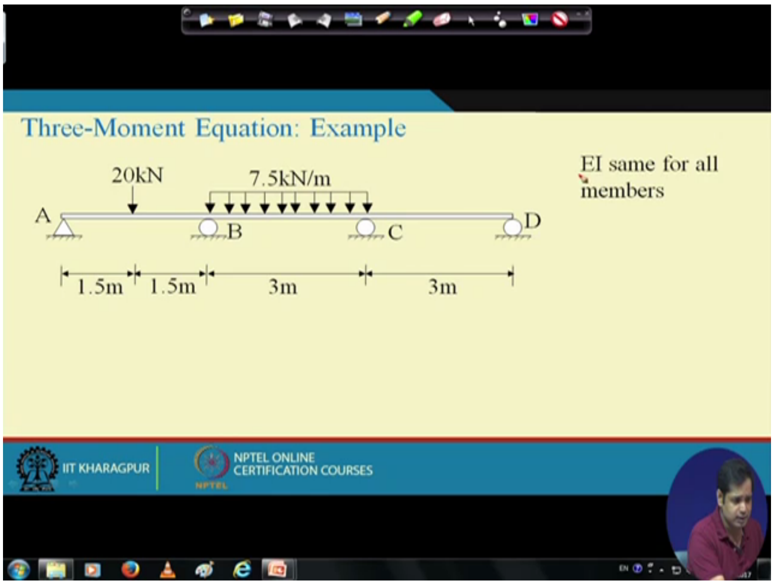
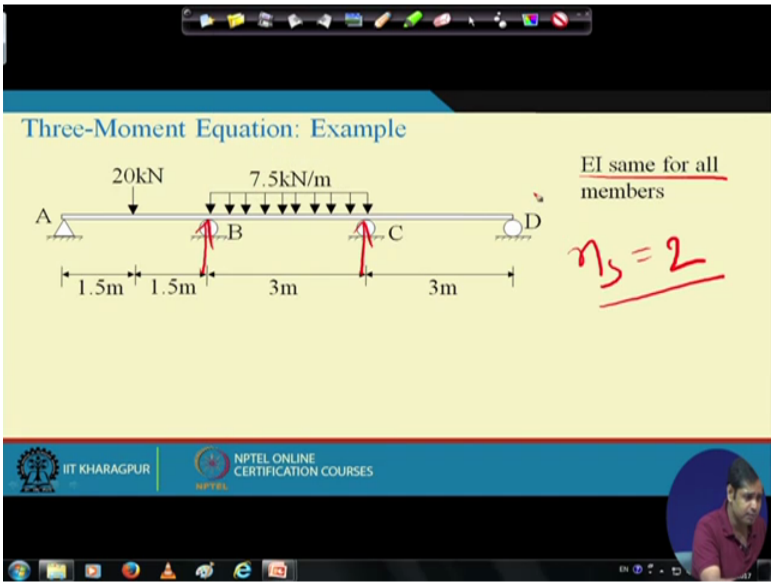
Structural Analysis I.
Professor Amit Shaw.
Department of Civil Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-43.
Analysis of Statically Intermediate Structures by Force Method (Continued).

(Refer Slide Time: 0:40)

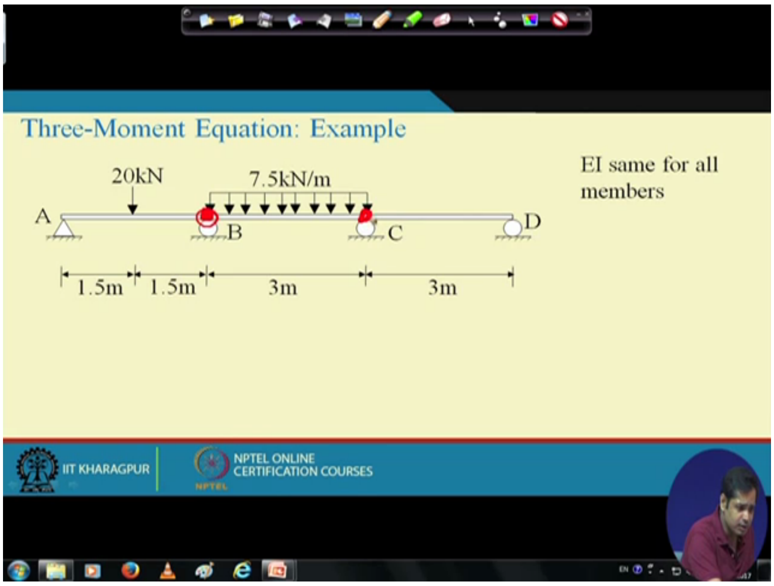


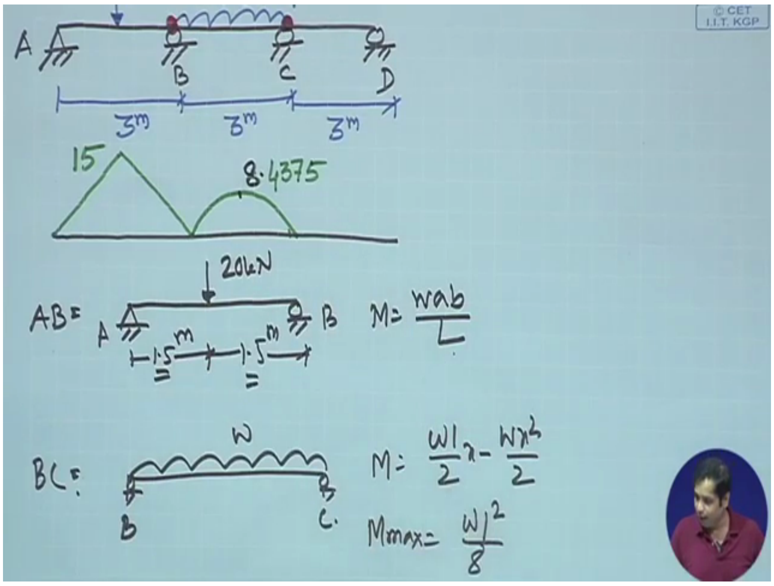


Hello everyone, last week, last class we discussed the derivation of Three-moment equation, what we will be doing today is we will demonstrate that derivation, demonstrate the concept through some examples. Let us take our 1st example, so this is, today's class is Three-moment equation and examples. Let us take the 1st example, these are continuous beams, 3 span continuous beams, A, B, C, D subjected to the loading condition that is shown here. Mth, fractural rigidity of all the EI is constant for all these all these members AB, BC and CD, fractural rigidity is constant, we will see some more examples today where fractural rigidity is not constant.

Okay, now this is an indeterminate structure and what is the degree of indeterminacy, we can see that that NS is equal to NS is equal to 2, reaction at, this reaction and this reaction because if we remove the support B and C, it becomes simply supported beam which is statically determinate structure. Now the presence of B and C, support at B and C make the structure statically indeterminate. So NS is static indeterminacy is equal to 2. Now so, okay, now let us then start this example, okay.

(Refer Slide Time: 2:11)





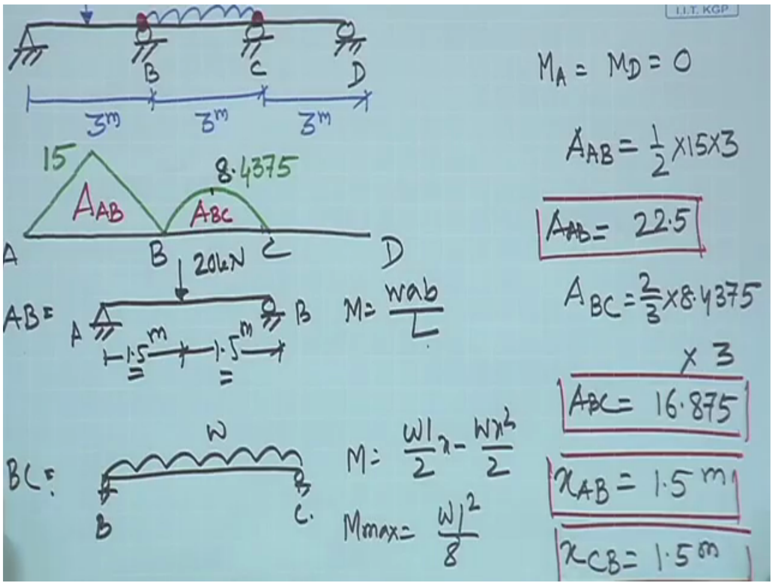
Now you see 1st part of, see the example is like this, this is a 3 span beam which is AB, this is A, then a roller support here, another support and then another roller support here, this is A, this is B, C and D and this is subjected to uniformly distributed load on span BC which is 7.5 kilos Newton per metre and then at midspan of A, B is a concentrated load which is 20 kilos Newton, okay. And these are all are 3 metre, 3 metre, 3 metre, this is at the midspan 1.5 meter. Okay. Now the 1st step is if we assume that there is a there is a hinge here, if you see we assume that there is a hinge here and there is a hinge here. So by inserting hinge, we are actually releasing the moment constraint at B and C, rotational constraint at B and C.

So static indeterminacy of this problem is 2, so we will insert one hinge at B, release 1 constraint and another hinge at C, release another constraint. So now this structure, after inserting 2 hinges at B and C, this structure becomes statically determinate structure, this is hinge, internal hinge, okay. Now we insert one hinge here at B and another hinge here at C. So essentially it becomes AB, BC, and CD, they all are simply supported, they, we can keep them as simply supported beam. Okay. Now so, let us let us find out the bending moment diagram for this structure, bending moment diagram is, if you, if you see for span AB which is subjected to a concentrated load at the midspan and the bending moment becomes, bending moment is this.

And then for band, for span BC, which is subjected to uniformly distributed load, bending moment, we know the bending moment is parabolic, it is like this and for span CD, there is no load, so there is no bending moment. So for span AB, for span AB, if we take, if we know span AB which is concentrated load, 20 kilos Newton, we know that bending moment will be this is 1.5 and this is 1.5 metre and we know the bending moment at this point will be WAB by L, in this case A is equal to 1.5, B is equal to 1.5, W is equal to 20, M is WAB by L, N is equal to 1.5 meter, B 1.5 meter, L is 3 metre, so this becomes 15, 15 kilos Newton metre. Okay.

And similarly if you take span BC, I am not writing the detail of these expressions for bending moment because we know how to do it. And then for span BC which is subjected to uniformly distributed load, this is AB, this is BC, BC, we know M is equal to, if it is W, then WL by 2X - WX square by 2, that is a question for bending moment and bending moment is maximum at the midspan and that value is WL square by M Max is WL square by 8. So L is equal to 3 metres, W is equal to 7.5 kilos Newton per metre and then this value becomes 8 point, this value becomes 8.4375 kilos Newton metre.

(Refer Slide Time: 6:53)



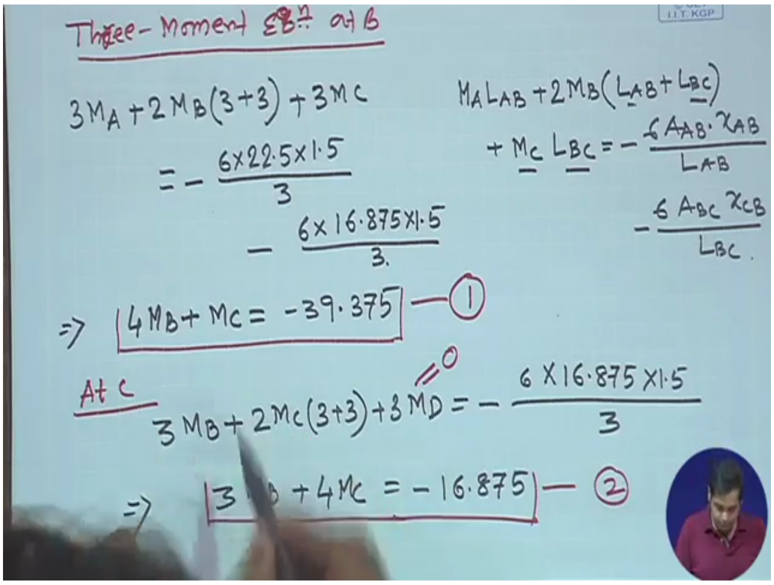
I am not writing the unit here, so this is the bending moment now if we see the, if we see equation for Three-moment, the equation is this, right. Now since E I is constant in this case, so this values are, these are all constants, okay. Now 1st let us apply Three-moments theorem at B, Three-moment theorem Three-moment equation at B means consider this span AB and then BC, gets equation and then we apply the Three-moment equation at C, means considering span BC and CD. Okay.

Now from this, from the configuration and support condition of this structure, we can say that the moment at A and moment at D is equal to 0, right. So MA, MA is equal to 0 and MD is equal to 0 because at this it is beam support at that point. Okay. Now we know that MA is equal to 0, MA is equal to MD is equal to 0. Right. Now in this equation, in this in these expressions and if you remember AB is the area of bending moment between A and B and ABC is equal to area of bending moment between B and C.

And XA B is the distance of Centre of that bending moment from A, bending moment M by I diagram from A and XCB is equal to the centroid of bending moment diagram between BC, BNC and this distance is measured from C. So let us let us now 1st determine what is that AB and this is then area AB, this is the area AB and this is the area area BC, area BC. So area AB is equal to half into 15 into 3 is the, then this becomes 22.5. Now, then what is area BC, area BC, that is equal to this area of this 2 3rd into this value 8.4375 and then into the length 3 which is 16.875. Okay.

So this is the area AB, we will be using these values, and this is the area BC. Okay. So we need to find out what is X AB, XA B is equal to, X A B which is the centroid of this diagram from A, so this is symmetric, this is 3 metre length, so X A B becomes 1.5 meter. Similarly X CB, it is the area, it is centroid of this bending moment diagram from C, again it is symmetric, so it becomes 1.5 meter. So these values we have already obtained. Okay. So this is A, B, C, D. Okay, now let us apply the Three-moment equation, 1st apply Three-moment equation at B, and then apply Three-moment equation at C.

(Refer Slide Time: 10:49)



So 1st apply Three-moment equation at B, okay, so, Three-moment equation at B, Three-moment equation at B. Now what is the expression, again the expression, if you if I write desperation, general expression for Three-moment, E I is constant, there is no support settlement, this question becomes MA LAB + 2MB into L AB + LBC + MC LBC is equal to -6 AB into X AB by LA B and 6 ABC into X CB by LBC. This is the Three-moment equation, we know this, right. Now if you apply it, then what we have is there is no EI EI term because the structural rigidity of span, this is constant, okay.

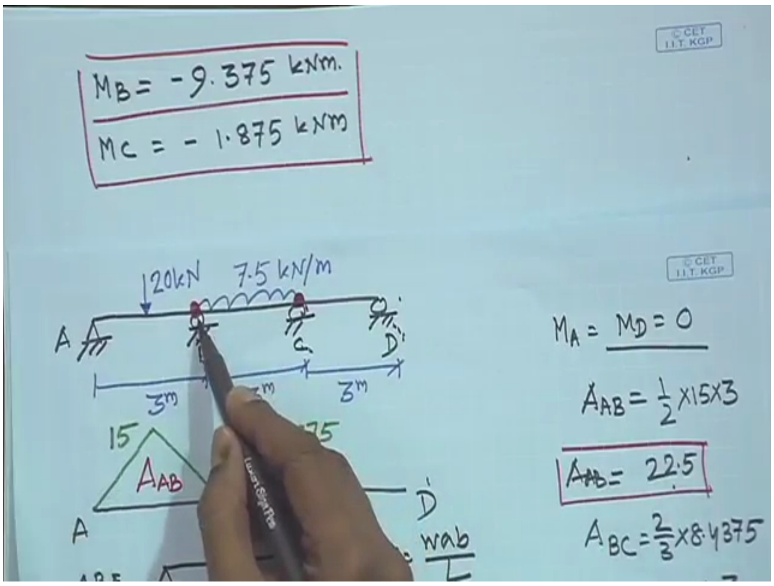
Now what will be LA B is equal to 3 metres, so this becomes 3 into MA then + 2MB, LAB LBC, all 3 metres, +3+3 and then + MC into 3 BC, LBC is equal to 3, 3M C, that is is equal to -6 into AB and if you remember AB we have determined, this is 22.5 and BC is 16.875, that is already determine, -6 into 5 XA B, again XA B is 1.5 and X CB is 1.5, so it is 1.5 and the length is 3 and then -6 into 16.875 into 1.5 by 3. So this becomes 4 MB + MC is equal to -39.375, you can do this calculation, okay. So this is equation number 1, okay. So this is equation number 1.

So we have relation between MB and MC and that relation we obtained by applying Three-moment relation at B. Now let us let us apply, so let us apply now Three-moment equation at C. If we apply the moment equation at C, then the expression becomes, similarly if we do the same exercise, the expression becomes at C, at C, this expression becomes 3 MB + 2MC into 3+3, when we apply at C, it means we have to consider, when you apply at C, we have to consider span BC and CD, means MB, MC and MD will come into this equation. Okay.

And then +3 MD, okay and that is is equal to, that is is equal to -6 ABC, ABC means 16.875 into 1.5 divided by 3, you check here, since there is the bending moment between C and D is equal to 0, so therefore on the right-hand side we will have just contribution from this only only for the for the span BC, okay. We already, we have already written here that MD is equal to 0 because it is simply supported, it is simply supported end, so it is hinged support we have it there, so MD becomes 0, so here MD becomes 0.

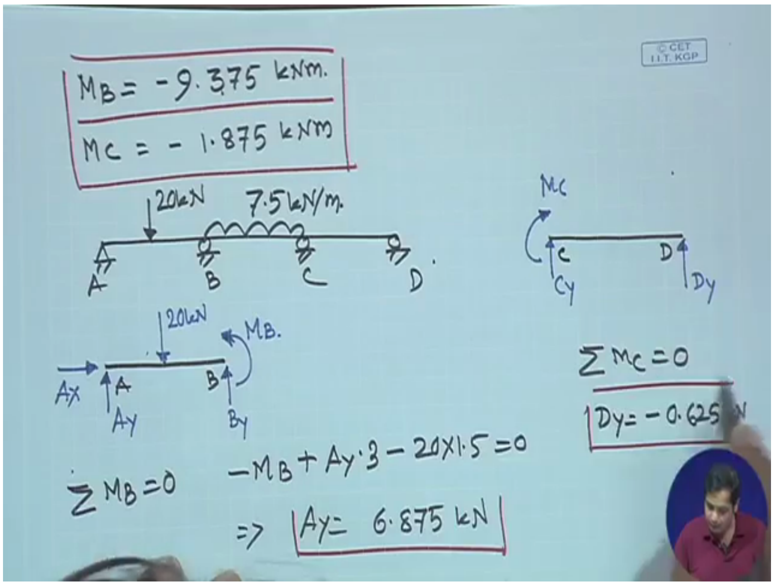
If MD is 0, then finally the equation becomes 3MB, 3MB +4 MC is equal to -16.875, okay. So this is the equation number 2, this is equation number 2. Now this equation is another relation between MB and MC, now we have 2 equations and number of unknowns are2, MB and MC, we can solve them and if we solve them the values that we get is this.

(Refer Slide Time: 16:10)



MB is equal to MB is equal to you can you can, I am not solving it, I am just writing the final result, -9.375 kilonewton meter and MC is equal to -1.875 kilonewton meter. So 2 constraints, 2 redundant forces were MB and MC and these redundant forces, the values of these redundant forces are these and we obtained these values by applying Three-moment equation at B and C. Now once we know, once we know the moments at B and C, then rest of the thing gives just take the free body diagram of different parts, apply the equilibrium condition and get the other, get the other forces.

(Refer Slide Time: 17:57)



And we can do that let us let us draw the free body diagram of the, free body diagram of MB, the actual of the actual structure because now we need to, we have already obtained the values at MB and MC, now we need to find out the reaction forces of the actual structure, not for this. So actual structure is this A, B, C, D and then we have force and then like this. This is 7.5 kilos Newton per metre and this is 20 kilos Newton. Let us draw the free body diagram of AB, if we draw the free body diagram of AB, then these will be the reactions.

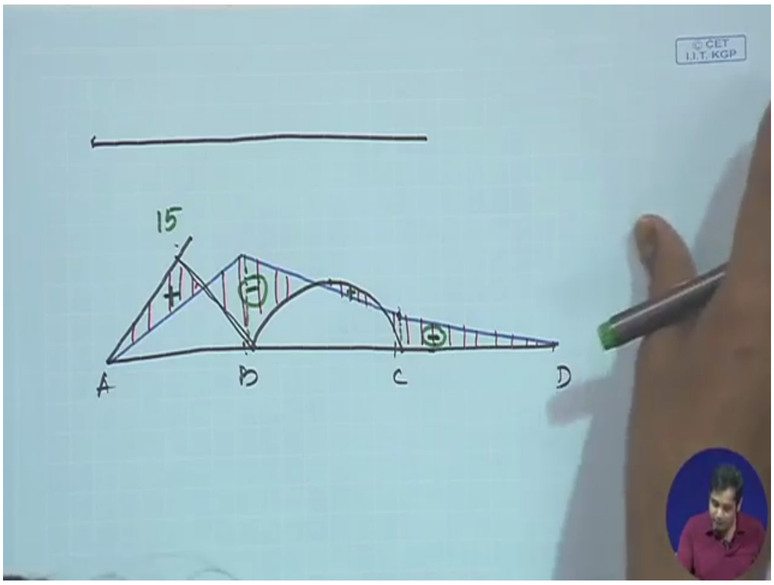
A Y, A X and then BY and then moment MB and concentrated load 20 kilonewton. Okay, not writing FX here because we are doing that there is no actual deformation. Now apply, apply summation of say M B is equal to 0, moment about B is equal to 0, this is A, this is B, what are the moments we have, we have - MB which is anticlockwise, so negative. Then then AY, AY will contribute and then AY will be AY into into 3, 3 is the length, and then AY, AY produces clockwise moment and then finally -20 into 1.5, this will produce anticlockwise moment and that is equal to 0.

MB already we obtained this, MB if you substitute in this equation, we will get AY is equal to 6.875 kilonewton, okay. Now this is, this is AY. Now similarly if we take free body diagram of CD, if we draw the free body diagram of CD and then what are the what are the reactions we have. We have support reaction DY and then CY and then MC. Okay. This is C, this is D.

Now take MC is equal to 0, if you substitute MC is, if you take moment about C is equal to 0, only forces will contribute DY and the MC, the value of MC already known and we can get, we can get the value of DY.

DY will be -0.625 kilonewtons. So similarly you can you can apply once you know D, similarly you can take the free body diagram of other parts, apply the equilibrium equations and get the other unknown. Because with the knowledge of binding moment at, moment of B and moment of C, the structure becomes determinate, all the other reaction forces or the internal forces that can be determined just by apply the equilibrium conditions, okay. So rest of the things are very straightforward, you can continue with that. You can you can compute the results for this. Okay. Now if we have to finally draw the bending moment diagram of this, okay.

(Refer Slide Time: 20:42)

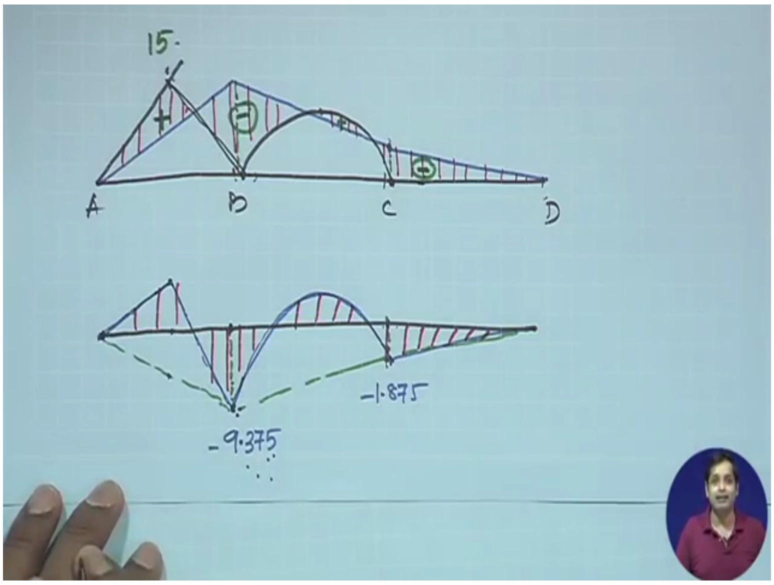


Now let us draw the bending moment diagram, bonding moment diagram of this will be the you see the things, the linear superposition theorem is valid, so bending moment diagram, this bending moment is assuming there is a hinged at B and C, by releasing the redundant forces and then, so this is the primary structure which is subjected to only the external loading. Then there will be another primary structure which is subjected to only the redundant forces. So total bend, total response of the structure will be, the response of the primary structure subjected loading last the response of the primary structure subjected to the redundant forces.

So total bending moment will be, or bending moment diagram will be this bending moment diagram + the bending moment diagram that you get because of these redundant forces, okay. Now there are 2 ways you can draw this bending moment diagram, 1st is the betting moment diagrams due to the, let us draw it here so that this, so 1st the pending moment diagram is due to the redundant forces, so this is A, B, this is C, and this is D. And then, okay, so this value is, this value is 50 and this value is we know 8.4 and there is no bending moment at C.

These are all positive bending moments right because the way we take the sign convention according to the these bending moments are positive. Now that we have MB and MC are negative and then support, moment at A and moment at B, they are 0 because it is hinged support. So MB is negative and this value is -9 point something, so it is negative, so this value is -9 point something and then at C it is -1.875 and then you are bending moment for the redundant forces becomes this, this and this. Okay. Now if we, so these bending moment on this is, this part will be cancelled, bending moment on this will be only this part. Okay, only this part.

(Refer Slide Time: 23:52)



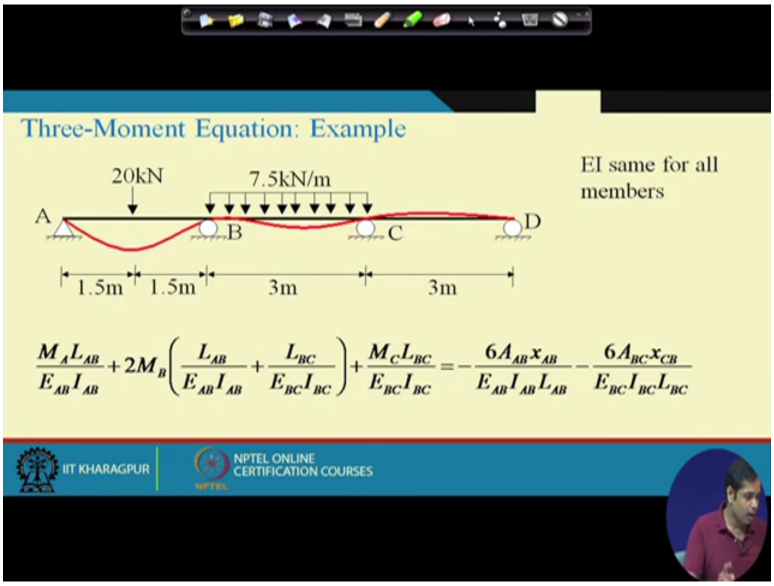
This will be the bending moment on this. And the values will be, this is positive, this is positive and this is negative, this is also negative and this is positive. This is one way you can show the bending moment diagram, now another, another approach I personally prefer the 2nd one, this approach. You draw this, draw the entire beam okay, as per our, according to our sign convention we draw the hop sagging bending moment on the top side and hogging bending moment at the bottom side, right. So we always know the bending moment as per our sign convention, we are trying the bending moment in the compression side, okay.

The 1st draw the negative bending moment, negative bending moments are MB, at MB it is -9 point something and at MC it is this value. So negative bending moment is this, okay. Then since the superposition is valid, so total bending moment at any section will be the bending moment to do the external load + bending moment to do the redundant forces. Now the bending moment due to the external load is this, you draw the bending moment at A is equal to 0, at B it is 0, at this point it is 15 but at this, this value is 9.375 and this is 1.875, there all negative values, they are all negative values.

So at the midspan it will be -9.375 divided by 2, so at the midspan, bending moment due to the primary load is 15 and the redundant force is this divided by 2, so the bending moment is somewhere here. Okay. And then at point B, bending moment due to external load is 0, so at point B, mainly bending moment, whatever we have that is due to the due to the redundant forces which is the values, okay. So this value and then this value. Then again go to C at point C, the bending moment is, bending moment is, it is 1.875 and similarly for external load, the bending moment at C is equal to 0 and it varies for the external load it varies quadratically.

And then this is, this varies quadratically, okay. And between C and D, there is no bending due to the external load, it is only the redundant forces. So the bending moment for this become, for this becomes this. The advantage of writing the bending moment, drawing the bending moment like this is again you do not have to mention the sign like this, it is always understood, if it is, because our sign convention is, we are drawing the bending moment on the compression side, so it is the sign, just by looking at the bending moment diagram we can understand the sign.

(Refer Slide Time: 26:55)



Okay, so this was the 1st problem, now quickly let us see what happens if the condition is slightly different. Now this is the deflected shape of this problem, we know the bending, we have just now drawn the bending moment diagram and this will be the deflected shape of this problem. This deflected shape is not the actual, it is a normalised deflected shape, scale deflected shape, the idea, main thing is to have an idea about the deflection of the bend. Okay. Now the important thing get is we assume in this example, EI is constant throughout this, throughout this plan, right.

But actually EI may not be constant and if there is no, if that is not constant, then how to take , how to, that variation we can take care, we can take that variation with the Three-moment equation itself, then another condition where, suppose in this example we have seen the, the structure is, the response of the structure is mainly due to the external loading. Loading is being applied in the form of some distributed force or distributed moments or concentrated force and moments. But as we have seen in the last class the external agitation may be given to the structure in the form of settlement of the support, okay.

Support may displace and that displacement of the support may cause, may induce stresses in the bending moments, shear force in the bend. Okay. So next example we will see is, 2 examples, the same example but with slightly changed , change the example by 1st thing by considering E I is not constant and then in addition to the these actual support, we will see that if one of these supports, they settle, then what happens to the deflectation of bending moments. Okay, that we will discuss in the next class, thank you.