Course on Structural Analysis I
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Lecture No 59
Direct Stiffness Method (continued)

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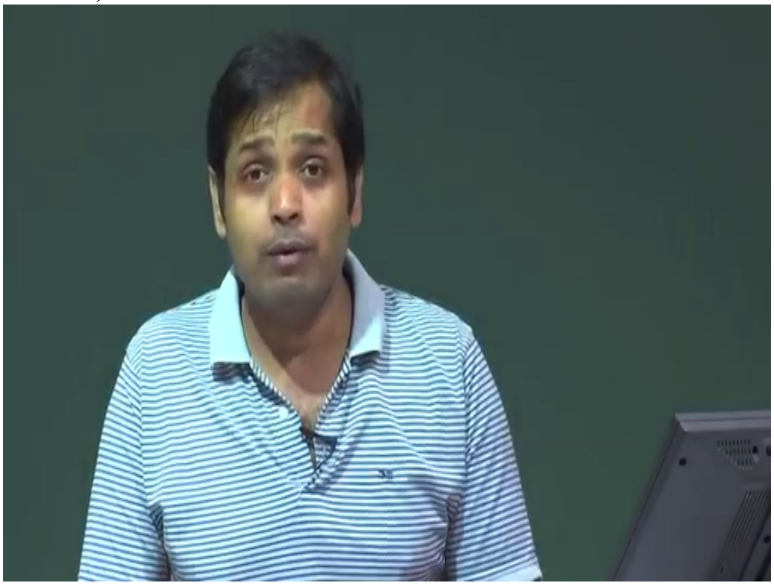

Hello everyone. Welcome what we do, we are going to do today is see have already discussed what is direct stiffness matrix, direct stiffness method and the, the, the concept behind it and general procedure of direct stiffness, direct stiffness method for truss, beam and frame problems. Now, then next today and next, next, next two, next two classes we will give the demonstration of the direct stiffness method to different problems, one problem that we are going to take today is truss problem and then next week, beam and then next week frame problem, Ok.

Now, so today,

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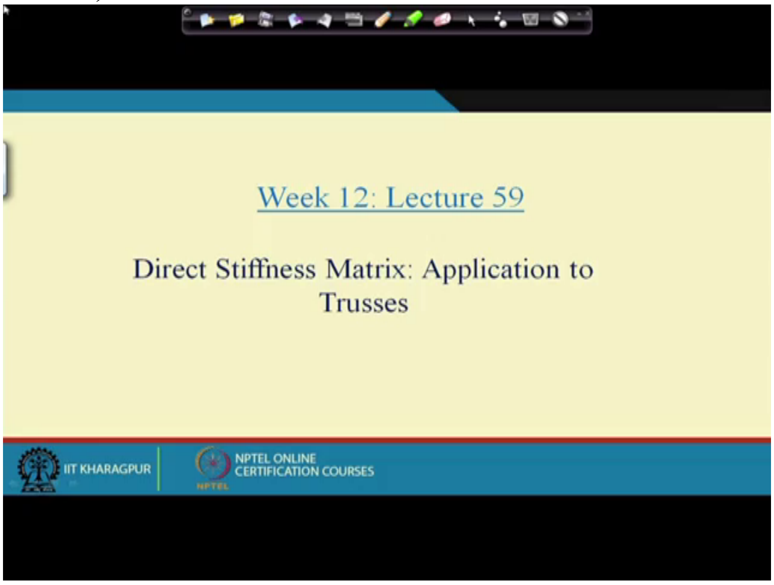

direct stiff, application of the direct stiffness matrix, direct stiffness method, please

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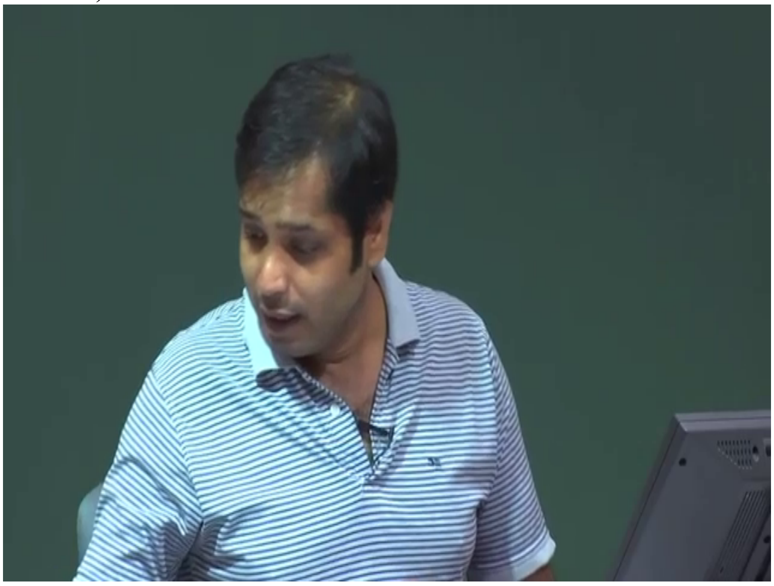

correct it, Ok, this is,

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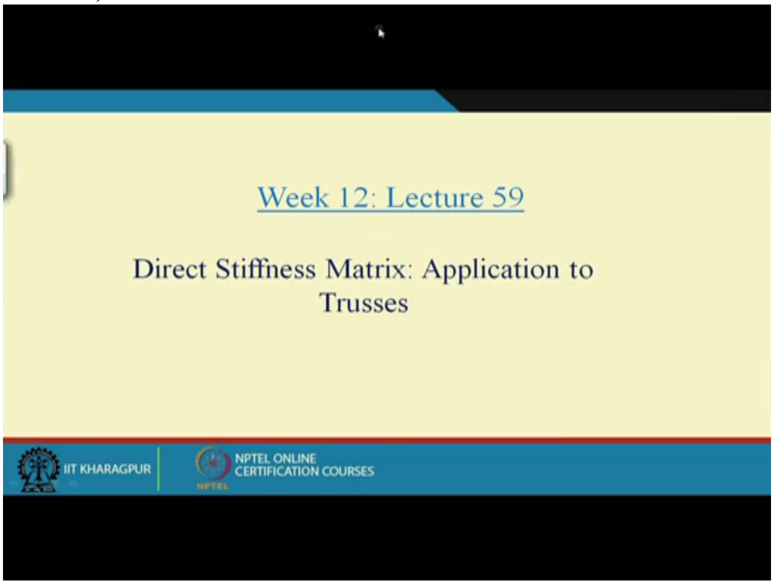

though it is not, not in a, in some it is not wrong because it is a matrix method, it is a matrix method of structure analysis only. So this is direct stiffness method,

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Ok. Now

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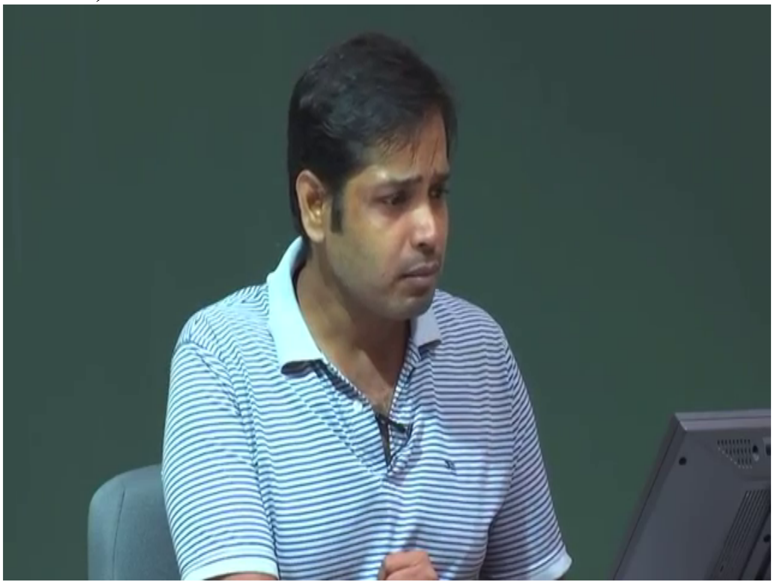

let us start with, let us

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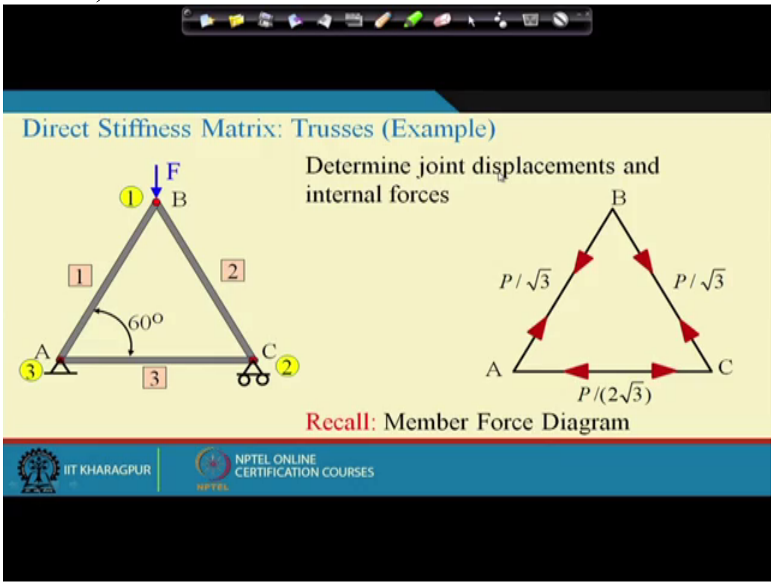

take this example. If you remember this example we have already taken while analyzing, analyzing truss, Ok.

Now we are, all the calculations

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that we are going to do today is manual calculation and this is the reason why I took a problem where the degrees of, total degrees of freedom are less. But as I said, this method is general and is suitable for computer implementation. So if you have the larger structures where many members and the space, space structure, space frame, then the concept can be applied, then the same concept can be applied but that we have to do with the use of computers, Ok. Now manual calculations would be difficult in those cases.

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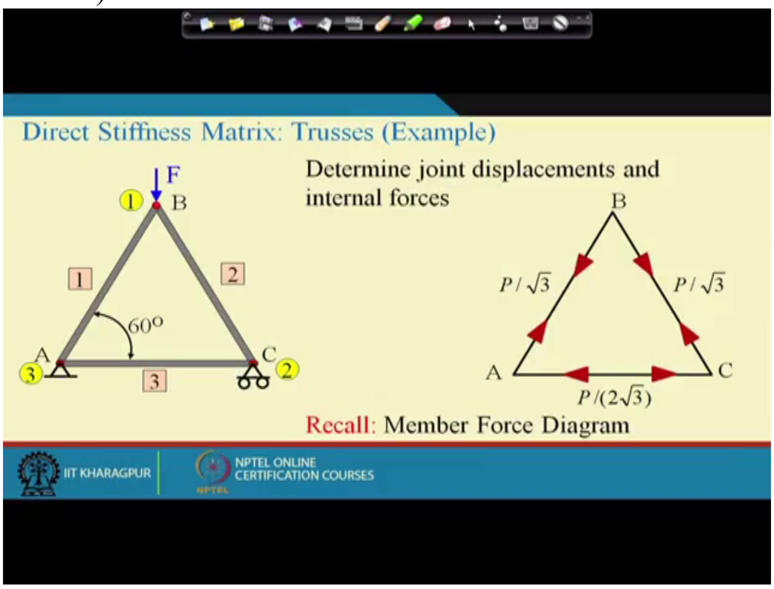

Great.

So let us take this example, we have already deter/determine, what we need to determine is determine the joint displacements and internal forces. We already determined the joint displacement, not joint displacement; we already determined the internal forces. If you go back to, to the lecture classes where we discussed different methods for analysis of truss and this is the result for this, Ok.

So this means, this, this point is in this, this is under compression. That is what our sign convention, this is under compression, this is under compression and this, this, this beam, this member is under tension.

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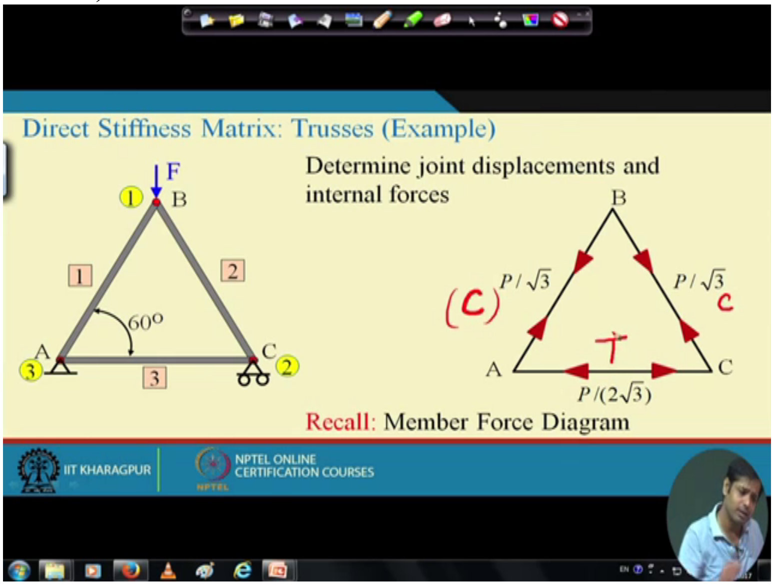

These are the values given, Ok. Now we will see whether we are going to get these values or not and how easily we are going to get these values. Great.

Now, so,

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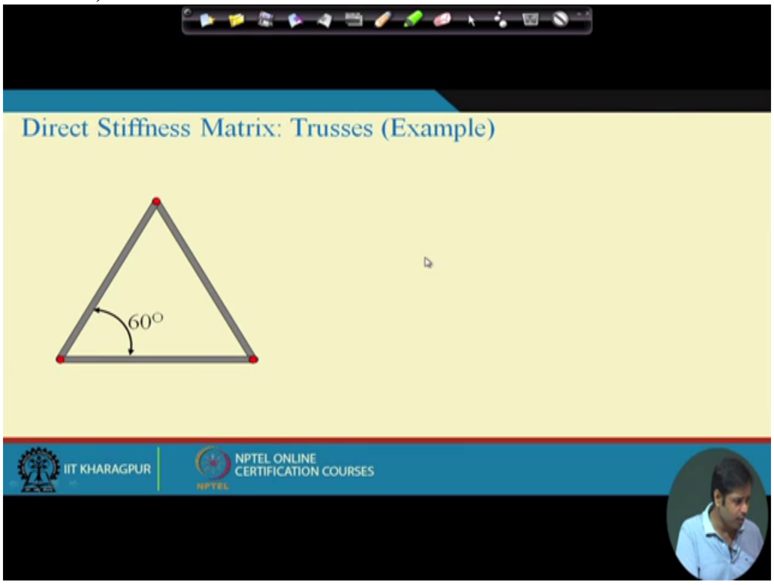

take this is the, this is the, this is the frame. You take the structure itself, remove it from the support and suppose

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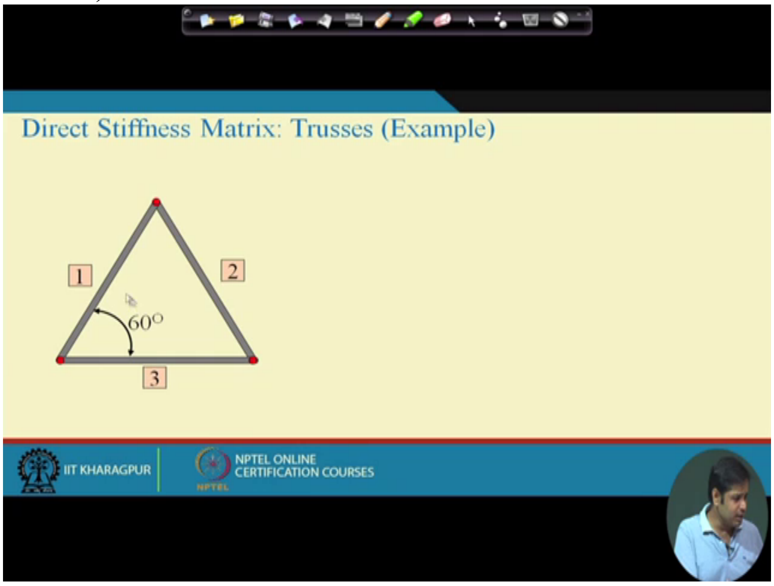

this member is 1, member 2, member 3. And

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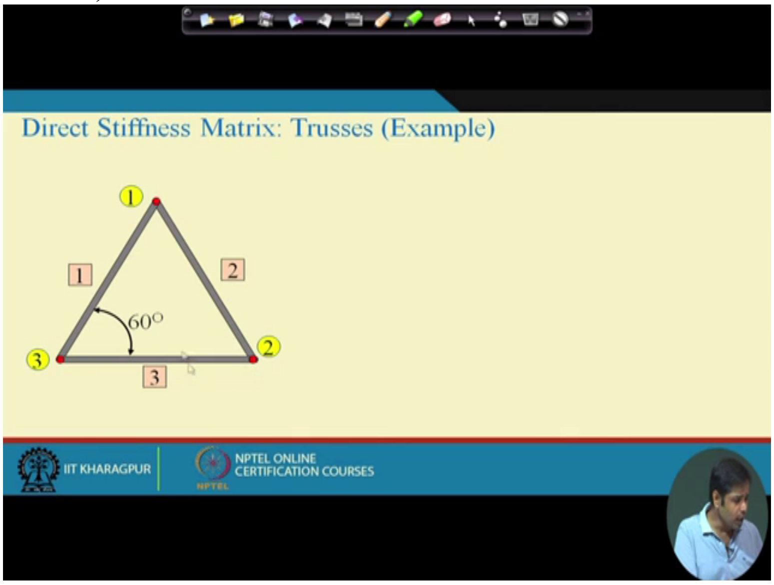

the joint number is 1, 2, and 3. Now here one thing please note that

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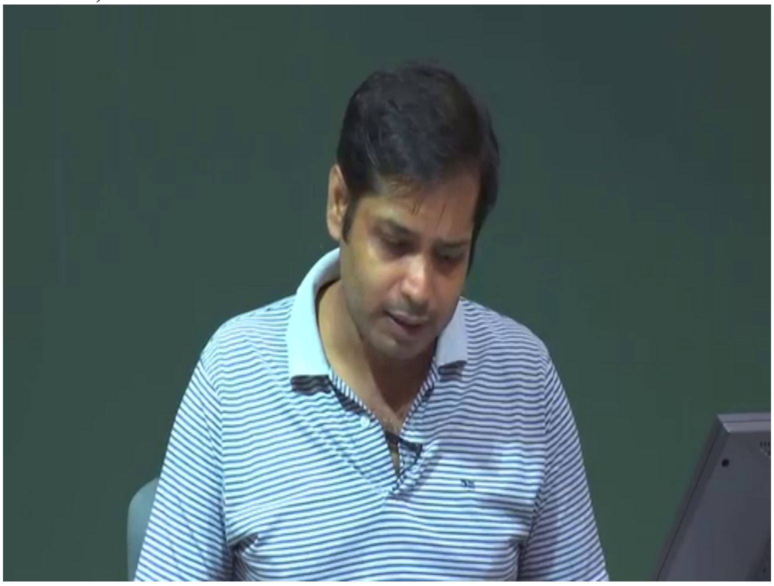

I say in the previous classes as well, how, how you give the joint number that is also a trick. Because at the end you need to solve a system of equations, right?

Now, now in a large, at the end you have an equation like A x is equal to b where A is a stiff/stiffness, where k x is equal to, say F where k is the stiffness matrix, x is the unknown, unknown displacement field and then F is the force vector, Ok. Now for small matrix, you can directly invert it but for larger matrix then inverting, direct inversion of the matrix is not a good idea to solve this equation. There are many, many methods to, to solve, solve, solve these equations. And I am sure in the first year engineering maths course, you have learnt some of those methods.

Now your computation becomes easier if, if, if you store the, the form of the matrix that you have. Most of the, most of the elements in this, in this, in this matrix will be zero. Some of the elements which are non-zero but how these non-zero elements in the matrix are distributed in the non-zero, are distributed in the matrix, depending on that, your computation depends on that as well. So always we want a matrix to be banded so that your computation becomes less, your computational effort becomes less.

So therefore in order to get those banded matrix, you have to number the node in a, in, in such a way that it, it leads to a banded matrix. So, but for this case, I have

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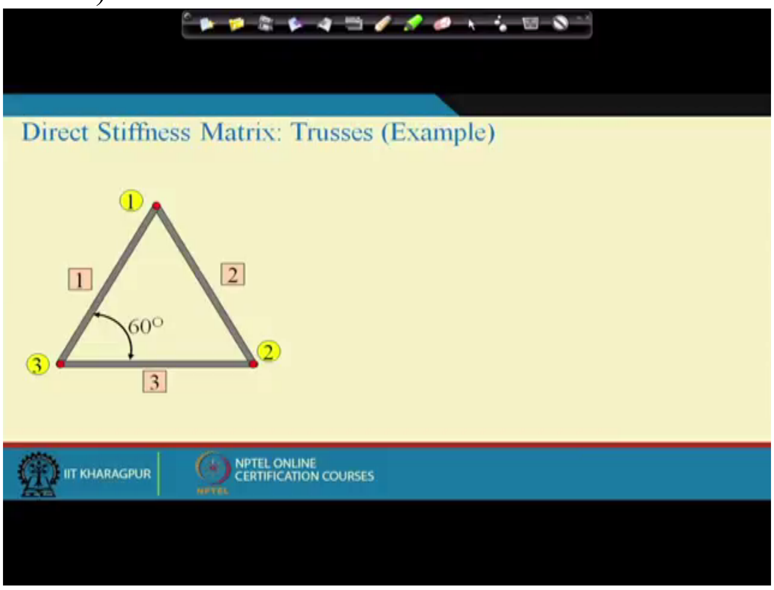

given the node number arbitrarily, not really arbitrarily. I will tell you why I have given 1, 2, 3 like this. But when you actually implement this, this, this method in computer please note that numbering of this node is also not arbitrary. You need to optimize

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to get the most optimized form of stiffness matrix; you need to number them accordingly. Ok. Now for the

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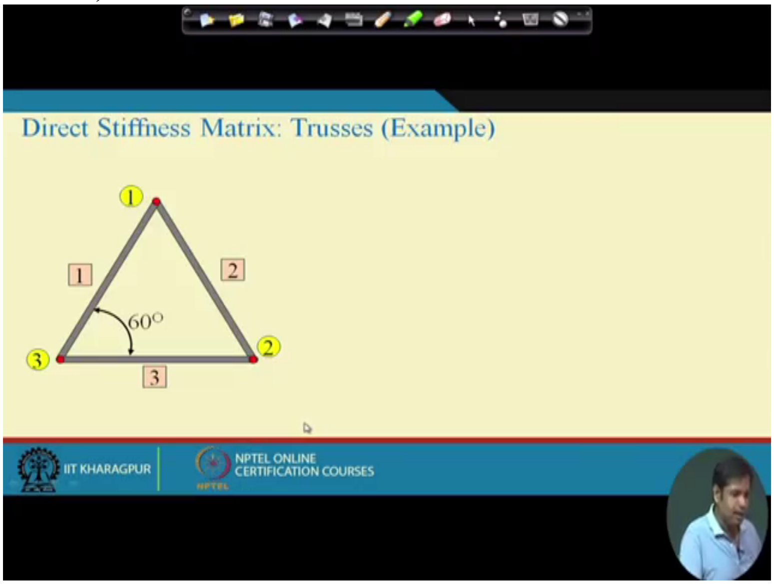

time being you don't have to bother right now. Ok now what we are interested now to, to understand the concept, to demonstrate the, demonstrate the concept, Ok. Now this is, this is, this is the member number and node number we have, then what we have is,

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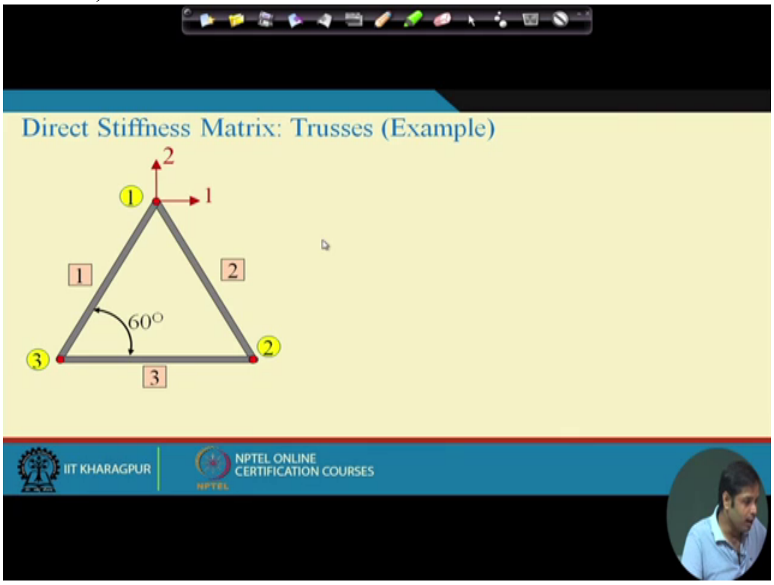

now next is suppose, you see at every joint we have 2 degrees of freedom, right? Every joint can move in x direction and can move in y direction. And all the degrees of freedom you have to number.

Now suppose the first degree is the, this is 1, this is 2, so whenever I say d 1, it means that the horizontal displacement of node, node 1, d 2 is horizon/horizontal, vertical displacement of node 2. Similarly

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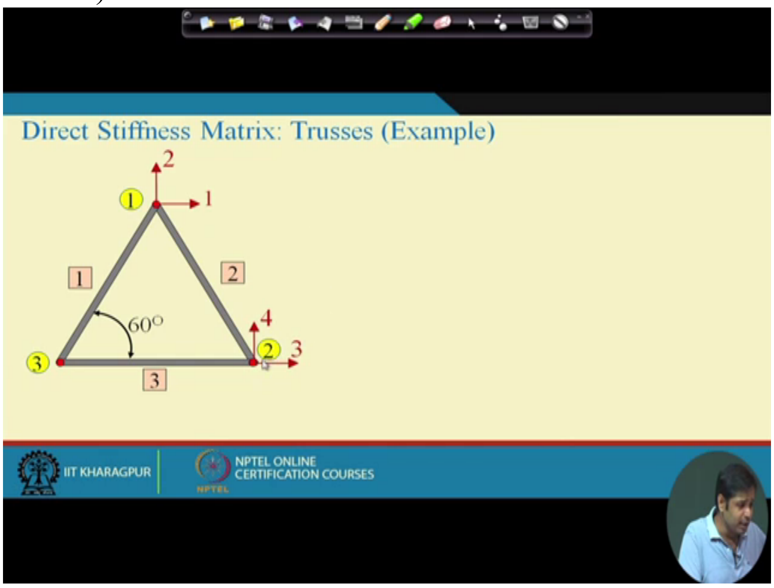

d 3 will be the horizontal displacement of node 2 and 4 will be the, d 4 will be the vertical displacement of node 4 and similarly

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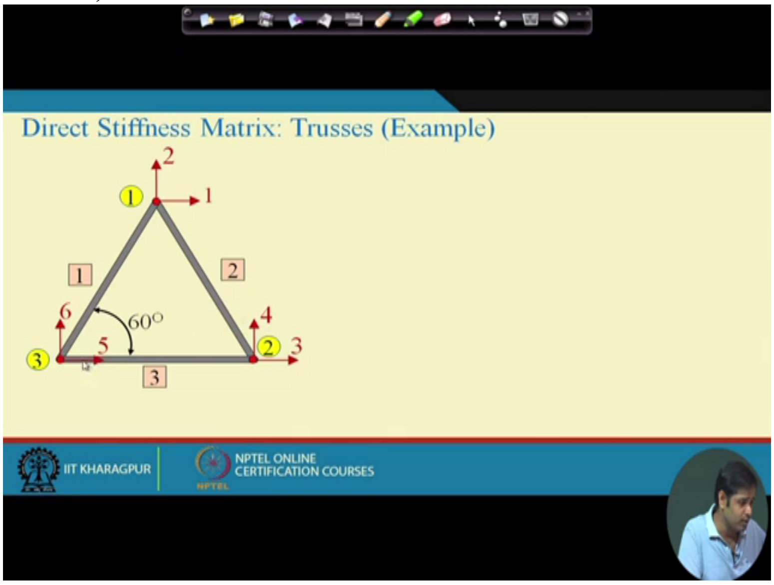

this is the fifth degree of freedom, this is the sixth degree of freedom, Ok. Now, the unknown joint displacement Ok. Now total we have, total we have 6 joint displacements.

Now, the next thing is as we discussed, next thing is we have to break this structure into pieces and, and, and get the, get the, get the

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stiffness matrix for each small sub-structure and then finally assemble them to get the global stiffness matrix of the entire structure. So now let us, now you have 3 members here. So let us break them, let us

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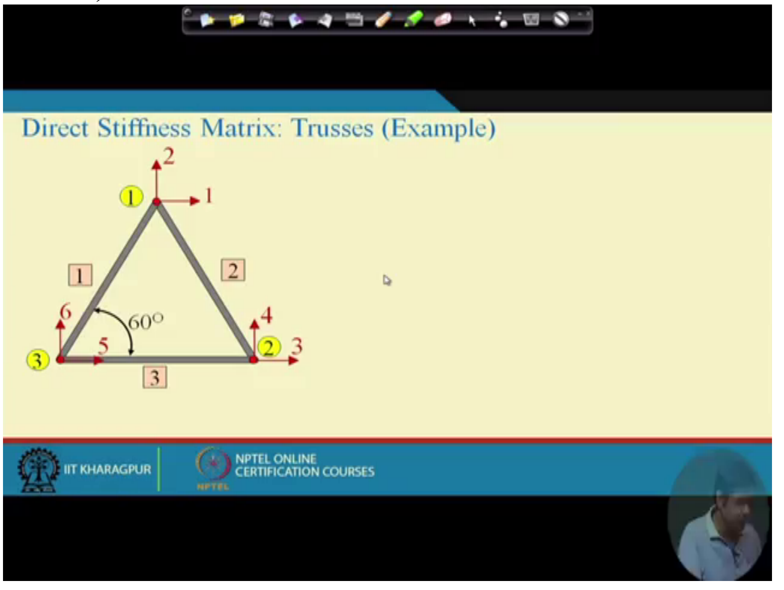

separate all these three members, so

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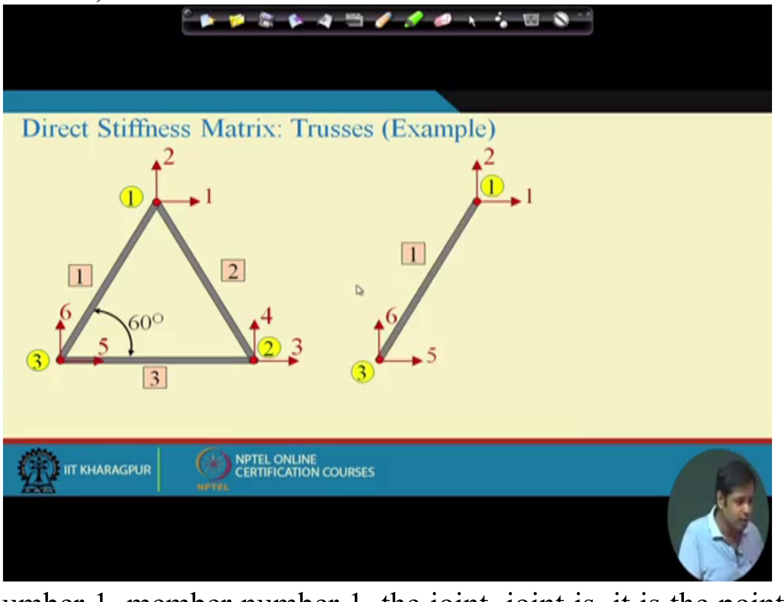

this is member number 1, member number 1, the joint, joint is, it is the point is 3 and 1, it is connected between joint 3 and 1 and these are the corresponding joint translation we have.

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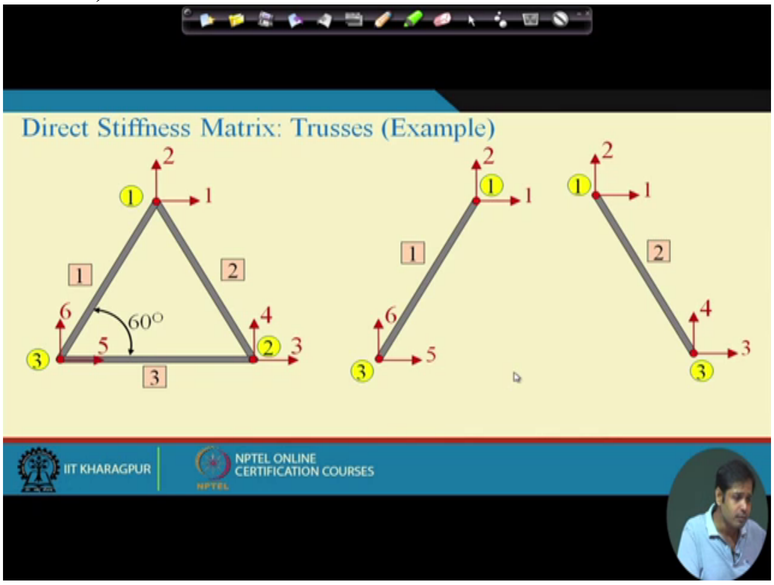

Similarly this is member number 2. It is between 3 and, again please correct it, this is not 3. This is 2, this will be, this will be 2.

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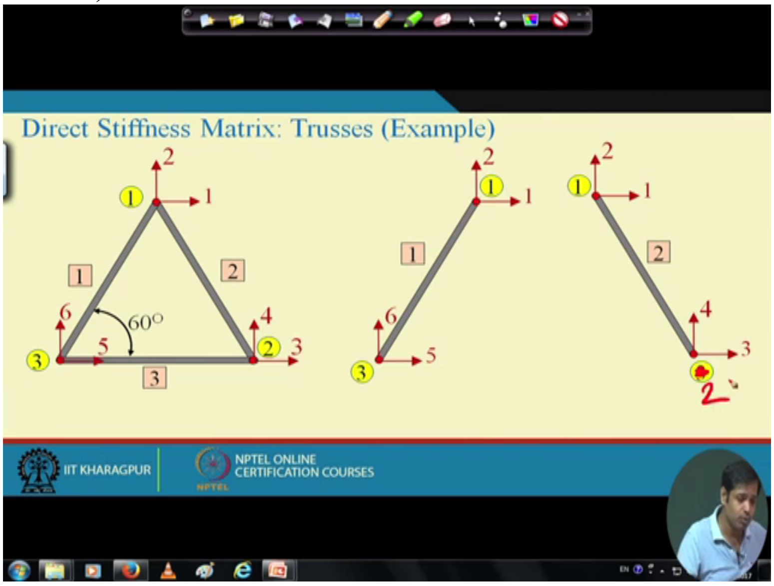

This will be 2 and then we have joint

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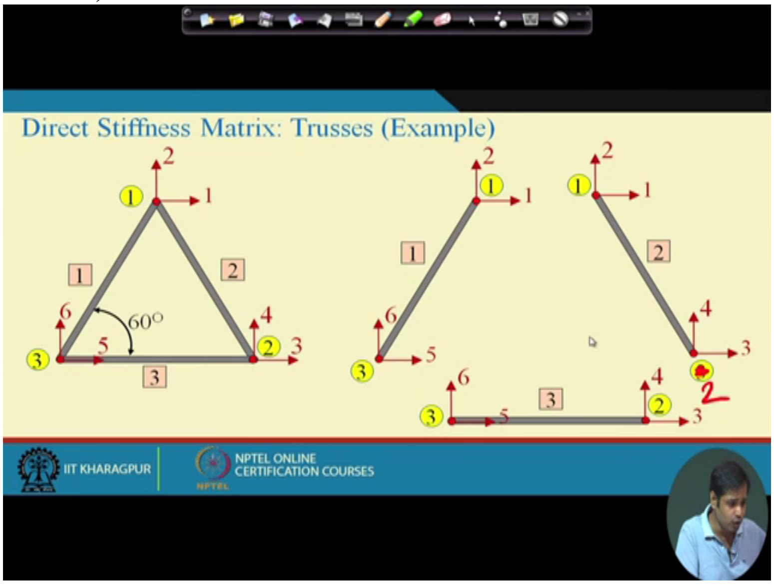

number, joint number 3, member number 3 which is connected between 3 and 2, Ok. So what we need to find out now is we need to find out element stiffness matrix for this, element stiffness matrix for, for member 1 which will be k, which will be k 1 and then

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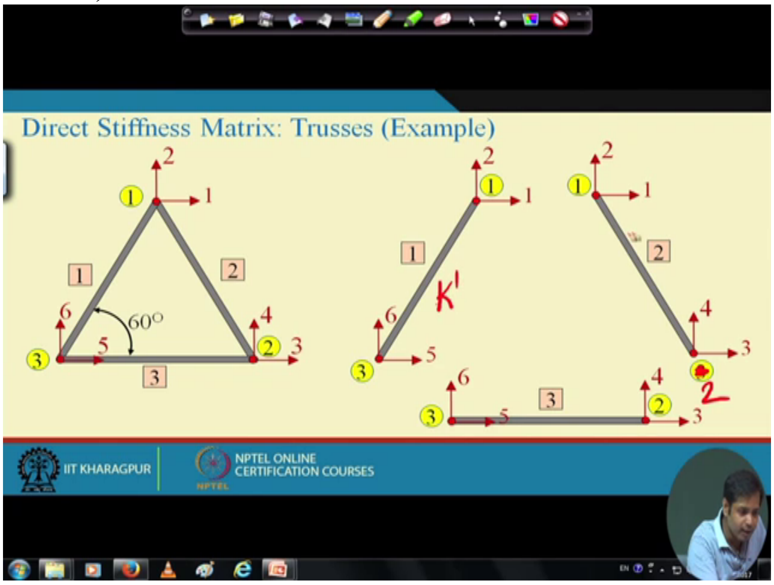

member 2 which will be k 2

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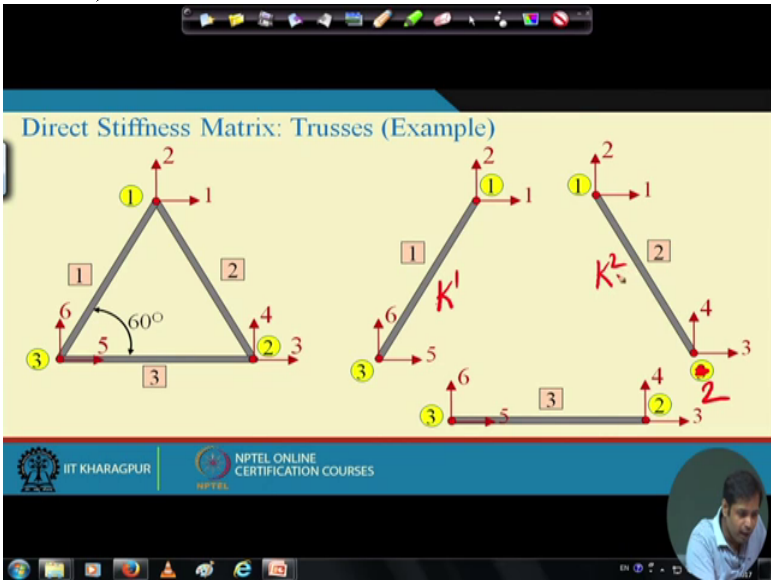

and then member 3 which will be k 3.

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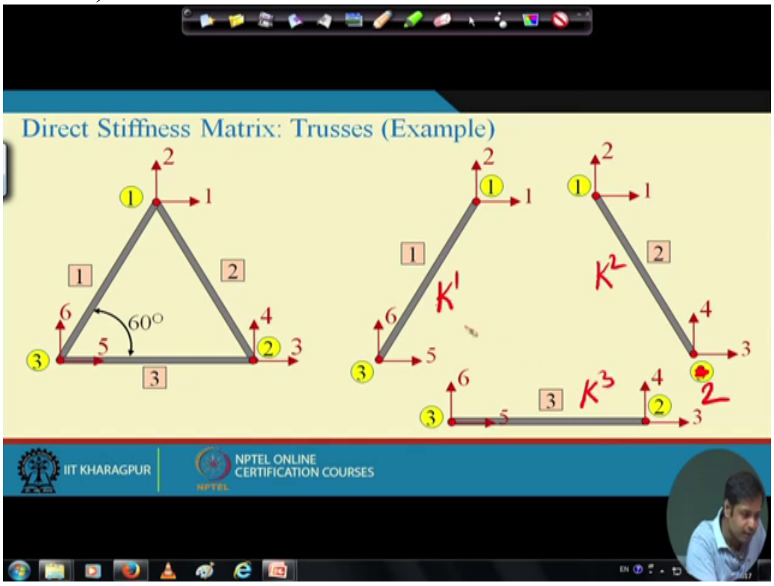

These are all element stiffness matrices, Ok or member stiffness matrices rather,

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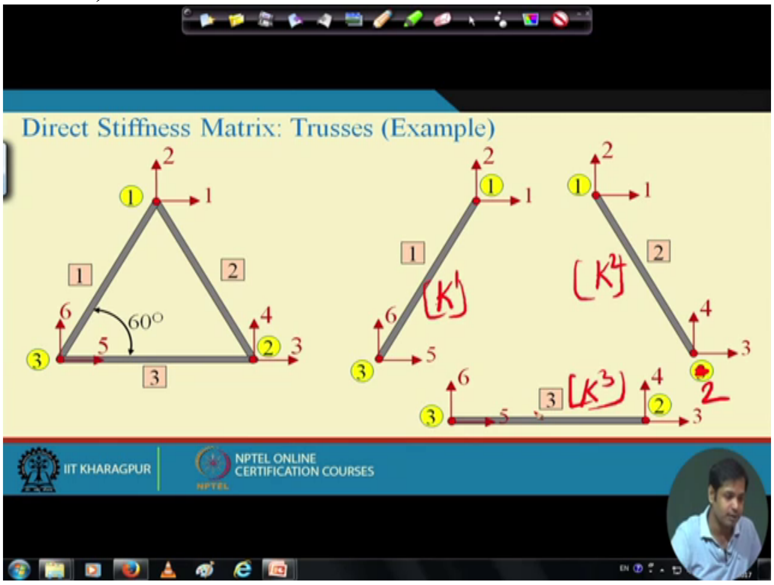

Ok.

And then we need to assemble them k 1, k 2 and k 3 to get the global stiffness matrix. So k 1, k 2, k 3 are the stiffness matrices for member 1, member 2 and member 3 when they are separated, Ok. Now let's see what, if you recall then

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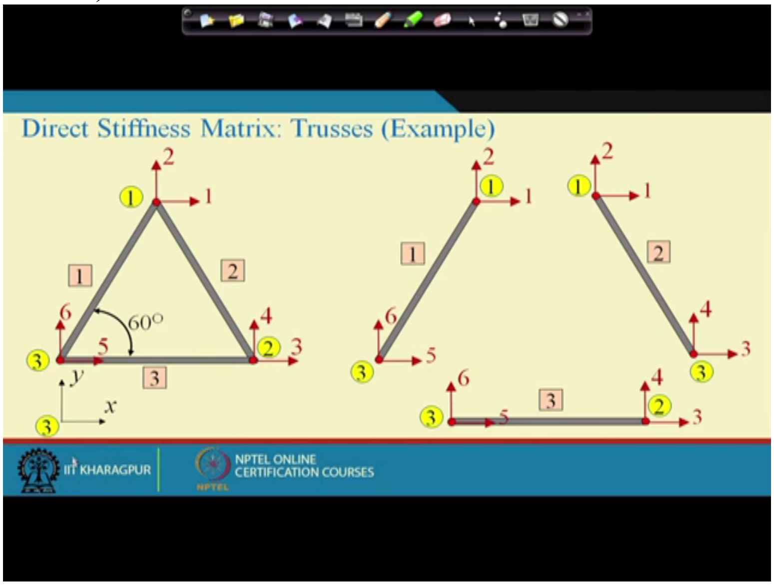

this is the coordinate system we take. x is equal to, x is in this direction, y is equal to this direction and origin we have taken, the node number 3 we have taken as the origin. This node number 3 is taken as the origin.

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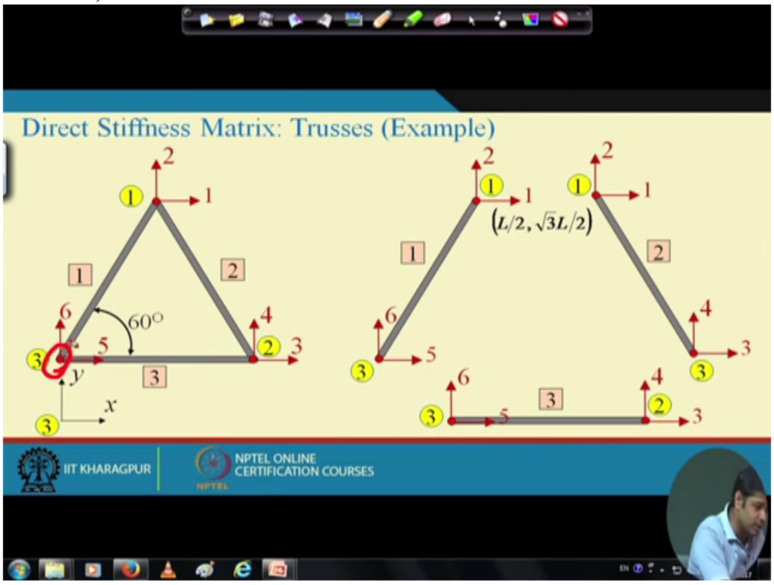

So therefore the coordinate of node number 3 is, coordinate of node number 3 is 0 0, 0 0

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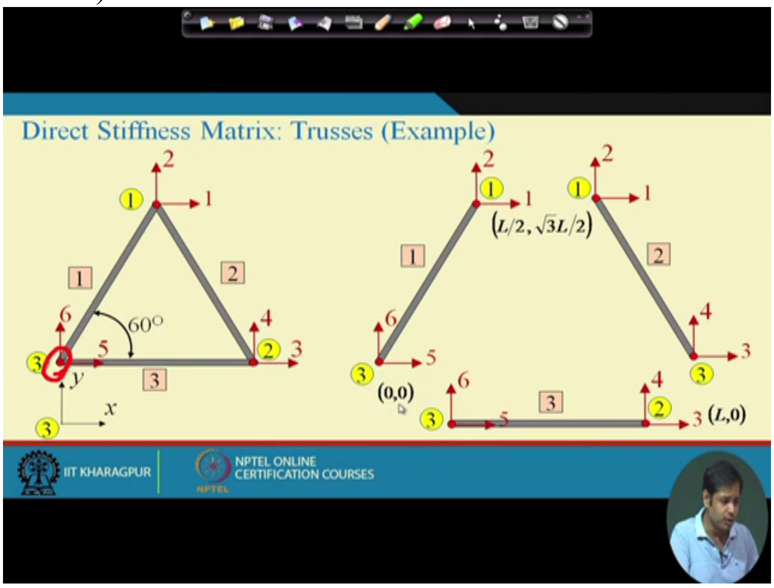

and then coordinate of node number 2 will be L 0, because this, all the lengths are L, these lengths are L, these lengths are L, this is

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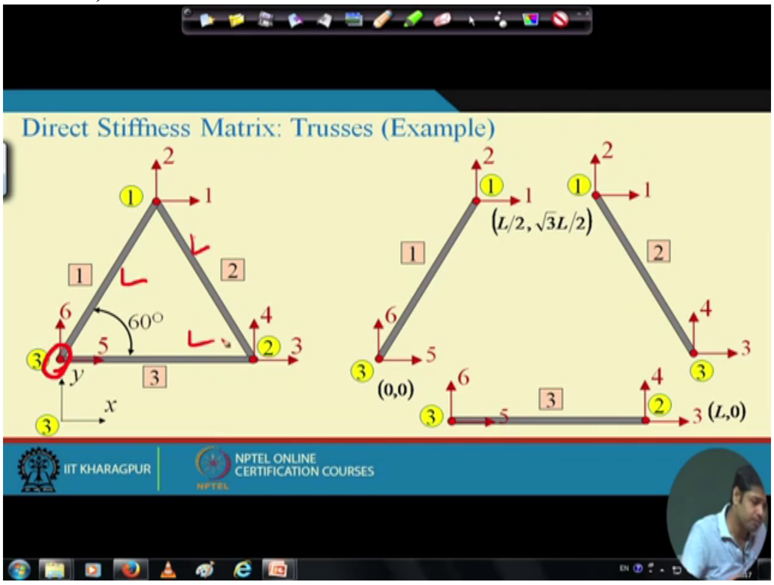

equilateral triangle and then coordinate of node, coordinate of point 1, joint 1 will be L by 2 root 3 L by 2, Ok.

Now next is, next if

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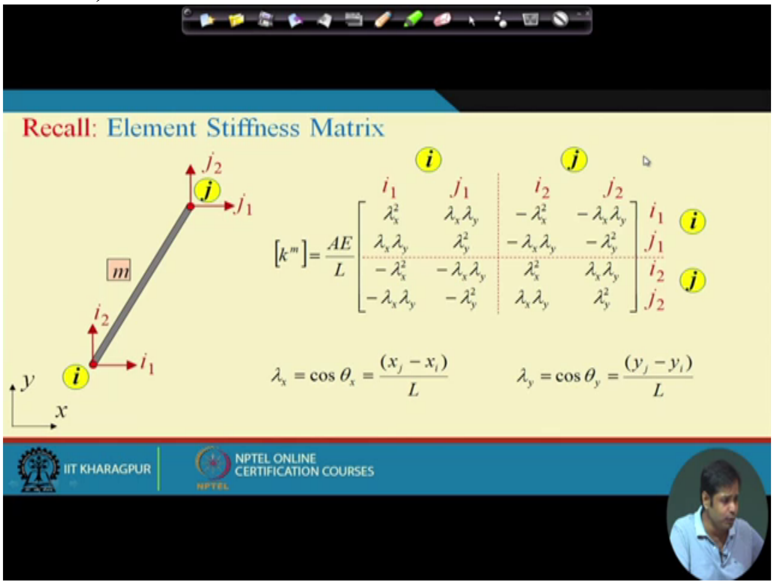

you recall that, for truss element if we take a member m which is connected between ith and jth point and the corresponding joint displacement at ith point is i 1, i 2 and j 1 and j 2, then the element stiffness matrix or the member stiffness matrix for this is A by L, L is the length of the member, A is the, A is the actual rigidity, it is the only deformation takes place actually so it is actual rigidity. Again we discuss why it is called actual rigidity because if A is more, then the stiffness will be more, deformation will be less. For, for the extreme case when A becomes infinite, just,

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just imagine then your stiffness becomes infinite and so there will be no deformation at all. The structure remember, behaves in a rigid way.

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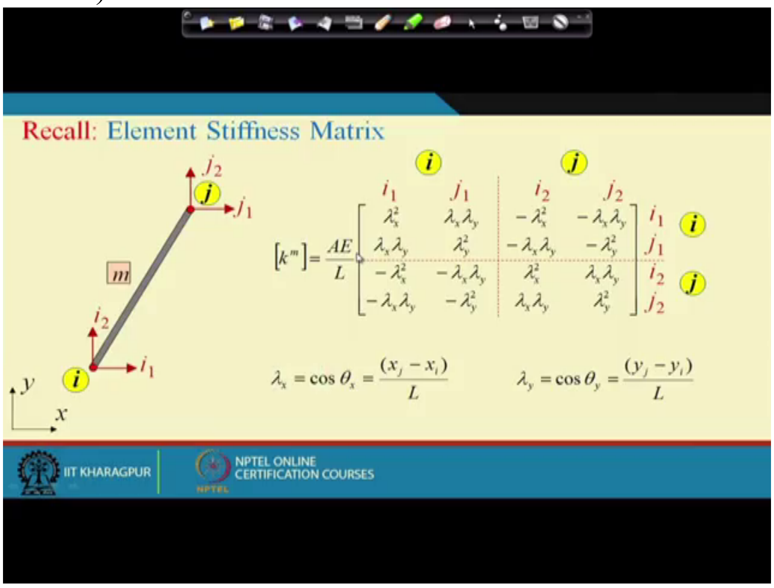

So that is why it is called rigidity.

Now and then this is the, this is the stiffness, this is the stiffness matrix and this is a symmetric stiffness matrix, positive definite stiffness matrix Ok and lambda x is essentially the, the angle that the member makes with x axis and the lambda y is the angle that the member makes with the y axis and if you write it in terms of coordinate this becomes this. So if you substitute lam/lambda for different members, if you substitute lambda x and lambda y into, into this, this expression we get the stiffness matrix.

Ok let's do that. Now we know that. Just now we have already, Ok, now let's do that.

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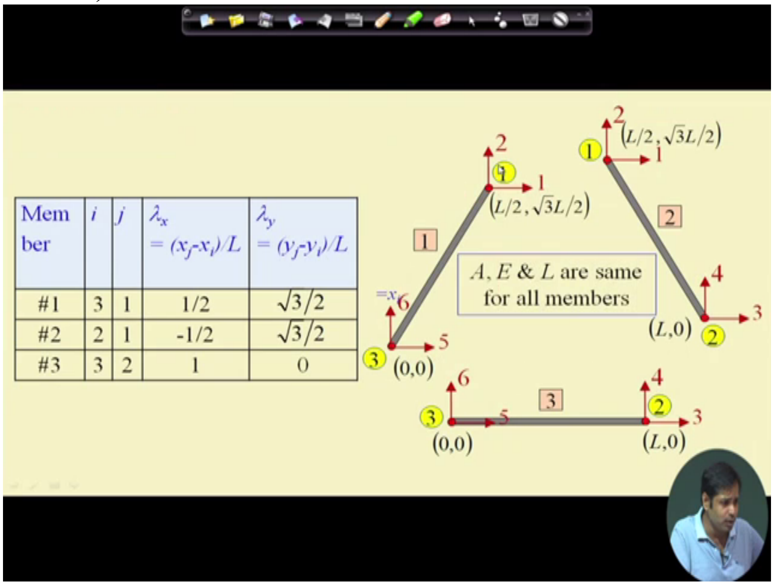

Member	i	j	$\lambda_x = (x_j - x_i)/L$	$\lambda_y = (y_j - y_i)/L$
#1	3	1	1/2	$\sqrt{3}/2$
#2	2	1	-1/2	$\sqrt{3}/2$
#3	3	2	1	0

This, this is the, these are the three members and these are the corresponding joints and to consider for this case, the cross-sectional area, Young's Modulus and length are same for all members. Then what we can do is, say member number 1, member number 2, member number 1 is connected between 3 and 1. So ith, i will be 3 and j will be 1.

This is very important, because you see you could have taken member number 1 connected between 1 and 3, means i is 1 and 3 is 1, and you are free to do that, but if you do that be consistent with the

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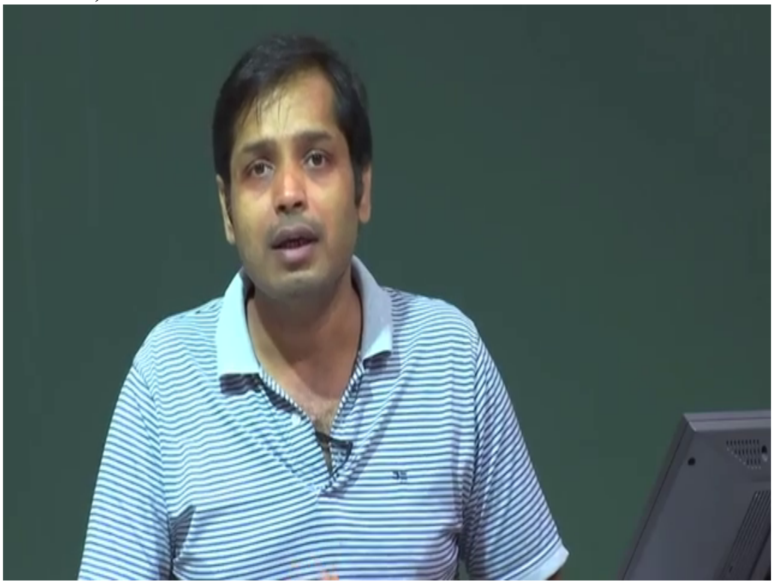

rest of the calculation, Ok.

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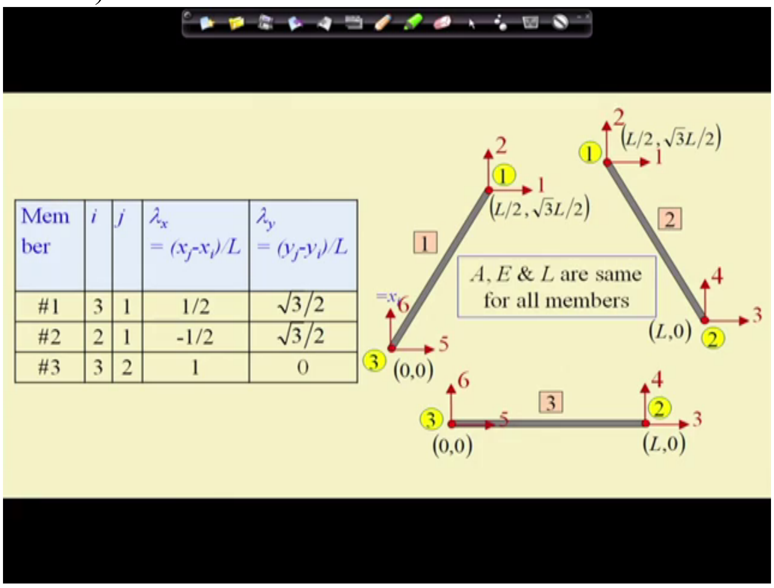

Now again the member number 2 is connected between 2 and 1, so your numbering is, in this case your numbering is in this direction, in this direction. In this case your numbering is

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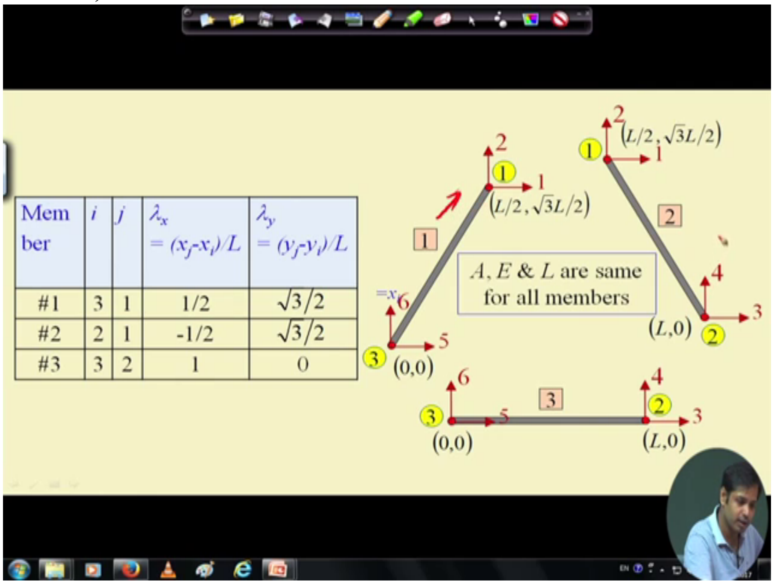

in this direction

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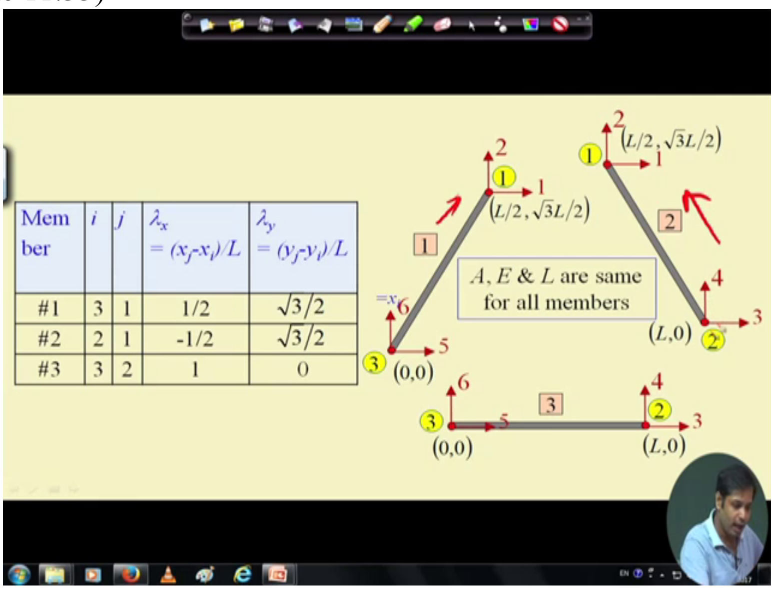

and finally member number 3 is connected between 2, connected between 3 and 2, so in this case your numbering is between this direction.

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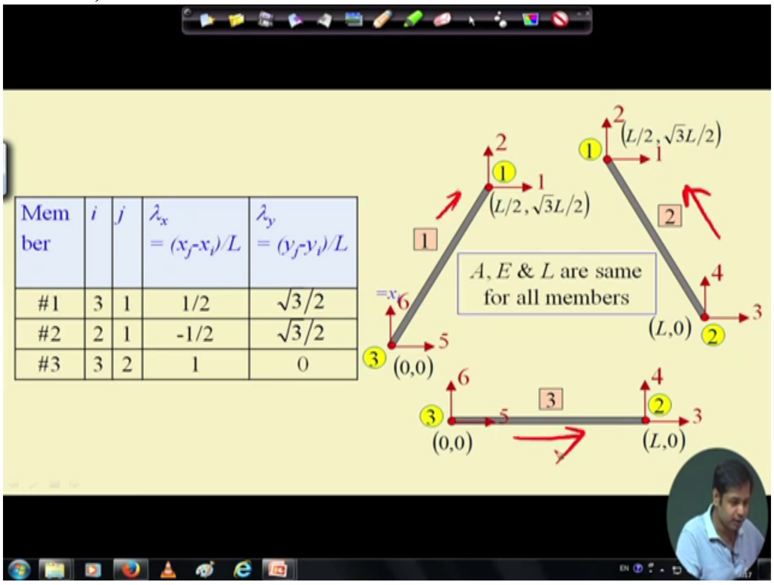

So for member 1 these become i, this is j. For member 2 this is i, this is j and for member 3 this is i and this is j. And we will be consistent with this notation throughout the calculation. And then you substitute x i, x j you get lambda x for different member this and lambda y for different members are this. Ok, now let us compute the, once you do that, then next what we need to do is, we need to apply, we need to substitute this lambda into this

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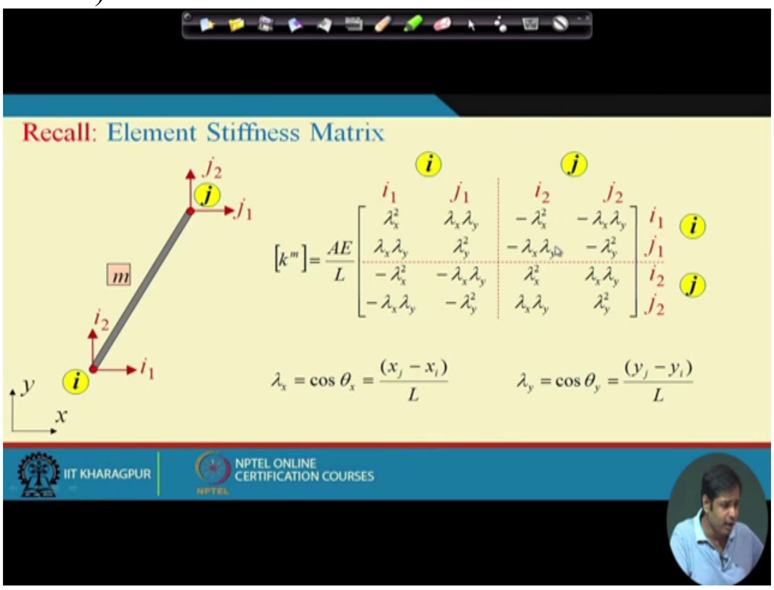

expression.

And if we get for different members, we have different lambda x and lambda y and then we get the element stiffness,

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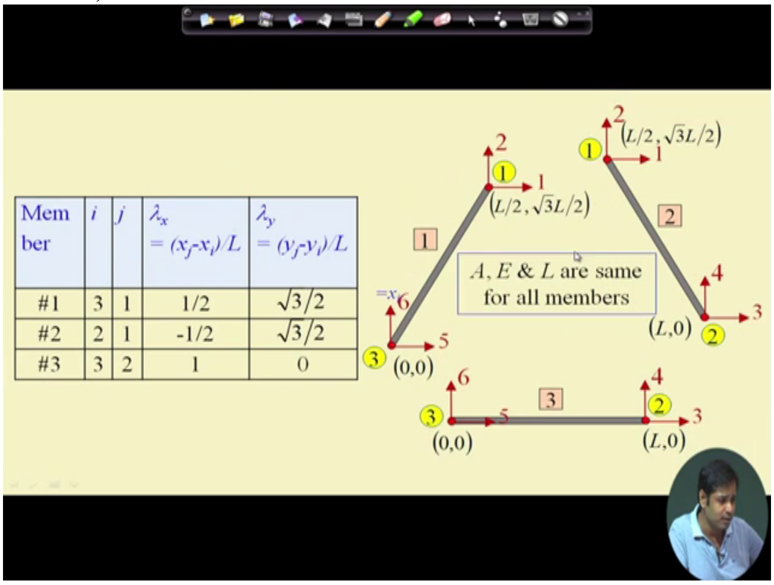

member stiffness matrix for different members and these matrices will be this.

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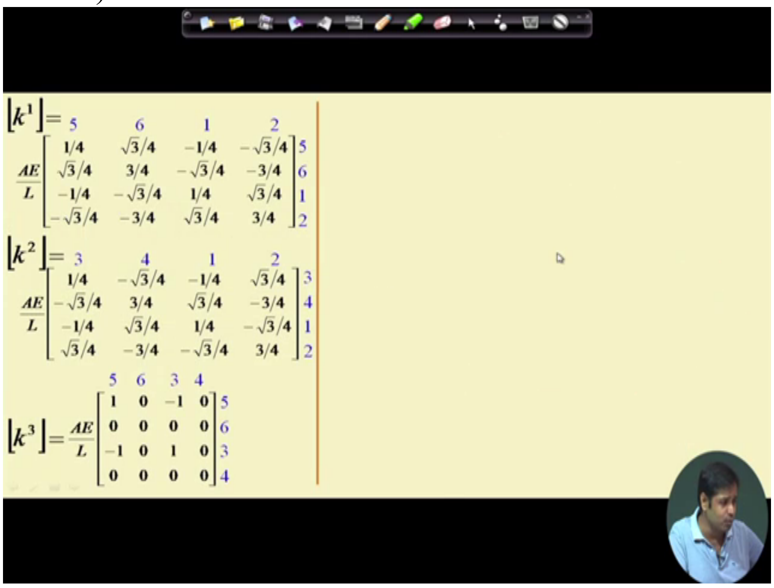

Now you see, this, this numbering is important here. Now you, you remember, remember it was, our, our truss was like this, Ok. Truss was like this and our degrees were, this was 1, this was 2 and this was 3, this was 4 and again this was, this was 5 and this was 6.

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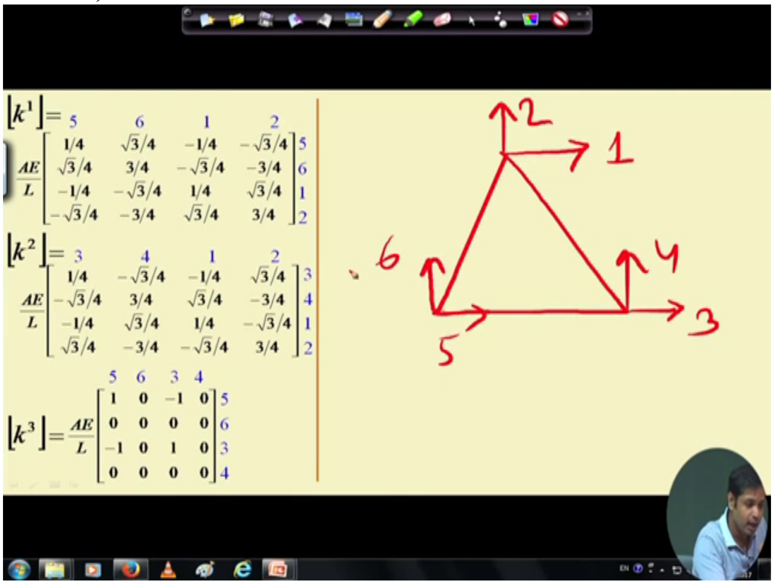

And this is member number 1, member number 2 and member number 3,

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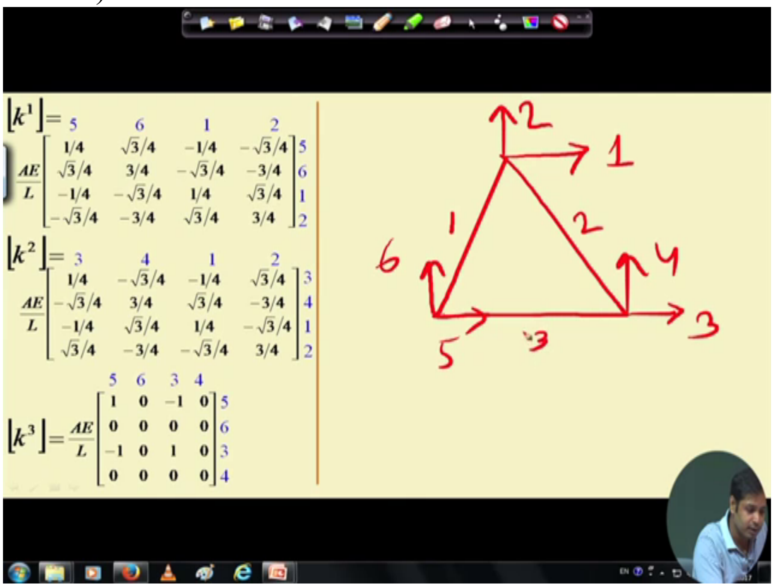

Ok member number 1, member number 2 and member number 3.

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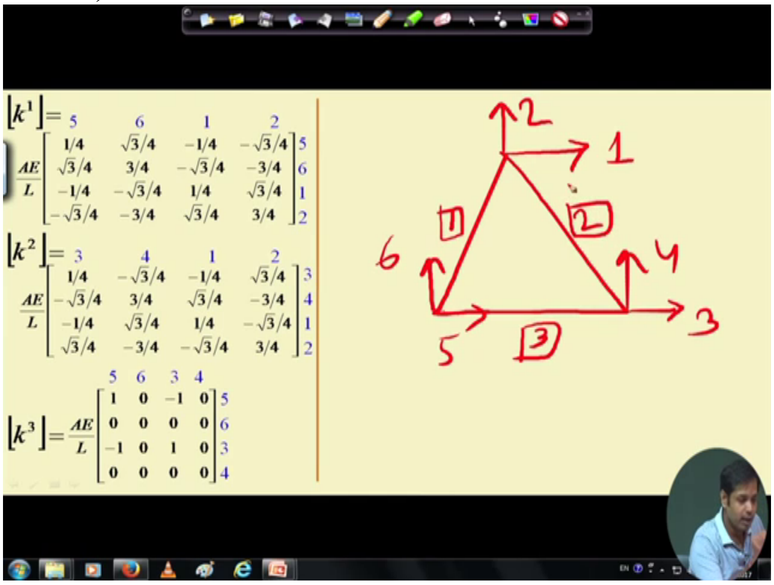

And in member number 1, we went in this

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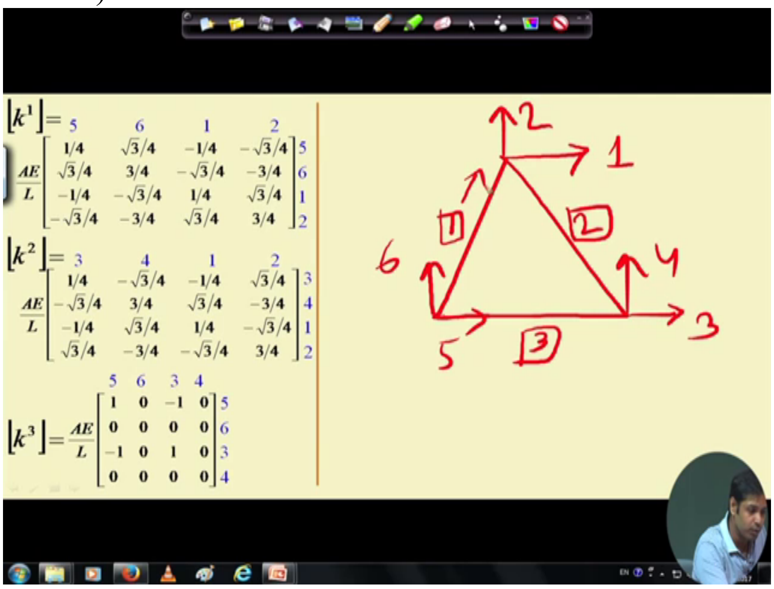

direction numbering and in this

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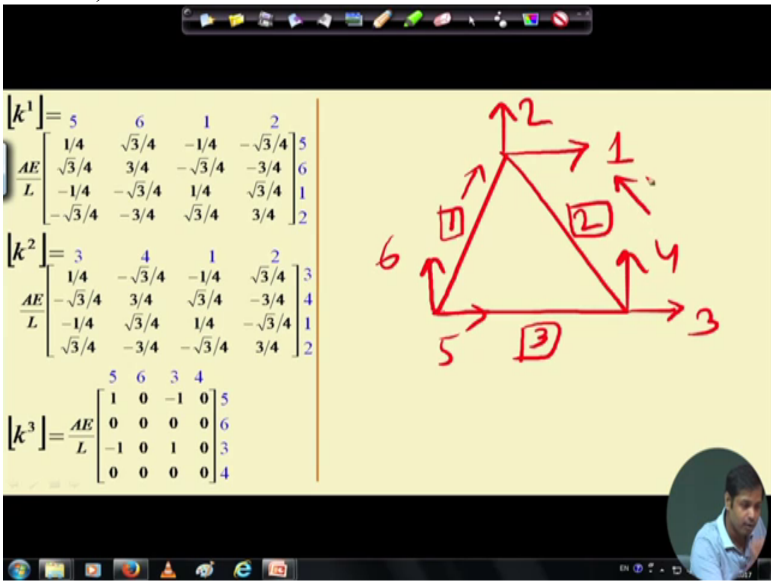

direction and in this direction.

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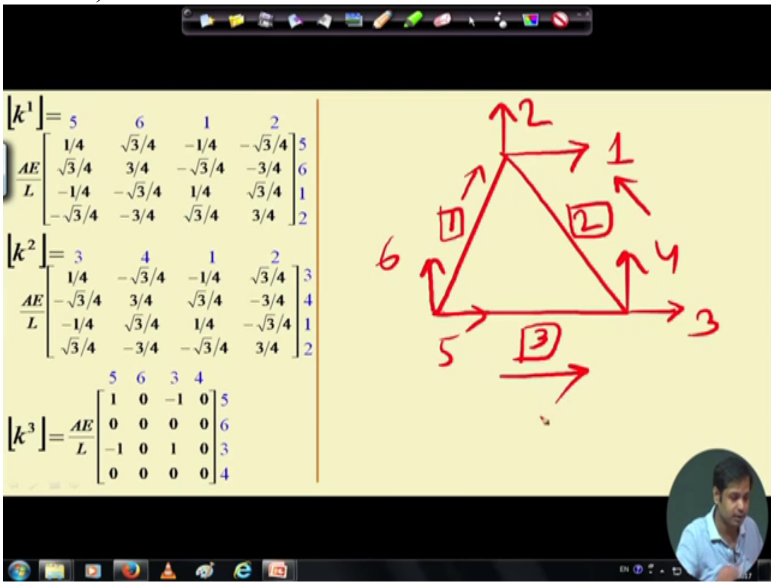

So member number, no then the, the, the stiffness matrix for, stiffness matrix for point number, stiffness matrix for point number 1 will be, then what are the degrees of freedom, we have 5, 6 and 1 and 2, Ok. And corresponding element will be this, this will be 5, this will be 6, this will be 1 and this will be 3.

The first column corresponds to 5, the second column 6, first column, because this is i, this is for node i and this is for node j.

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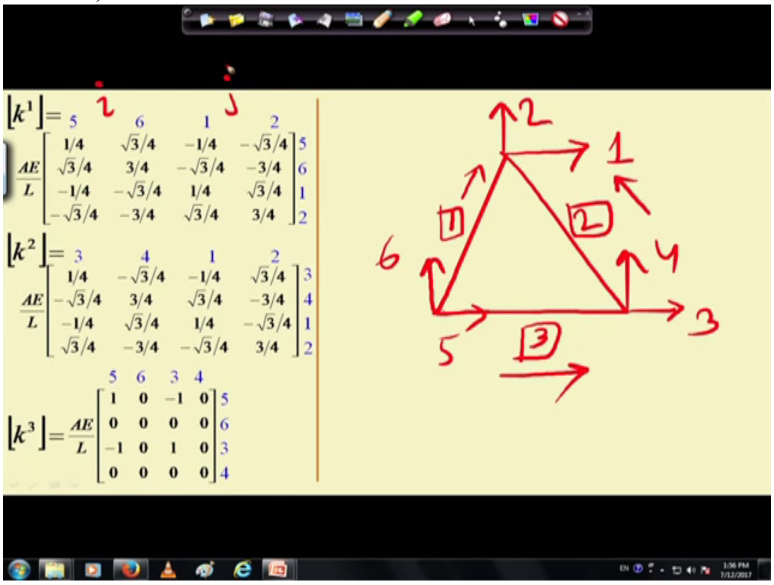

Since for this member we are going in this direction, node i becomes, node i is this point is the node i, and this point is the node j. So first 2 column will be the degrees of freedom at node i and second will be degrees of freedom at node j. The first node, i degrees of freedom are 5 and 6 at node j 1 and 2. So that's why it is 5, 6, 1, 2 and similarly this is 5, 6, 1, 2, Ok. Ok for member number 2, member number 2 you went in this direction so we have 3, 4 and then 1, 2. 3, 4, 1, 2; 3,4,1,2 and member number 3 we went from this to this, so it is 5, 6, 3, 4. So it is 5, 6, 3, 4; 5, 6, 3, 4.

This is very important because if this is wrong

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your assembled stiffness matrix will be wrong, Ok, so be careful about it.

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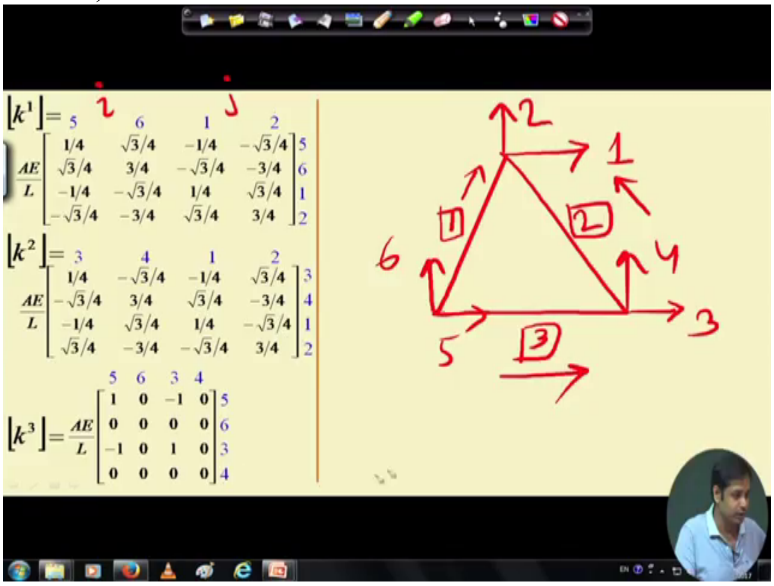

Now, so once we, once we have that what we have to do is we have to assemble them. Now every member, the, for every member

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here stiffness matrix is 4 by 4 stiffness matrix because every member has 2 joints and 2 degrees of freedom per joint, total degrees of freedom per member is 4. Therefore we have 4 by 4 stiffness matrix.

Now if you look at the actual structure, the actual structure

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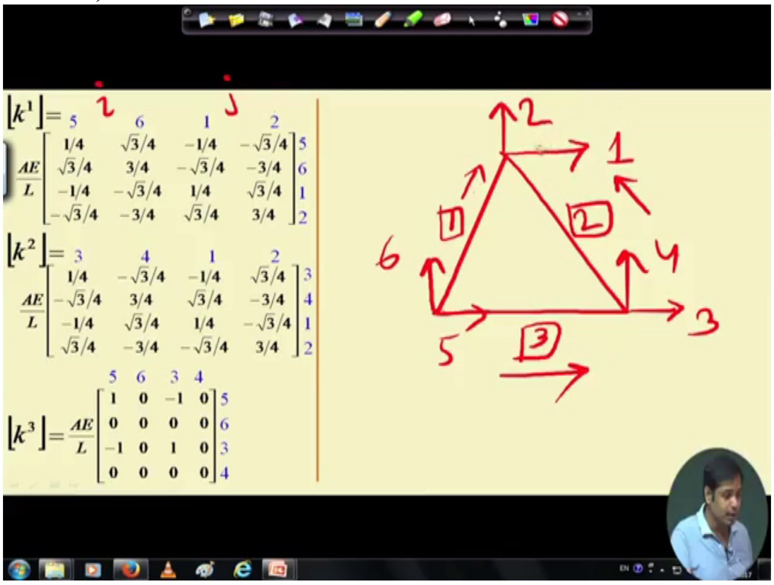

stiffness matrix, stiffness matrix will be 6 by 6 because it has 6 degrees of freedom, Ok. Now the assembling is a very, very important step in this, in this process. Then how to do this assembly? Now if you, if you, if you are writing a code, then also you can follow this process. The process that we are using for manual calculation as well. The first thing we know that the stiffness matrix is, stiffness matrix will be 6 by 6.

First you get a stiffness matrix, initialize a stiffness matrix which is, which is, 1 2, these are the

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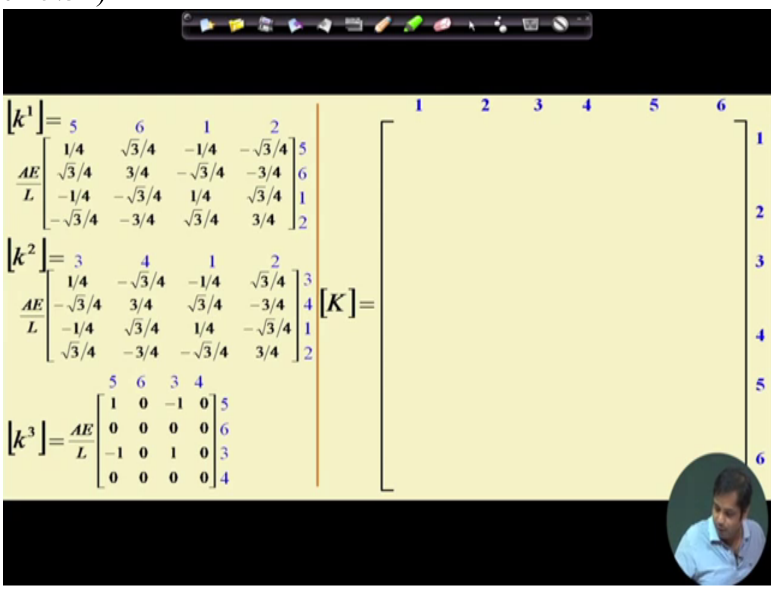

corresponding degrees of freedom, Ok. This is 1, 2, 3, 4, 5, 6. Also write 1, 2, 3, 4, 5, 6. So first column is for degrees of freedom 1, second for 2, and correspondingly first row for 1, first and second for 2 and so on. And now initialize them all, initialize them. So this is for what, this is your

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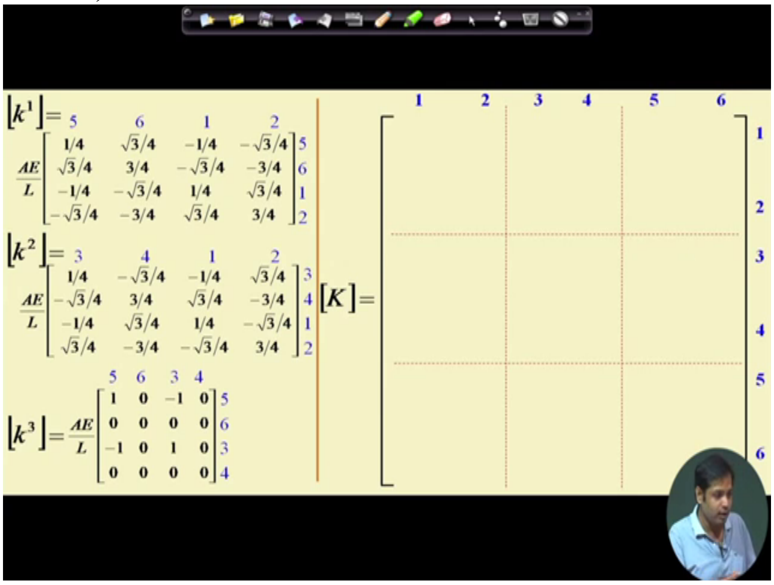

1,2 means it is for joint number, it is for joint number 1,

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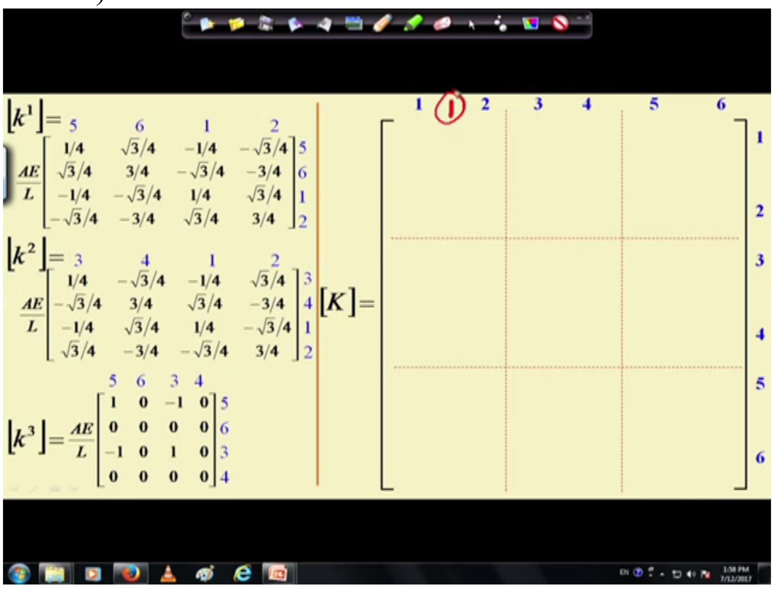

and 2,3, 4 means it is

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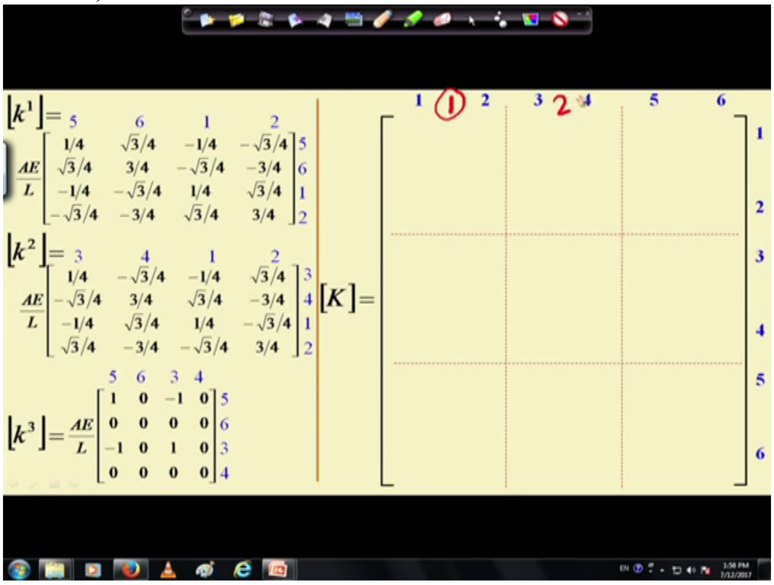

for joint number 2 and this is for joint number 3.

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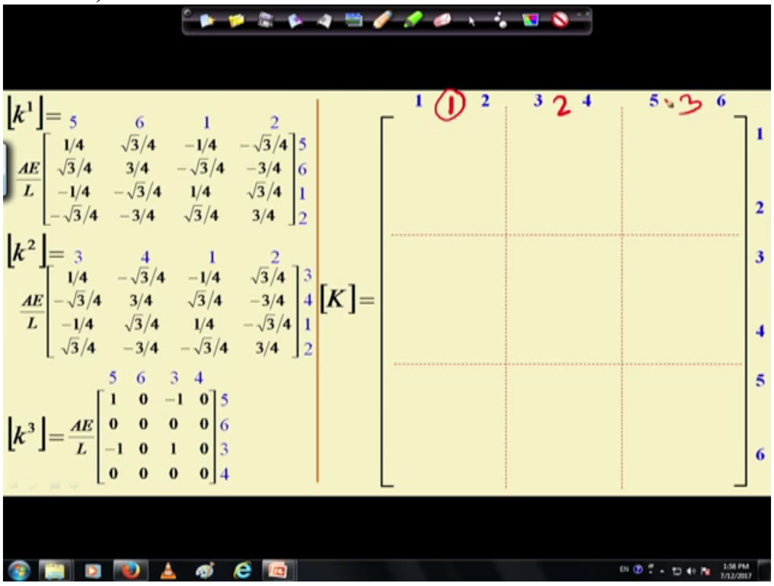

Similarly this is for joint number 1, this is for

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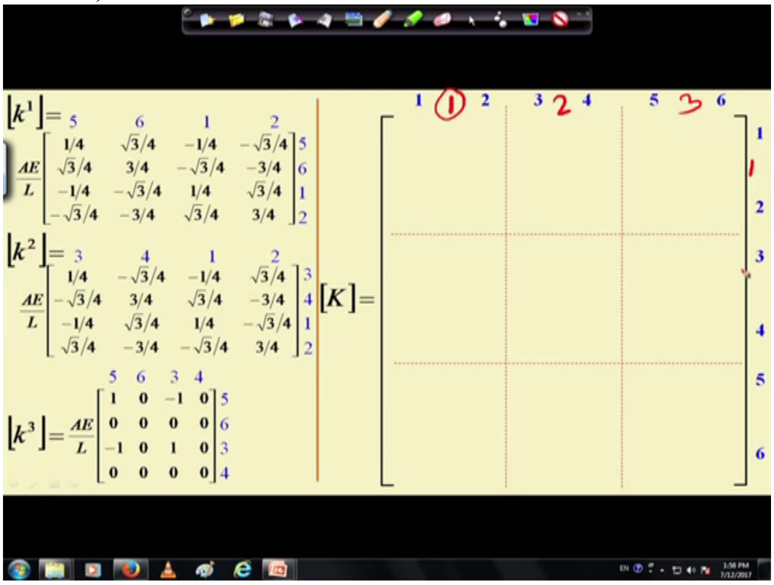

joint number 2, and this is

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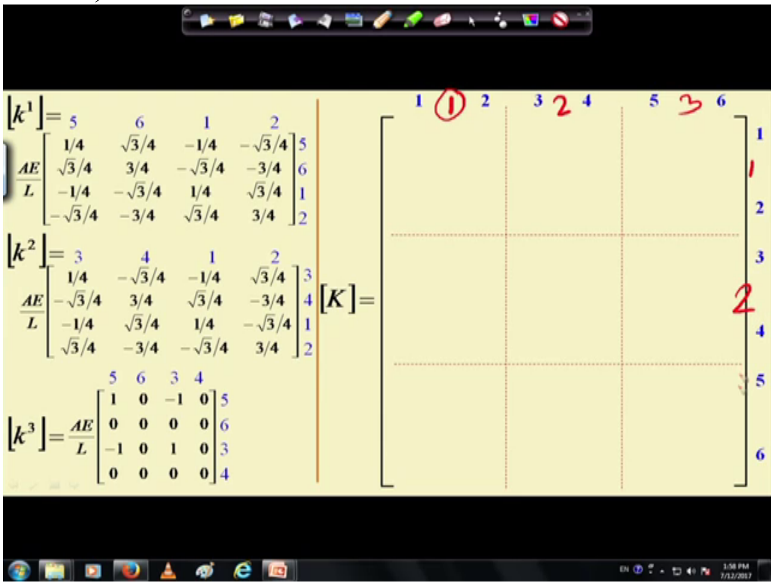

for joint number 4.

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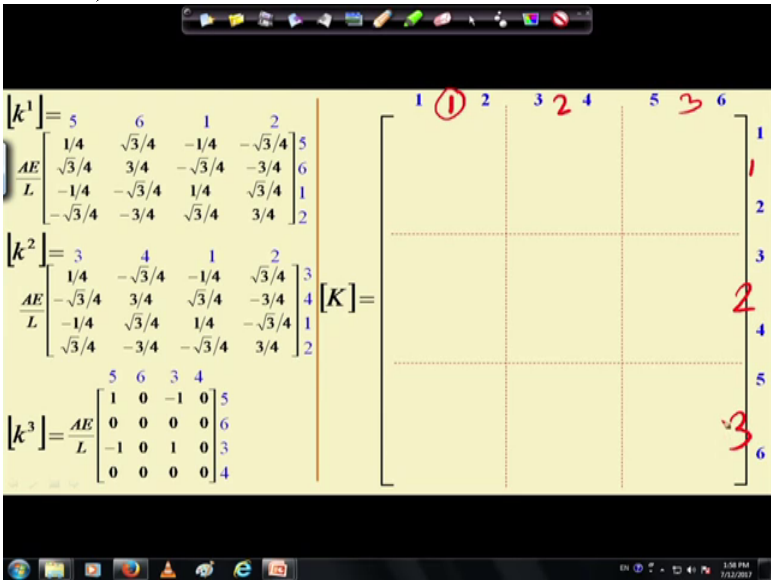

So this block is essentially joint number 1, joint number 1. This block will, essentially will be the how the joint number 2 and joint number 1 are, are related to each other. And this will be how the joint number 3 and joint number 1 is relating to each other. Similarly this gives you joint number 2 and joint number 1, this joint number 2 with joint number 2, joint number 2 with joint number 2, this is joint number 3 with joint number 2, and this is joint number 1 with joint number 3 and so on, Ok.

Now next is we, we initialize

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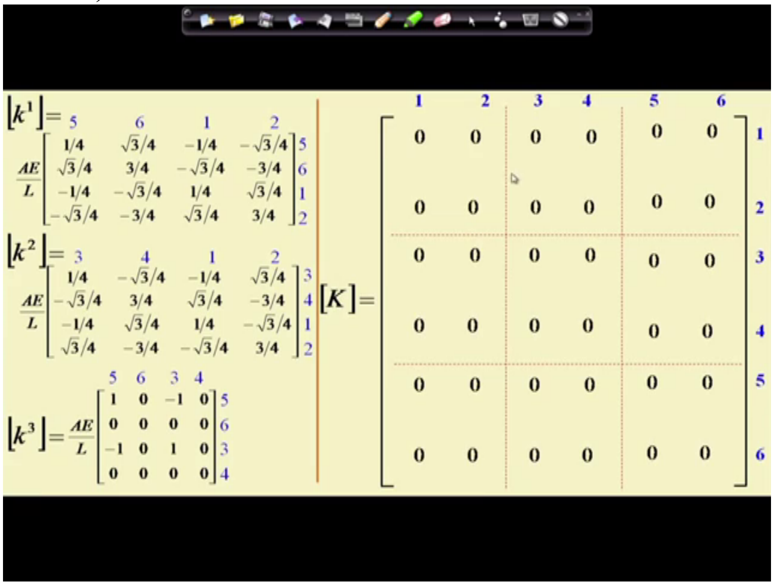

all the values as zero. Initially all the values

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are zero. Now then what we do,

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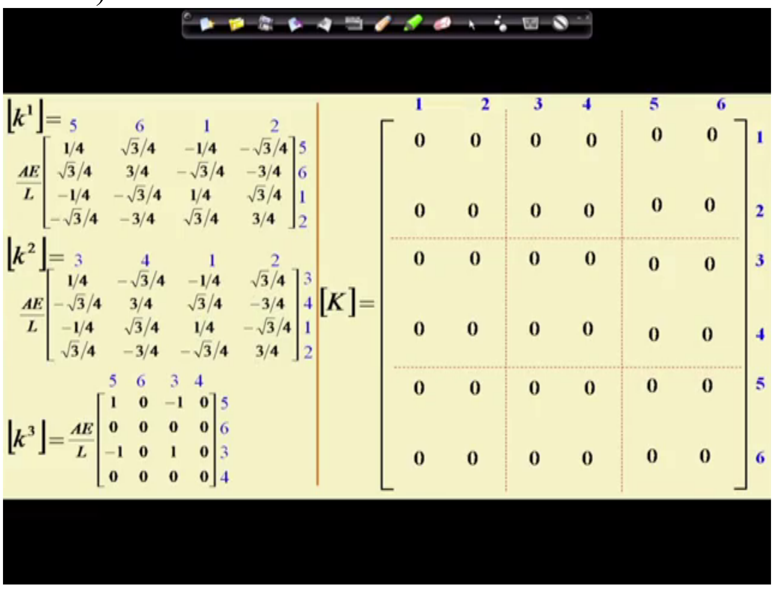

we have 9 blocks here, right, 9 blocks here. What we do then, we will assemble, we look at each block separately and then, and then substitute them, Ok. Now let us first see block 1, block first. The block first is what, block first we have 1, 2 here and then 1, 2 here. So what are the things we have in 1, 2 and 1, 2? Here we have, this is 1, 2, this is, this is 1, 2 versus 1, 2.

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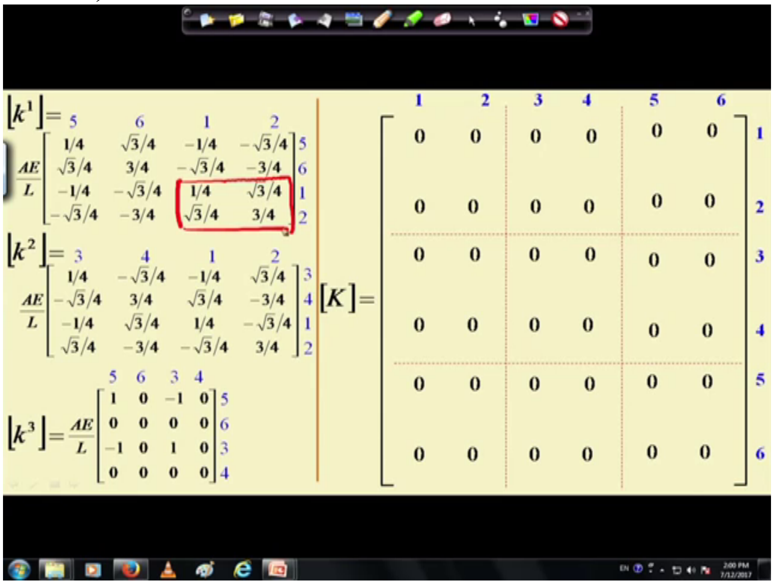

This is 1, 2 versus 1, 2. And then this is also 1, 2 versus 1, 2.

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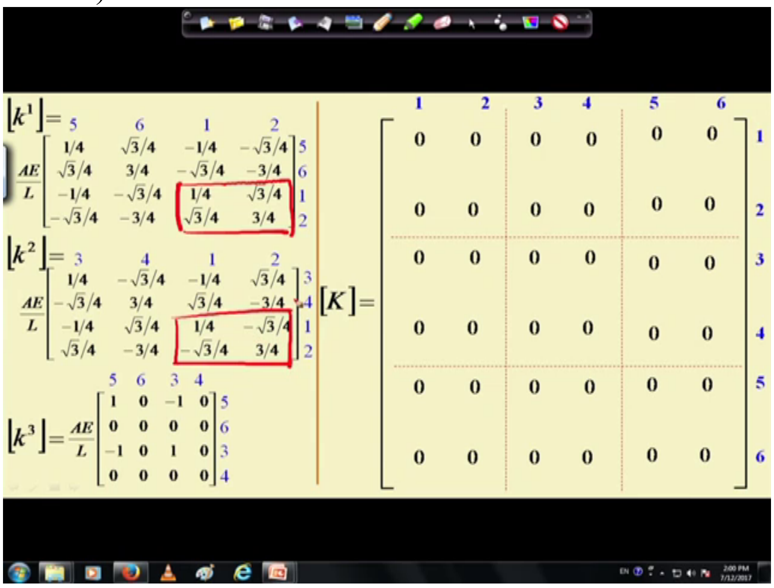

There is no 1, 2 versus 1, 2. So this will be, this plus this, so this block

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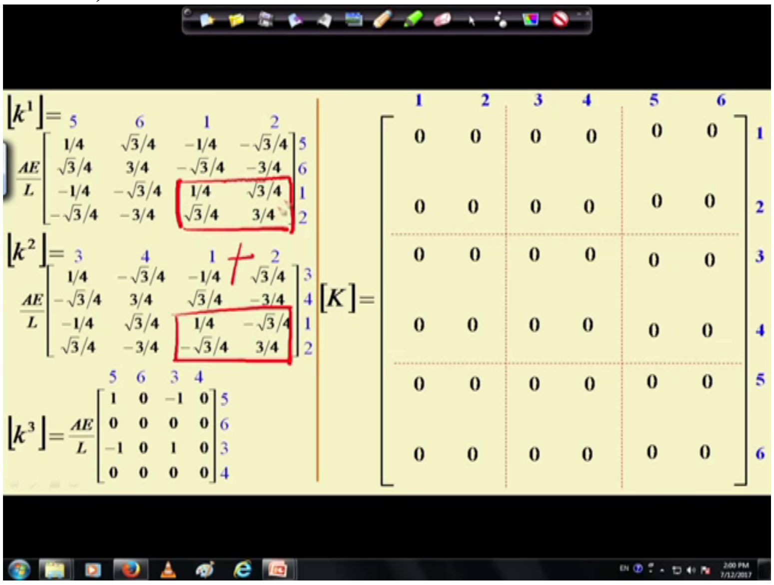

plus this block goes here. So this will be 1 by 4 plus 1 by 4 and there is root 3 by 4 minus root 3 by 4, root 3 by 4, 3 by 4 plus, plus 3 by 4.

And this becomes,

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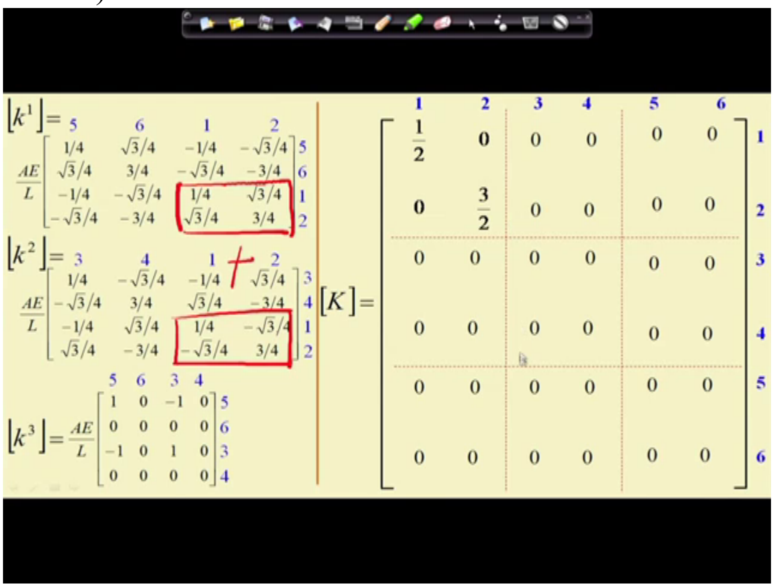

this becomes this, you see. So this is, becomes 1 by, 1 by 4 plus 1 by 4, half, this is for block number 1, block number 2. Now similarly let's see for block number, block number, we can see for block number, let's see this is, this is, this is 1, 2 and 3, 4. 1,2,3, 4 means again there is no 1,2,3, 4 here. In this case, in this case, this is 1,2, this is 3,4 so the column is 3,4 and row is 1,2. Column is 3, 4

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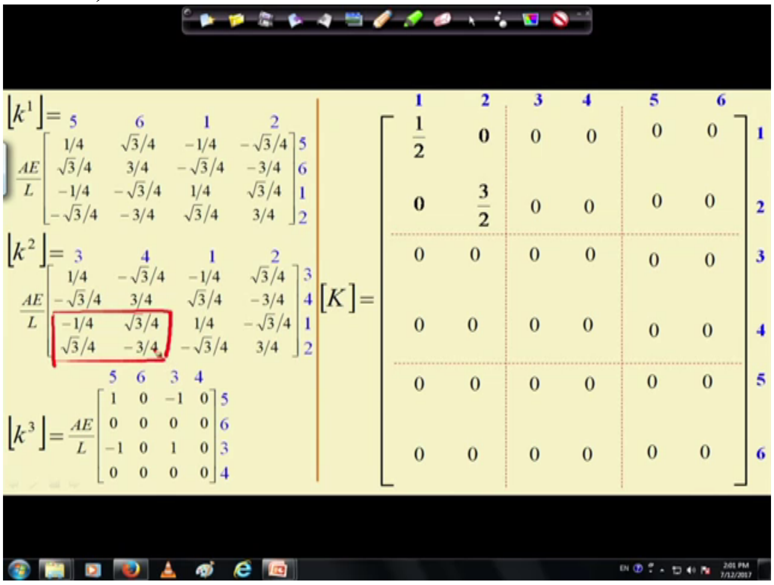

and row is 1, 2 and here also there is nothing 1, 2, and 3, 4.

So this, this block

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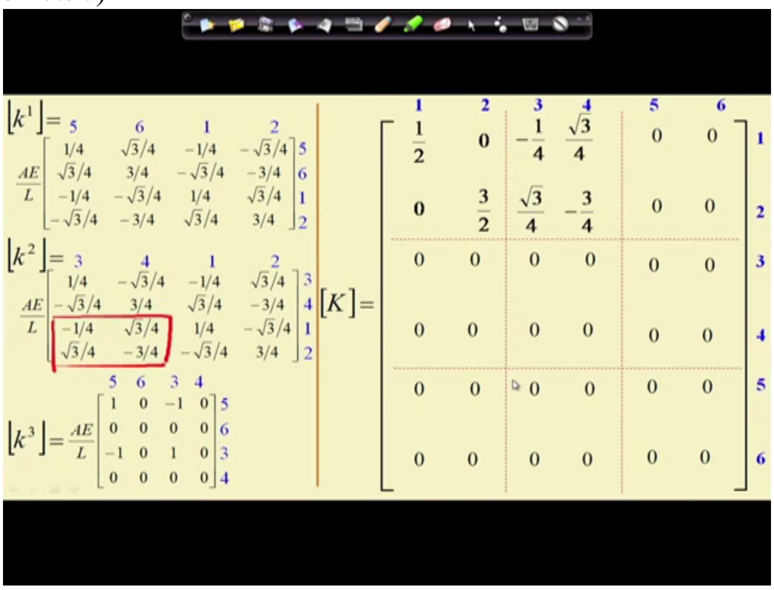

goes, this block directly

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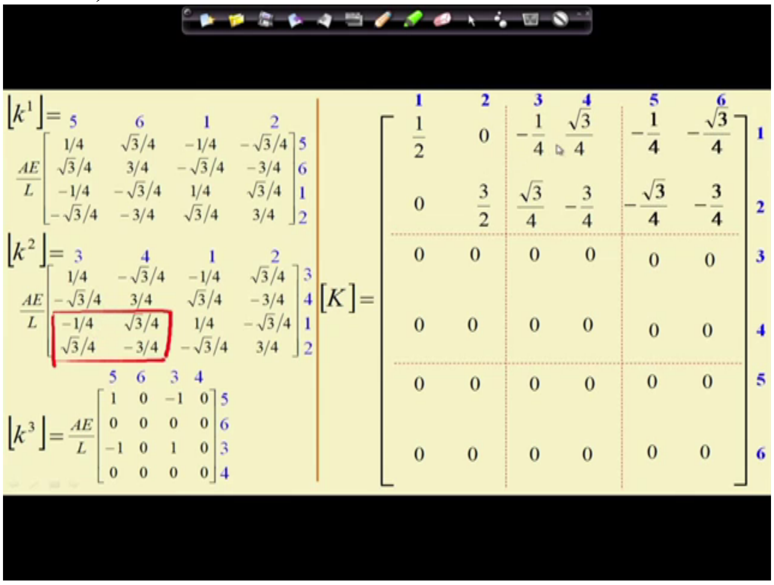

goes here, Ok.

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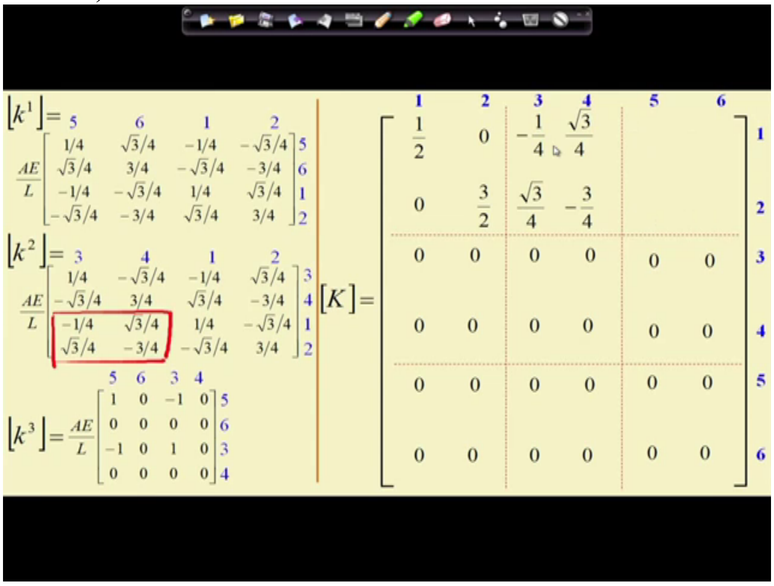

These blocks directly goes here. Similarly it is 1, 2 and 5, 6. 1, 2 and 5, 6, column is 5, 6, column is 5, 6, row is 1, 2. So this is the block,

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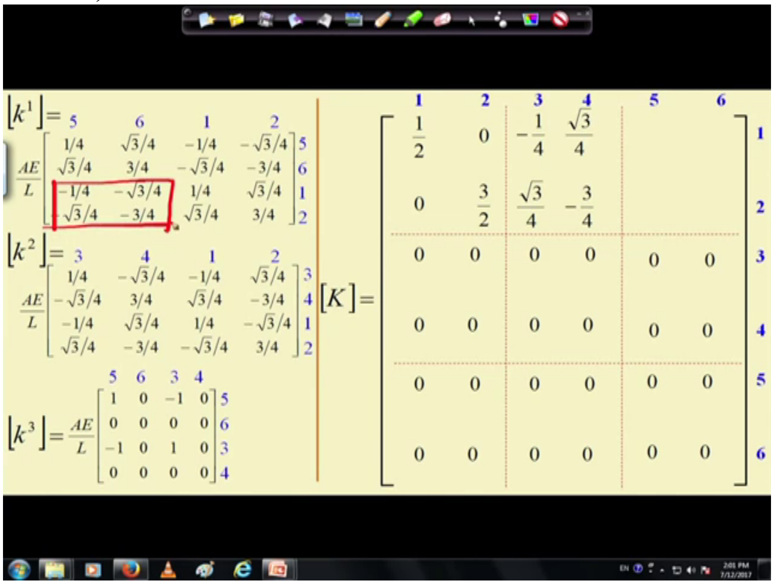

column is 5, 6, row is 1, 2, is there anything? Column is 5, 6 row is, nothing. So this directly,

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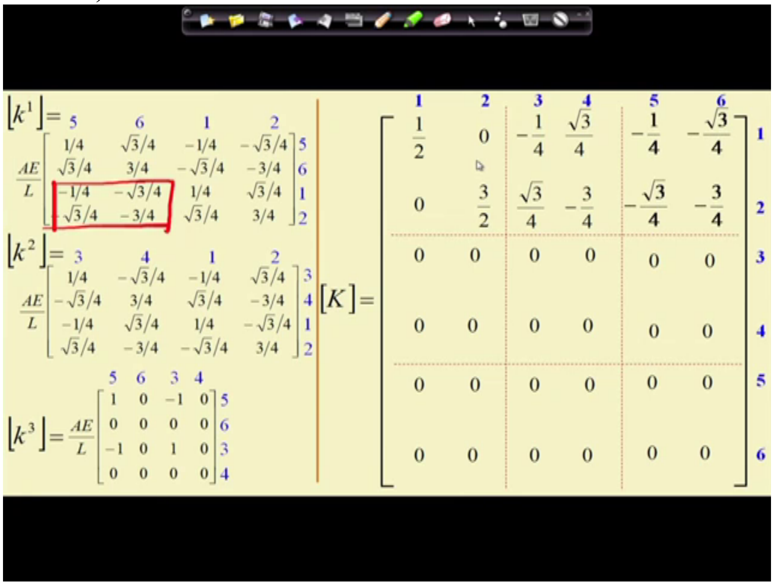

this part directly, this sorry,

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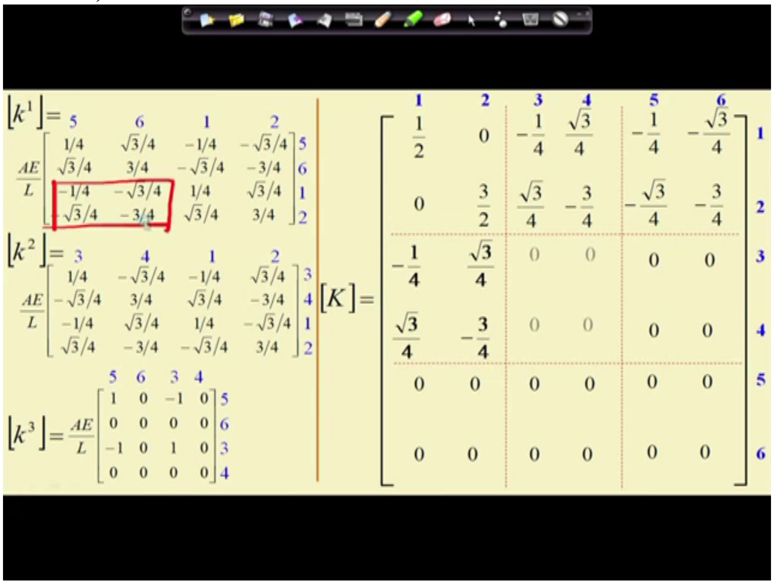

let us not,

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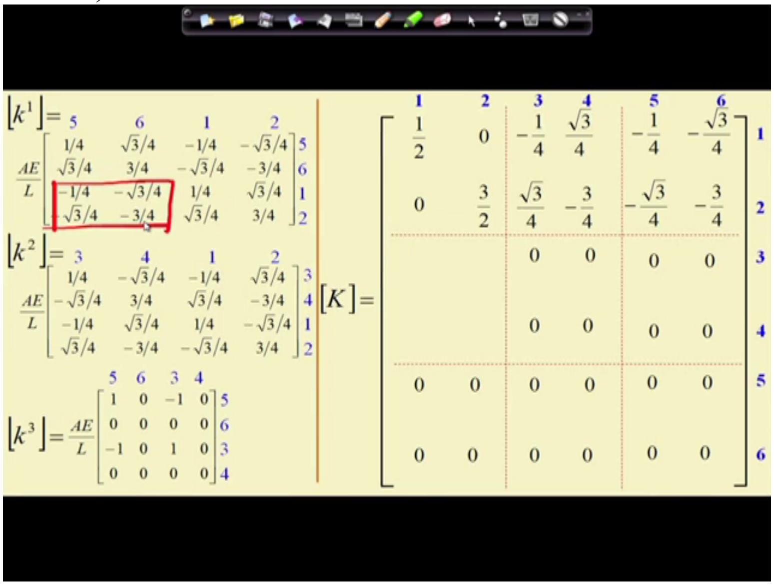

Ok, this part directly goes here,

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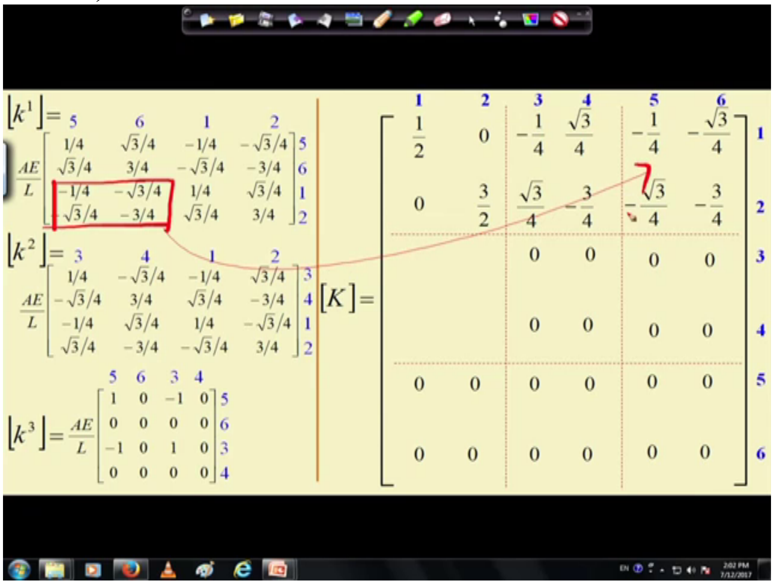

Ok.

Now similarly let us see, let us see this block. This block,

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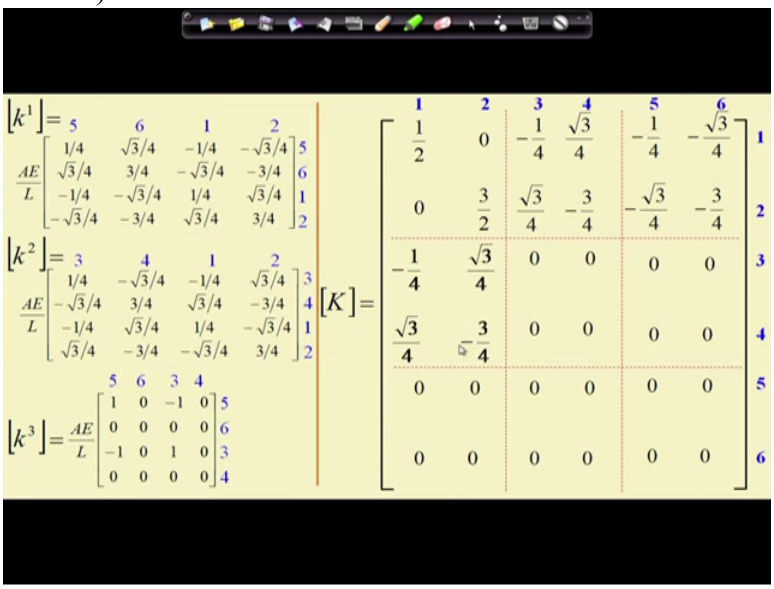

this block is what, this block is, this block is 1,2,3,4. 1, 2 there is nothing 1,2,3,4. Row is 1, 2, column is 3, column is 1, 2, row is 3, 4. So column is 1, 2, row is 3, 4. Column is 1, 2, row is 3, 4 is this part.

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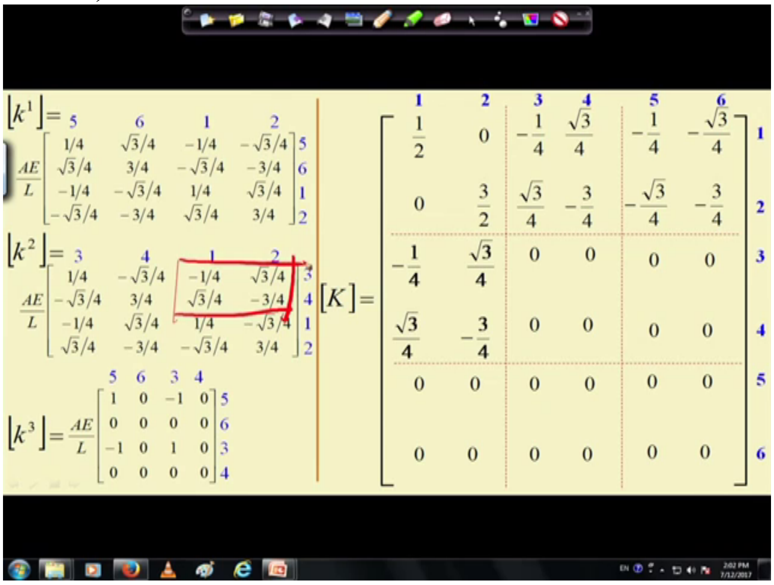

So this directly goes here

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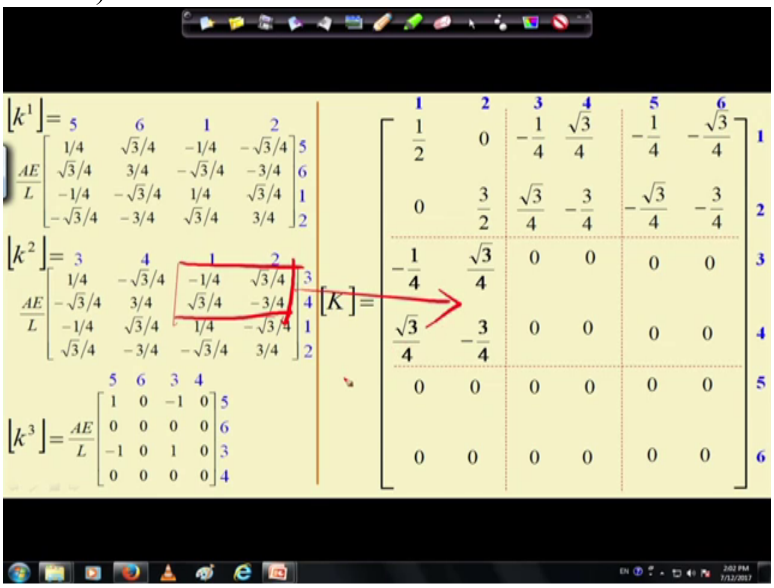

Ok. And then next is, next is column is 3, 4, row is 3, 4.

Now this is, column is 3, 4, row is 3, 4, this part and

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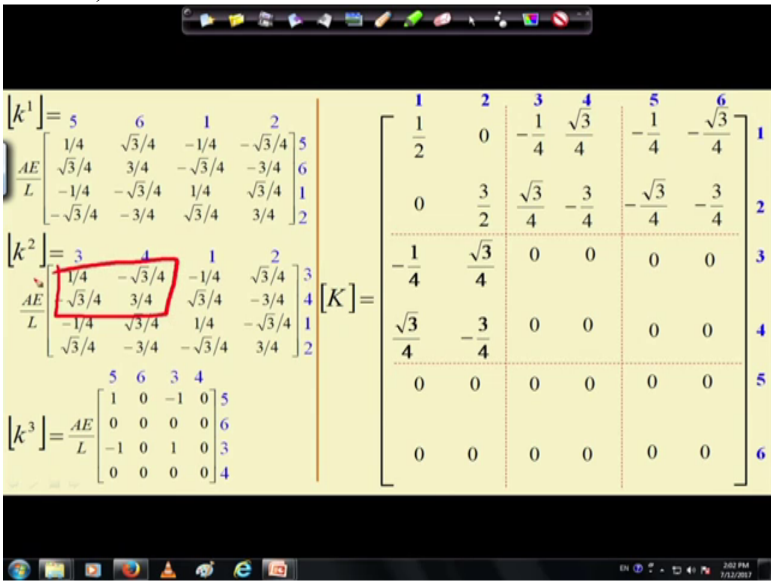

anything else, yes, column is 3, 4 and row is 3, 4 this part.

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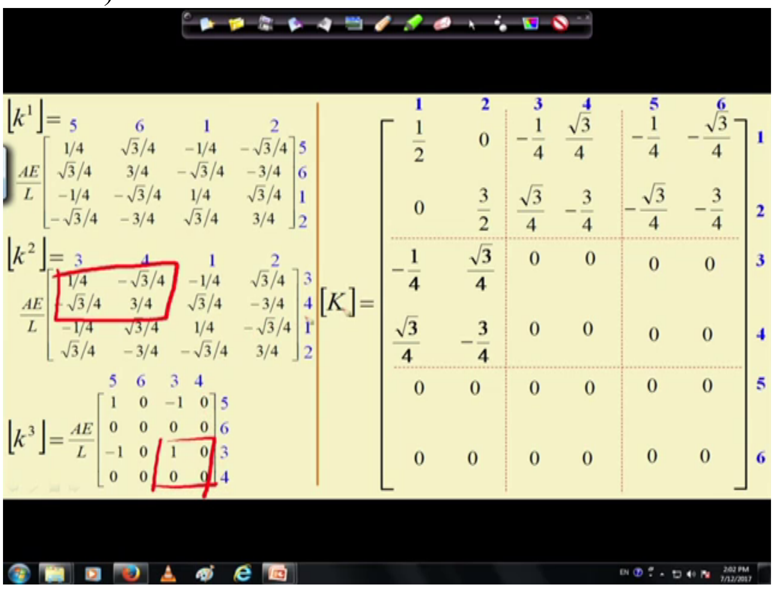

So this will be this plus this. So 1 by 4 plus 1 means this is 5 by 4 and then, and then it becomes,

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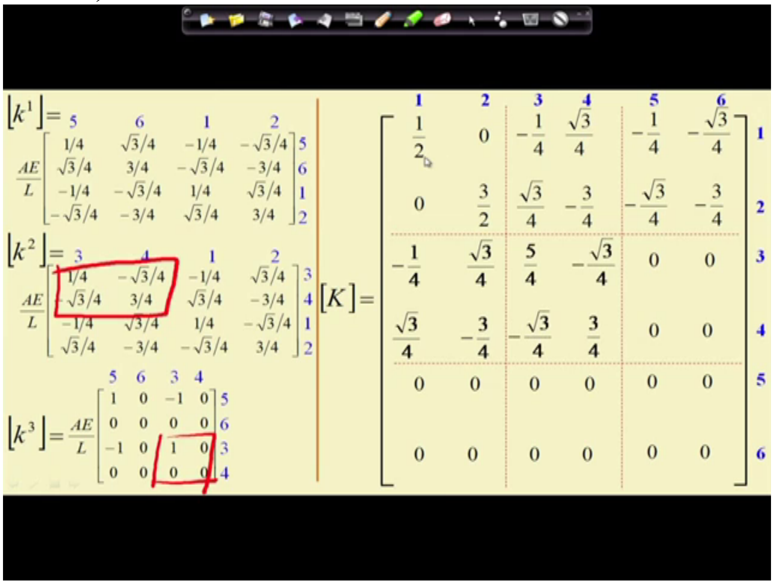

, it becomes this, Ok.

Now then this part will be, column is 5,6 row is 3,4, column is 5,6 row is 3,4, column is 5,6 row is 3,4 only this part, Ok

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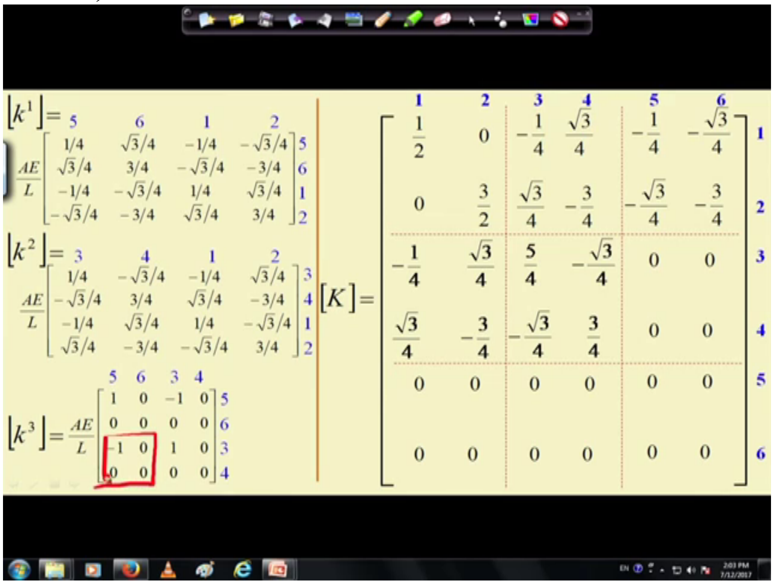

So this part goes here, this part goes here. And this is 1, 2 and 5, 6 and similarly if you take from this, you will get this part is this

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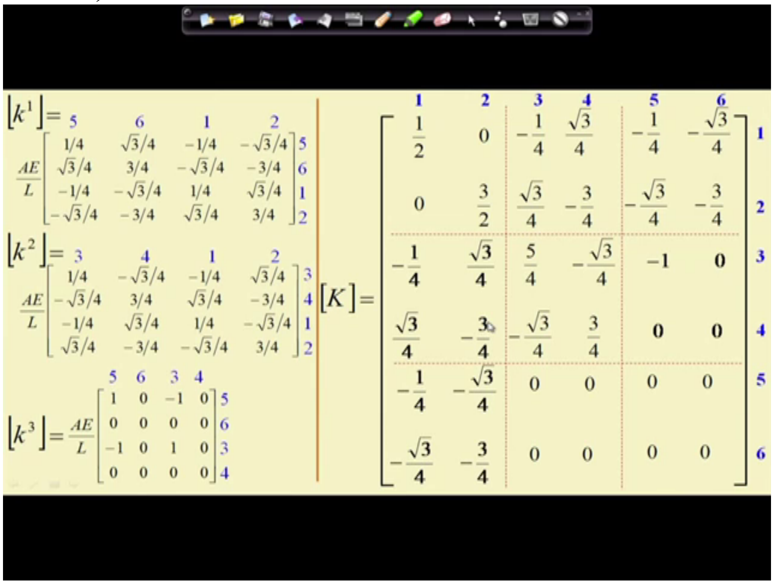

and then this is 3, 4 and 5, 6, this part is this

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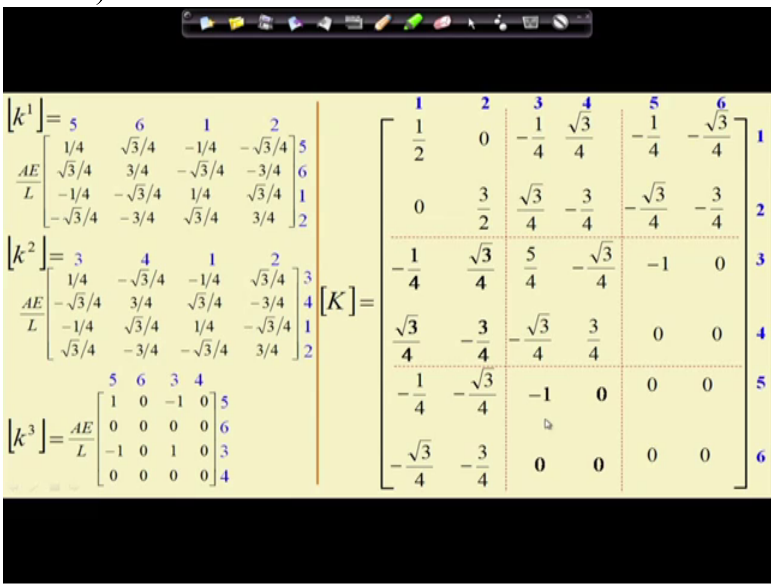

and corresponding

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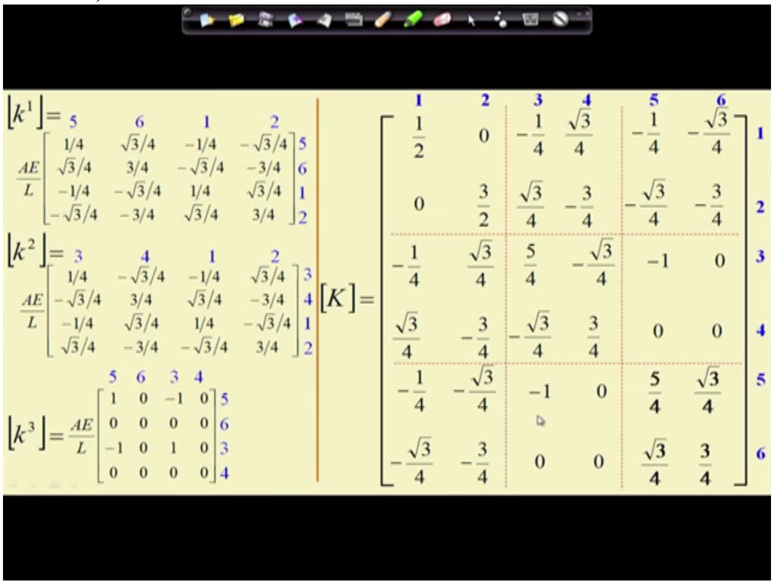

this part is this.

Now you can check that, we check all these diagonal terms are positive, yes

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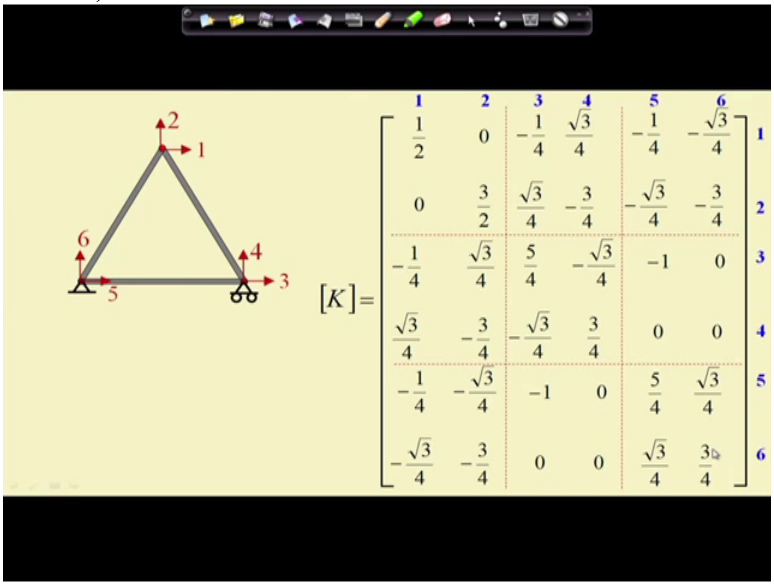

they are positive, fine and

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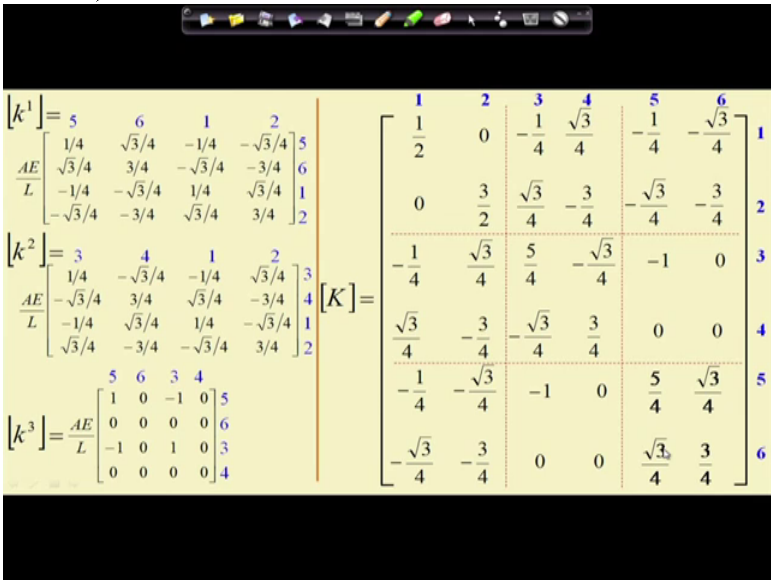

then the matrix is symmetric, check matrix is symmetric or not, matrix is symmetric. Now one exercise first you do is you try to find out determinant of this matrix, Ok. Find out

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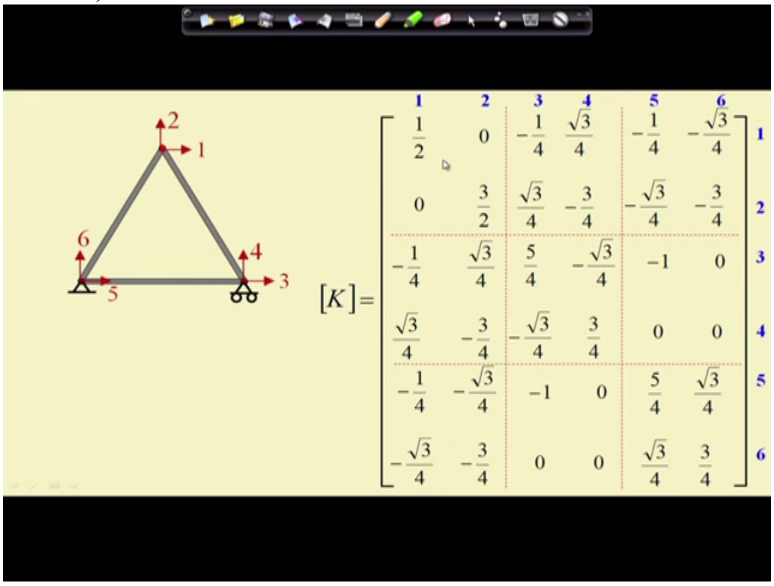

the determinant of this, of this matrix. You find out determinant, you find out determinant

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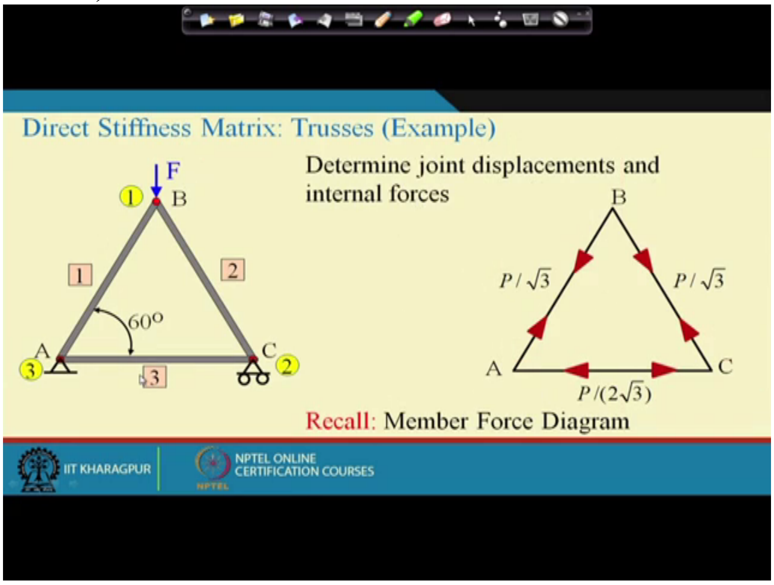

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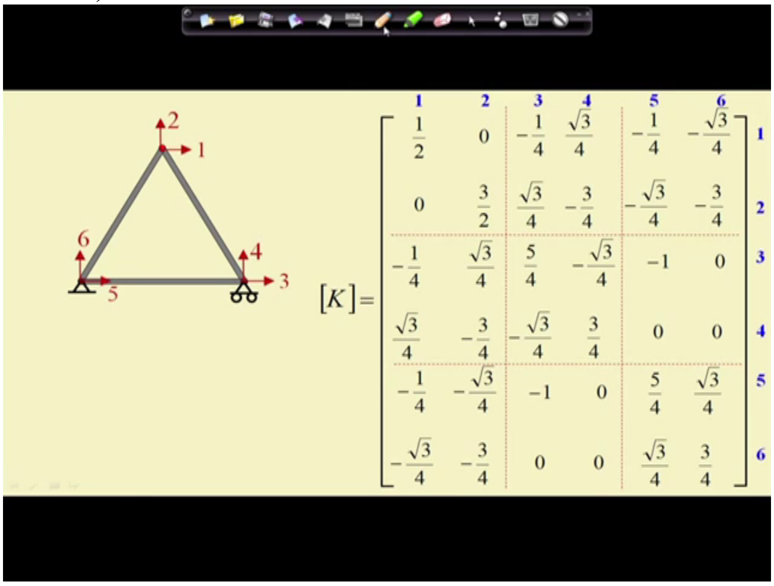

of this matrix K, determinant of this matrix K and you check what

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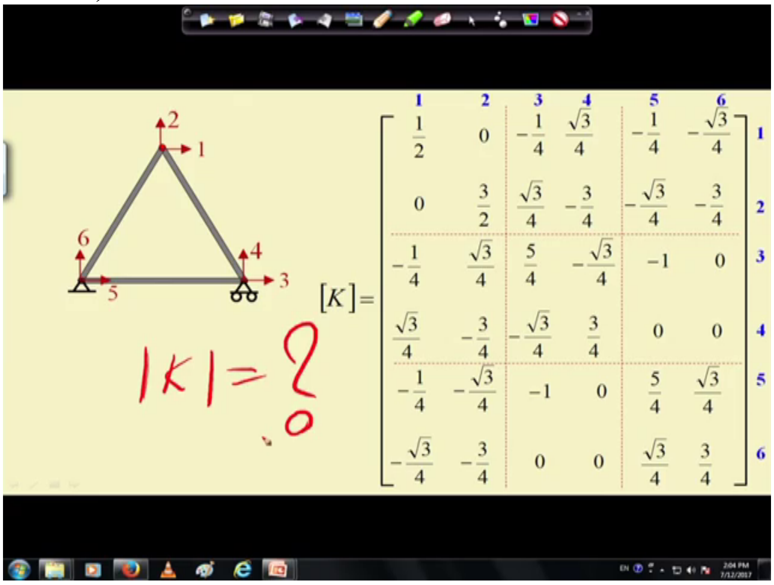

is this value, Ok and, and as we discussed what should be the determinant of this

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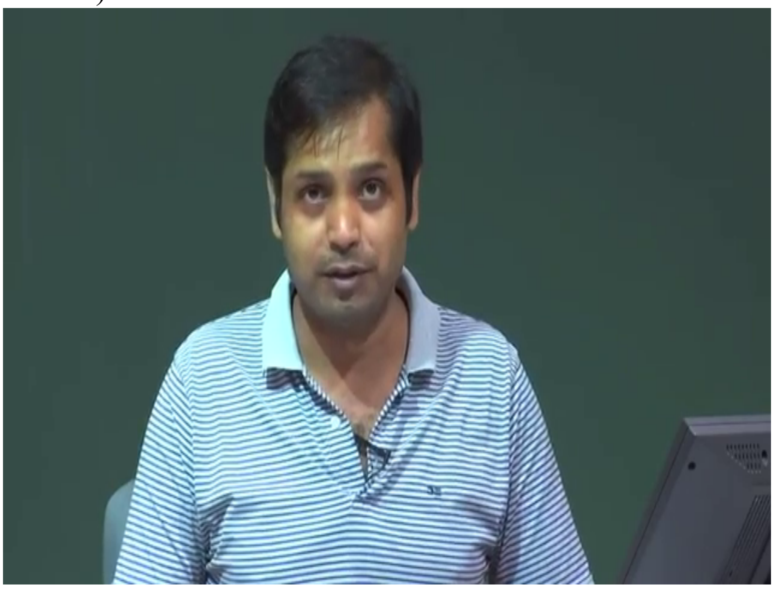

matrix in the previous classes, global stiffness matrix without, without imposing any boundary conditions, what would be the determinant, you check that and, and, and

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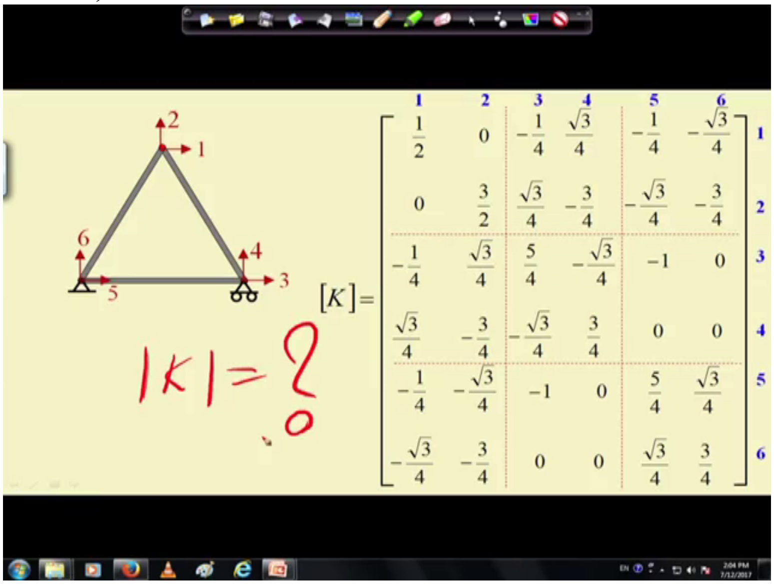

then see whether you are getting that, the

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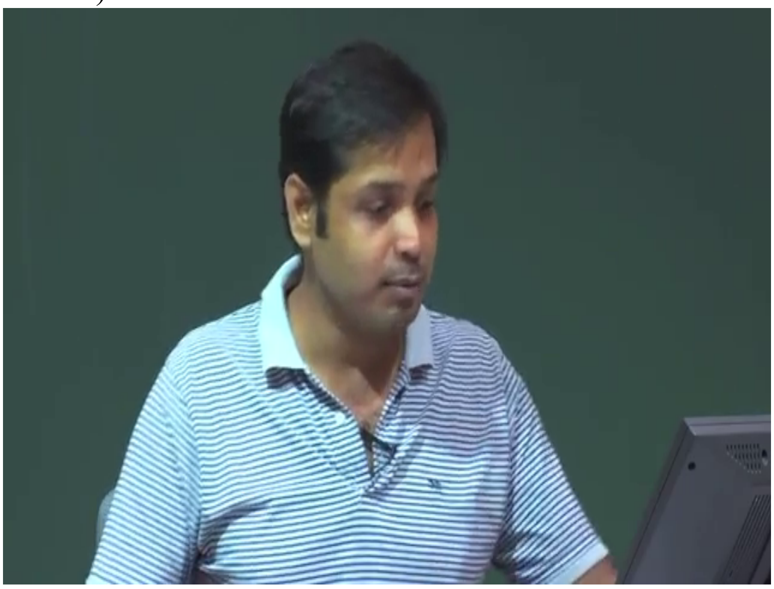

determinant of this matrix will be zero. Ok.

Now

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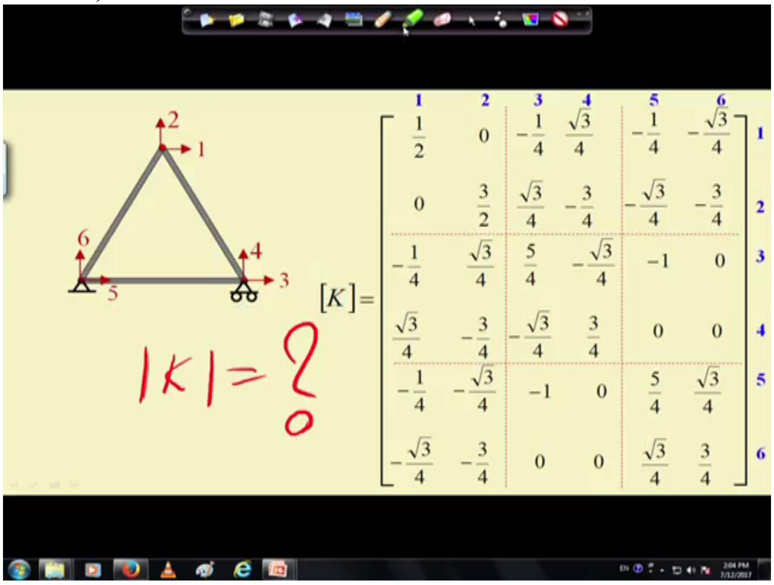

once we have the global stiffness matrix, next part is to calculate the, calculate the, apply the boundary conditions. Now how you do that? If you remember, in this case, you know, in this case what are the things we know and what are the things we do not know. What are the unknown, unknowns are, unknown d is, unknown displacements are, this is unknown, this is unknown, this is unknown,

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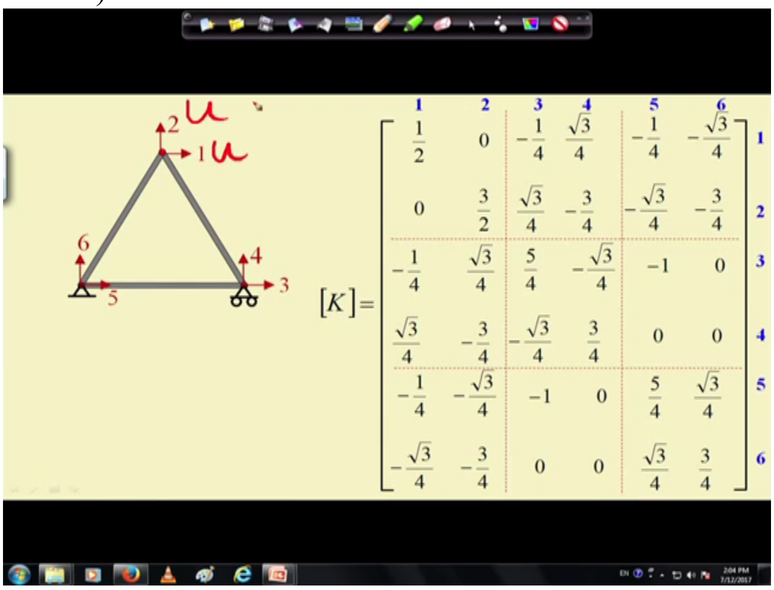

and this 3 is unknown. And then this is known, this is known,

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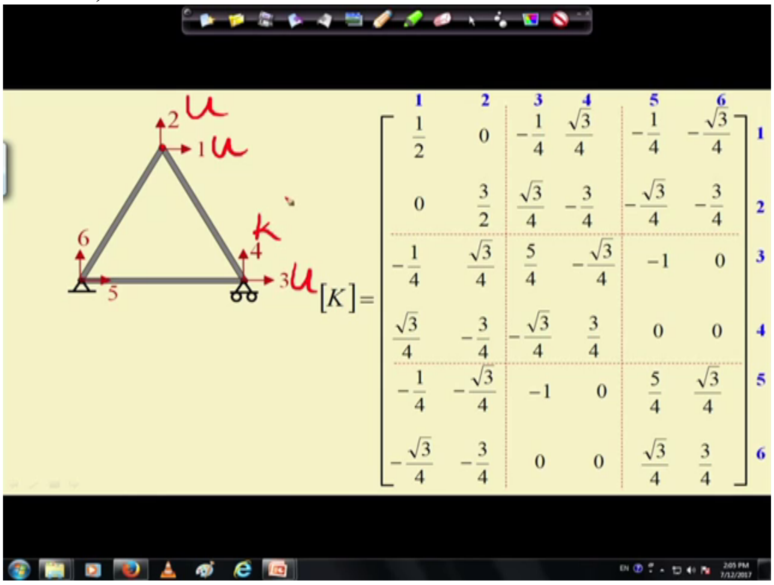

this is known and this is known.

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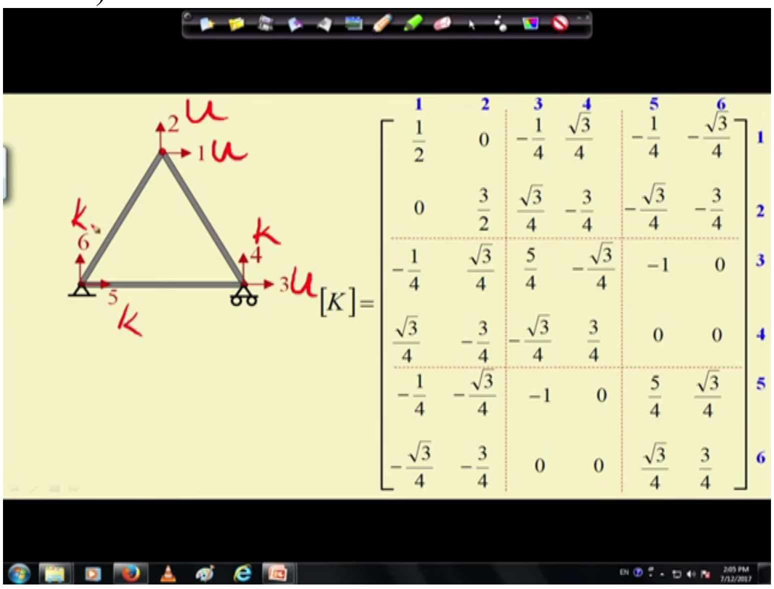

What we need to find out is now is we need to, we need to, we need to write this stiffness, we need to write it and, we need to write it so what will be the, let us write the, what will be the deformation vector, deformation D will be, then this is D 1, D 2, D 3 then D 4, D 5, D 6, right?

And D 4

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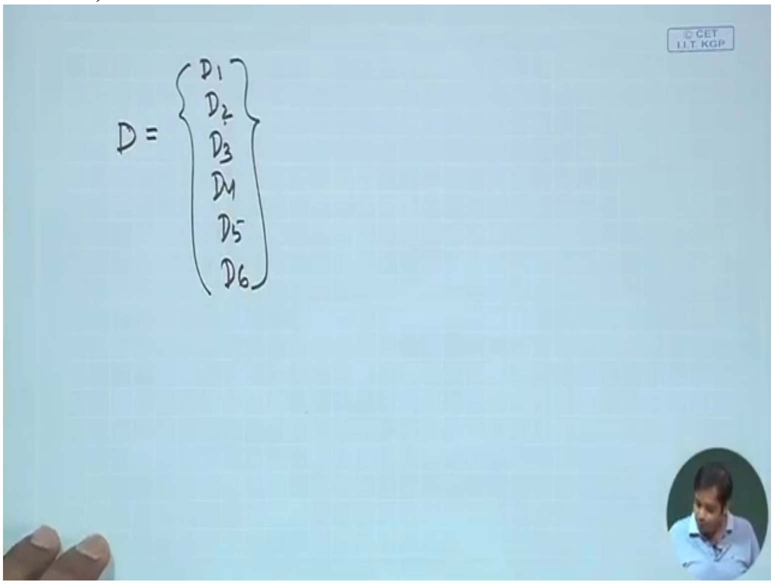

you look at our, our, our, our, our problem was like this, if you remember and this was this, this is 1, 2, 3, 4 then 5, 6. And we have, this was singly supported and this was roller supported.

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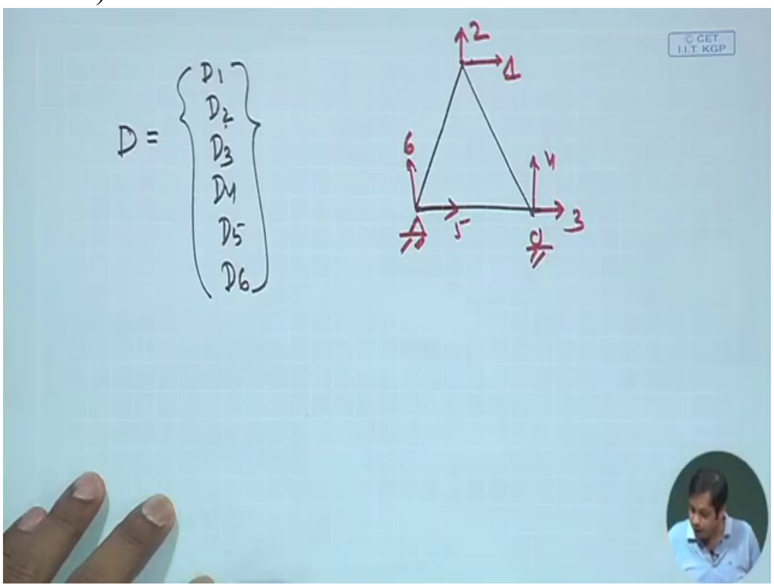

So boundary condition is D 5 is equal to D 6 is equal to D 4, they are all, they are all zero.

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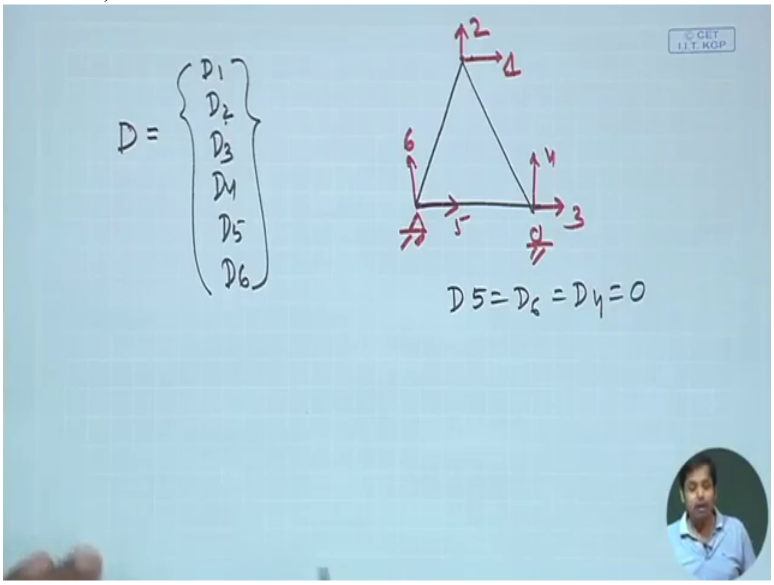

So only unknown, unknown are this is unknown. So this is unknown, right. So this we can write as D unknown and D known, Ok

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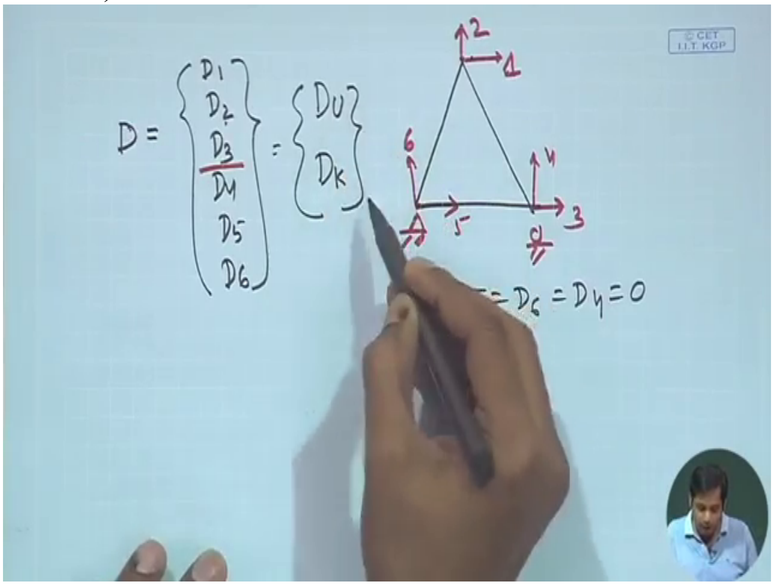

where known is D 4, D 5, D 6 and D unknown is D 1, D 2, D 3, D 4 Ok. Now if you remember we need to write then K as like this, K Ok, now then what we can, what we can, what we can write is K will be, and then what are the force vectors, let us write the force vector, F vector is, F vector will be F 1, F 2, F 3, F 4, F 5 and F 6, right?

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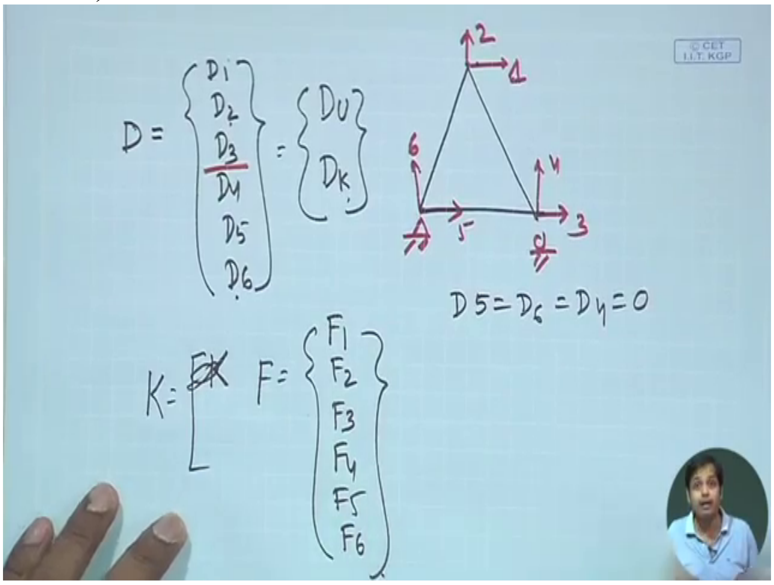

And what are the forces we know in this case? If you, we have only vertical load in this direction. At this point we have a vertical load P.

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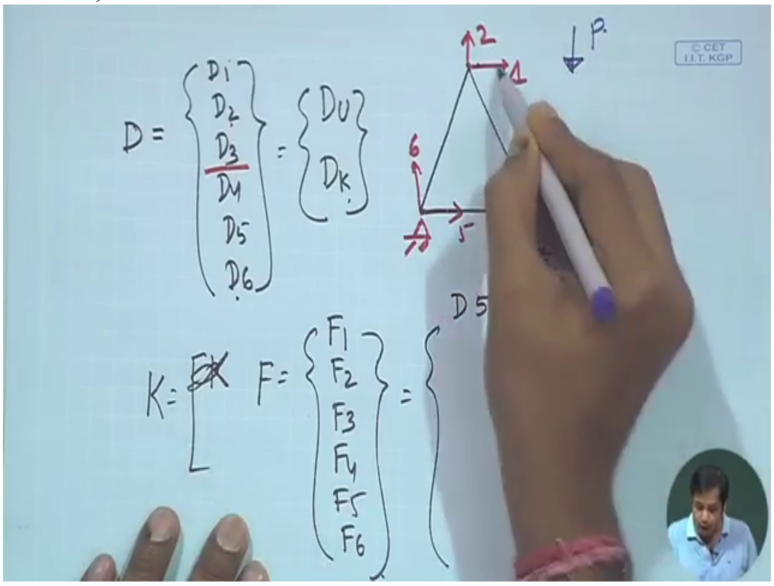

All other joints there is no load. So the force will be, F 1 is zero, F 2 will be minus P and this is zero, zero, zero, zero. This is,

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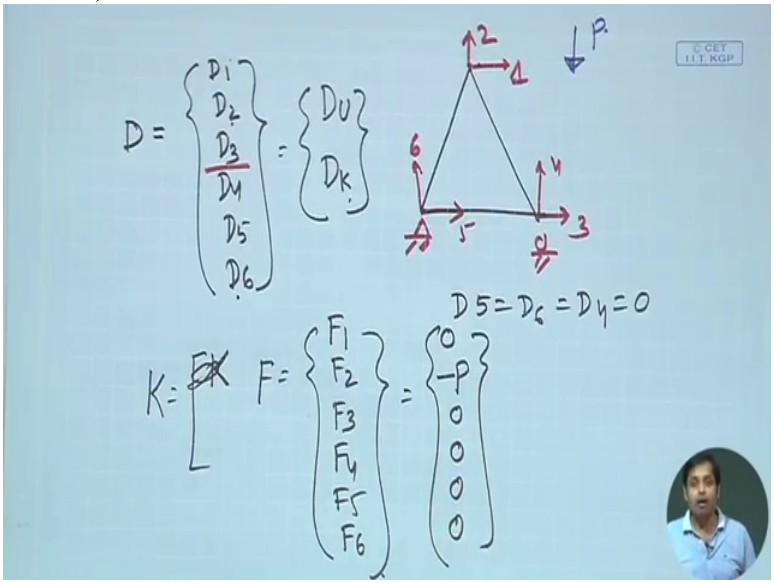

this the entire force field. Ok so this is the displacement, this is the force field, Ok. Now again corresponding, corresponding, since this corresponding force will also, we can, we can, we can partition like this, Ok.

Now, now you see if you, if remember we discuss

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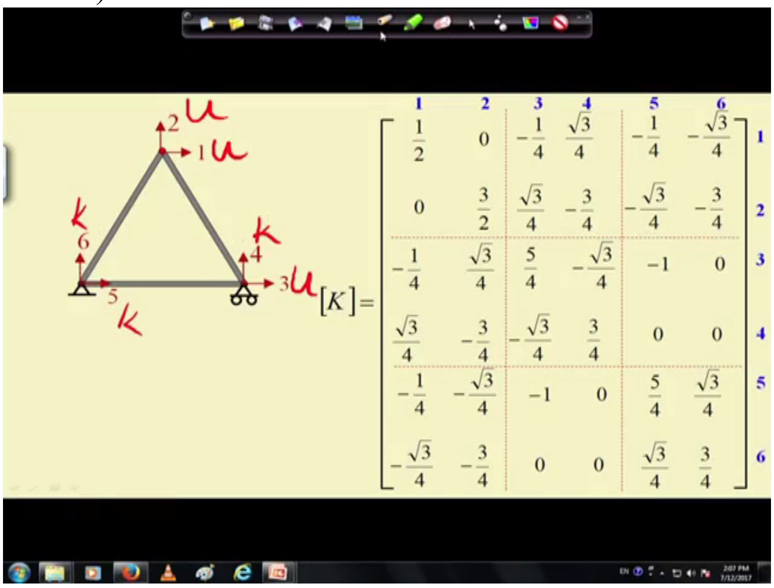

the partition also, now point number 1, 2, 3 they are, their displacement is to be determined and point number 1,2,3 we need to determine the displacement. So the matrix can be partitioned like this.

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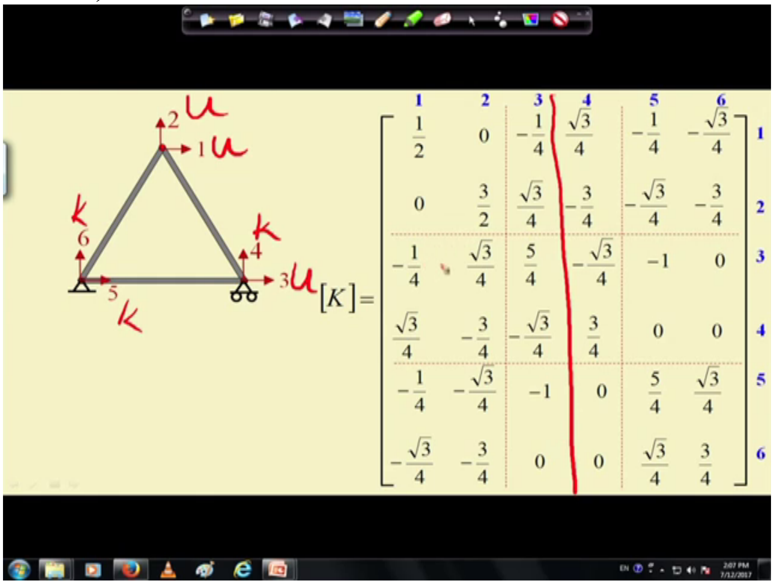

Matrix can be

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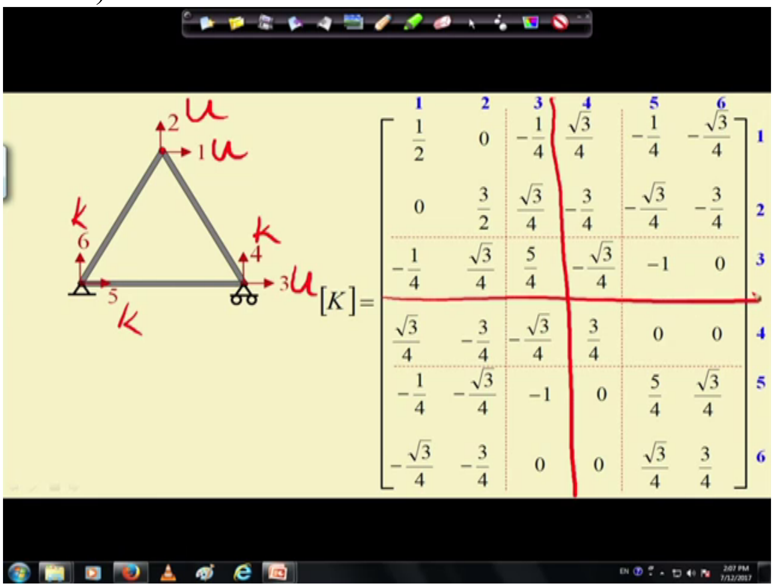

partitioned like this, Ok. Now this, so what will be our, what will be, what will be, so what, matrix will be, if we, if we partition it, the matrix will be,

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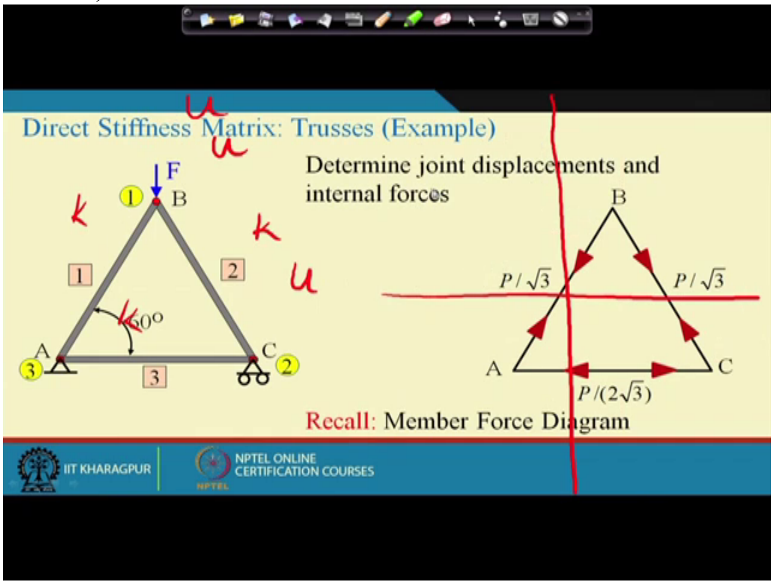

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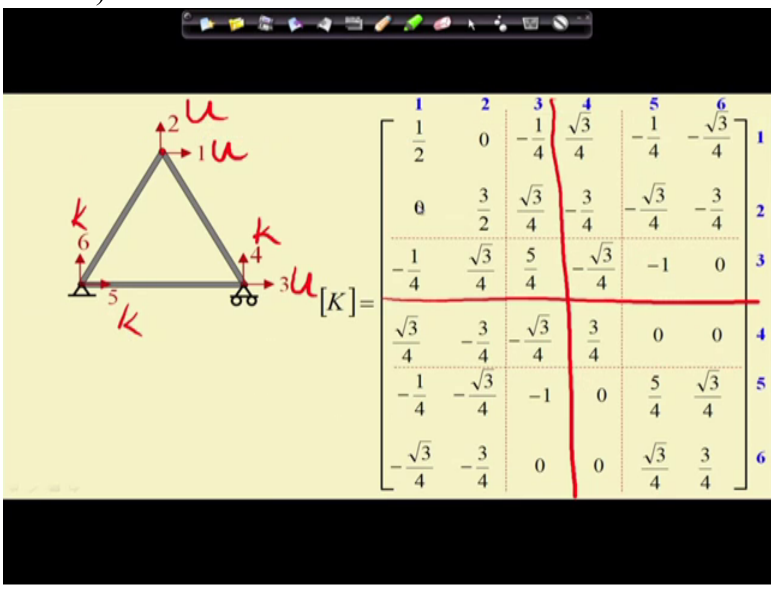

now what is then our force displacement relation?

This is the, this is the matrix K and then this is D 1, D 2, D 3. This is the, this is the, this is the displacement, unknown displacement and correspondingly it will be F 1, F 2 and F 3 right? And what is K?

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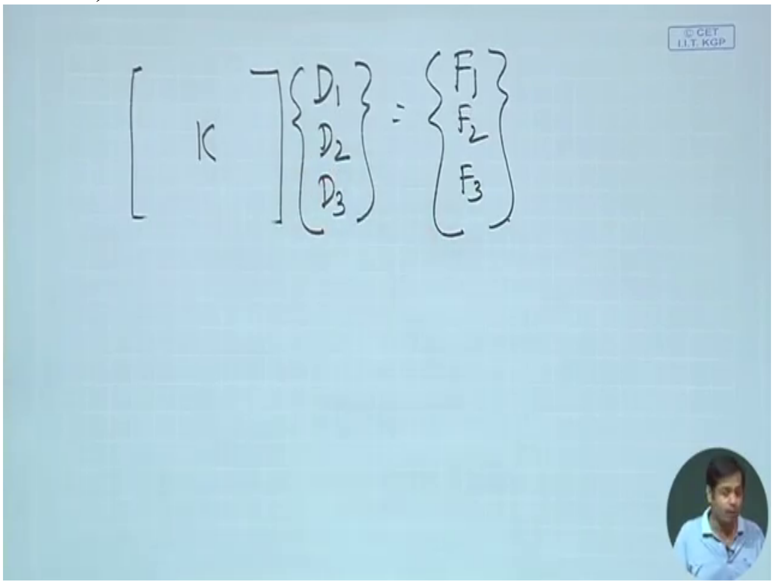

K will be, if you look at there,

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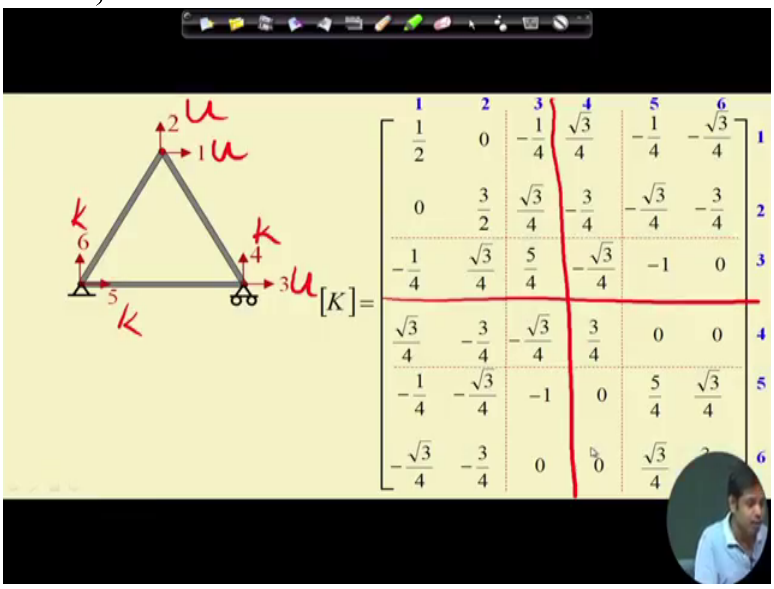

K will be this part. K will be this part.

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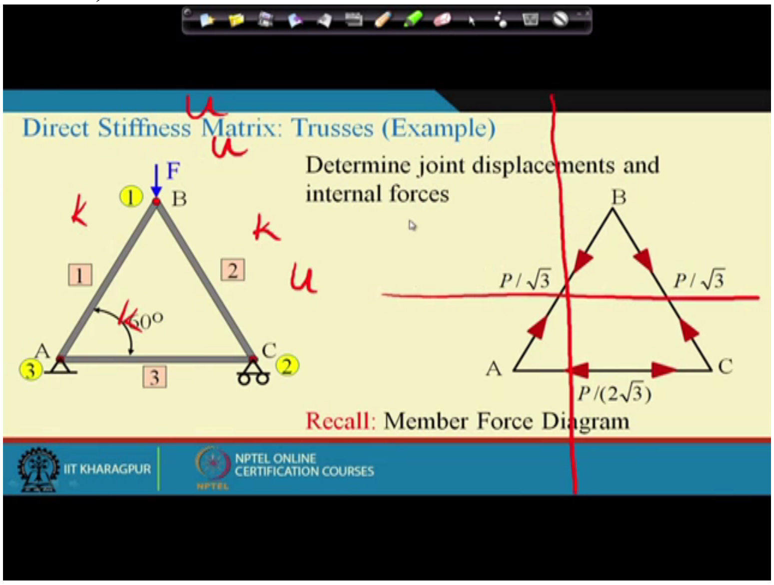

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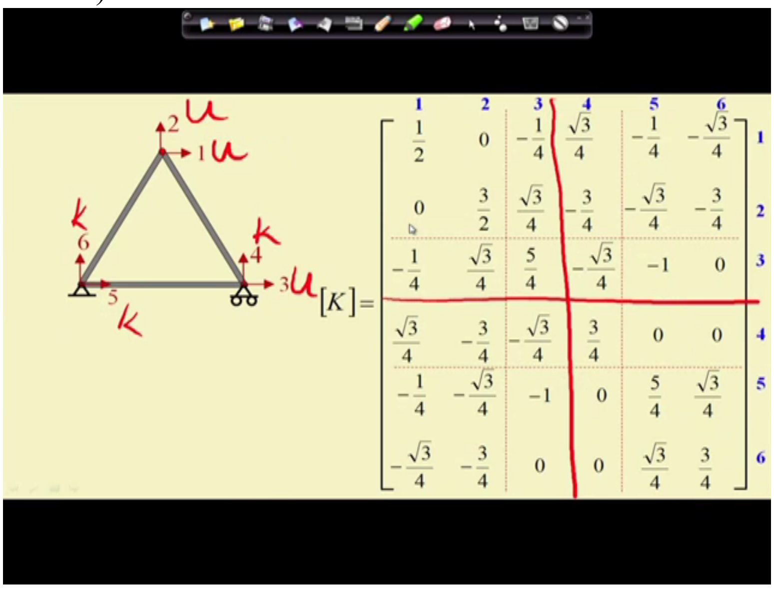

K will be only, only this part, Ok. Now

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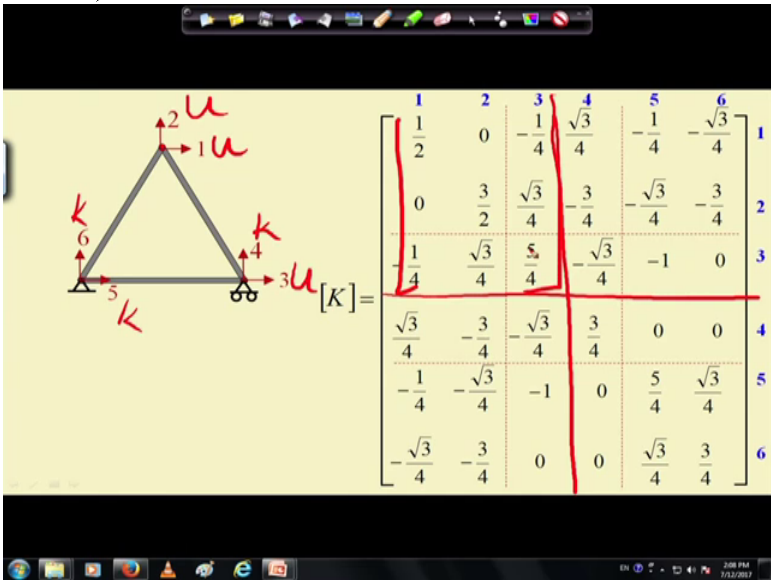

how to do that?

So final expression, final expression is that U, general expression is F unknown is equal to F, F known and F unknown, F unknown is equal to K sub matrix K 1 1, K 1 2 or you can write K k k, K k u and then K u k and K u u, unknown, unknown and that is equal to D unknown and D known,

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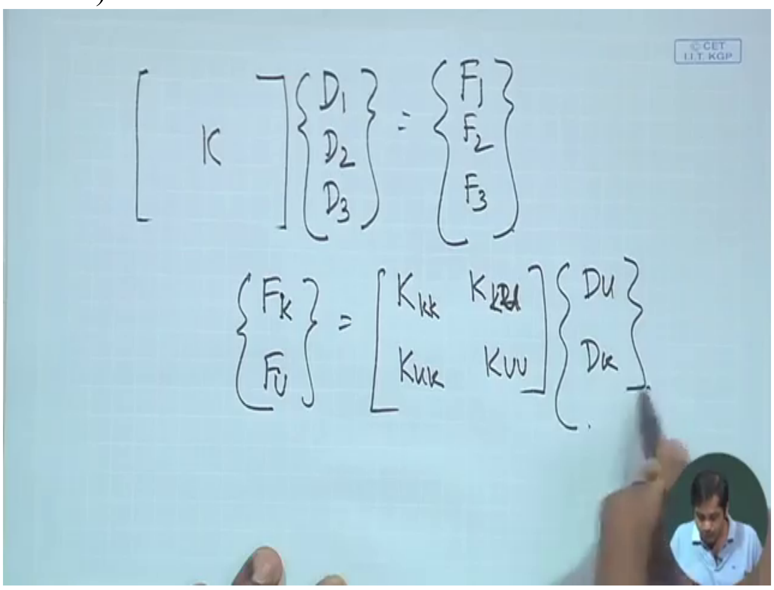

Ok. So what we, what, from that what we can write is F unknown, F known is equal to, F known is equal to K k k into D known.

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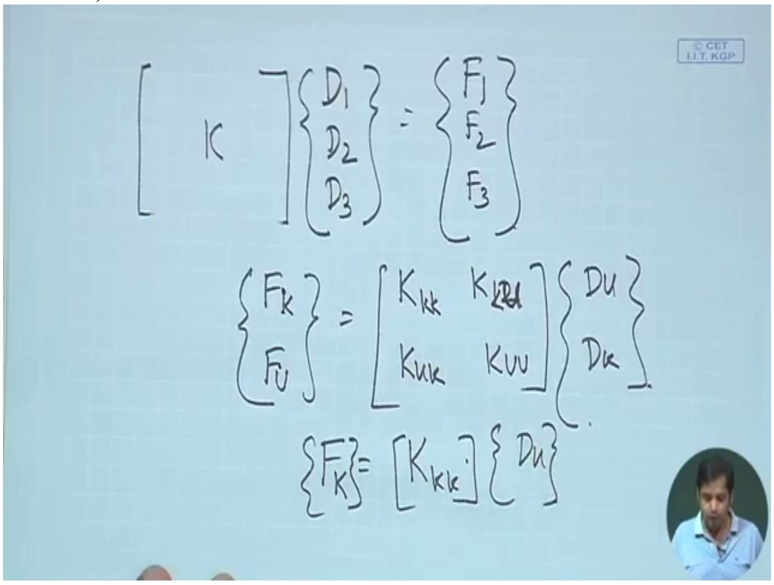

Here this, this, this, this, this, this

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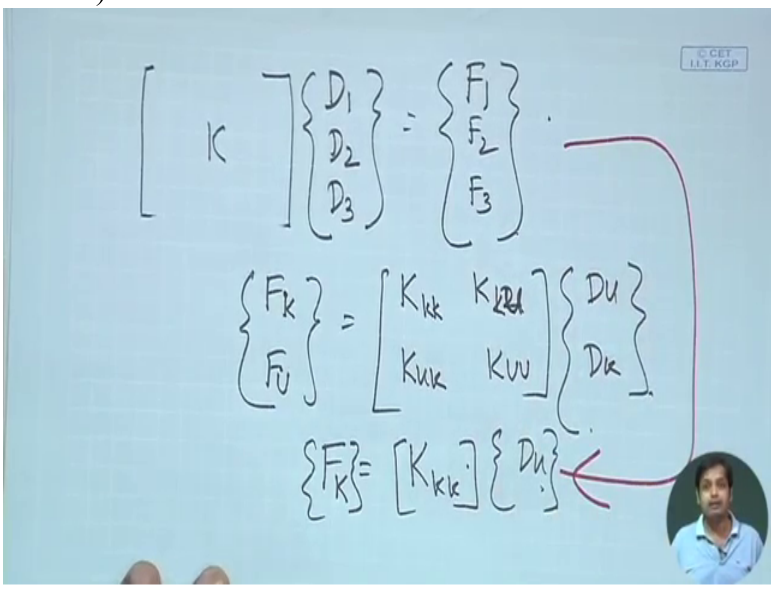

same Ok. Now K K is known. And then you, if you write it, let us, let us write it quickly then the matrix will be, the final matrix is something like this. Here it is half, then zero, minus 1 by 4, then zero, 3 by 2, root 3 by 4, and then this is minus 1 by 4, root 3 by 4 and then 5 by 4. This is, and displacements are D 1, D 2, D 3 and unknowns are zero, minus P, zero. This is the force field.

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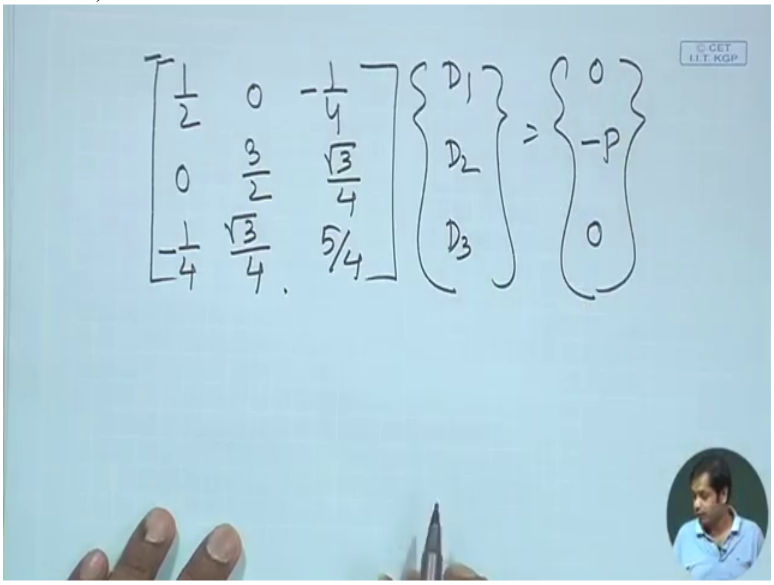

And if we, if we, if we calculate them then of course, there will be, there will be A, there will be A E by L as well, there will be

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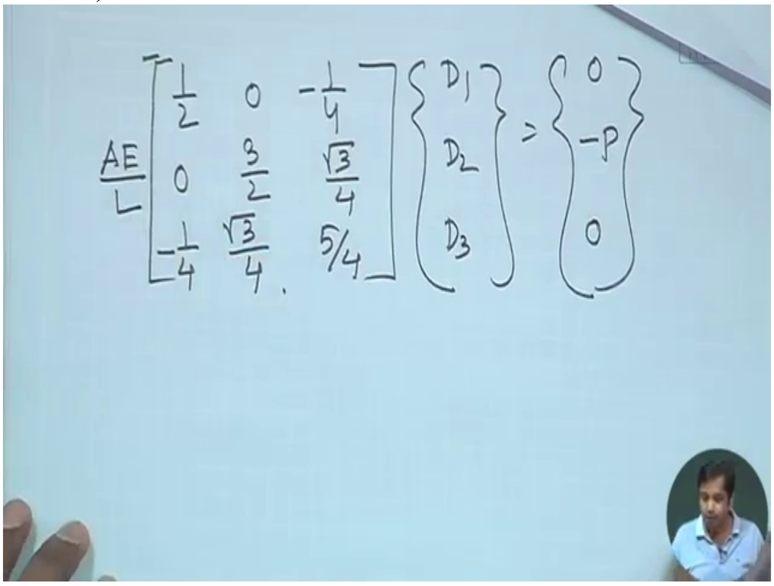

A E by L as well. So then D 1, D 2, D 3 will be D 1 will be zero point 1 4 3 3 P L by A E and

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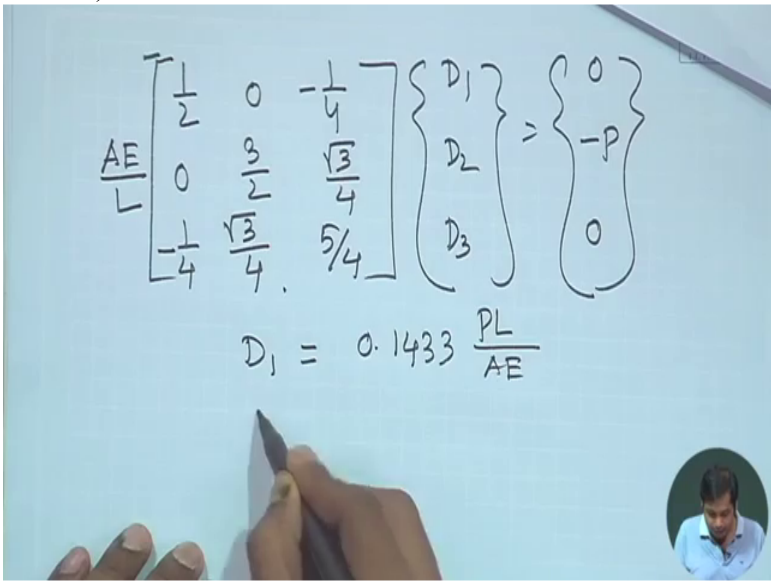

then D 2 will be minus zero point 7 5 zero P L by A E.

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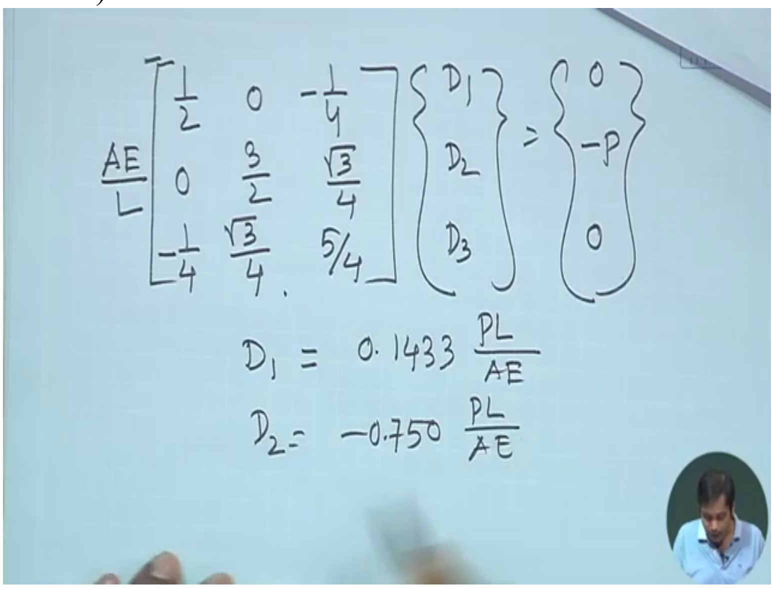

And then D 3 will be zero point 2 8 7, 2 8 8 7 P L by A E. This is

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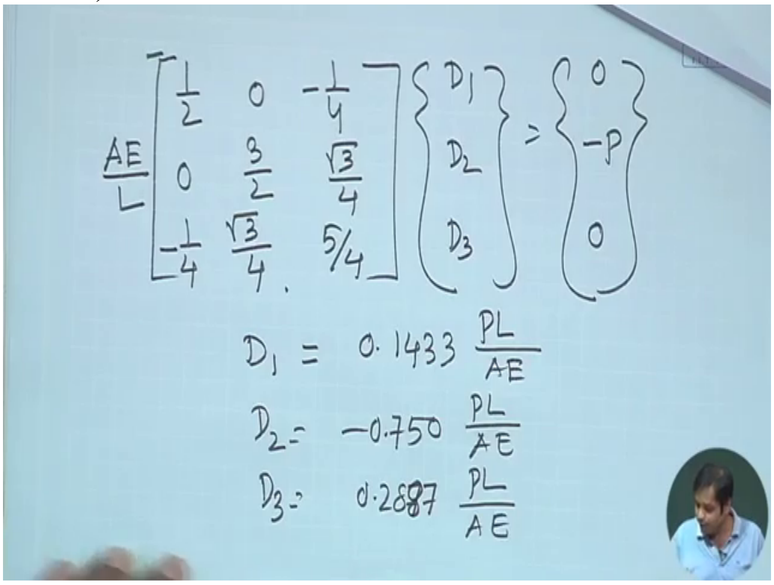

the final solution. This is the final solution.

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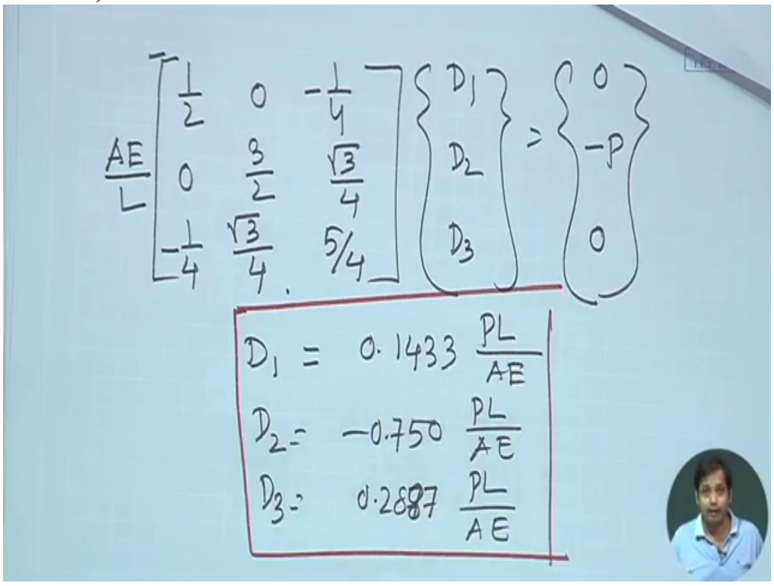

So these are the displacements, at, at various joints

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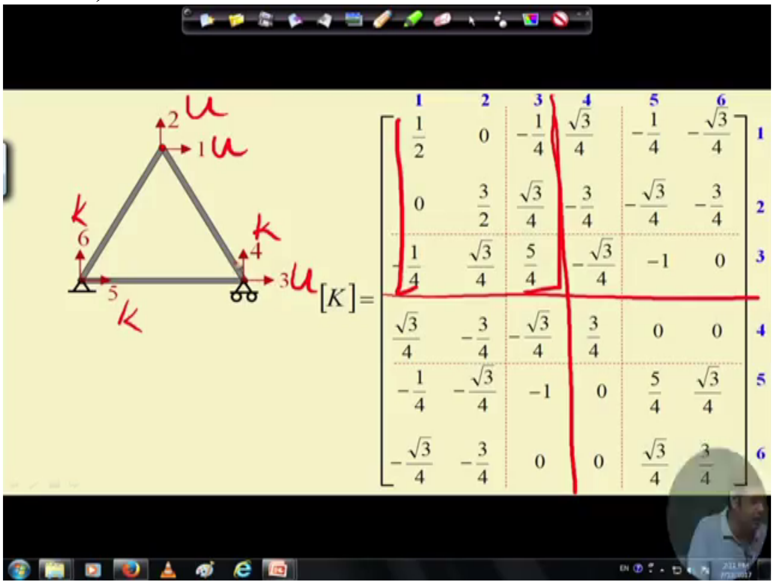

and D 4, D 5, D 6 will be zero and this is D 1, D 2, D 3 are these values. So we have already obtained what are the forces for, what are the forces,

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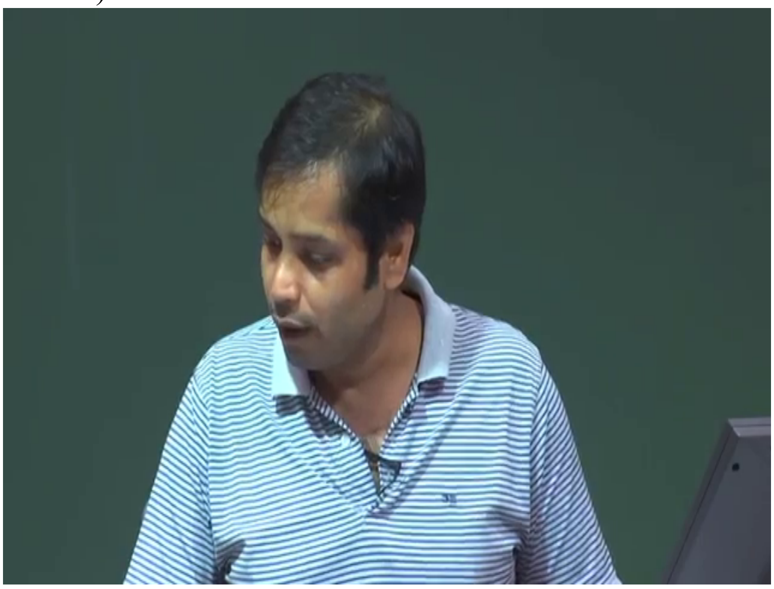

what are the displacement, what are the displacement, Ok at various joint. Now once we know the displacement, you see, we need to find out, we need to,

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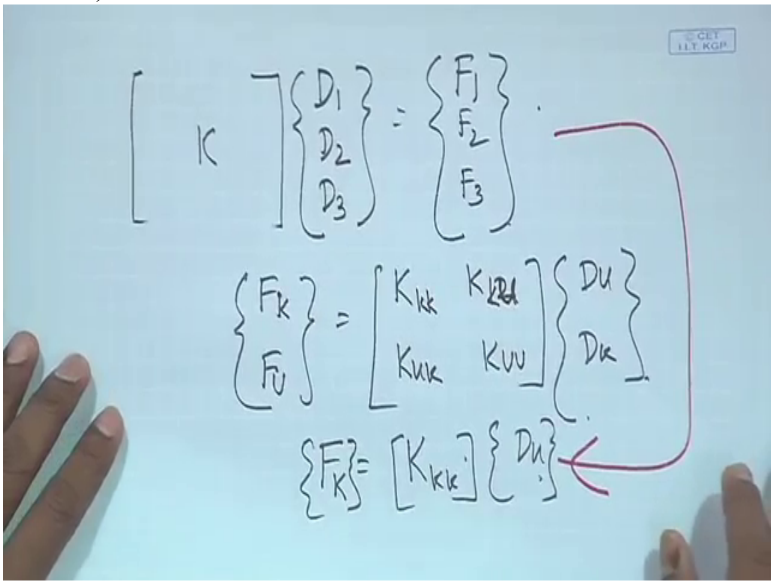

what we need to find out, we need to find out the

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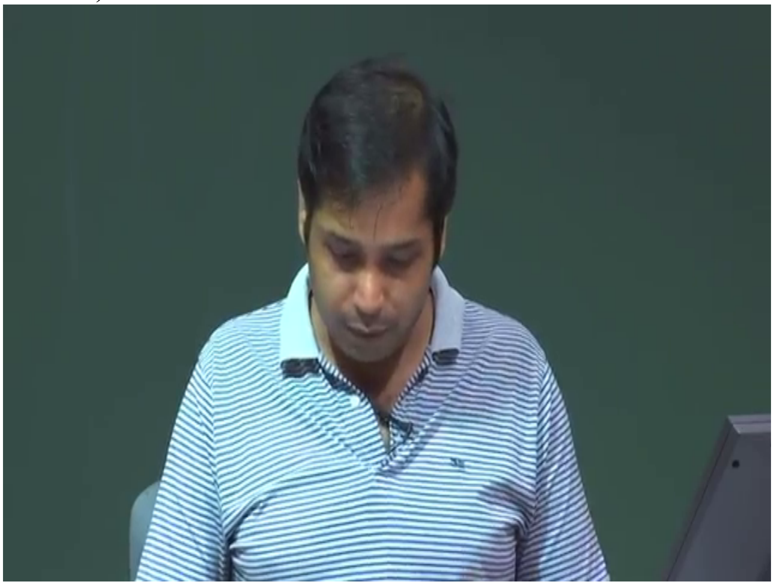

joint forces as well.

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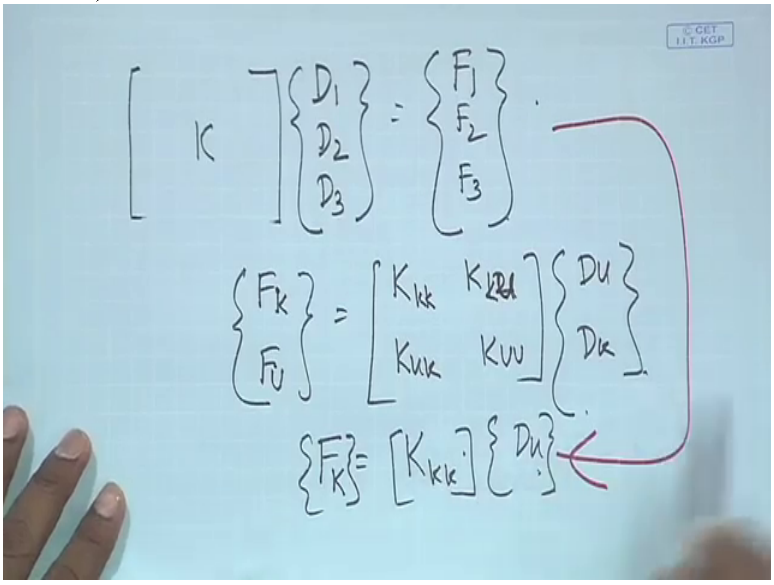

So joint forces means F u u we need to find out, F u u we need to find out. So what will be F u u?

F u u will be, F u will, this is the force known, the forces at joint known, and this force at the joint we need to determine and then F u will be, this is the partitioned, after partition and from that what we can write is F u will be, F u will be that K, K u k into D u plus K u u into D u. Now D u, D k, so this is, this was

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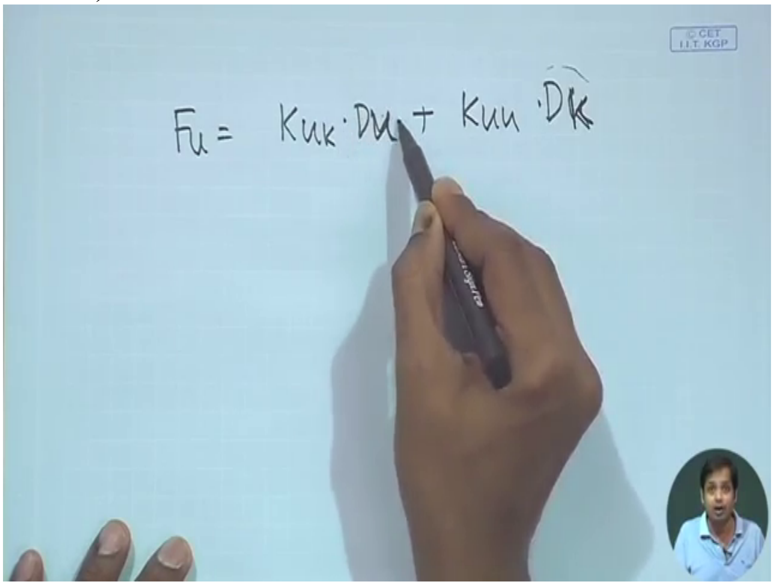

the unknown. Now we have already obtained what is the value of D unknown. The value of D unknown is now this. Value of D unknown

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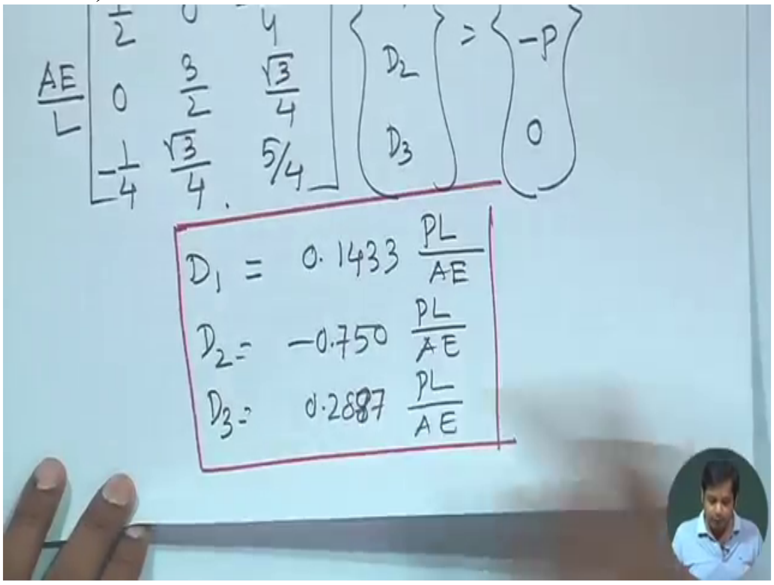

is this is the, this is, entire thing is D u. And then

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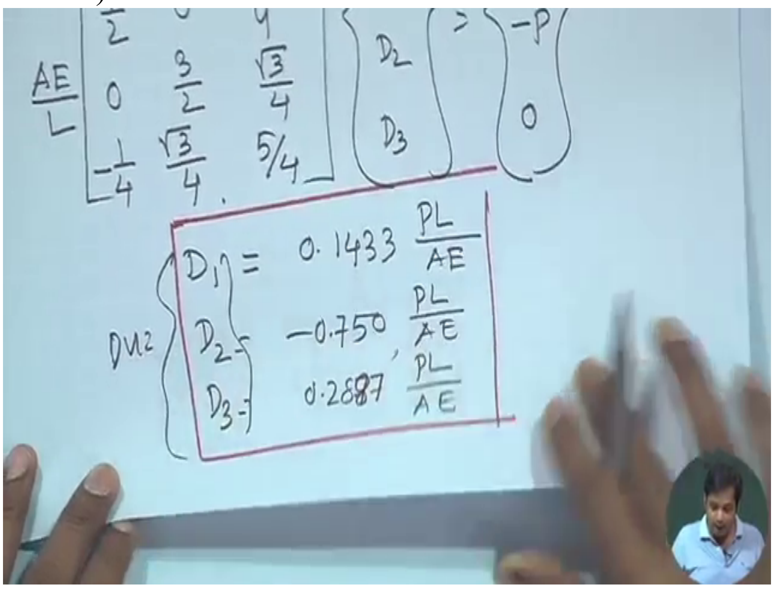

D k is zero.

D k is zero means because at those points your displacements are zero because it is supported, so F unknown. F unknowns are, F 4, F 5, F 6 is equal to this into D 1, D 2, D 3, D k is zero. This is zero, and what is this, this matrix?

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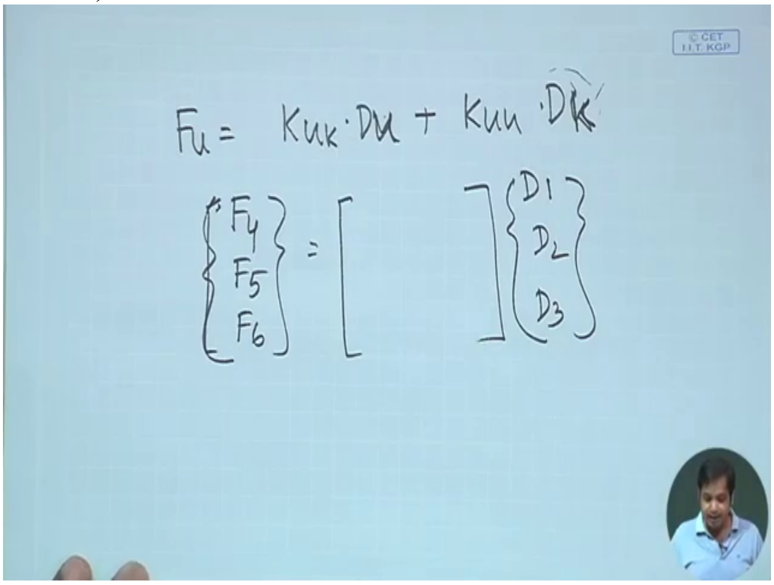

This matrix will be this one. If you look at this expression, this matrix

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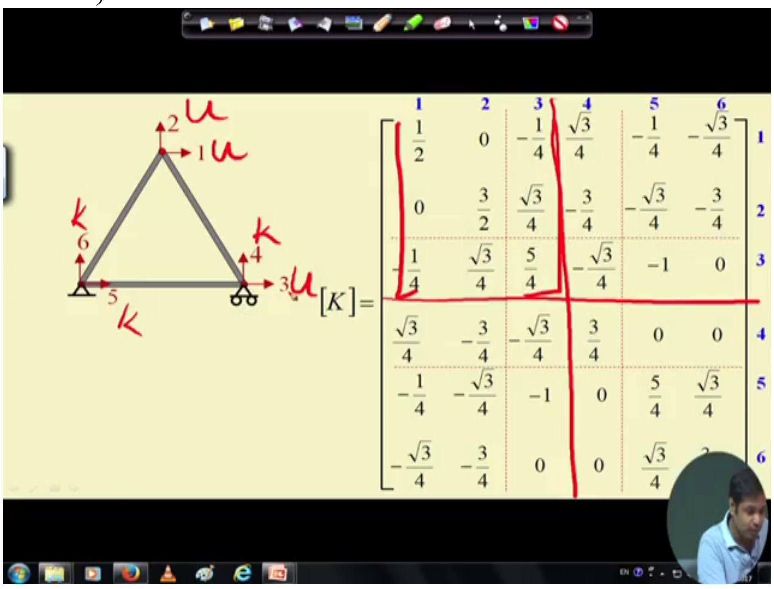

will be this one. This

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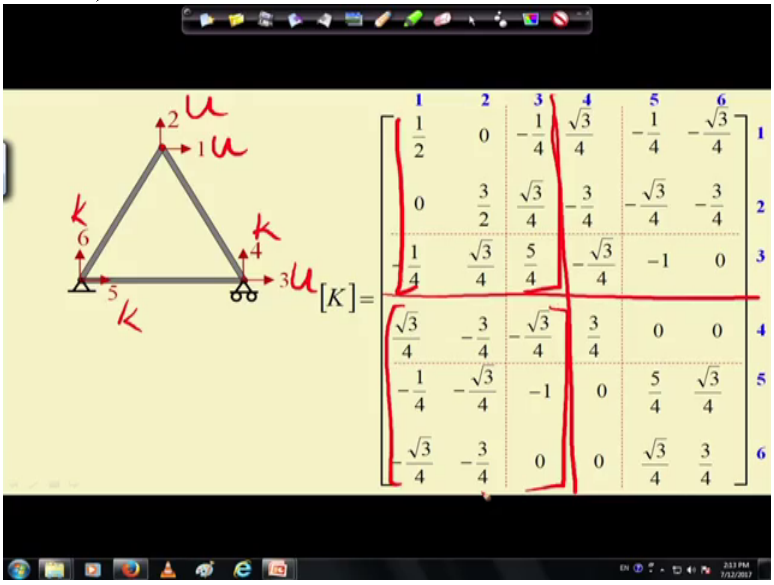

matrix is this one.

Now if you substitute that, you get F 4, F 5,

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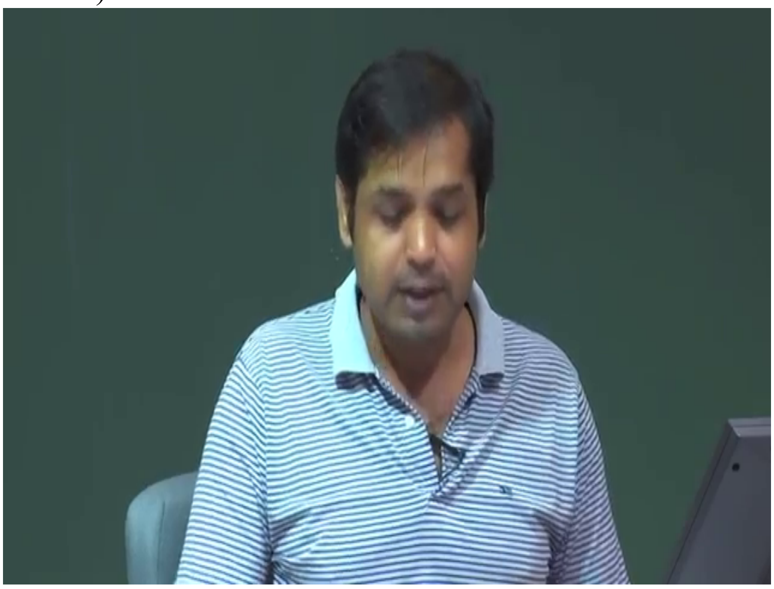

F 6. What are the

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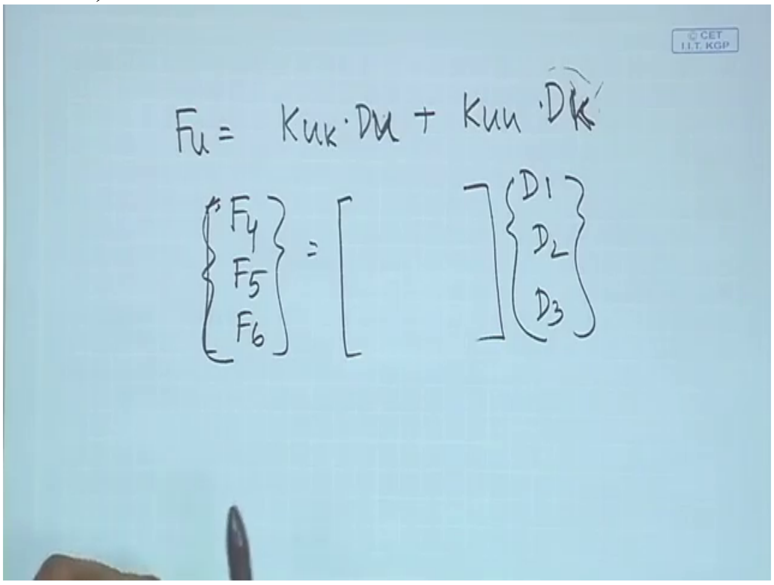

F 4, F 5, F 6; F 4, F 5, F 6,

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at the support reactions. So we have already obtained displacement, support reactions and then quickly what we need to find out is the member forces. Now member forces, if we remember, the member forces are, the member 1, the member q 1 will be the force in member 1 if, or N 1 if you write, N 1 is the force in member 1, this will be A E by L, A E by L into minus lambda x, minus lambda y, lambda x lambda y into your, degrees of, degrees of, degrees of freedom in various joints.

The member 1 is, this is the

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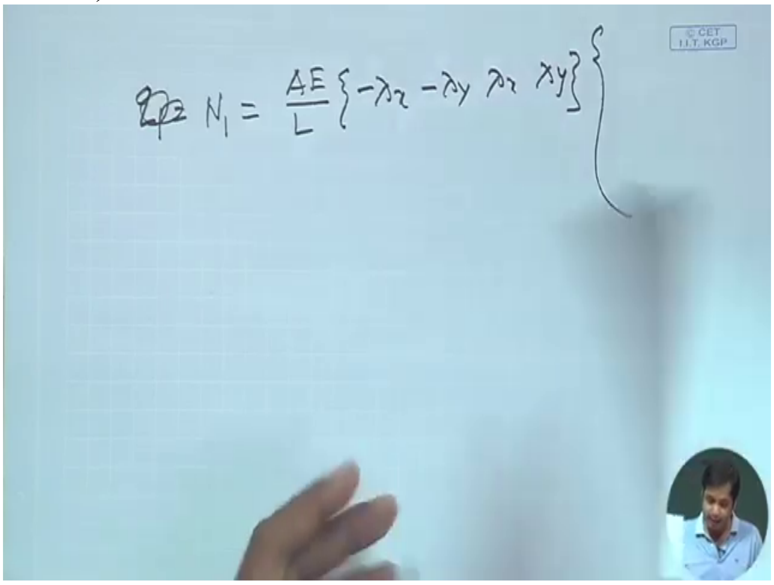

member 1, this is member 1, so member 1 is, we have connected

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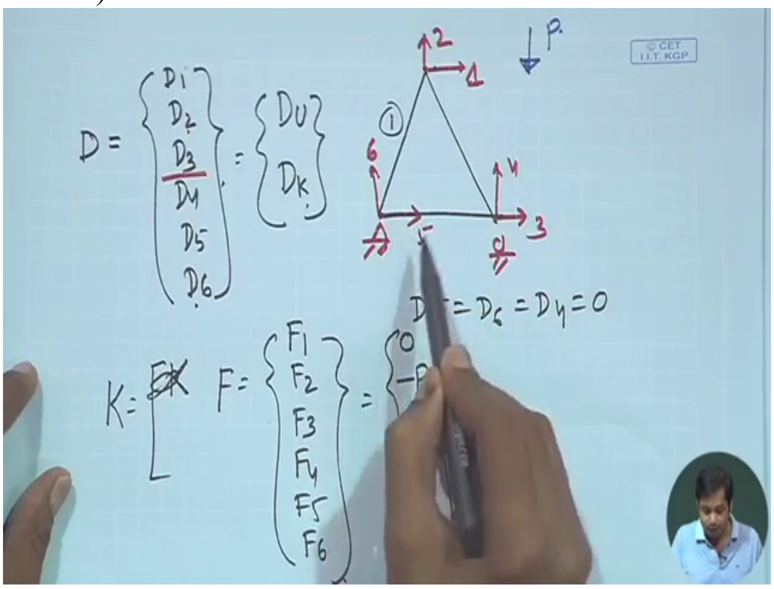

degrees, the joint, the displacement at the two ends are D 5, D 6, D 1, D 1 and D 2. So it will

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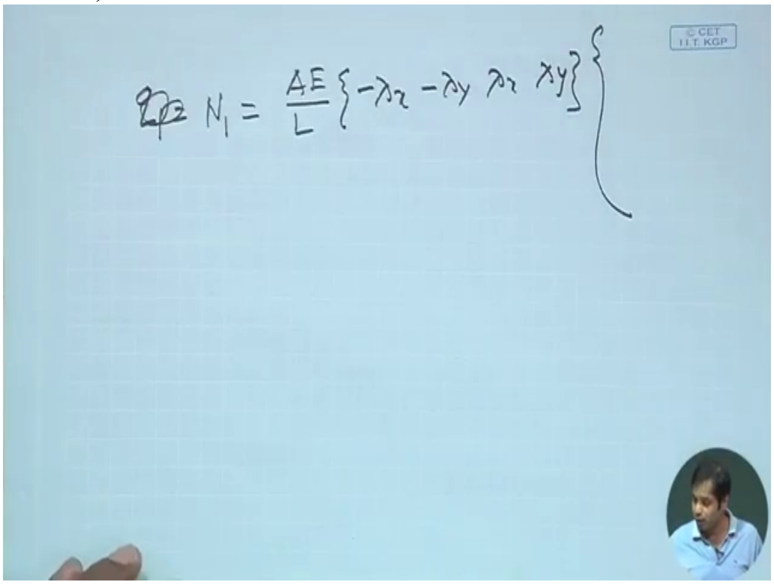

be D 5, D 6, D 1 and D 2, Ok.

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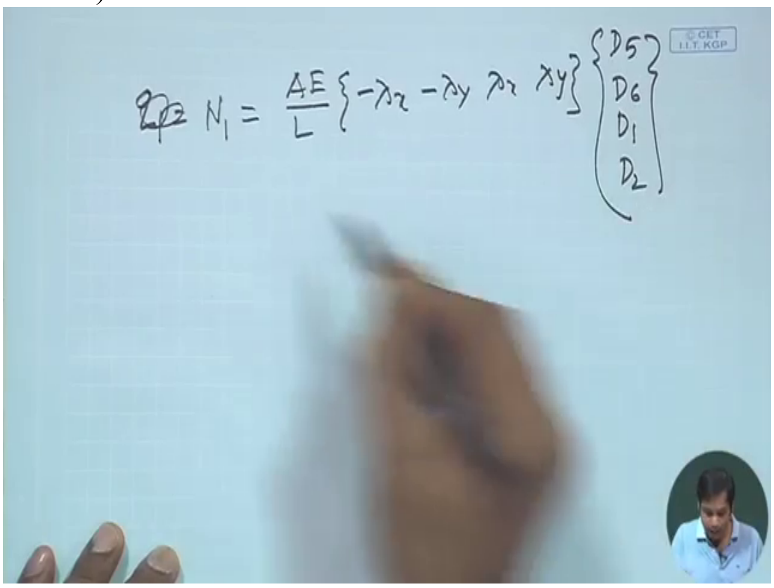

Now lambda 1 and lambda x we have already obtained. D 5, D 6, D 1; D 1, D 2 is already obtained and these are the values for D 1, D 2. D 1, D 2 is already obtained and these are

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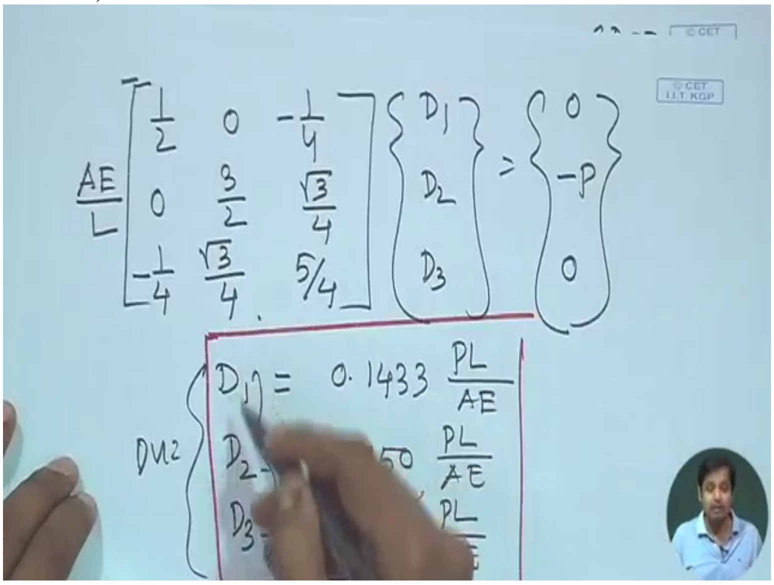

the values. And for D 5 and D 6, D 5 and D 6 will be

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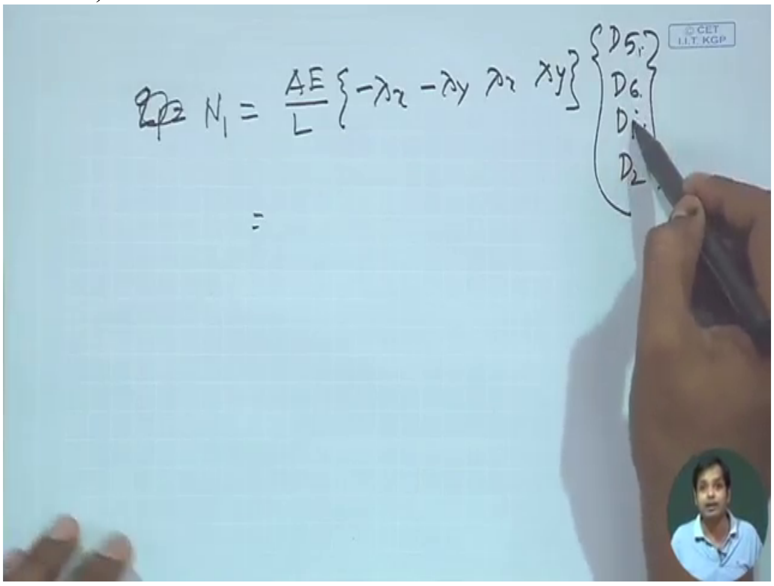

zero because it is the displacement at the hinged point which is supported point, which is zero and from there if you multiply you will get N 1.

Similarly, if you get N 2, N 2 will be A E by L, again lambda x, lambda x, lambda y, lambda x lambda y into, now N 2 is member 2, member 2

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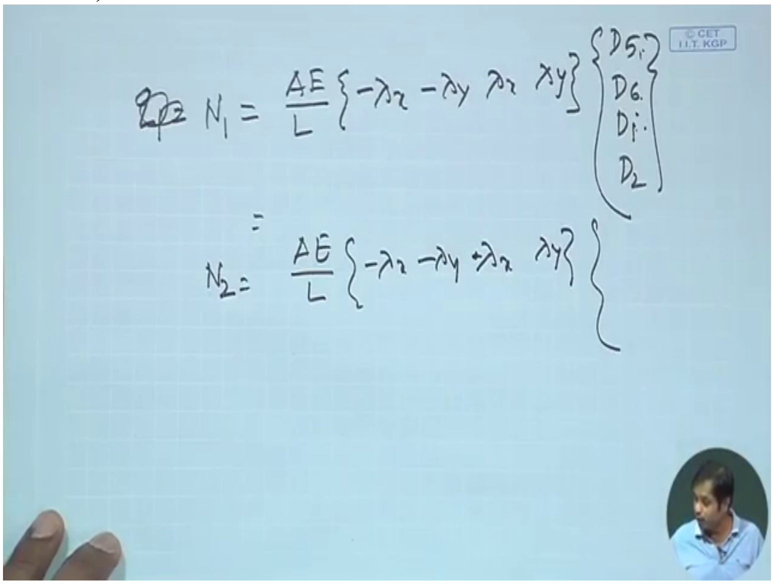

is, this is member 2,

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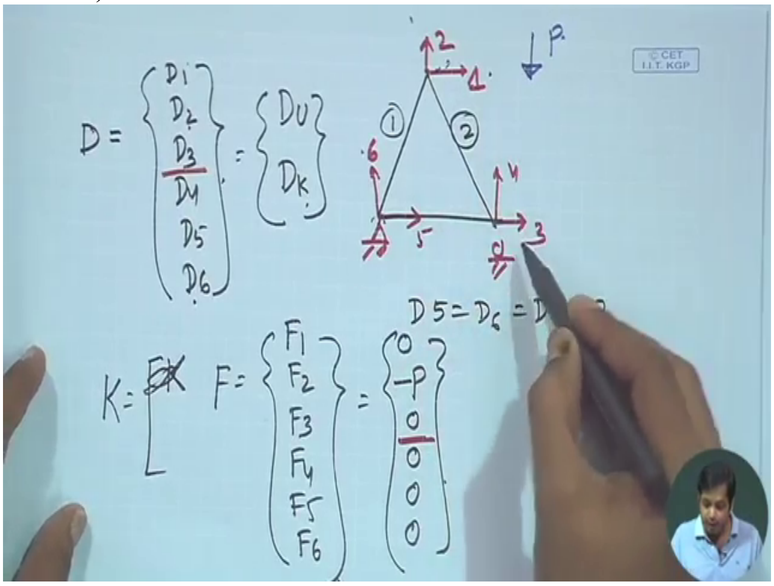

member 2 is connected between this point and this point, so degrees of freedom are D 3, D 4, D 1, D 2. So it will be D 3, D 4, D 1, D 2. Again D 3 is already calculated. D 1 is already calculated, D 2 is calculated, D 4 is zero. So we can substitute

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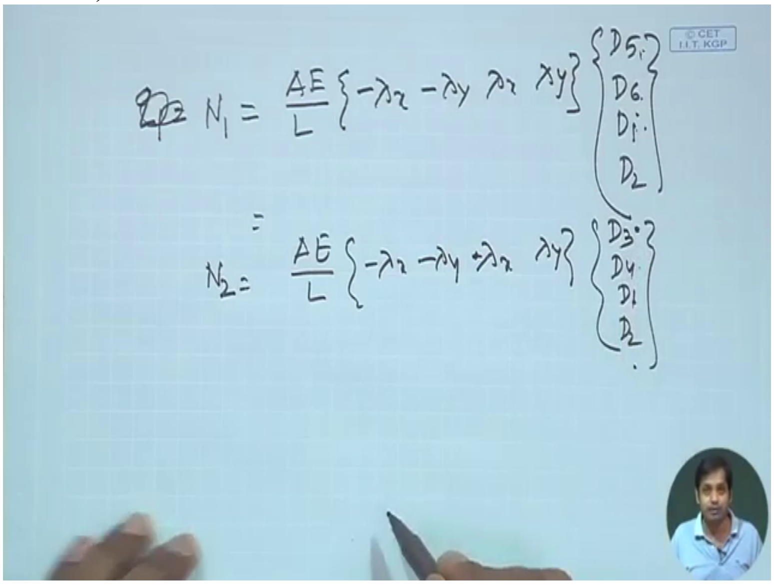

these values.

Similarly N 3 will be A E by L and then same thing, minus lambda x, minus lambda x, lambda x and lambda x and finally

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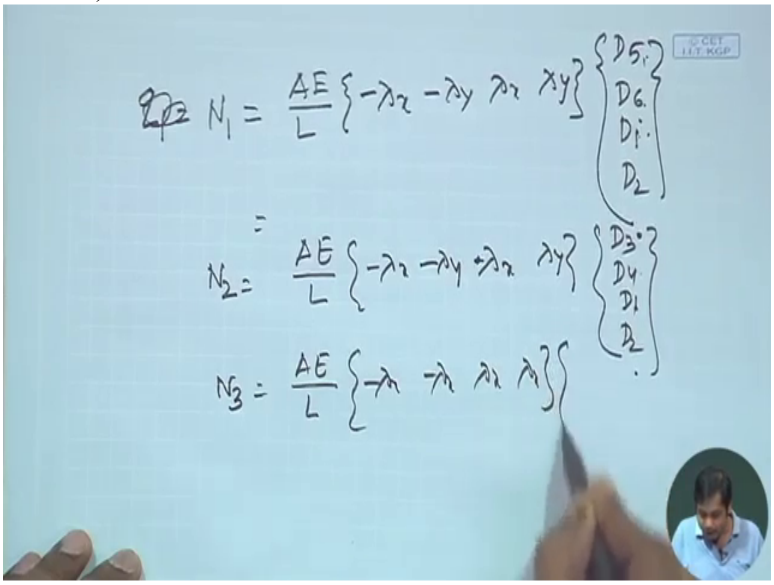

N, this is the point

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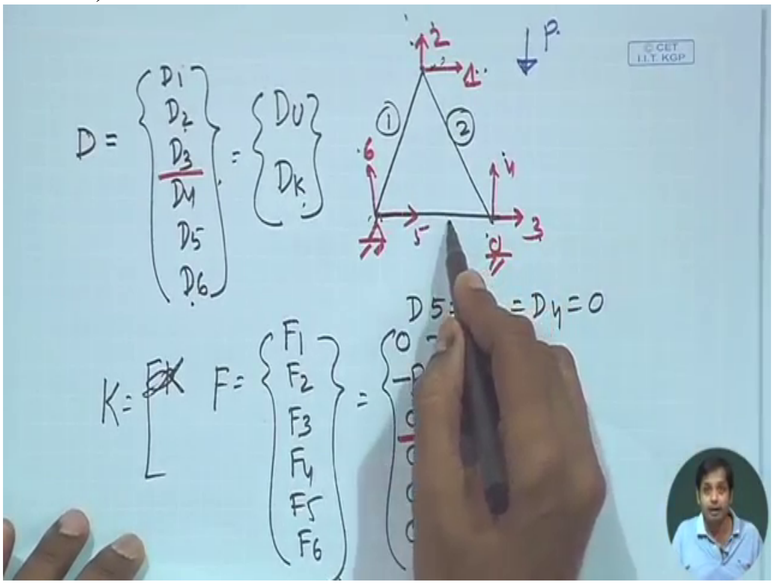

which is connected between this and this, this will be 5, 6, 3, 4. So D 5, D 6, D 3, D 4. So D 3 is calculated, D 4,

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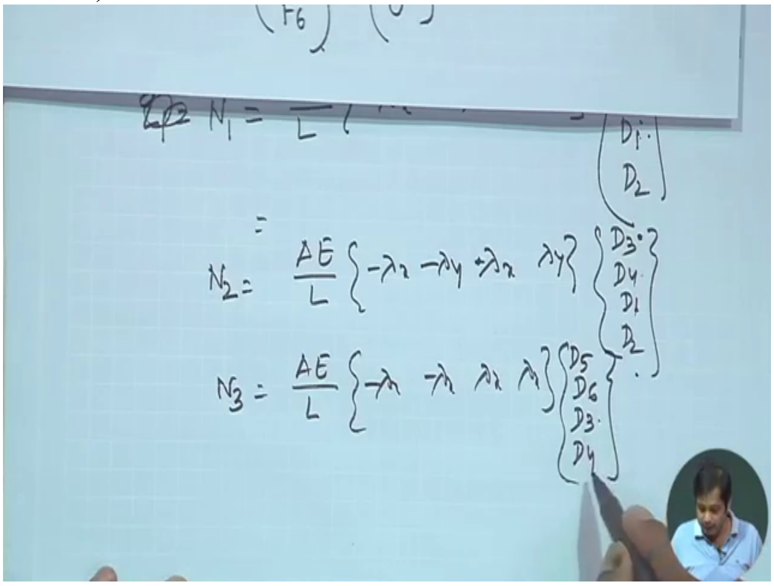

D 5, D 6 they are all zero. You can calculate N 3, values of N 3 and if we do that, let us, let us, do it for first. Let us do it for first.

For N 1, N 1 is A E by L, lambda 1 we have already calculated, that is minus half, then it is minus root 3 by 2, then again half, and root 3 by 2, root 3 by 2, Ok. Now D 5, D 6 they will be zero, D 5, D 5 and D 6 they will be zero and D 1

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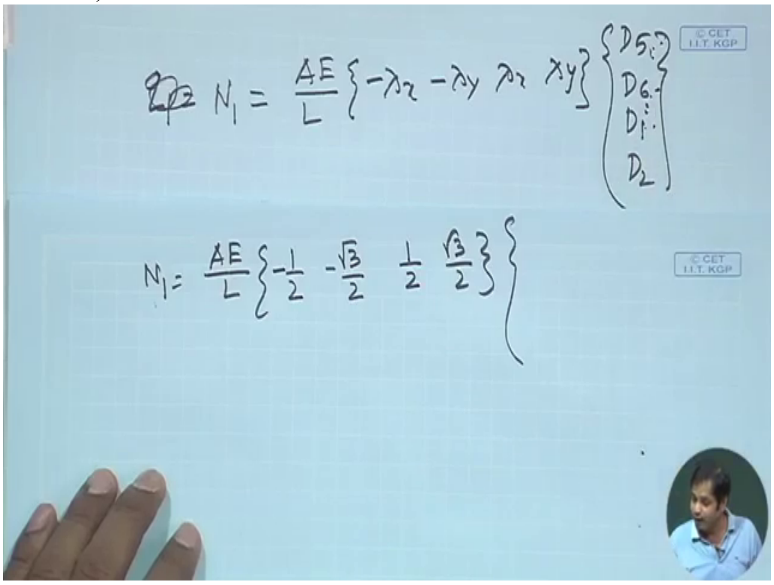

and D 2, they are, we have already calculated D 1 and D 2 as (()), D 1 and D 2 is (()).

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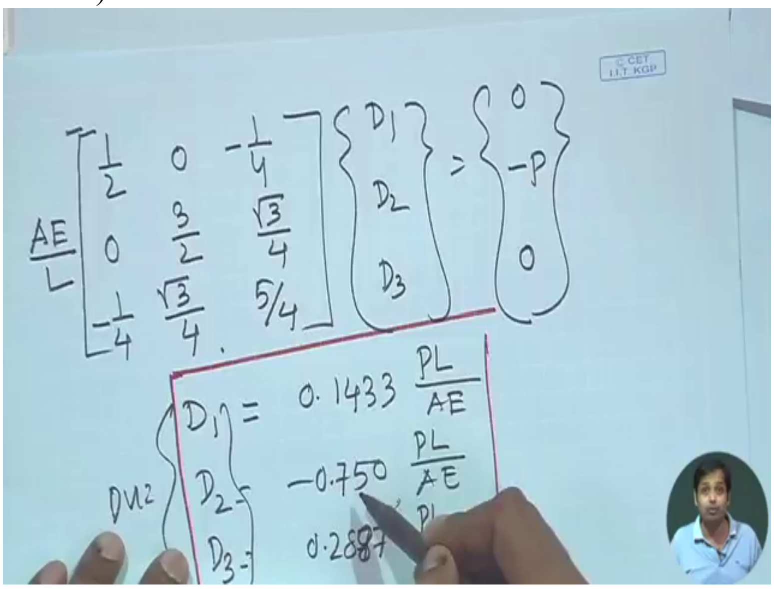

If we substitute that zero, zero and this is zero point 1 4 3 3 and minus zero point 7 5 0 and of course P L by A E, P L by A E, that L A E, L A E

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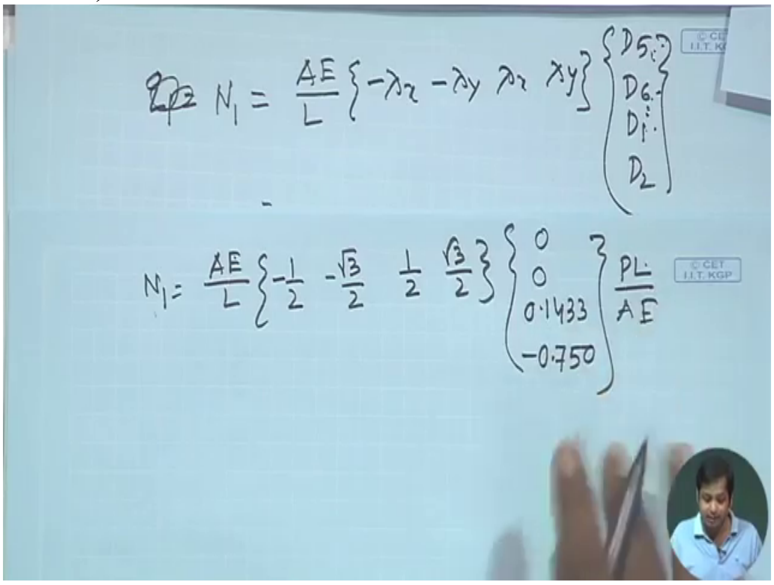

they will cancel each other and these values will get minus zero point 5 7 7 4 which is actually minus 1 by root 3. This will be P, minus 1 by root 3 P,

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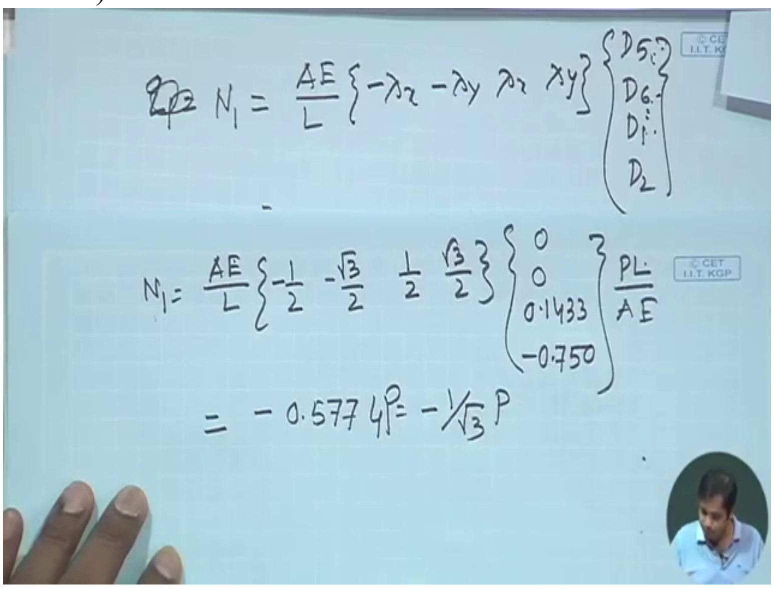

Ok,

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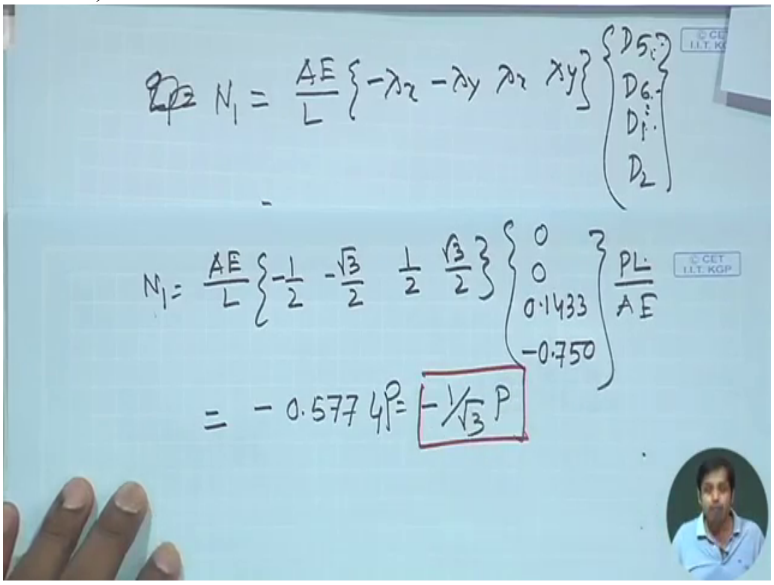

minus 1 by root 3 P.

And then similarly if we do it for, do it for N 2 and we can do it

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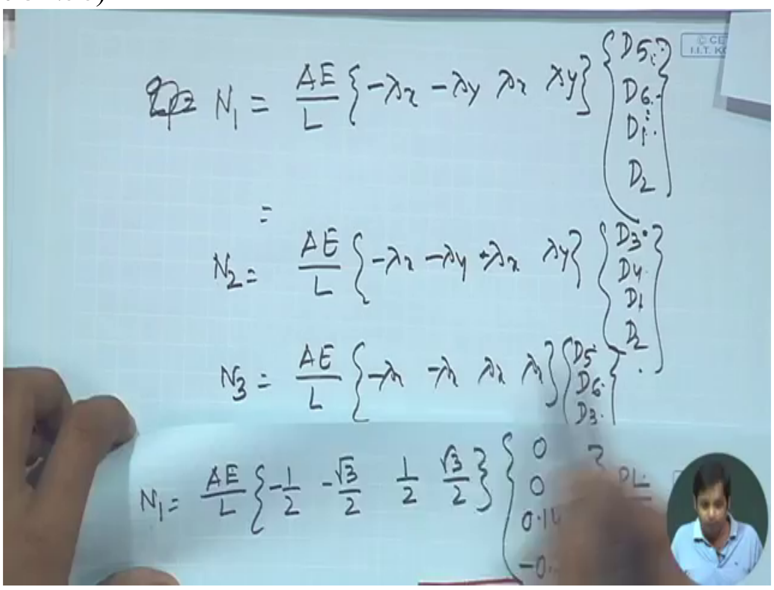

for N 3 and we get N 2 is equal to, N 2 is equal to minus, minus 1 by root 3 P, root 3 P

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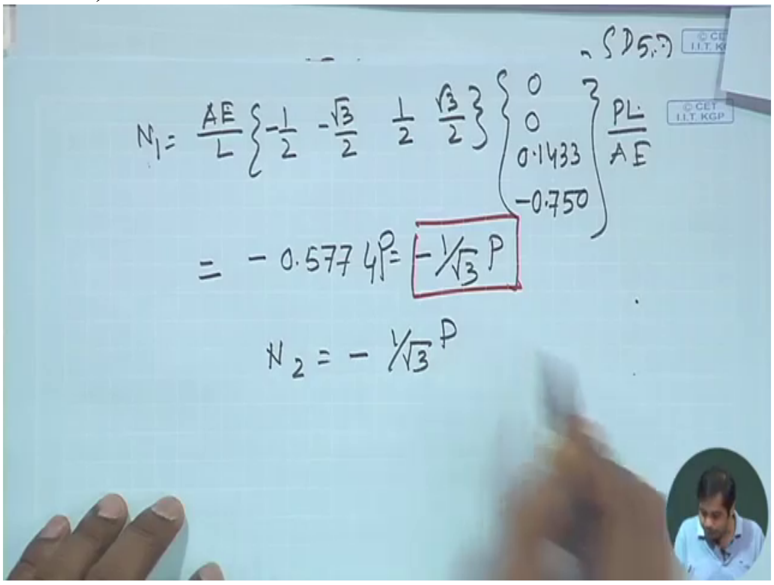

and N 3 is equal to, is equal to 1 by P by 2 root 3,

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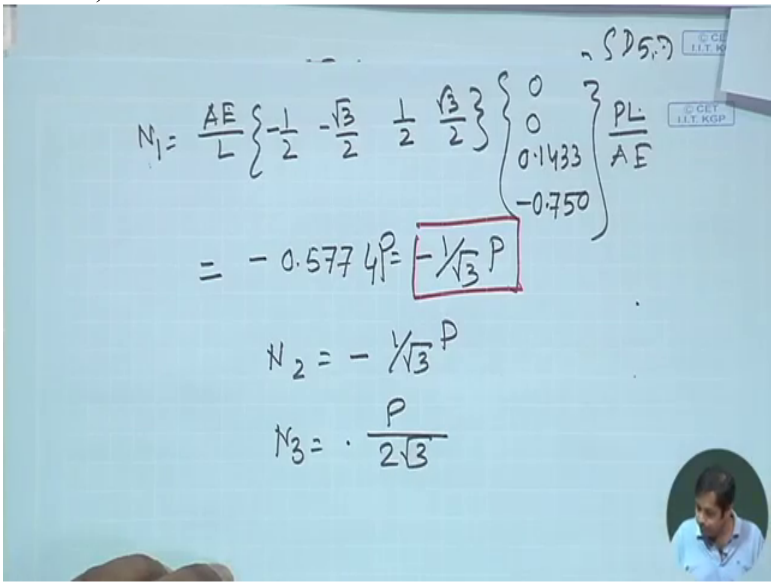

P by 2 root 3, P by 2 root 3,

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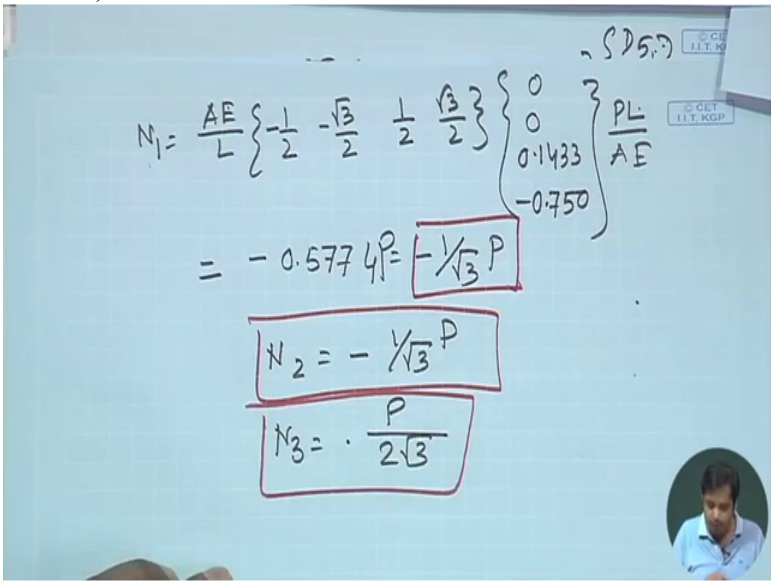

Ok. Now let us see the results. The results are,

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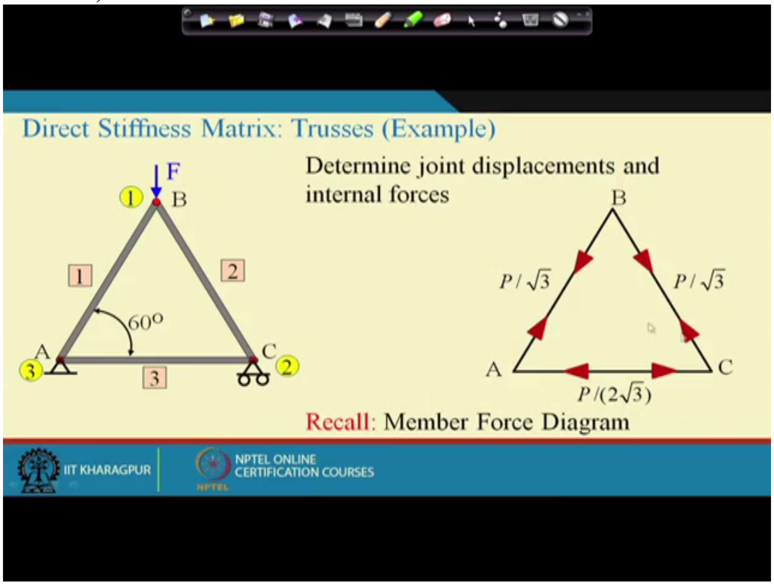

this is the result we have, right. This is compression, the compression that is why you got minus here, and then you got minus here that is why this is compression, this is compression and well this is positive.

So this is, this is the

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single method. You can get the displacement, joint displacements, you can get internal forces, you can get support reactions everything. So this is a very small example of direct stiffness method for truss. And believe me, no amount of, no amount of lecture will help you to, lectures will help you to understand the concept but to, to make yourself comfortable with the different steps involved in this method there is no way other than, other than practice.

So please, please apply these methods to some simple problems. 2 things you please do, you take, first you take a smaller problem which has 3, 4 members with a total number of joints, unknown members, number of unknown degrees of freedom are less, restricted to 3 so that you can, you can, 3 or 4 and get, apply all the steps the way we have applied now and get the final solution. This is the manual, you do these steps manually.

Then next thing you should do, you write a code where all these steps you are doing manually, you write a code for that. Then you, first thing you validate your code using, using a very standard benchmark problem, smaller problem with hand calculation and what code, what result your code gives you, and then you, you take a bigger problem where your number of unknowns are more and the final matrix that, the final matrix that you get, that will be of a larger size which involves so many manual calculation, you do that problem using, using, write a computer, small computer program and you will see and compare this method with the methods that we learnt in previous weeks like slope deflection method, moment distribution method where, where the, where, may be those methods were developed in those time where there was no computers but, but those methods required huge manual calculations, right? But these methods is not require, and it is, it is suitable, see at the end of the day what is important is whether a method can be translated, translated into, into, into, into an, into usable code or not, Ok.

If a method is rigorous, sophisticated, very sound mathematical background but it is difficult to translate them into a code which can be used very easily, then, then the method doesn't become that much successful, Ok. So it is, it is a good thing about this method that it can be translated into code and that please, you, you try yourself and, and, and, and, and, and then only can appreciate in a better way. Ok, then quickly next week what we do is we will apply direct stiffness matrix for, direct stiffness again, direct stiffness method

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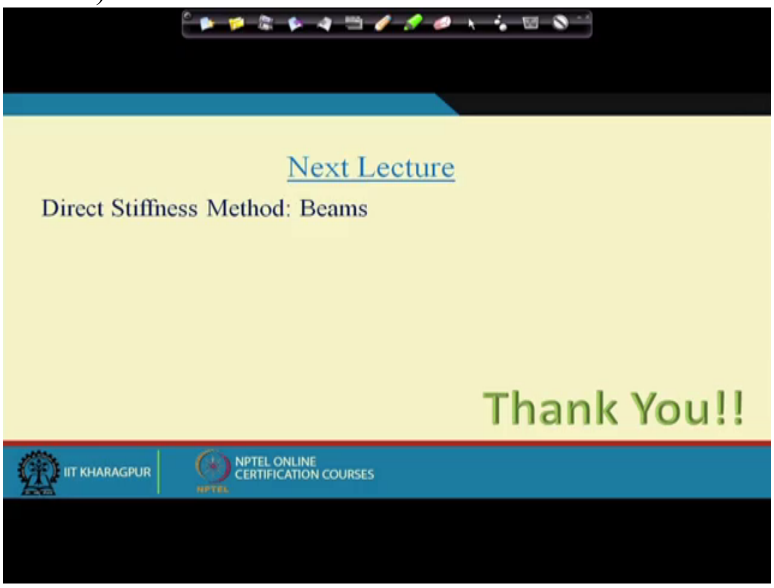

for beams

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and then subsequent weeks we will see how to apply direct stiffness method for frames, Ok. Then we will stop here. See you in the next class, thank you.