Structural Analysis 1
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Lecture 5
Review of Bending Moment and Shear Force Diagram of Beam

Hello welcome to 5th lecture of module 1. You see in this lecture we will review the concept of bending moment and shear force diagram in beam. I believe you have studied bending moment and shear force diagram either in mechanics course or (00:39) course. What we will do is here briefly review the concept, because the concept of bending moment and diagram we will be using throughout this lecture. Okay.

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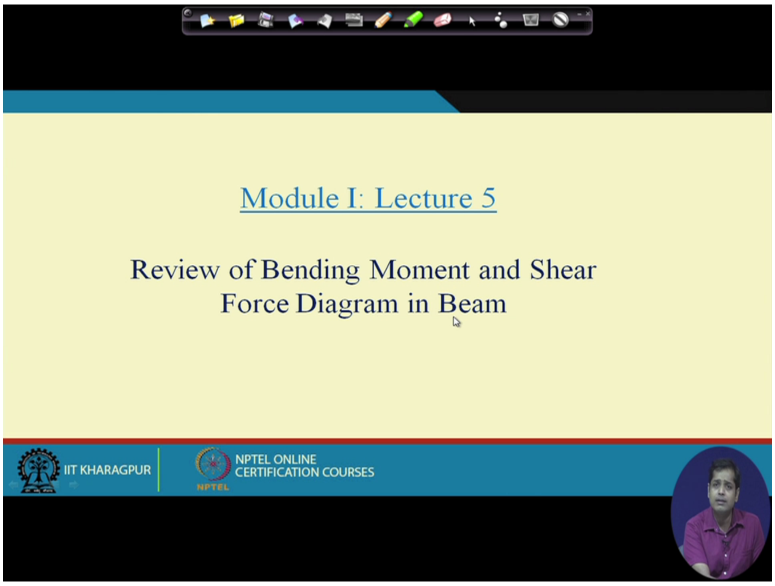

Now what is beam? Beam is essentially a structural element that primarily resists load applied transverse to its longitudinal axis. Now if you see this is simply supported beam, this is the longitudinal axis, okay and this structure element is the subjected to load which is transverse to this axis. This kind of structural component is called beam.

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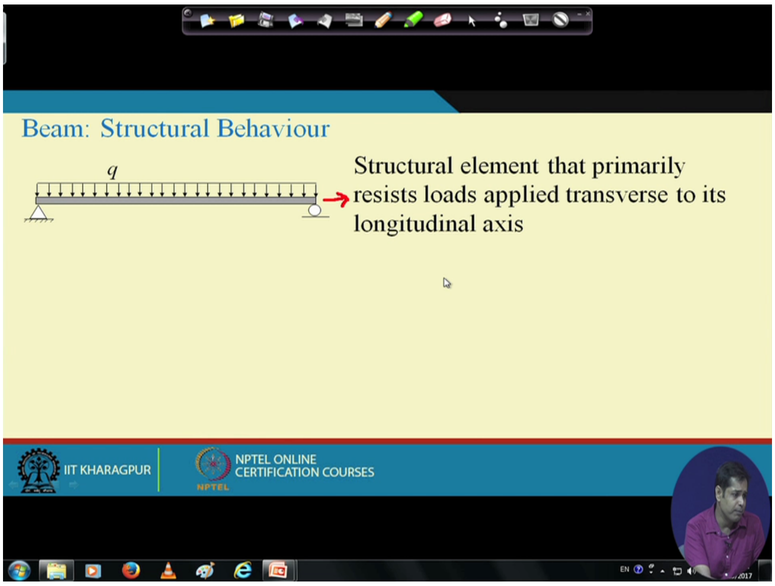

And when it is subjected to this kind of loading or any kind of transverse loading, it undergoes bending, right? Like this.

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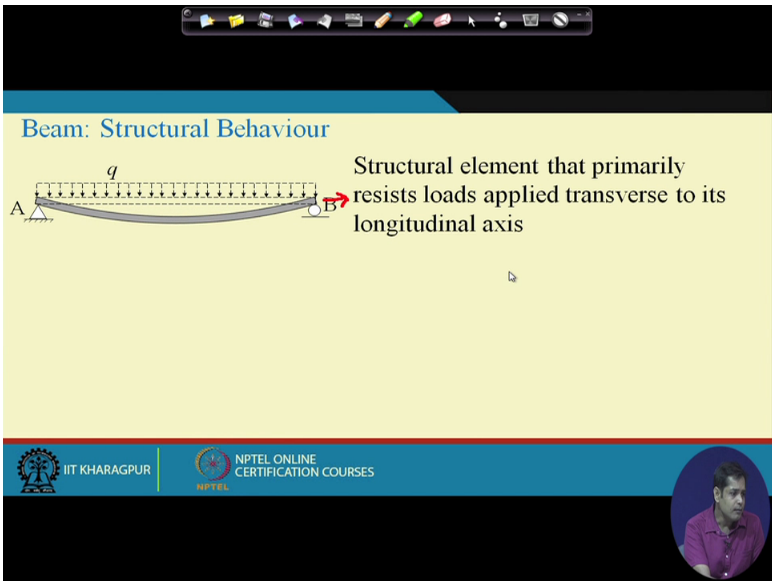

Now you know the internal forces developed in the beam is bending moment and shear force. Now let us see what is bending moment and shear force. If you see if I take one section at a distance x from A and draw the free body diagram of this part and this part then what happens?

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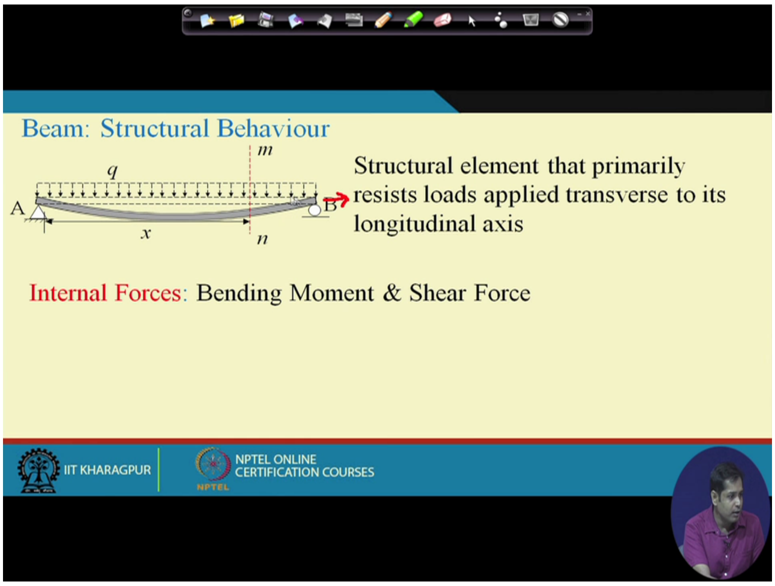

This is the free body diagram of this part and then again this is a free body diagram of this part.

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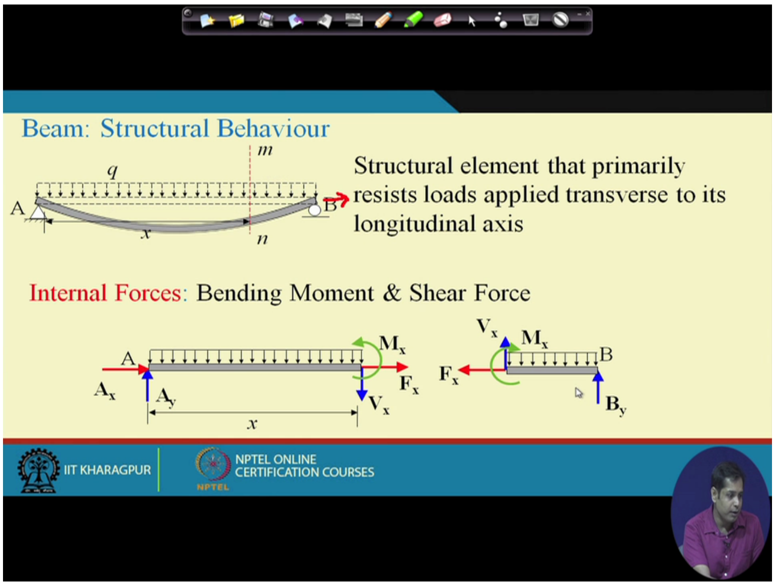

Now this is hinge support so it is replaced by two reactions Ax and Ay. Now at this point the internal forces are horizontal direction in this case Fx, then vertical Vx and then moment Mx.

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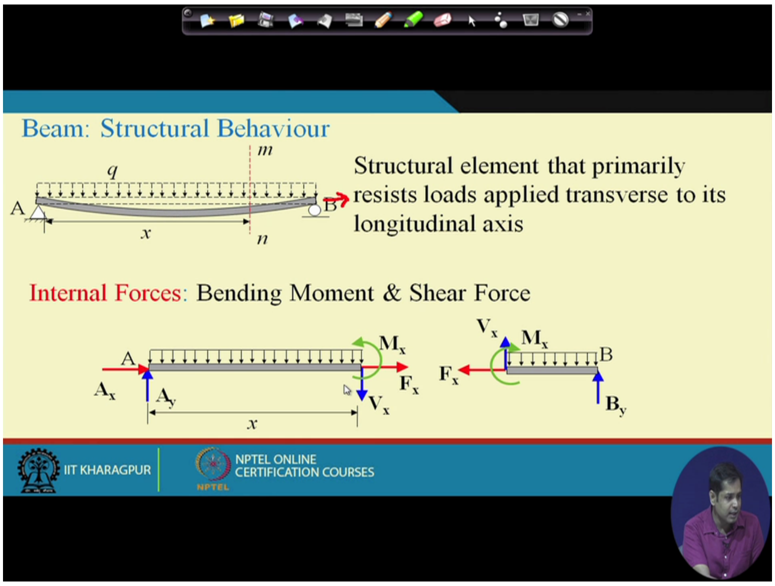

Similarly in this case this roller support is replaced by reaction By and these are the internal forces. If you remember in the last class we said whenever you break a structure and that continuity of that point in a free body diagram needs to be represented by the internal forces, right?

Now when you draw the internal forces, sign of the internal forces needs to be taken very carefully. For instance when I say the objective is in equilibrium, every point of the object is in equilibrium. So this point was an equilibrium, right?

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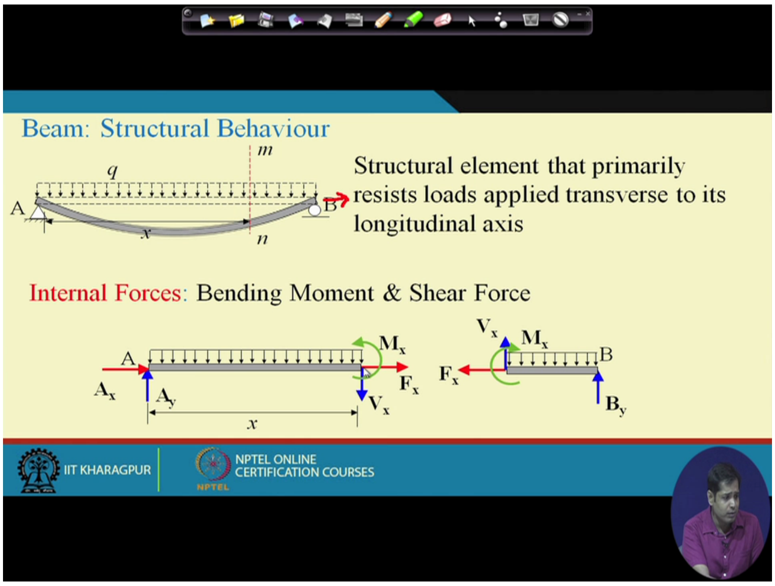

It means the net force in this point, net internal forces need to be zero. So therefore if you are showing Fx in this direction for this part, you need to show Fx in the opposite direction for this part. Similarly for Vx, similarly for moment. Now this Vx is shear force, Mx is bending moment and Fx is actual force. Now what we assume here is there is no actual force. So beam only resists the external forces which is acting in a transverse direction by bending moment and shear force.

In a practical purpose we will see there are many cases where any structure component may be subjected to all. It may be subjected to shear force, maybe bending moment actual force together but analysis of those kind of structure component we will be discussing later. Okay. But at least for this case there is no actual force, right? Now let us define the sign convention. You see this is our sign convention we used for bending moment and shear force.

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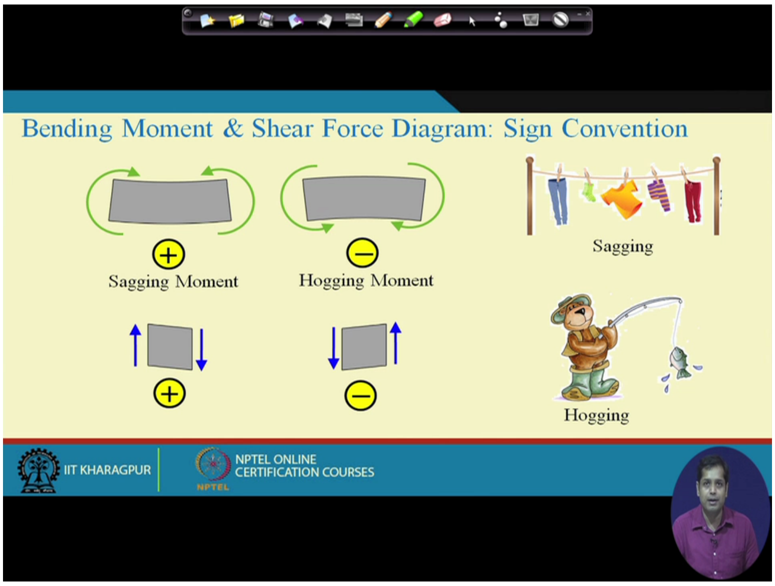

Then what it says? You see there are two signs convention. Okay. One sign convention is the sign convention we used in the bending moment and shear force diagram and another sign convention is sign convention that we used when we do algebraic operation between moments and shear forces. Okay. Now let us first discuss what sign convention we will use in the bending moment and shear force diagram.

The sign convention in bending moment shear force diagram is this. You see this kind of movement is called sagging moment. Okay. Because it causes sagging in the structure. Okay.

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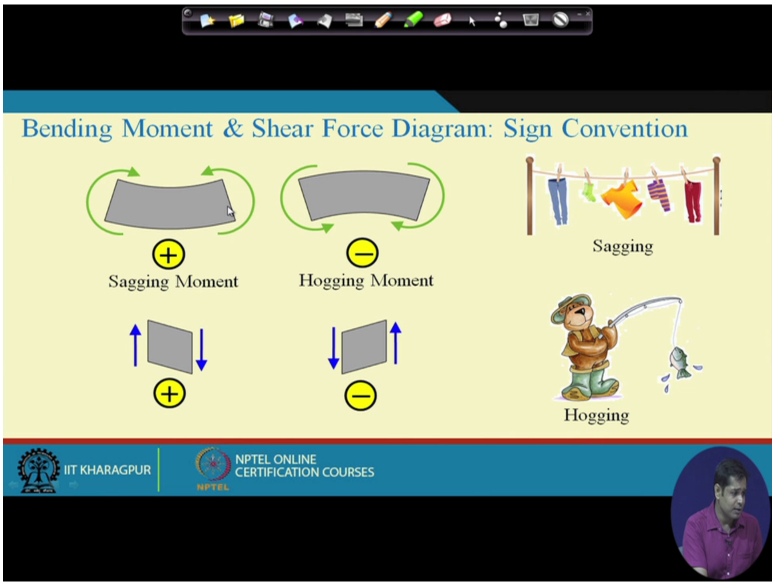

And this kind of movement is called hogging moment. Now (sa) one example for hogging moment is when a fisherman holds a fishing stick then the fishing sticks bends like this. Okay. This moment generated in the fishing stick is hogging moment.

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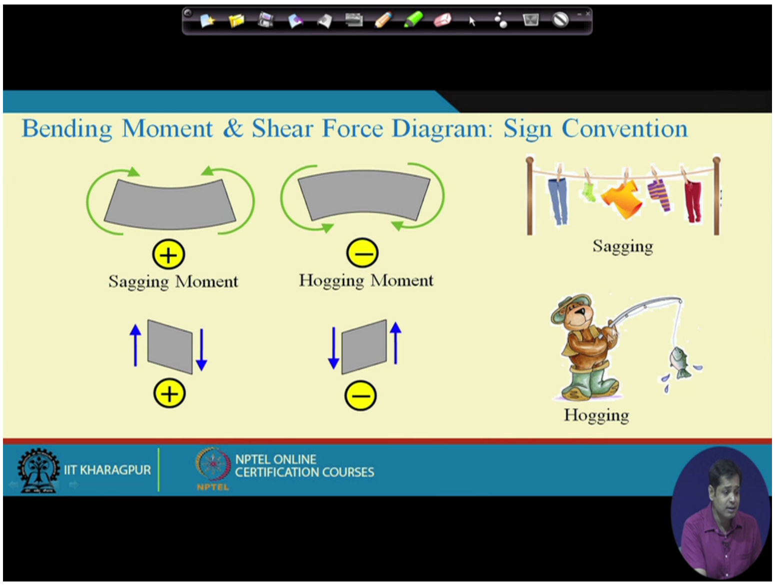

Now example of sagging moment is if you take a rod and which is subjected to or very common cloth hanging rod that we can see in our household. Then you might have observed it that rod experience is sagging like this. In this case the movement generated or the movement causes this is called sagging moment.

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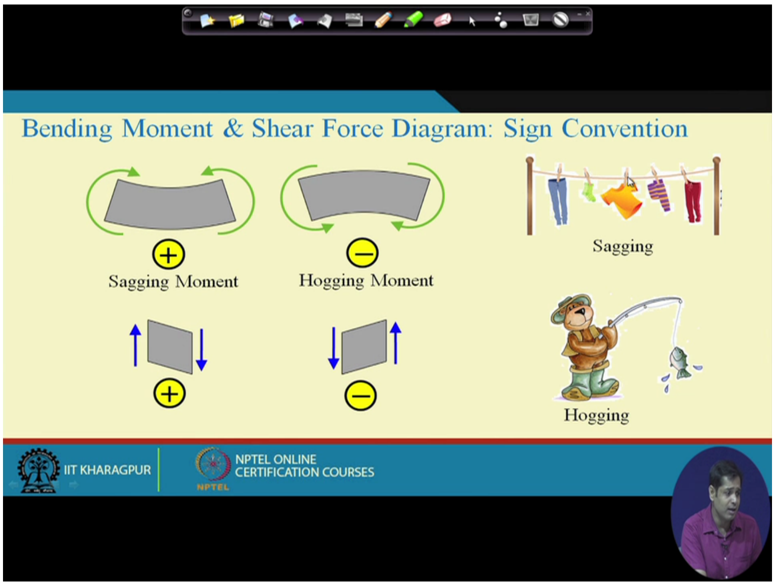

Now for (shea) bending moment and shear force diagram this is positive and this is negative. Similarly for shear force, the shear force which produces clockwise couple is considered as positive, shear force which produces anticlockwise couple considered as negative. Now if you remember throughout this course for all (alb) algebraic operation we have been using clockwise moment as positive and anticlockwise moment as negative and vertical direction force as positive and horizontal direction force as negative.

Means we took clockwise moment as positive and this is positive direction, right?

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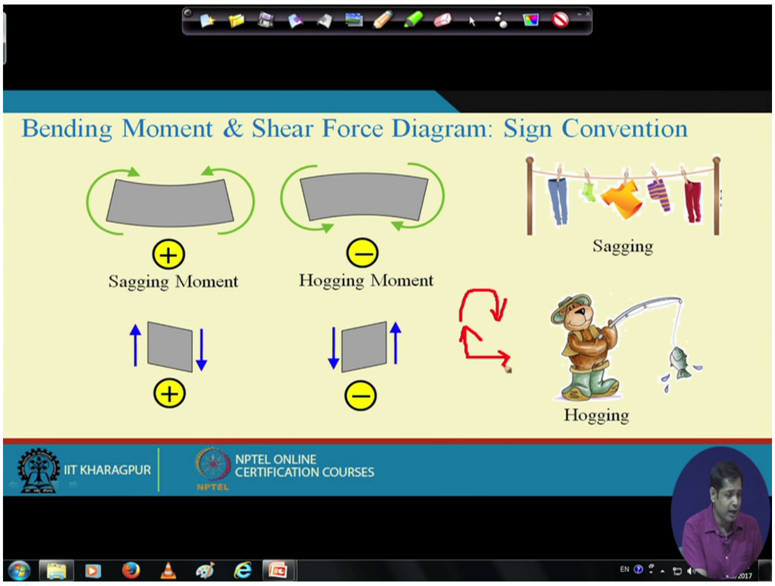

But this sign convention we used for algebraic operation. But when we draw the bending moment and shear force direction we use this sign convention. Okay.

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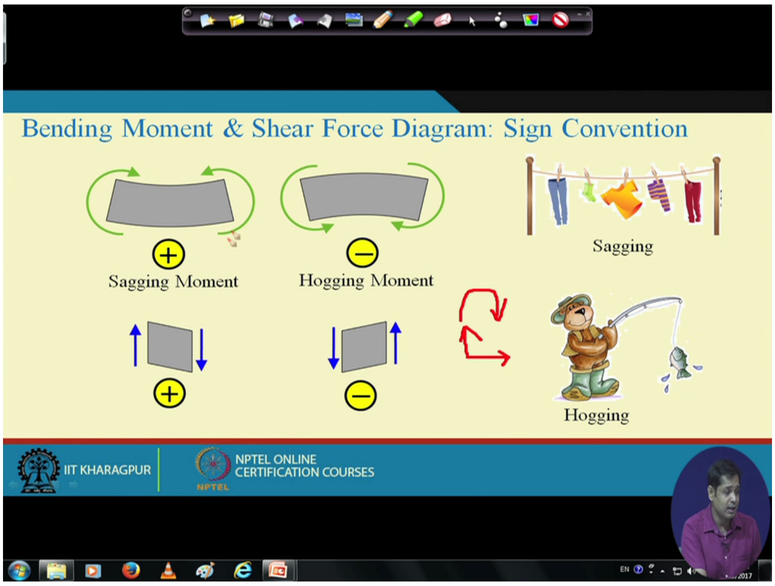

Whether it is a sagging moment or it is hogging moment. Okay. Now this is the two sign convention for bending moment diagram. This is sign convention bending moment and shear force and for all algebraic operation this sign convention is positive. Okay.

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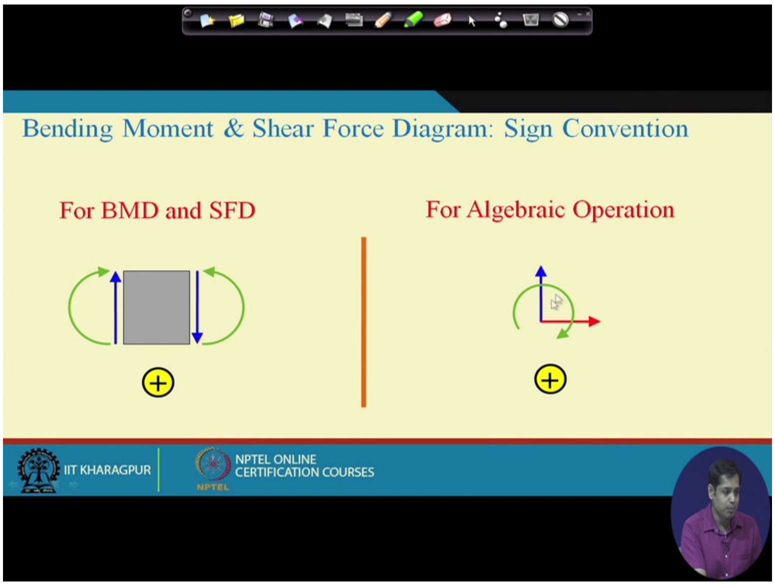

Now let us demonstrate through some examples. Okay. Now the first example is if you take it is a simply supported beam which is subjected to uniformly distributed load. Okay.

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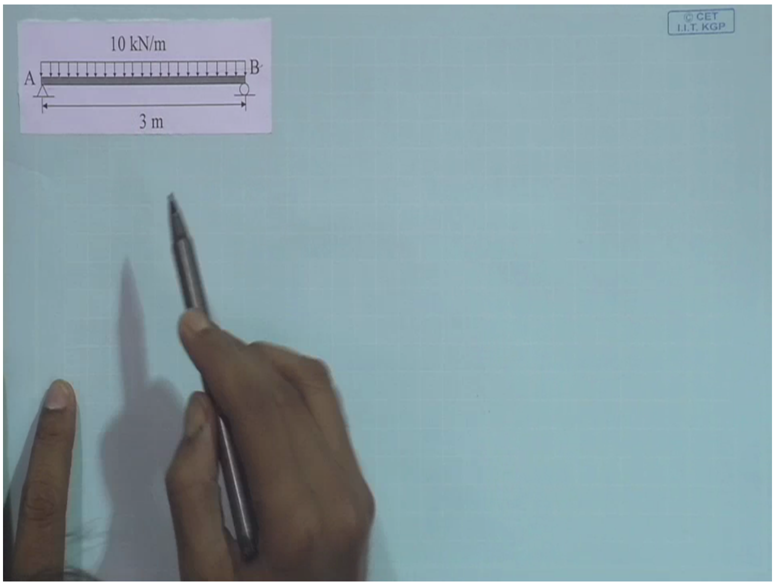

Now what we have to do is we have to first find out the reactions at the support, then we need to determine the bending moment. The bending moment diagram is essentially the graphical representation of moment and shear force which tells you how the moment and shear force varies along the length of the beam. Let us find out that. The first is we need to determine the support reaction. Let us draw the free body diagram of the entire structure.

Free body diagram will be, this force will be Ay and then this is Ax, this is By and this is the very standard representation of uniformly distributed load which is 10 kilo Newton per metre and this length is 3 metre. This is the free body diagram. Okay.

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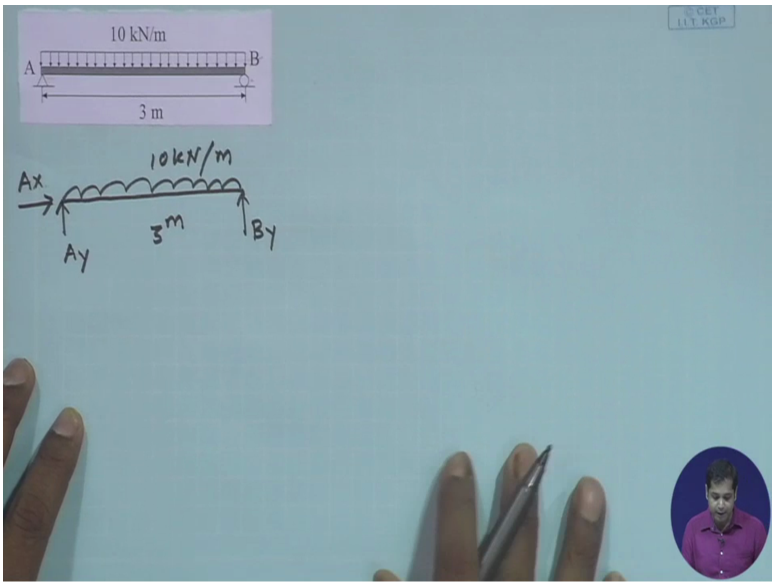

Now the first is summation of Fx is equal to zero which immediately gives me Ax is equal to zero, right? Then next is if you take summation of moment at A is equal to zero which gives what? Which gives you anticlockwise moment due to By minus 3 into By and then plus clockwise moment due to external load. This load will be the total load is 3 into 10 multiplied by the distance and the centre of application of the result and it is 3 by 2. That is equal to zero. So this gives me By is equal to 15 kilo Newton.

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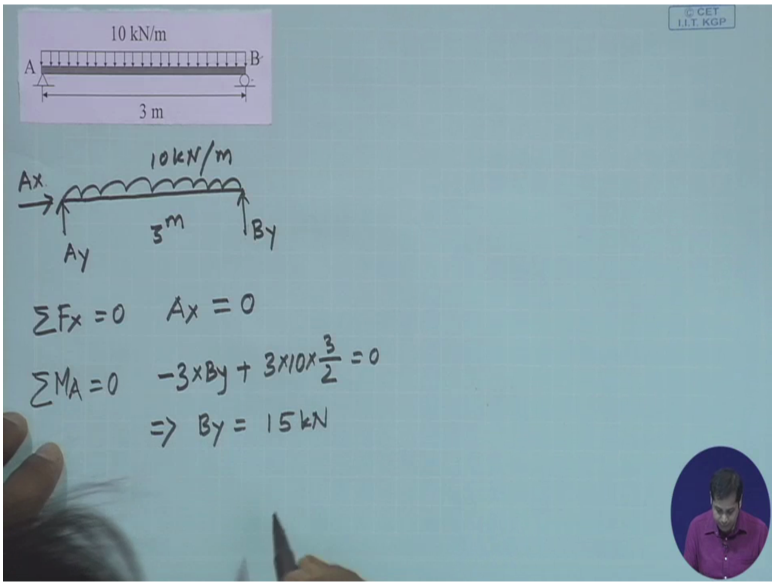

And similarly summation of Fy is equal to zero which gives me Ay plus By is equal to total downward load 3 into 10. And from this equation if we substitute By is equal to 15, we will get Ay is equal to again 15 kilo Newton. So we have support reaction Ax is equal to zero and then By is equal to zero and then Ay is equal to zero, right?

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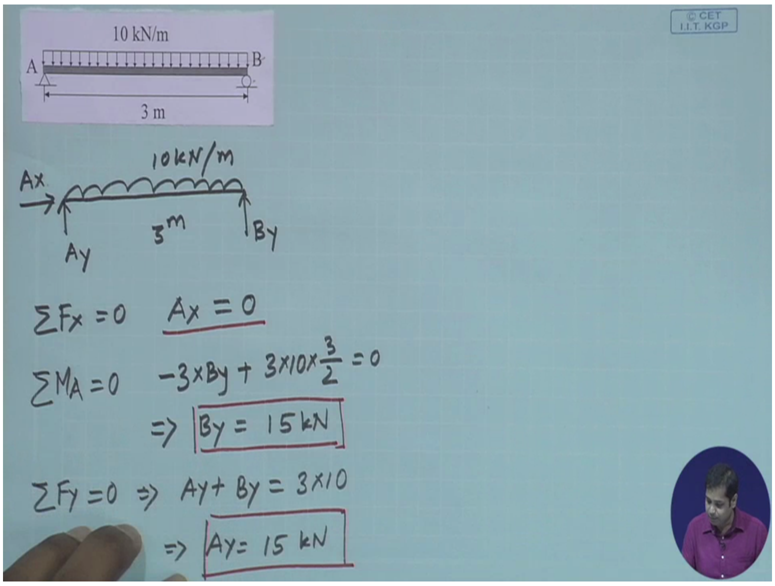

Let us find out bending moment diagram. Now to do so, let us take one section here at a distance from A. Okay. And draw the free body diagram of this part.

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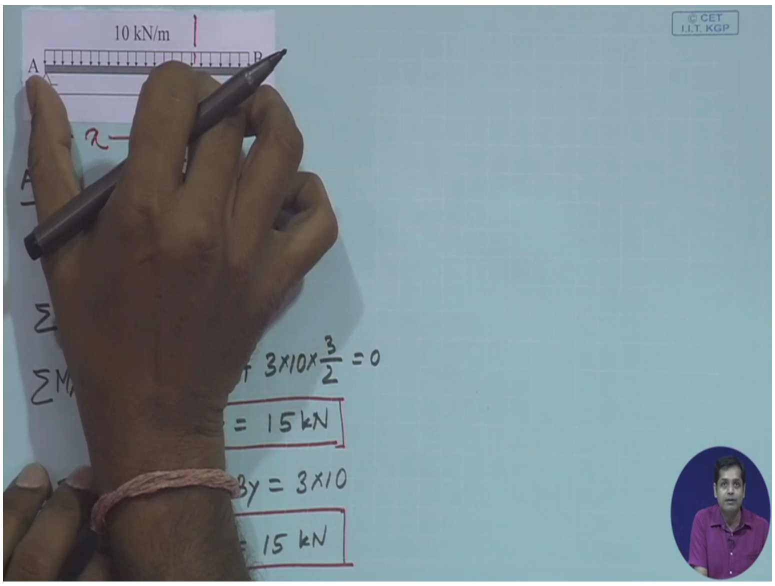

Now if you draw the free body diagram of this part, this become this. This is Ay. Ax is equal to zero that is why there is no point to show that. Then uniformly distributed load 10 kilo Newton per metre. This distance is x and then if you remember our sign convention was this moment is positive and for shear force, this shear force and this shear force is positive, right?

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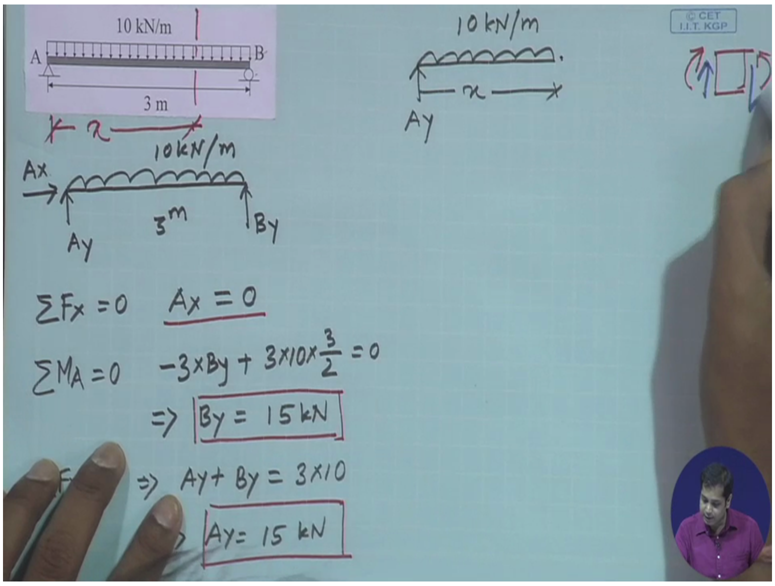

So the direction of force is this and this. Okay. So this is Vx which is the shear force at (dis) x and this is Mx which is moment at a distance x from A.

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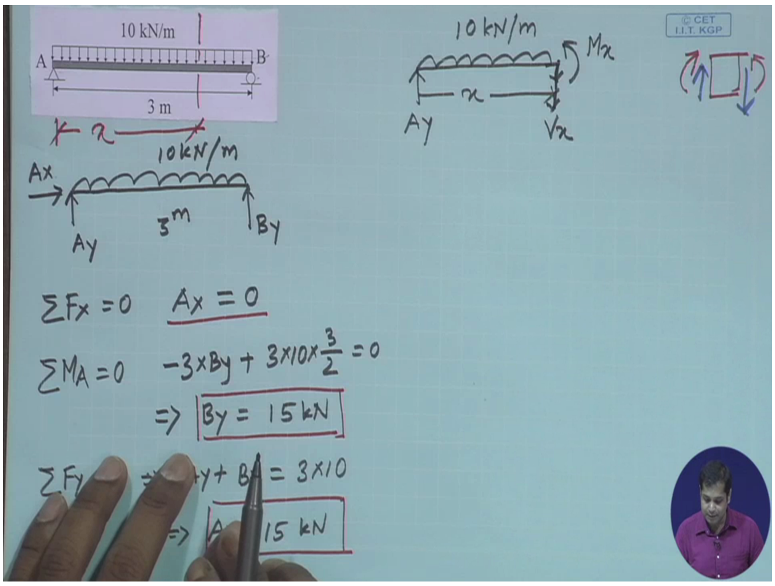

Now take summation of Fy is equal to zero. What are the forces you have? Ay upward direction plus minus 10 into x which is externally applied uniformly distributed load. And then minus Vx is equal to zero, right? And Ay is equal to 15 already we determined and this gives me Vx is equal to 15 minus 10x. So this is how the Vx varies with x.

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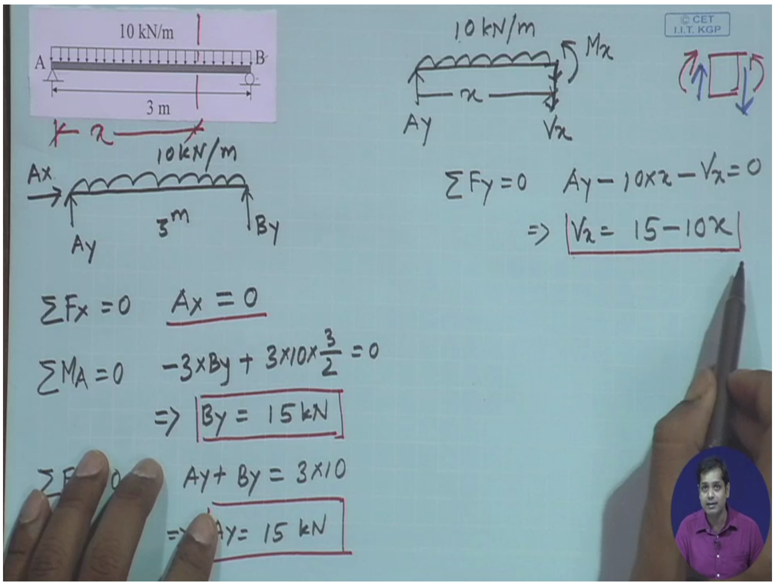

Now then summation of say Ma is equal to zero. This gives me what? Only forces is or you can take summation of Mx is equal to zero, moment about this point. And this gives me, Vx will not contribute, Ay will contribute, the external load will contribute and the Mx itself. This gives me, Mx is anticlockwise direction, this is negative if you remember for all algebraic operation we use whether the sign convention will be anticlockwise or clockwise.

So Mx is minus Mx then plus Ay into x. And then minus 10 into x is the force which is at a distance x by 2. That is equal to zero. Okay.

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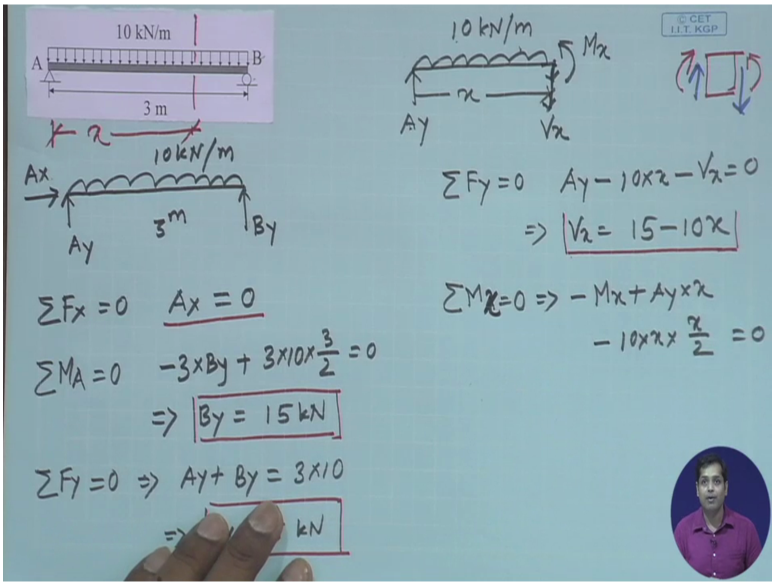

Ay is equal to 15 we know. And this gives you Mx is equal to 15x minus 5x square. Okay. So this is how Vx varies with x and this is how Mx varies with x.

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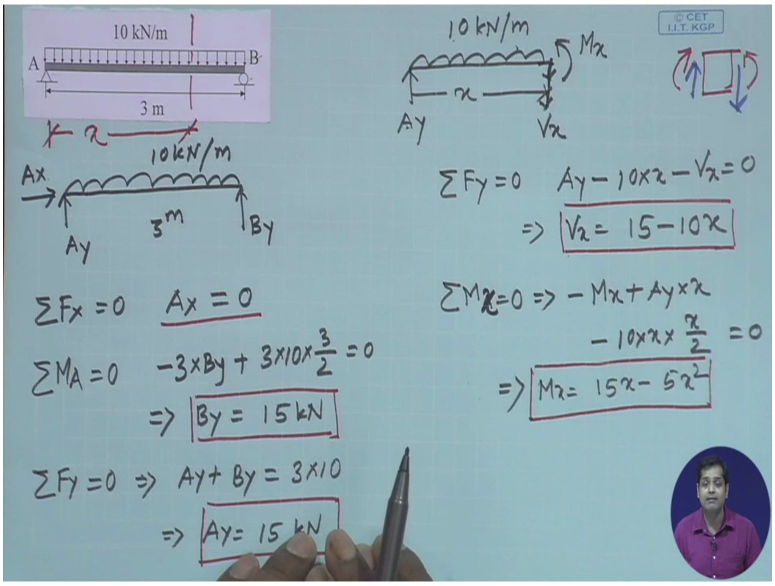

Now let us draw the diagram. Bending moment and shear force diagram will be graphical representation of this equation along this length and bending moment diagram will be (ver) graphical representation of this equation along this length.

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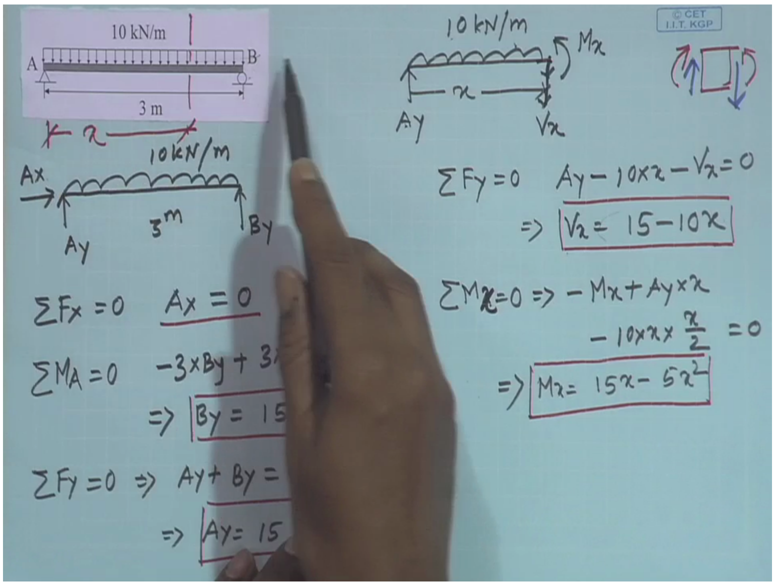

Now if I do that then that was the problem, okay. That was the AB. Now Vx we obtained. Vx is equal to 15 minus 10x and Mx is equal to 15x minus 5x square.

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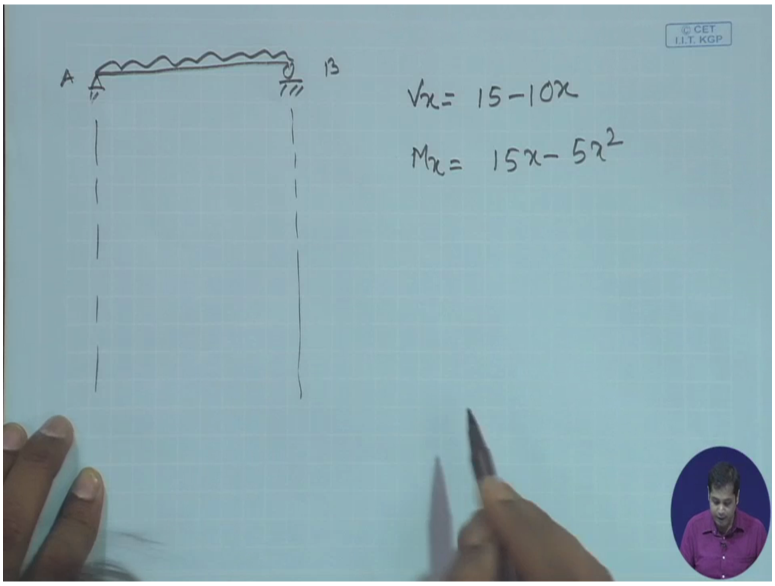

Let us first draw shear force diagram. You know one check is you should immediately check by substituting the value of x here which is then you can check whether it is consistent with the problem and the support condition.

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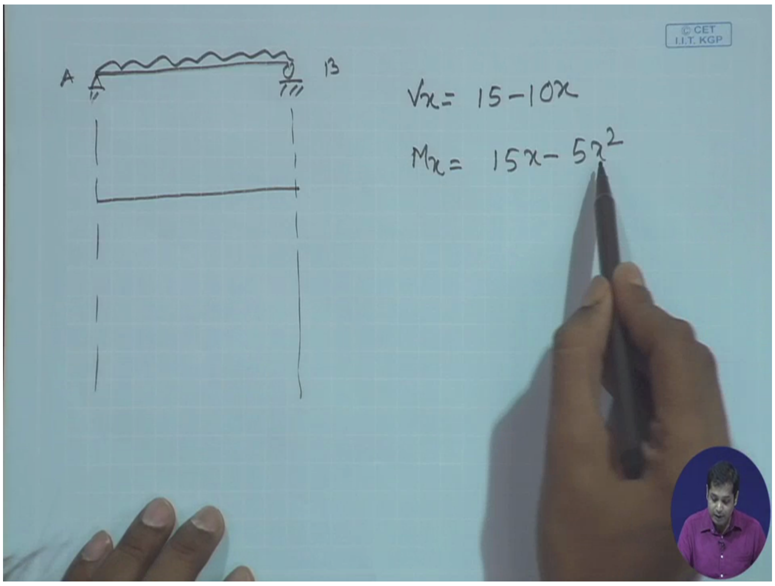

For instance you see this distance is simply supported beam. Therefore at this point and at this point your bending moment will be zero. Let us see if I substitute x is equal to zero, Mx is equal to zero. If I substitute x is equal to 3 here at B then also Mx is equal to zero. So this expression is consistent with this.

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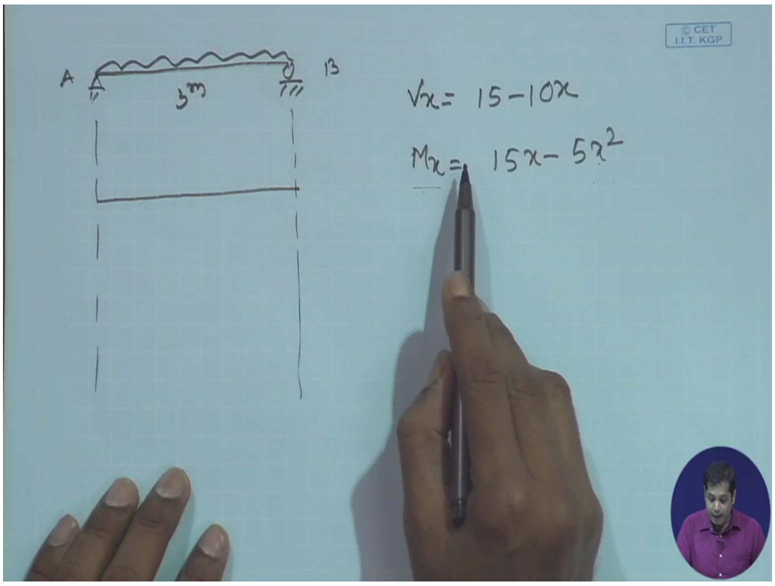

Let us see whether this is consistent whether not? You see if I substitute x is equal to zero which gives you Vx is equal to 15. If you remember 15 was the (rea) support reaction at A. If I substitute x is equal to 3 here, this becomes minus 15. So again if you remember this reaction at support B was 15 which is shown vertically upward.

But while representing the shear force we used this as positive. That is why it is negative. So these equations are consistent with the support reactions and the conditions here.

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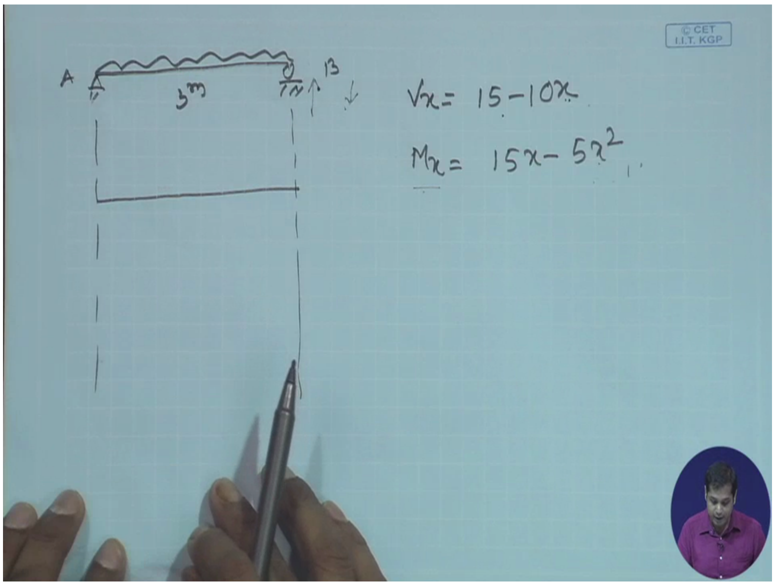

Now x is equal to zero, therefore bending moment is 15 and x is equal to 3 shear force is minus 15 and throughout this length bending moment varies linearly. So this will be the shear force diagram. This is 15 this is also 15. I am just writing the absolute value because below this it is understood, it is negative. Okay.

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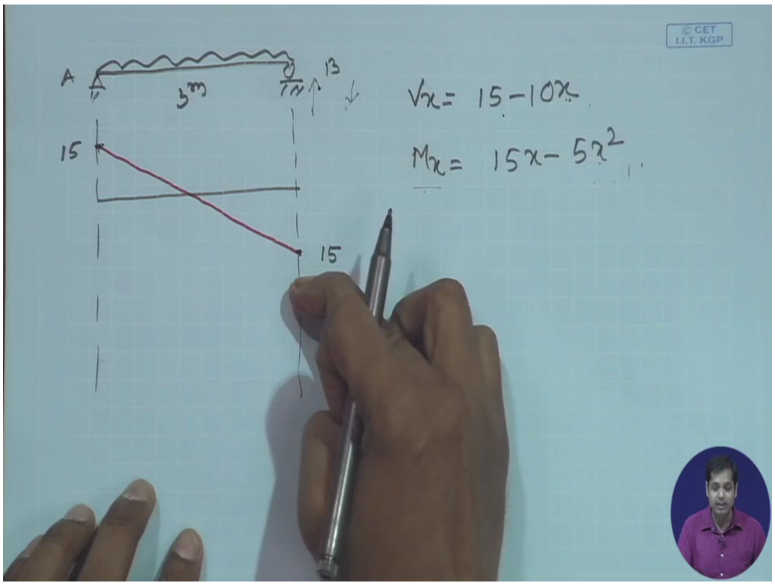

Now let us draw the bending moment diagram. X is equal to zero and x is equal to 3. Bending moment is zero and then between x is equal to zero and between A and B bending moment varies. As per these equations if you plot it, this equation will become something like this. Okay.

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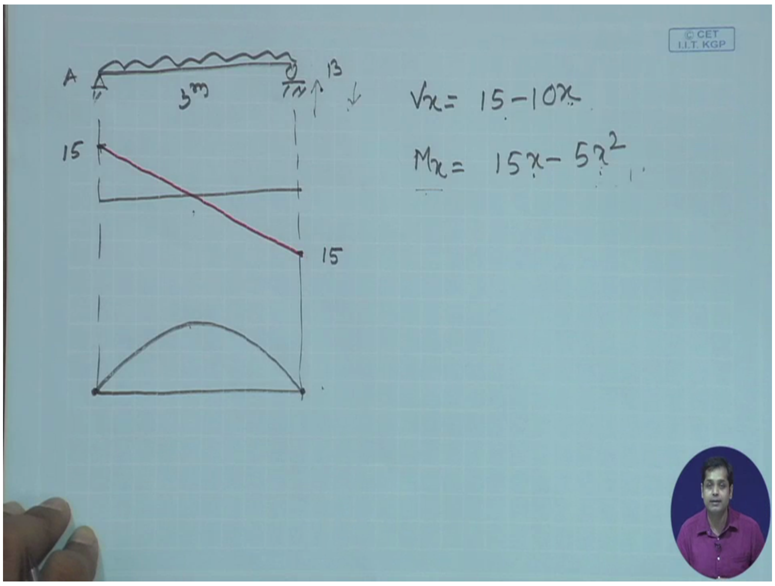

So this is positive, this is negative, right? This is shear force diagram and this is bending moment diagram.

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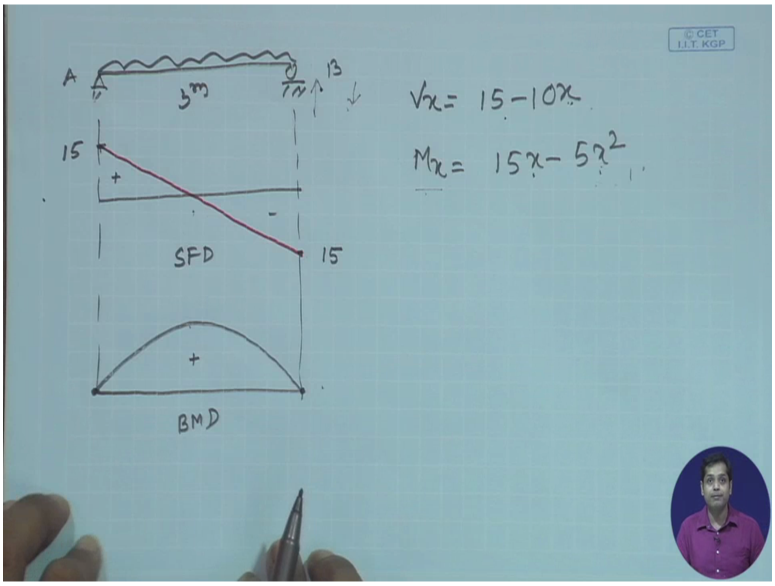

Now one observation here please note that in this equation your shear force is linear and bending moment is quadratic. We are not going to make any conclusion based on this observation. Let us see other problems and how the order varies in those cases as well.

Now suppose if you have to find out what is the maximum bending moment here, maximum bending moment will be where dM dx is equal to zero and if you substitute that you will get x is equal to 1 point 5 metre. And if you substitute x is equal to 1 point 5 metre in here, you get M max will be 11 point 25 kilo Newton metre. So this will be 11 point 25 kilo Newton metre.

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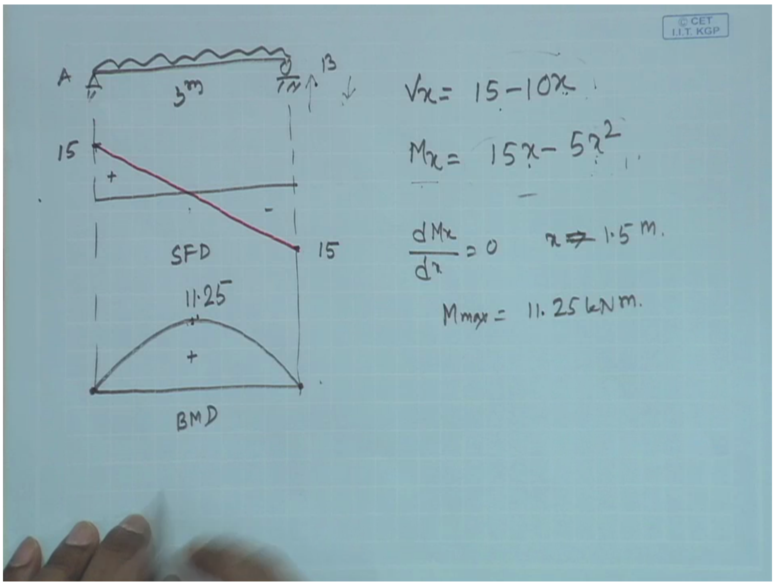

Another observation here at the middle the shear force is zero and at the middle bending moment is maximum. So two observation we have here is shear force is linear and the bending moment is one order higher quadratic.

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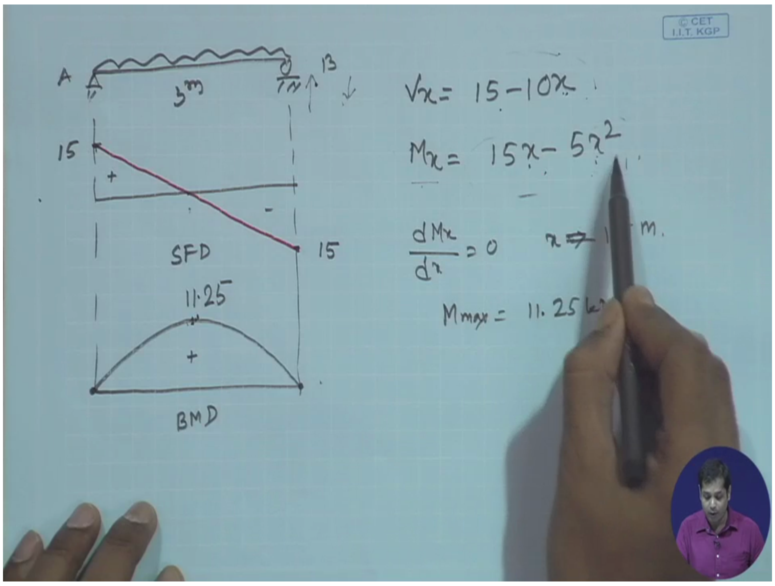

Here shear force is zero and at the same point the bending moment will be maximum.

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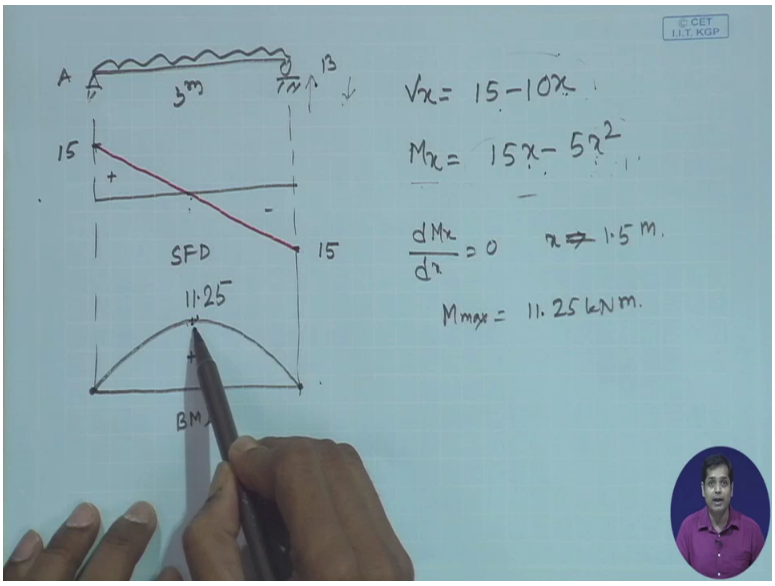

Let us see whether similar observation we have another problems as well. Let us do one more example. Okay. Now this was probably the simplest example of bending moment and shear force, the first one. And if you take any book the first example gives you a simply supported beam subjected to uniformly distributed load.

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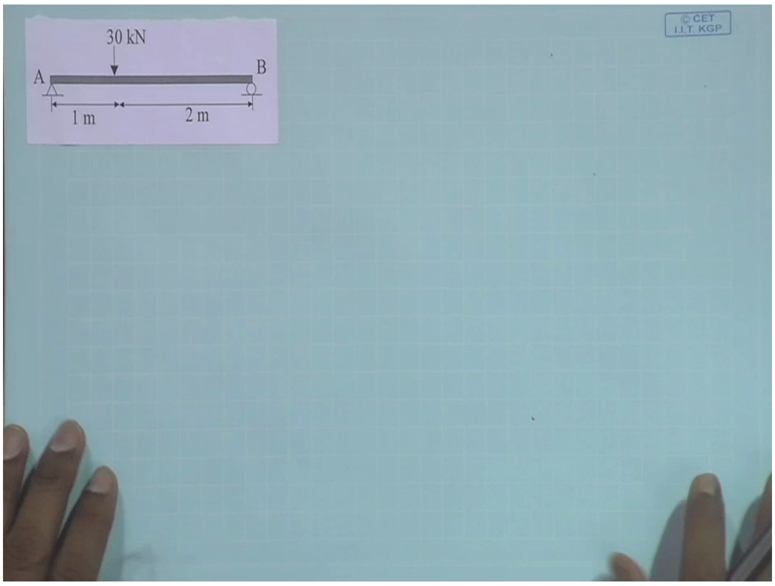

Let us take one more example here. It is again a simply supported beam but it is subjected to concentrated load. Okay. Now again draw the free body diagram. This is Ax, Ay, then By, this is 30 kilo Newton, this is 1 metre, this is the free body diagram of AB. Okay.

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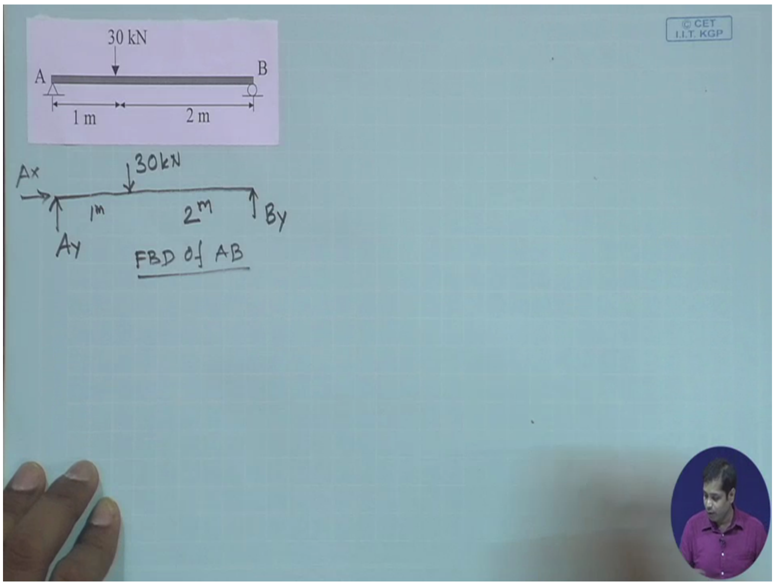

Now again Ax will be zero that we get directly from the first equation. Summation of Fx is equal to zero. And then if you take moment about A that is equal to zero, then this gives you 30 into 1 which is clockwise moment minus By into 3 is equal to zero. This gives you By is equal to 10 kilo Newton. And similarly if you take Fy is equal to zero, Ay plus By is equal to 30. This gives you, substitute By is equal to 10. Ay is equal to 20 kilo Newton.

Whenever you write force and moment please write unit also because this is a common mistake I have seen that you write the value but then forget to mention the unit. So these are the support reactions, okay. And Ax is equal to zero is another support reaction. Okay.

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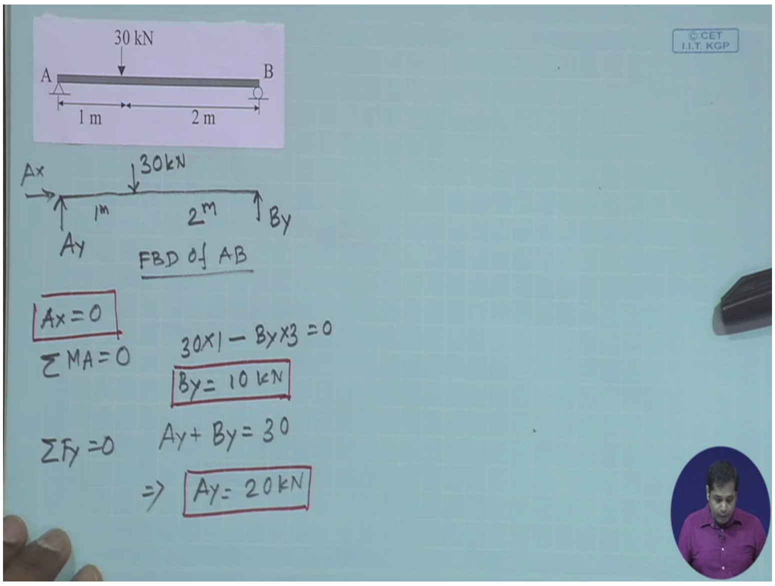

Now you see this we need to find out the bending moment diagram and shear force diagram. First you take one section here at a distance x from A. This section is between point A and point C. So the first case Ax varies from zero to 1, okay.

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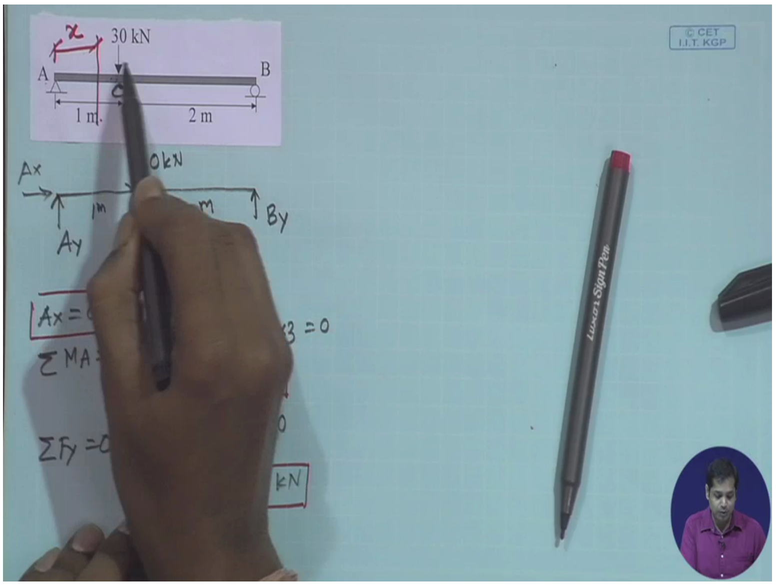

Now first is Ax varies from zero to 1, okay. Now draw your free body diagram. This is Ay, Ax is equal to zero not showing explicitly. This is Vx and this is Mx, okay and this distance is x. No other external load between A to C.

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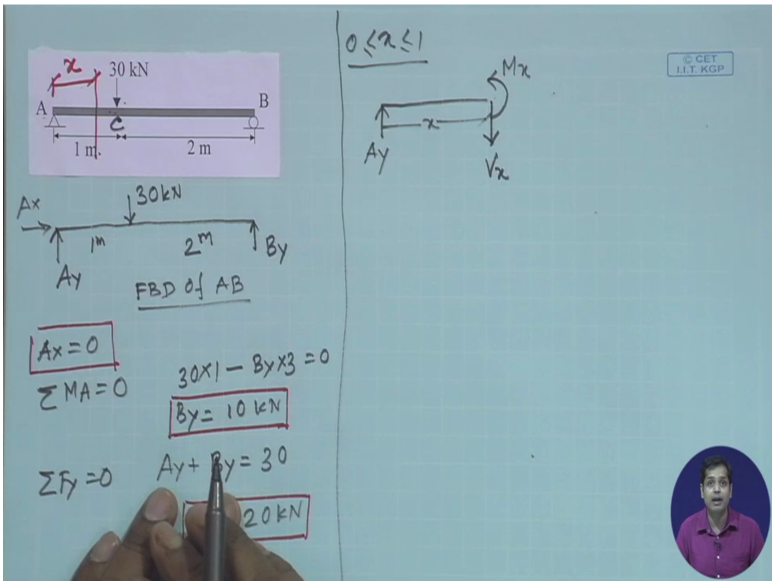

Now this give you summation of Fy is equal to zero, this gives you Vx is equal to Ay. And Ay we already obtained 20. So this gives you Vx is equal to 20 kilo Newton. So this is how Vx varies between A to C.

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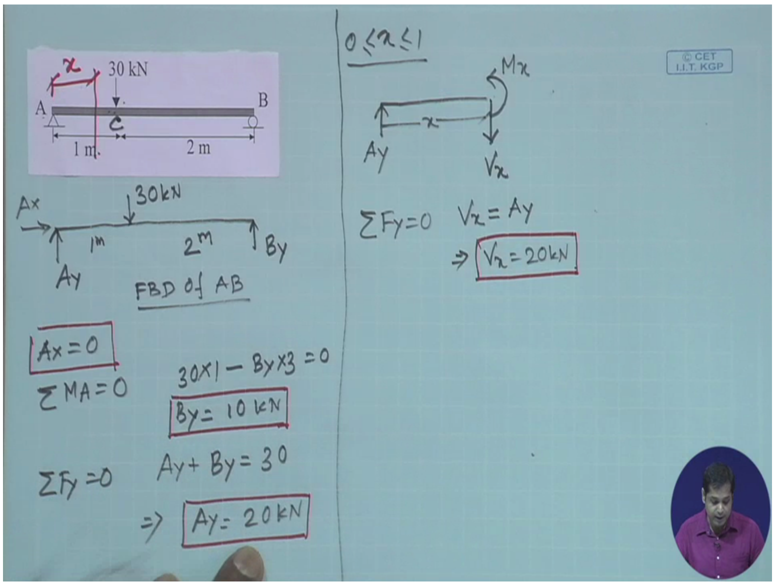

Let us take moment at x is equal to zero. So moment at x is equal to zero will be minus Mx because Mx is anti clockwise direction. Again and again I am repeating that when you do the algebraic equation check whether it is clockwise or anticlockwise. Then plus Ay into x. That is equal to zero, okay. And this gives you Mx is equal to, you see Ay is equal to 20, this gives you 20x. So this is how bending moment varies between 1 to C.

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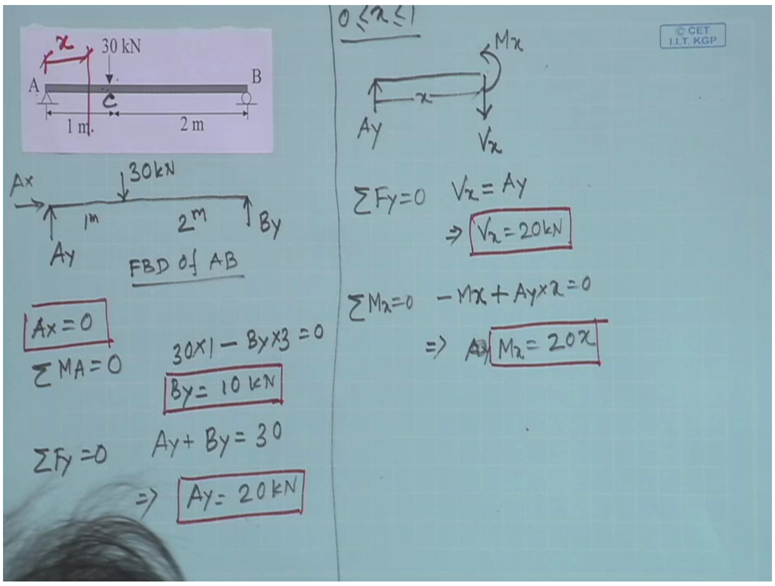

Let us see what happens when you take one more section it is section 1, then one more section here. This is again at a distance x from the x but now in this case x varies from C to B. Means x varies from 1 metre to 3 metre.

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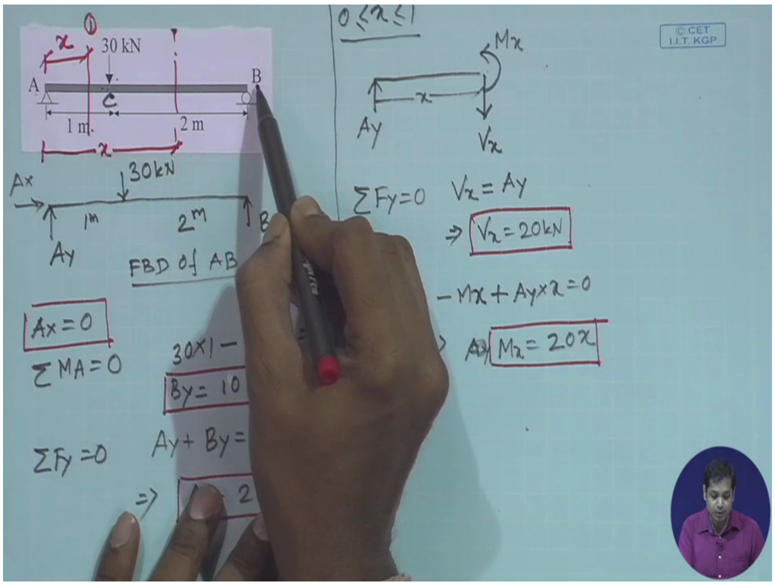

So if I draw the bending moment this is again Ay , this is 30 kilo Newton, this is 1 metre and then again this is Vx and then this is Mx, right?

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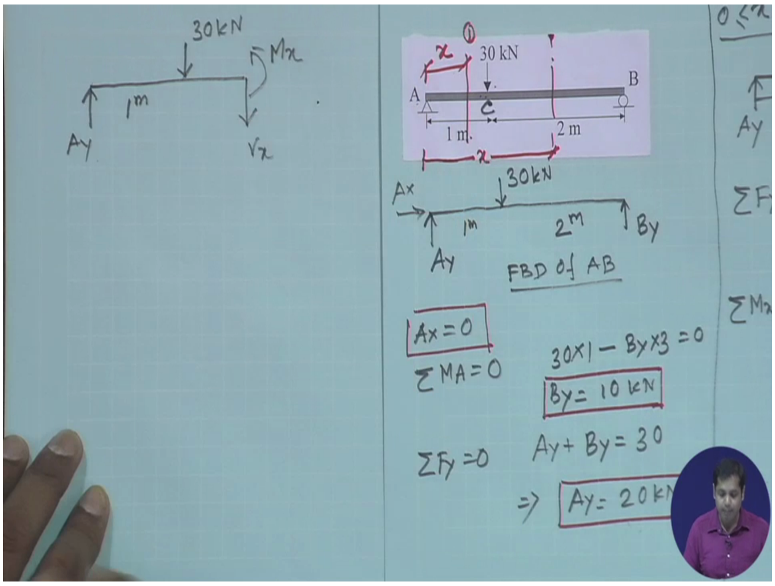

Now summation of Fy is equal to zero. This gives you Ay minus Vx minus 30 is equal to zero. And Ay is equal to 20 we have already obtained, this gives you Vx is equal to minus 10 kilo Newton. So this is how Vx varies between C to B. And take Mx is equal to zero and Mx, Ay into x, this distance is x.

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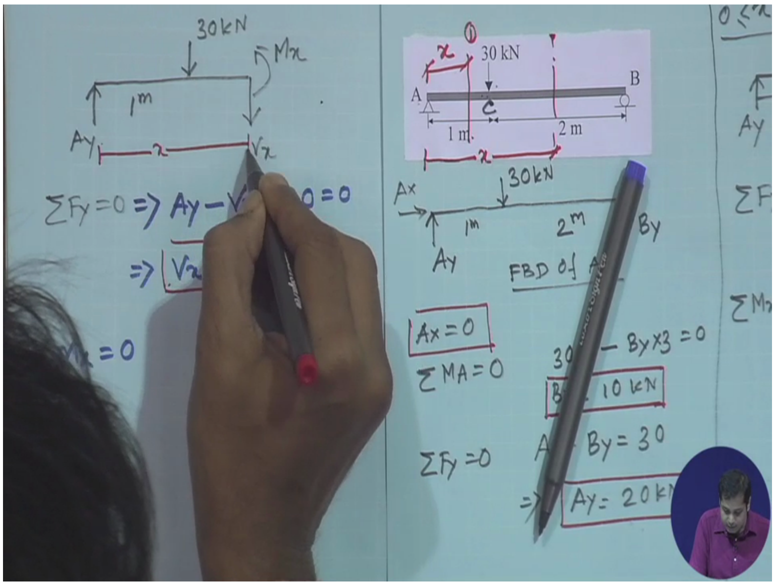

Ay into x which is clockwise. Then minus 30 into x minus 1 which is anticlockwise. And again minus Mx which is anticlockwise, this is zero. And this gives you, Ay is equal to 20, so this gives you 20x minus 30x plus 30 minus Mx is equal to zero. And Mx is equal to minus 10x plus 30.

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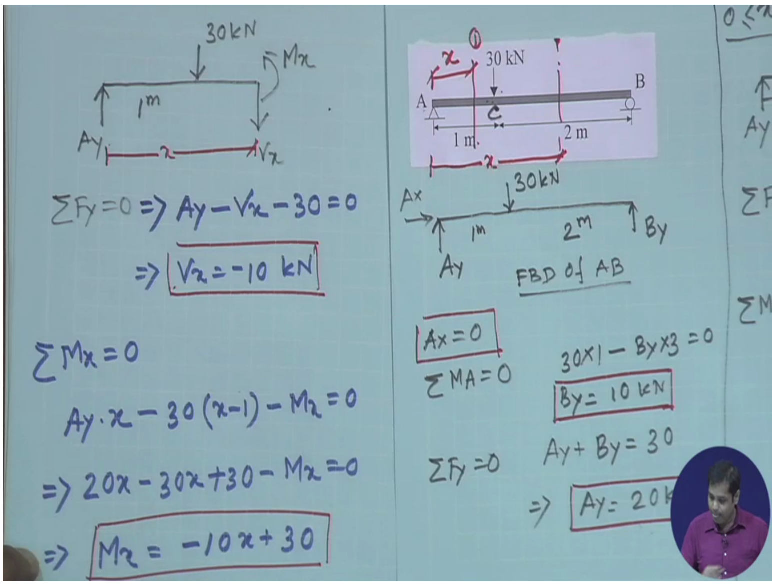

Now this one is x from 1 metre to 3 metre. Now if you see we have two results. This one is for x between zero to 1 and this is between 1 to 3.

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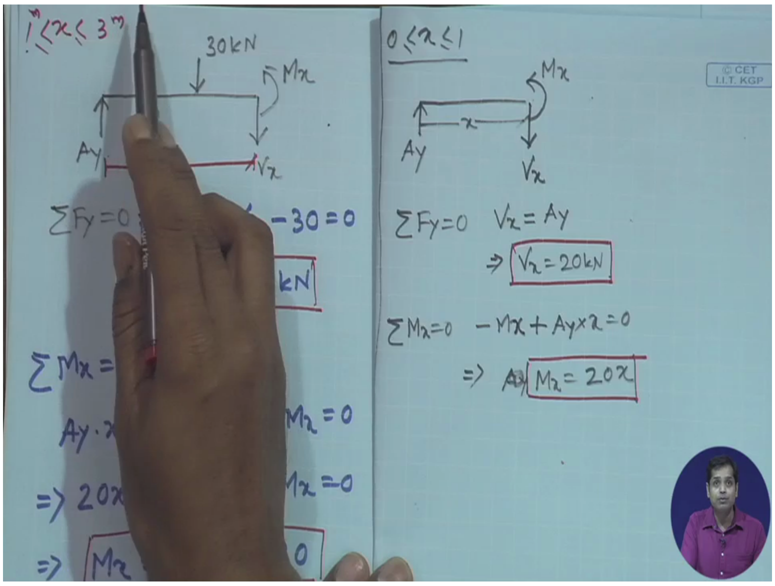

Now so your distribution of shear force and bending moment between zero to 1 is as per these equations.

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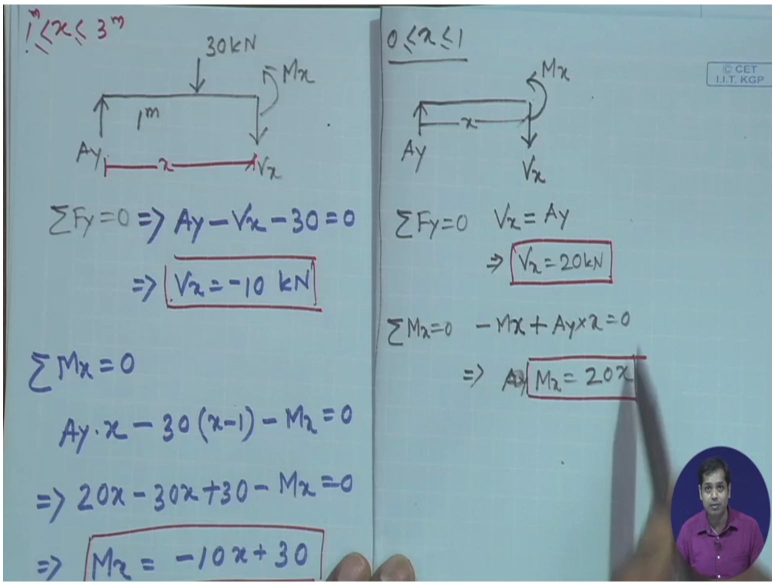

And distribution of bending moment and shear force between 1 to 3 metre are as per these equations.

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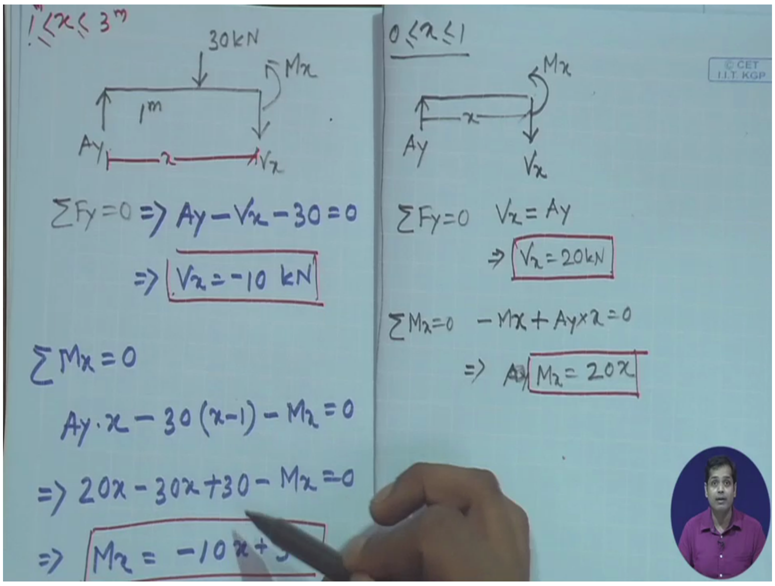

So let us draw the bending moment and shear force. So this was the problem. Now it is always good practice to draw bending moment and shear force diagram just below the structure so you can relate bending moment and shear force to the structure. First draw the shear force diagram. You see between this point to this point this is A, B, C.

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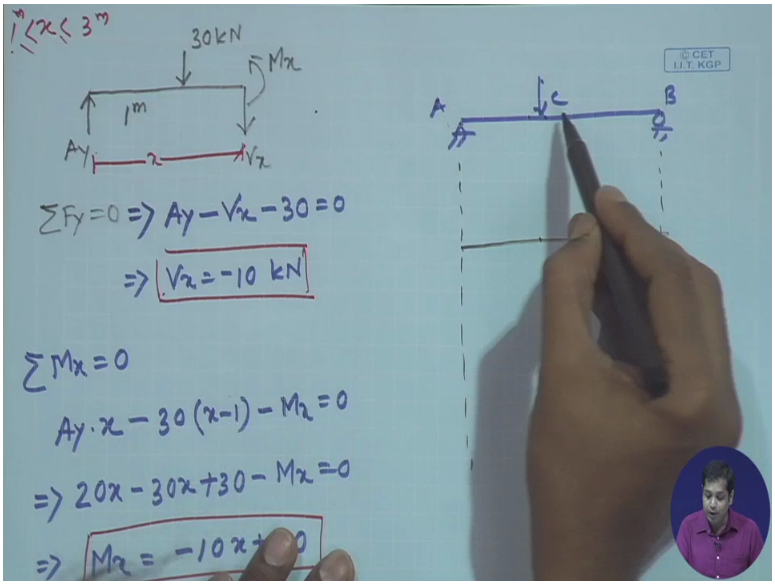

Between this point to this point, shear force was 20 kilo Newton positive.

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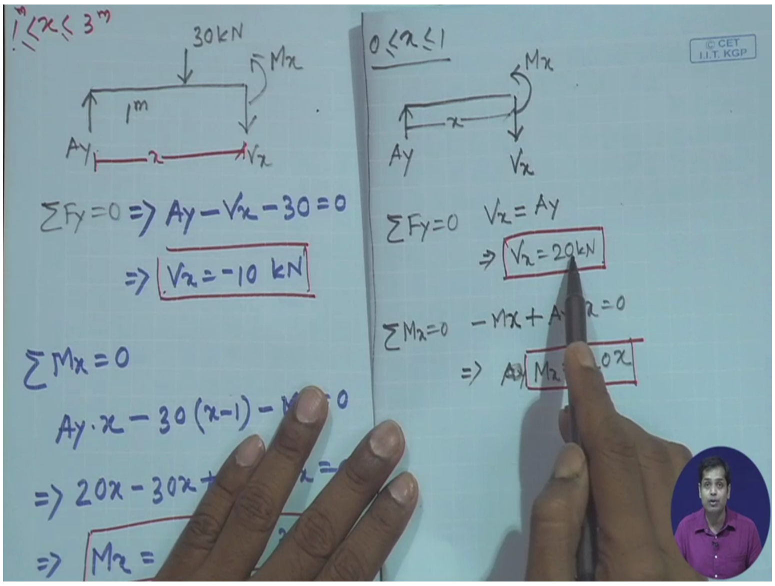

And then between this point to this point shear force is minus 10 kilo Newton.

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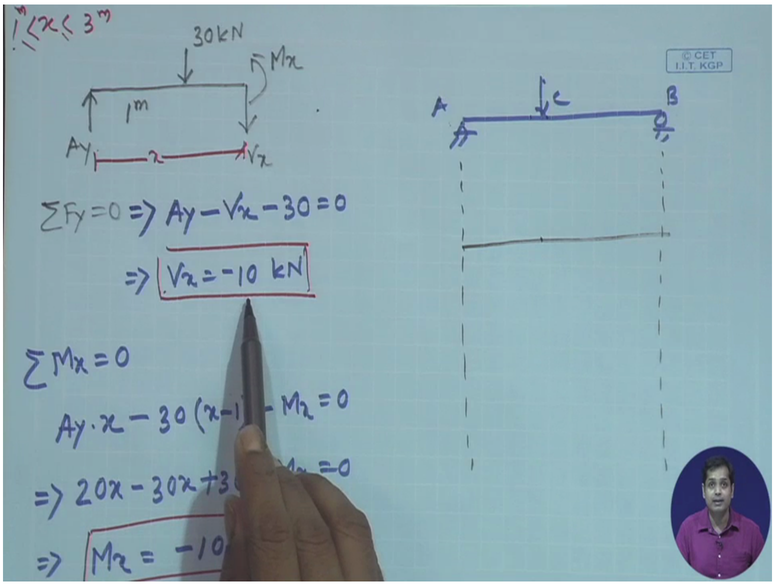

So (diag) shear force diagram it is constant between this point. And then here it is minus 10.

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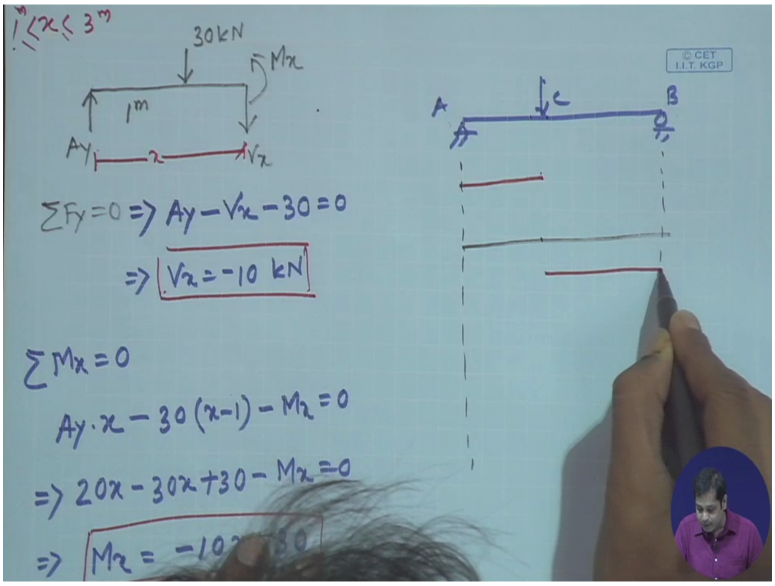

So sometimes you can join them also. This is 20 and this is 10, okay. Now draw the bending moment diagram. Again if you remember this between A to C it is 20x.

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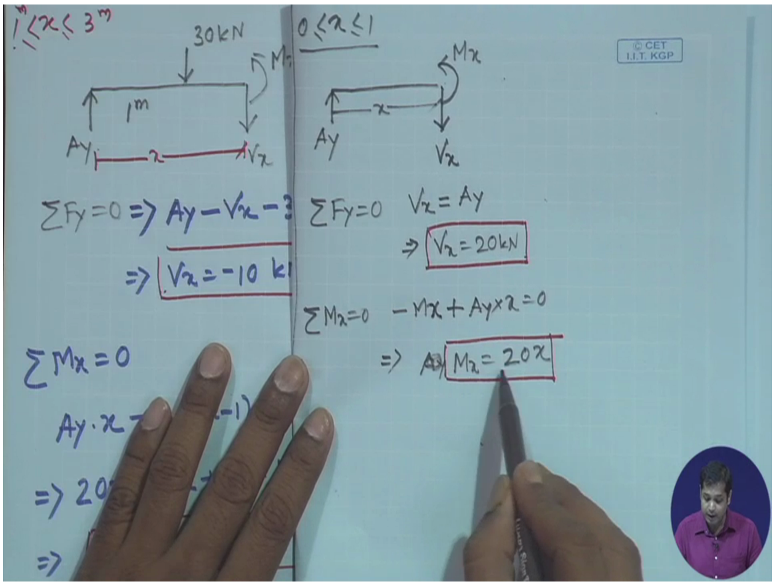

So x is equal to zero, bending moment is zero. And x is equal to 1, bending moment becomes 20. So this is 20.

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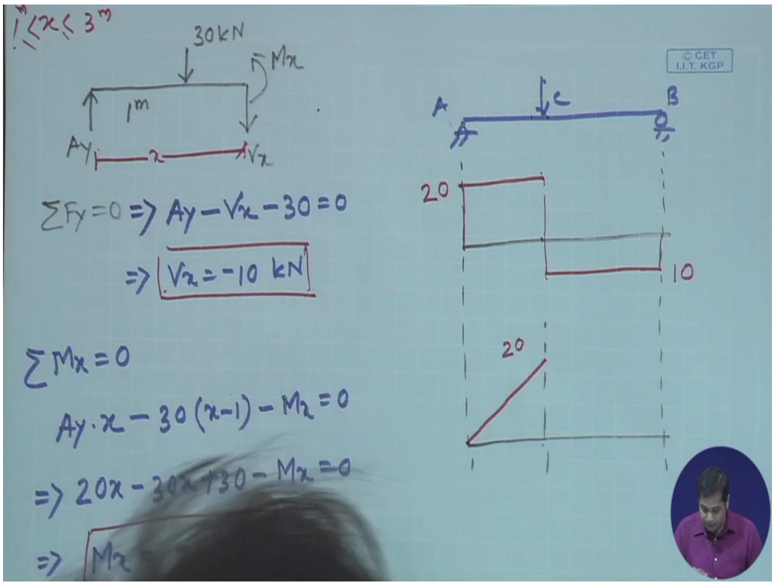

And again between this to this bending moment diagram is this, this equation is valid for this range. So if you substitute x is equal to 1, you get Mx is equal to 20 which is again consistent. So there is a continuity maintained. And x is equal to 3 if you substitute then Mx is equal to zero, it is simply supported beam, so bending moment will be zero, okay.

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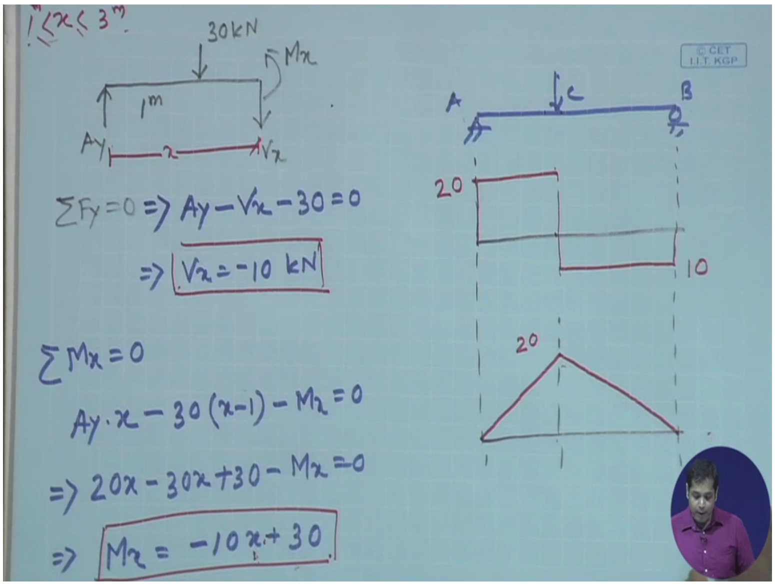

Now this is bending moment diagram and this is shear force diagram, okay.

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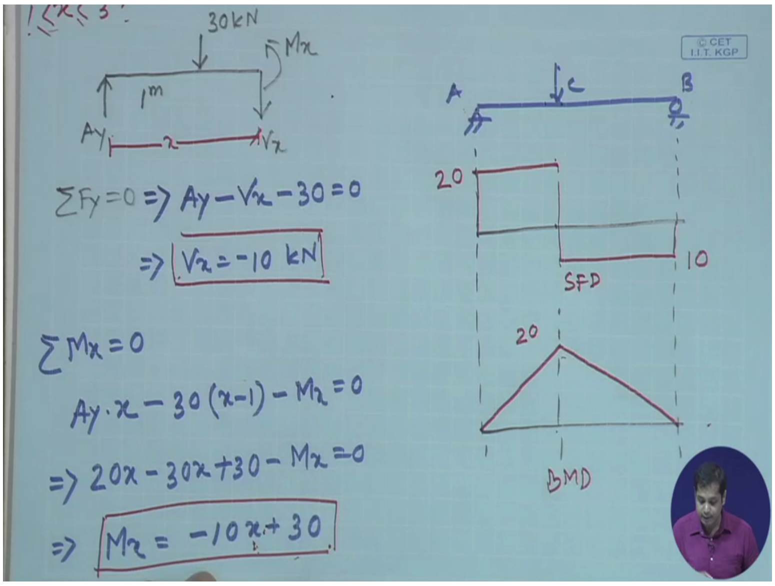

Now let us see what is the observation? Shear force is constant, bending moment is one order higher linear. In this case shear force is constant, bending moment is one order higher linear.

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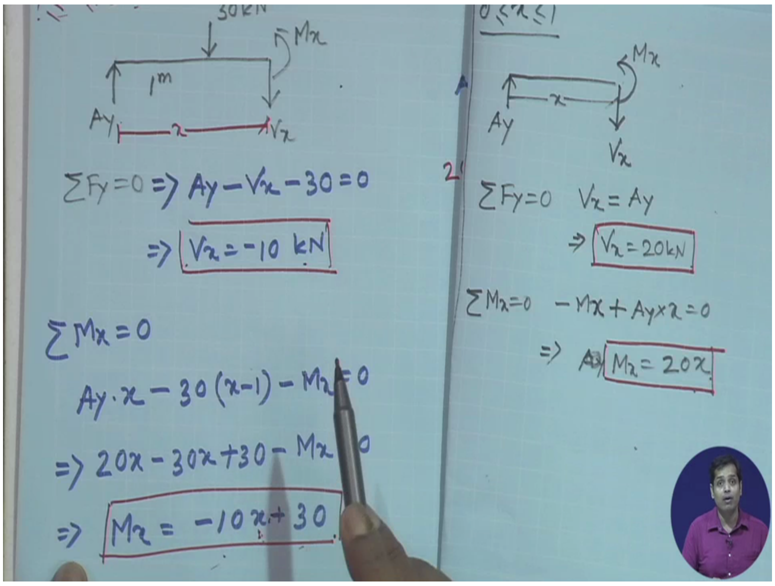

Again bending moment and shear force diagram you see, at this point your bending moment is positive.

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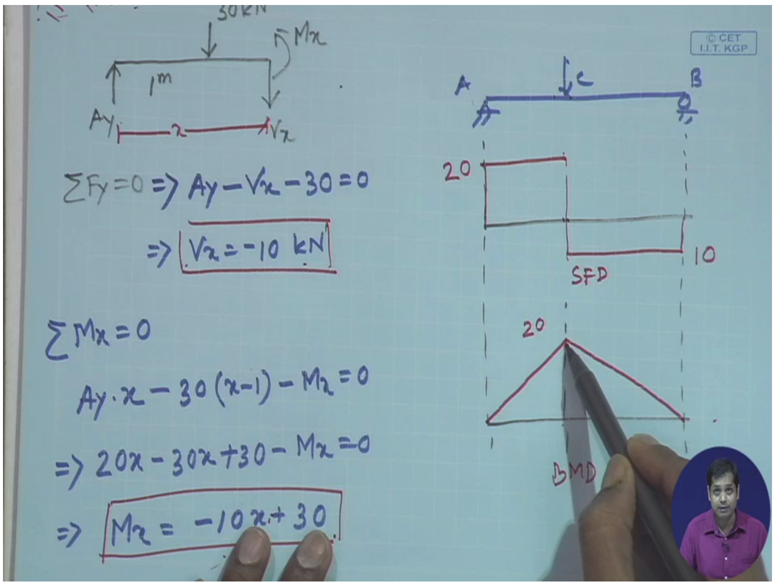

And then bending moment is maximum. Here at this point your shear force is changing sign, okay.

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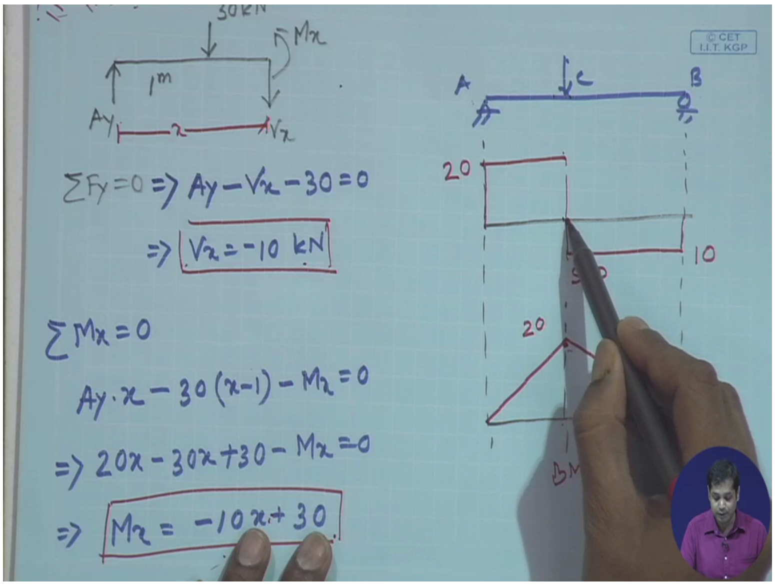

One another point to be noted as I said, (al) it is a good practice to draw bending moment and shear force diagram just below the structure. You see what was the value of this force? It was 30 kilo Newton, right? 30 kilo Newton which is downward. You see this is 20 and this is 10, 20 plus 10 is equal to 30 kilo Newton.

So C is a concentrated load here and because of that load shear force is constant throughout and suddenly you have a concentrated load and therefore there is sudden change in sign of bending moment or sudden drop in or sudden increase in shear force and that change will be according to the value of this load.

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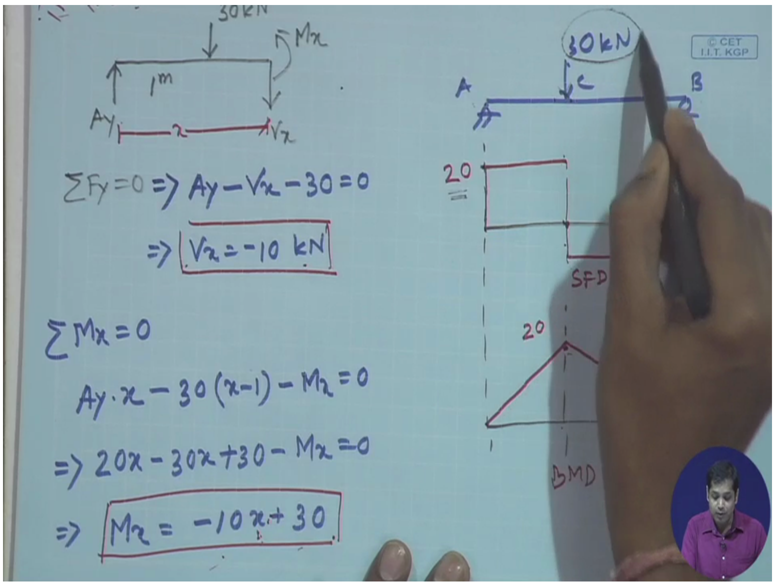

So this is reflected in shear force diagram, okay. Now for today that is all. We will have one tutorial class, we will try to have some more example on bending moment and shear force diagram. Thank you.