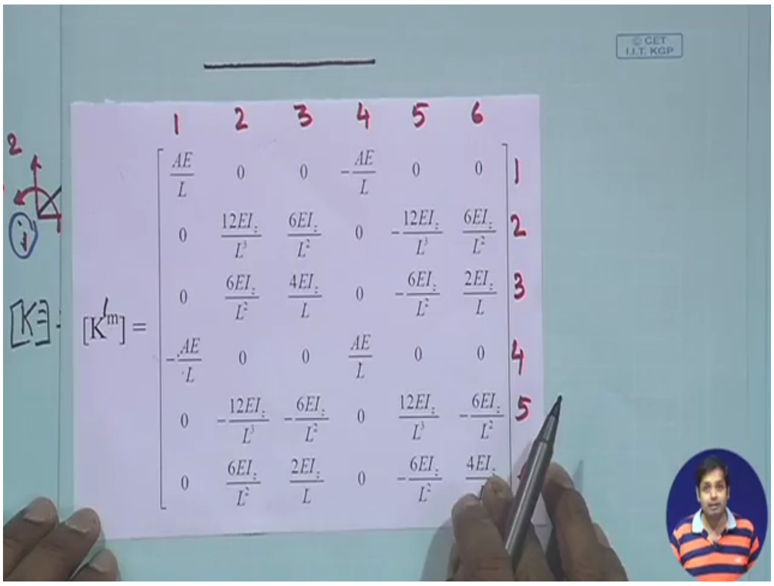

We can get the transformation matrix from this and then we know the properties of this member as well length Young's modulus and second moment of area we can get K dash from this and once we have it and get the element members stiffness matrix by this, okay.

So next this then getting the members stiffness matrix is again a straightforward, similarly for member 2 is Theta is 0, it is going in this direction Theta is 0. So l becomes 1 and m becomes 0 again do the same thing. Again do the same thing you see if for member l, if l becomes 1 then m becomes 0, so this becomes a unique matrix, right?

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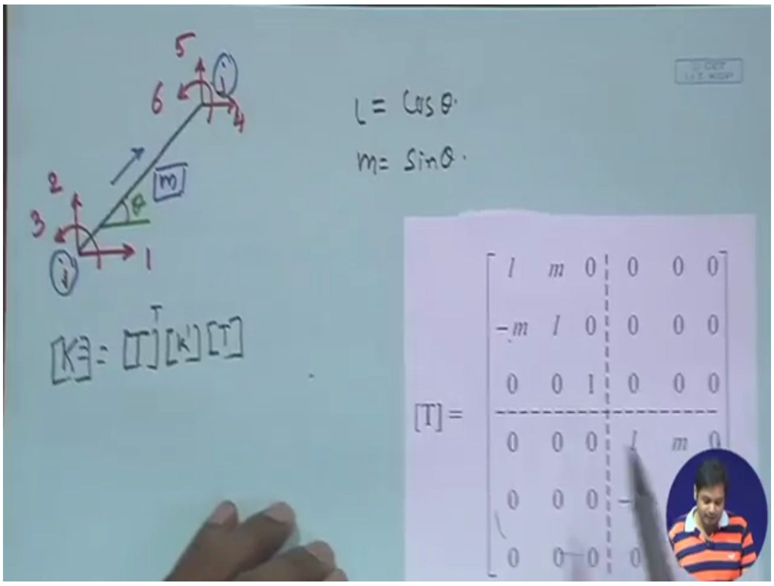

So all the elements at 0 on the diagonal elements are 1. So it is same as when you transformation matrix like this means essentially if you apply this what you end up? you get K is equal to K dash. So whatever your stiffness matrix K dash you obtain after transformation also it will remain same and that was same thing was happening for beam.

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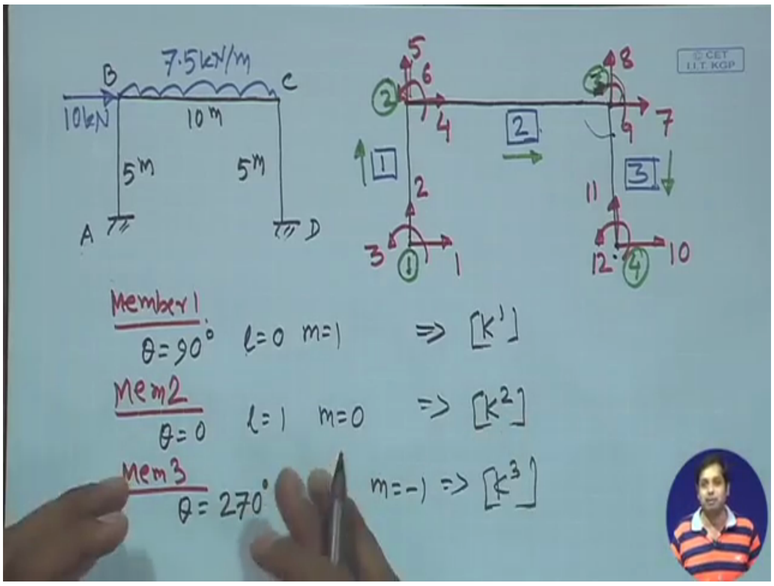

So beam whatever stiffness matrix we had that we use, we need not to transfer them because your Theta is 0 in that case, okay. Now so member 2 we can obtain, similarly for member 3. Now number 3 is interesting, member 3 you see member 1 Theta was 90 degree but member 3 it is also oriented vertical direction but Theta is not 90 degree.

Why Theta is not 90 degree? Because our representation of this member from this to this, okay. So your angle become this, this is the angle. So Theta becomes 3, 270 degree, okay. Now if you consider the representation like this, from this point to this point then you can take Theta is equal to 90 degree, okay. But since our representation is from this point to this point your Theta becomes 270 degree.

So please be careful with this, this is 270 degree this convention. So if it is 270 degree l becomes 0 and m becomes minus 1. You see in this case though they are both are vertically oriented in this case m is 1, in this case m is minus 1, okay. Now great, so again we can calculate, so we know what is we can this will give us K1 and this will give us K2 and this will give us K3. So we have elements stiffness matrices.

Once we have the element stiffness matrix we have to assemble them. Again assembling, the process of assembling we have discussed 2, we have discussed in the case of truss, we have also discussed in the case of beam the process remains same. You write it in a different, suppose just for one example. Suppose for member 2, okay.

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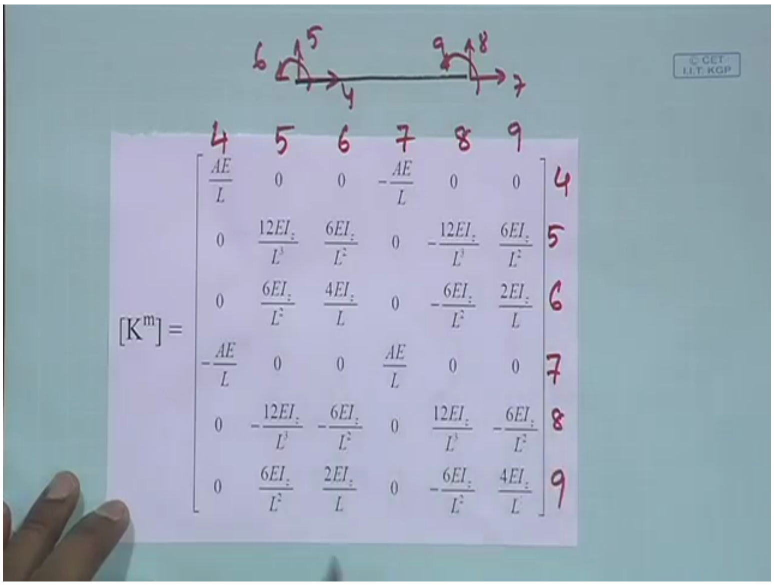

Member 2 then suppose this is your element stiffness matrix, okay. Now member 2 is oriented like this and corresponding degrees of freedom are 4, 5, 6 -7, 8, 9. So this is 4, this is 5, this is 6, this is 7, this is 8 and this is 9, okay. So means it is for 4, 5, 6, 7, 8, 9, this is 4, 5, 6, 7, 8 and then 9. Similarly you get for other 2 members as well and then what you have to do?

When you globalise it you take element by element and then sum them and put that in the corresponding position in the matrix. Better way is when you are actually writing when you are writing a code (()) (16:29) assembling better thing is as we discuss in the truss, we know that in this case your global stiffness matrix will be since you have 12 degrees of freedom global stiffness metrics will be 12 by 12.

You start with a matrix of 12 by 12 but all elements at 0 and then what you do? You just substitute and then keep on adding and keep on at every position you keep on adding corresponding points from corresponding stiffness matrix and then you get the global stiffness matrix, okay. Now once you get the global, that exercise we are not doing it here, so please do that.

So once we get the global stiffness matrix, next step is to get the Load vector, okay. Now again the process is again same, first we need to find out what are the fixed end reactions and then we need to find out what is the corresponding equivalent joint Load?

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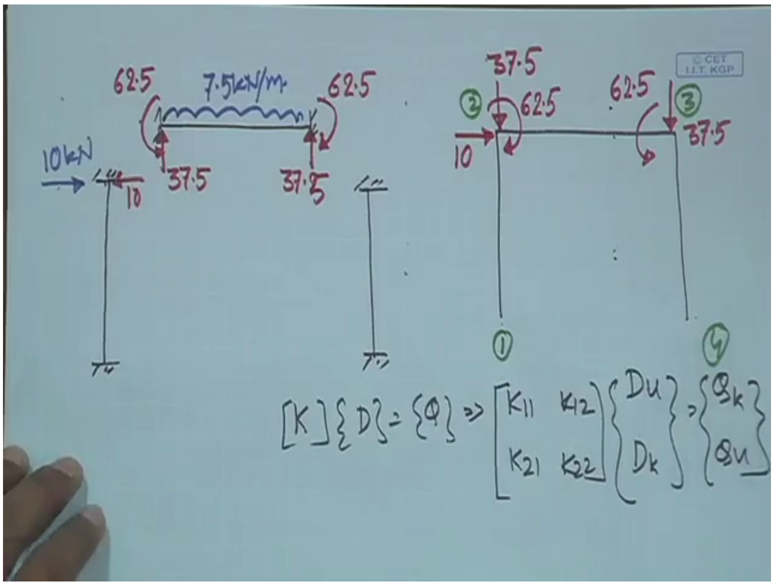

Okay, now first is considering these joints are fixed and to get, so we assume that these joints are fixed, these joints are fixed, these joints are fixed, these joints are fixed and on we have and uniformly distributed load here 7.5 kilo newtons per meter and then we also have 10 kilo newtons load, right?

Now what would be the equivalent then? It is a fixed beam which subjected 2 uniformly distributed to equivalent the (()) (18:02) moment will be WS square by 12 and then this will be this direction; this will be this direction WS square by 12 becomes 62.5, okay. I am not writing the unit here otherwise things become very congested.

Now then there will be vertical reaction as well, this is and this vertical reaction, vertical reaction is half of this and this length is 10 meter, so total 75 by half, this will be 37.5 and this will be 37.5, okay. Now then there is no moment here because there is no externally applied load here. So these moments and reactions all are 0.

Now in this case load is acting at the joint, so it will not create any moment or any reaction to other joints only thing this there will be a reaction likes this which is 10, okay. So this is the reactions if we considered this as fixed end if we consider each member as beam. Now find out equivalent joint Load.

Equivalent joint Load will be, all the loads at this point and at this point they all will be 0, right? So there is no equivalent joint Load at this point and this point, this is node number 1, node number 2, node number 3 and this is node number 4, okay. Now what is equivalent joint

Load here? Equivalent joint Load will be the total, if we sum them whatever we get just the opposite that.

In order to, now when you consider them are fixed whatever moments are reactions we got if we sum them at the point there will not be equilibrium, seeing how to satisfy the equilibrium? We have to apply just opposite moment and opposite force that is the equivalent joint Load. So let us there is no moment here 1 and 4 there will be no equivalent joint Load whereas considered joint 2.

Join 2, horizontal direction we have 10 which is negative in this direction, so there will be equivalent joint Load will be a positive in this direction which is 10, okay. Now the vertical direction at 37.5 which is upward direction then here also we have 37.5 but t it will be in downward direction then there is anticlockwise moment of 62.5 there will be a clockwise moment of 62.5 , so this is equivalent joint Load at node 2.

Similarly at node 3 there is a clockwise moment of 62.5, so there will be an anti-clock wise moment of 62.5 and then a vertical direction 37.5 there will be downward direction 37.5. So this is equivalent joint Load and the next step is once we have this next step again we know that this is K into D is equal to Q this is the stiffness relation Load displacement relation and then we need to partition it.

Partition it means those D which are known and those displacement degrees of freedom which are known. So if we partition it then after that we get K11, K12, K21 and K22 and the partitions are like this which is now D unknown and then D known, okay. And then this becomes Q known and Q is known, okay. Now what are the Ds we know and what are the Ds we do not know? So let us do it here, okay. So what are the Ds we know and what are the Ds we do not know. Now you see at this point and at this point the reactions all the displacements at 0.

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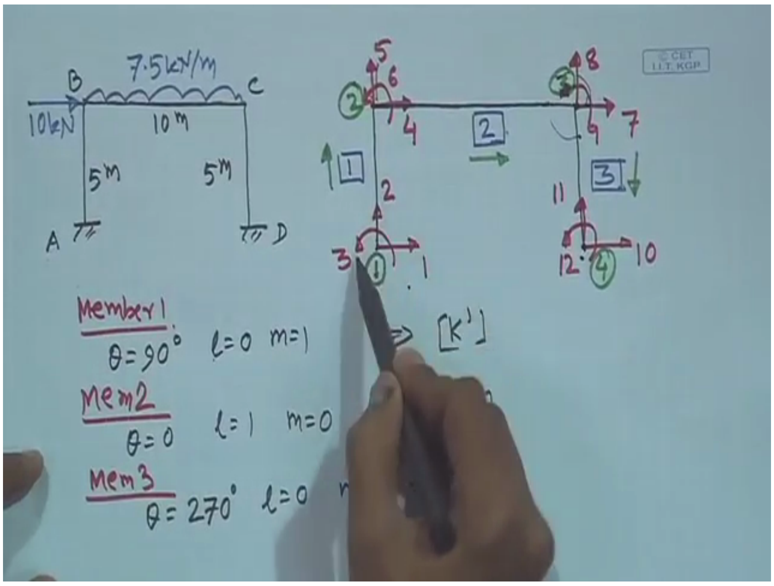

At this point we have D1, D2, D3 which at 0 and then D11, D12 and D10 they are 0. So only unknown Ds are 4, 5, 6 and 7, 8, 9.

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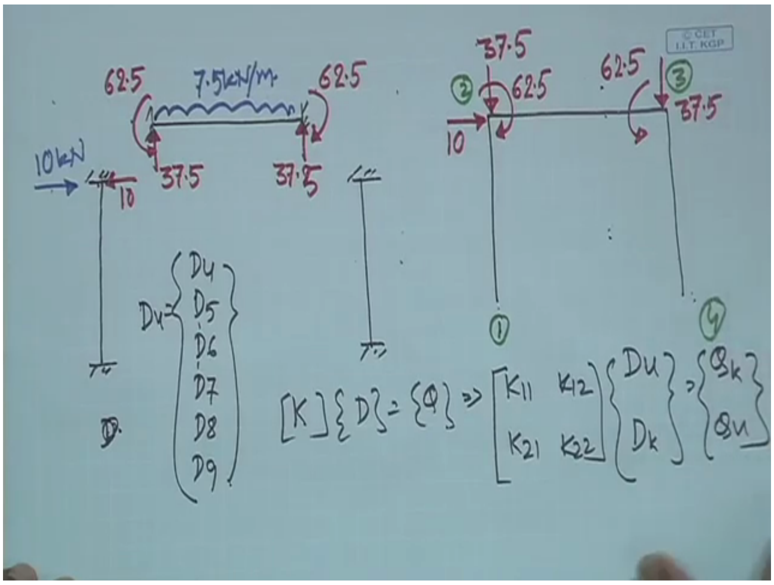

So D unknown are   let us write it here , D unknown are D4, D5, D6, D7, D 8 and D 9 and all other Ds are known and those are 0, okay. And then what is Q unknown?

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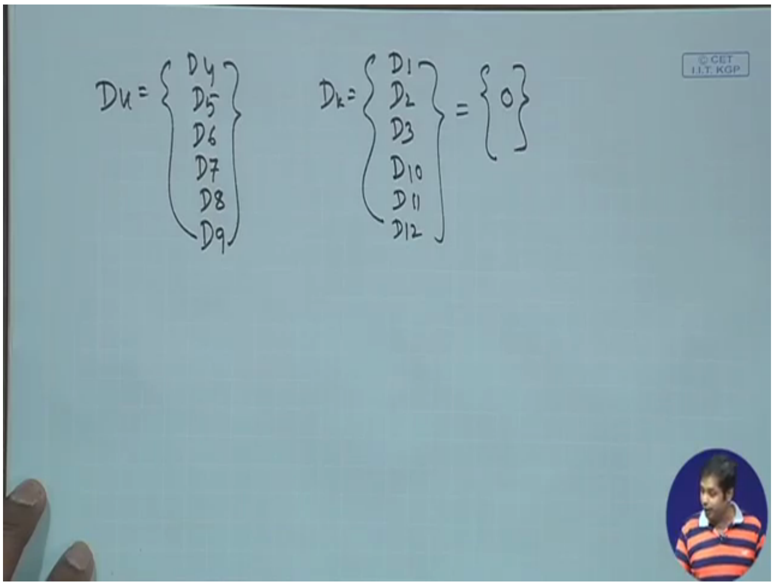

So let us write in a different page, so D unknown is equal to D4, D5, D6, D7, D8 and then D 9, okay. D 9 and then D known is equal to D1, D2, D3 which are at joint number 1 and then D10, D11, D12 which are at joint number joint number 4 these are all 0, okay. Now similarly we need to find out Q known and Q unknown, okay.

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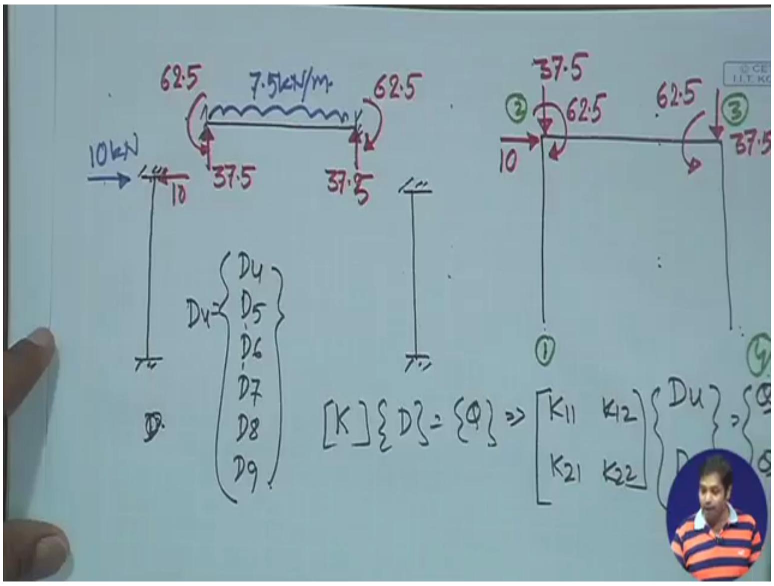

Q known are what these, we know these Q known here, okay. And what is Q known?

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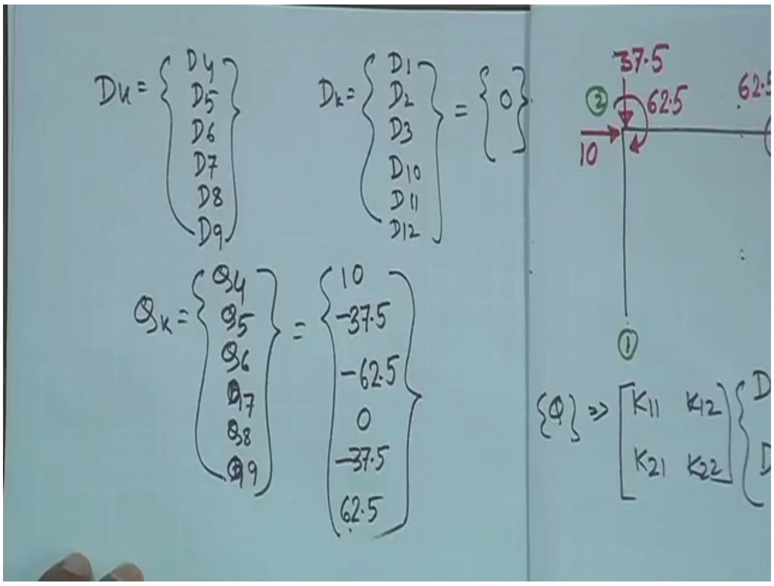

Q known is, if we have to write Q known is equal to what, now you see we know these as supports we know Q 4, Q 5, Q 6 then 7, Q 8 and Q 9, okay. And so Q 4, Q 5, Q 6, Q 7, Q 8 and Q 9 means these are at unconstrained these Qs are the equivalent joint Load at the unconstrained joint, okay. And what are these values? These values are you see it is this is Q 3; Q4 is 10, so this is 10.

And then we need to find out what is Q 5? Q5 is minus 37.5 minus 37.5 and Q 6 is minus 62.5 minus 62.5, minus because it is clockwise as per the sign convention for the stiffness matrix expression views anticlockwise as positive and then take Q 4, 5, 6, Q 7 is horizontal load at joint which is 0 here and then Q8 is this Load which is again minus 37.5 and then finally Q 9 is 62.5 which is anticlockwise as well as positive 62.5, okay. So this is Q known.

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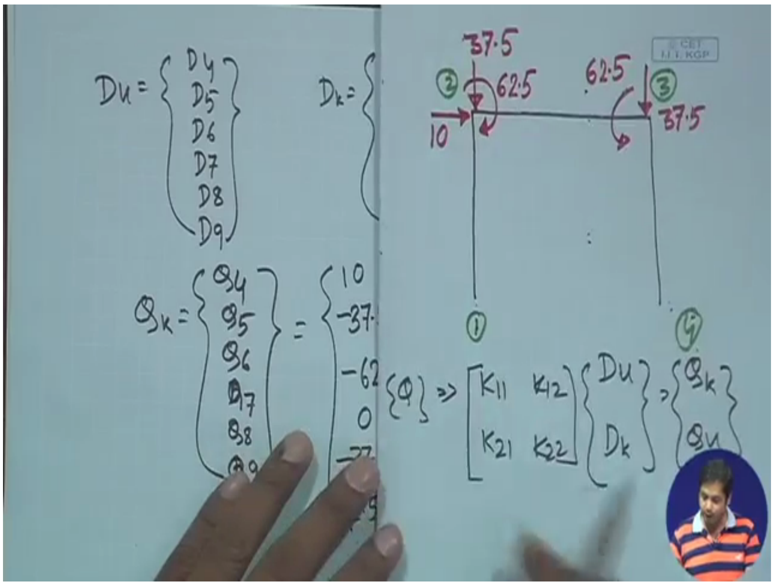

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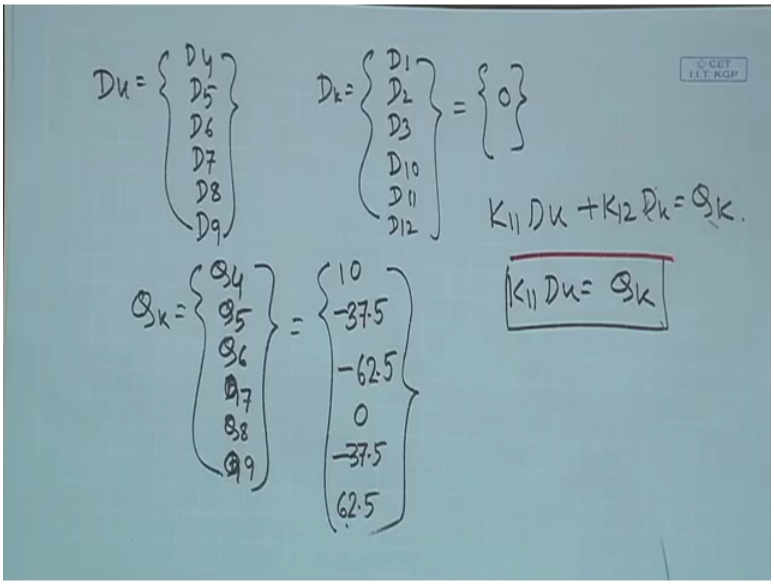

Now what we need to find out? We need to find out the, if we look at the partition, if we look at this partition then these become this K11 D unknown plus K12 D known is equal to Q known or in other way you can interpret that degrees of freedom at unconstrained joint, degrees of freedom at constraint joint and this is equivalent joint Load at unconstrained joint, okay.

Now this is 0 that we have already obtained, so K11 D U is equal to QK, so from this expression we can calculate QK, okay. So this will give us the unknown Qs. Now once we know the unknown Q then, this will give us unknown displacement.

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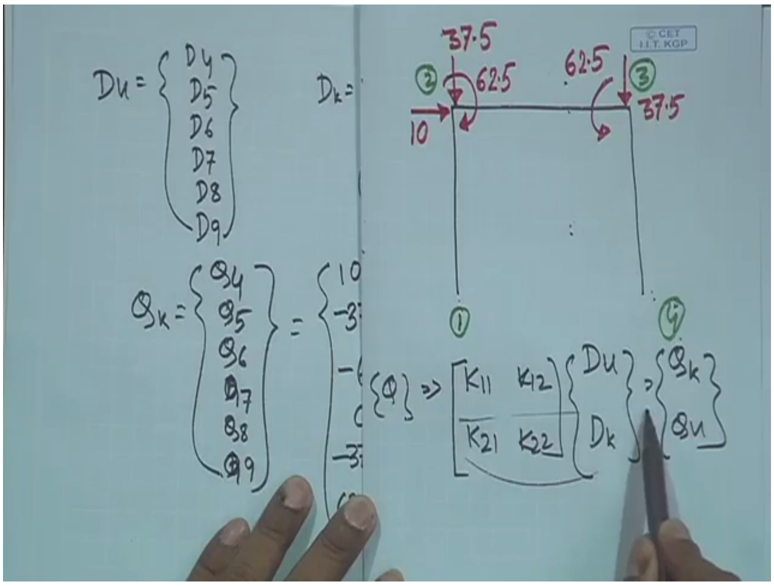

Now then what we have to find out? We need to find out unknown Q, then unknown Q will be again then for that case needs to take the second part of this equation, okay. This is the second part of this equation which is K 21 QK, K22 K21 DU, K22 DK is equal to QU.

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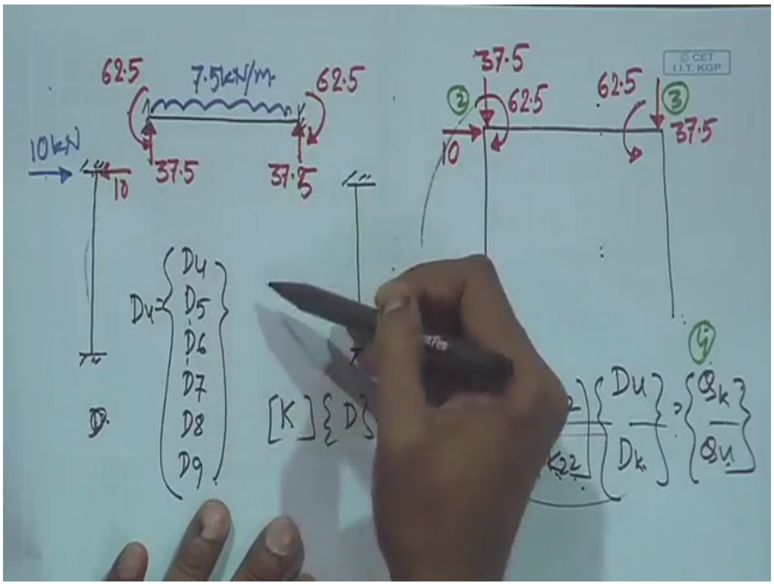

So QU will be, but this will give us QU this will give us due to displacement but total will be the displacement whatever reactions you have because of the displacement plus whatever reactions we have because of this part assuming this constraint. So the total will be the fixed end constraint and plus displacement in this.

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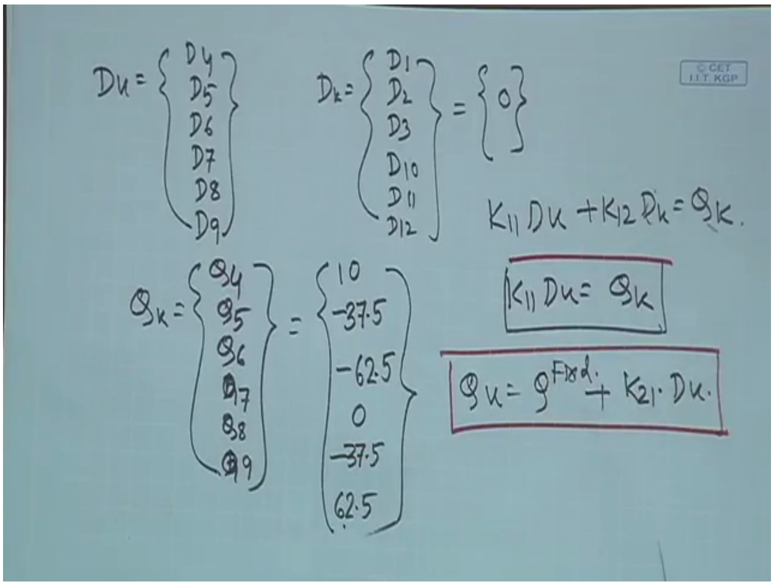

So total will be Q assuming fixed assume for fixed end plus it will be K21 into D unknown, okay. D known is constraint in 0 this will be the unknown reactions, okay. Now once you have the unknown reactions take every element separately, apply it will be the same procedure that we have already done for beam to get the forces in different members, okay. So this is how you can analyze frame.

Now this same concept with the demonstration is done using, we have we have demonstrated the concept using all the structure the planar structure but this same concept can be extended to space structure as well. For instance if it is not plain frame, if it is a space frame then what happens? The first is at every points now we have 3 degrees of freedom if you remember the first week itself we discussed when we degrees of freedom, any point in a space we have 6 degrees of freedom, 3 translation and 3 rotation.

And if it is plain then we have 3 degrees of freedom, to translation and one rotation and since all the structure we are trying to analyze here this is a planar structure that is why for frame at every point we have the degrees of freedom. Now if you have a space structure at every point you have 6 degrees of freedom and therefore in any element we have any element connecting 2 points the total degrees of freedom will be 12.

6 at ith point and 6 at jth point, so elements stiffness matrix the member stiffness matrix will be 12 by 12 matrix, okay. This is first difference and the second difference will be, now when the Orientation, in this case the orientation, now in the orientation is still always in a plane therefore we had in a transformation matrix, we had just transformation can matrix can be represented by just one parameter Theta.

Now in space frame the transformation is in 3-D, the orientation now is in 3-D so you are transformation matrix will be different because then this transformation is to define with 3 Theta, okay. So you transformation matrix will be now different. But the concept once you have the transformation matrix, member stiffness matrix then getting the elements stiffness matrix for the actual orientation of the member that step is same.

Again once you have the member stiffness matrix then assembling process is again same ones you have the assemble stiffness matrix, rest of the process is again same even if you want to apply for 3-D structure, okay. But you see direct stiffness method is what we have discussed in one way it is just the purpose was to introduce direct stiffness matrix because this week the direct stiffness method, this is a transition from your basic level structure analysis course this course and then slightly advanced structure analysis course, this is a transition from basic to slightly advanced structure analysis course.

So here the object even has been to introduce the method but in order to understand the method a full-fledged a one semester or half semester course is required and I believe that you will be having those courses in your subsequent semester, okay. Now this is the last class not last class, if we see from Syllabus point of view then yes this is the last class of this course but we will be having one more class where we try to summarize what we have learnt and what we yet to learn, okay. That we would be discussing in the next class which in some way closure of this entire course. Okay then I stop here today see you in the next class, thank you.