Structural Analysis 1
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Lecture 3
Static Equilibrium

Hello, welcome to lecture 3 of module 1. Todays topic is equilibrium. You see before we start todays topic let us quickly review what we have done so far.

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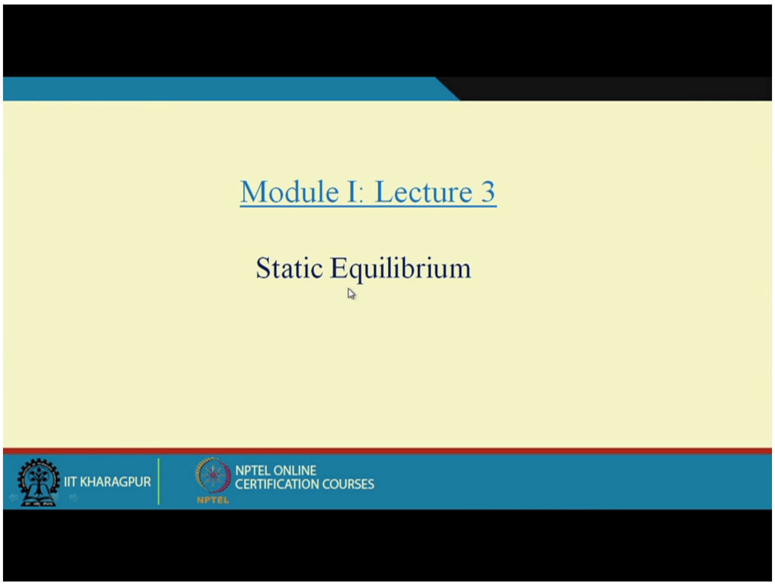

You see in the last class we stopped here and this is the list of our first two classes. What we have learnt is that analysis essentially is a process where we determine response of a structure subjected to certain threat. We also learnt that when we do structural analysis then we need to idealize the system, idealization of structure and as well as idealization of threat. Now we will be discussing in this course two different idealizations. One is plane truss and plane frame.

A beam is also, concept of beam is important. So we will also review the concept of beam. And as far as idealization of threat is concerned, we discussed that any threat can be translated into concentrated and distributed force or concentrated and distributed moments. So structure plane truss and plane frame subjected to this and then the response will be looking for is internal force and deflection. Internal force from safety point of view and deflection from serviceability point of view.

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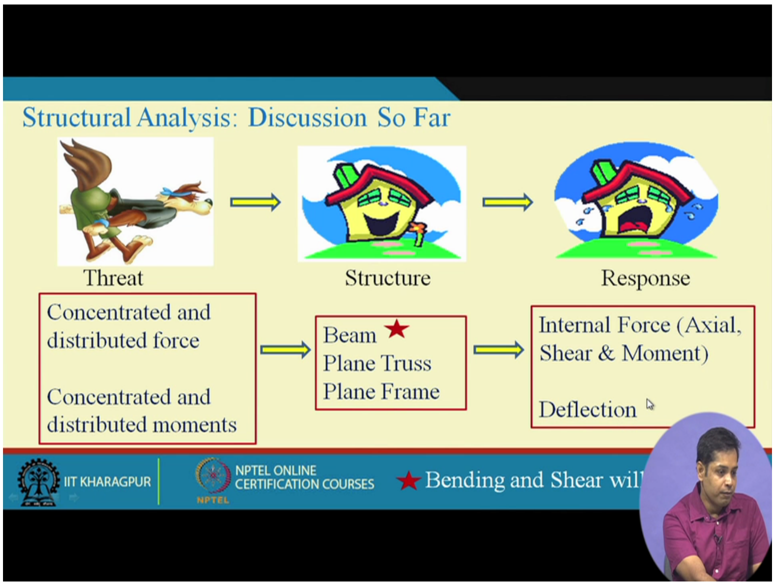

So what we will do today? Today we will take the first step towards that, okay. Specifically in this lecture today we will discuss degrees of freedom, supports and support reactions, static equilibrium equations, determinate and indeterminate structures and determination of support reaction using static equilibrium equation, okay.

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Degrees of freedom. What is degrees of freedom? Before I give you formal definition of degrees of freedom let us try to understand. You see as name suggests it is a freedom of any object to move in three dimensional space. For instance if I take an object like this.

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Now what are the possible ways that this object can move? First is this object can move in this direction, then this object can move in this direction and then this object can move in this direction. So this object can translate in x direction, y direction and then again z direction. Now this object has three translation degrees of freedom. Now apart from this translation degree of freedom this object can rotate also. For instance this object can rotate like this, then this object can rotate like this and then this object can rotate like this.

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So this object has three rotational degrees of freedom. Now therefore any object in three dimensional space has six degrees of freedom, three translations and three rotations. Now let us see for two dimensional problem any object, how many degrees of freedom it has. In two dimensional problem this object can move in this direction, this object can move in this direction and then this object can rotate like this.

So this object can have three degrees of freedom, two translations and one rotation. Probably I am going to show you some animation for that and things will be cleared after seeing those animations. Now if I have to formally define what is degree of freedom then it says that degrees of freedom of a mechanical system is the number of independent coordinates required to completely specify the configuration of the system.

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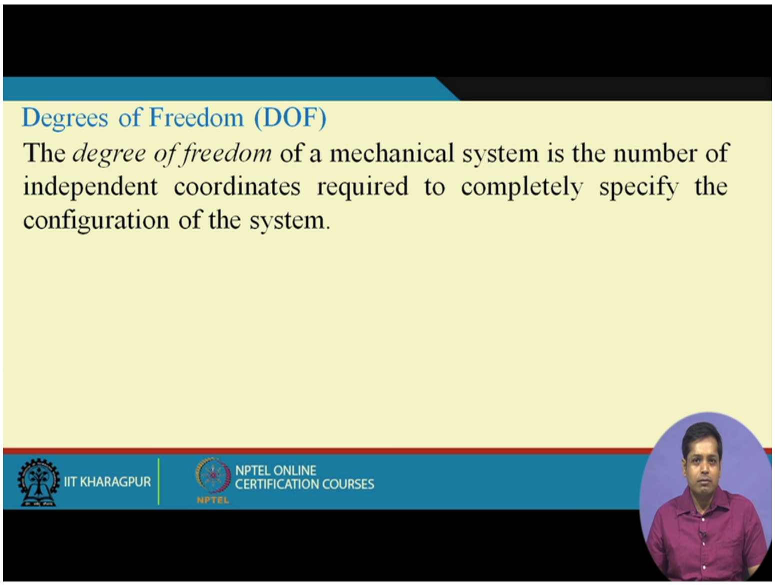

Now just now the way we explained degrees of freedom and the formal definition you will see that both definitions are consistent. Now to appreciate in a better way let me show you some animation. You see this is the degrees of freedom in three dimension.

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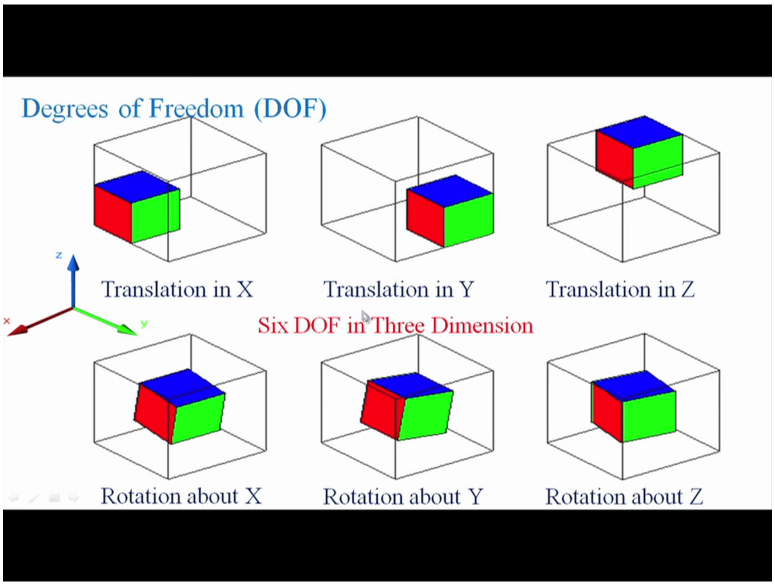

You consider any object. This cube is an object, okay. Now this is the coordinate system x, y, z. This object can translate in x direction, this object can translate in y direction and this object can translate in z direction. So these are the three translation degrees of freedom. Similarly this object can rotate about x axis, this is object can rotate about y axis and the object can rotate about z axis. So these are three (trans) rotational degrees of freedom. So total this object has six degrees of freedom.

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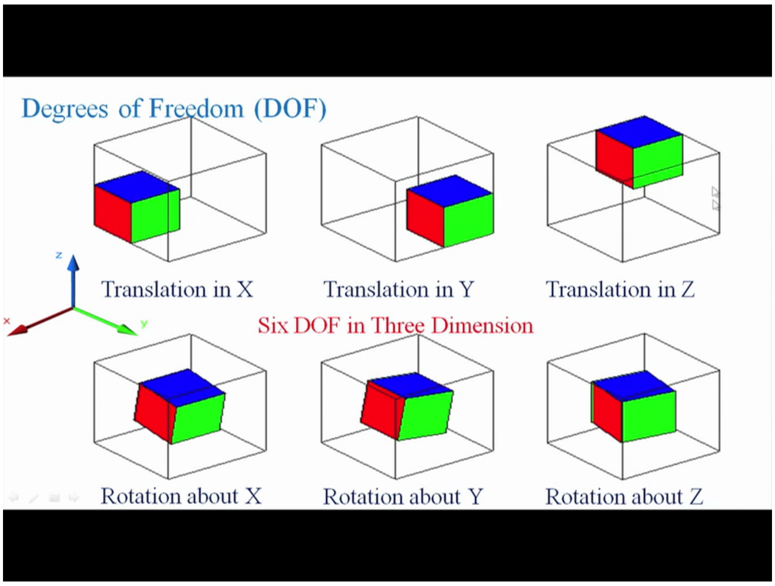

Now we will see that if we want to define the motion of an object in three dimensional space then these six degrees of freedom is enough to describe its motion. Now let us see in two dimension. This is x coordinate, this is y coordinate, now z coordinate is suppressed. So this object can move in x direction, this object can move in y direction and this object can rotate in z direction, right?

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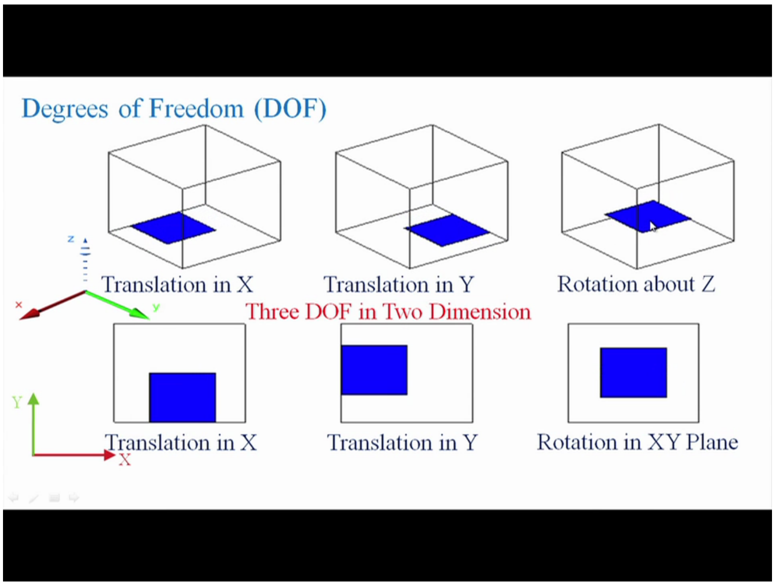

So this view is the panel view of three dimensional view. So this is x direction, this is y direction and this rotation is about y axis. So any object in two dimensional space has three degrees of freedom. Now you see what is external stability of a structure? You see if we consider any structure, for instance if we consider this structure, okay.

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If you remember we talked about idealization of a chimney where the idealization is the chimney itself is model idealized as one dimensional element and then support. Now what we have here if you see there are three components of the structure. One is this is the member and then this is the support and then another thing we can have component which is the joint.

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Now for instance you take another structure like this. Now it has two members and these two members are connected here.

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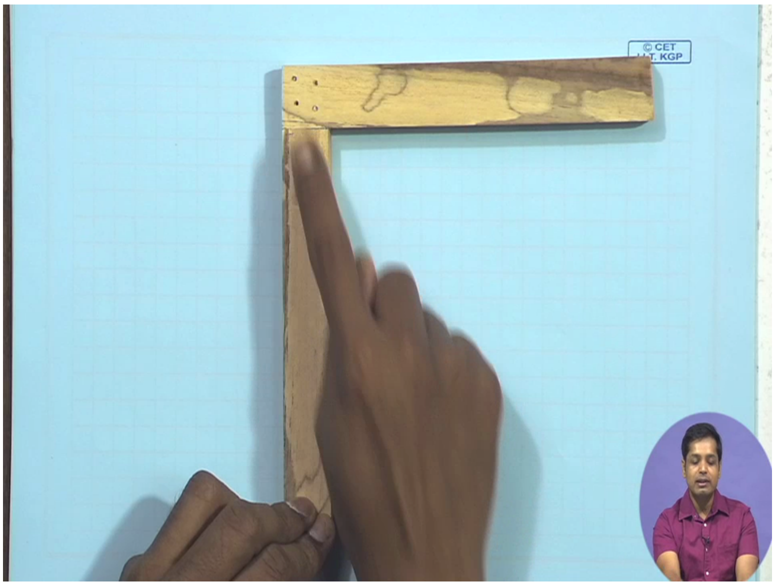

And then in addition to that we have another support here. This structure is supported at somewhere. So if we see this components of structure there are majorly three components. One is the member and the joints. Joints means the connection of those members and then the support. Support means the structure is either fixed at ground or fixed at some other structure so that it restricts the motion of the structure.

Now today we will be talking about importance of support. Now you see what happens if any structure is not supported then what happens? The structure becomes unstable. For instance you take any object which is just kept on the ground. There is no connection, there is no support between the object and the ground.

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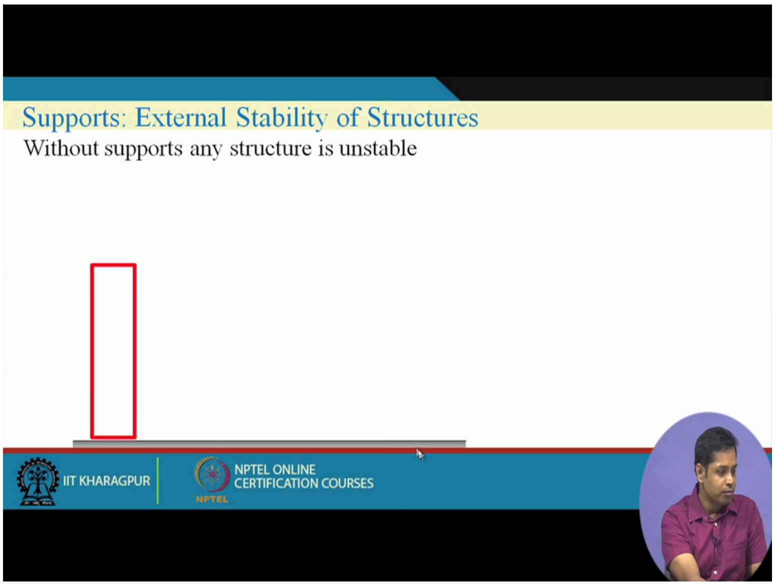

Then if the structure is subjected to some load, horizontal load for instance, then what happen, this structure becomes unstable. You see, so because there is nothing to hold the structure to the ground. Now therefore without support any structure becomes unstable.

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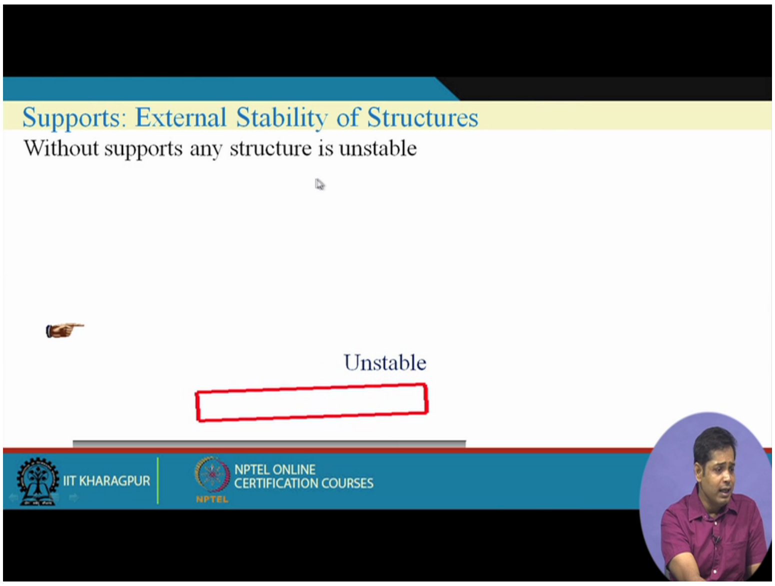

Now what the support does? Support provides constraint and reduces number of degree of freedom. You see in this case it is a two dimensional problem. Now how many degrees of freedom this object has? This object has three degrees of freedom. One is translation in this direction, translation in this direction and rotation about z axis, rotation in this plane.

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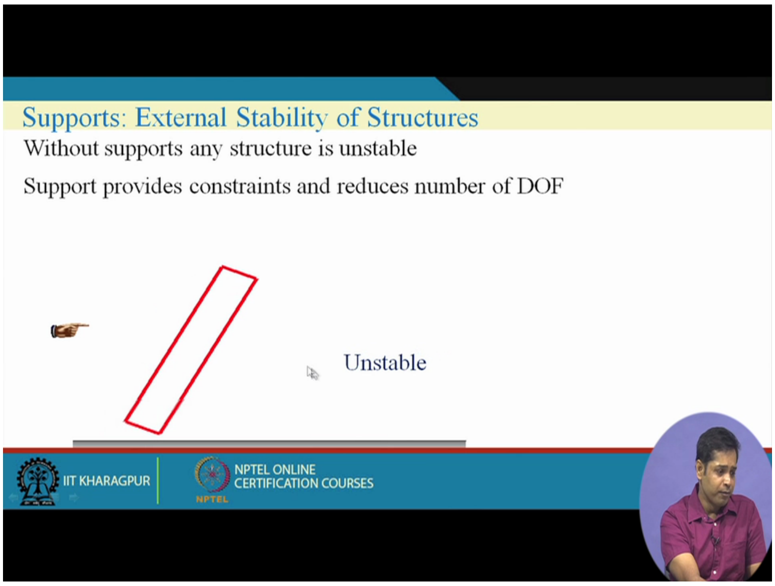

And these three degrees of freedom completely describes the motion of this object. Now when this object is not supported then means this object has all three degrees of freedom. Therefore when it is subjected to any kind of load or any small hesitation this object will undergo motion which can be described by all these three degrees of freedom. Now you see if we support the structure.

Suppose in this case the structure is not supported in the ground. Then what happens if I apply the load? Then because of the support this object will not move the way it was moving when it was unsupported.

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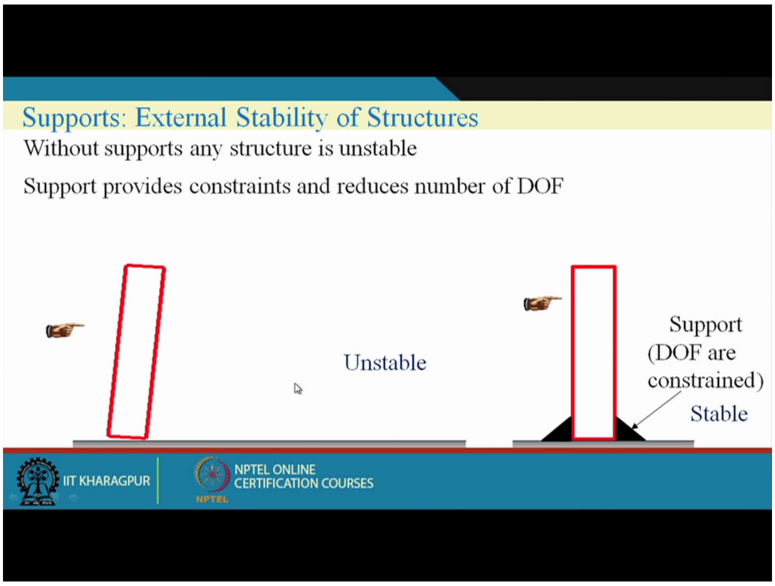

So what does it means? It means that when we apply a support then this support restricts some degrees of freedom of this object. Now restrict degrees of freedom means the object is not now free to move. The freedom of the object is restricted. Freedom of the object to move is restricted. So one of the major role that support plays the major role support plays is, it restricts the degrees of freedom and makes this structure stable. Now as I said here there are 3 degrees of freedom.

Now the question comes when a structure is supported then what degrees of freedom is restricted? Is it the translation degrees of freedom restricted or it is rotational degrees of freedom restricted? Even if it is translation degrees of freedom then translation in which direction is restricted? So depending on what kind of support you are giving, depending on what kind of degrees of freedom you want to restrict, you can have different kinds of support.

Now let us see some examples. For instance the first kind of support is fixed support. Fixed support means as name itself say that it is fixed to the ground or fixed to any object such that it cannot move. When I say it cannot move means all degrees of freedom are restricted. For instance if you see this is the idealization of the chimney and this is the fixed support.

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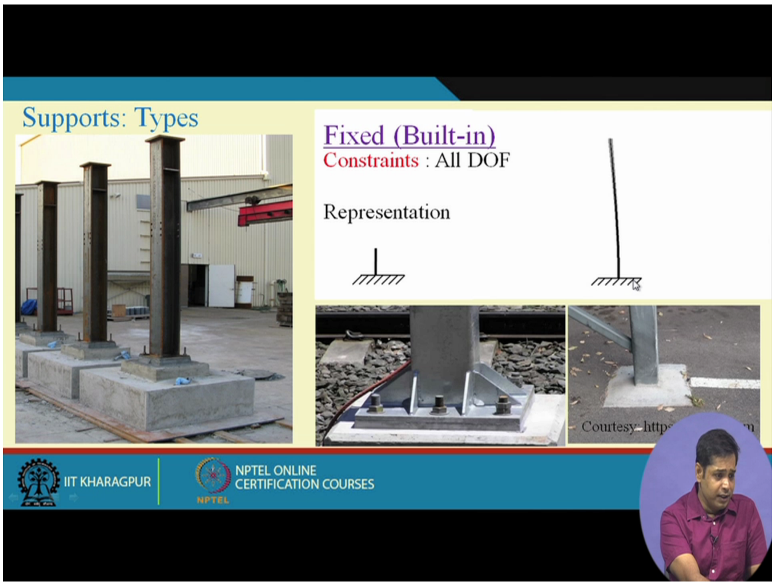

Now since this is fixed at this point, all the degrees of freedom are restricted. Means this point cannot move in this direction, this point cannot move in this direction and this point cannot rotate also and which is evident from this figure. You see this is obvious this point is not moving either in this direction or this direction. But another important thing is that you look it carefully, if you draw a straight line and see the slope then you will see the slope at this point is zero.

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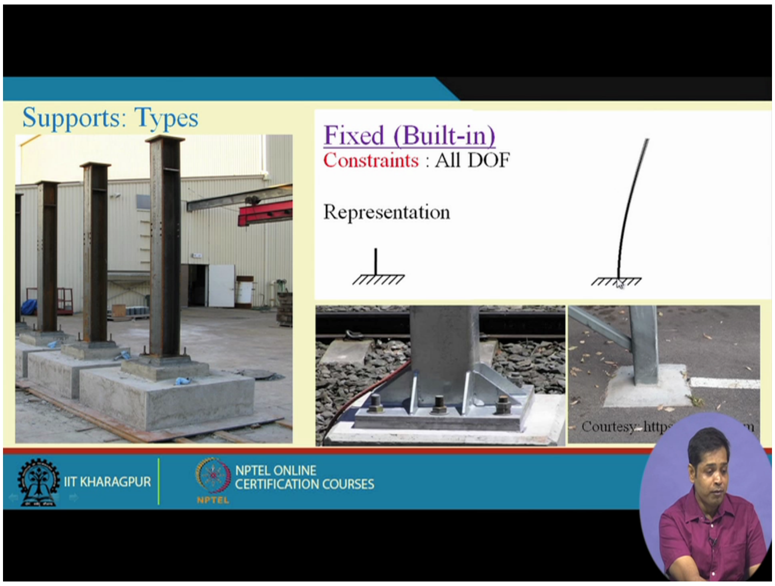

It means the object is not rotating. This is called fixed support. Now the fixed support is represented like this. There could be different ways. If you see any book but in throughout this course whenever we use this representation it means it is fixed support. Here are some of the examples of fixed support. This fixed support is also sometime called built in support.

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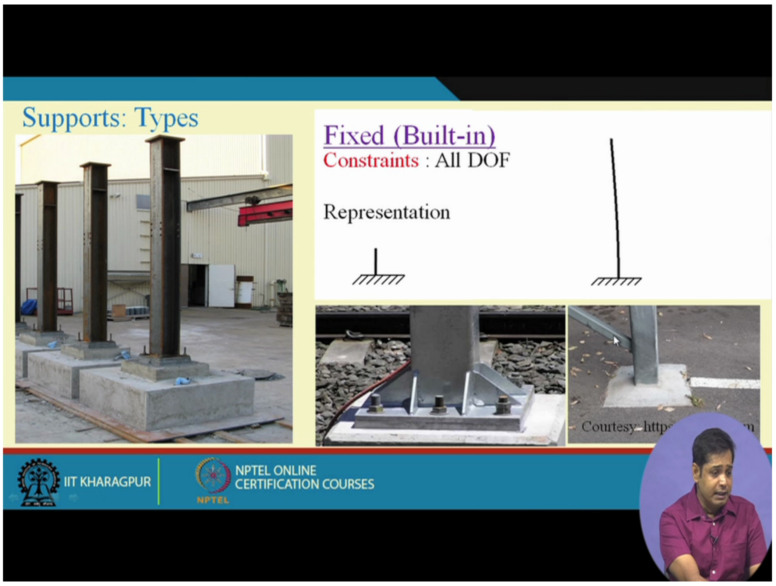

Now in this case all degrees of freedom are restricted. Now instead of restricting all degrees of freedom, restrict only the translation and allow the rotation. And if you do so then we have the next kind of support that is called hinged support or pinned support. There are some examples of pinned support.

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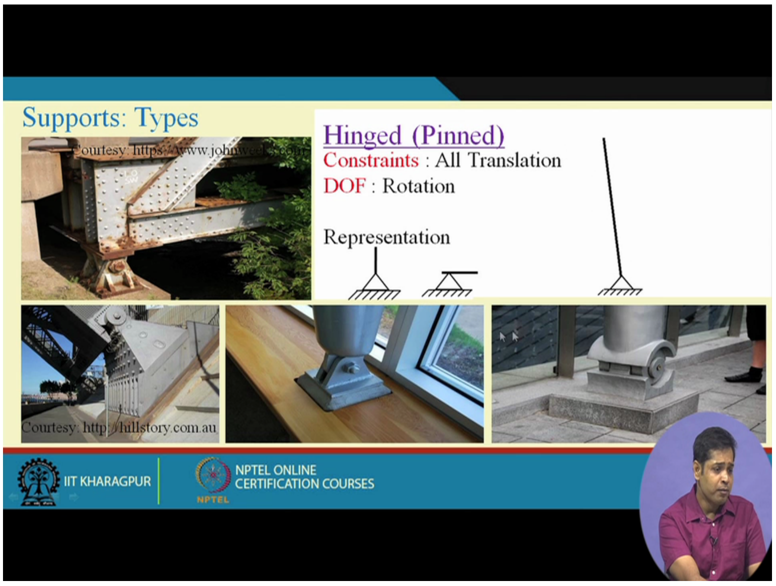

You see again this is evident this point is not moving anywhere so translation degrees of freedom are restricted. But this entire object is free to rotate about this point and therefore if you see the slope, slope at this point is not zero.

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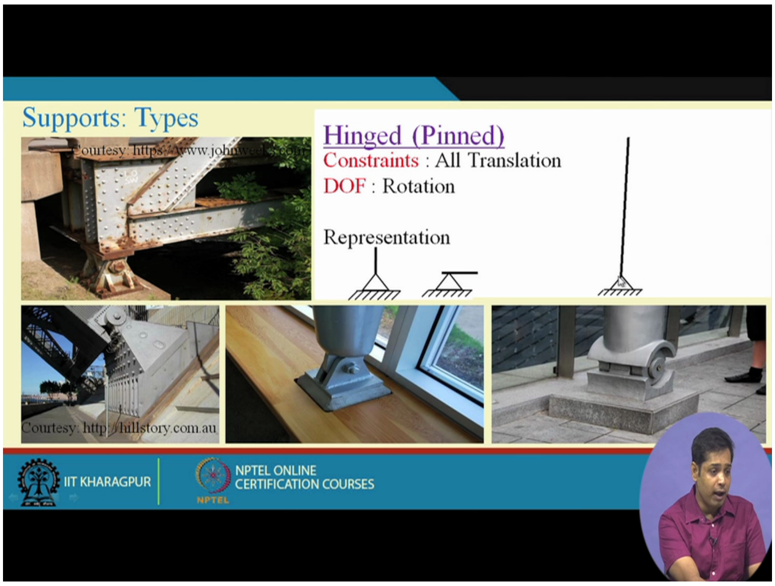

So this kind of support is called pinned support or hinged support. Representation, whenever we use this symbol it means that it is hinged support. Now so in this case what we have done is we allowed the object to rotate. So degrees of freedom the object has now is or at this point the degrees of freedom allowed is only one which is rotational degrees of freedom and provided two translations are constraint. Now let us allow one translation also. Meant, rotation plus one translation.

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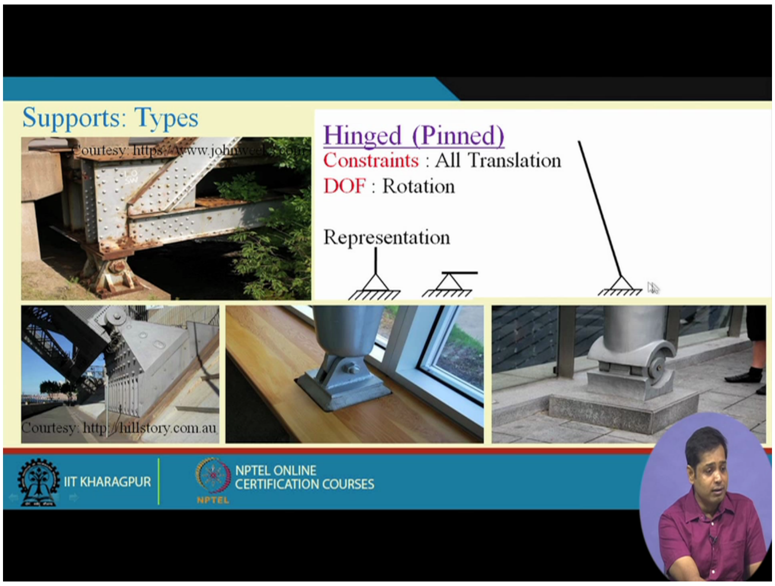

Then what happens? Then next kind of support we have is roller support or sticky roller support. Why it is called sticky roller support? I will tell you shortly. Now you see here what is happening? This point is also translating in a particular direction.

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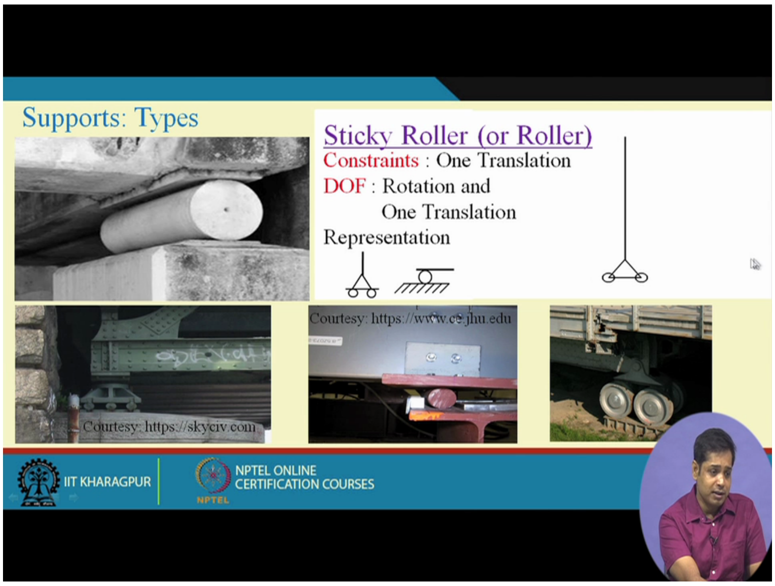

So this object has degrees of freedom in this direction, okay. So these degrees of freedom is not constraint. But why it is called sticky roller? Because it will not leave the ground. Means these degrees of freedom is not allowed. This is constrained. The only degrees of freedom it has in this direction.

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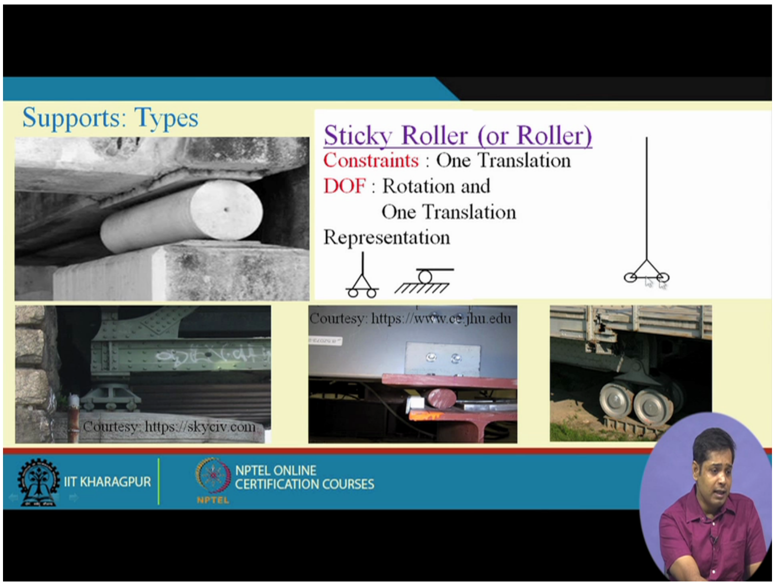

So this is roller support. And representation, representation is whenever we use this symbol this means it is roller support.

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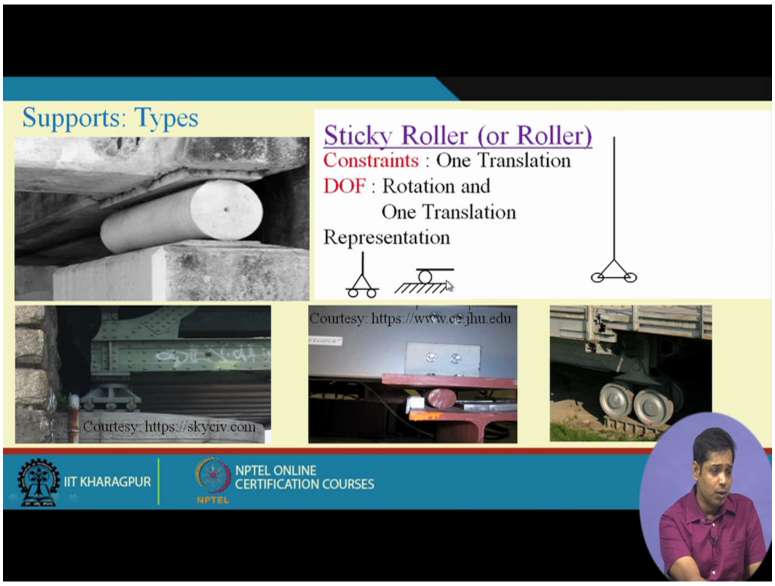

Now let us show you, I have some model for these different kinds of supports. I will show you those models. First the fixed support. You see suppose this is the support and this is the member and this is fixed here.

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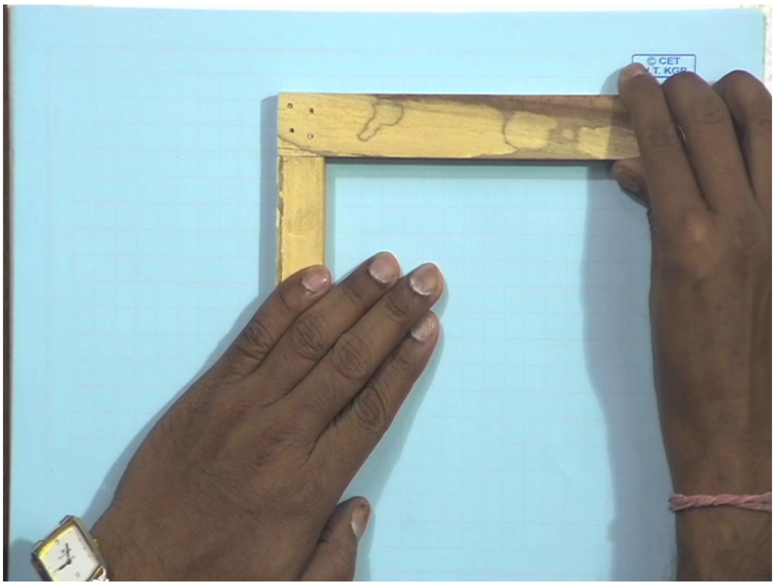

If you apply any load then this (ob) point neither it translates nor it rotates. So this support is called fixed support. There is no moment at this point, okay.

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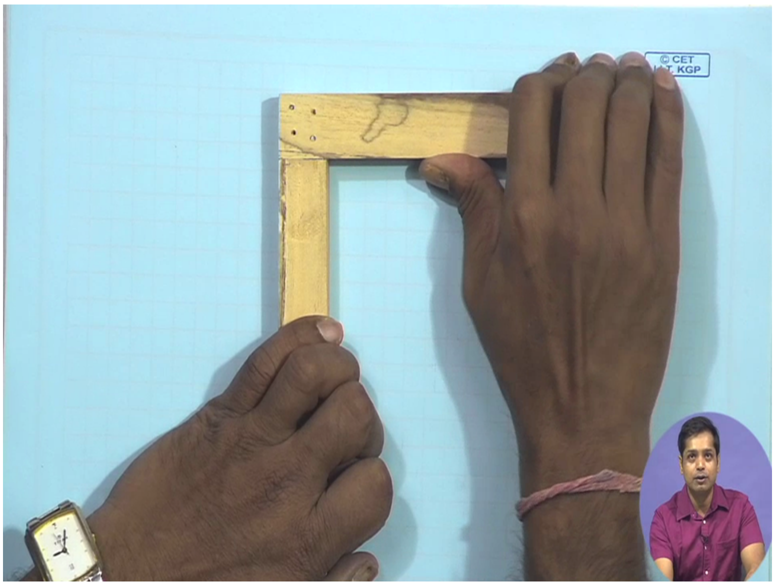

Now next is we have hinged support. You see suppose this is the object, again the idealized structure and it is hinged here.

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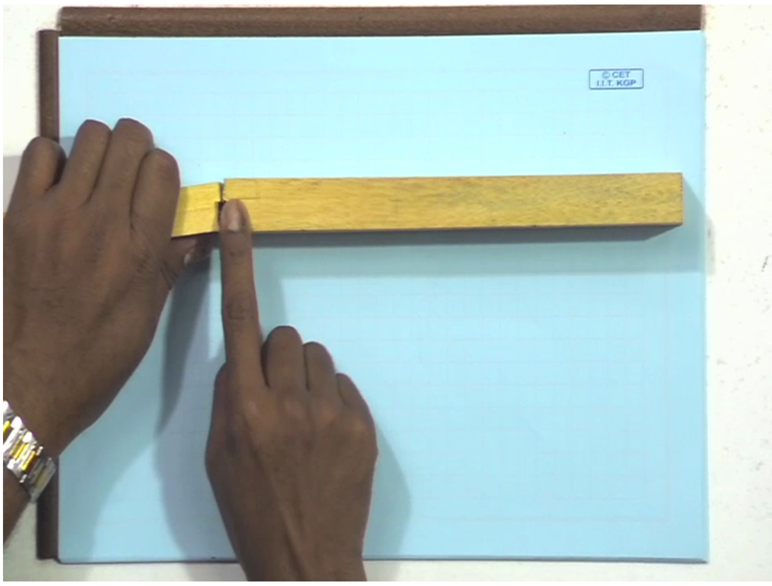

Now it is free to rotate, you see. But the translation degrees of freedom, they are restricted. The only degrees of freedom it has is rotate. It can rotate. So this is the characteristic of hinged support.

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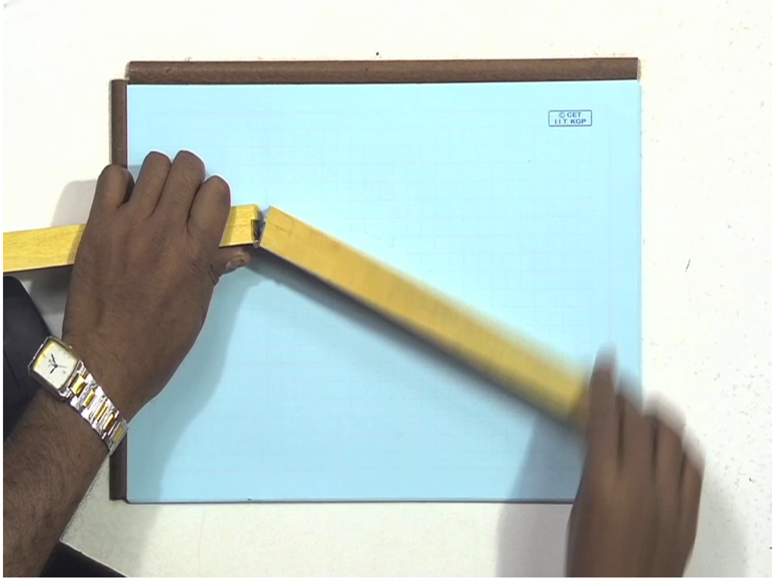

Now the next support is roller support where only one translation is allowed. This is roller support if you see. Now it can slide over this support you see.

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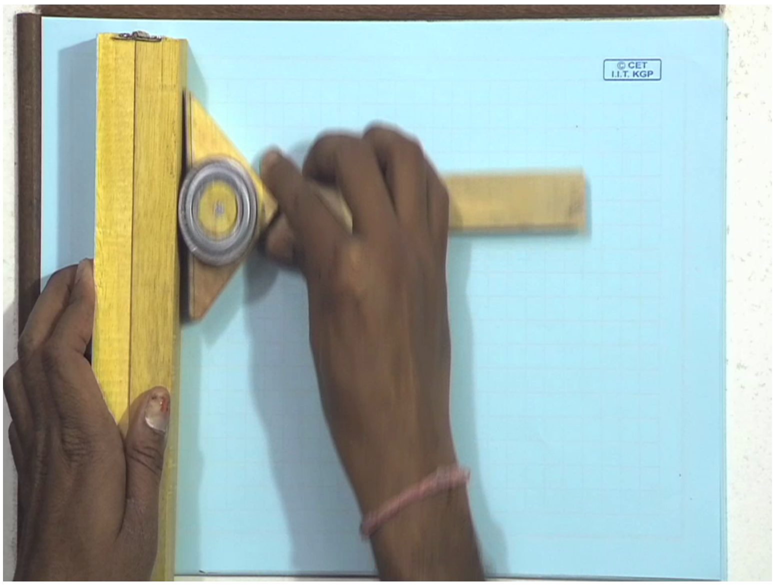

So this translation is allowed but it will not leave the ground. That is why it is sometime called sticky roller. So this is the example of roller support. Now if you see this is some of the figures which can tell you how the roller support may look like, okay.

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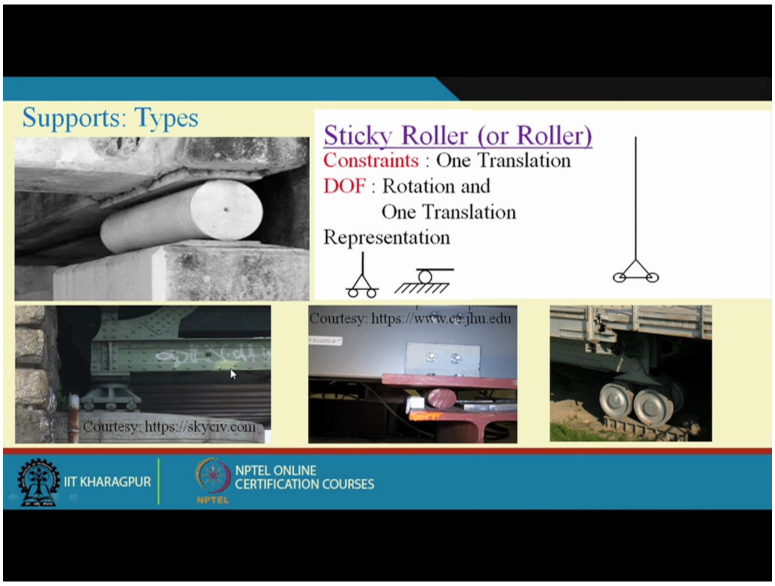

So these are different kinds of support we can have in a structure. Now if the structure is in three dimensional then also the concept of fixed support, hinged support and roller support are equally valid. But in that case what happens, when you talk about translation you have to talk about three translation, when you talk about rotations, there are three rotations.

So then your characteristic support will based on three translations and three rotations but the concept, the definition of these different kinds of supports will remain same, okay. Now what happens when a structure is supported? You see as of now it is clear to us that if the structure is not supported then it becomes unstable, right? So what support does? Support is because (unst) without support object is unstable because it has all degrees of freedom, okay.

It can move freely in space. Now when that object is supported, support provides constraint in certain degrees of freedom. Therefore the object is allowed to move in a particular manner but other moment is restricted and in doing so we provide stability to the structure, okay. Now but what exactly support does? Okay, now take example of this.

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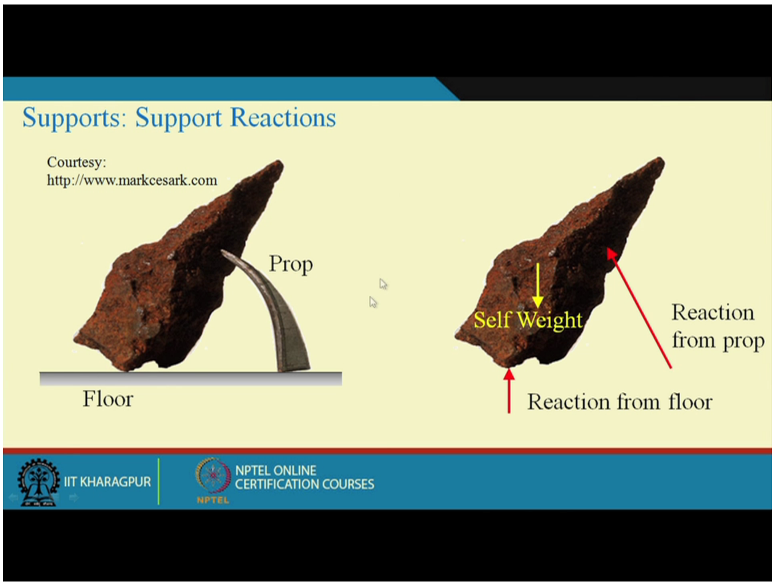

You see this is an object. This object is supported at two places, one is at the ground and one is using a prop. If you do not provide support then what happens? For instance if you take this thing, if you do not provide support. Now this is supported here.

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So if I remove the support then it will fall. Then what I can do is I need to provide support, right? Now if I do this, this means this is supported here and this is supported here. This is hinge here and this is maybe roller or hinge at this point, okay.

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Now what exactly I am doing here? This object is under its self weight, okay. So if I do not provide any support, because of the self weight it will fall down. Now what exactly we are doing here? We are applying a force at this point, we are applying a force at this point, okay. And this force and this force, they balance the weight of the structure, okay. And therefore the structure is stable, okay.

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Now exactly this is happening here. Now this is the object which is supported at the ground and also with a prop, okay.

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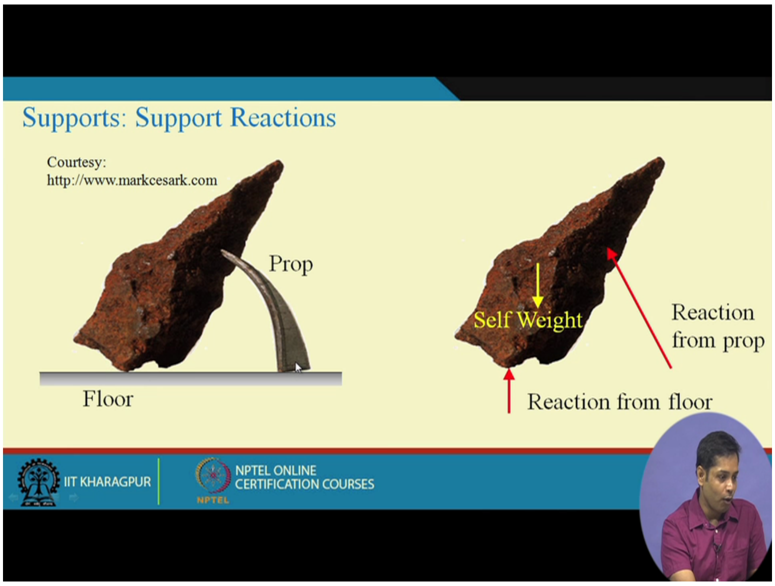

Now what this floor is actually doing? Floor is actually providing a force to this object or instead of force we will call reaction to this. This is under self weight. So how this floor will react to this? The floor will react by providing a force at this point. And that is why this is called reaction from the floor. Reaction from the floor is (supp) force in this direction.

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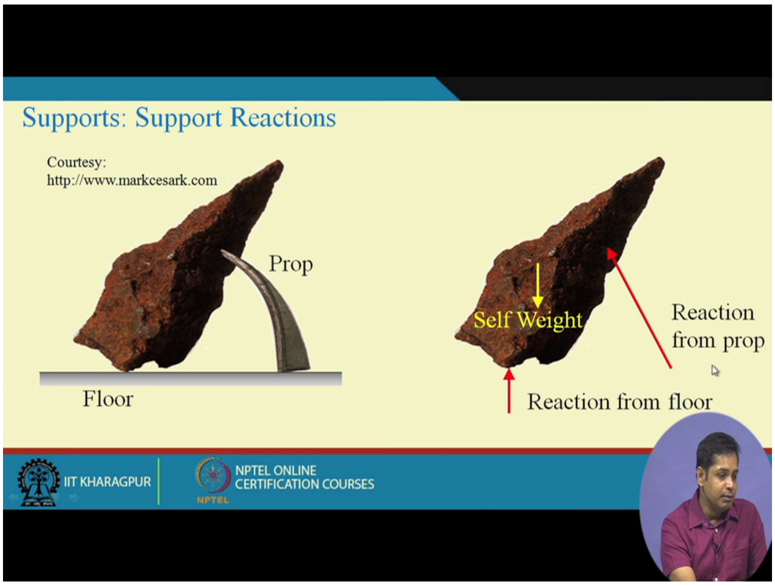

Similarly what this prop will do? Prop will also react to this and what will be the reaction? Reaction will be applying a force like this in this direction, okay. This is a reaction from the prop.

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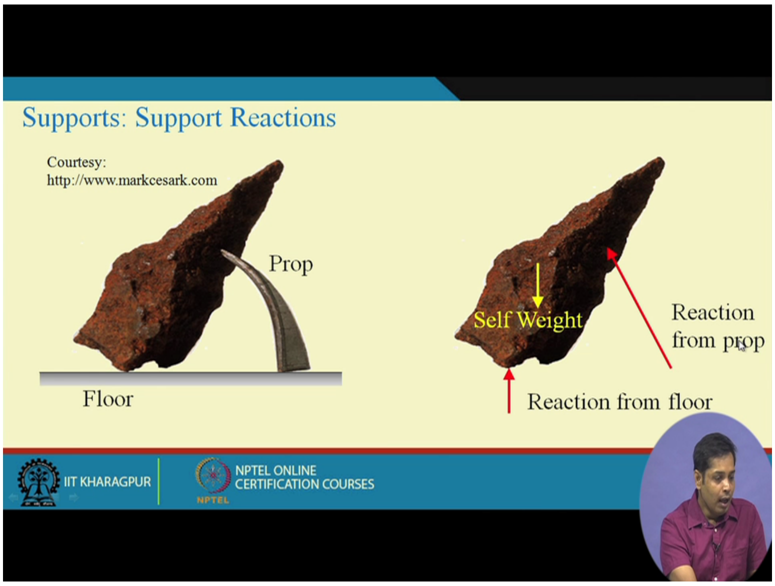

Now what happens, intuitively we can say that this object will be stable when the reaction from the floor and the reaction from the prop, they both balance the self weight of this object. If they cannot balance the self weight of the object then this object will fall down and therefore in order to make the object stable this force and this force total reaction must balance the self weight.

If we tell in a very general way then this two reactions must balance the external load applying on this object and to make the object stable. Now what is the difference between this figure and this figure?

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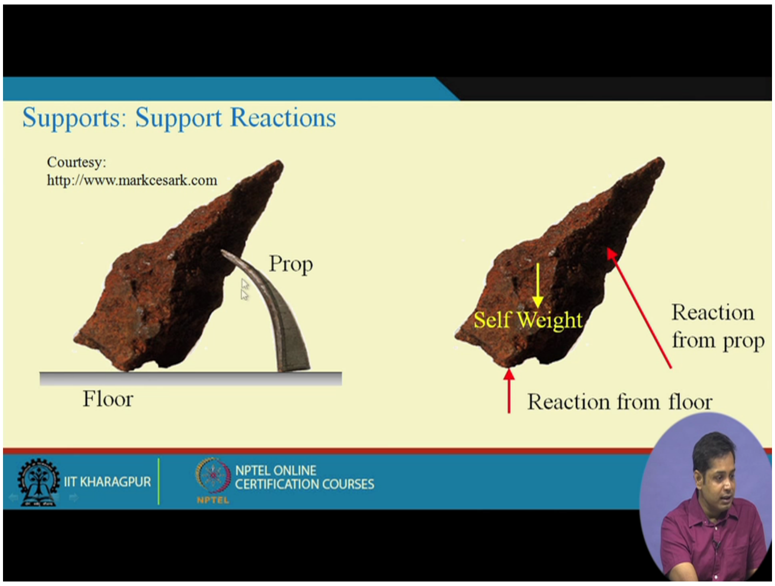

The difference is, object remains same but here the object is made free from the support, it is taken out from the support and this supports are replaced by equivalent reaction. And this is the self weight of the object. So this drawing is called free body diagram. Free body diagram because it is now free from all the support and all the supports are represented by the reaction.

I believe that some concept of free body diagram you already have in your mechanics course and solid mechanics course but we will review the concept once again in this course. So this is the free body diagram.

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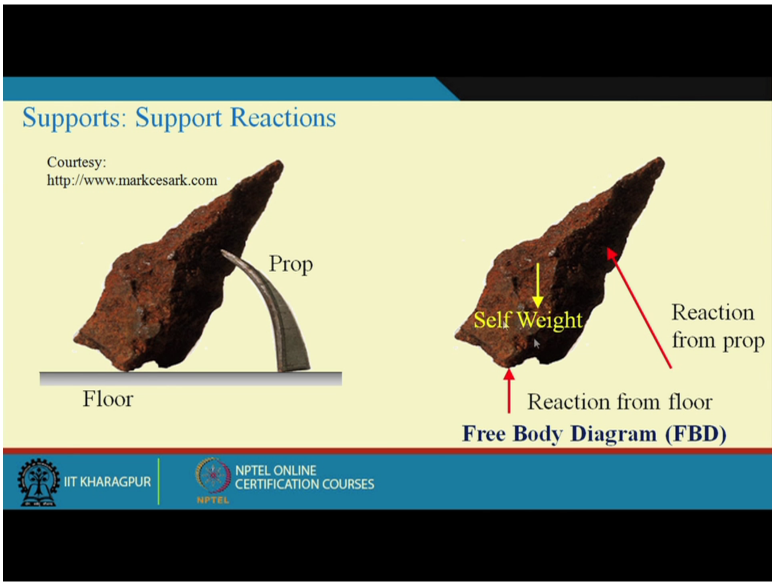

Now so you see this table gives you the characteristics of different kinds of support. Now let us talk about fixed support. Fixed support is what? Fixed support, it restricts all degrees of freedom. It does not allow translation and rotation. Now how it restricts the translation? It restricts translation by applying a force in that direction. Now since it has two translations means this support restricts the translation in this direction and this direction by applying reaction in this direction and this direction.

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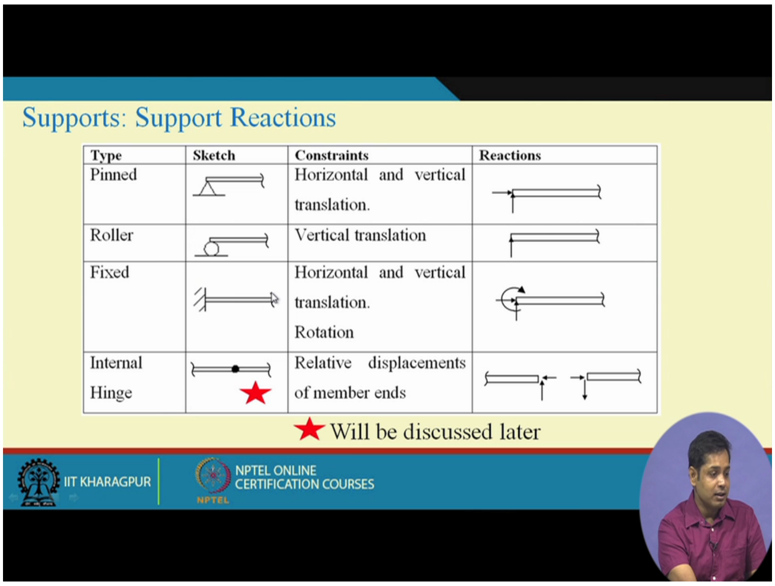

So the restriction in the translation is due to this reaction force and this reaction force.

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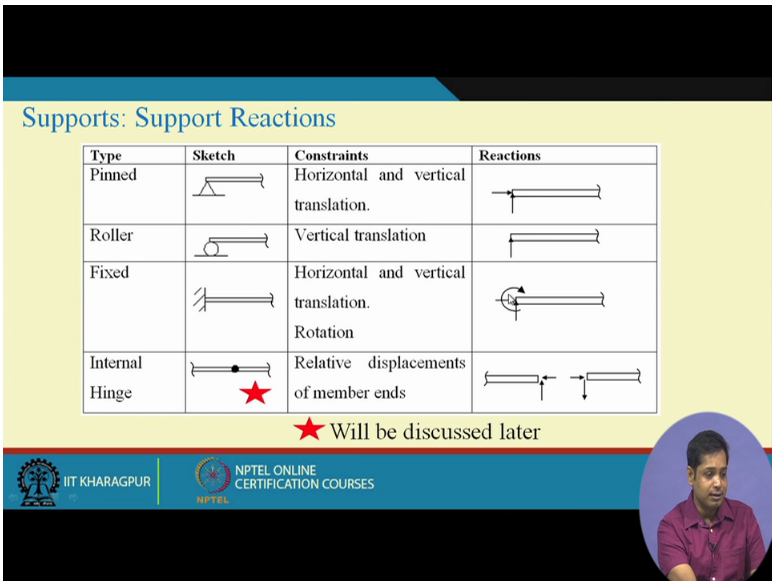

Now this support will also restrict, since is it is a fixed support, this also restrict the rotation. How the rotation is restricted? Rotation is restricted by applying a reaction moment in a particular direction. So if I have to remove this support then this support has to be represented by two reaction force and one moment. So by combination of these three forces, two forces and one moment, all the degrees of freedom at this point is restricted, okay.

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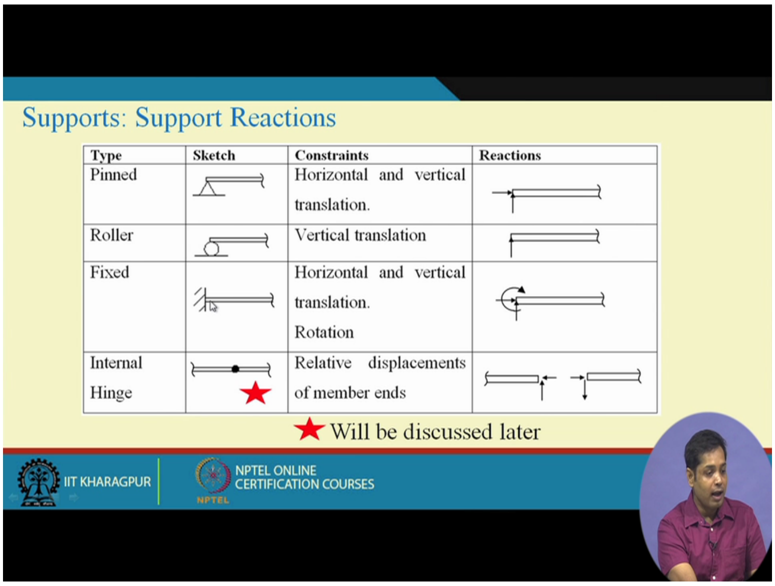

So this is the (charac) characteristic of fixed support. Now let us see the pinned support. Pinned support allows rotation but it restricts translation. How it restricts translation? It restricts translation by applying reaction in those directions. So therefore if we remove the support means that has to be represented by two forces in two reactions. Since it allows rotation means there is no restriction in rotation therefore there will be no resistive moment. There will be no reaction moment at the support. This is the (characteris) characteristic of pinned support.

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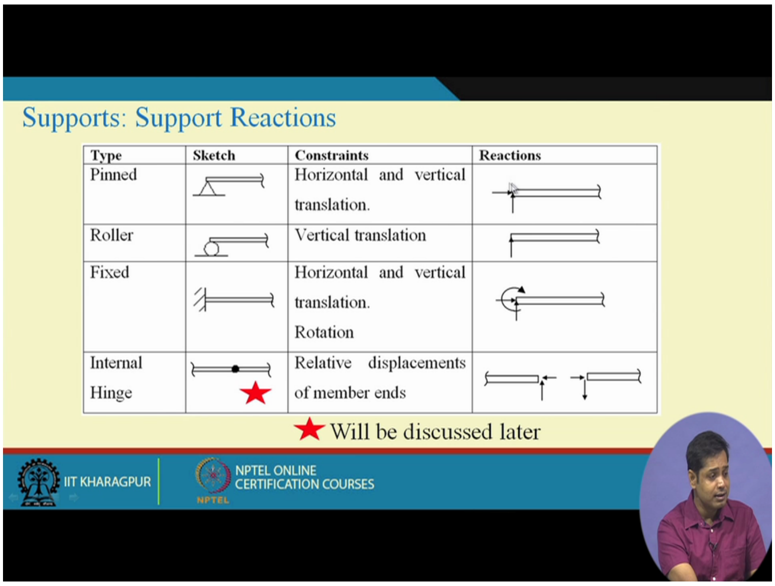

Now roller support. Roller support allows translation in one direction and also rotation. And it restricts translation in only one direction. Suppose in this case the translation in vertical direction is restricted. Therefore how it will be restricted? It will be restricted by applying a reaction force here, okay.

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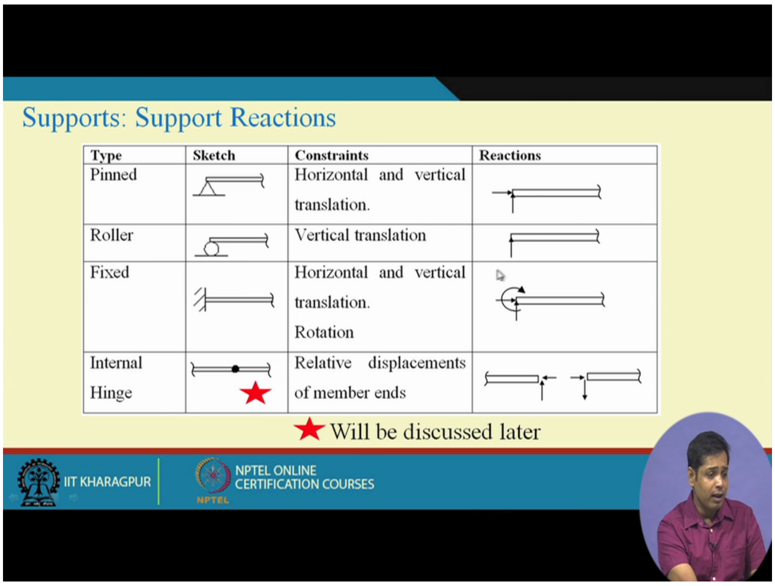

So if you remove this support than that has to be represented by reaction force in vertical direction. But there will be no reaction in the horizontal direction because the object is free to move in horizontal direction. There will be no movement there will be no moment because the object is again free to rotate at this point.

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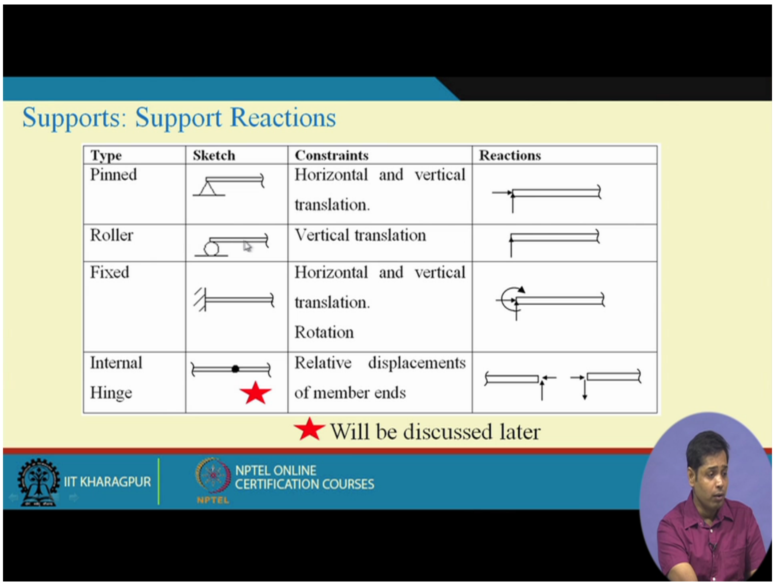

So these are the major three kinds of support and their free body diagram. So this is essentially the free body diagram of this support, this is free body diagram of this support and this is free body diagram for this support. We do have another kind of, not support, yes you can say support in some way that is internal hinge. Let us not discuss internal hinge here right now. While attempting to solve the problems we will discuss internal hinge and what is their effect.

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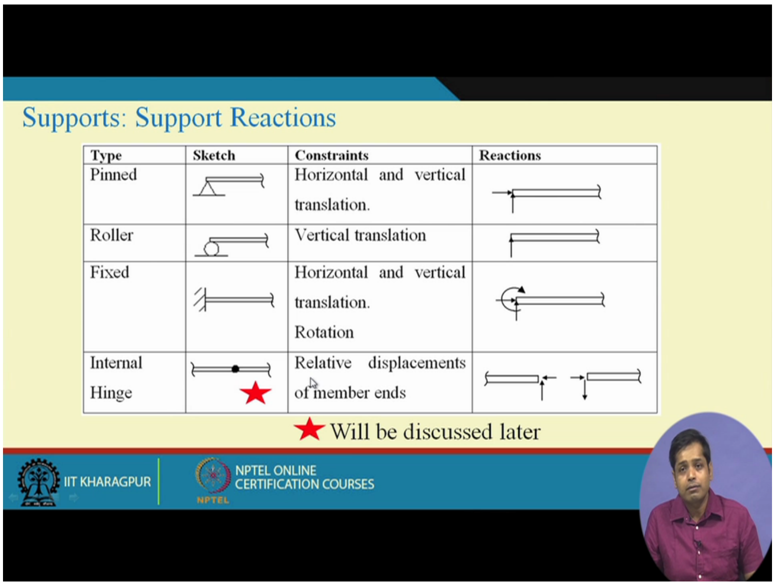

Another important thing majorly we discussed here three kinds of support. One is fixed support, roller support and hinge support. There could be different other supports as well. For instance one support is called the support on elastic foundation, okay. Those things we will discuss later. But for the time being we have only three supports, fixed, pinned and roller, okay. Now let us discuss the free body diagram of some problem. You see this is a simply supported beam subjected to concentrated load. This is hinge support here and this is roller support here.

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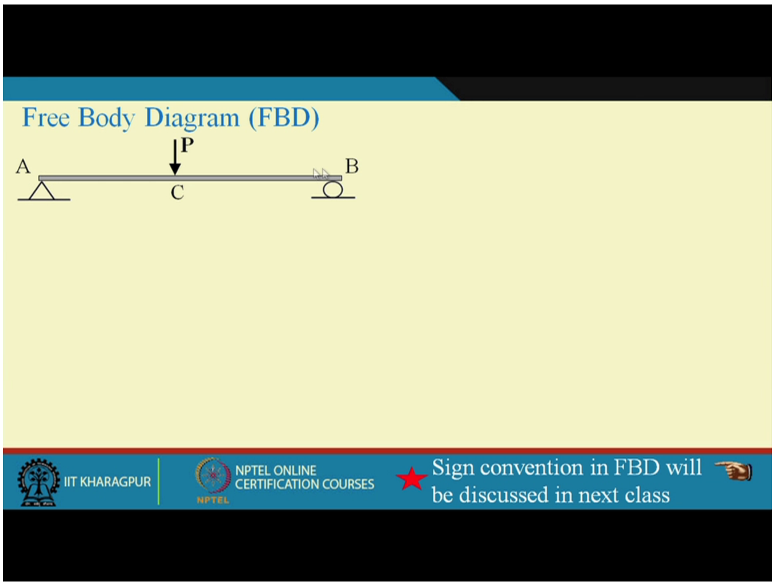

Now let us draw the free body diagram of this object, okay. Now free body diagram means first it has to be taken out from the support system. It has to be free and then all those supports need to be represented by the equivalent forces. Now, just now we saw the roller support means is apply only reaction force in one direction normal to this support. So this support has to be replaced by equivalent vertical force.

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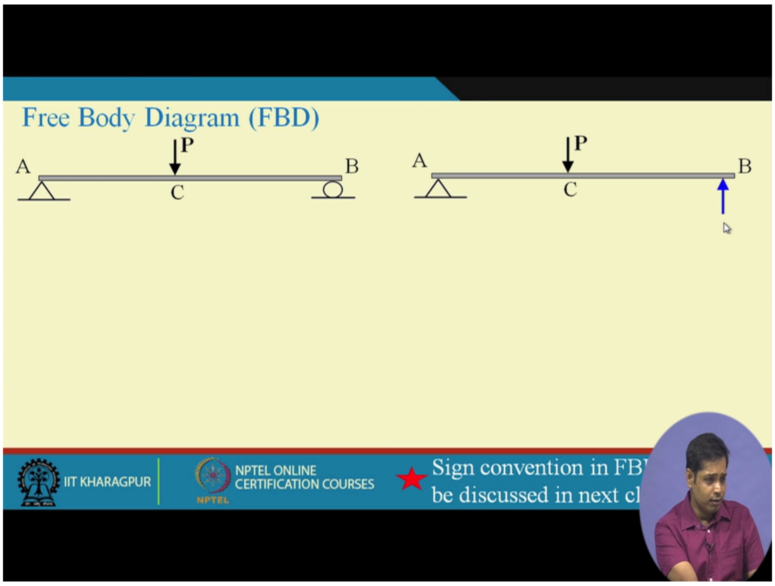

Now this is hinged support. The characteristic is one horizontal force and one vertical force and these two forces represent the hinge support.

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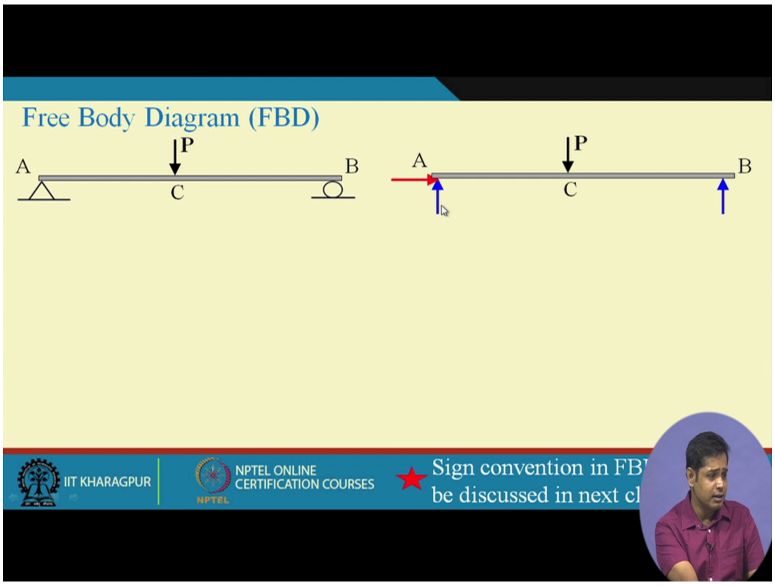

Now you give some name. The naming system we will use here is By means it is reaction force at B in y direction. It is reaction force at A in y direction. It is reaction force at A in x direction. So this is a free body diagram of beam AB, okay.

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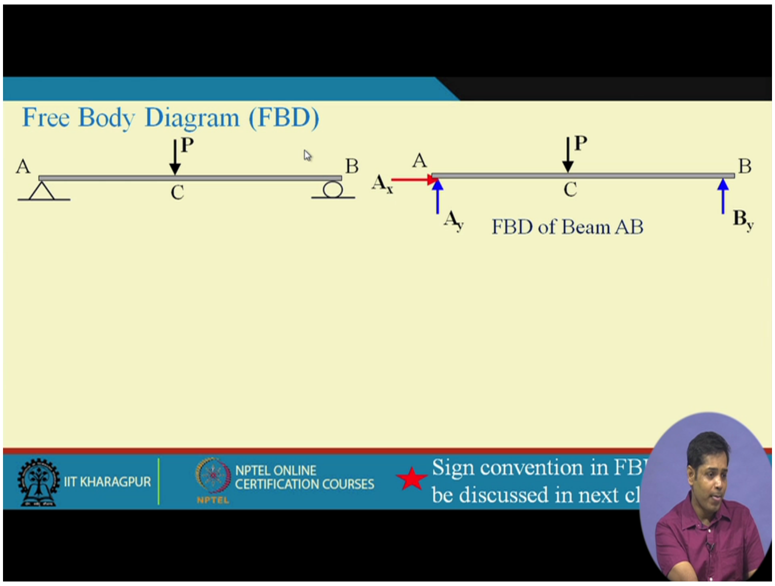

Now let us take one more example. The same, now this hinge support is replaced by fixed support. Let us draw the free body diagram of this. The same thing, roller support replaced by vertical support and fixed support is replaced by two horizontal force and one moment and this becomes the free body diagram of this beam. Again the same naming system and Ma means moment at point A.

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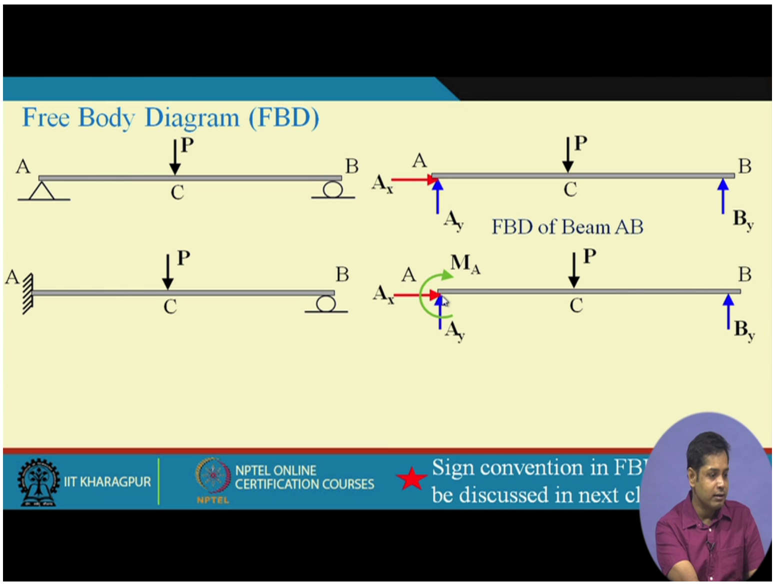

Now suppose if I take a section here, okay. Now if I take a section here then we have two parts. One is this part and another one is this part.

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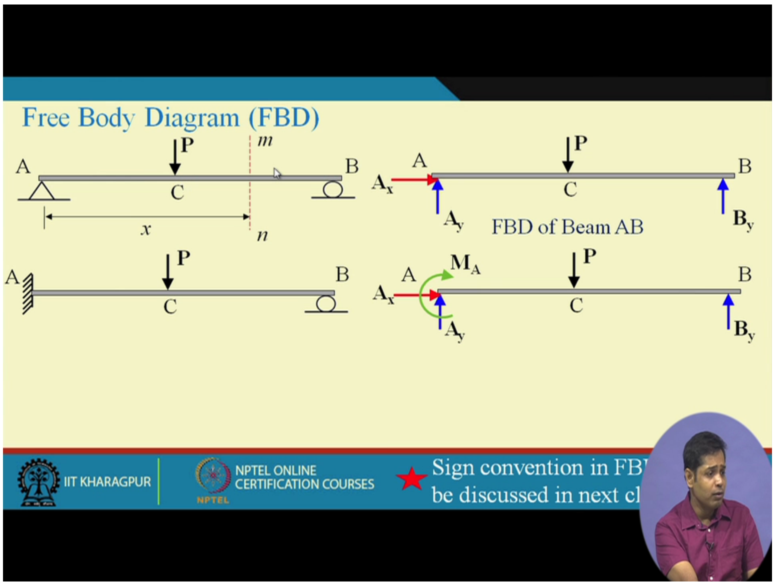

Now suppose I need to draw the free body diagram of this part and this part separately. Now you take this part and then this hinge is replaced by equivalent forces and this part will be treated same as fixed support. These things will be more clear when we will talk about internal hinge.

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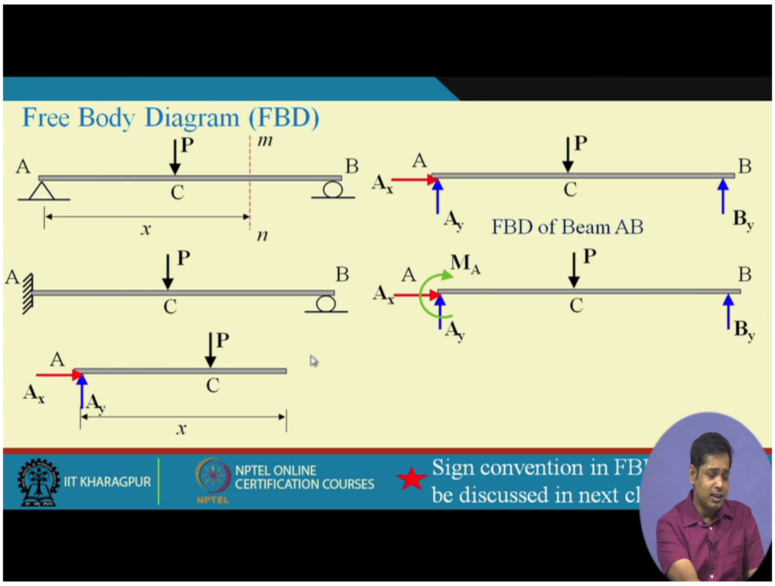

Now since if this section is treated as same as fixed support means if we cut this member at this section means the internal forces will be vertical force, one is vertical direction, one is horizontal direction and one is moment. So this is the free body diagram of this part.

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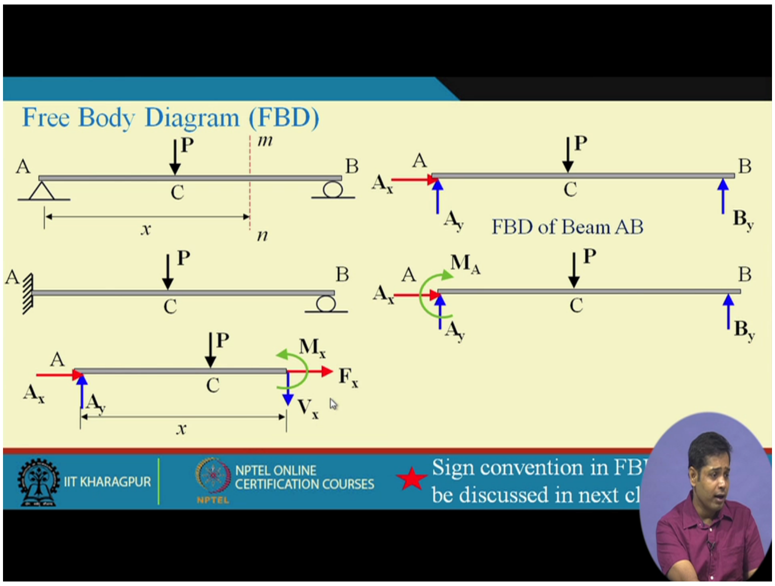

Now similarly if I take the free body diagram of this part then again this will be replaced by vertical force and this is represented by two forces and one moment. You see one thing is highlighted here. The sign convention in free body diagram will be discussed in next class. You may ask that why in all other cases you have given force in upward direction. In this case you have given moment like this but why in this case you have given moment like this, force in a downward direction.

But when we draw the free body diagram of this part we use horizontal force in this direction, vertical force is again in other direction, moment in other direction. What sign convention we will use here? That we will discuss in the next class when we talk about beams, okay. But here the point what we are going to make is the concept of the free body diagram, okay.

Free body diagram is, if you want to draw the free body diagram the system has to be free, this system has to be taken from its support system and all the supports need to be replaced by their equivalent forces, okay. Now so if you take the free body diagram in this case, say in this case. Now this is the external load and this is the support reaction, right?

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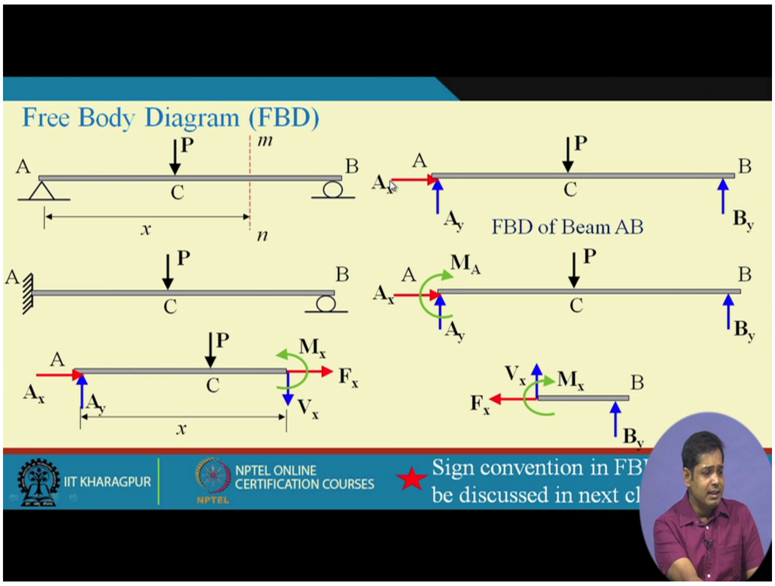

In this case this is the external load and these are the support reactions and these are the internal forces, okay. Now in order to make the structure stable what we need is all these are essentially reactions, right? Reactions to this external force. What is (im) needed for the structure to be stable is all these reaction must balance the external force, okay. And that property is called equilibrium. So what is equilibrium? Equilibrium is you see you have an external load on a structure and the structure will respond to that.

When structure respond to that means the internal forces in the member will respond to the external load. Now when these internal forces, they balance the external load, then we say this object is in equilibrium, okay. And the object has to be in equilibrium for stability. Otherwise the object will not be stable.

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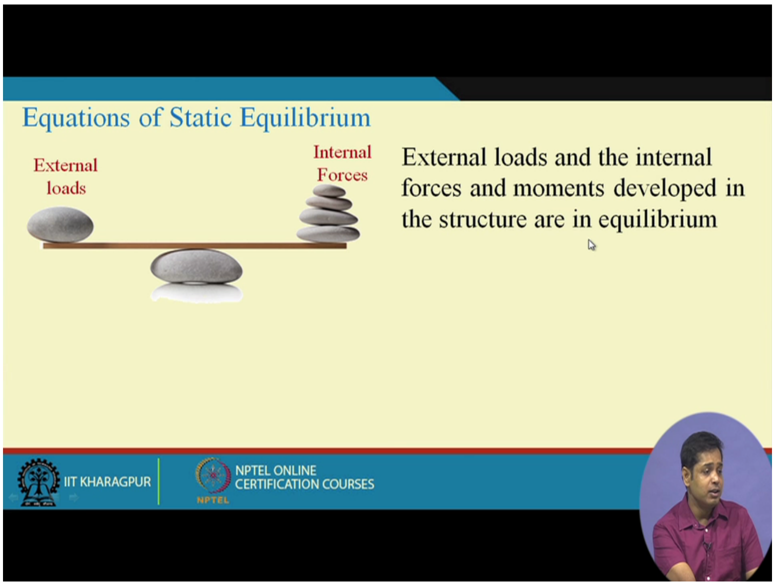

Now then what is equilibrium? Equilibrium will be summation of external forces has to be equal to summation of all internal forces, summation of external moments has to be equal to all internal moments, okay. Now in three dimension how many forces we can have? We have three forces and we can have three moments because just now we said that in three dimension we have six degrees of freedom, three translation and three rotations.

So three translation means three forces and three rotations means three moments. So summation of forces in any particular direction or summation of moment in about any particular axis have to be zero. That is the equilibrium equation.

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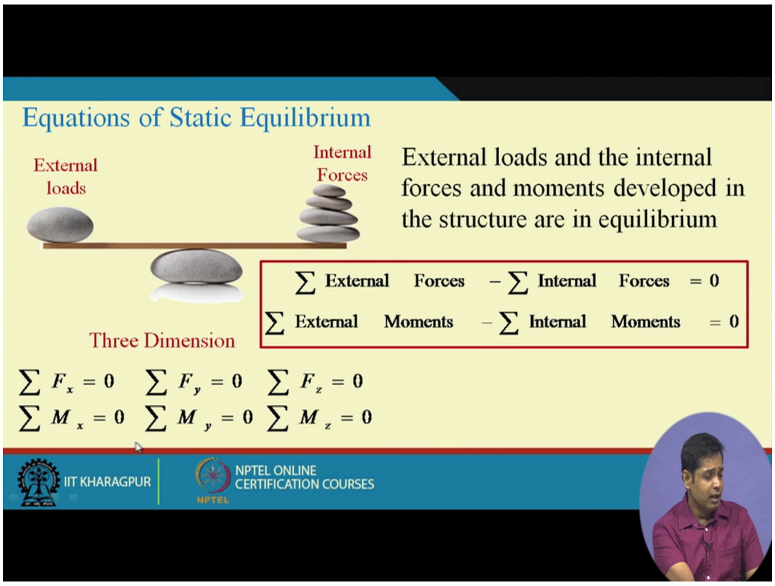

And similarly (ext) for two dimension case you have two translations means two forces and one rotation means one moment and this is the equilibrium equations for two dimension, okay.

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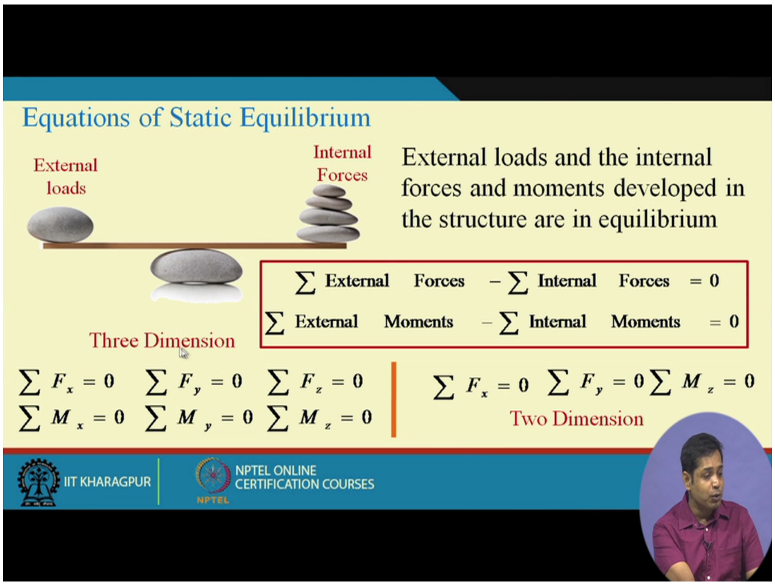

Now when this equilibrium equations are satisfied then we can say the object is stable and the object will not move in uncontrollable manner, okay. Now you see one point is very important here. For instance if I give you an example, suppose an aircraft is flying or an aeroplane is (fying) flying, it is moving, right? Or you fire a bullet and the bullet travels through here. So this bullet is also moving, okay.

Now the aeroplane is flying means it is in motion, right? Does it mean that it is not in equilibrium? It is also in equilibrium but that equilibrium will be different. You see in this case when these equations are satisfied this equilibrium is called (equi) static equilibrium, okay. Means your underlying assumption is the object is in static condition. So it is not moving, okay. Now this is static equilibrium and these equations are called equilibrium equations, okay.

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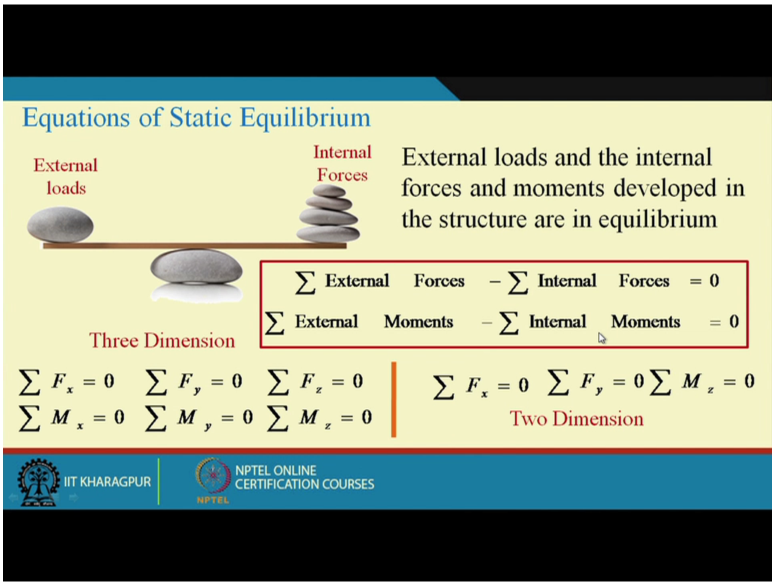

Just quickly let me give you some demonstration of static. What are the usefulness of static equilibrium equations? You see when we demonstrated that usefulness of static equilibrium equation this is a sign convention we are using. But remember, this sign convention and the sign convention I mentioned while drawing free body diagram, they are different.

Free body diagram sign convention we will discuss later and in subsequent classes. But for this the sign convention is this. Forces in this direction and forces in this direction is taken as positive and clockwise moment is taken as positive.

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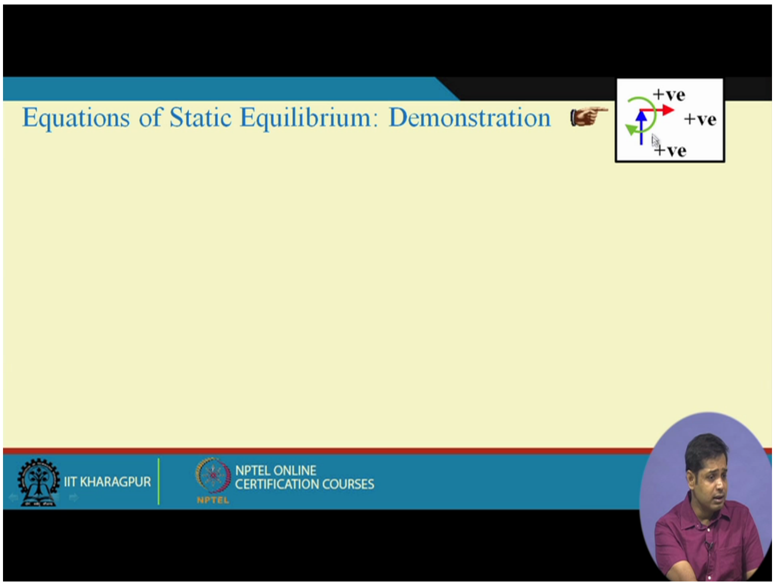

Quickly suppose this is a simply supported beam subjected to vertical concentrated load at C and naturally this is the equilibrium equation, right? Now this is a free body diagram.

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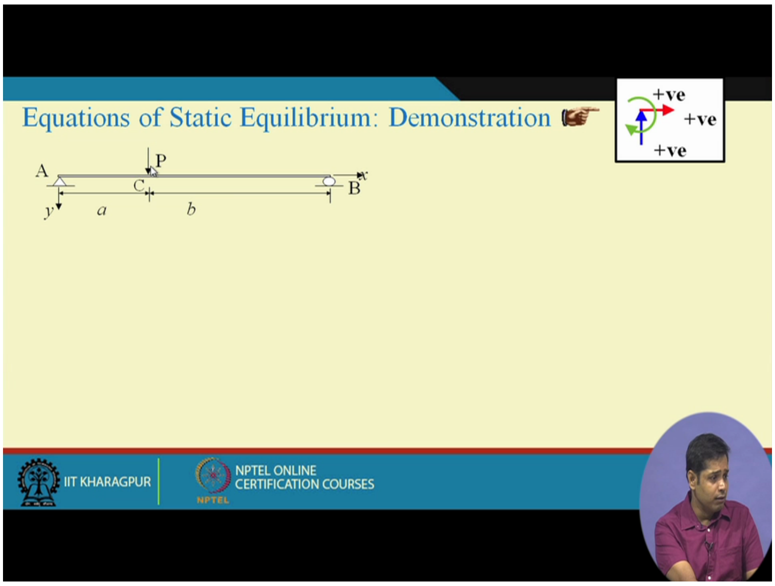

Now what are the equilibrium equations? Equilibrium equation says that in two dimension, summation of forces in y direction is zero, summation of force in x direction is zero and summation of moment at any point will be zero, right?

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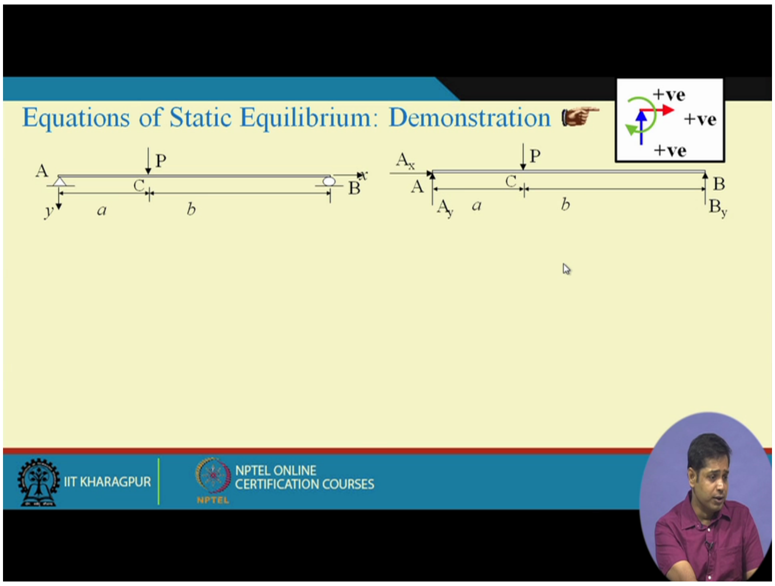

So if you take summation of forces in x direction is zero then the horizontal component of force is Ax. So Ax will be zero in this case.

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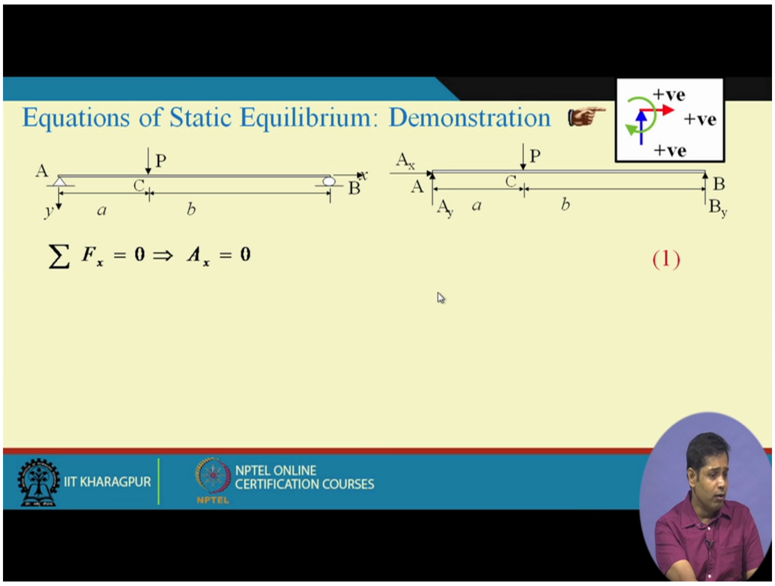

Now take summation of forces in y direction zero. What are the forces we have in y direction? Ay By which is upward direction means positive, then P downward direction negative. So Ay plus By minus P is equal to zero. It gives you Ay plus By is equal to P.

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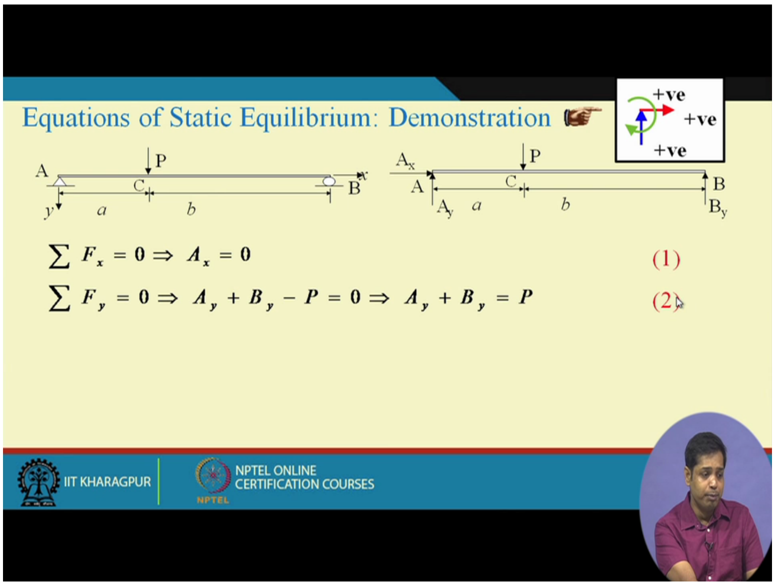

Now then summation of moment is equal to zero. You take moment about B, it gives you that Ay into A plus B which is clockwise direction, then if we take moment about B, this force would be in clockwise direction that is why it is positive. Then there will be moment for P as well and this moment will be anti clockwise direction that is why negative and you have another Ay is equal to this and substituting Ay here you will get By is equal to zero.

So all the reactions are Ay is equal to this, Ax is equal to and By is equal to this. Now you see applying equilibrium equation we could determine the reaction forces, right?

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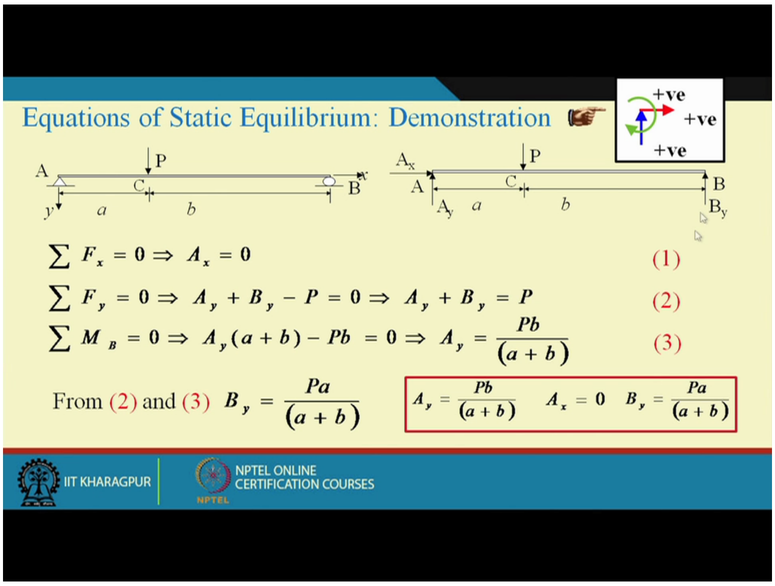

Okay, we will stop here today. Some more examples will be shown in the tutorial class. Thank you.