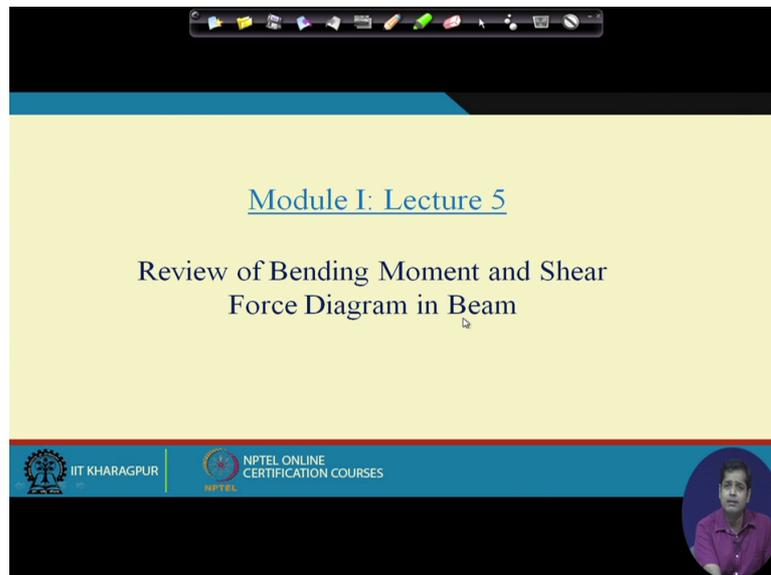


Structural Analysis 1
Professor Amit Shaw
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 5
Review of Bending Moment and Shear Force Diagram of Beam

Hello welcome to 5th lecture of module 1. You see in this lecture we will review the concept of bending moment and shear force diagram in beam. I believe you have studied bending moment and shear force diagram either in mechanics course or (00:39) course. What we will do is here briefly review the concept, because the concept of bending moment and diagram we will be using throughout this lecture. Okay.

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Now what is beam? Beam is essentially a structural element that primarily resists load applied transverse to its longitudinal axis. Now if you see this is simply supported beam, this is the longitudinal axis, okay and this structure element is the subjected to load which is transverse to this axis. This kind of structural component is called beam.

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

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And when it is subjected to this kind of loading or any kind of transverse loading, it undergoes bending, right? Like this.

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

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Now you know the internal forces developed in the beam is bending moment and shear force. Now let us see what is bending moment and shear force. If you see if I take one section at a distance x from A and draw the free body diagram of this part and this part then what happens?

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

Internal Forces: Bending Moment & Shear Force

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This is the free body diagram of this part and then again this is a free body diagram of this part.

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

Internal Forces: Bending Moment & Shear Force

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Now this is hinge support so it is replaced by two reactions A_x and A_y . Now at this point the internal forces are horizontal direction in this case F_x , then vertical V_x and then moment M_x .

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

Internal Forces: Bending Moment & Shear Force

The slide contains two diagrams. The first diagram shows a beam of length l with a pin support at A and a roller support at B. A uniformly distributed load q is applied downwards. A section x is taken at a distance x from support A. The internal forces at this section are shown as a bending moment M_x (counter-clockwise), a shear force V_x (downwards), and an axial force F_x (to the right). The second diagram shows the same beam with the internal forces at section x represented as a bending moment M_x (clockwise), a shear force V_x (upwards), and an axial force F_x (to the left). The slide also features the IIT Kharagpur and NPTEL logos and a small video inset of a presenter.

Similarly in this case this roller support is replaced by reaction B_y and these are the internal forces. If you remember in the last class we said whenever you break a structure and that continuity of that point in a free body diagram needs to be represented by the internal forces, right?

Now when you draw the internal forces, sign of the internal forces needs to be taken very carefully. For instance when I say the objective is in equilibrium, every point of the object is in equilibrium. So this point was an equilibrium, right?

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Beam: Structural Behaviour

Structural element that primarily resists loads applied transverse to its longitudinal axis

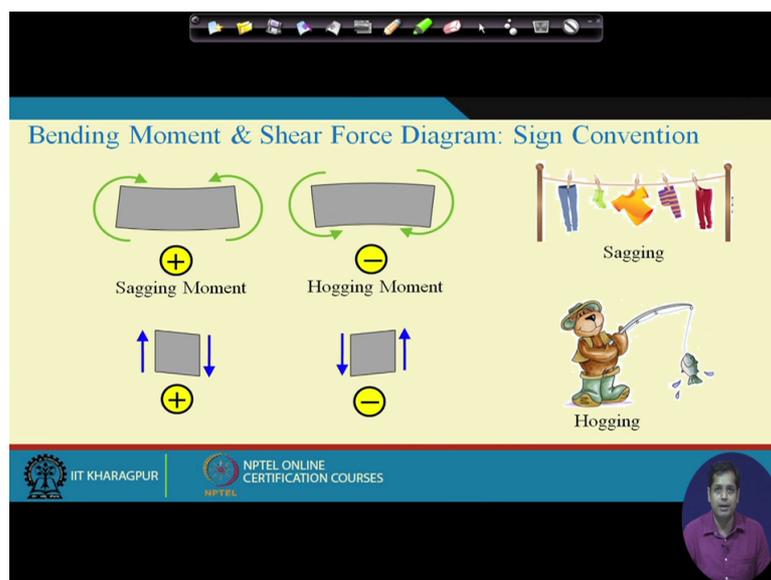
Internal Forces: Bending Moment & Shear Force

The slide contains two diagrams. The first diagram shows a beam of length l with a pin support at A and a roller support at B. A uniformly distributed load q is applied downwards. A section x is taken at a distance x from support A. The internal forces at this section are shown as a bending moment M_x (counter-clockwise), a shear force V_x (downwards), and an axial force F_x (to the right). The second diagram shows the same beam with the internal forces at section x represented as a bending moment M_x (clockwise), a shear force V_x (upwards), and an axial force F_x (to the left). The slide also features the IIT Kharagpur and NPTEL logos and a small video inset of a presenter.

It means the net force in this point, net internal forces need to be zero. So therefore if you are showing F_x in this direction for this part, you need to show F_x in the opposite direction for this part. Similarly for V_x , similarly for moment. Now this V_x is shear force, M_x is bending moment and F_x is actual force. Now what we assume here is there is no actual force. So beam only resists the external forces which is acting in a transverse direction by bending moment and shear force.

In a practical purpose we will see there are many cases where any structure component may be subjected to all. It may be subjected to shear force, maybe bending moment actual force together but analysis of those kind of structure component we will be discussing later. Okay. But at least for this case there is no actual force, right? Now let us define the sign convention. You see this is our sign convention we used for bending moment and shear force.

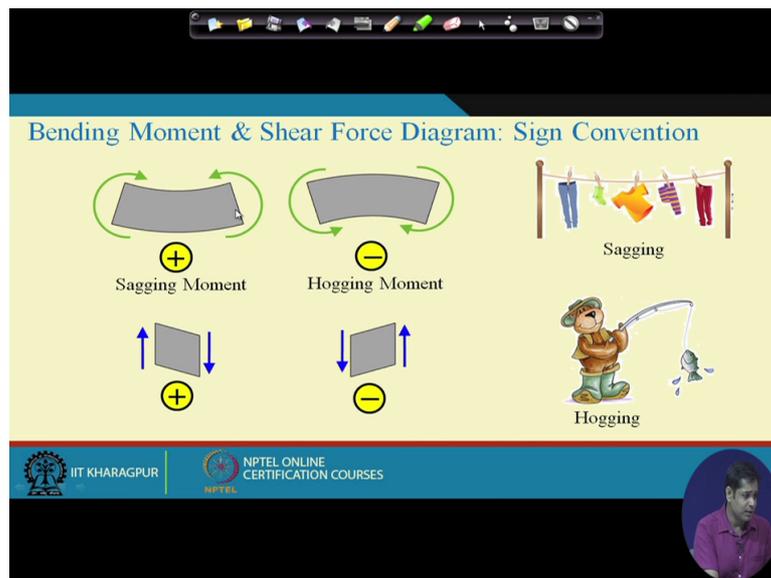
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Then what it says? You see there are two signs convention. Okay. One sign convention is the sign convention we used in the bending moment and shear force diagram and another sign convention is sign convention that we used when we do algebraic operation between moments and shear forces. Okay. Now let us first discuss what sign convention we will use in the bending moment and shear force diagram.

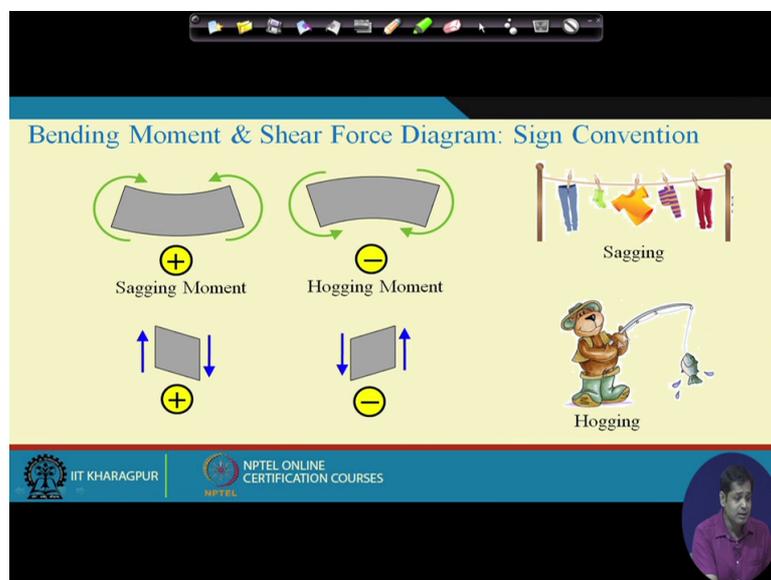
The sign convention in bending moment shear force diagram is this. You see this kind of movement is called sagging moment. Okay. Because it causes sagging in the structure. Okay.

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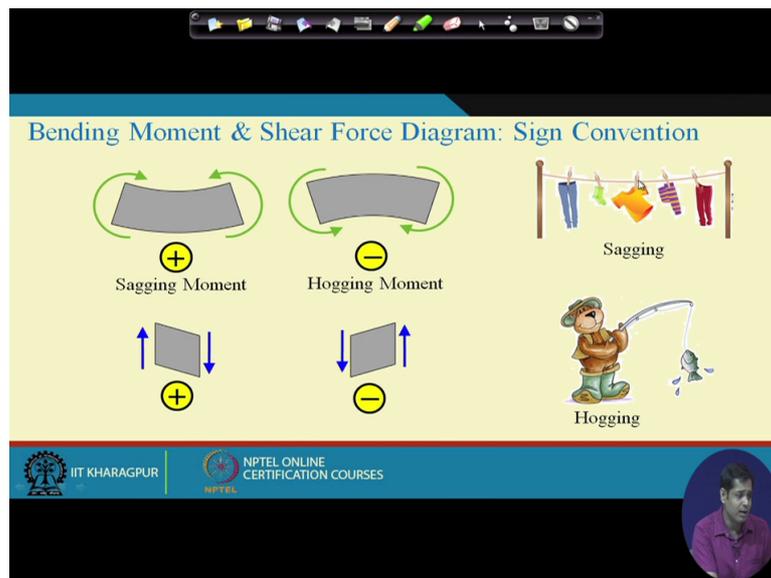
And this kind of movement is called hogging moment. Now (sa) one example for hogging moment is when a fisherman holds a fishing stick then the fishing sticks bends like this. Okay. This moment generated in the fishing stick is hogging moment.

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Now example of sagging moment is if you take a rod and which is subjected to or very common cloth hanging rod that we can see in our household. Then you might have observed it that rod experience is sagging like this. In this case the movement generated or the movement causes this is called sagging moment.

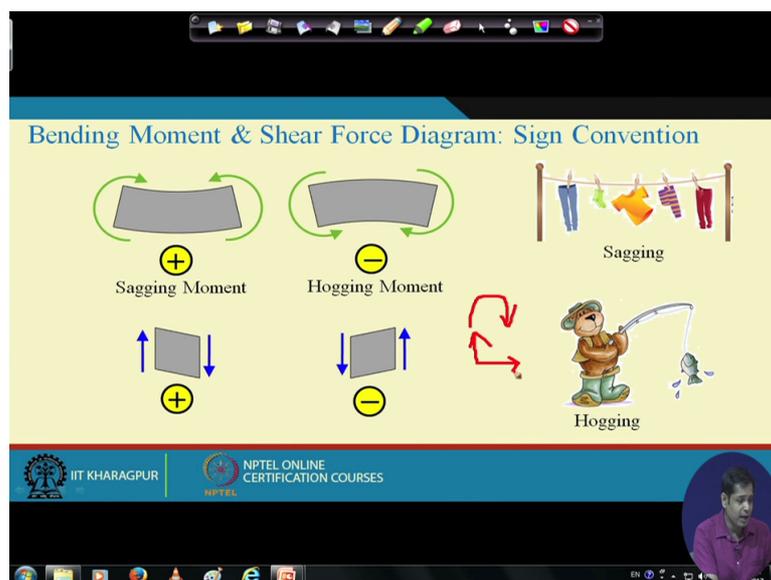
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Now for (shea) bending moment and shear force diagram this is positive and this is negative. Similarly for shear force, the shear force which produces clockwise couple is considered as positive, shear force which produces anticlockwise couple considered as negative. Now if you remember throughout this course for all (alb) algebraic operation we have been using clockwise moment as positive and anticlockwise moment as negative and vertical direction force as positive and horizontal direction force as negative.

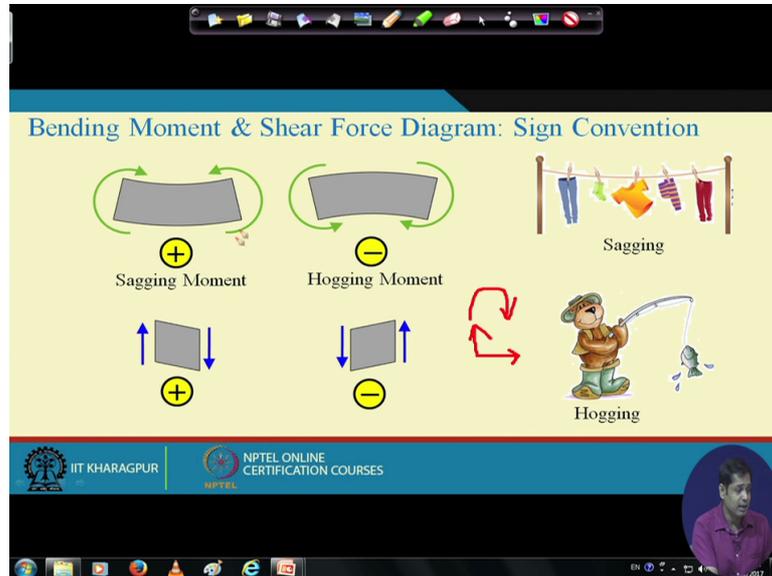
Means we took clockwise moment as positive and this is positive direction, right?

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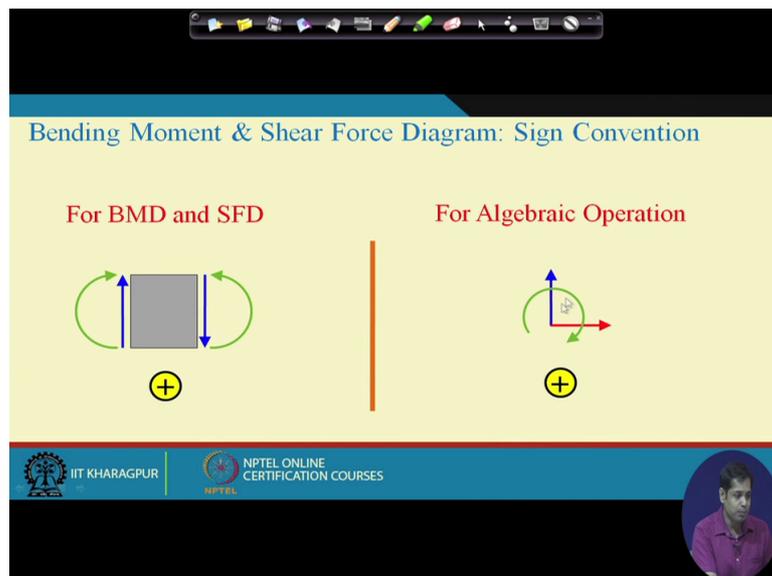
But this sign convention we used for algebraic operation. But when we draw the bending moment and shear force direction we use this sign convention. Okay.

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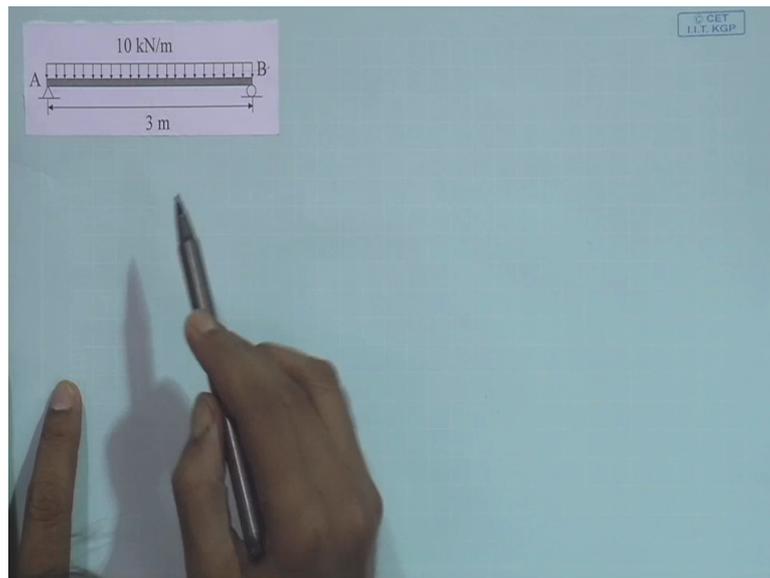
Whether it is a sagging moment or it is hogging moment. Okay. Now this is the two sign convention for bending moment diagram. This is sign convention bending moment and shear force and for all algebraic operation this sign convention is positive. Okay.

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Now let us demonstrate through some examples. Okay. Now the first example is if you take it is a simply supported beam which is subjected to uniformly distributed load. Okay.

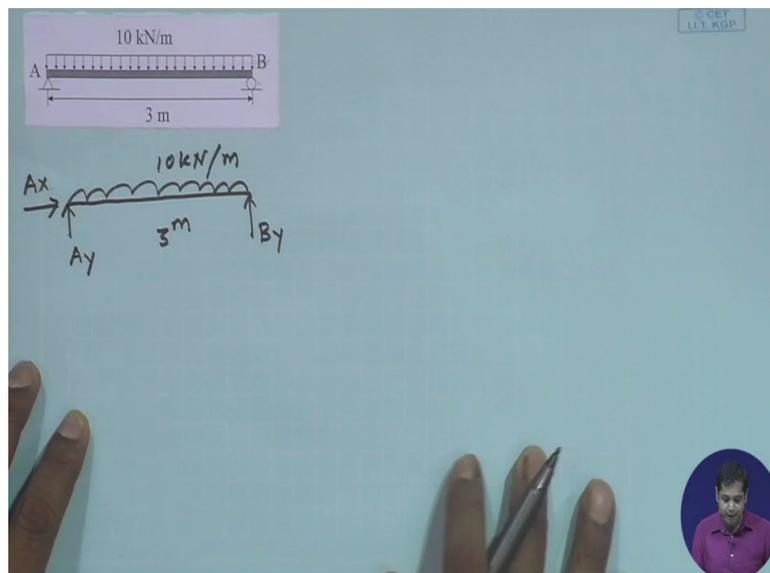
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Now what we have to do is we have to first find out the reactions at the support, then we need to determine the bending moment. The bending moment diagram is essentially the graphical representation of moment and shear force which tells you how the moment and shear force varies along the length of the beam. Let us find out that. The first is we need to determine the support reaction. Let us draw the free body diagram of the entire structure.

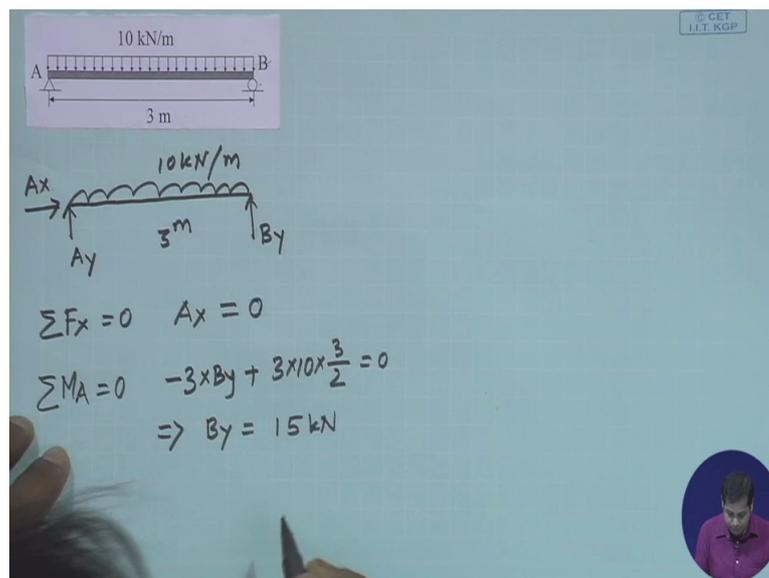
Free body diagram will be, this force will be A_y and then this is A_x , this is B_y and this is the very standard representation of uniformly distributed load which is 10 kilo Newton per metre and this length is 3 metre. This is the free body diagram. Okay.

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Now the first is summation of F_x is equal to zero which immediately gives me A_x is equal to zero, right? Then next is if you take summation of moment at A is equal to zero which gives what? Which gives you anticlockwise moment due to B_y minus 3 into B_y and then plus clockwise moment due to external load. This load will be the total load is 3 into 10 multiplied by the distance and the centre of application of the result and it is 3 by 2. That is equal to zero. So this gives me B_y is equal to 15 kilo Newton.

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And similarly summation of F_y is equal to zero which gives me A_y plus B_y is equal to total downward load 3 into 10. And from this equation if we substitute B_y is equal to 15, we will get A_y is equal to again 15 kilo Newton. So we have support reaction A_x is equal to zero and then B_y is equal to zero and then A_y is equal to zero, right?

(Refer Slide Time: 09:50)

10 kN/m

A B

3 m

A_x

A_y

10 kN/m

3 m

B_y

$\sum F_x = 0 \Rightarrow A_x = 0$

$\sum M_A = 0 \Rightarrow -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$

$\Rightarrow B_y = 15 \text{ kN}$

$\sum F_y = 0 \Rightarrow A_y + B_y = 3 \times 10$

$\Rightarrow A_y = 15 \text{ kN}$

Let us find out bending moment diagram. Now to do so, let us take one section here at a distance from A. Okay. And draw the free body diagram of this part.

(Refer Slide Time: 10:12)

10 kN/m

A B

x

A_y

$\sum M_A = 0 \Rightarrow -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$

$B_y = 15 \text{ kN}$

$B_y = 3 \times 10$

$A_y = 15 \text{ kN}$

Now if you draw the free body diagram of this part, this become this. This is A_y . A_x is equal to zero that is why there is no point to show that. Then uniformly distributed load 10 kilo Newton per metre. This distance is x and then if you remember our sign convention was this moment is positive and for shear force, this shear force and this shear force is positive, right?

(Refer Slide Time: 10:50)

The slide shows a beam of length 3 m with a uniformly distributed load of 10 kN/m. The beam is supported by a pin at A and a roller at B. The distance from A to the center of the beam is 3 m. The reaction forces are A_x and A_y at A, and B_y at B. The equations shown are:

$$\sum F_x = 0 \quad A_x = 0$$
$$\sum M_A = 0 \quad -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$$
$$\Rightarrow B_y = 15 \text{ kN}$$
$$\sum F_y = 0 \quad A_y + B_y = 3 \times 10$$
$$\Rightarrow A_y = 15 \text{ kN}$$

There are also diagrams showing a free body diagram of a section of length x from A, with reaction A_y at the left end and a uniformly distributed load of 10 kN/m over the length x . A small inset shows a person in a purple shirt.

So the direction of force is this and this. Okay. So this is V_x which is the shear force at (dis) x and this is M_x which is moment at a distance x from A.

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The slide shows the same beam as slide 10:50, but with a section of length x cut out. The reaction forces are A_x and A_y at A, and B_y at B. The equations shown are:

$$\sum F_x = 0 \quad A_x = 0$$
$$\sum M_A = 0 \quad -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$$
$$\Rightarrow B_y = 15 \text{ kN}$$
$$\sum F_y = 0 \quad A_y + B_y = 3 \times 10$$
$$\Rightarrow A_y = 15 \text{ kN}$$

There are also diagrams showing a free body diagram of a section of length x from A, with reaction A_y at the left end, a uniformly distributed load of 10 kN/m over the length x , and internal forces M_x and V_x at the right end. A small inset shows a person in a purple shirt.

Now take summation of F_y is equal to zero. What are the forces you have? A_y upward direction plus minus 10 into x which is externally applied uniformly distributed load. And then minus V_x is equal to zero, right? And A_y is equal to 15 already we determined and this gives me V_x is equal to 15 minus 10x. So this is how the V_x varies with x .

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$\sum F_x = 0 \Rightarrow A_x = 0$
 $\sum M_A = 0 \Rightarrow -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$
 $\Rightarrow B_y = 15 \text{ kN}$
 $\sum F_y = 0 \Rightarrow A_y + B_y = 3 \times 10$
 $\Rightarrow A_y = 15 \text{ kN}$

For a section of length x :
 $\sum F_y = 0 \Rightarrow A_y - 10x - V_x = 0$
 $\Rightarrow V_x = 15 - 10x$

Now then summation of say M_a is equal to zero. This gives me what? Only forces is or you can take summation of M_x is equal to zero, moment about this point. And this gives me, V_x will not contribute, A_y will contribute, the external load will contribute and the M_x itself. This gives me, M_x is anticlockwise direction, this is negative if you remember for all algebraic operation we use whether the sign convention will be anticlockwise or clockwise.

So M_x is minus M_x then plus A_y into x . And then minus 10 into x is the force which is at a distance x by 2 . That is equal to zero. Okay.

(Refer Slide Time: 12:56)

$\sum F_x = 0 \Rightarrow A_x = 0$
 $\sum M_A = 0 \Rightarrow -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0$
 $\Rightarrow B_y = 15 \text{ kN}$
 $\sum F_y = 0 \Rightarrow A_y + B_y = 3 \times 10$
 $\Rightarrow A_y = 15 \text{ kN}$

For a section of length x :
 $\sum M_x = 0 \Rightarrow -M_x + A_y \times x - 10 \times x \times \frac{x}{2} = 0$
 $\Rightarrow V_x = 15 - 10x$

A_y is equal to 15 we know. And this gives you M_x is equal to $15x$ minus $5x$ square. Okay. So this is how V_x varies with x and this is how M_x varies with x .

(Refer Slide Time: 13:21)

The image shows handwritten notes on a light blue background. On the left, there is a diagram of a beam AB of length 3m with a uniformly distributed load of 10 kN/m. Reaction forces A_x , A_y , and B_y are indicated. Below the diagram, the following equations are written:

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum M_A = 0 \Rightarrow -3 \times B_y + 3 \times 10 \times \frac{3}{2} = 0 \Rightarrow B_y = 15 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow A_y + B_y = 3 \times 10 \Rightarrow A_y = 15 \text{ kN}$$

On the right, a free body diagram of a section of length x is shown. It includes the reaction A_y , the shear force V_x , and the bending moment M_x . The following equations are derived:

$$\sum F_y = 0 \Rightarrow A_y - 10x - V_x = 0 \Rightarrow V_x = 15 - 10x$$

$$\sum M_x = 0 \Rightarrow -M_x + A_y x - 10x \times \frac{x}{2} = 0 \Rightarrow M_x = 15x - 5x^2$$

A small circular inset in the bottom right corner shows a person's face.

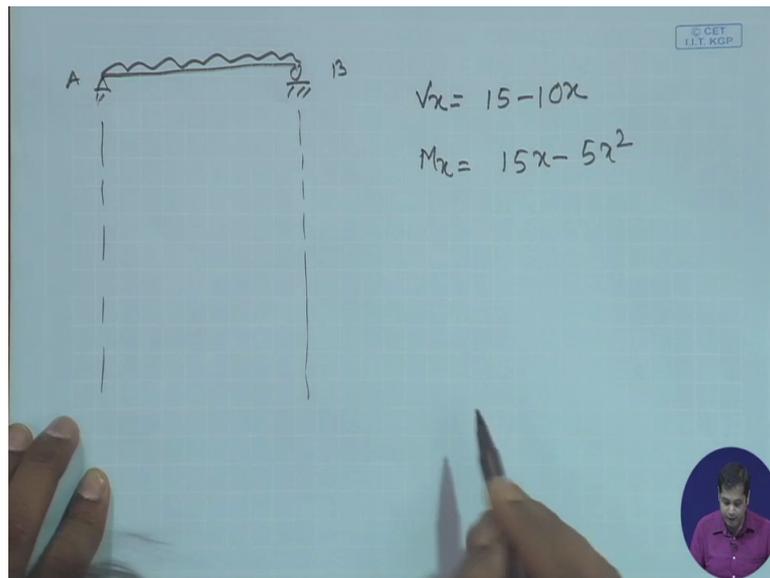
Now let us draw the diagram. Bending moment and shear force diagram will be graphical representation of this equation along this length and bending moment diagram will be (ver) graphical representation of this equation along this length.

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This image is a duplicate of the previous one, showing the same handwritten notes and diagrams for the beam analysis. It includes the same diagrams of the beam, the free body diagram, and the derived equations for reaction forces and internal forces.

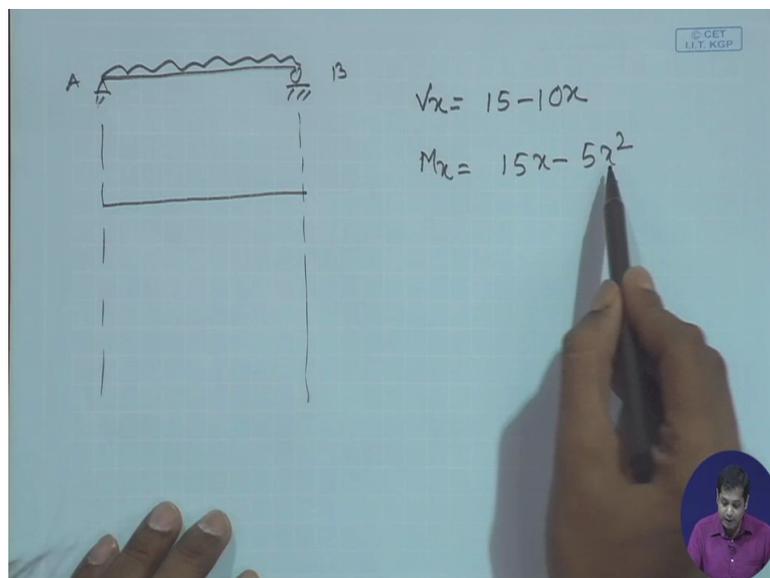
Now if I do that then that was the problem, okay. That was the AB. Now V_x we obtained. V_x is equal to 15 minus 10x and M_x is equal to 15x minus 5x square.

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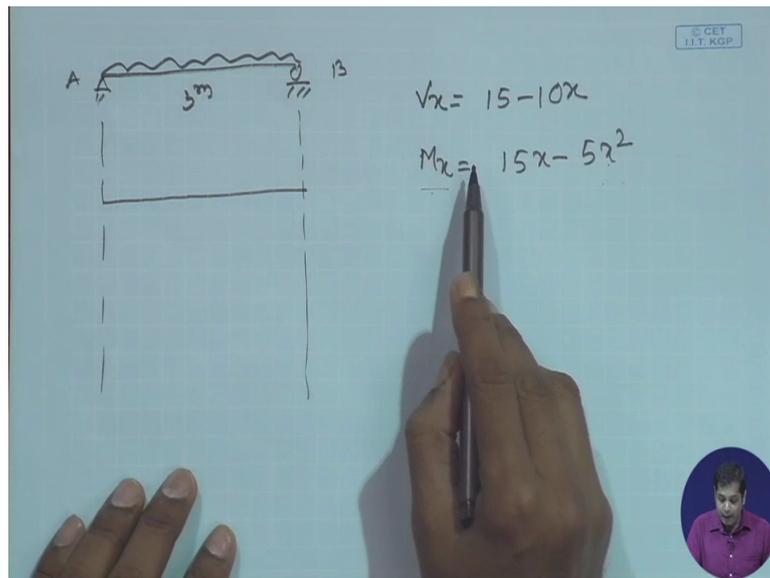
Let us first draw shear force diagram. You know one check is you should immediately check by substituting the value of x here which is then you can check whether it is consistent with the problem and the support condition.

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For instance you see this distance is simply supported beam. Therefore at this point and at this point your bending moment will be zero. Let us see if I substitute x is equal to zero, M_x is equal to zero. If I substitute x is equal to 3 here at B then also M_x is equal to zero. So this expression is consistent with this.

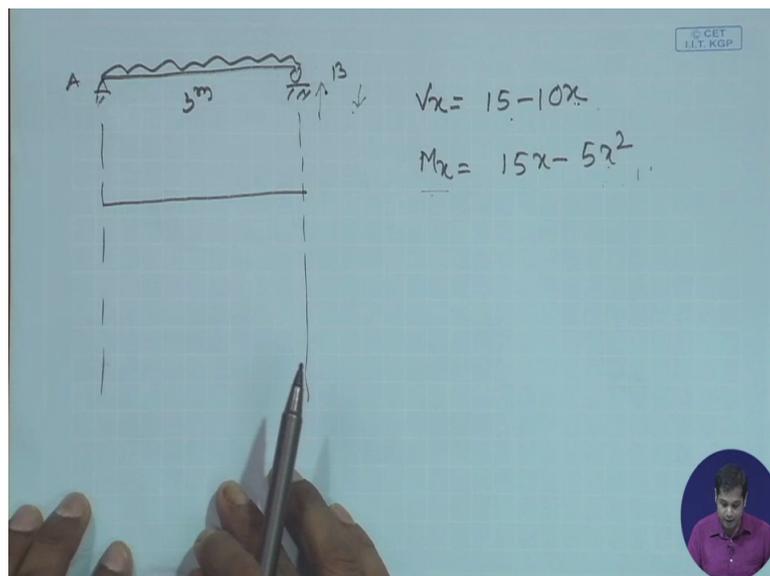
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Let us see whether this is consistent whether not? You see if I substitute x is equal to zero which gives you V_x is equal to 15. If you remember 15 was the (rea) support reaction at A. If I substitute x is equal to 3 here, this becomes minus 15. So again if you remember this reaction at support B was 15 which is shown vertically upward.

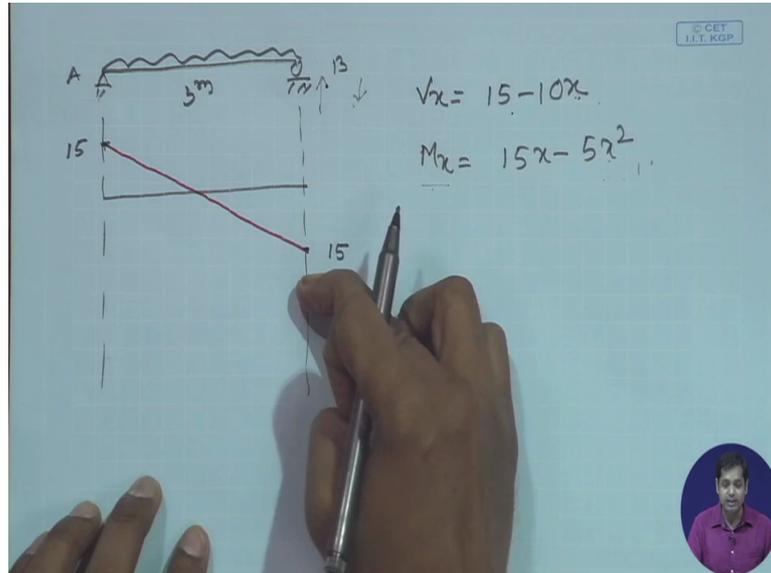
But while representing the shear force we used this as positive. That is why it is negative. So these equations are consistent with the support reactions and the conditions here.

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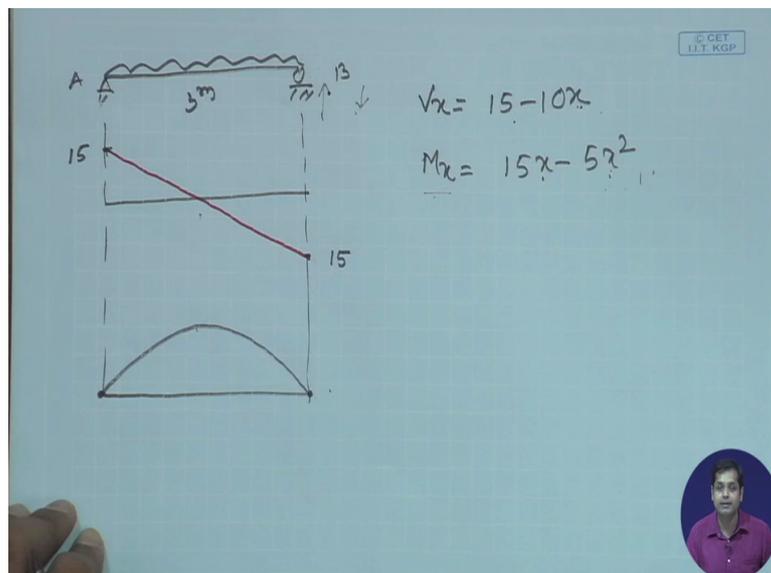
Now x is equal to zero, therefore bending moment is 15 and x is equal to 3 shear force is minus 15 and throughout this length bending moment varies linearly. So this will be the shear force diagram. This is 15 this is also 15. I am just writing the absolute value because below this it is understood, it is negative. Okay.

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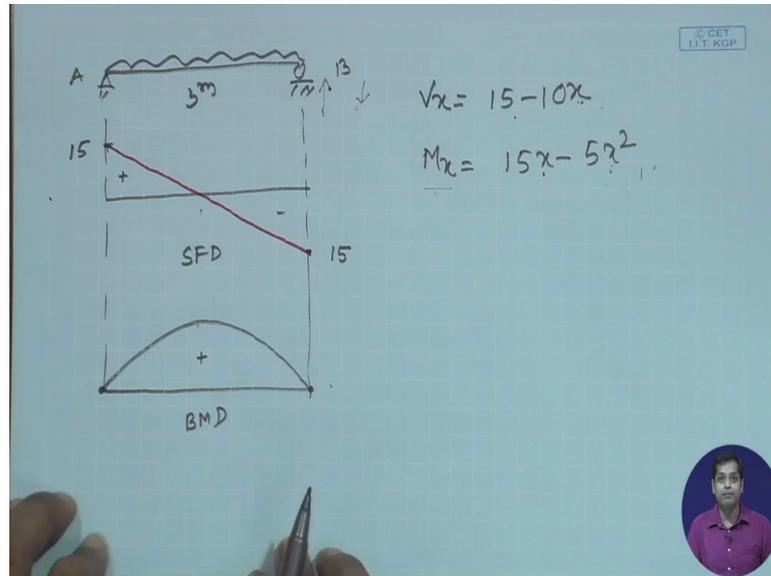
Now let us draw the bending moment diagram. x is equal to zero and x is equal to 3. Bending moment is zero and then between x is equal to zero and between A and B bending moment varies. As per these equations if you plot it, this equation will become something like this. Okay.

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So this is positive, this is negative, right? This is shear force diagram and this is bending moment diagram.

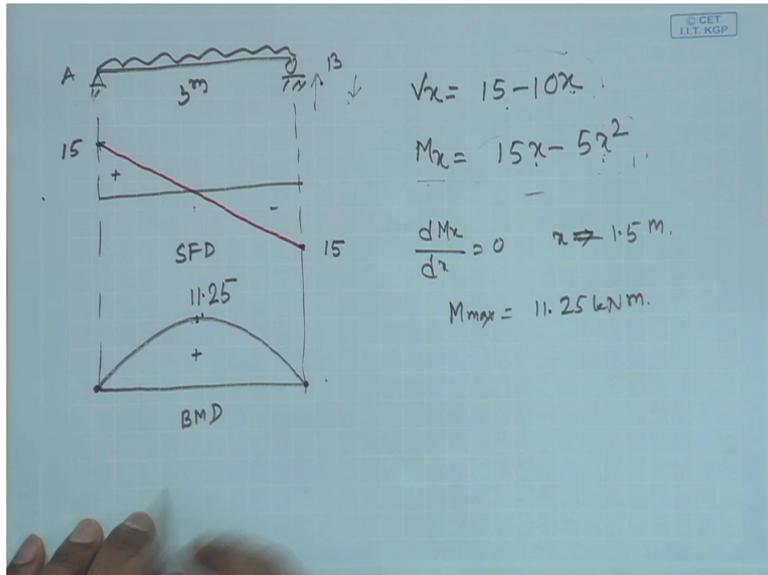
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Now one observation here please note that in this equation your shear force is linear and bending moment is quadratic. We are not going to make any conclusion based on this observation. Let us see other problems and how the order varies in those cases as well.

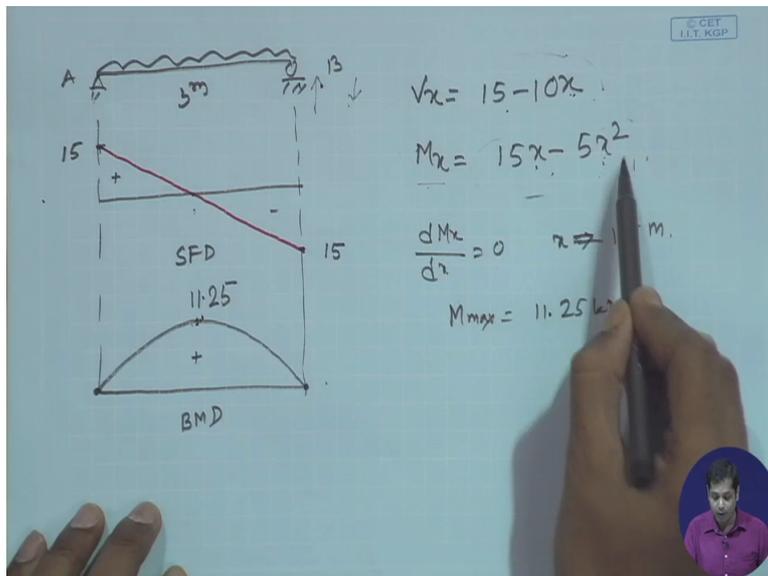
Now suppose if you have to find out what is the maximum bending moment here, maximum bending moment will be where dM/dx is equal to zero and if you substitute that you will get x is equal to 1 point 5 metre. And if you substitute x is equal to 1 point 5 metre in here, you get M_{max} will be 11 point 25 kilo Newton metre. So this will be 11 point 25 kilo Newton metre.

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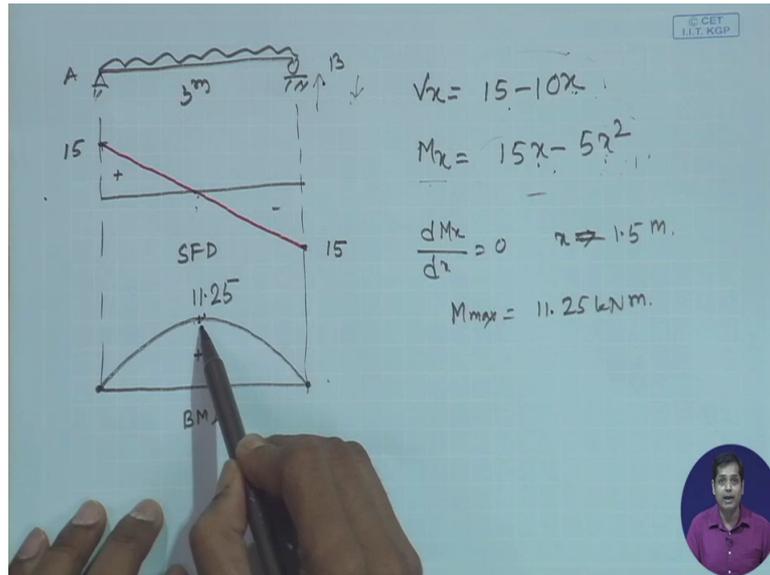
Another observation here at the middle the shear force is zero and at the middle bending moment is maximum. So two observation we have here is shear force is linear and the bending moment is one order higher quadratic.

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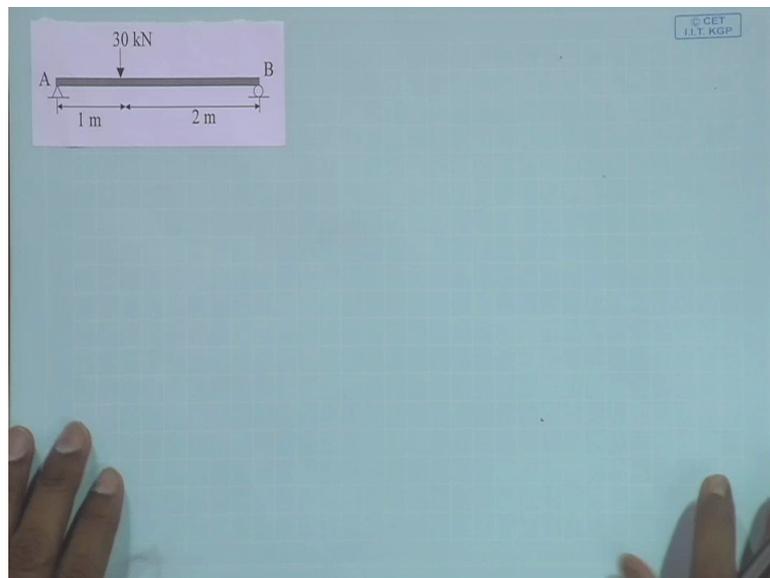
Here shear force is zero and at the same point the bending moment will be maximum.

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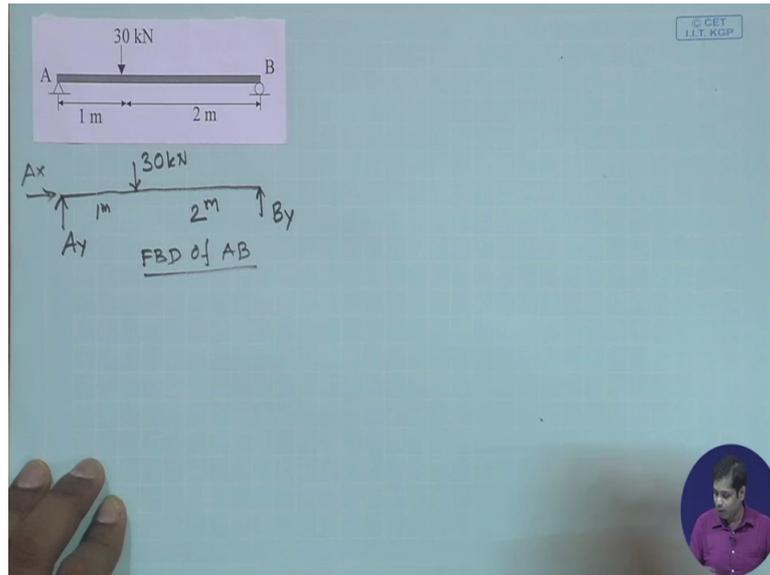
Let us see whether similar observation we have another problems as well. Let us do one more example. Okay. Now this was probably the simplest example of bending moment and shear force, the first one. And if you take any book the first example gives you a simply supported beam subjected to uniformly distributed load.

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Let us take one more example here. It is again a simply supported beam but it is subjected to concentrated load. Okay. Now again draw the free body diagram. This is A_x , A_y , then B_y , this is 30 kilo Newton, this is 1 metre, this is the free body diagram of AB. Okay.

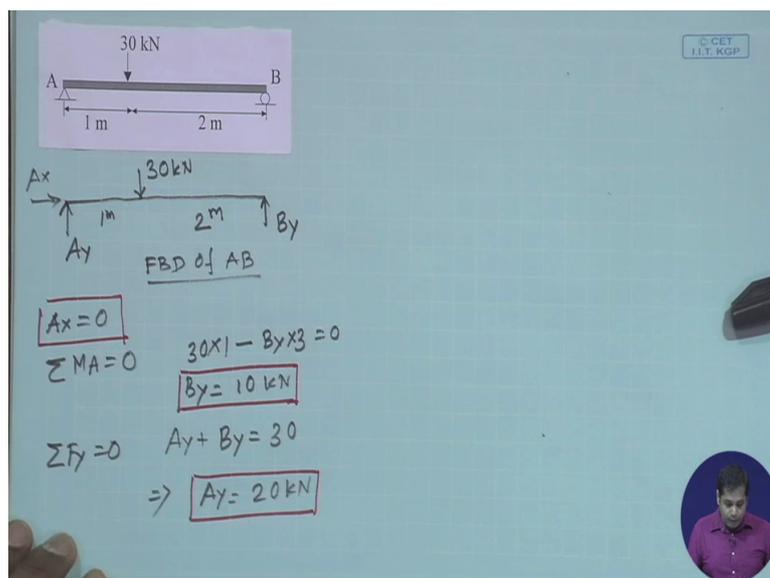
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Now again A_x will be zero that we get directly from the first equation. Summation of F_x is equal to zero. And then if you take moment about A that is equal to zero, then this gives you 30×1 which is clockwise moment minus $B_y \times 3$ is equal to zero. This gives you B_y is equal to 10 kilo Newton. And similarly if you take F_y is equal to zero, $A_y + B_y$ is equal to 30. This gives you, substitute B_y is equal to 10. A_y is equal to 20 kilo Newton.

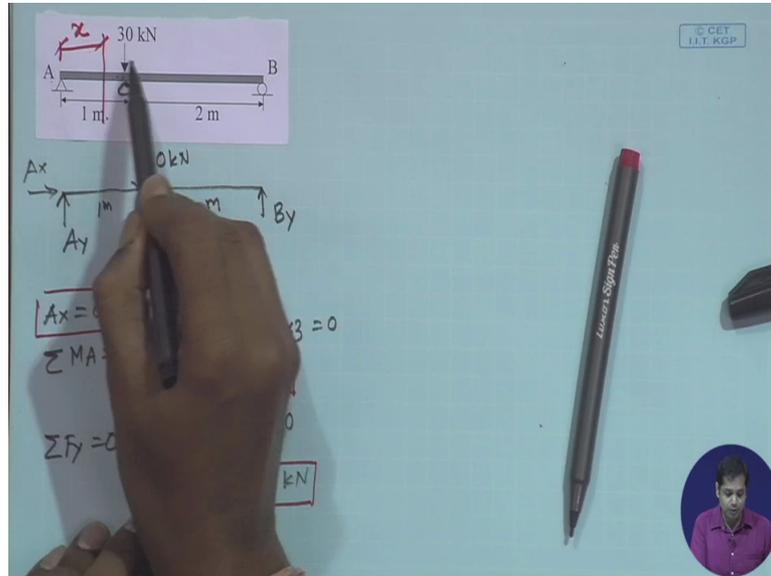
Whenever you write force and moment please write unit also because this is a common mistake I have seen that you write the value but then forget to mention the unit. So these are the support reactions, okay. And A_x is equal to zero is another support reaction. Okay.

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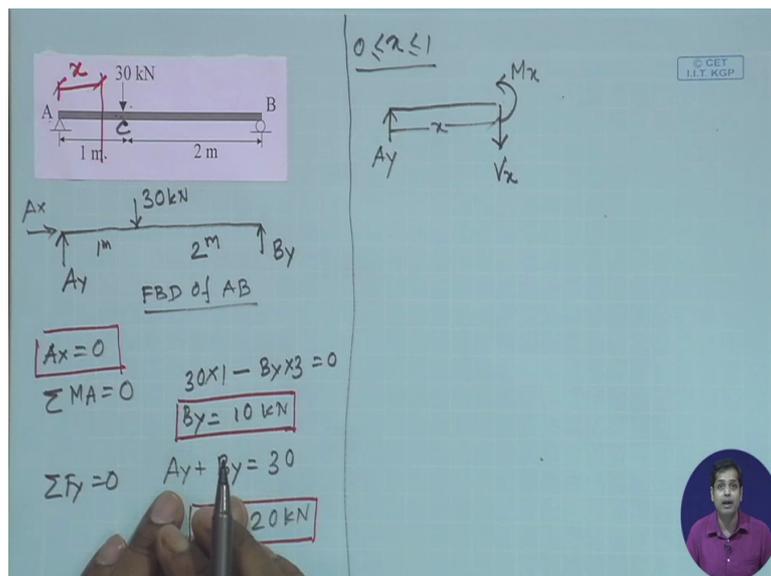
Now you see this we need to find out the bending moment diagram and shear force diagram. First you take one section here at a distance x from A. This section is between point A and point C. So the first case Ax varies from zero to 1, okay.

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Now first is Ax varies from zero to 1, okay. Now draw your free body diagram. This is A_y , A_x is equal to zero not showing explicitly. This is V_x and this is M_x , okay and this distance is x . No other external load between A to C.

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Now this give you summation of F_y is equal to zero, this gives you V_x is equal to A_y . And A_y we already obtained 20. So this gives you V_x is equal to 20 kilo Newton. So this is how V_x varies between A to C.

(Refer Slide Time: 22:05)

The image shows handwritten engineering notes on a blue background. On the left, there is a diagram of a beam AB of length 3m, with a pin support at A and a roller support at B. A 30 kN downward point load is applied at point C, which is 1m from A and 2m from B. Below this is the free body diagram (FBD) of segment AB, showing reaction forces A_x (horizontal, right), A_y (vertical, up) at A, and B_y (vertical, up) at B. The 30 kN load is shown as a downward arrow at 1m from A. The equations for equilibrium are written as follows:

$$\sum M_A = 0 \quad 30 \times 1 - B_y \times 3 = 0$$

$$\Rightarrow B_y = 10 \text{ kN}$$

$$\sum F_y = 0 \quad A_y + B_y = 30$$

$$\Rightarrow A_y = 20 \text{ kN}$$

On the right side, the notes are for a section cut at a distance x from A, where $0 \leq x \leq 1$. The FBD of this section shows a pin support at A with reactions A_y (up) and A_x (right). At the cut, there is a shear force V_x (down) and a bending moment M_x (clockwise). The equilibrium equations are:

$$\sum F_y = 0 \quad V_x = A_y$$

$$\Rightarrow V_x = 20 \text{ kN}$$

A small circular inset photo of a man in a purple shirt is visible in the bottom right corner of the slide.

Let us take moment at x is equal to zero. So moment at x is equal to zero will be minus M_x because M_x is anti clockwise direction. Again and again I am repeating that when you do the algebraic equation check whether it is clockwise or anticlockwise. Then plus A_y into x . That is equal to zero, okay. And this gives you M_x is equal to, you see A_y is equal to 20, this gives you $20x$. So this is how bending moment varies between 1 to C.

(Refer Slide Time: 22:53)

This slide is similar to the previous one, showing the same beam diagram and FBD of segment AB. The equilibrium equations for the beam are repeated:

$$\sum M_A = 0 \quad 30 \times 1 - B_y \times 3 = 0$$

$$\Rightarrow B_y = 10 \text{ kN}$$

$$\sum F_y = 0 \quad A_y + B_y = 30$$

$$\Rightarrow A_y = 20 \text{ kN}$$

On the right side, the notes are for a section cut at a distance x from A, where $0 \leq x \leq 1$. The FBD of this section shows a pin support at A with reactions A_y (up) and A_x (right). At the cut, there is a shear force V_x (down) and a bending moment M_x (clockwise). The equilibrium equations are:

$$\sum F_y = 0 \quad V_x = A_y$$

$$\Rightarrow V_x = 20 \text{ kN}$$

$$\sum M_x = 0 \quad -M_x + A_y \times x = 0$$

$$\Rightarrow M_x = 20x$$

Let us see what happens when you take one more section it is section 1, then one more section here. This is again at a distance x from the x but now in this case x varies from C to B. Means x varies from 1 metre to 3 metre.

(Refer Slide Time: 23:16)

$0 \leq x \leq 1$
 $\sum F_x = 0 \Rightarrow V_x = A_y$
 $\Rightarrow V_x = 20 \text{ kN}$
 $-\sum M_x + A_y \times x = 0$
 $\Rightarrow M_x = 20x$

$A_x = 0$
 $\sum M_A = 0 \Rightarrow 30 \times 1 - B_y \times 3 = 0$
 $B_y = 10$
 $\sum F_y = 0 \Rightarrow A_y + B_y = 30$
 $\Rightarrow A_y = 20$

So if I draw the bending moment this is again A_y , this is 30 kilo Newton, this is 1 metre and then again this is V_x and then this is M_x , right?

(Refer Slide Time: 23:50)

$0 \leq x \leq 2$
 $\sum F_x = 0 \Rightarrow V_x = A_y$
 $\sum M_x = 0 \Rightarrow 30 \times 1 - B_y \times 3 = 0$
 $B_y = 10 \text{ kN}$
 $\sum F_y = 0 \Rightarrow A_y + B_y = 30$
 $\Rightarrow A_y = 20 \text{ kN}$

Now summation of F_y is equal to zero. This gives you A_y minus V_x minus 30 is equal to zero. And A_y is equal to 20 we have already obtained, this gives you V_x is equal to minus 10

kilo Newton. So this is how V_x varies between C to B. And take M_x is equal to zero and M_x , A_y into x , this distance is x .

(Refer Slide Time: 24:50)

$\sum F_y = 0 \Rightarrow A_y - V_x = 0 = 0$
 $\Rightarrow V_x = A_y = 20$

$A_x = 0$
 $\sum M_A = 0$
 $\sum F_y = 0$
 $\Rightarrow A_y + B_y = 30$
 $B_y = 10 \text{ kN}$
 $A_y = 20 \text{ kN}$

A_y into x which is clockwise. Then minus 30 into x minus 1 which is anticlockwise. And again minus M_x which is anticlockwise, this is zero. And this gives you, A_y is equal to 20, so this gives you $20x$ minus $30x$ plus 30 minus M_x is equal to zero. And M_x is equal to minus $10x$ plus 30.

(Refer Slide Time: 25:42)

$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$
 $\Rightarrow V_x = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_x = 0$
 $\Rightarrow 20x - 30x + 30 - M_x = 0$
 $\Rightarrow M_x = -10x + 30$

$A_x = 0$
 $\sum M_A = 0$
 $\sum F_y = 0$
 $\Rightarrow A_y + B_y = 30$
 $B_y = 10 \text{ kN}$
 $A_y = 20 \text{ kN}$

Now this one is x from 1 metre to 3 metre. Now if you see we have two results. This one is for x between zero to 1 and this is between 1 to 3.

(Refer Slide Time: 26:11)

$0 \leq x \leq 3^m$
 $\sum F_y = 0 \Rightarrow -30 = 0$
 $\sum M_x = 0 \Rightarrow -30 \cdot x + M_x = 0$
 $M_x = 30x$

$0 \leq x \leq 1$
 $\sum F_y = 0 \Rightarrow V_x = A_y$
 $\Rightarrow V_x = 20 \text{ kN}$
 $\sum M_x = 0 \Rightarrow -M_x + A_y \cdot x = 0$
 $\Rightarrow M_x = 20x$

Now so your distribution of shear force and bending moment between zero to 1 is as per these equations.

(Refer Slide Time: 26:20)

$1 \leq x \leq 3^m$

$\sum F_y = 0 \Rightarrow A_y - V_2 - 30 = 0$
 $\Rightarrow V_2 = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_2 = 0$
 $\Rightarrow 20x - 30x + 30 - M_2 = 0$
 $\Rightarrow M_2 = -10x + 30$

$0 \leq x \leq 1$

$\sum F_y = 0 \quad V_2 = A_y$
 $\Rightarrow V_2 = 20 \text{ kN}$

$\sum M_x = 0 \quad -M_x + A_y \cdot x = 0$
 $\Rightarrow A_y M_2 = 20x$

And distribution of bending moment and shear force between 1 to 3 metre are as per these equations.

(Refer Slide Time: 26:26)

$1 \leq x \leq 3^m$

$\sum F_y = 0 \Rightarrow A_y - V_2 - 30 = 0$
 $\Rightarrow V_2 = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_2 = 0$
 $\Rightarrow 20x - 30x + 30 - M_2 = 0$
 $\Rightarrow M_2 = -10x + 30$

$0 \leq x \leq 1$

$\sum F_y = 0 \quad V_2 = A_y$
 $\Rightarrow V_2 = 20 \text{ kN}$

$\sum M_x = 0 \quad -M_x + A_y \cdot x = 0$
 $\Rightarrow A_y M_2 = 20x$

So let us draw the bending moment and shear force. So this was the problem. Now it is always good practice to draw bending moment and shear force diagram just below the structure so you can relate bending moment and shear force to the structure. First draw the shear force diagram. You see between this point to this point this is A, B, C.

(Refer Slide Time: 27:11)

$0 < x < 3^m$

$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$
 $\Rightarrow V_x = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_x = 0$
 $\Rightarrow 20x - 30x + 30 - M_x = 0$
 $\Rightarrow M_x = -10x + 30$

$0 \leq x \leq 3$

A_y , V_x , M_x , 30 kN , 1 m , x , A , B

Between this point to this point, shear force was 20 kilo Newton positive.

(Refer Slide Time: 27:14)

$0 < x < 3^m$

$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$
 $\Rightarrow V_x = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_x = 0$
 $\Rightarrow 20x - 30x + 30 - M_x = 0$
 $\Rightarrow M_x = -10x + 30$

$0 \leq x \leq 1$

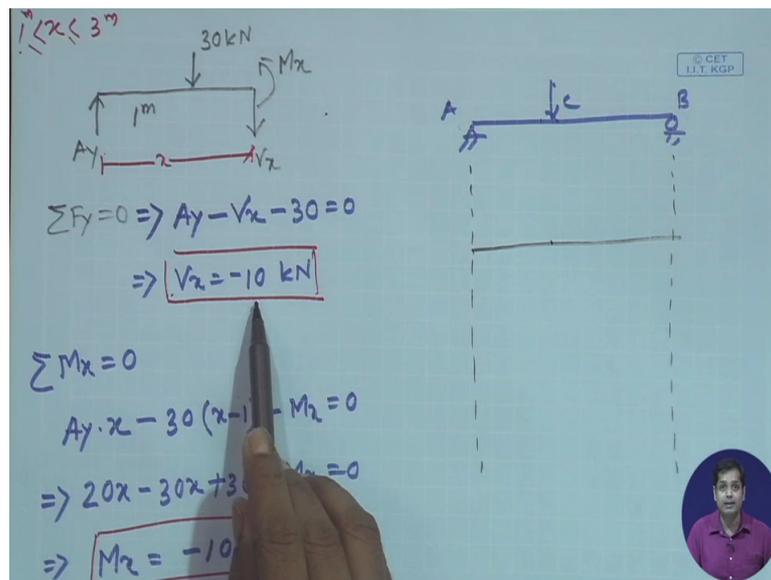
$\sum F_y = 0 \Rightarrow V_x = A_y$
 $\Rightarrow V_x = 20 \text{ kN}$

$\sum M_x = 0 \Rightarrow -V_x \cdot x + A_y \cdot x = 0$
 $\Rightarrow M_x = 20x$

A_y , V_x , M_x , 30 kN , 1 m , x , A , B

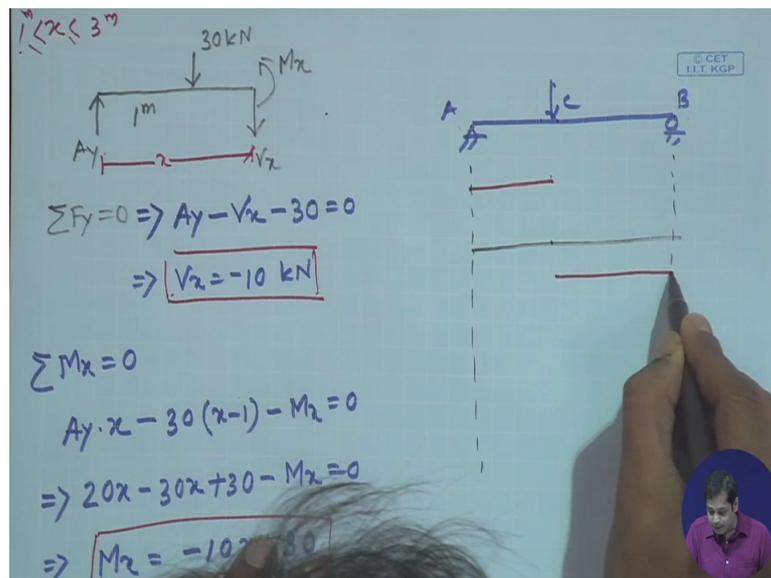
And then between this point to this point shear force is minus 10 kilo Newton.

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So (diag) shear force diagram it is constant between this point. And then here it is minus 10.

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So sometimes you can join them also. This is 20 and this is 10, okay. Now draw the bending moment diagram. Again if you remember this between A to C it is $20x$.

(Refer Slide Time: 27:53)

$0 \leq x \leq 3^m$
 $0 \leq x \leq 1$

$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$
 $\Rightarrow V_x = -10 \text{ kN}$

$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_x = 0$
 $\Rightarrow 20x - 30x + 30 - M_x = 0$
 $\Rightarrow M_x = 20x - 30x + 30$

$\sum F_y = 0 \Rightarrow V_x = A_y$
 $\Rightarrow V_x = 20 \text{ kN}$

$\sum M_x = 0 \Rightarrow -M_x + A_y \cdot x = 0$
 $\Rightarrow M_x = 20x$

So x is equal to zero, bending moment is zero. And x is equal to 1, bending moment becomes 20. So this is 20.

(Refer Slide Time: 28:05)

$0 \leq x \leq 3^m$

$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$
 $\Rightarrow V_x = -10 \text{ kN}$

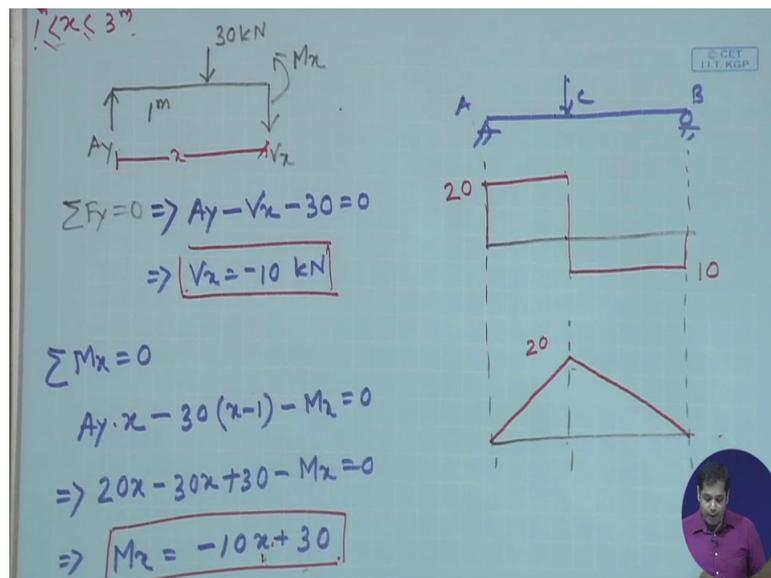
$\sum M_x = 0$
 $A_y \cdot x - 30(x-1) - M_x = 0$
 $\Rightarrow 20x - 30x + 30 - M_x = 0$
 $\Rightarrow M_x = 20x - 30x + 30$

Shear Force Diagram: A horizontal line at 20 kN from x=0 to x=1, then a vertical drop to -10 kN at x=1.

Bending Moment Diagram: A linear increase from 0 at x=0 to 20 at x=1.

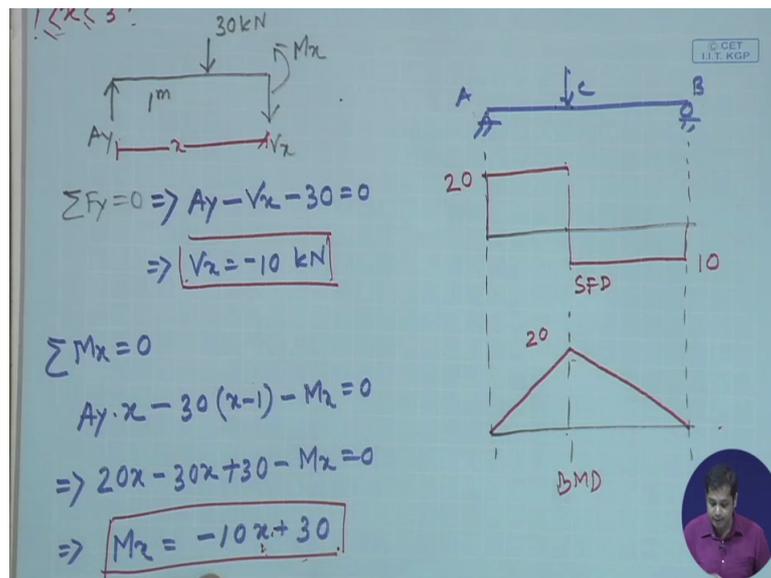
And again between this to this bending moment diagram is this, this equation is valid for this range. So if you substitute x is equal to 1, you get M_x is equal to 20 which is again consistent. So there is a continuity maintained. And x is equal to 3 if you substitute then M_x is equal to zero, it is simply supported beam, so bending moment will be zero, okay.

(Refer Slide Time: 28:34)



Now this is bending moment diagram and this is shear force diagram, okay.

(Refer Slide Time: 28:41)



Now let us see what is the observation? Shear force is constant, bending moment is one order higher linear. In this case shear force is constant, bending moment is one order higher linear.

(Refer Slide Time: 28:57)

The image shows handwritten notes on a whiteboard. On the left, a free body diagram of a beam segment of length x is shown. It has a pin support at A with reaction A_y and a shear force V_x at the right end. A 30 kN point load is applied at $x=1$ m. The bending moment at the right end is M_x . Below the diagram, the equilibrium equations are written:

$$\sum F_y = 0 \Rightarrow A_y - V_x - 30 = 0$$
$$\Rightarrow V_x = -10 \text{ kN}$$
$$\sum M_x = 0$$
$$A_y \cdot x - 30(x-1) - M_x = 0$$
$$\Rightarrow 20x - 30x + 30 - M_x = 0$$
$$\Rightarrow M_x = -10x + 30$$

On the right, a similar free body diagram is shown for a segment of length x from the left end. It has a pin support at A with reaction A_y and a shear force V_x at the right end. The bending moment at the right end is M_x . The equilibrium equations are:

$$\sum F_y = 0 \Rightarrow V_x = A_y$$
$$\Rightarrow V_x = 20 \text{ kN}$$
$$\sum M_x = 0 \Rightarrow -M_x + A_y x = 0$$
$$\Rightarrow M_x = 20x$$

A small circular inset in the bottom right corner shows a man in a purple shirt.

Again bending moment and shear force diagram you see, at this point your bending moment is positive.

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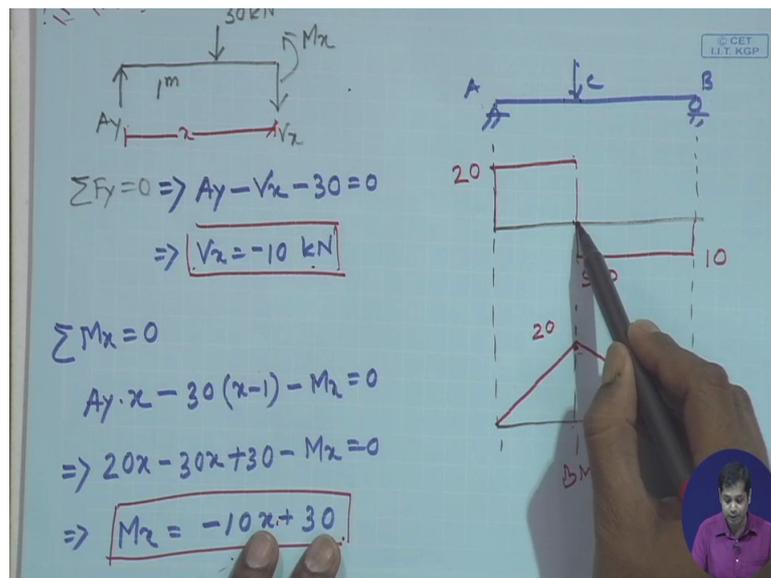
The image shows handwritten notes on a whiteboard. On the left, the same free body diagram and equilibrium equations as in slide 28:57 are shown. On the right, the complete Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) are plotted for a beam of length 2 m between points A and B. A 30 kN point load is applied at $x=1$ m.

The SFD shows a constant shear force of 20 kN from A to $x=1$ m, and a constant shear force of -10 kN from $x=1$ m to B. The BMD shows a linear increase in bending moment from 0 at A to 20 kNm at $x=1$ m, and a linear decrease from 20 kNm at $x=1$ m to 0 at B.

A small circular inset in the bottom right corner shows a man in a purple shirt.

And then bending moment is maximum. Here at this point your shear force is changing sign, okay.

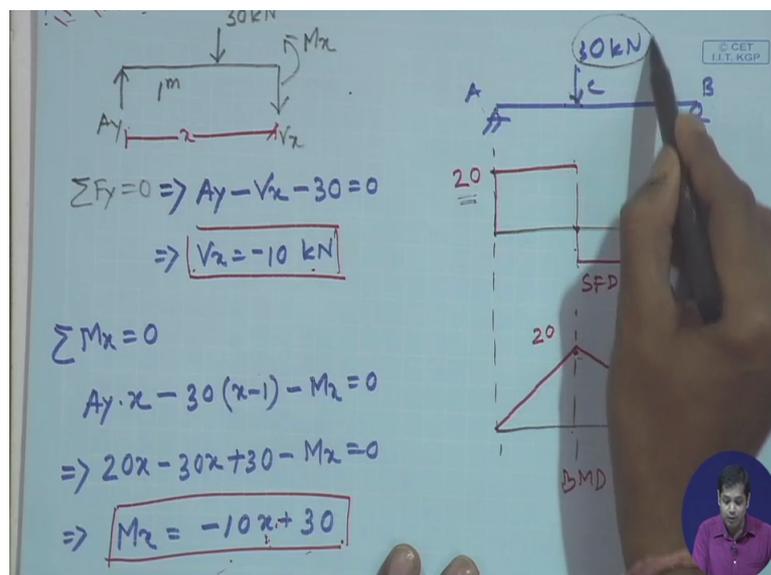
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One another point to be noted as I said, (al) it is a good practice to draw bending moment and shear force diagram just below the structure. You see what was the value of this force? It was 30 kilo Newton, right? 30 kilo Newton which is downward. You see this is 20 and this is 10, 20 plus 10 is equal to 30 kilo Newton.

So C is a concentrated load here and because of that load shear force is constant throughout and suddenly you have a concentrated load and therefore there is sudden change in sign of bending moment or sudden drop in or sudden increase in shear force and that change will be according to the value of this load.

(Refer Slide Time: 30:00)



So this is reflected in shear force diagram, okay. Now for today that is all. We will have one tutorial class, we will try to have some more example on bending moment and shear force diagram. Thank you.