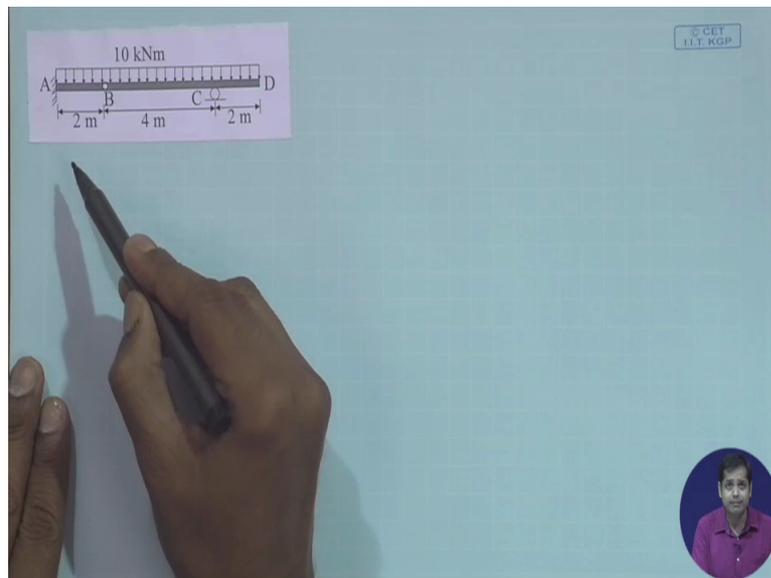


**Structural Analysis 1**  
**Professor Amit Shaw**  
**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 6**  
**Tutorial - 1**

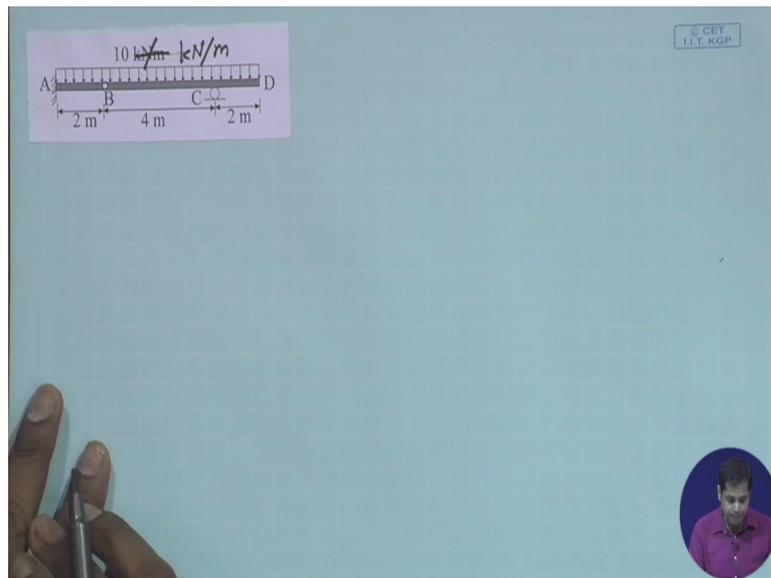
Hello. Today is the tutorial class. We have been trying to see several problems in bending moment and shear force diagram of beams. Let's see some more examples in this class. You see now let's start with this problem. If you see this is a beam where your support is fixed support. And then we have a roller at C and then in addition to that you have an internal hinge at B, right?

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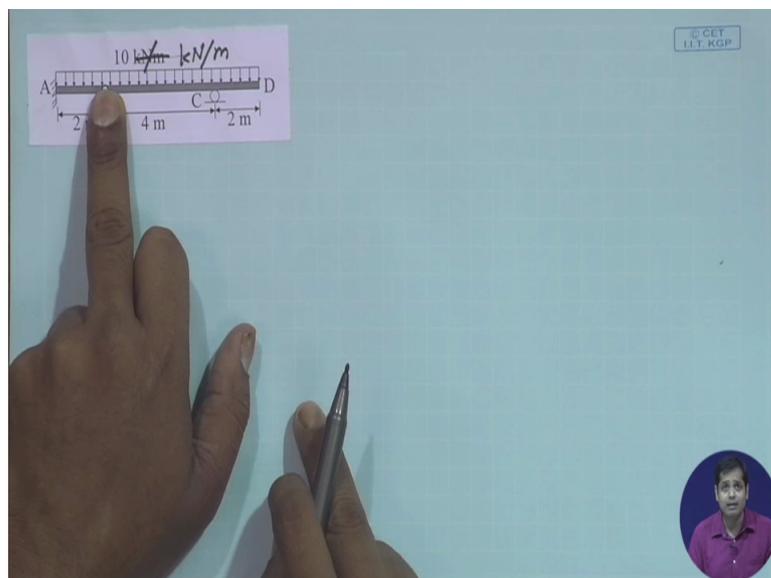
And then beam AD is subjected to uniformly distributed load of kilo Newton per meter. This is kilo Newton per meter.

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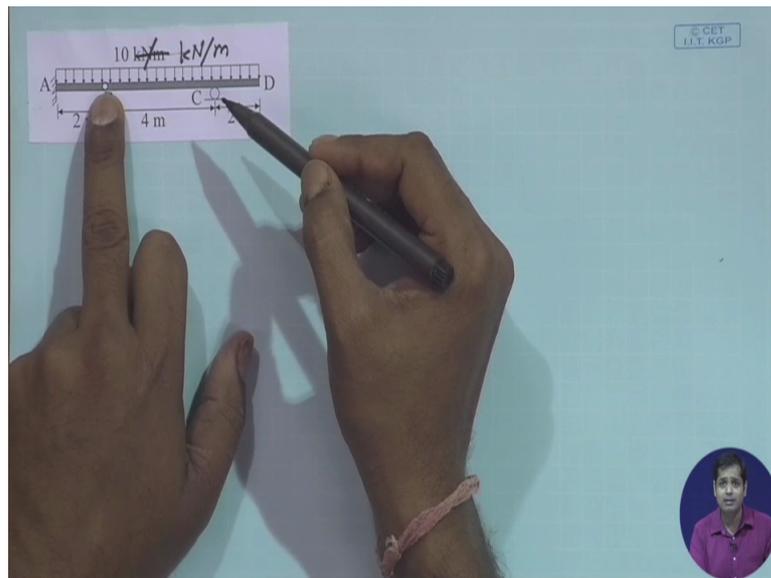
Now you see if you remember what we discussed during static equilibrium conditions and indeterminate structure. If you do not have these internal hinge then the structure is statically indeterminate structure.

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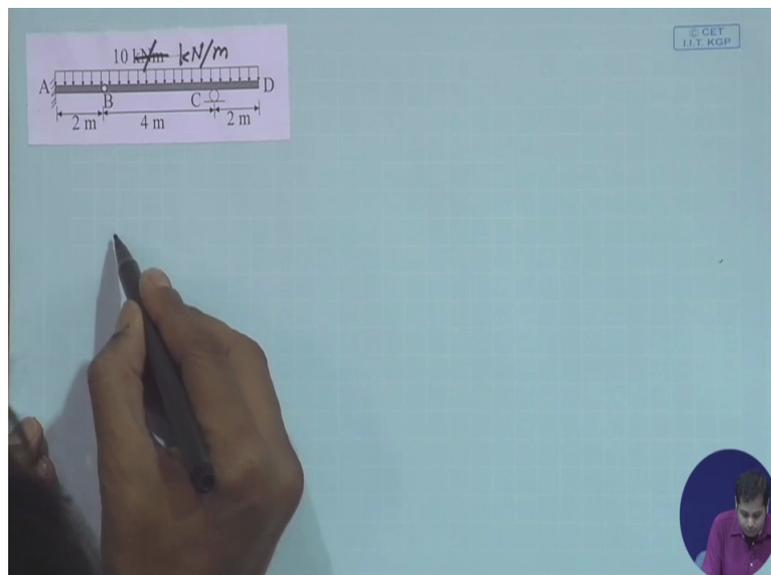
Because this is fixed support where your number of reaction components are 3. And then in addition reaction component at roller support which is 1. So the total reaction are 4. Total reactions are 3 plus 1, 4. And then number of equations you have is 3. So this is a statically indeterminate structure. But now if you put one hinge here then this hinge will give you one more condition. And the condition is, the moment at this location, the hinge location is zero.

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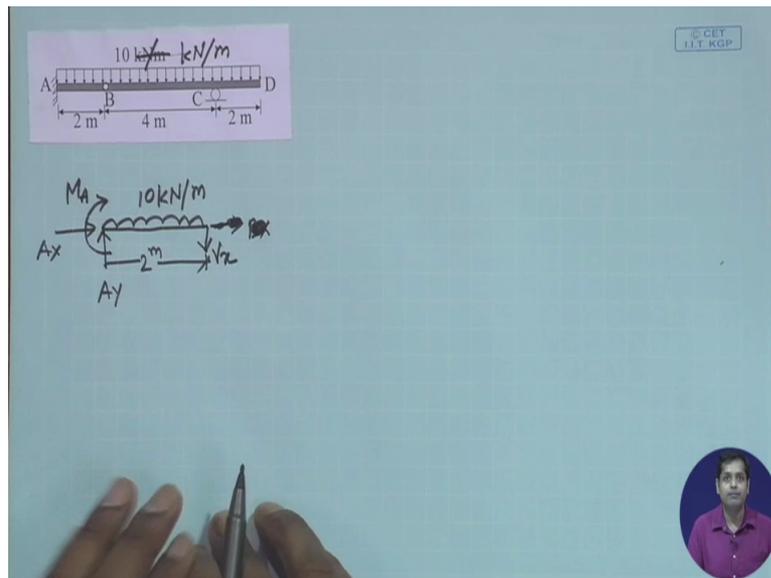
And that condition gives you one more equation and that equation can be used along with other equilibrium equation to solve for the unknown. Now we will just going to demonstrate that here. Now first what you do is, we divide the entire beam into two parts. One is AB, this side of the hinge. And then BD, the other side of the hinge.

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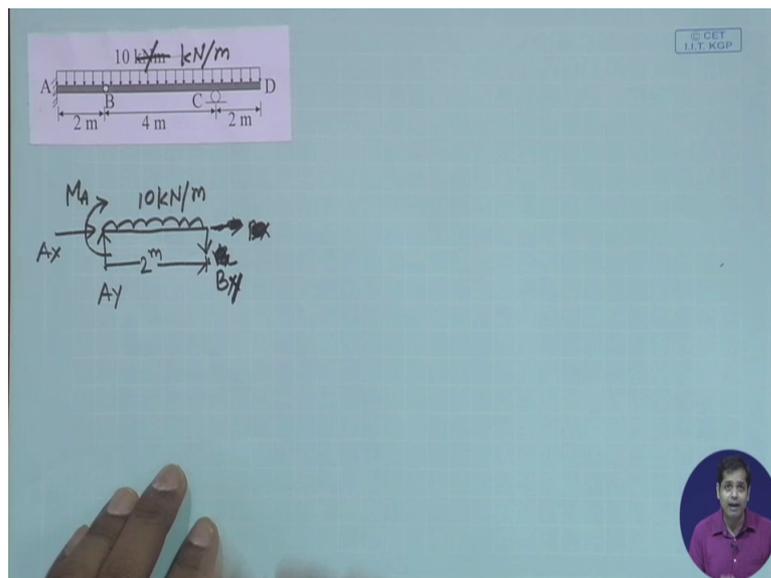
Now let's draw the free body diagram of this part AB. Free body diagram of AB will be, at this point. This is  $A_y$  reaction, then  $A_x$  reaction and sagging moment positive. This is  $M_a$ . This is subjected to uniformly distributed load in kilo Newton per meter. This distance is 2 meter. And at this point there will be no moment. Only it will be  $B_x$  and then we are not considering any actual force. So there will be no  $B_x$ . Only force will be  $V_x$ , the shear force.

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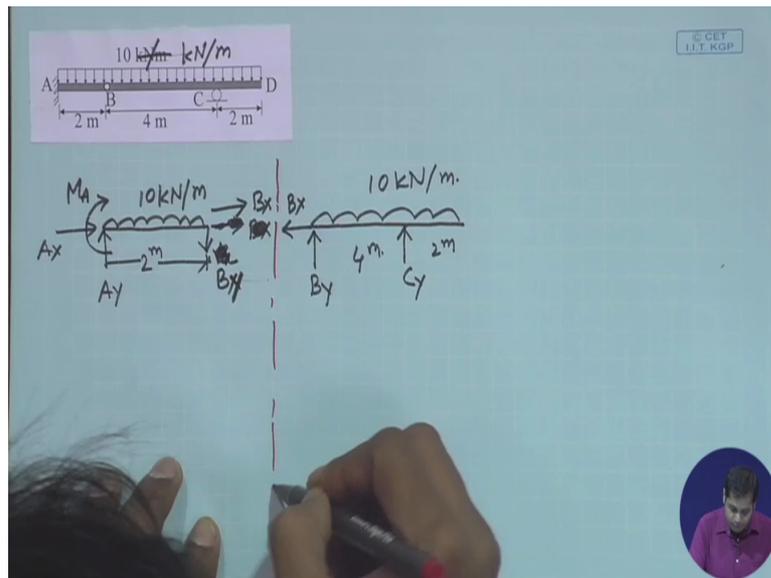
Or let us not write  $V_x$  here. Let us denote it as  $B_y$ . Which says that the hinge reaction at B.

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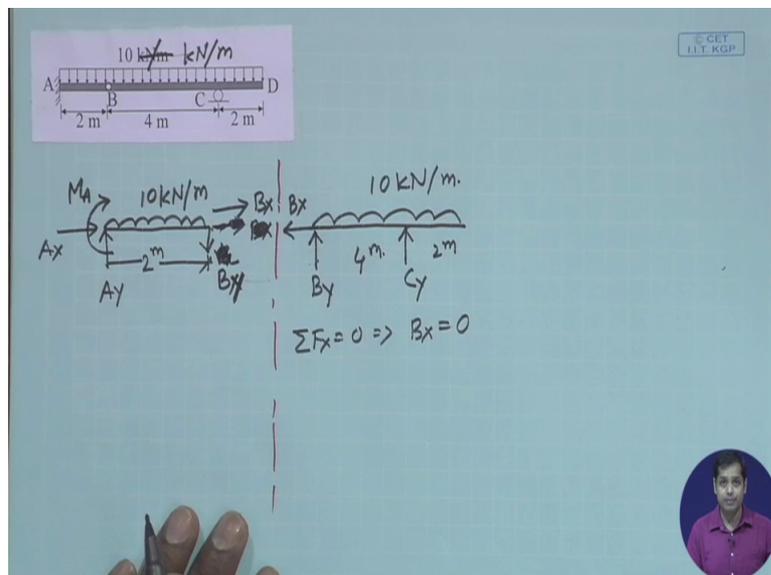
Now if I draw the free body diagram of the other part, the other part will be this and this at C will be vertical component which is  $C_y$ . Then uniformly distributed load which is 10 kilo Newton per meter. This is 2 meter. This is 4 meter. And then since  $B_x$  is shown downward here, here we have to show  $B_y$  upward to satisfy the equilibrium. And let's take  $B_x$  and  $B_x$  here which is in this direction. And here it should  $B_x$  in this direction. So this is a free body diagram of this part, this is the free body diagram of this part.

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So hinge reactions are  $B_x$  and  $B_y$  and this case again  $B_x$  and  $B_y$ . Then but here  $B_x$  and  $B_y$  and  $B_x$  and  $B_y$  here is shown in the opposite direction, right? Now once the free body diagram is drawn, next is apply the equilibrium condition. Now from this if I apply summation of  $F_x$  is equal to zero, so this gives me  $B_x$  is equal to zero, right?

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Then take moment about B, summation of  $M_b$  is equal to zero. And what are the forces we will contribute?  $C_y$  and this force and  $C_y$  is anticlockwise. So  $C_y$  into 4 minus  $C_y$ , because it is anticlockwise. Then plus moment due to 10 kilo Newton per meter uniformly distributed load. It is distributed over length 6 meter and since it is uniformly distributed, line of action

was the resultant, these are the meet position. So total load will be 10 into 6 meters and then line of action is 6 by 2. So this is zero. And this gives me  $C_y$  is equal to 45 kilo Newton.

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Diagram showing a beam AD with a uniformly distributed load (UDL) of  $10 \text{ kN/m}$  from A to D. The beam is divided into segments AB (2m), BC (4m), and CD (2m). A hinge is located at B, and a roller support is at C. Reactions are  $A_x$ ,  $A_y$  at A;  $B_x$ ,  $B_y$  at B; and  $C_y$  at C. The calculations shown are:

$$\sum F_x = 0 \Rightarrow B_x = 0$$

$$\sum M_B = 0 \Rightarrow -C_y \times 4 + 10 \times 6 \times \frac{6}{2} = 0$$

$$\Rightarrow C_y = 45 \text{ kN}$$

Now once we have  $C_y$ , now let's use another equilibrium equation. Summation of  $F_y$  that is equal to zero. That gives us  $B_y$  plus  $C_y$  is equal to total load which is 6 into 10.  $C_y$  obtained already, 45 kilo Newton meter. And we will get  $B_y$  is equal to 15 kilo Newton. So this gives you 15 kilo Newton.

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Diagram showing a beam AD with a uniformly distributed load (UDL) of  $10 \text{ kN/m}$  from A to D. The beam is divided into segments AB (2m), BC (4m), and CD (2m). A hinge is located at B, and a roller support is at C. Reactions are  $A_x$ ,  $A_y$  at A;  $B_x$ ,  $B_y$  at B; and  $C_y$  at C. The calculations shown are:

$$\sum F_x = 0 \Rightarrow B_x = 0$$

$$\sum M_B = 0 \Rightarrow -C_y \times 4 + 10 \times 6 \times \frac{6}{2} = 0$$

$$\Rightarrow C_y = 45 \text{ kN}$$

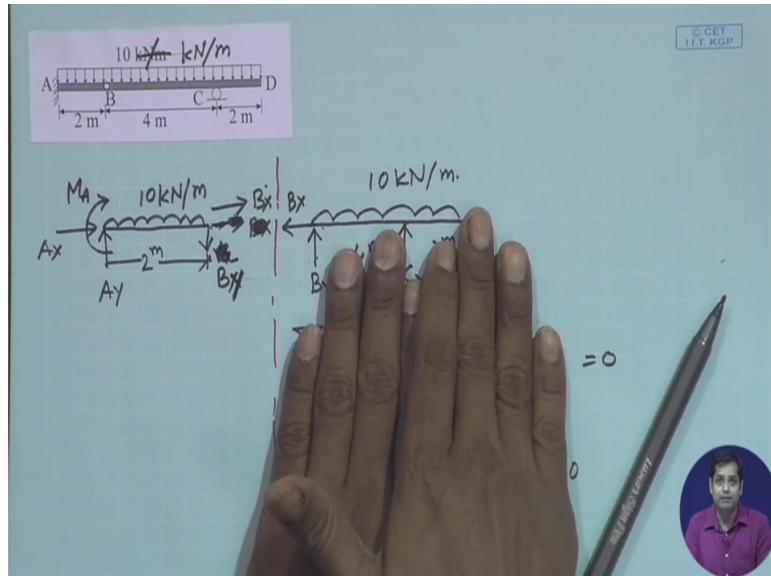
$$\sum F_y = 0 \quad B_y + C_y = 6 \times 10$$

$$B_y = 15 \text{ kN}$$

So support reactions here and hinge reactions are determined. Now you come to this free body diagram,  $B_x$  is already determined here,  $B_y$  is already determined here, only thing we need to

determine is  $A_x$  and  $A_y$ . Now here one point to be noted. You see here we started from this free body diagram.

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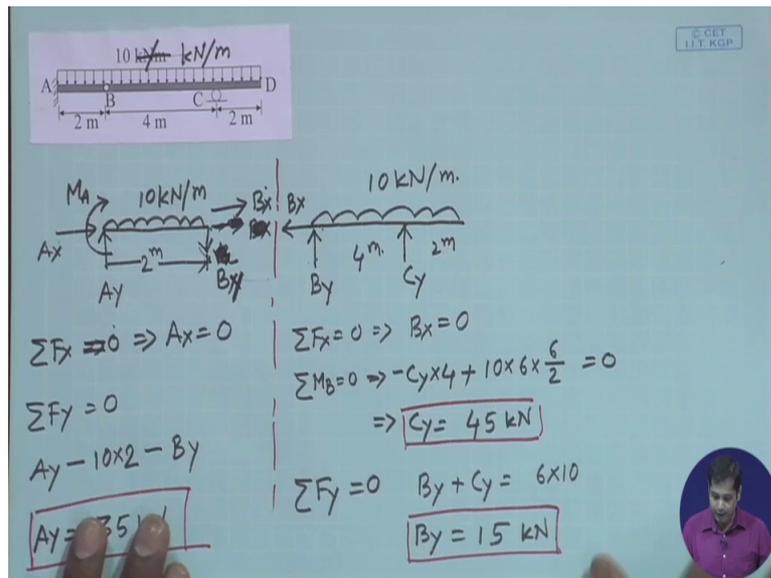


Now instead of that if you start from this free body diagram, what happens? The number of unknown you had is, if you do not have calculated the support reactions from these free body diagram. If you start solving the problem with this free body diagram, your number of unknowns are  $A_x$ ,  $A_y$ ,  $B_x$  and  $B_y$ . So 4 unknown. And numbers of equation you can have at most 3. So you cannot solve it.

Therefore even if you have divided the structure into several parts, which part to take first that we need to decide depending on the problem. There is no standard specification which can tell you that. You have to learn that through practice. Only thing is you need to keep in mind that the number of unknown should not be more than the number of equations in any free body diagram. So  $B_x$  and  $B_y$  are already obtained. Only thing we need to obtain is  $A_y$ .

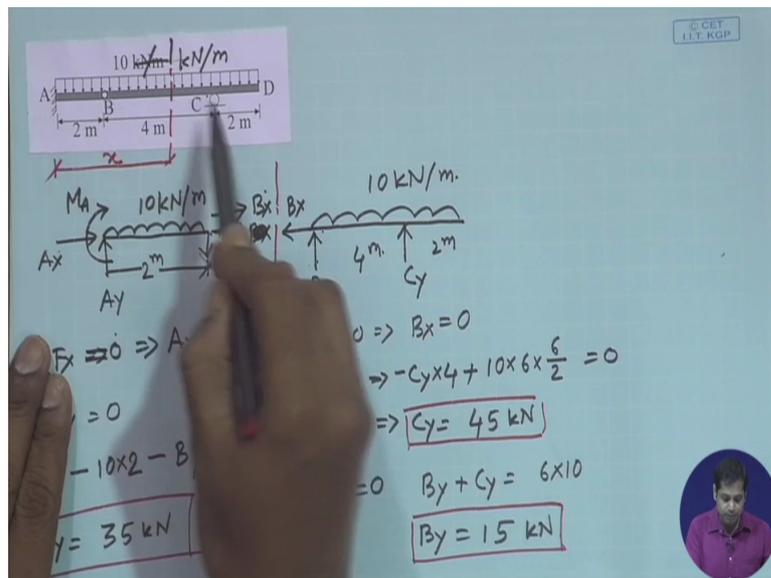
$B_x$  is equal to zero here. So again summation of  $F_x$  is equal to zero, gives us  $A_x$  is equal to zero. And then if you take summation  $F_y$  is equal to zero, this gives us  $A_y$  minus  $10 \times 2$  minus  $B_y$ . Please note here  $B_y$  is in opposite direction. And  $B_y$  is already 15, we obtained. And then finally  $A_y$  will be 35 kilo Newton. So these are support reactions.  $C_y$ ,  $B_y$  and  $A_y$ . Next is draw the bending moment and shear force diagram.

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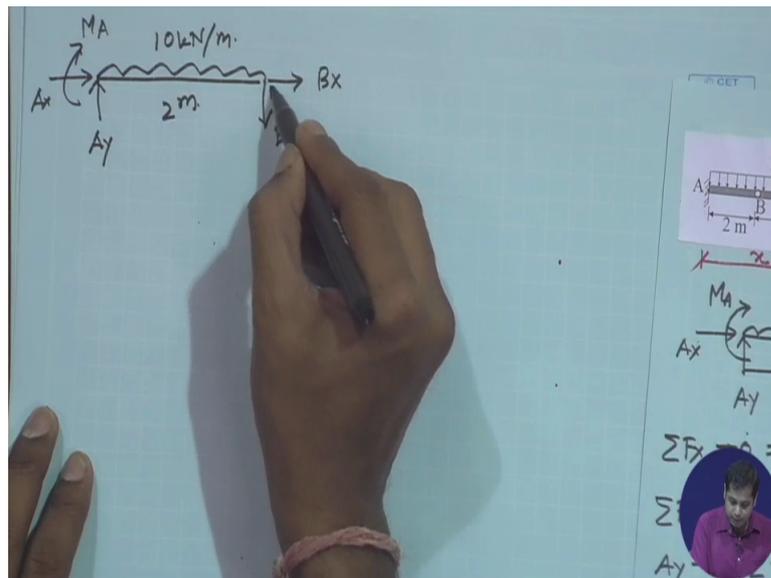
Now let's do that. First what you do is, first you take a section here at a distance  $x$  from A. And this  $x$  varies from A to C. Means  $x$  varies from zero to 6.

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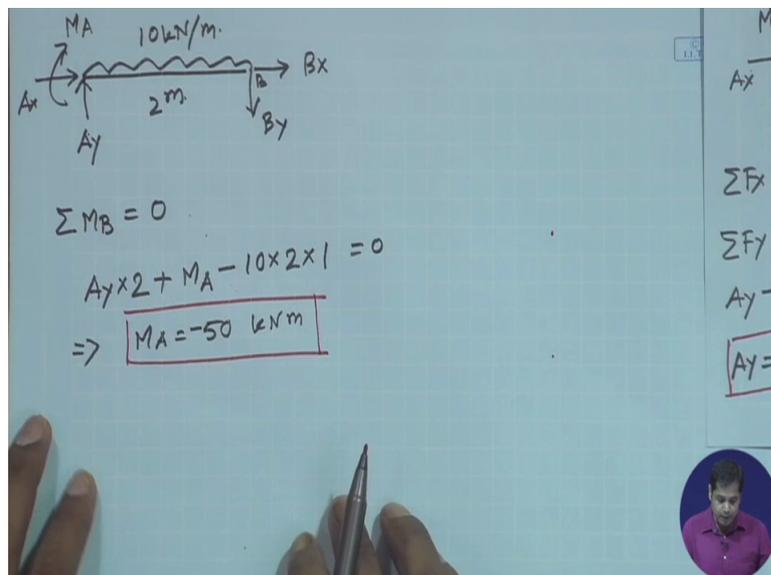
If we take that, then what would be the free body diagram? The free body diagram will be, this is  $A_y$ . Another thing we have not yet determined, that is moment at A. Moment at A you need to determine by taking moment about A. If you take moment about A is equal to zero or you can take moment about B is equal to zero. Because this is hinge, so there will be no moment. Once again let me draw the free body diagram here. This is  $M_A$ , this is  $B_x$  and then  $B_y$ . Now this is B.

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You take moment about B is equal to zero. Now forces will contribute is Ay, Ma and this. So Ay into 2 which is clockwise, then again Ma clockwise and then minus 10 into 2 into 1 which is anticlockwise. That equals to zero. Ay, we already obtained 35. And so if you substitute that, you will get Ma is equal to minus 50 kilo Newton meter.

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So now all reactions are determined, right? You have Ay, Cy, By and then Ma is 50 kilo Newton meter. Now let's find out the bending moment and shear force diagram. Take a section here at a distance x from A. And this x varies from A to C. Means X varies from zero to 6 meter. And the free body diagram will be like this. Ay, Ax I am not showing because Ax is zero, this point is B. We have internal hinge.

This is 10 kilo Newton per meter and this distance is  $x$ . At this point shear force is  $V_x$  and bending moment is  $M_x$ . Hogging bending moment and shear force which provides clockwise couple.

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$\sum M_B = 0$   
 $A_y \times 2 + M_A - 10 \times 2 \times 1 = 0$   
 $\Rightarrow M_A = -50 \text{ kNm}$

Now take summation of  $F_y$  is equal to zero. Which gives me  $A_y$  which is positive. Then minus  $V_x$ , then minus  $10x$  is equal to zero. 10 is the intensity of the uniformly distributed load.  $A_y$  is already obtained as 35 kilo Newton. So from that  $V_x$  becomes 35 minus  $10x$ . So this is how  $V_x$ , the shear force varies from point A to C.

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$\sum M_B = 0$   
 $A_y \times 2 + M_A - 10 \times 2 \times 1 = 0$   
 $\Rightarrow M_A = -50 \text{ kNm}$

$\sum F_y = 0$   
 $A_y - V_x - 10x = 0$   
 $\Rightarrow V_x = 35 - 10x$

Now let's see bending moment. Take moment at this point. Summation of  $M_x$  is equal to zero. Moment at a distance  $x$  and this gives you a moment  $M_x$ . This is anticlockwise direction. And then plus  $A_y$  into  $x$ .  $A_y$  produce clockwise direction moment and then minus  $10$  into  $x$  into  $x$  by  $2$ . That is a loadmoment due to external load Newton. That is equal to zero. And from there we get  $M_x$  plus  $A_y$  is equal to  $35x$  minus  $5x$  square is equal to zero. So this gives  $M_x$  is equal to  $35x$  minus  $5x$  square.

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The image shows two diagrams of a beam and their corresponding equilibrium equations. The left diagram shows a beam of length 2m with a uniformly distributed load of 10 kN/m. Reactions are labeled as  $M_A$  (counter-clockwise moment at A),  $A_y$  (upward force at A),  $B_x$  (rightward force at B), and  $B_y$  (downward force at B). The right diagram shows a section of length  $x$  with reactions  $M_A$  (counter-clockwise moment at A),  $A_y$  (upward force at A),  $V_x$  (downward force at the section), and  $M_x$  (counter-clockwise moment at the section).

Left diagram equations:

$$\sum M_B = 0$$

$$A_y \times 2 + M_A - 10 \times 2 \times 1 = 0$$

$$\Rightarrow M_A = -50 \text{ kNm}$$

Right diagram equations:

$$\sum F_y = 0 \quad A_y - V_x - 10x = 0$$

$$\Rightarrow V_x = 35 - 10x$$

$$\sum M_x = 0 = 0$$

$$-M_x + A_y \cdot x - 10 \cdot x \cdot \frac{x}{2} = 0$$

$$\Rightarrow -M_x + 35x - 5x^2 = 0$$

$$\Rightarrow M_x = 35x - 5x^2 - 50$$

You see this is the distribution of moment between A to C. So we have taken one section here, that section gives you distribution and we have calculated shear force and bending moment at this point and that gives me how the shear force and bending moment varies between A to C.

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Diagram of a beam AD with a pin support at A and a roller support at B. A uniformly distributed load of  $10 \text{ kN/m}$  is applied from B to D. The beam is divided into segments AB (2m), BC (4m), and CD (2m).

Free body diagram for segment BC shows reaction forces  $A_x$ ,  $A_y$ ,  $B_x$ ,  $B_y$ , and  $C_y$ .

Equilibrium equations for segment BC:

$$\sum F_x = 0 \Rightarrow B_x = 0$$

$$\sum M_B = 0 \Rightarrow -C_y \times 4 + 10 \times 6 \times \frac{6}{2} = 0 \Rightarrow C_y = 45 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow B_y + C_y = 6 \times 10 \Rightarrow B_y = 15 \text{ kN}$$

Let's take one more section between C to D. that section is this. Now you have several subsystems, right? Now all the subsystem should satisfy the equilibrium, right? Nowsometime what happens, in the same example if you see herewhen we started, we divide the entire beam into two parts. One is AB and BD. We started our calculation with this part.

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Diagram of a beam AD with a pin support at A and a roller support at B. A uniformly distributed load of  $10 \text{ kN/m}$  is applied from B to D. The beam is divided into segments AB (2m), BC (4m), and CD (2m).

Free body diagram for segment AB shows reaction forces  $A_x$ ,  $A_y$ ,  $B_x$ , and  $B_y$ .

Equilibrium equations for segment AB:

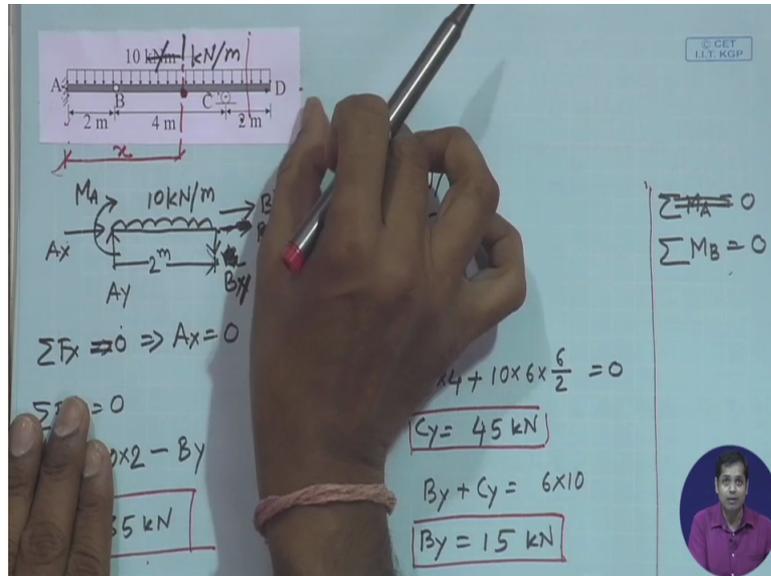
$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow -10 \times 2 - B_y = 0 \Rightarrow B_y = 35 \text{ kN}$$

And then once this is done we move to this part. Because the number of this was easier. Otherwise if we start with this, then the number of unknown will be more. So as I have already said this before, which part to take? That you need to decide based on the problem. And also sometimes what happens both parts may give you the information but probably one part give you the information. Dealing with one part will be much easier than the other part.

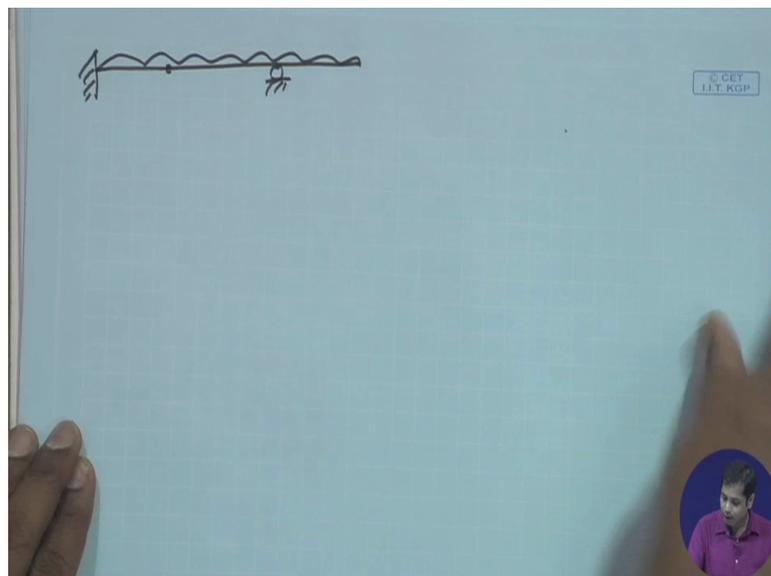
For instance, if I take this beam and take a section between C to D. Now this section divide the entire beam into two parts. One is this and one is this.

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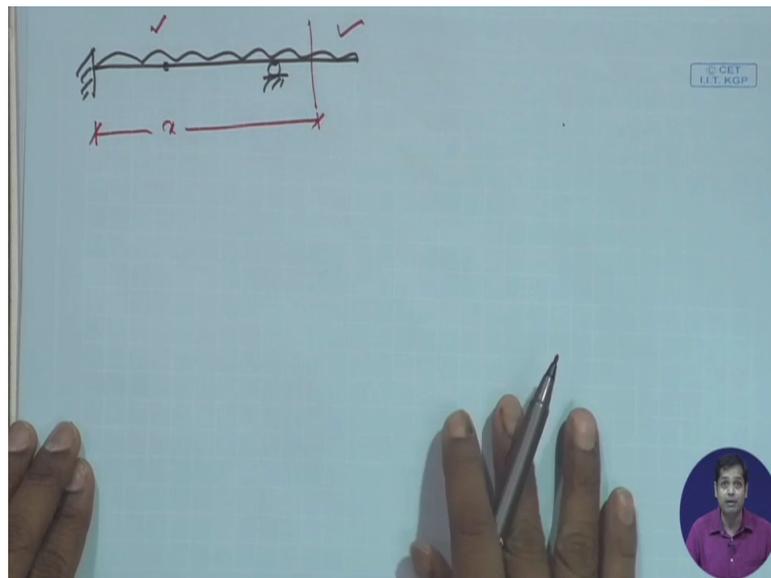
Let's draw these two parts separately. Now this two parts will be, the first will be this. This is the entire beam. This is fixed and then you have internal hinge here and then roller and uniformly distributed load.

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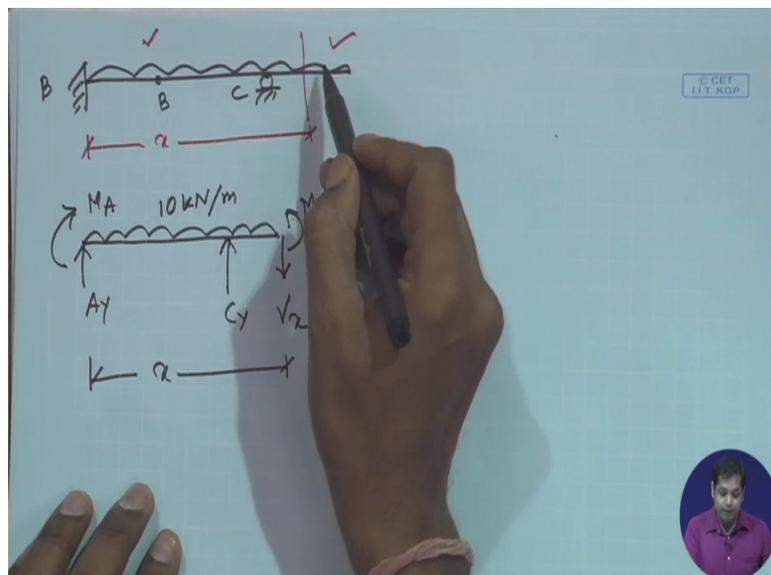
Now we take a section here and this distance is x. Now we have two part, one is this part and another one is this part.

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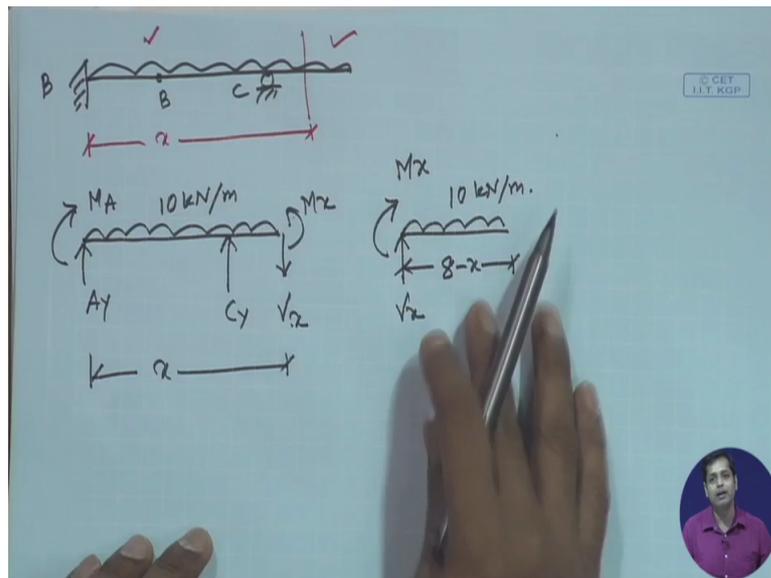
Let's first draw the free body diagram of this. Free body diagram of this part will be  $A_y$  and then  $C_y$ , uniformly distributed load. This is  $V_x$  and this is  $M_x$ . And this distance is  $x$ . This is a free body diagram of this part. This is 10 kilo Newton per meter. Now if you draw the free body diagram of this part.

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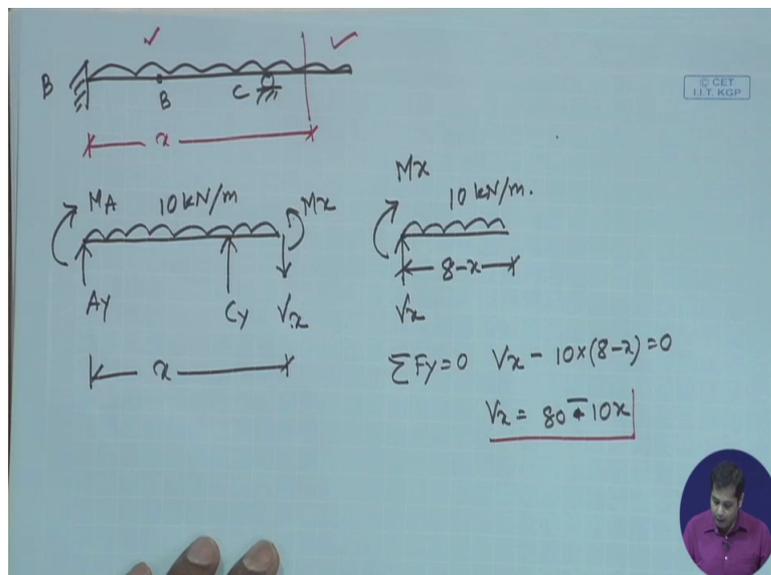
Free body diagram of this part will be this. This  $V_x$  and sagging moment positive this and this distance will be total distance is 8 meter. So this distance will be 8 minus  $x$ . And this is 10 kilo Newton per meter. You see if I calculate  $V_x$  and  $M_x$  from this free body diagram and  $V_x$  and  $M_x$  for this free body diagram, both will give me the same expression, similar expression.

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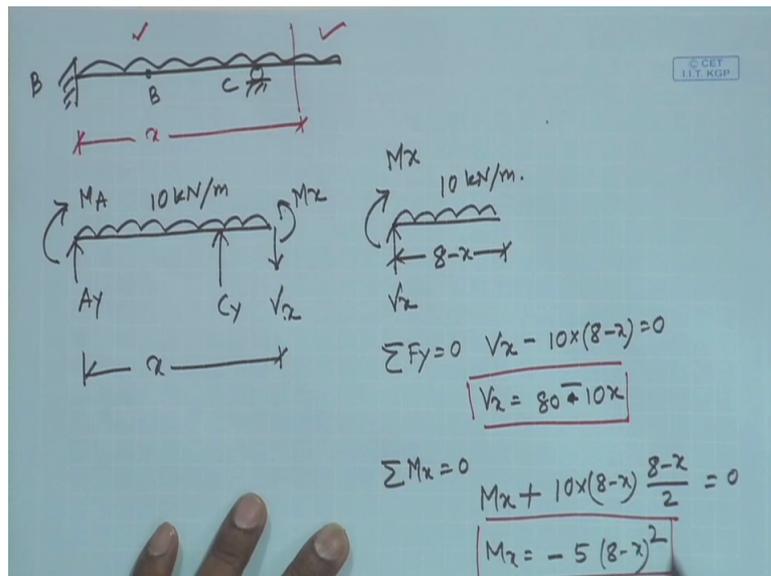
But computation with this free body diagram is easier as compared to this. Why? Because the number of forces, number of reactions are less here. Now if I take this, then summation of  $F_y$  is equal to zero. This gives me  $V_x$  minus 10 into 8 minus x is equal to zero. So  $V_x$  is equal to 80 plus 10x. So  $V_x$  is equal to 80 minus 10x.

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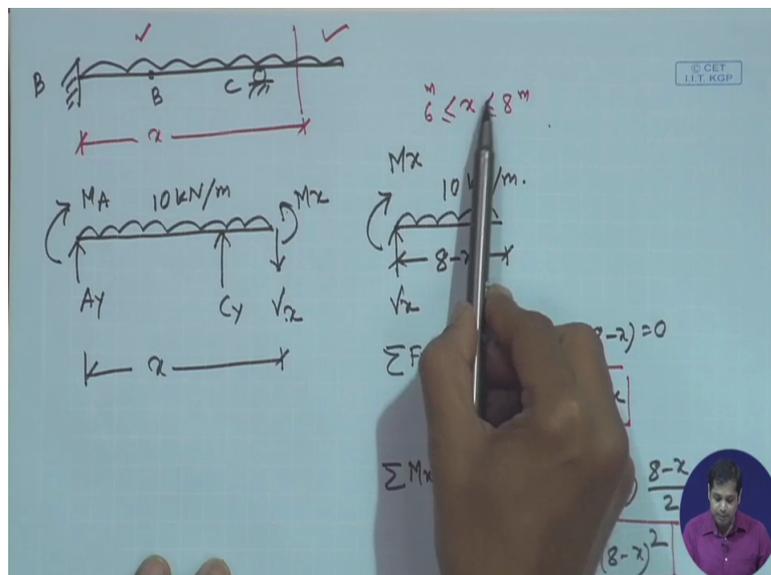
Now this is the shear force and if I take summation of  $M_x$  is equal to zero. This gives  $M_x$  which is clockwise, plus again this is clockwise, 10 into 8 minus x is total length. And this is by 2 is equal to zero. And this gives me  $M_x$  is equal to minus 5, 8 minus x whole square.

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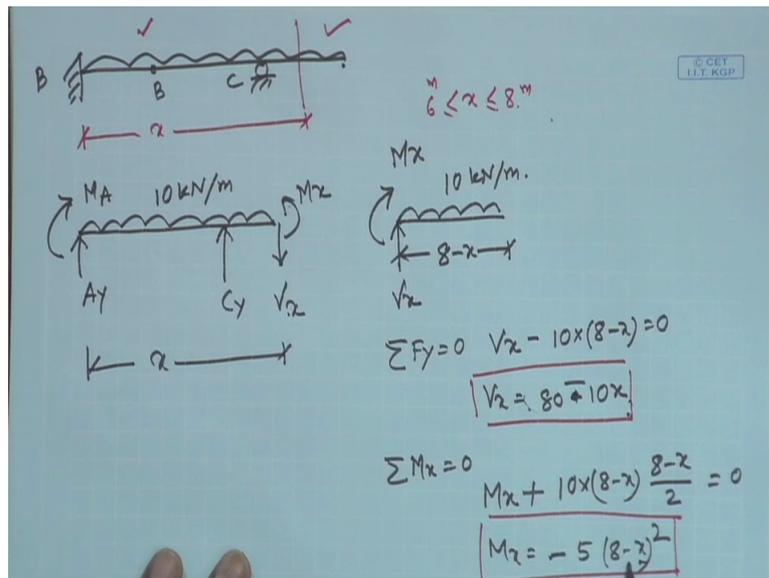
You see, so this is a distribution of shear force and distribution of moment in this part. Now let's see quickly some observation whether this distribution is consistent with the problem. You see, in this case  $x$  varies from 6 to 8 meter, right? Now if I substitute  $x$  is equal to 8 which is at the free end, your shear force and bending moment both should be zero.

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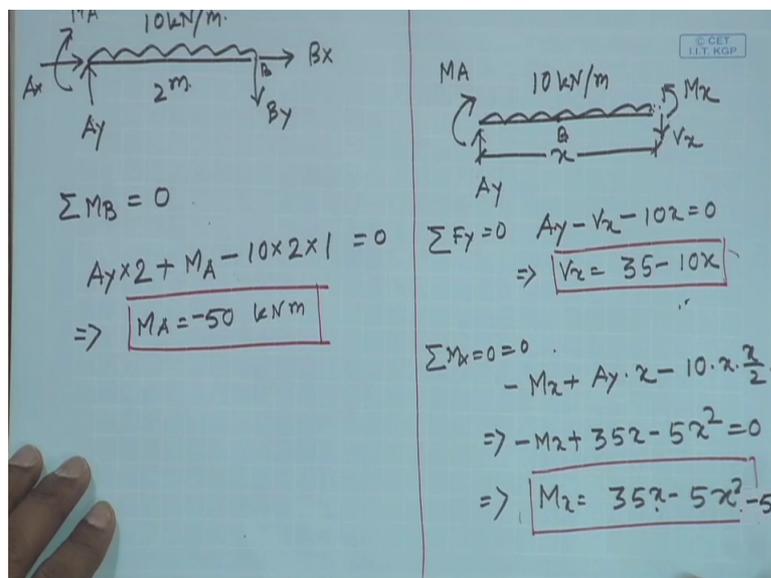
Substitute  $x$  is equal to 8 here this is zero, substitute  $x$  is equal to 8 here, bending moment is equal to zero.

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So this is consistent with the problem, right? Now another observation, shear force is linear and bending moment is quadratic. Bending moment is one order higher. In the previous part between B to C if you remember, this was the variation of shear force and variation of bending moment. So variation of shear force is linear and variation of bending moment is quadratic one order higher.

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Now if I draw the bending moment and shear force diagram for the same beam, it will be like this. This is a problem. A, C, D. First draw the shear force diagram. Now between A to C shear force distribution was this.

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$\sum M_B = 0$   
 $A_y \times 2 + M_A - 10 \times 2 \times 1 = 0$   
 $\Rightarrow M_A = -50 \text{ kNm}$

$\sum F_y = 0$   
 $A_y - V_x - 10x = 0$   
 $\Rightarrow V_x = 35 - 10x$

$\sum M_x = 0 = 0$   
 $-M_x + A_y \cdot x - 10 \cdot x \cdot \frac{x}{2}$   
 $\Rightarrow -M_x + 35x - 5x^2$   
 $\Rightarrow M_x = 35x - 5x^2$

And between C to D, shear force distribution was this.

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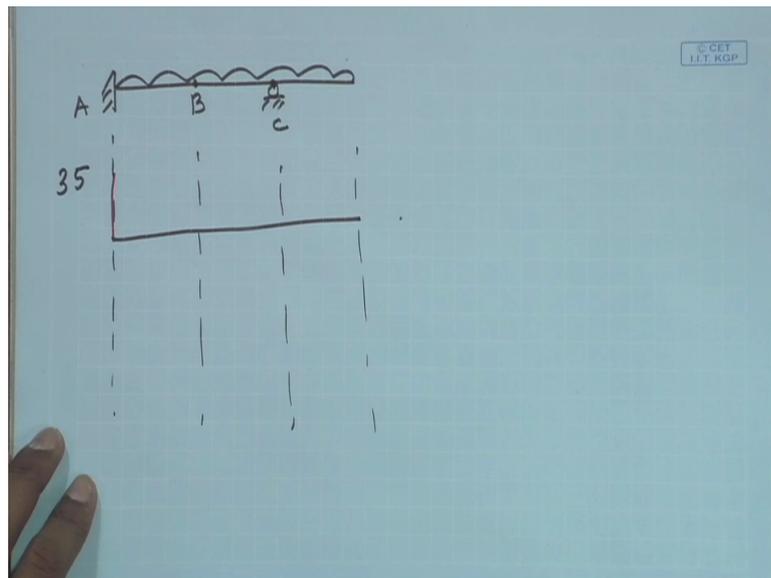
$6 \leq x \leq 8 \text{ m}$

$\sum F_y = 0$   
 $V_x - 10x(8-x) = 0$   
 $V_x = 80 - 10x$

$\sum M_x = 0$   
 $M_x + 10x(8-x) \cdot \frac{8-x}{2}$

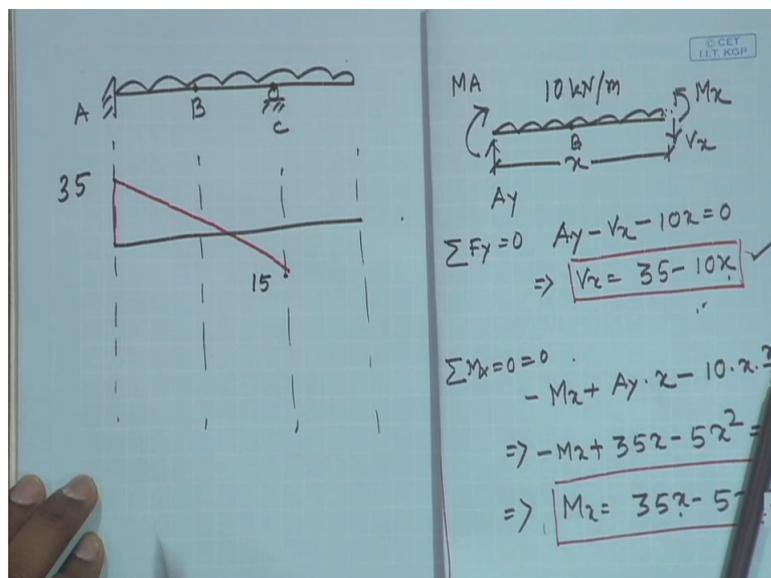
Now bending moment and shear force diagram will be. You can verify this. This is linear. This value is 35 at x is equal to zero.

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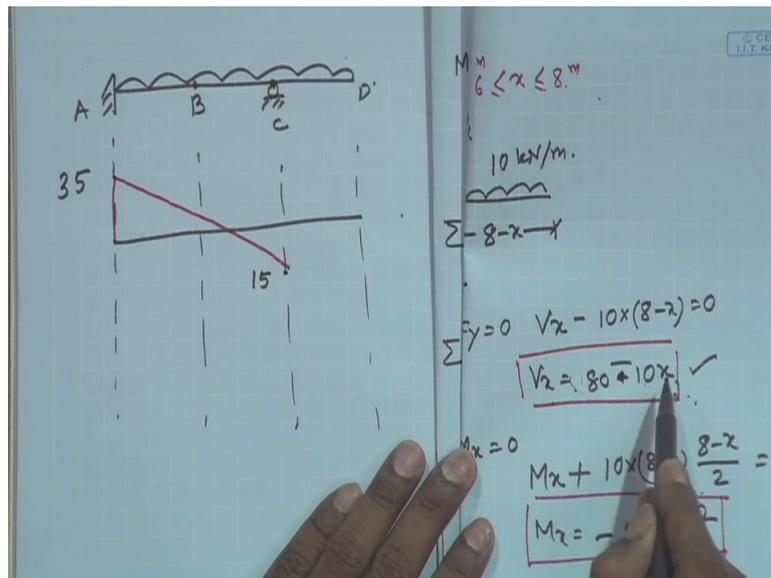
Show you. This is  $x$  is equal to zero, it is 35 and then it varies linearly. And  $x$  is equal to 6, it becomes minus 15. So  $x$  is equal to 6, it becomes minus 15. And between that it is linear. So it is 15. This value is 15.

(Refer Slide Time 23:33)



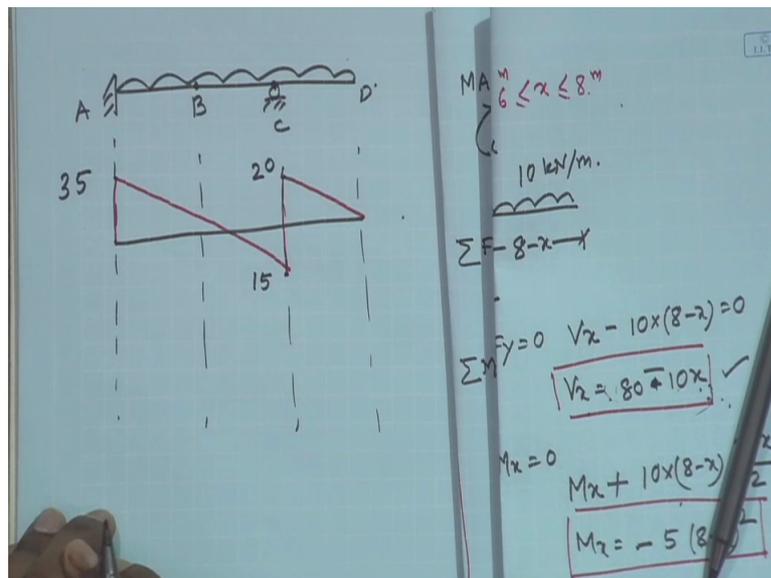
Now let's see the shear force diagram for this part. Shear force distribution is this where  $x$  changes from C to D. Substitute  $x$  is equal to location at C means 6, it becomes 80 minus 60, 20 and this is positive. So this is 20.

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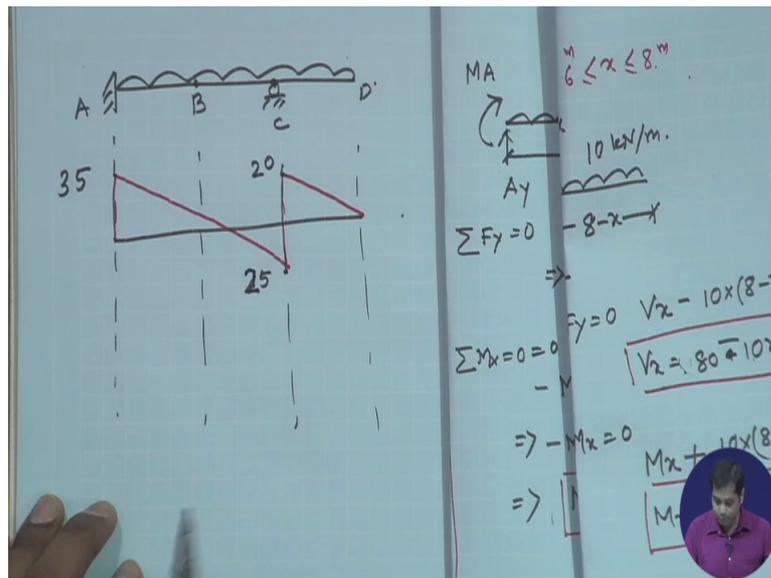
So and then again it decreases linearly. So this decreases linearly. So this is your bending moment. This is your shear force diagram. And this value is 20.

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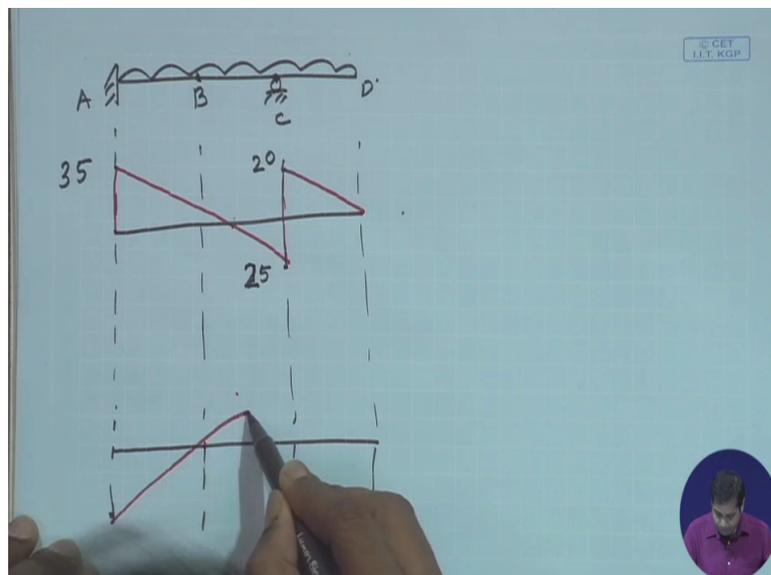
Now if you remember, this value is 25. Now this is the shear force diagram.

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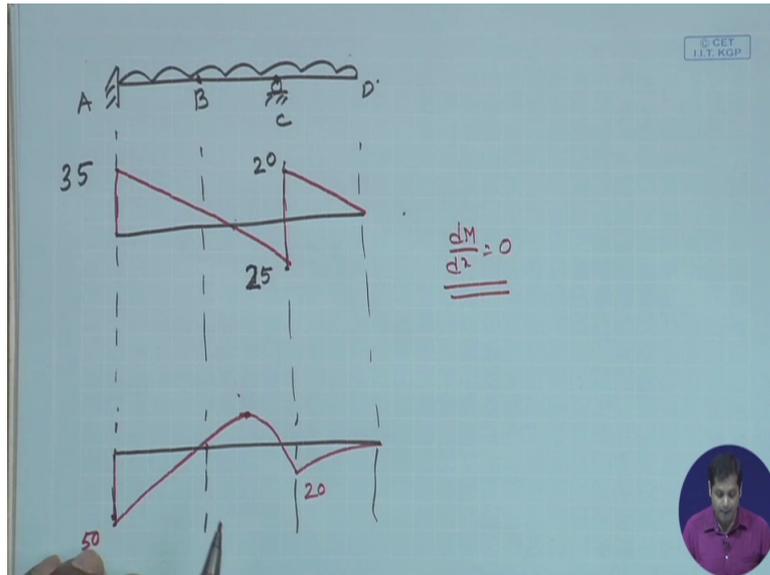
Let's draw the bending moment diagram. Once again now your bending moment distribution was this for this part. X is equal to zero, bending moment at x. 35 minus 35. You know this part where is that? If you draw this bending moment diagram, your bending moment diagram will be something like this. It will be, this at B it is hinge. So it should be zero here.

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And then it is like this and this. Your bending moment diagram will be zero. And these values will be 50, then 20. And you can determine again what is the maximum bending moment by  $dM/dx = 0$ . And get the value of x where your bending moment is maximum.

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So this is how you can draw the bending moment and shear force diagram. Now things will be more clear when you do more exercises. If you take any structural analysis book, there are many solved examples are given, many exercise problems are given. I suggest you to go through those value problems and attempt them. And another thing, by the end of this course you see for instance, if I take a cantilever beam like this.

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Now apply a load here, then this will deflect like this.

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And in order to say that this will deflect like this, we do not need any structural analysis course. Just by intuition we can say that yes, if the load is applied downward, the deflection will be like this. So these we can say just by intuition, right? Structural analysis course is required to determine the value of the deflection. Determine the value of internal forces. Now by quoting this example what point I want to make is this.

By the end of this course if you solve more problem, you should be able to draw bending moment and shear force and diagram based on your intuition. Just by looking at the problem. And you should reach that level. Now what will be the values of bending moment and shear force for the diagram? For this you need equilibrium equation and solution of linear system.

But just to draw a typical bending moment and shear force diagram for a given structure, you should be able to do that from your engineering sense. And that sense you need to develop and for that you need to solve several examples. Because of the time restrictions, we could do few problems in the class. But I suggest you to go through the problems given in the books. Thank you. See you next week.