Structural Analysis 1
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Lecture 4
Determinate and Indeterminate Structures

Welcome to lecture 4 of module 1. In this module we will try to understand the concept of determinate and indeterminate structures. Specifically what we will do is in the last class we discussed the concept of static equilibrium, I will give some more applications of static equilibrium and then through this (opplica) application we will try to understand that what is determinate and what is indeterminate structures, okay.

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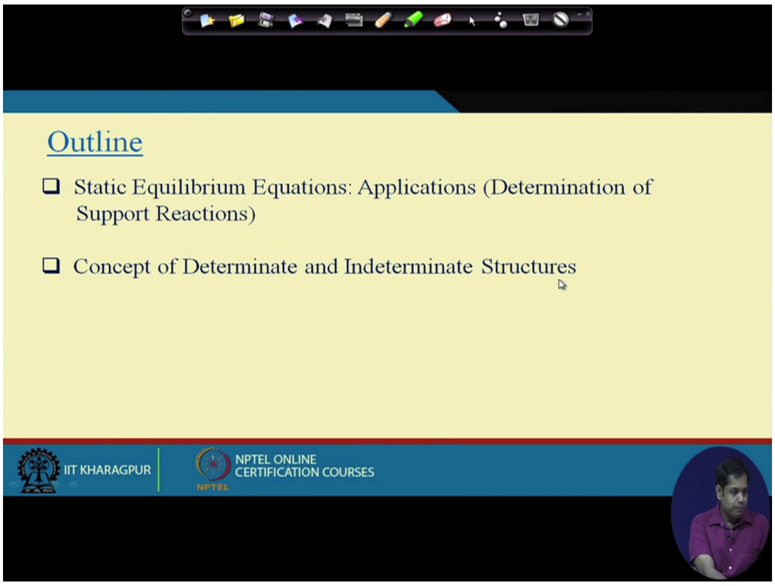

Let us quickly revise what we have discussed in the last class. What is equilibrium? We said that equilibrium is when any object subjected to some external load and then object is the reaction to the external load. Some internal force is generated in that object. And we say that object is in equilibrium when this external load and internal load balance each other. And this equilibrium can be represented through some equation which is called equation of static equilibrium.

And this equation says in three dimensions that summation of forces in each direction is equal to zero and summation of moment about any axis is equal to zero. And the two dimensions this equation becomes summation of forces in x and y direction is equal to zero and summation of moment about z axis is equal to zero, okay.

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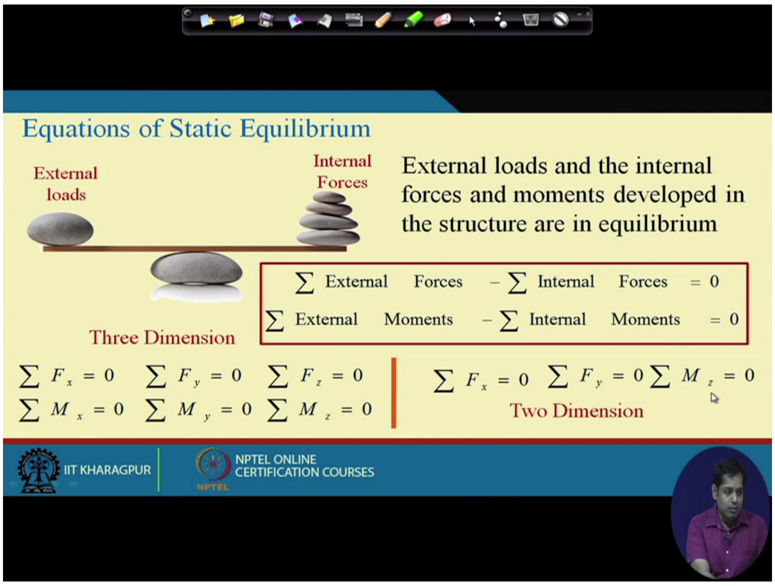

Before we start giving some more applications of equilibrium equations and the determinate and indeterminate structure let us see what are the properties that these equations should satisfy? you are engineering maths course, you see these are the equations, right?

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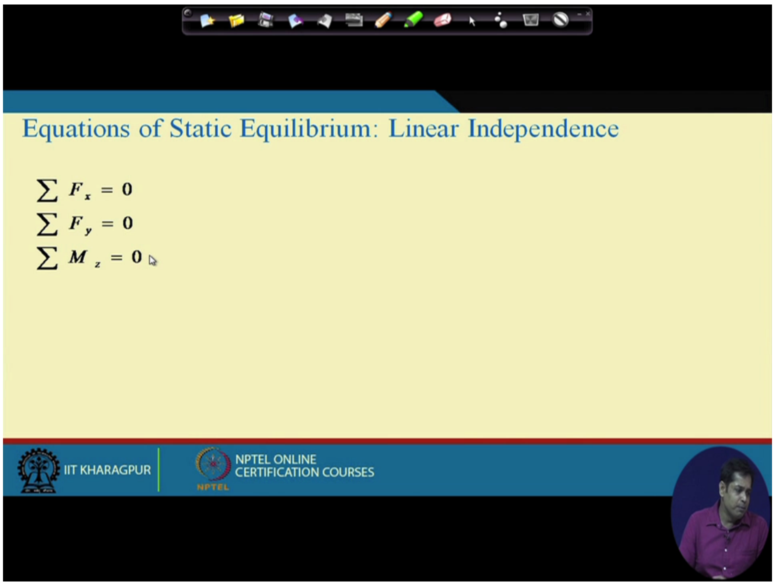

If you remember towards the beginning of this module we said that any physical system needs to be transmitted through some equations that could be an algebraic equation, differential equation or integral equation and then once this equations are obtained, representing the physical process we need to solve these equations, right? Now in this case in the present context those equations are equilibrium equations.

So these equilibrium equations essentially are representation of a physical process, right? Now at the end we will see that most of these equations can be translated into a system of linear equation like this, okay.

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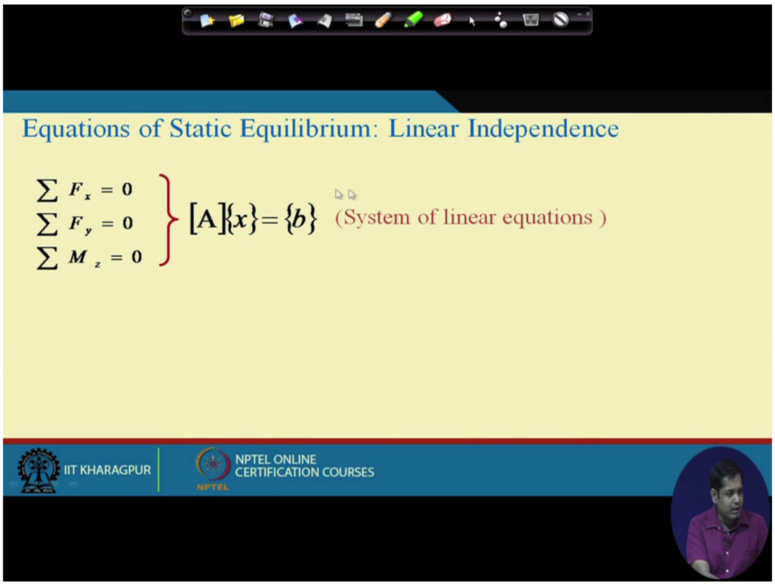

Now the problem of understanding a physical process is reduced to a problem of solving a system of linear equation. But linear because in this course we assume the structure behaviour is linear. The relation between force and displacements are linear. We will understand the linearity in more better way as we go through this journey. The solution of this equation actually gives you the understanding of the physical process, right?

Now these equations needs to satisfy certain properties so that we can have the solution in a unique way. And the property is that this linear equation should be independent. By independent means that none of the equation can be derived from others. Just to better understanding of this line let us give some example. Like such three equations, okay. And we have three variables x, y, z.

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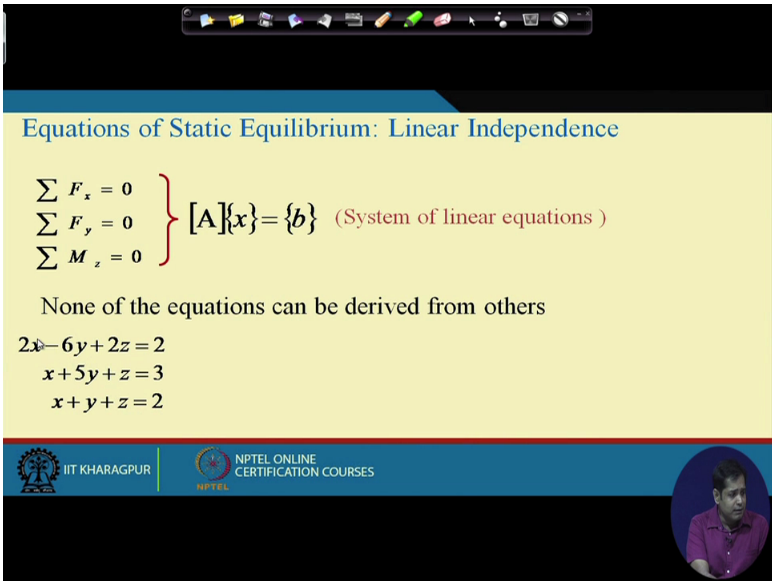

Suppose this equation you obtain from equilibrium. Now you see this equation representing a matrix form like this.

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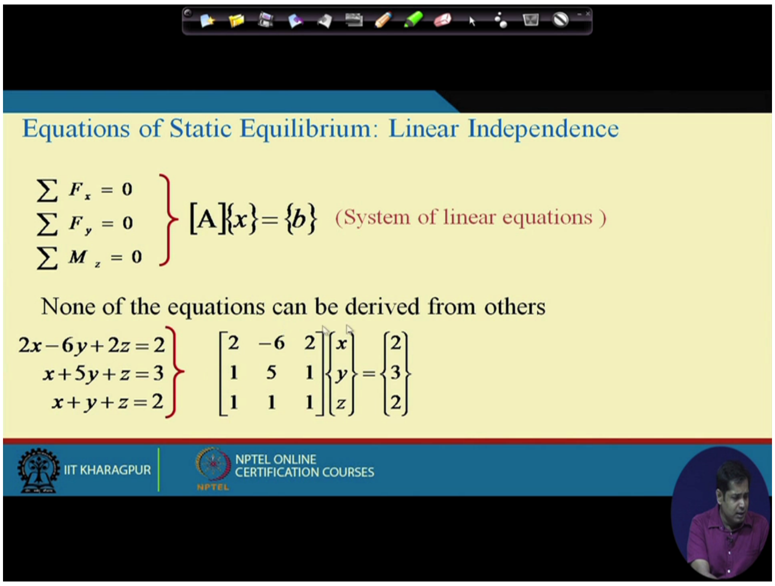

If you look at these equations closely they are not linearly independent because you see the third equation can be derived from this two.

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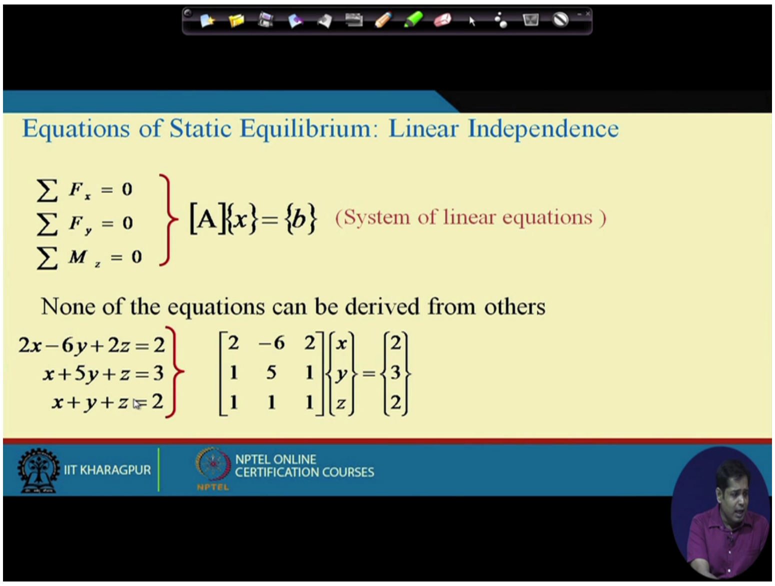

For instance if you take this and then add two times of this equation you get this equation.

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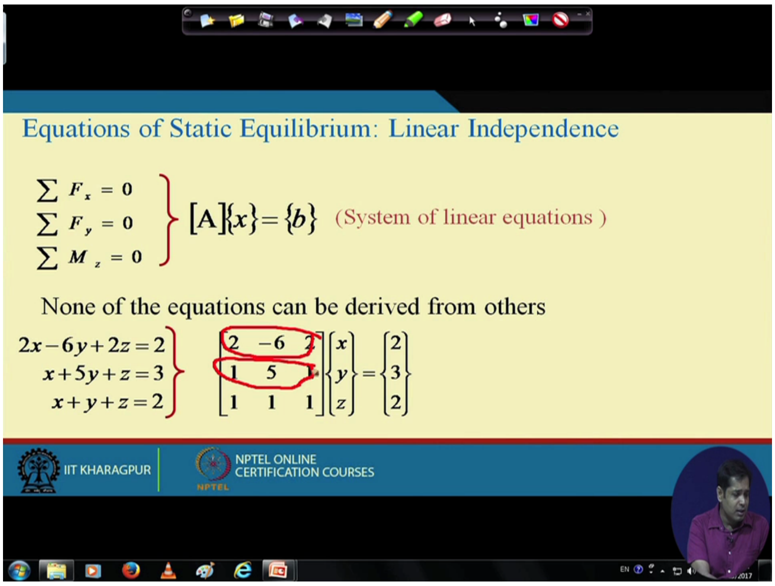

So these equations are not linearly independent. So if the equations are not linearly independent then what happen? You cannot have a unique solution of the system, right? For instance in this system all these three equations are satisfied with these values of z and so on. You can have many values of this combination which can satisfy this equation.

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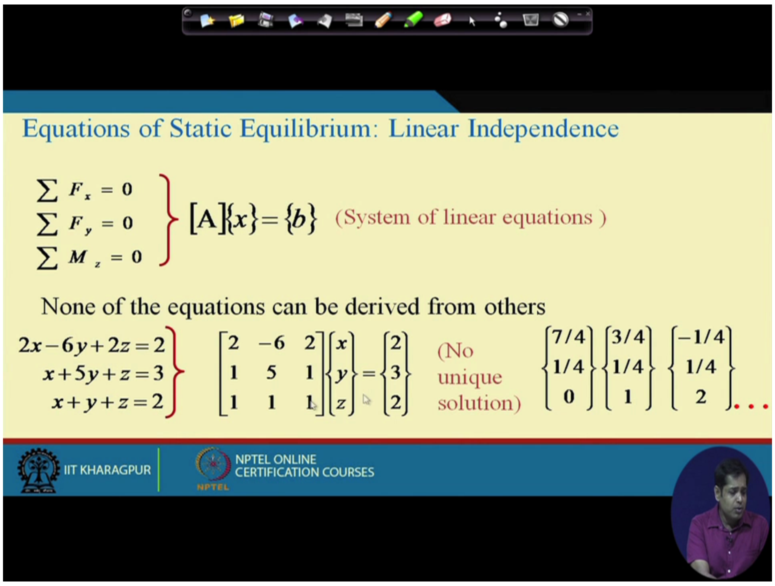

So you cannot have a unique solution. Just for better understanding suppose I want to locate a point in a three dimensional space. So what information do I need? I need the position along x axis, position along y axis and position along z axis. Now if I give you position along x and y axis then you cannot uniquely determine the point.

You see when I say that x coordinates of a point is this, y coordinate of a point is this and z coordinate of a point is this means I am giving you some new information in the form of coordinates in any particular direction I am giving you some new information about the location of the point and with that information you can uniquely look at that point. Similarly all these equations give some information about these variables.

Now this information should come from independent sources and if they do not come from independent sources then the equation become dependent and you cannot solve those equations. Now in that case what we say is the system is not determinate or another way the system is indeterminate. Now we will see the similar thing in the context of (struc) structural mechanics but before that let us see some example or application on equilibrium equations, okay.

Now let us take one cantilever beam which is subjected to a concentrated load at point C. What we need is we need to determine the reactions at the support A. So the first step is to draw the free body diagram of the entire structure. We discussed how to draw free body diagram. First is the object needs to be made free from the support and then the support should be represented from their equivalent forces. Now you see this is the free body diagram of the beam AB.

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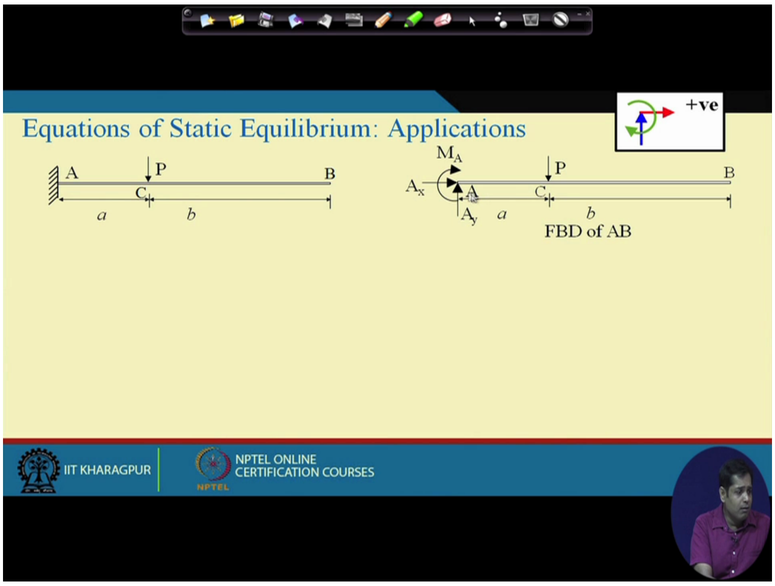

Now since this is characteristic of fixed support it has constraint in horizontal motion, vertical motion and then rotation. So equivalently you have horizontal reaction, vertical reaction and the moment.

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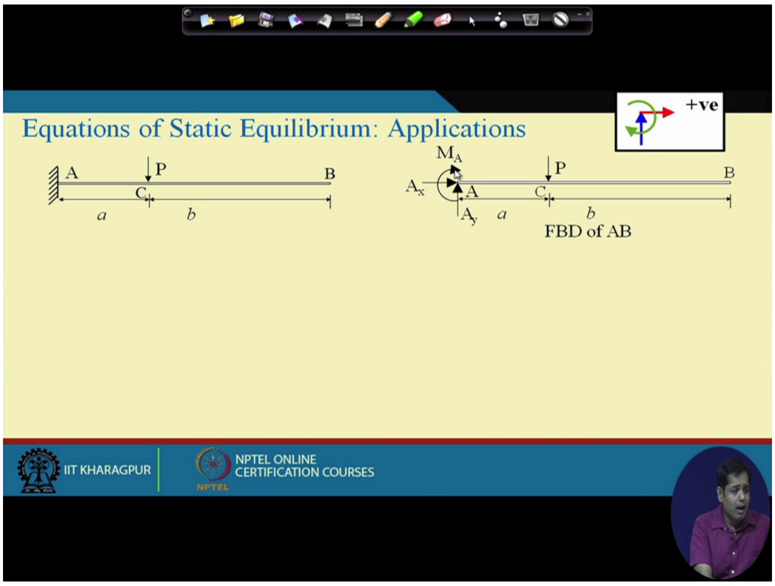

Now you apply the equilibrium equation here. Now applying the static equilibrium equation, the first equation is summation of forces in x direction is equal to zero.

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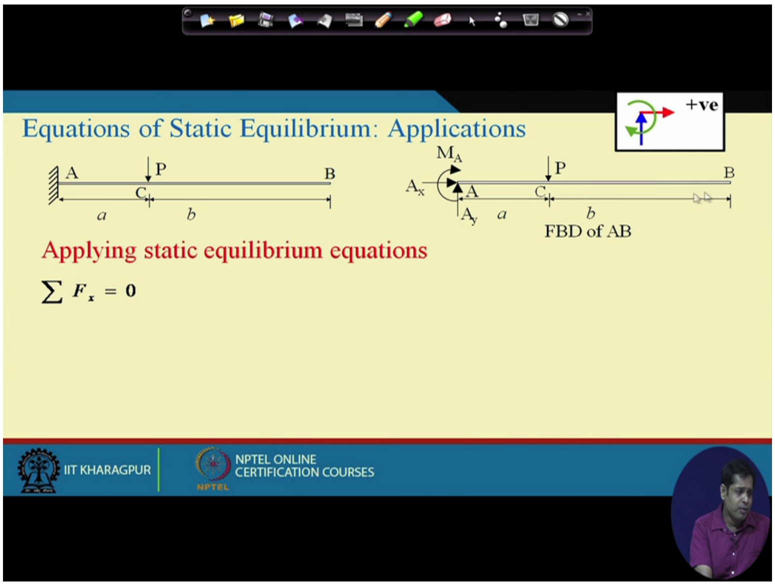

Now only component of force you have in x direction is Ax. So naturally this condition gives you Ax is equal to zero.

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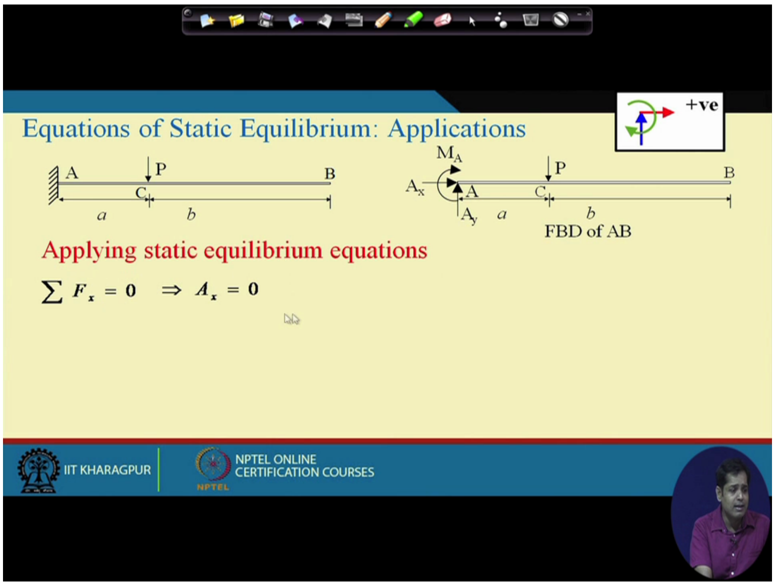

Now next equilibrium equation is summation of force along y direction is equal to zero. Now along y direction what are the forces you have? You have Ay and then P. Now you see this is our sign convention. When we subtract moment or subtract forces or add forces add moments, this is the sign convention we will use.

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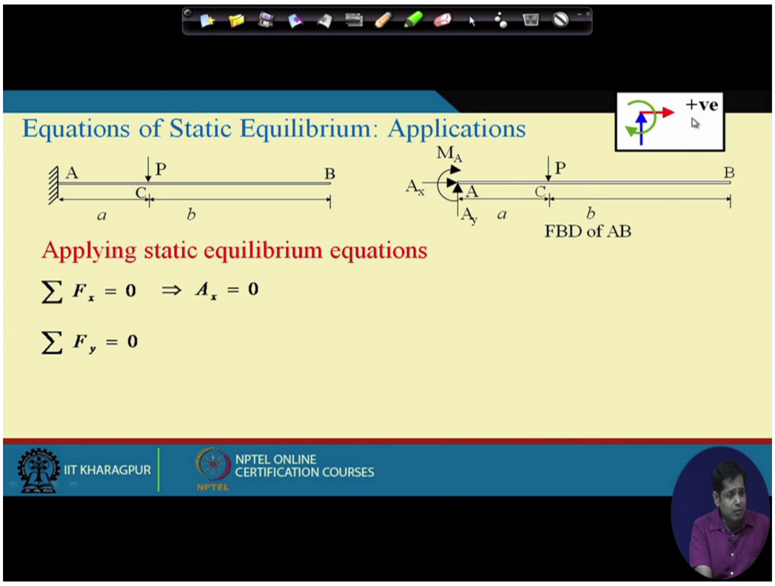

Vertical force and horizontal force in these directions are positive and clockwise moment is taken as negative. But in the next class when we talk about bending moment and shear force of beam we will discuss another sign convention but as far algebraic calculation is concerned algebraic operations are concerned this is our sign convention. You are free to use any sign convention but whatever sign convention we have used please be consistent with that.

But throughout the study throughout this course what we do is we will use this sign convention. Now if we write the equilibrium equation is in the summation of forces in y direction so Ay which is upward according to our sign convention is positive, then P downward vertically downward it is negative according to our sign convention, Ay minus P is equal to zero. It directly gives you Ay is equal to P.

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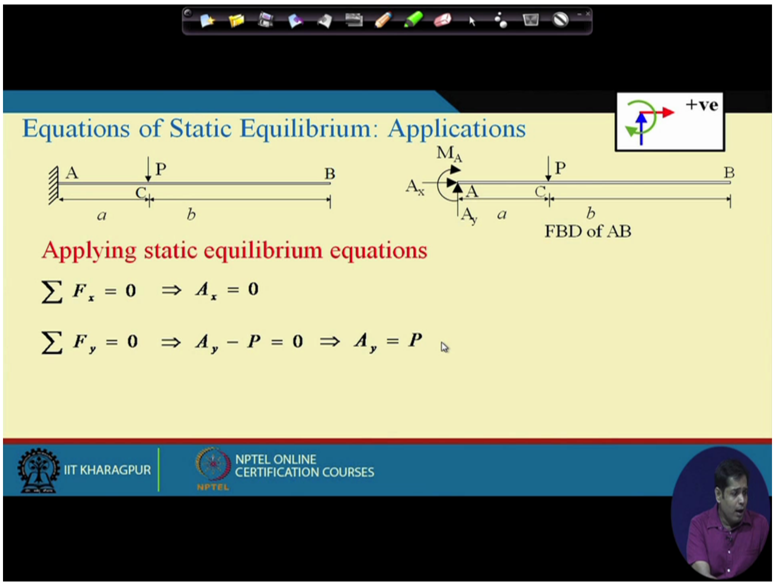

Now in the third equilibrium equation we see is the (mone) summation of moment at any point is equal to zero. Take moment at point A. So summation of moment at point A is equal to zero. So what you have is Ax and Ay will not contribute to this moment because we are taking this moment about point A and Ax and Ay they pass through point A.

So what are the moments you have? The Ma itself, moment at A and the moment due to force P. Now moment at A is in clockwise direction, it is positive. And then moment of P will be P into A. Again this is in clockwise direction. So this is positive. So this is the third equilibrium equation which directly tells you Ma is equal to minus Pa. And these are three support reactions.

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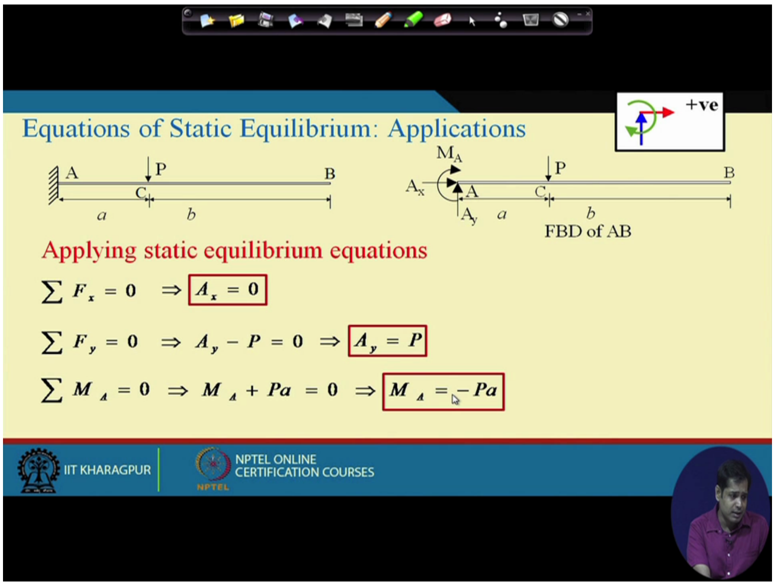

Now you see all these equations they give you some information about the system. The first equation tells you that Ax is equal to zero. Second equation tells you how the vertical reaction is and the third equation tells you how the moment is. Now these equations may be couple maybe decouple but this equation need to be linearly independent.

In this case we have seen that these equations are independent means this equation gives you some new information and those information will help you to find out the unknown. Let us take the similar example once again but apply one prop at point B. So it is now a propped cantilever beam.

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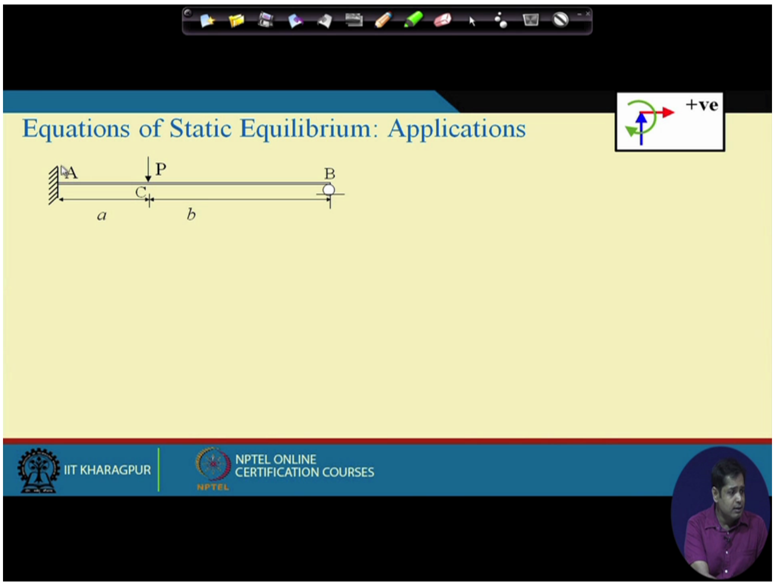

So at point A you have fixed support and point B the roller support. In the previous case when there was no roller support at point B your number of reactions were horizontal reactions at A, vertical reactions at A and moment at A. But now how many reactions we have? Three reactions at A, an additional vertical reaction at B. Now let us see what happens if we try to find out these reactions using equilibrium equations, okay. Now first draw the free body diagram of the system. Now this will be the free body diagram of the system.

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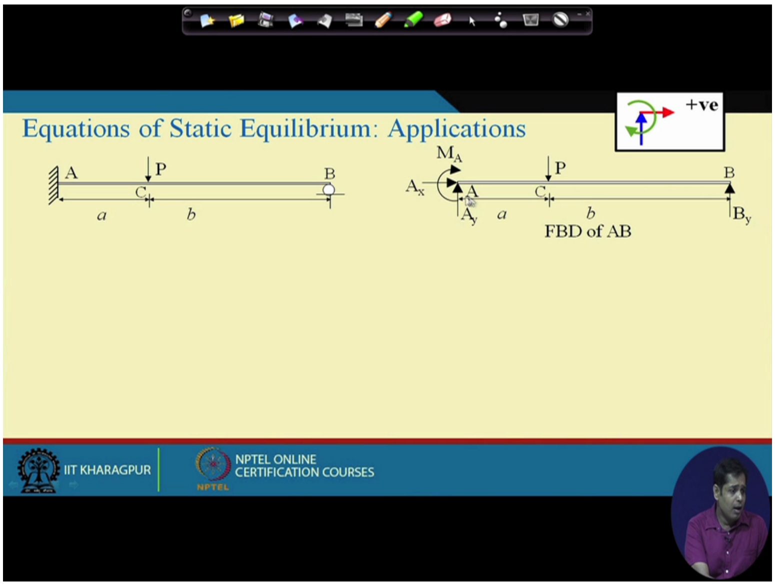

The characteristic of roller support is it provides constant in any particular direction or in any particular transition. So this is By reaction at B and Ax, Ay and Ma are the reactions at A. Now first equation is summation of forces in x direction is equal to zero. So naturally only component of force you have in x is Ax. So Ax is equal to zero. Then next equation is summation of forces in y direction is equal to zero. Now what are the forces we have in y direction? Ay reaction at A, then By reaction at B and then external load P at C.

So Ay and By are vertically upward as per our sign convention this is positive and P is vertically downward so it is negative. So this summation of Fy is equal to zero leads to another equation Ay plus By is equal to P. Total support reaction should balance the external load P vertical direction.

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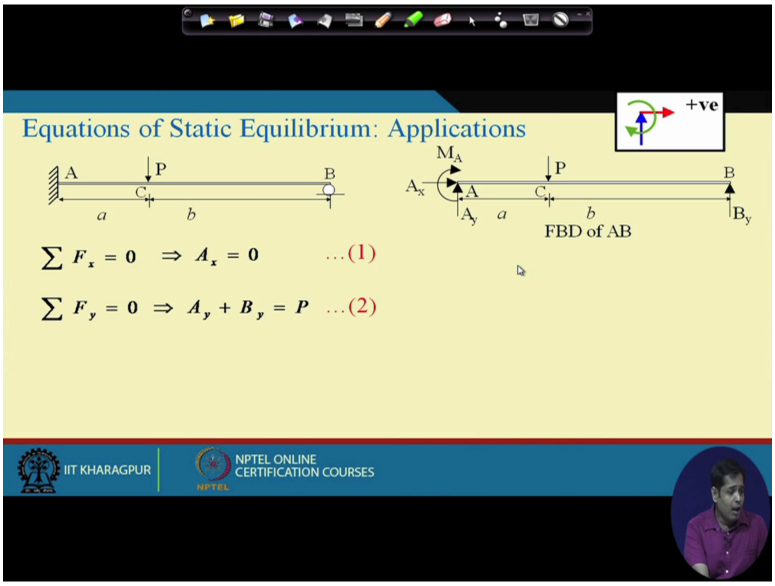

Now the next equilibrium equation is moment at A is equal to zero. Now moment at A is equal to zero, what are the moments you can have? The moment A itself and then clockwise moment due to P and then anticlockwise moment due to B. So Ma which is clockwise that is positive and By into A plus B which is anti clockwise negative and then P into A which is again clockwise that is why positive. So this is the equation you have when you take summation of moment is equal to zero.

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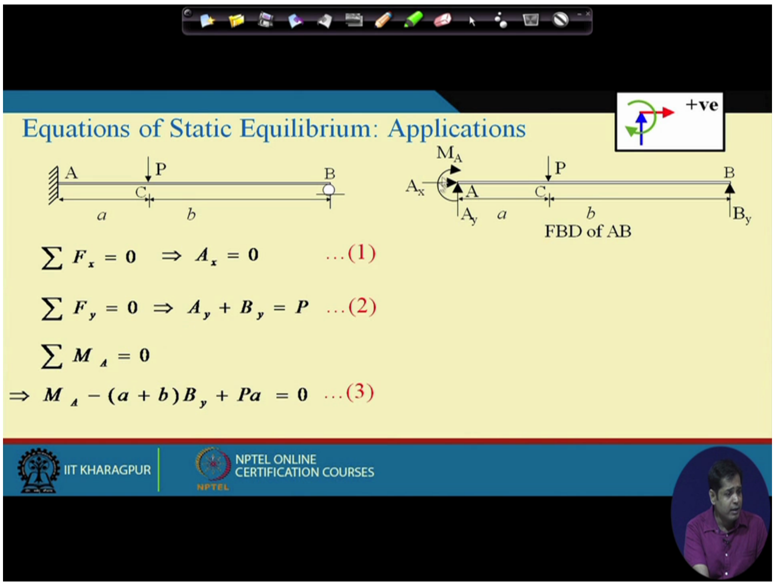

Now you see how many unknown you have here? We have, Ax is equal to zero anyway it is determined so you do not need to determine once again Ax is equal to zero. Ay is unknown, then By is unknown and then Ma is unknown. Now in order to do so we have three equations, the three unknowns, right?

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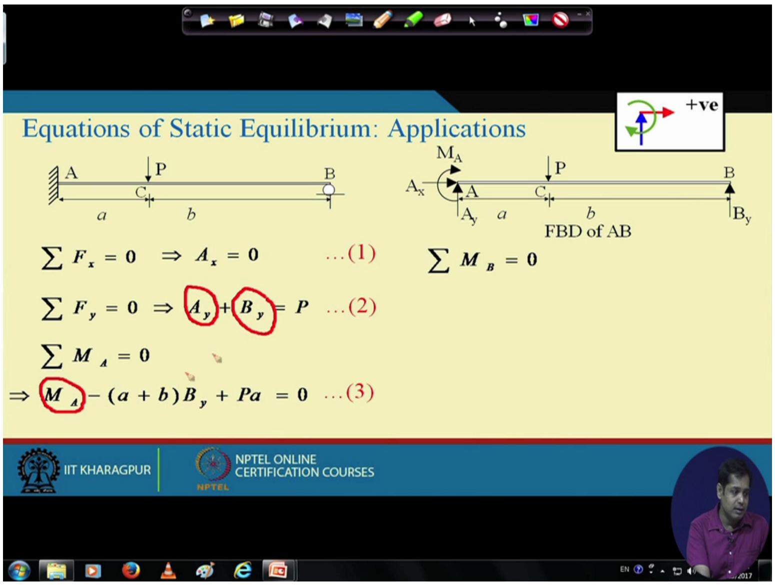

Now how many equations we have? Equation one we will not consider because equation one directly give you horizontal force is equal to zero. Now only we have equation two and equation three. Now how many unknown we have? We have (un) three unknowns. So you see the number of equation are now less as compared to the number of unknown.

Now you may say that fine then like we have taken moment at A is equal to zero, we will take moment at B is equal to zero which give you another unknown another equation and that equation can be used to solve three unknown. Let us see if I take moment at B is equal to zero that gives me an independent equation or not, okay.

Let us take moment B is equal to zero and if you take moment at B is equal to zero what are the forces we will contribute in this moment? Ma moment at A itself, then anticlockwise moment due to P and then clockwise moment due to Ay. So you have moment A which is in clockwise direction then due to force Ay it is Ay into A plus B again clockwise direction positive and then moment due to P (antoclo) anticlockwise direction P into B which is minus P is equal to zero.

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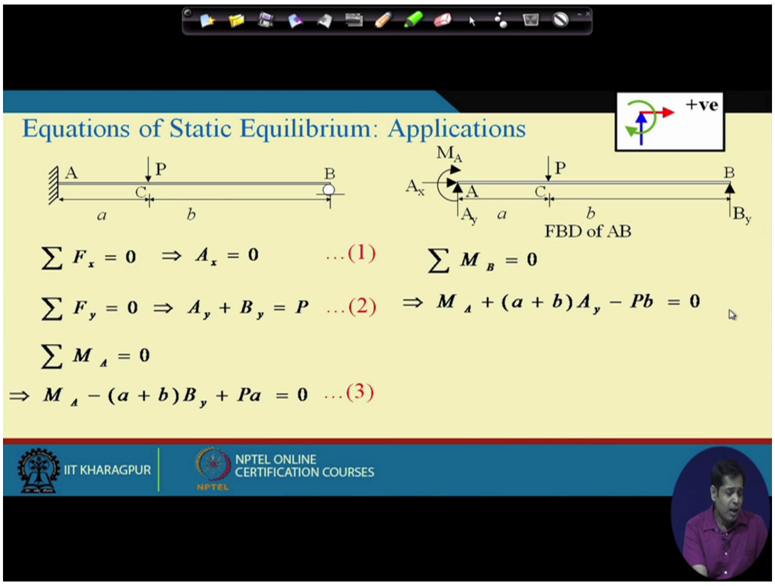

Now this is additional equation. But really this equation is independent from this equation and this equation. now you see from equation two we know Ay plus By is equal to P which gives me Ay is equal to P minus By. If I substitute Ay here P minus By what equation we get is this which is very similar to equation three.

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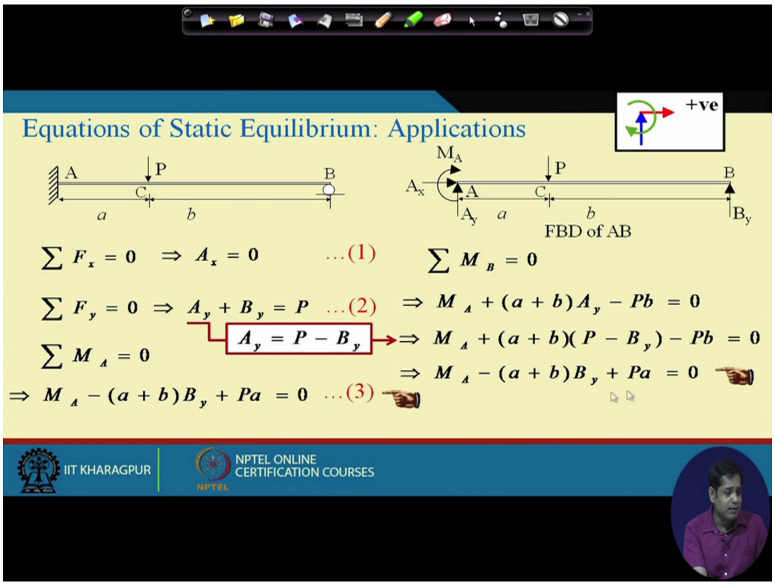

So even if you take moment at B is equal to zero that eventually will give you equation three. So essentially this situation is combination of equations two and three. So this is not an independent equation. Now therefore at least in this problem what we have seen is the number of unknown is more than the number of equations. So here the only unknown is the support reaction. as we progress we will see that there may be different unknowns in the form of member forces.

And in this case unknown is only support reaction. Now here the number of unknown support reactions are more than the number of equation. Why the number of support reactions are more? Because you see the cantilever beam itself this support itself enough to provide stability to the structure.

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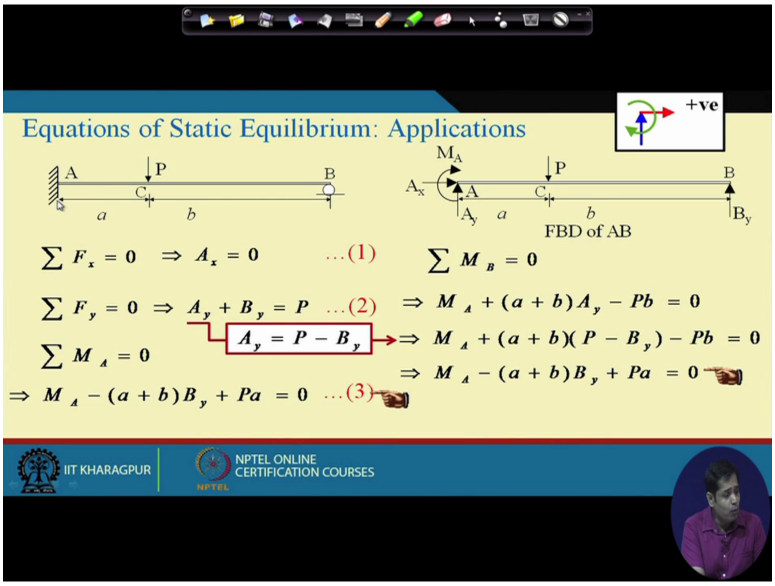

But in addition to that we provided this additional roller support which may not be required as far as the stability of the structure is concerned, right? Now what we have done here is we have provided one more constraint to the structure and therefore we do not have a sufficient equation to determine those constraint. This kind of structure is called indeterminate structure.

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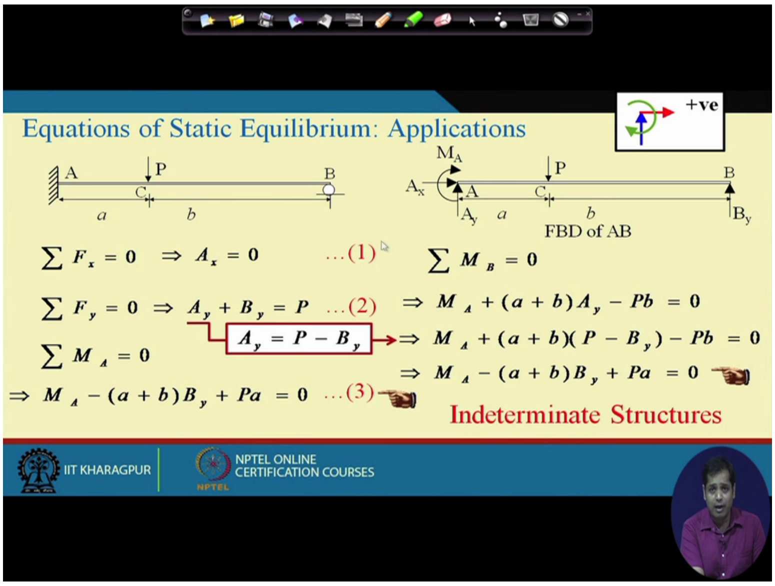

Now let us see one more example. Now before that let us see in the same structure we provide one internal hinge here, okay.

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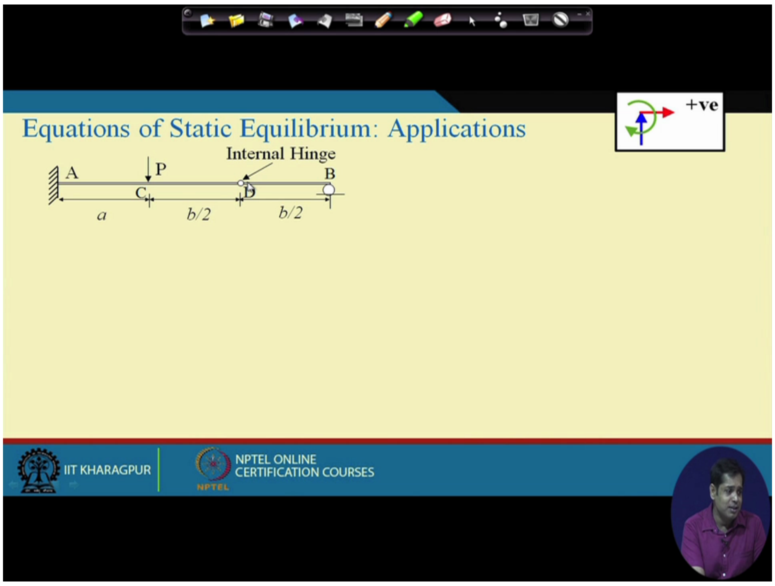

Let us see what happens if we provide internal hinge. Now again draw the free body diagram of this part AD and then free body diagram of this part BD.

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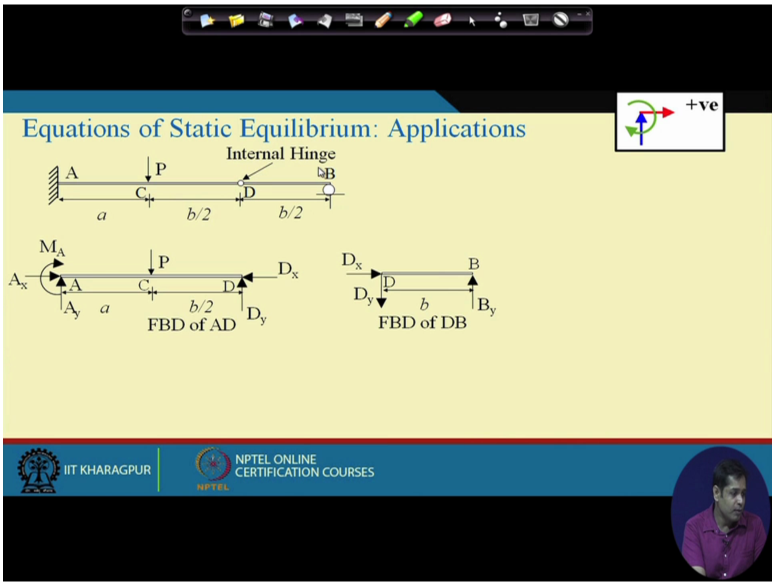

Now what will be the free body diagram of this part? This support will be represented by this three forces and then this is hinge. We know that which cannot provide any resistance to rotation. If you remember this we showed that if this is hinge then this can free to rotate, right? It is free to rotate.

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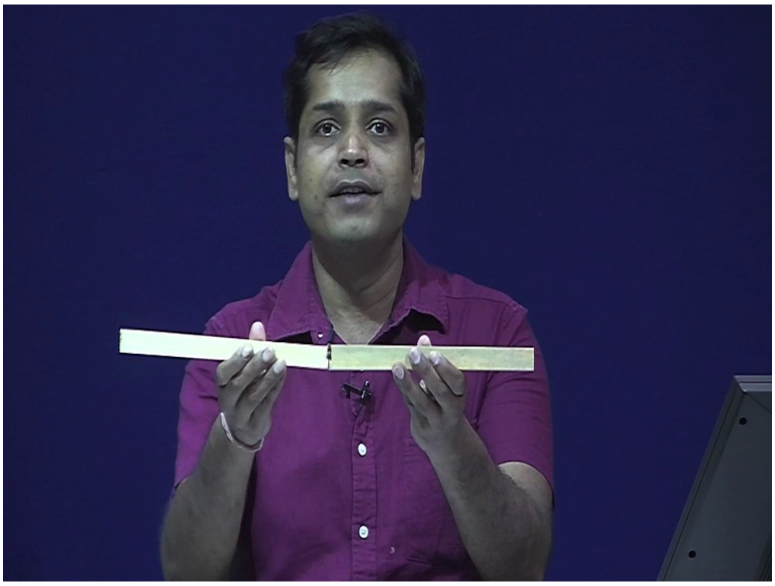

But if you take a cantilever beam like this, in this cantilever beam this is the fixed support and this is stable, right?

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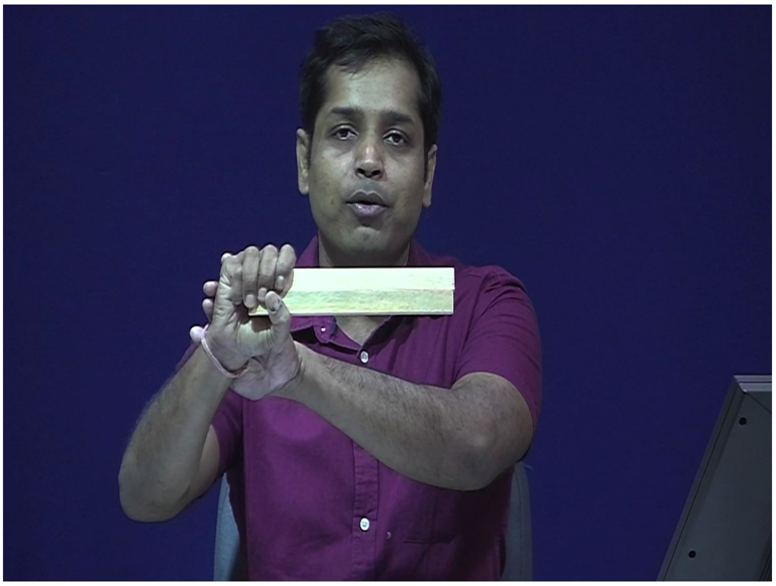

Now this was our first example. The second example was we provided one additional support here, the structure is still stable.

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But because of this additional roller support here the structure become indeterminate. Now this example is this. It is a fixed support here.

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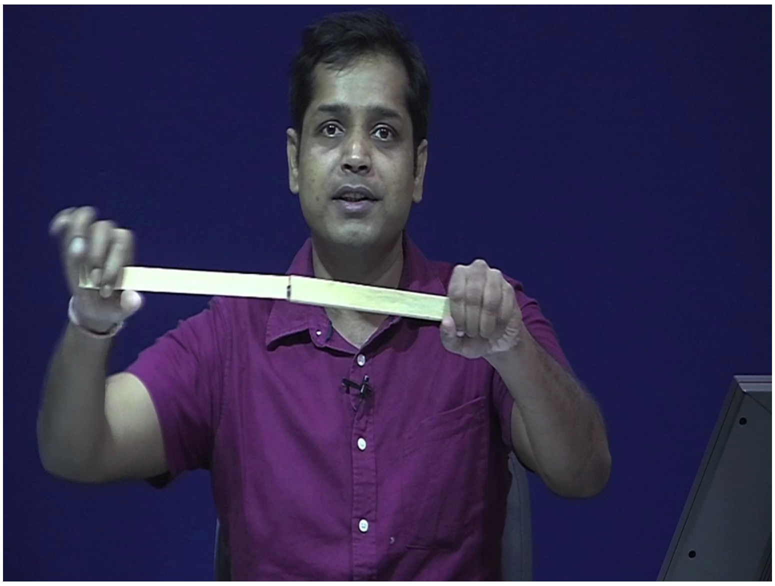

And then it provides one hinge.

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Now another roller support here, okay. So this is the structure here.

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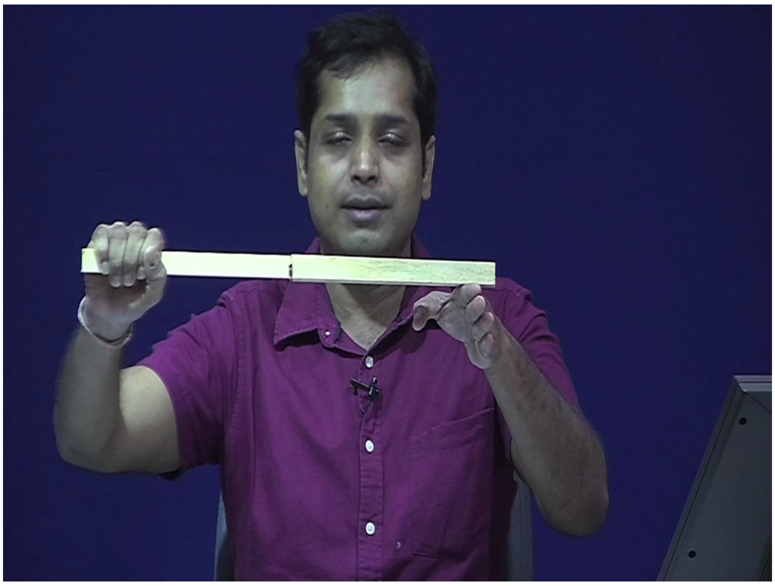

Now what we do now is we decompose the structure, (wa) we divide the structure into two part. One is this part and another one is this part and the connection between these two parts is hinge, okay.

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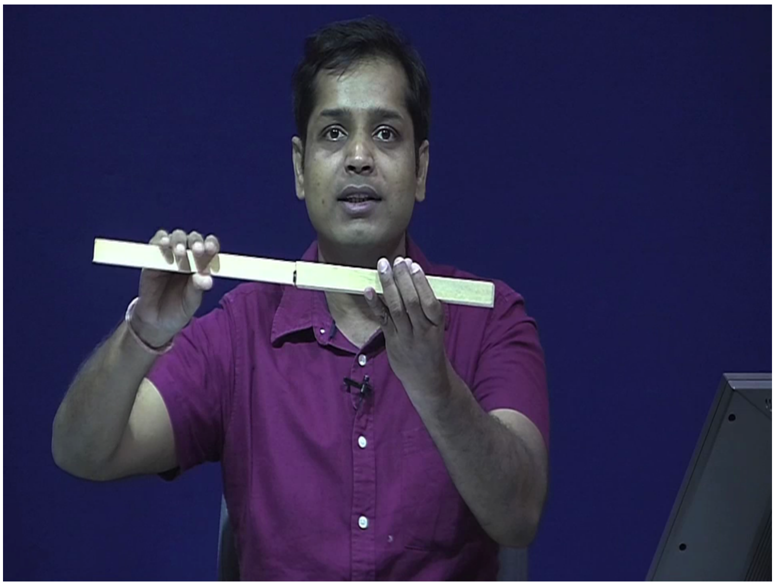

And which cannot provide any resistance against rotation.

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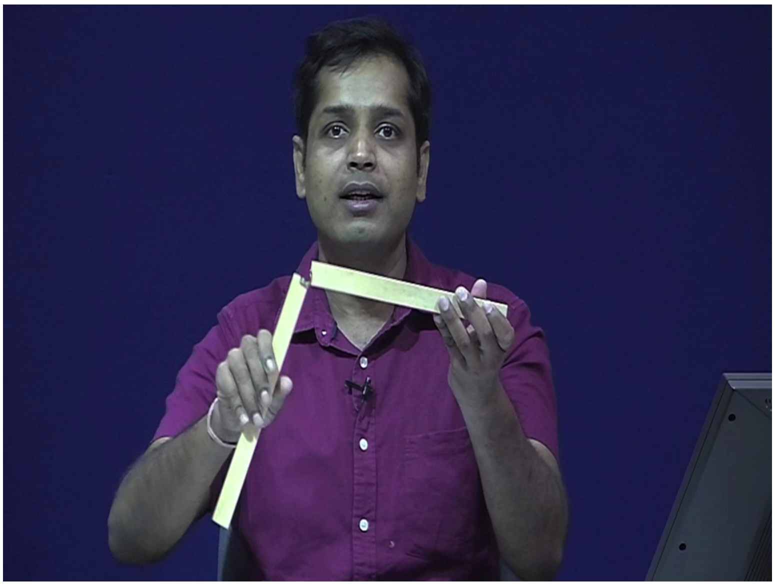

So since this cannot give any resistance to rotation there will be no moment here. That is the characteristic of hinge support or pin support. Now this is the free body diagram for this part and similarly the free body diagram of this part is this.

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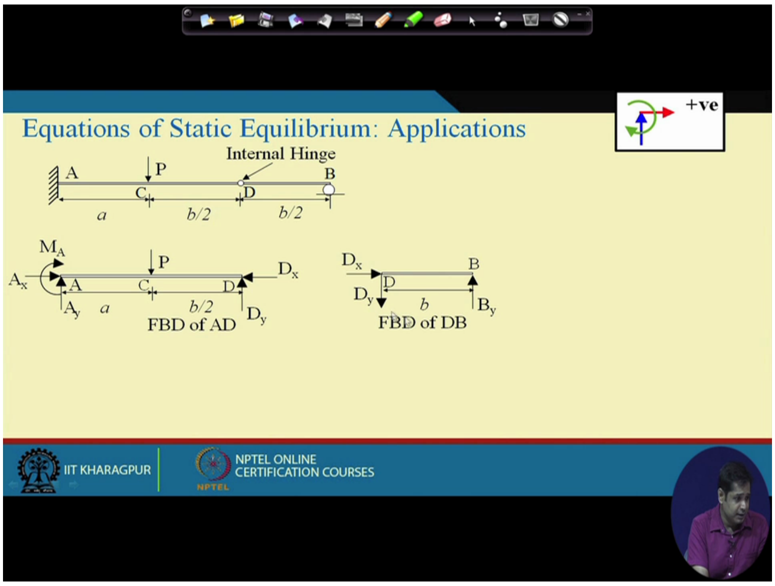

Now one thing please note that when we when the free body diagram of AD is drawn the vertical forces and the horizontal forces are shown in this direction, okay.

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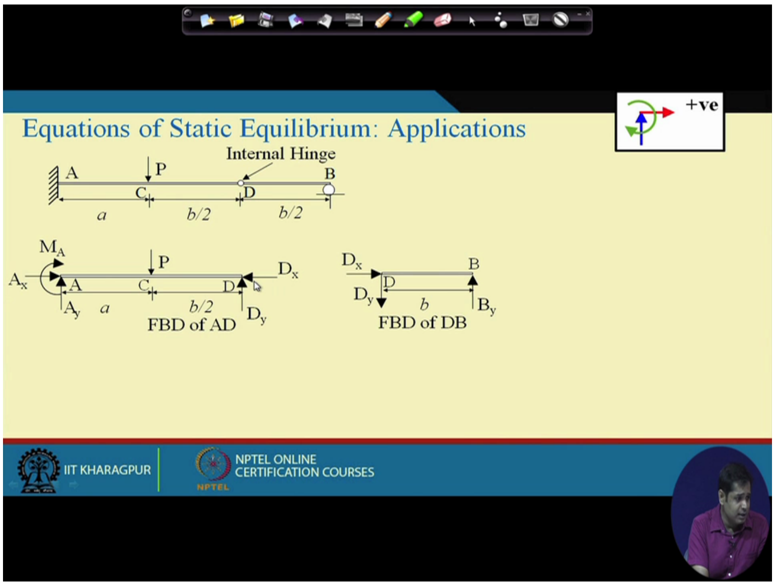

And when the same joint D when it belongs to part DB the horizontal force and vertical forces they are shown in opposite direction. The reason because when we join them at point D they should satisfy equilibrium. And in order to satisfy the equilibrium these two force they should be equal and opposite and these two forces should be equal and opposite. So that at the hinge support there will be no net force, right?

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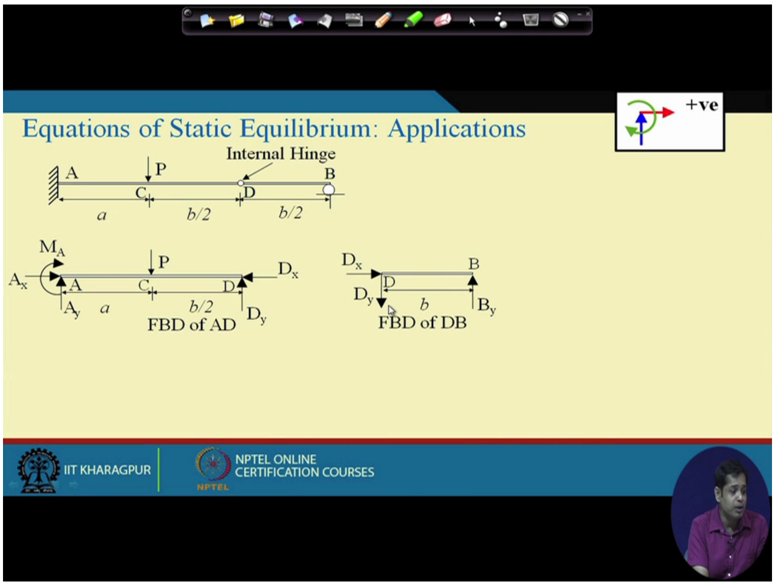

So please be careful when you draw a free body diagram specially those which contain hinges. Now next is apply the equilibrium condition. The first equilibrium you see the moment at D is equal to zero. You do not have to show them. But just for the demonstration I am showing it here. But when you do the calculation you do not have to show as it is zero in the free body diagram.

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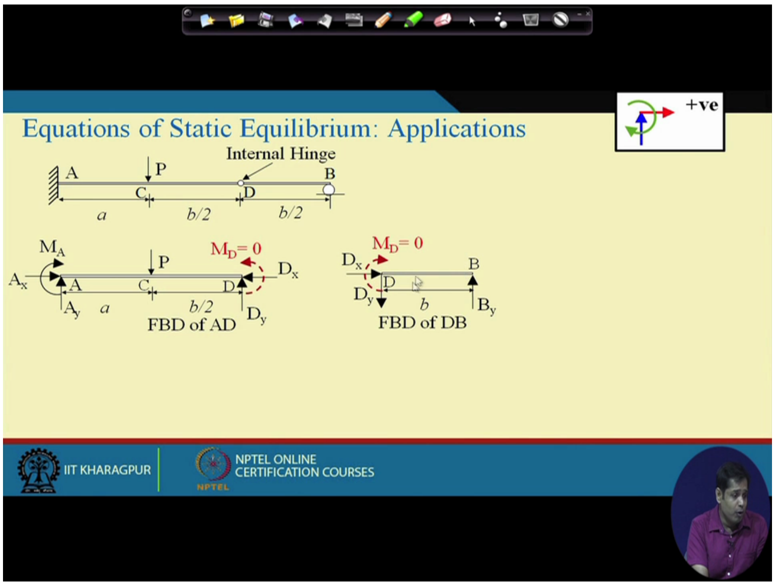

Now let us apply the equilibrium conditions. Summation of Fx is equal to zero and which gives you Ax is equal to zero as the previous example. So I am not writing it explicitly here. So summation of Fy is equal to zero. Summation of Fy is equal to zero means what it gives that Ay plus By that should be equal to the applied load P. It is the same (exam) equation as the previous example, okay.

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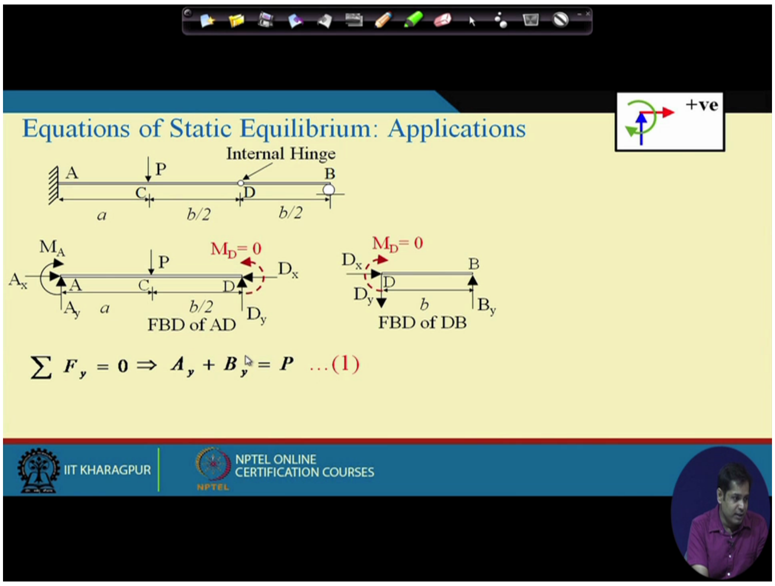

And you take summation of Ma is equal to zero and this equation is again similar. If you take moment about A, what are the forces which contribute to the moment? The force P into A which is clockwise direction and the moment due to B which is in anticlockwise direction .

Now but we have seen in the previous example that these two equations cannot give you sufficient information so that all these unknowns Ay, By and Ma can be determined, right? instead of B by 2 it should be B here, okay.

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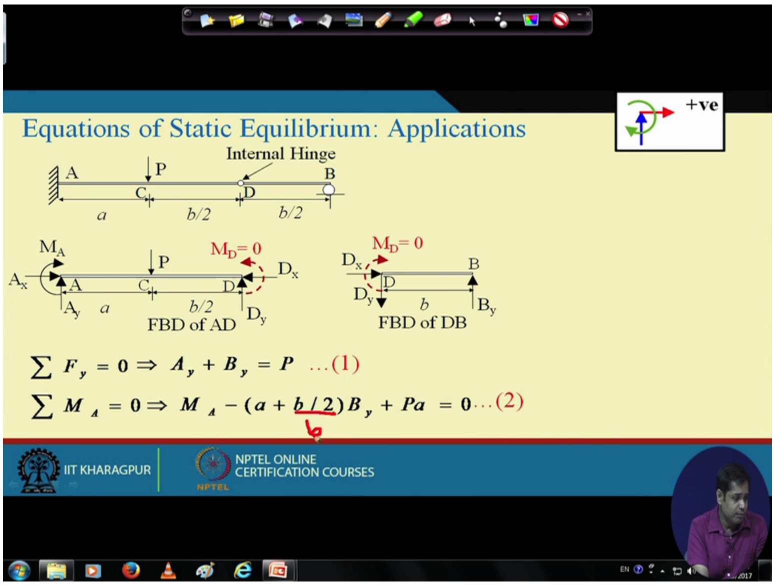

Now next we need additional equation because we have three unknown and two equations. Now you see where we get the additional equation? Additional equation can be obtained by this hinges. What condition we know at this hinges that at this point MD is equal to zero, right? Because that is the characteristic of hinge support.

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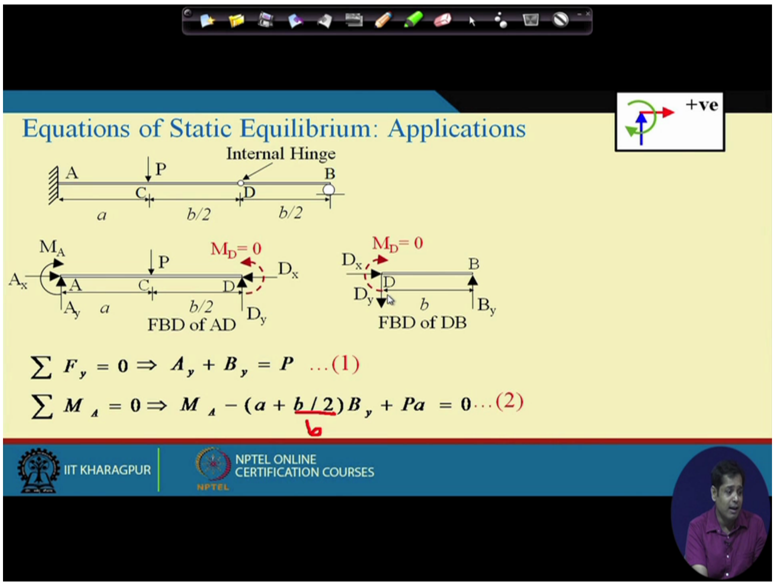

Now from the free body diagram of this if you take moment about D is equal to zero this give you B into By is equal to zero, By is equal to zero.

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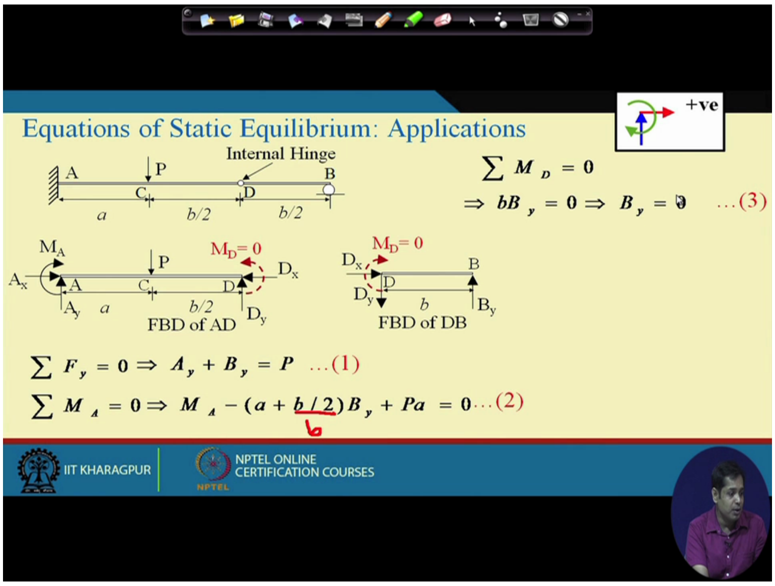

Now substitute this By in this equation you get Ay is equal to P. Again substitute By is equal to zero in this equation you will get Ma is equal to minus Pa. So this is the solution of this equation.

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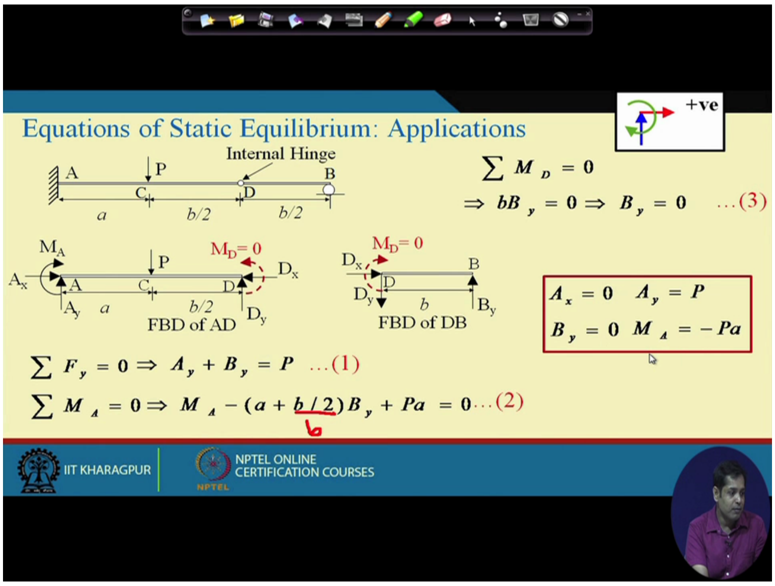

Now what happens when we provide hinge? When a hinge is provided at the same propped cantilever beam essentially then we have one more condition and the one more condition is, at that hinge your moment is equal to zero and this condition will give you an additional equation which will help you to solve the unknown. Now your equation will be two equations that we obtained (previ) in the previous example plus one more equation obtained by enforcing the condition that moment at D is equal to zero. Now by providing hinge, the structure becomes statically determinate structure, right?

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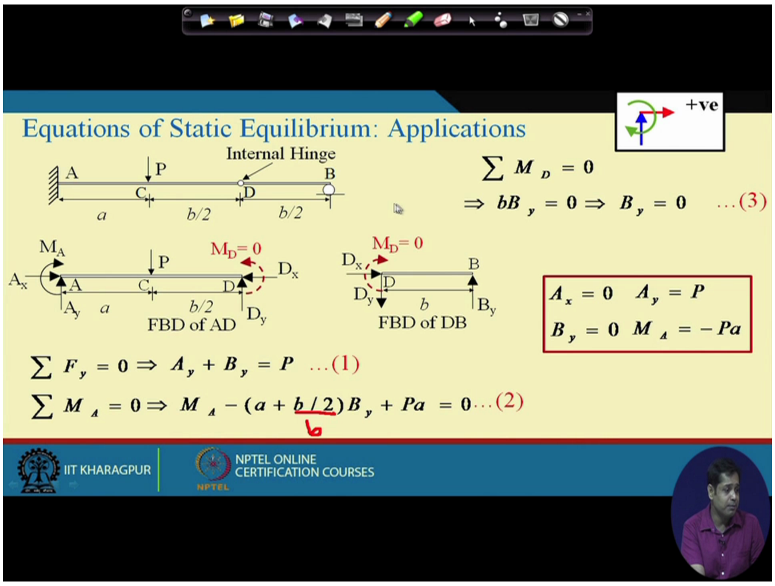

Means here all the unknown we have in this equation, those unknown can be determined from the equilibrium equations, okay. Let us take one more example. Now this is an arc and this is hinge support and this is also hinge support here.

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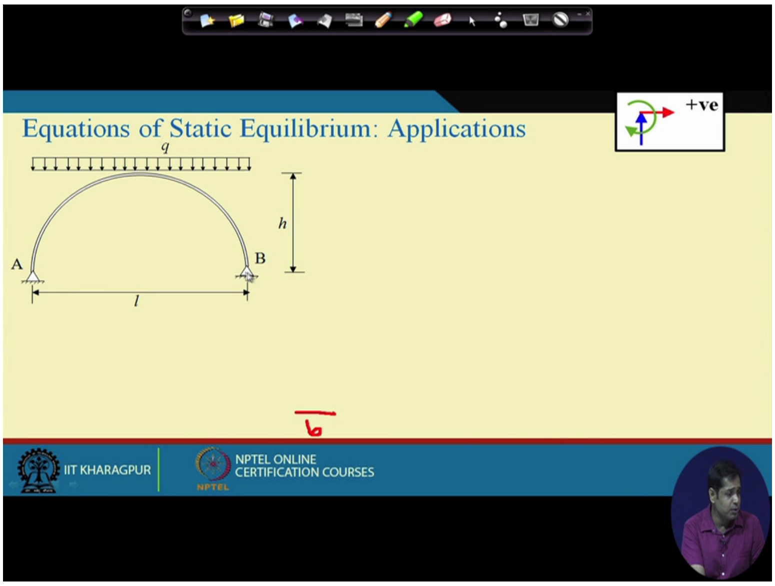

Let us find out the support reactions at A and B. Similarly first draw the free body diagram. Now at point A since it is hinge support there will be no moment here, only at B horizontal and vertical reaction. And this is uniformly distributed load, q is the intensity of the load. Intensity means unit of q is your force per unit length, okay.

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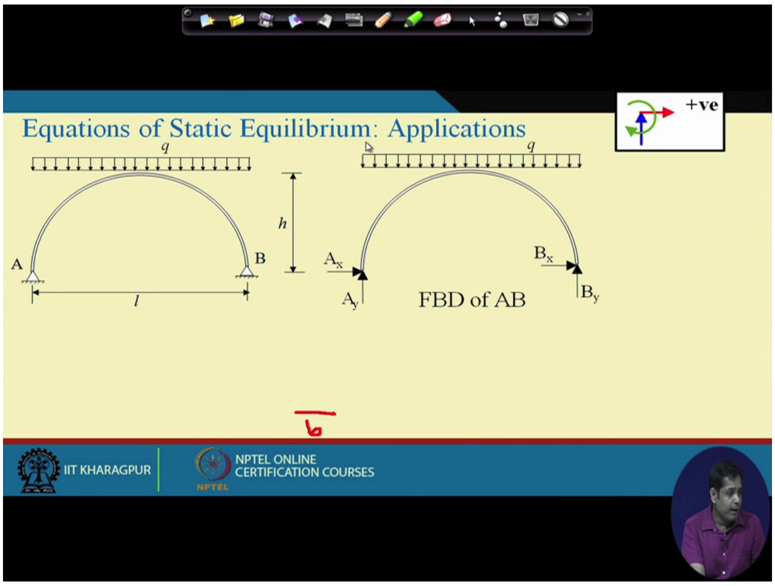

Now let us write the equilibrium equation. First equilibrium equation is summation of forces in x direction is equal to zero and the forces we have in x direction is Ax is equal to Ax and Bx and it gives me Ax plus Bx is equal to zero. This is first equation .

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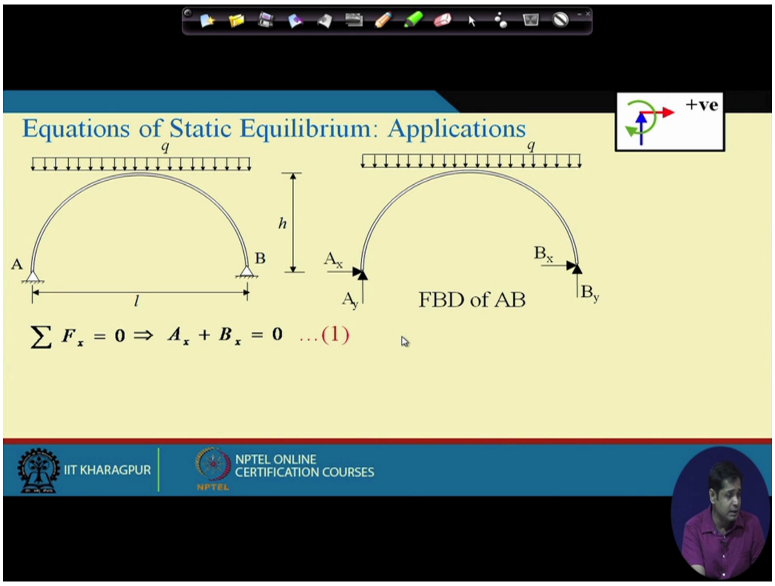

Now the next equation is again summation of Fy is equal to zero. So Ay and then By is equal to zero. Ay plus By is equal to qL. L is the total length of the length of the arc and intensity is q. This is acting downwards so negative and this is acting upwards so positive. So this is another equation, right?

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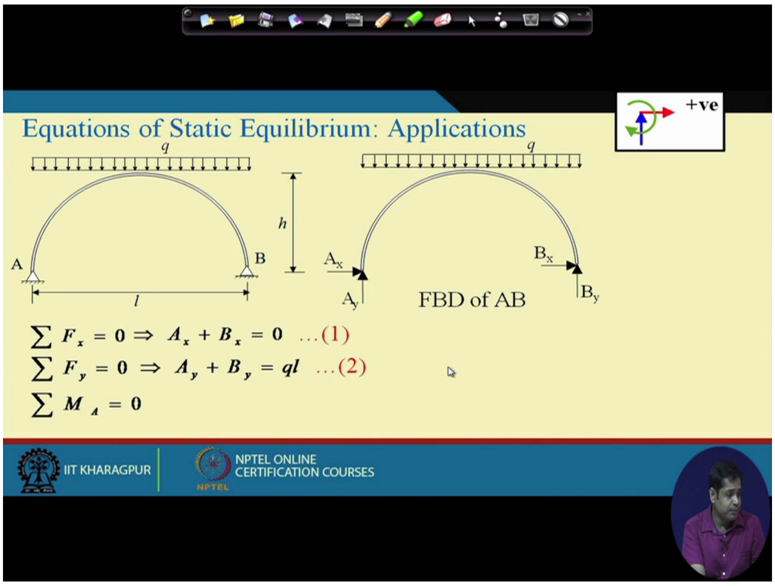

Take summation of Ma is equal to zero. And if you take summation of Ma is equal to zero then what are the forces we will contribute? Ax and Ay will not contribute, Bx will not contribute because Bx also passes through this point. The line of action of B passes through this point.

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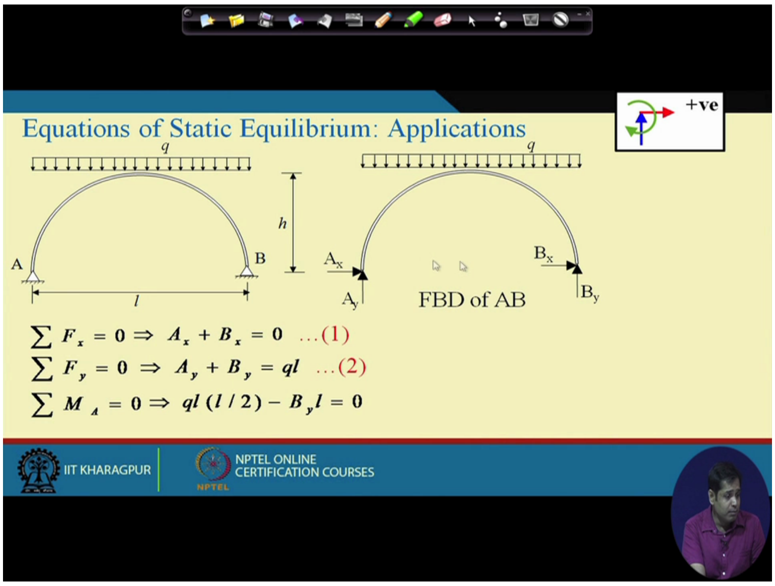

Only reaction component that will contribute to this moment is By and then external uniformly distributed load q. Now moment due to B will be anticlockwise. By into L anticlockwise negative and then moment due to q will be clockwise and what would be the moment? The total force will be q into L and then since it is uniformly distributed load their resultant will be at the middle of this entire length.

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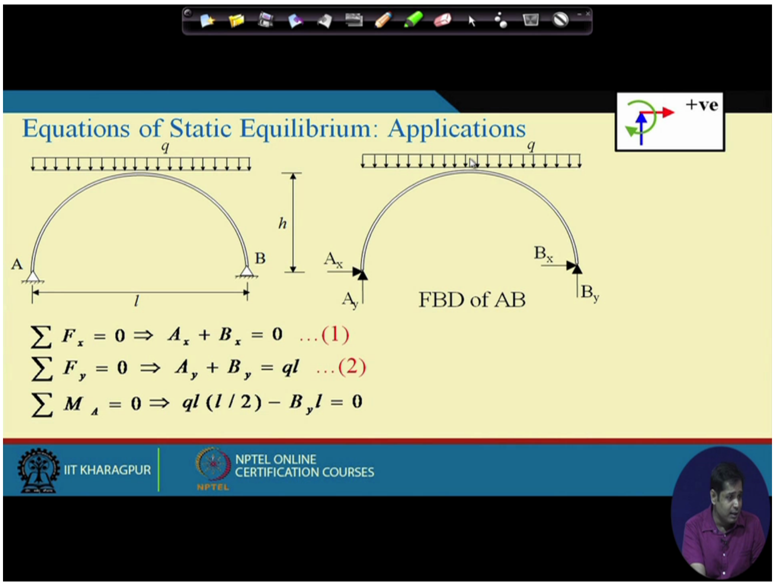

So total force will be qL and their resultant will be at the distance L by 2 from support A. So this is the equation we get from Ma is equal to zero. This gives you By is equal to qL by 2.

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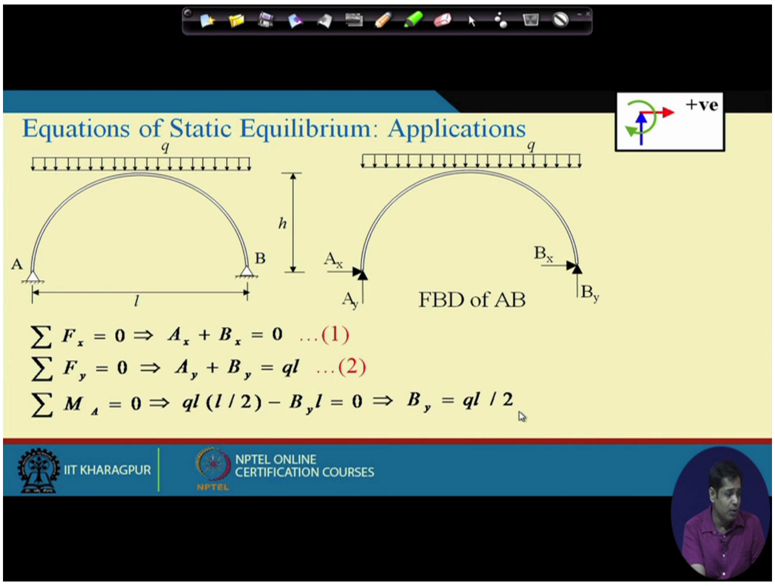

If you substitute By here you get Ay is equal to qL, okay. Now you could obtain reaction Ay and reaction By, right? But with this we do not have any information about Ax because if you see in this example only equation we have is Ax plus Bx is equal to zero. Even if you take moment about B you will not get any additional equation which can tell you or which can give you some information about Ax and Bx.

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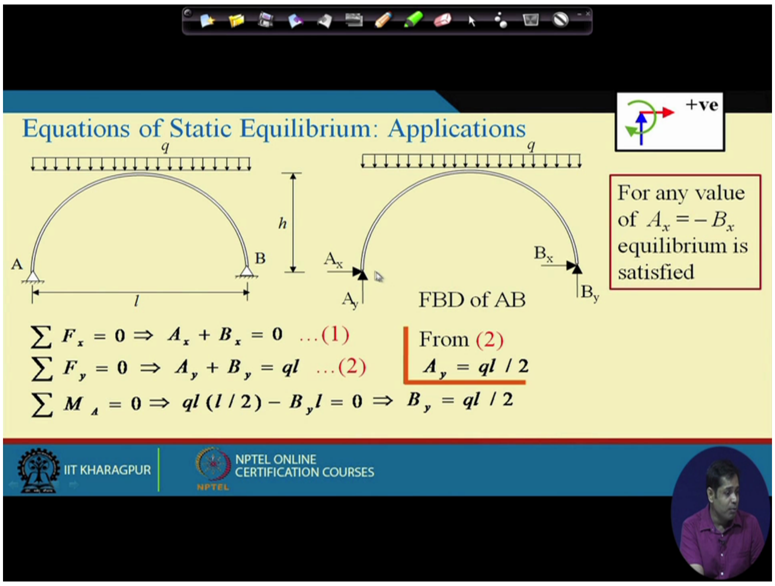

Now this equation is satisfied for any value of Ax and Bx. So if you take Ay and By is equal to qL by 2 and any value of Ax and Bx we satisfy Ax is equal to minus Bx, your equilibrium is satisfied, okay.

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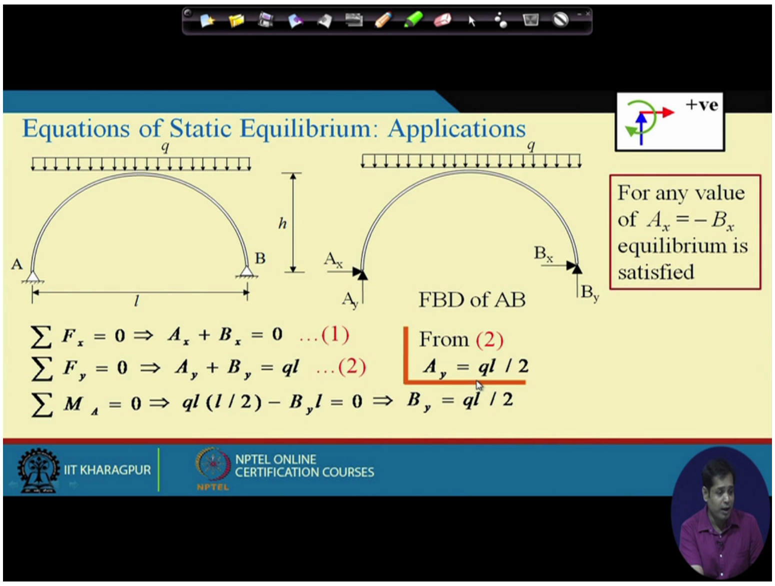

Now but does that mean that Ax and Bx can have any value? Actually not because in addition to the static equilibrium or equally static equilibrium equation there is another condition that need to be satisfied is kinematic admissibility. Now you put a star here.

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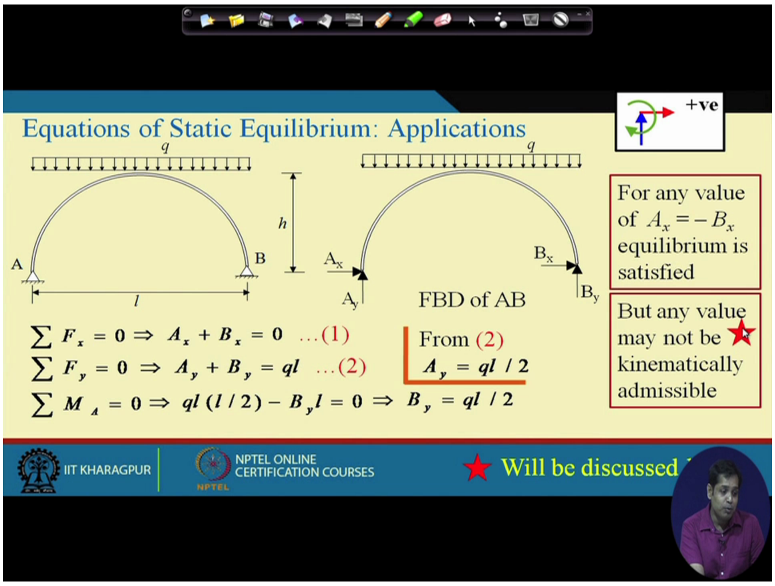

Kinematic admissibility we will discuss when we will talk about indeterminate structure in a more detailed way. Now so what this example tells you? Example tells you that yes we could solve the problem partially, we could determine some of the support reaction but in order to get all the information about the support reaction, the equilibrium equations are not sufficient. Now like the previous example if we put one hinge here, okay.

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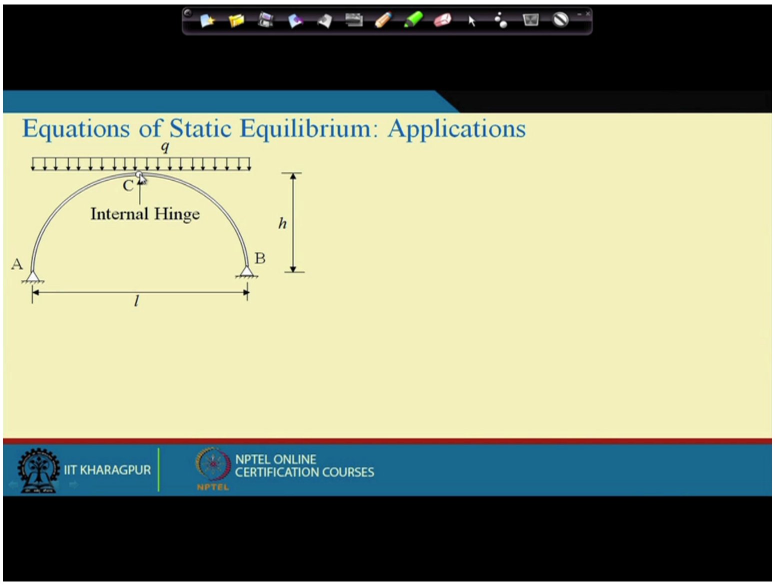

One hinge here means then we are providing one condition. What would be the condition? The moment at C will be equal to zero. Now this is the free body diagram.

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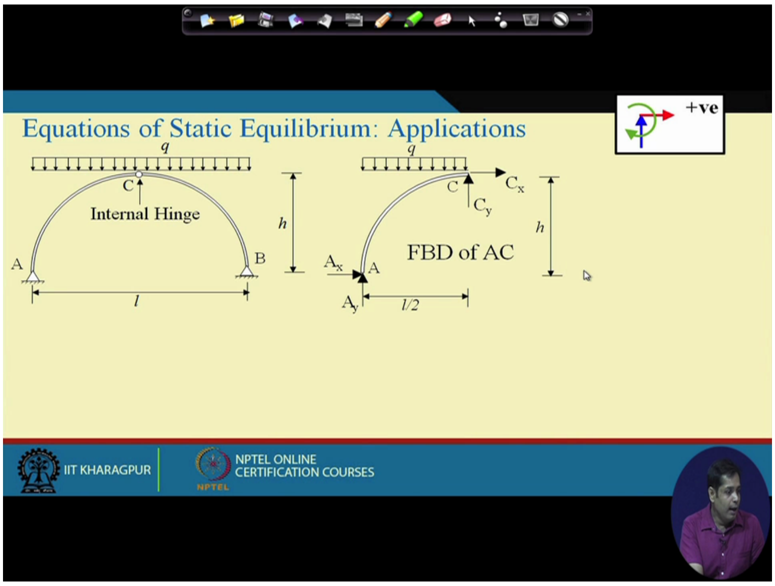

Now moment at C will be zero. That is why it is not explicitly shown here. Ay is equal to By is equal to qL by 2 that we have already obtained and now take Mc, moment at C is equal to zero. And this gives us an additional equation and from this additional equation you can get Ax is equal to this.

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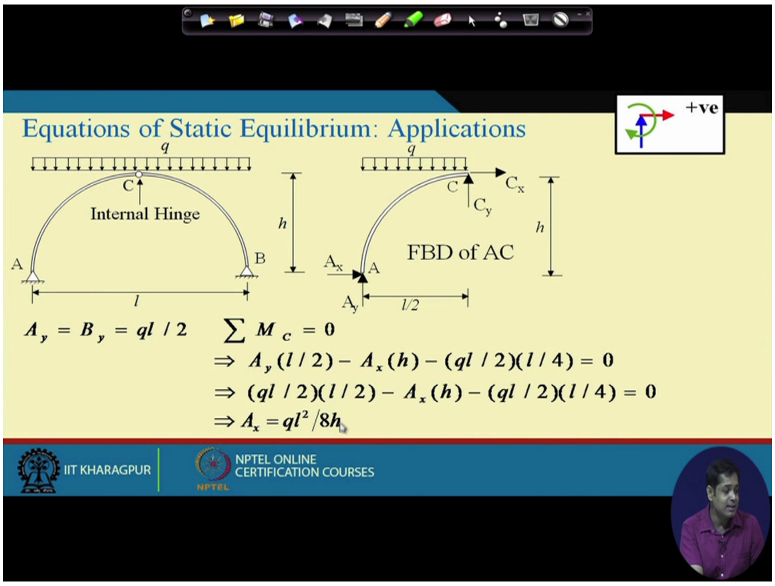

And similarly since Ax is equal to minus Bx and so Bx is equal to this. And again this will be Ay and this will be By, okay.

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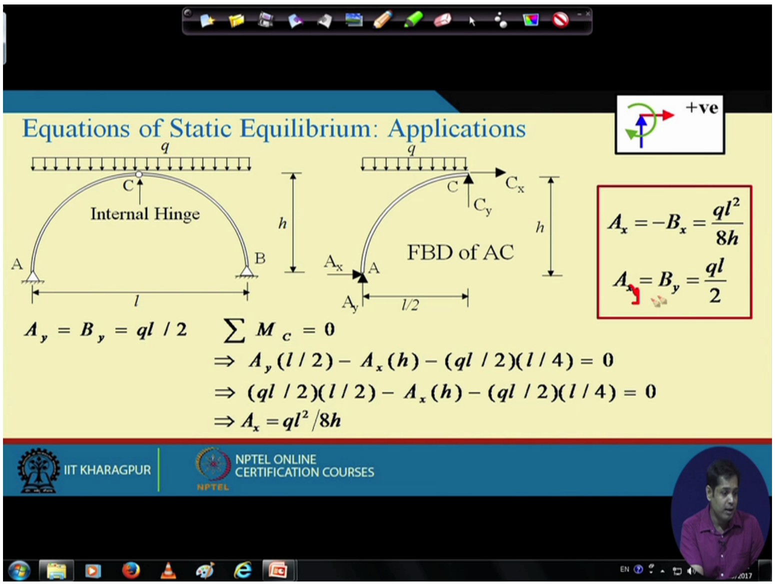

Now again what we have done? When we provide a hinge, means essentially we are giving additional information. What is that additional information? That moment at that particular hinge is equal to zero. And that additional condition will give you one more equation which can be used along with other equilibrium equation to determine the support reactions, okay.

Now if I have to summarise the entire thing. The structure where static equilibrium conditions alone are insufficient for determining support reactions on internal forces, right?

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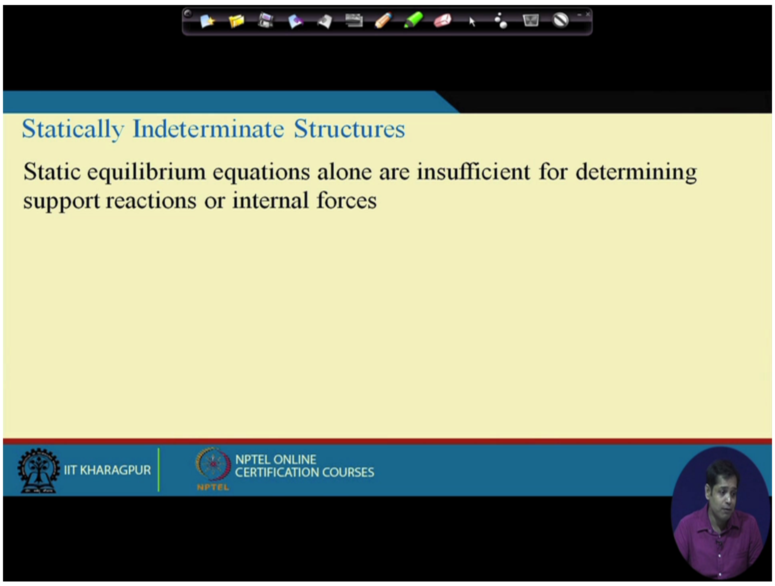

Now you have two parameters here. One is the number of constraint or number of unknown. In this case the unknowns are reactions. But as I said number of unknown could be internal forces as well. We will see later. So you have number of unknown one parameter and then you have another parameter, the number of equations, okay.

Now there are three cases possible where, when the number of unknown is equal to number of equation and then another case is number of unknown is more than the number of equation and another case theoretically number of unknown is less than the number of equation. Let us see in all three cases what happens. At the first case is number of unknown is equal to number of equilibrium equation. For instance this case, this is a simply supported beam.

One in hinge and another one is a roller here. Two reactions here and one reaction here, total three reactions and then number of equations we can have is three. So this structure is stable structure, okay.

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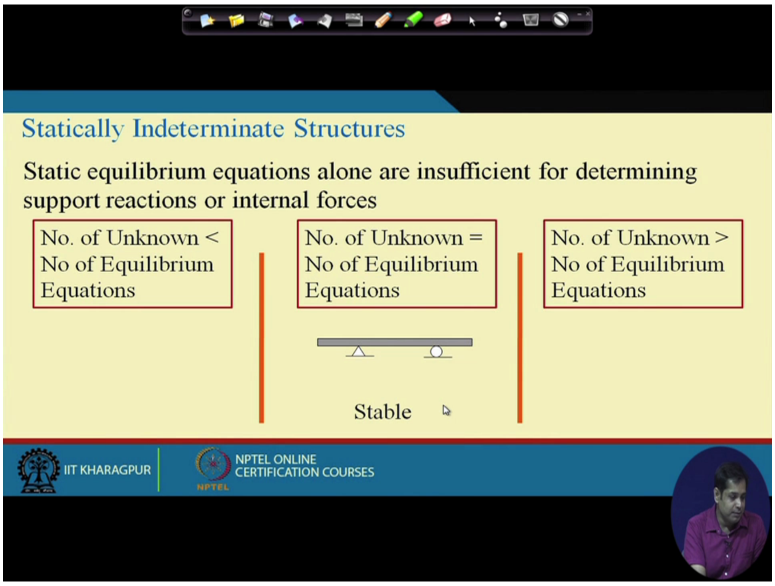

Now where it is stable structure what happens when a structure is unstable? We will just show you shortly. And the next case is when the number of unknowns are more than number of equations. For instance there are two unknown here and two unknown here, two reaction horizontal vertical, two reaction horizontal and vertical, total four reactions. Number of equations available is three. Therefore this structure is stable but the structure is indeterminate structure.

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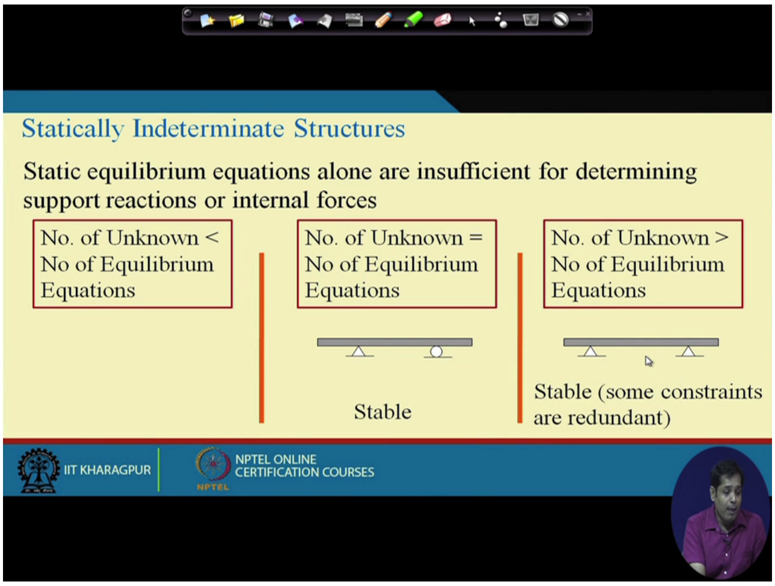

Here we will see some of the constraints are redundant. By redundant means for instance difference between this structure and this structure is, the roller support is now replaced by hinge support which (pro) was not required if we looked at from the stability point of view. Because when we replace roller support by hinge support then we are giving one more constraint. What is that constraint? One more constraint in horizontal direction which is redundant here which was not necessary from the stability point of view of the structure.

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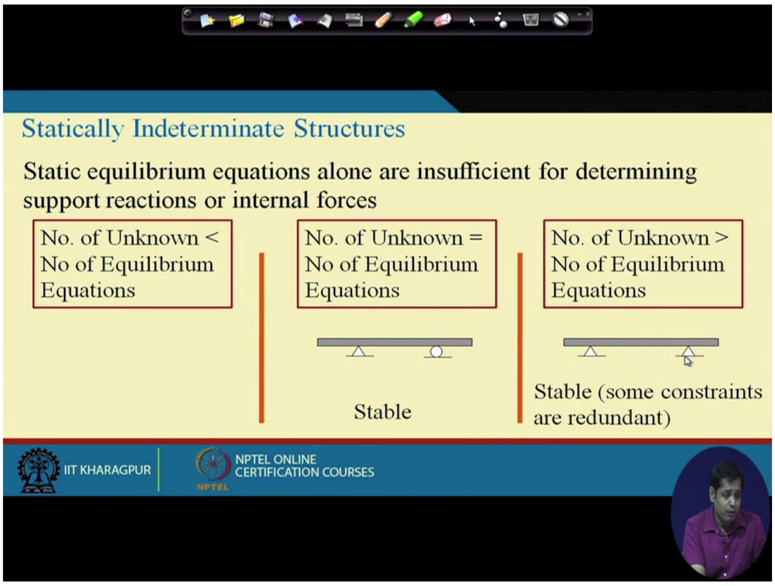

Now let us see the third case. Third case is when the hinge support is replaced by roller support.

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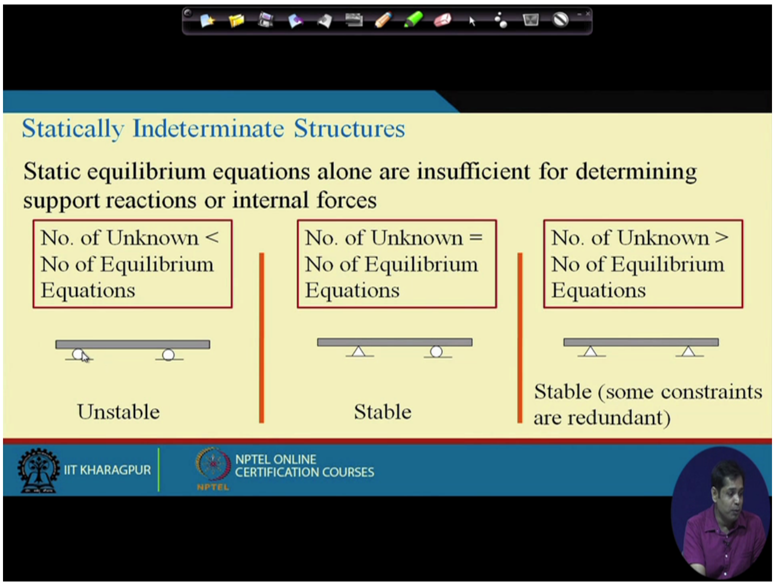

Now what happens in this case? This structure is not stable when it is subjected to horizontal load. Then it may move like this because roller support cannot provide you horizontal constraint, right? So this structure becomes unstable.

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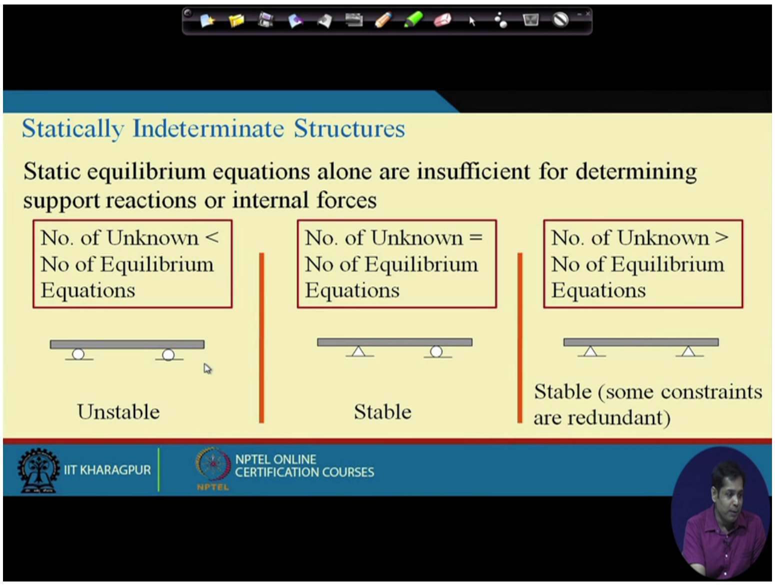

These are the structures which are determinate and then from seventh to eleventh week we will see different methods to analyse indeterminate structure.

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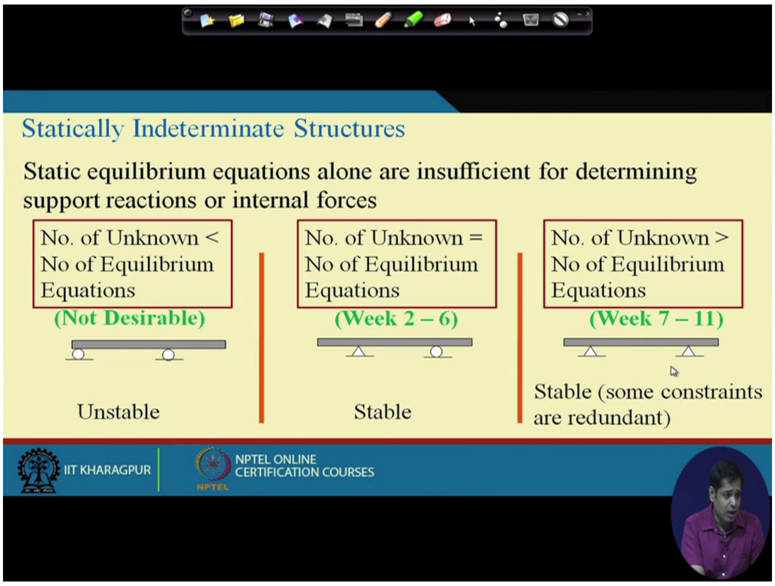

We will discuss the different kinds of indeterminacy, different sources of indeterminacy in a detailed way as you proceed in our journey. But for the time being at least for this week indeterminacy means or indeterminate structure is, the number of reaction are more than the number of available equilibrium equations, okay. And we will stop here. Next class what we will do is we will briefly review this concept of shear force and bending moment diagram of beams, okay. Thank you.