Structural Analysis 1
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Lecture 24
Deflection of Beams and Frames (Contd.)

Hello welcome, this is lecture number 24. What we will do is we introduced the concept of conjugate beam method in the last two classes. Today we will see some example, okay.

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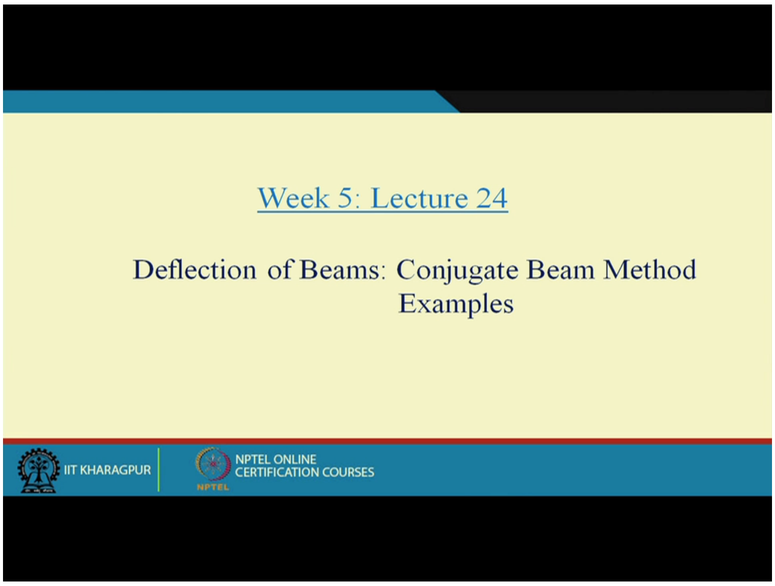

The first example is this example, cantilever beam subjected to tip load.

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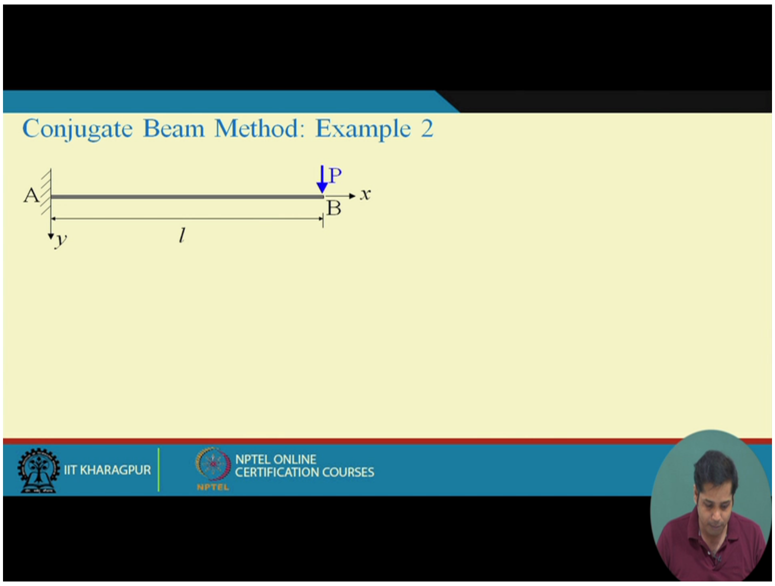

You remember in the last class through this example we demonstrated applying the force on the conjugate beam as bending moment diagram M by curvature diagram of the real beam. In addition to that you have to take care of support as well because if the conjugate beam is not chosen properly then it may give you wrong result.

So while demonstrating that what we did is we kept the beam as it is but did not change the support at this point and at this point, okay.

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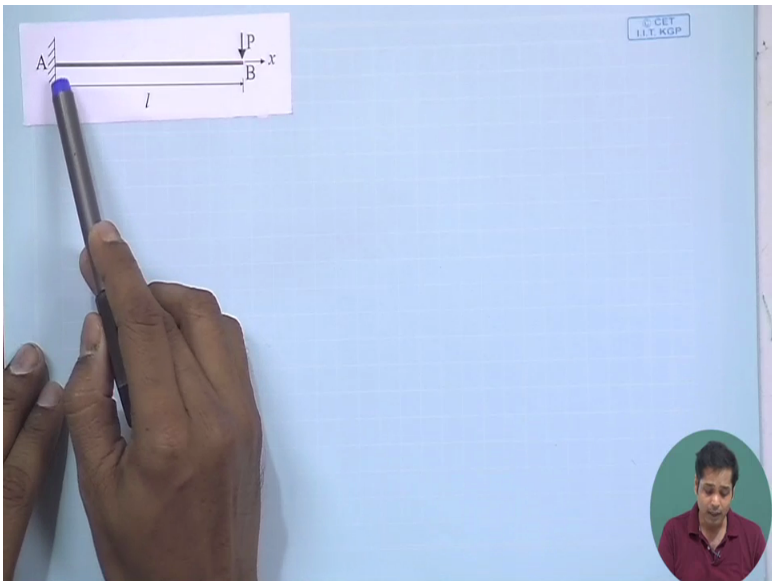

Now let us see this example once again with applying both loading and support of conjugate beam, okay. Now first draw the bending moment diagram of the beam. The bending moment diagram becomes like this. This cause hogging in the beam so bending moment will be negative and this is PL and this is P into L minus x. X is measured from A, okay. This is bending moment diagram.

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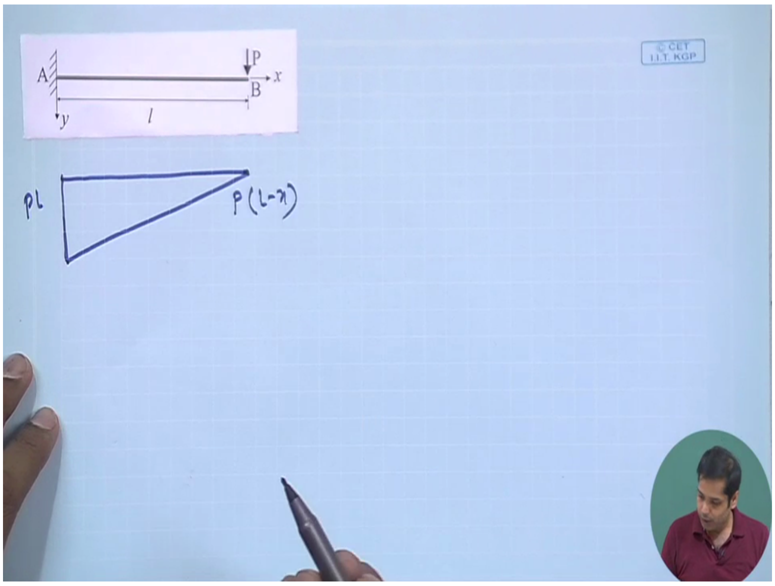

So now let us draw the conjugate beam. Length of the conjugate beam remains same. This is the length of the conjugate beam and the loading on the conjugate beam is this diagram divided by EI.

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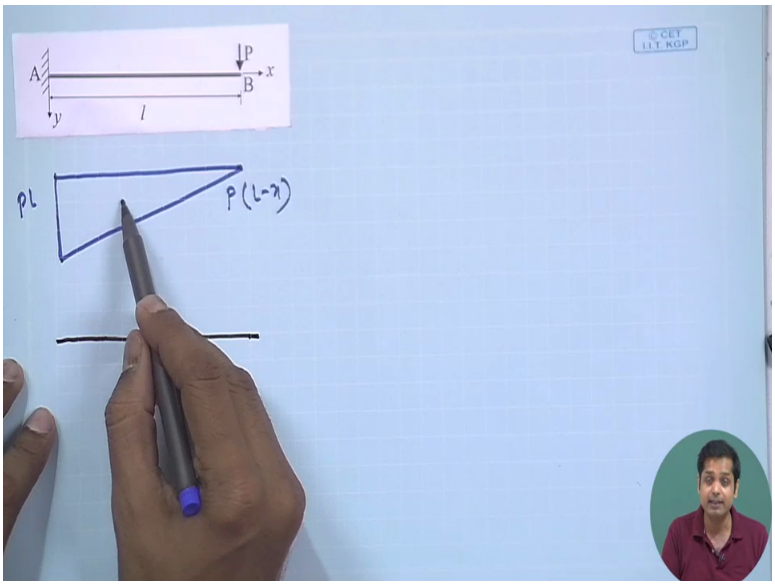

So this is acting upward because bending moment is negative. This we have seen in the last class, okay. Now so this is P L minus x divided by EI. Assume EI constant for this beam.

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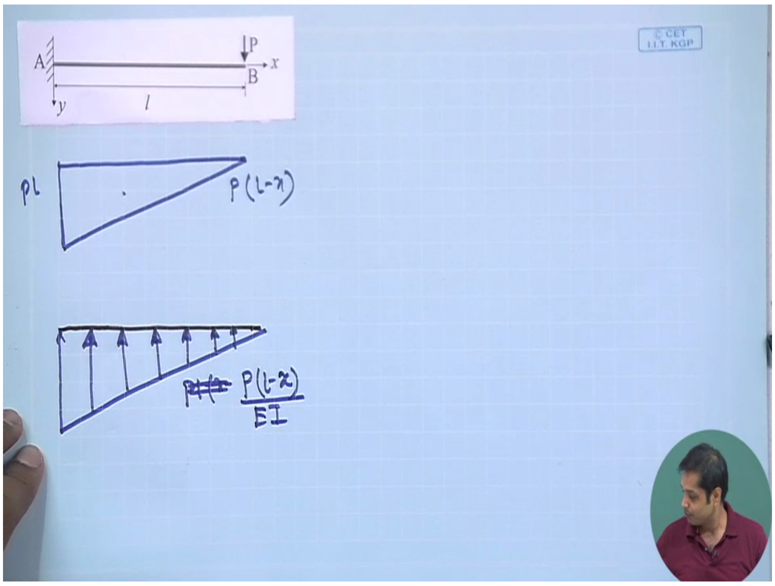

Now if you recall this table, if the fixed end in real beam becomes free in the conjugate beam and the free end in the real beam becomes fixed in the conjugate beam. And so if the beam is like this then the conjugate beam becomes.

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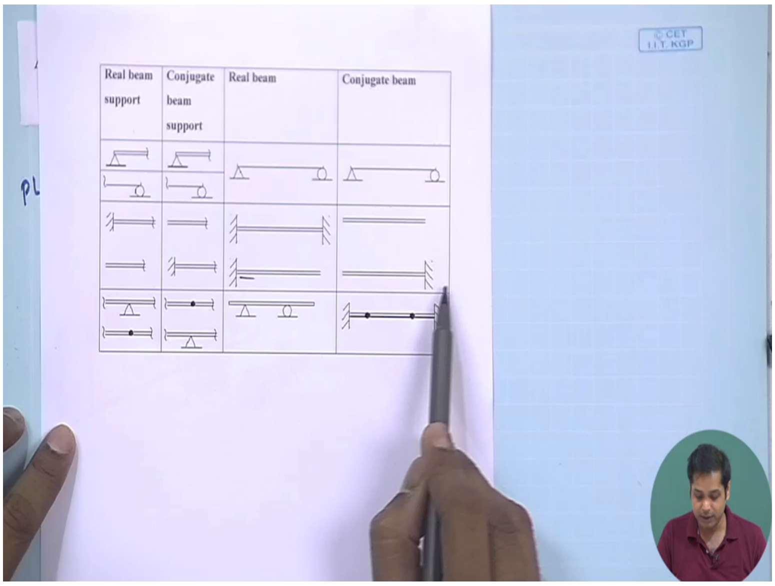

This becomes free and this free end becomes fixed. So what would be the support condition? Support condition for this beam will be, this end will be fixed and this end will be free. So this is A and this is B.

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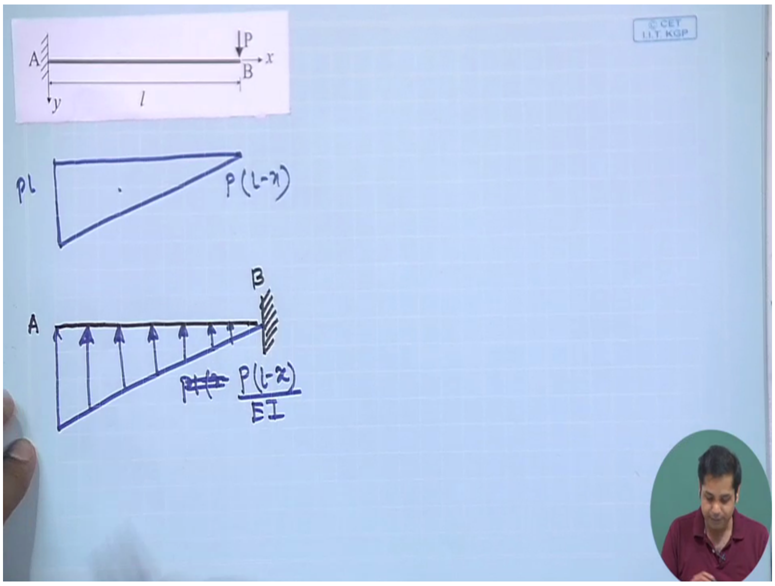

So this is real beam, this is bending moment diagram and this is conjugate beam, okay.

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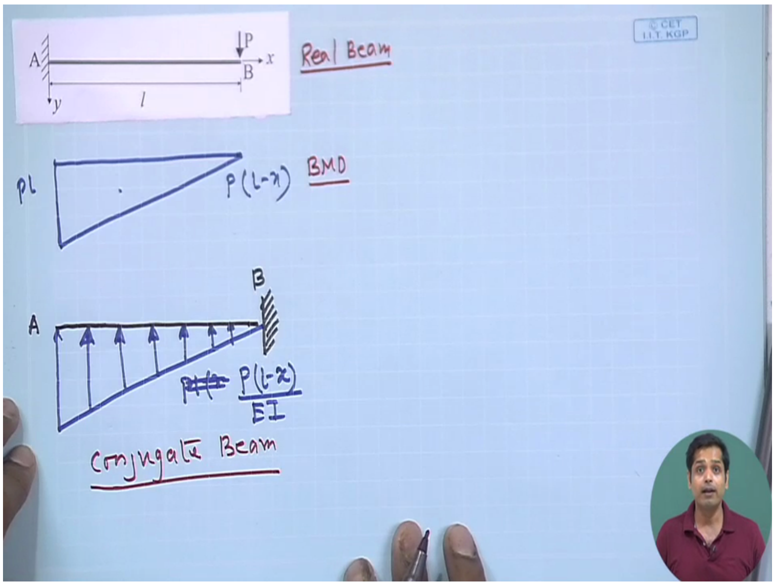

Now what next? Next determine the internal forces in this beam. So since in this example we are asked to determine deflection and slope at B so what we have to find out here, what would be the shear force and bending moment at B? And shear force will give you the (rotat) slope and bending moment will give you the deflection. So first draw the free body diagram of the entire structure. The free body diagram of entire structure will be this, apply the load on this and then fixed end we have say this is Bx.

Let us use different pencil. This is Bx and then this is By and moment MB, okay. And this is A and this is B. This length is L, okay.

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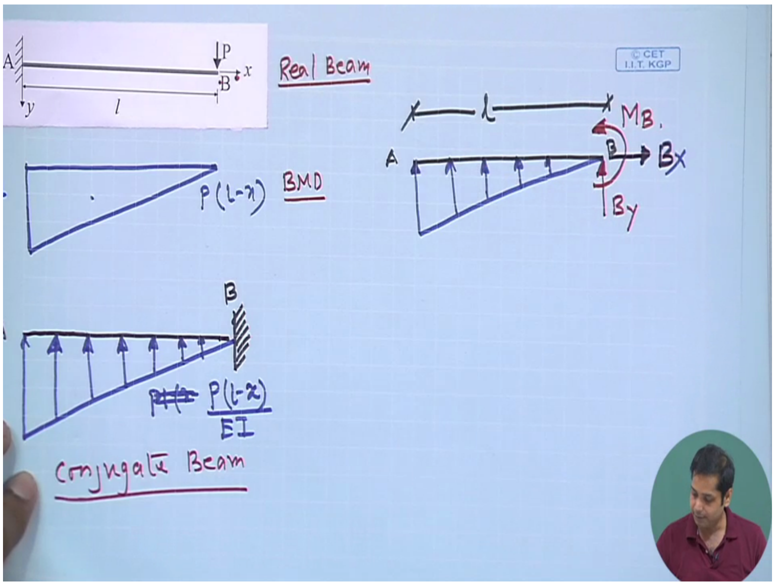

Now apply summation of Fx is equal to zero, Ax is equal to zero. You may (ne) pass this step because just by looking at this problem we can say there is no horizontal force here and the support condition is also such that the horizontal reaction will be zero. But just for the completeness I am writing it say Bx is equal to zero.

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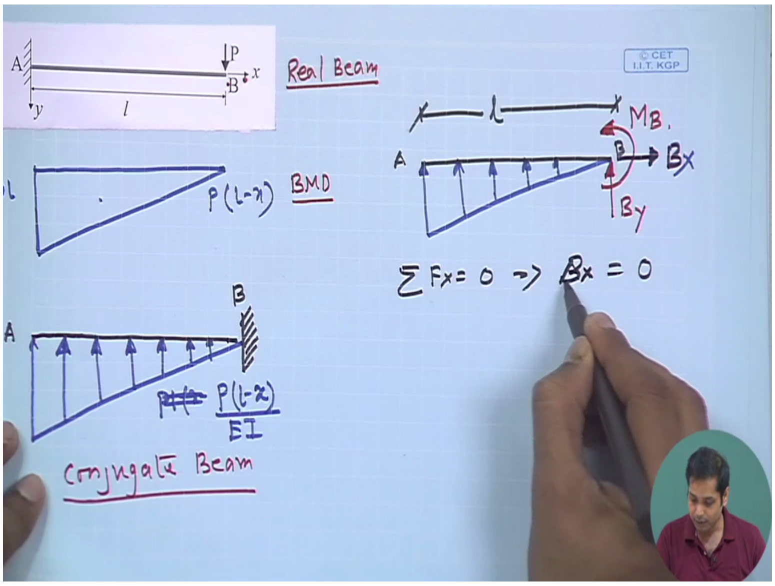

So then take summation of Fy is equal to zero and this gives us By which is upward direction so positive as per as algebraic operation is concerned. Then plus the entire load is also upward so that is (po) positive half into this is PL by EI into L that is equal to zero, okay.

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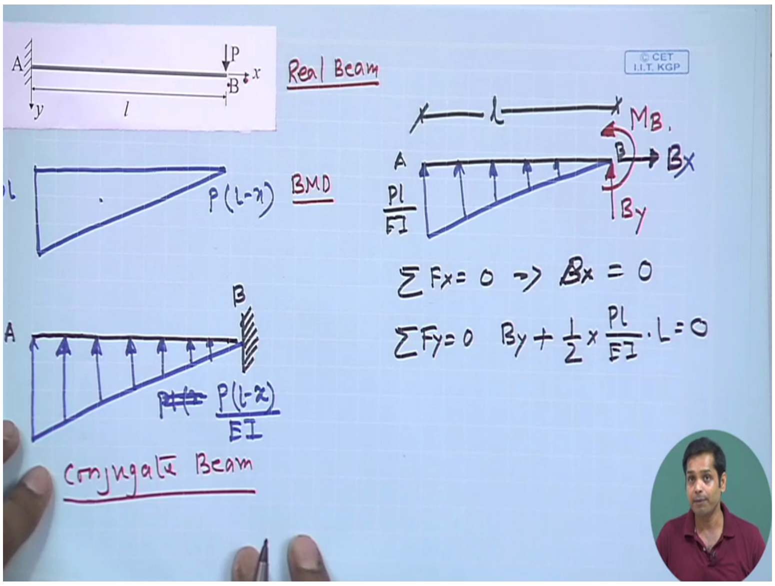

So this gives me By is equal to minus PL square by 2 EI, okay. By is equal to this, okay.

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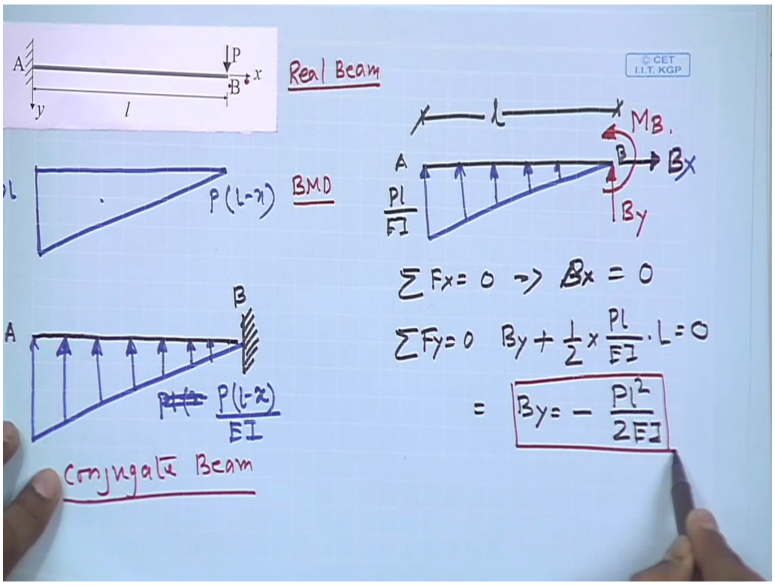

Now what is MB? Take summation of MB is equal to zero and MB is positive so MB is anticlockwise negative and then this By will not contribute and then plus this is clockwise, this becomes half PL by EI into L into 2L by 3. And MB is equal to PL cube by 3 EI, okay. PL cube by 3 EI, okay.

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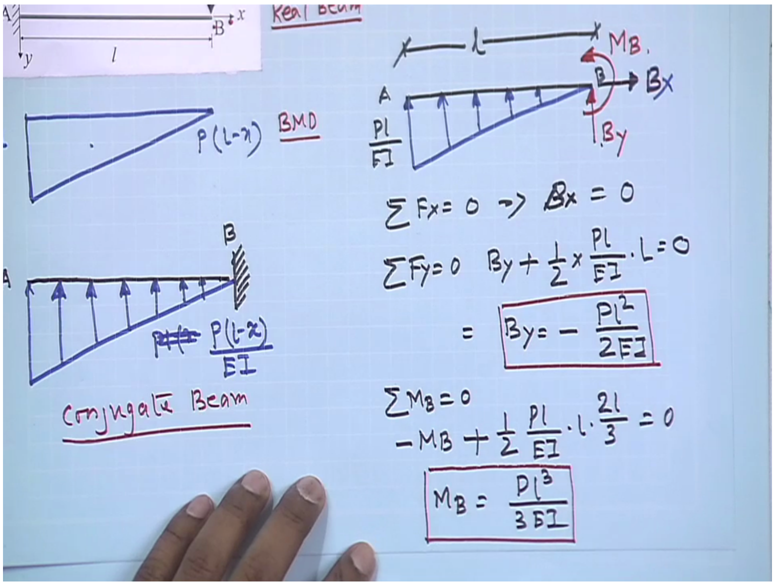

So what we know is the shear force at B will be slope at B, bending moment at B will be deflection at B. What is this shear force at B? Suppose By is the reaction at B. And if you remember our sign convention was this. If I write our sign convention somewhere here, if this is a small segment of the beam then this is clockwise couple, this is positive. And then this is our sign convention positive, okay.

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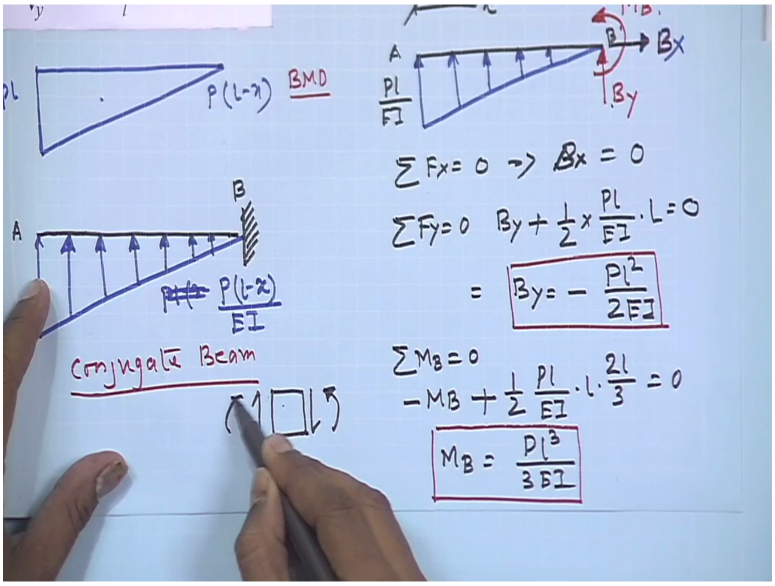

So what will be delta B in this problem? Then let us write. Delta B is equal to this.

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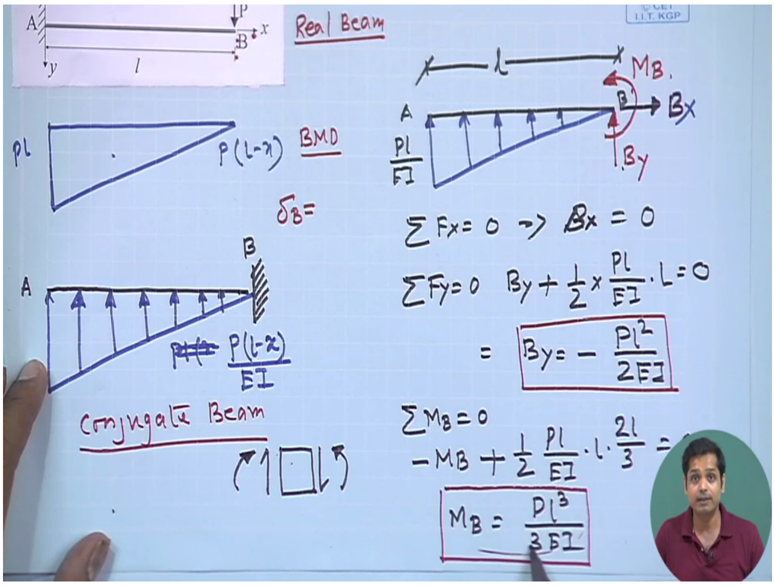

This is PL cube by 3 EI and absolute value of theta B if you write then this becomes PL square by 2 EI, okay. I have written the absolute value because you give the sign as per your convention. So this is a result, okay. So delta B is PL cube by 3 EI and slope is PL square by 2 EI.

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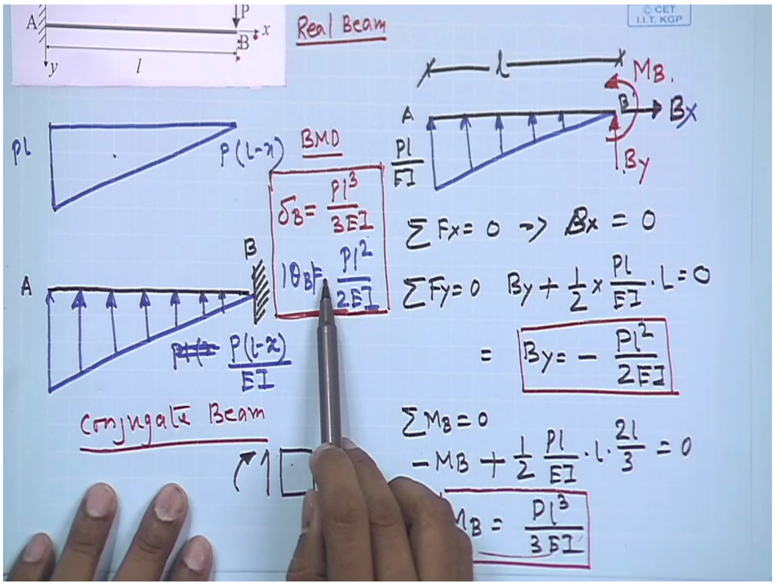

Now if you remember in the direct integration method and moment diagram method we (ob) obtained similar result, okay. Now if we see this example and what is the deflected shape of this beam, then the deflected shape becomes like this, okay. So at this point your slope is zero and at this point your deflection is this and you take a tangent at this point and this angle will be slope.

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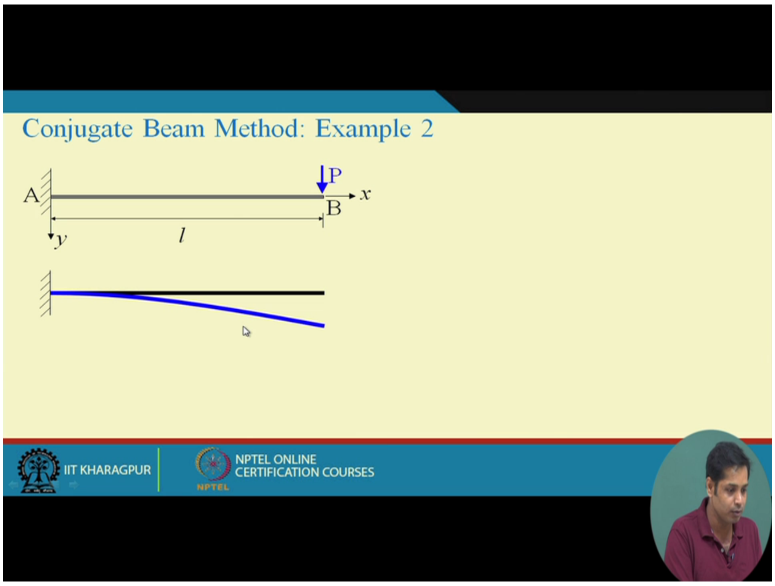

Every time when you determine the deflection even at any point try to understand by intuitively what would be the probable deflected shape of a beam, okay. And then see that deflected shape is consistent with the bending moment you get and the sign convention you used. Okay, this was the first example.

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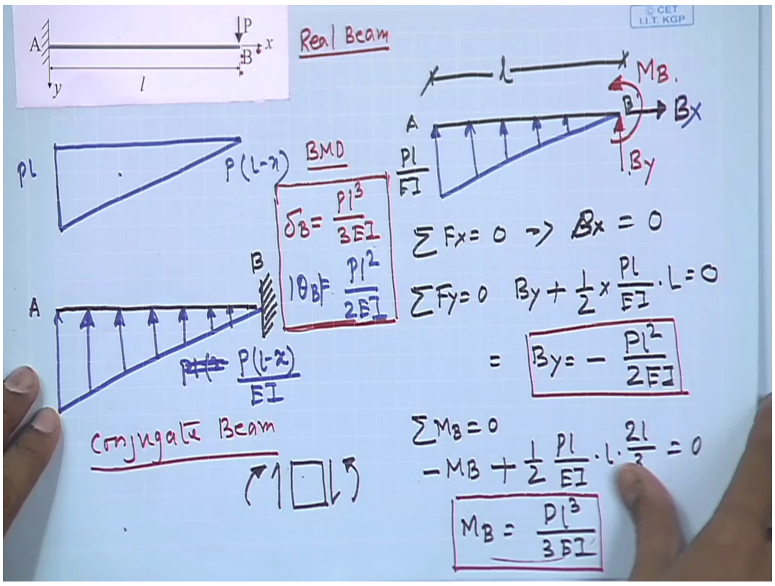

Now the second example is this is a simply supported beam but with an over angle of length L by 2 and then we have a concentrated load at the tip of the over angle, okay. What we need to do is we need to determine what is the deflection and slope at point B, okay.

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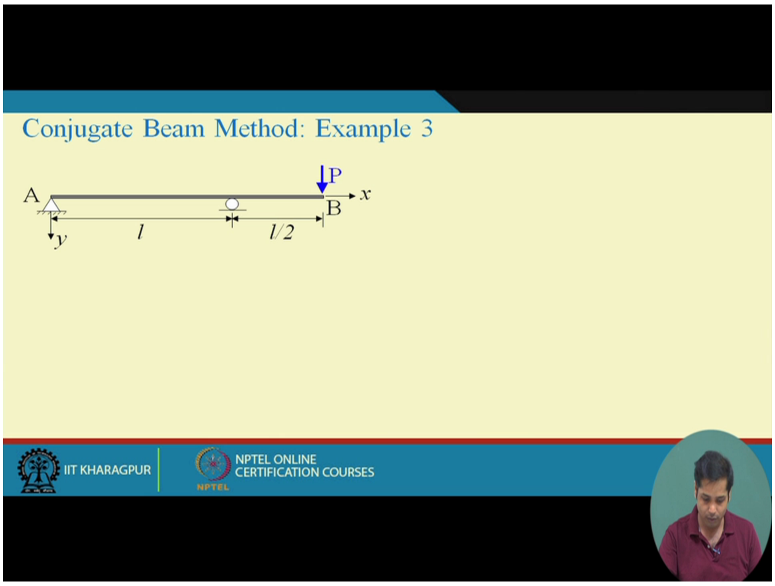

Now so this is the problem. For instance say this point is C, okay. Now let us first find out the bending moment diagram.

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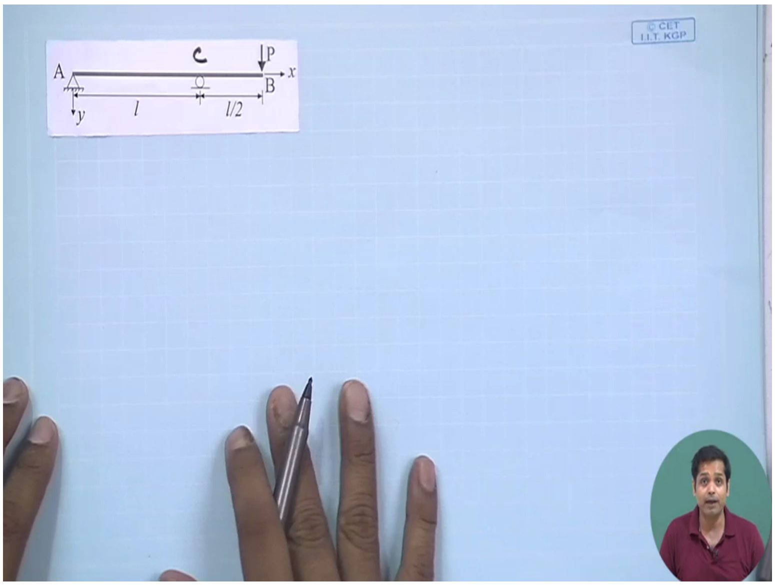

So you see all the methods that we have been discussing, the bending moment diagram plays a very important role so you must be very comfortable in drawing bending moment diagram of any beam. And before you draw the bending moment diagram try to understand what would be the bending moment diagram just by looking at beam based on your intuition and then check whether your intuition is consistent with the results you obtain from structural analysis or not, okay.

So this is the real beam. Now draw the bending moment diagram. Without any calculation how can we find out the bending moment diagram? This is point A, this is point C, this is point B, okay. So point A is hinge support so naturally bending moment will be zero here and point B is a tip loaded. There is no externally applied load here so naturally bending moment will also be zero here, okay.

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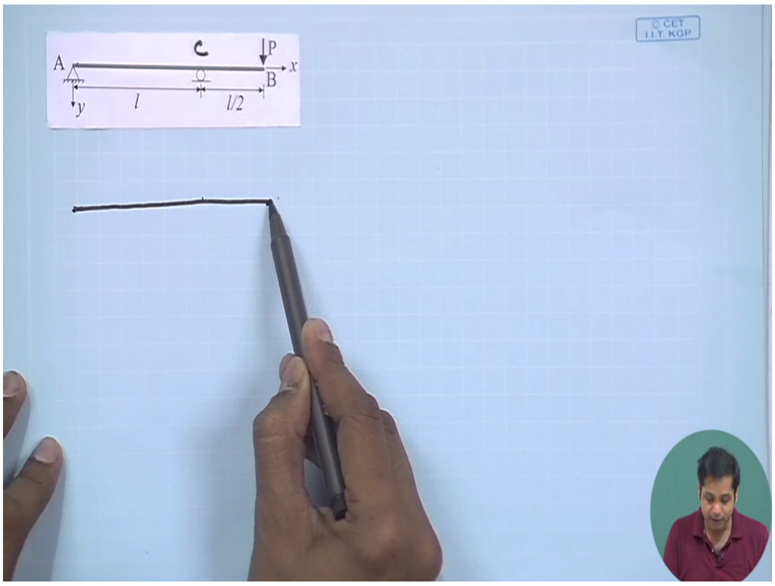

Now in between C and B there is no other load. So if you take any section the bending moment at this section will be due to this force P. So e if you take any section here, the bending moment at this section will be P multiplied by this distance, okay.

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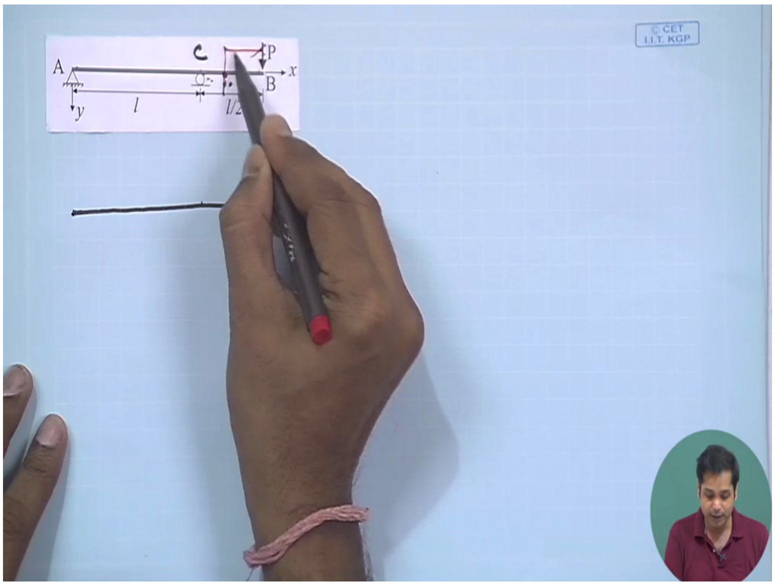

So bending moment is linearly varying between B to C. Now what is the sign of this bending moment? Sign of this bending moment is by just looking at this diagram we can say that the deflected shape at least for this CB part will be something like this.

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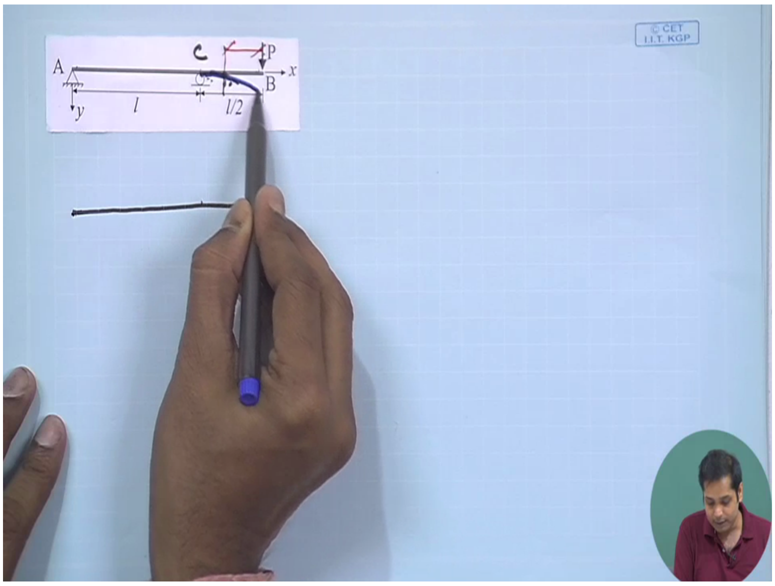

So naturally this is hogging so as per our sign convention this is negative. So between B to C your bending moment varies linearly. And what would be the bending moment at C? It will be PL by 2, okay, because this is L by 2. So this is bending moment diagram.

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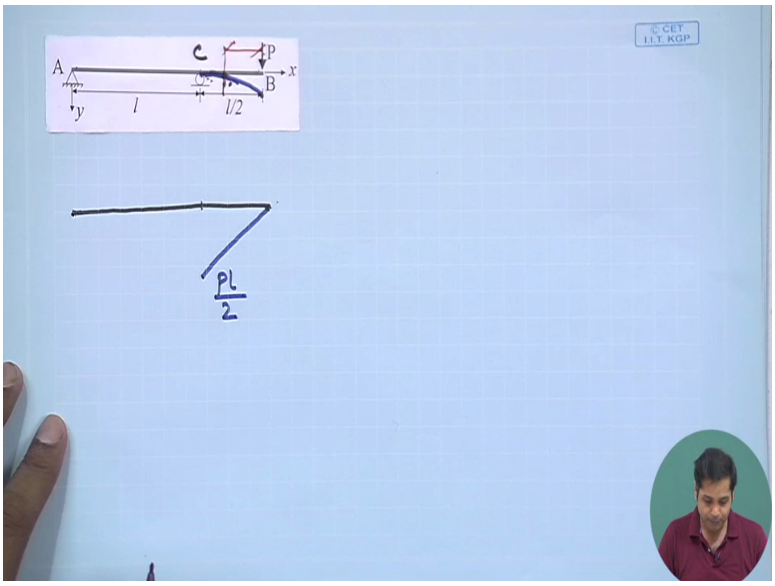

Again between these two there is externally no other load so whatever bending moment if you take any section here the bending moment at that section will be due to the reaction force here.

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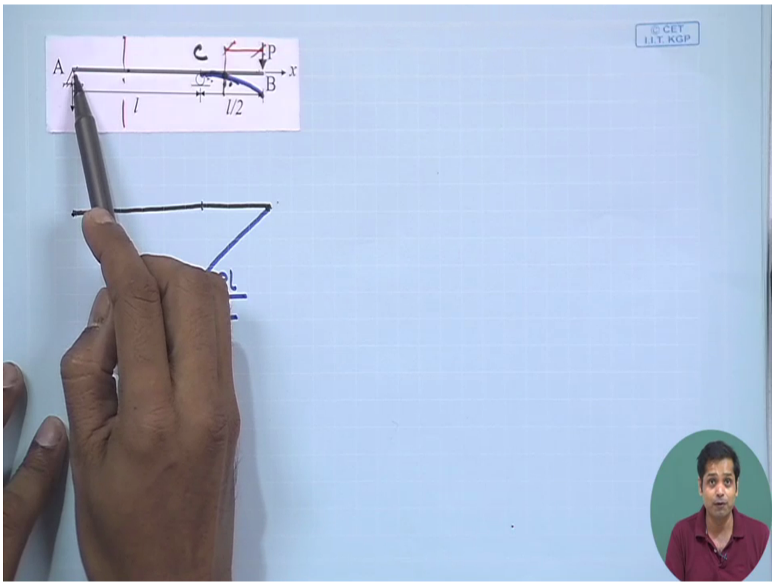

And what would be the (react) bending moment? This reaction force multiplied by this distance.

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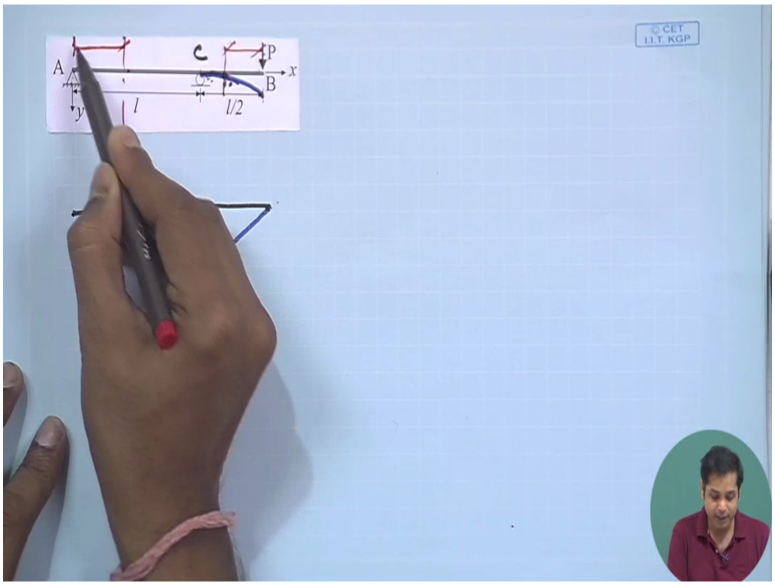

So bending moment will be linearly varying between A and C, okay. And since there is no externally applied moment here, okay, so bending moment will be continuous at C. So what will be the bending moment diagram? Then the bending moment diagram is this. This is the bending moment diagram and as per our sign convention this is negative, okay.

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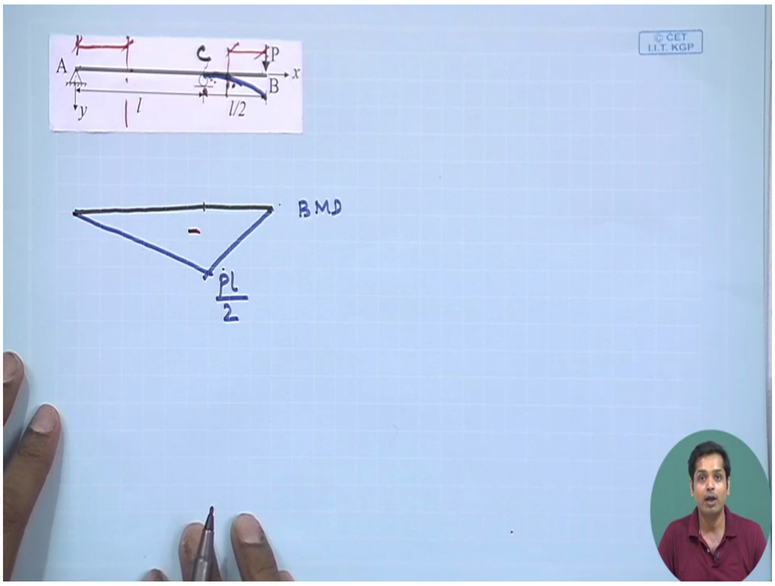

So without any calculation we could draw the bending moment diagram for this, okay. Now let us construct the conjugate beam. Conjugate beam will be real beam. The length remains same as the real beam. This is the real beam, okay.

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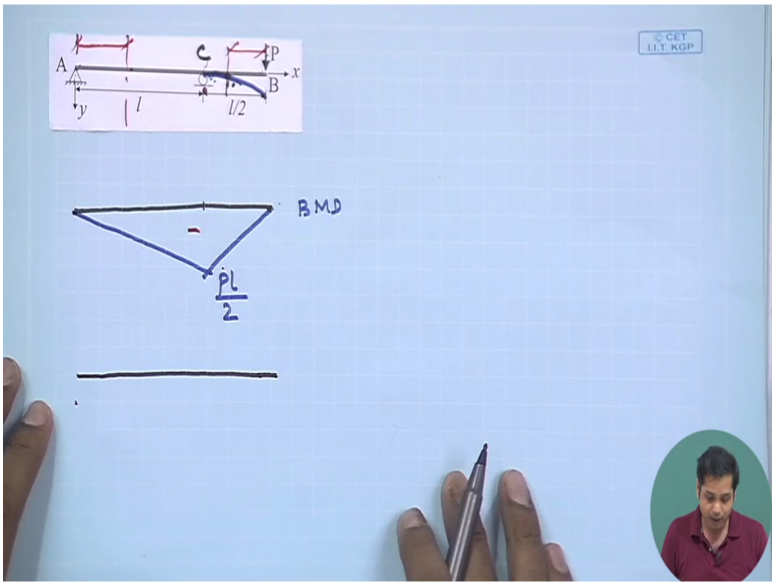

And the next step is the bending moment curvature diagram of the real beam will be the loading on the conjugate beam. So curvature diagram will be this by EI. So loading will be like this. This is PL by 2 EI and this is upward because bending moment is negative, okay.

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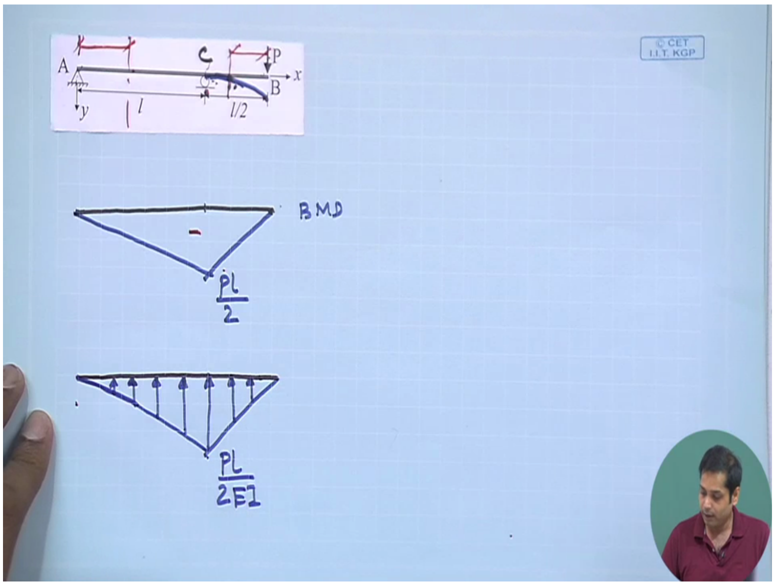

Now another important part is the support condition. Again go back to the support condition, the list of support condition what we have? That if the free end becomes fixed, fixed becomes free and if you have any internal support that internal support becomes hinge, okay.

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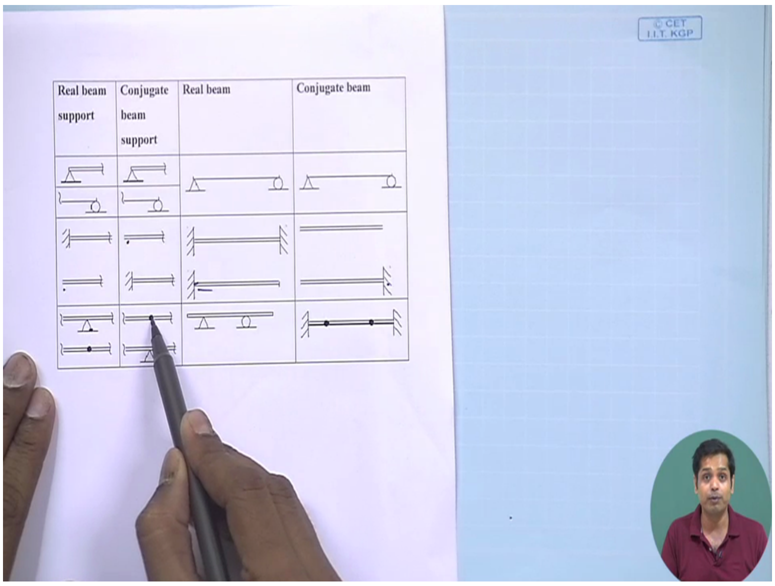

So let us keep here. So this is free end so this becomes fixed and this was hinge so it remains hinge.

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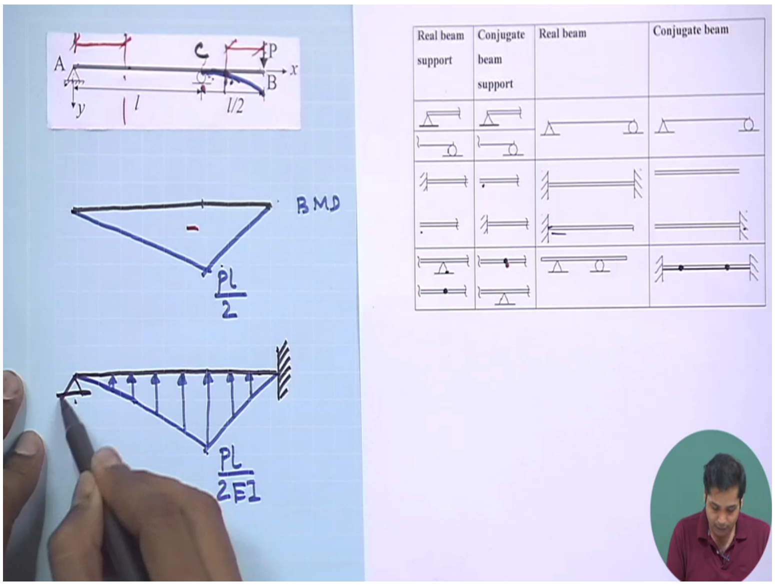

And there was an internal support here so any internal support it becomes hinge in the conjugate beam. So we have one hinge at this point, an internal hinge at the conjugate here, okay.

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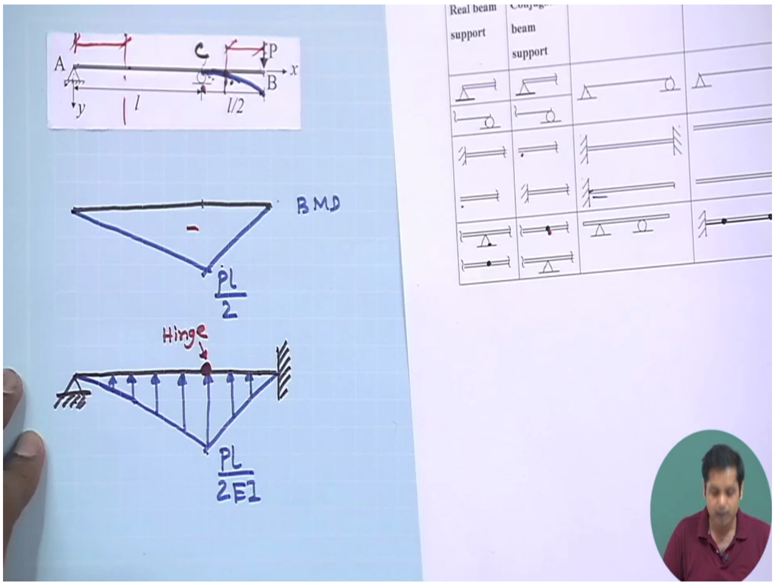

Now we have a beam. This is A, this is C and this is B. So in this ABC which is subjected to this kind of loading and their support conditions are given here, okay. And again the problem of kinematics now reduced to problem of statics. Here we need to find out what would be the internal forces.

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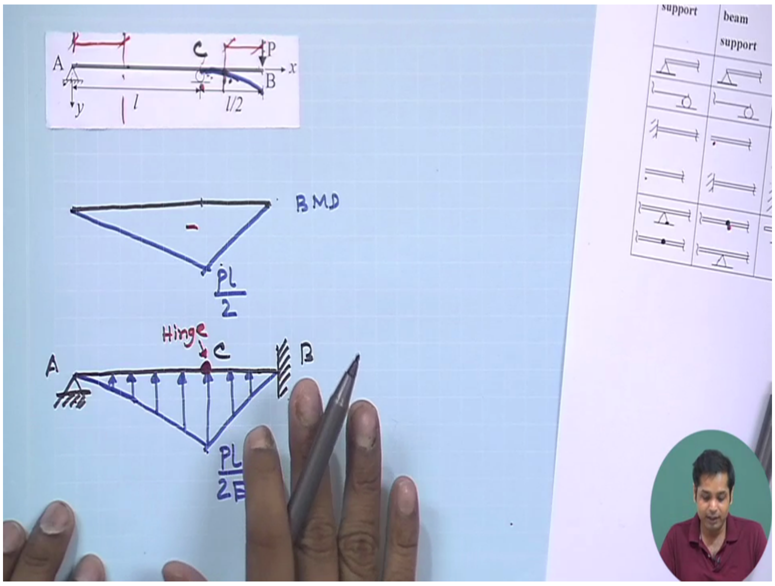

Now since we are asked to find out slope and deflection at theta what we need to find out is what is the shear force and bending moment at point B? Now you can take the free body diagram of part AC separately, free body diagram of CB separately, apply the equilibrium condition and find out what is the bending moment. You know this so we will not do this exercise here. I will just give you the final result.

Final result is delta B that was we are asked that is equal to PL cube by 8 EI. This is the final result, okay. So this is acting downward so this is positive.

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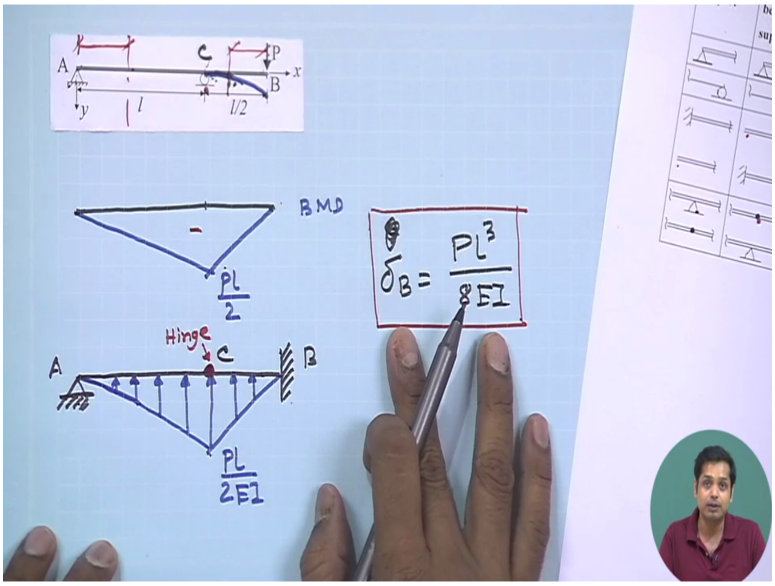

So your job is to please verify this, okay. Now let us see what would be the possible deflected shape of this beam. Now let us find out what could be the deflected shape here. You see this part will go down because this will behave as regular cantilever beam subjected to tip load.

This will go down and there is no load here so since this will go down in order to maintain the continuity this beam will deflect like this. So if you see, yes. So this part will go down and at this point slope has to be continuous and you have to maintain the continuity and this will go down, okay. This will go down.

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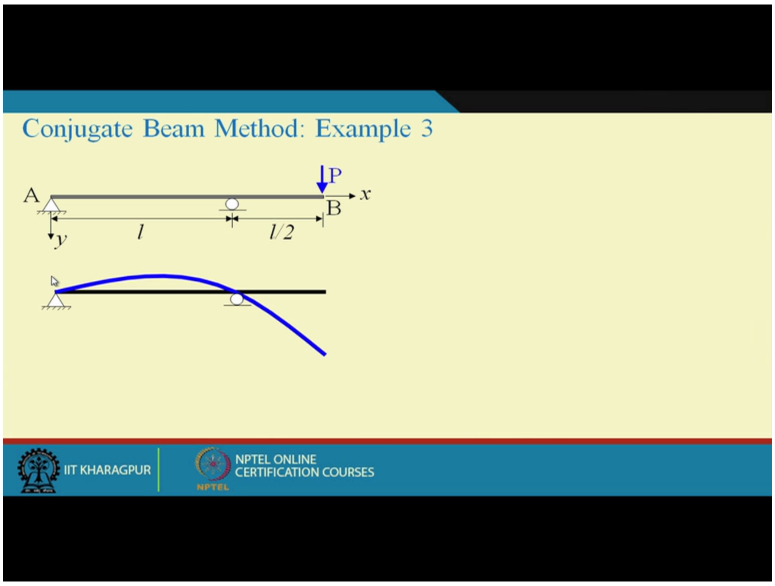

Now this is the deflected shape. Again for any problem it is good to see the deflected shape as well. Now next problem is we have one more problem and this problem is this, okay. The deflected shape is already shown here.

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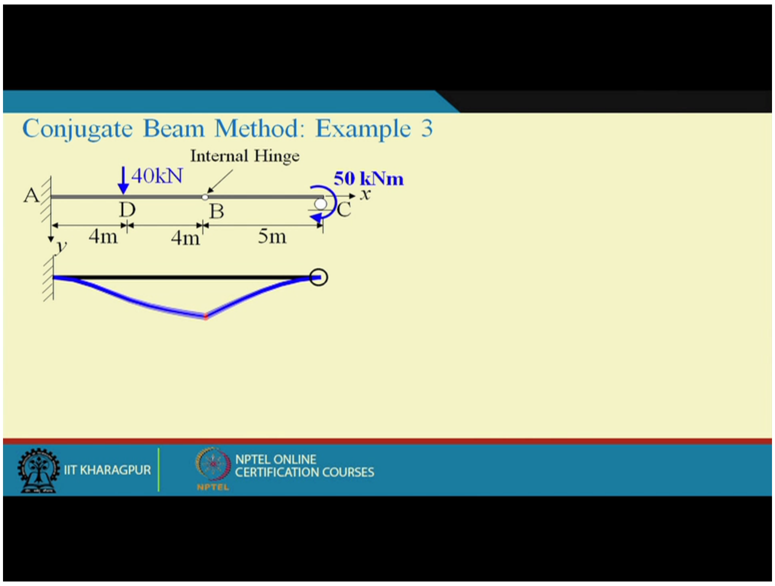

Now the problem is this, okay. Now you remember in one of the earlier lecture we introduced the concept of superposition. Since linearity is one of the important assumption in all our analysis we assume that the relation is linear, the stress strain relation is linear.

So what you can do is we can divide the problem into small problem and find out the response of each sub problem separately and then combine them to get the response of the actual problem. And (com) combine them means just adding them by algebraic addition will give us the response of the actual system.

And this step is sometimes very useful because suppose in a structure if you have a very complicated loading condition there are different kinds of loading so what you can do, but if you look at each load separately then probably those analysis with those loads are not difficult. But since they all are acting together that is why the problem seems to be very complicated or the calculation seems to be very tedious.

So this is one example where we can show how we can demonstrate two things, one is the principle support position and also the conjugate beam theory, okay. Now suppose this is the problem.

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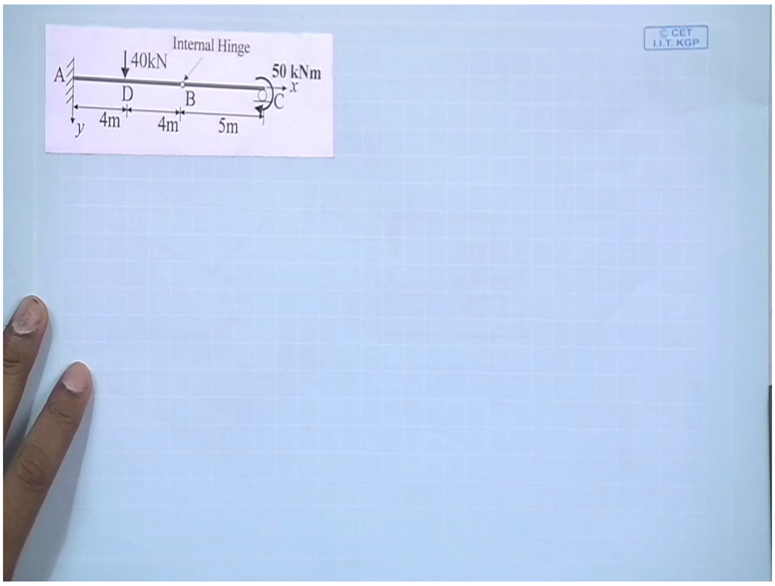

Now it is a fixed end and then this is roller support and one internal hinge here and then lengths are given. This problem is taken from the structural analysis book by Hibbeler. So this is the load, okay. And the interesting thing is you check there is an internal hinge here at B, okay.

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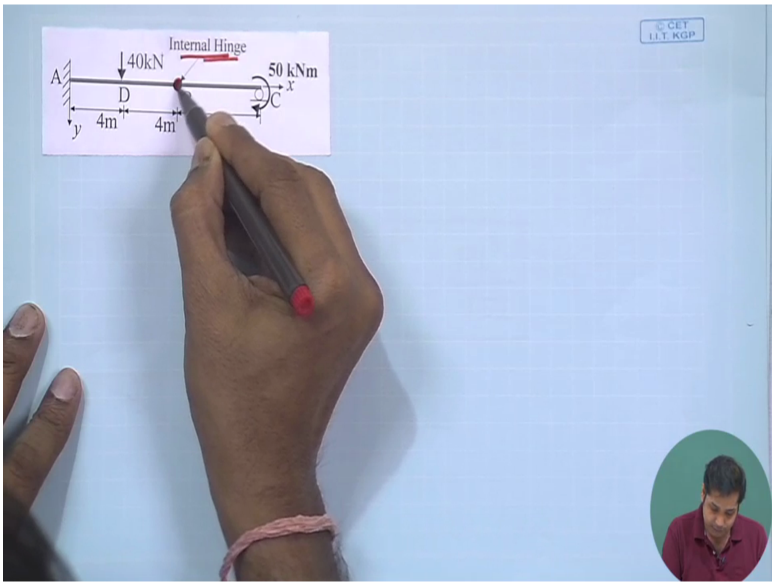

Now what we do first? We first decompose the problem into two. How we can decompose the problem into two? First is the same beam with 40 kilo Newton load and then again the same beam with this 50 kilo Newton per metre moment, okay.

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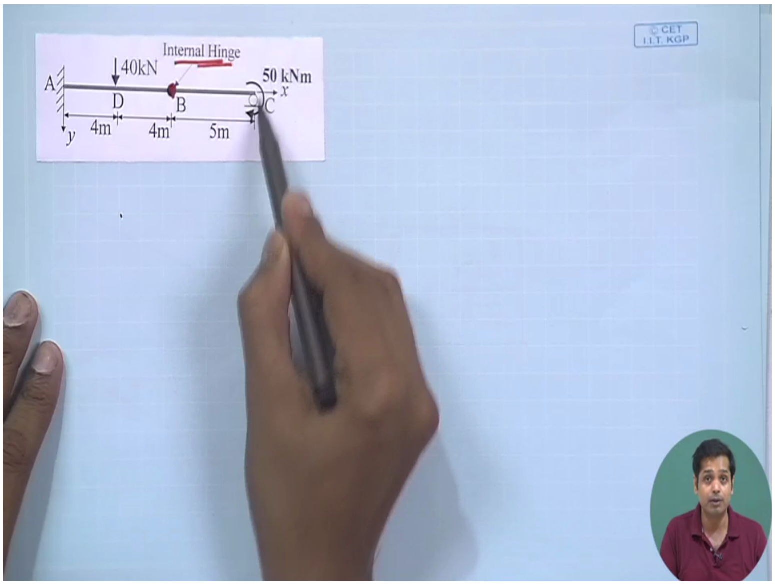

So the first part is the same beam, one end fixed and then we have an internal things here and then roller support and then externally applied load 40 kilo Newton, okay. And this is A, B, C and this D. This is first case.

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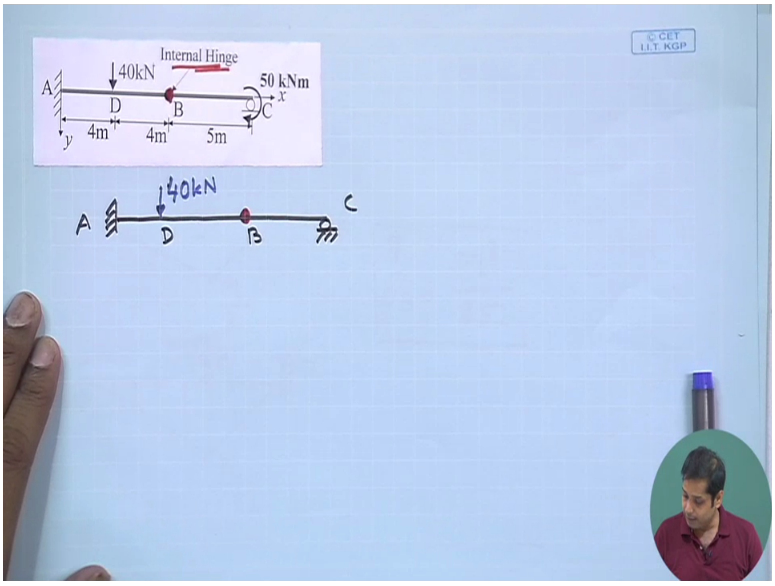

And the second part is again the same beam, same support condition and then another internal hinge here. This is A, B, C and at C we have additional moment say this is 50 kilo Newton, okay.

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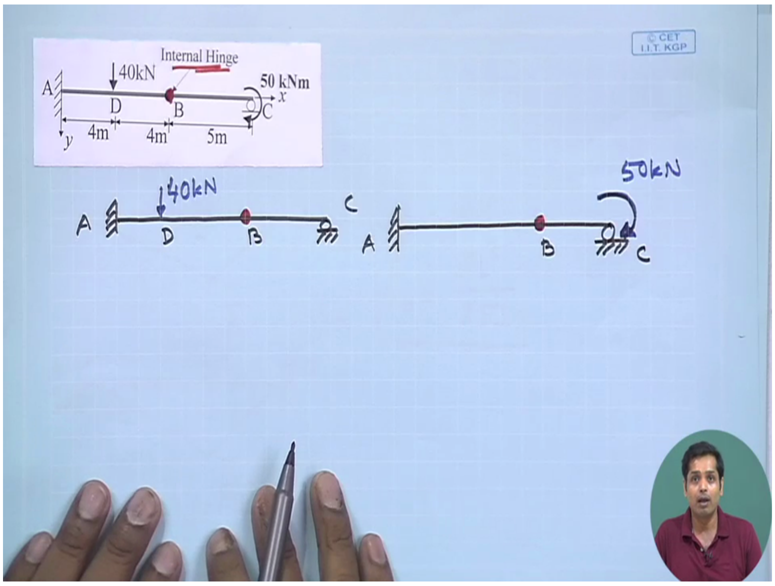

Now so this will be equal to this plus this. What you will do is now find out what is the deflection at B? And so first from this we have to find out the deflection at B, from this we have to find out deflection at B and then if this is system 1 and this is system 2 and then delta B1 plus delta B2 will be equal to delta B of the real structure, okay.

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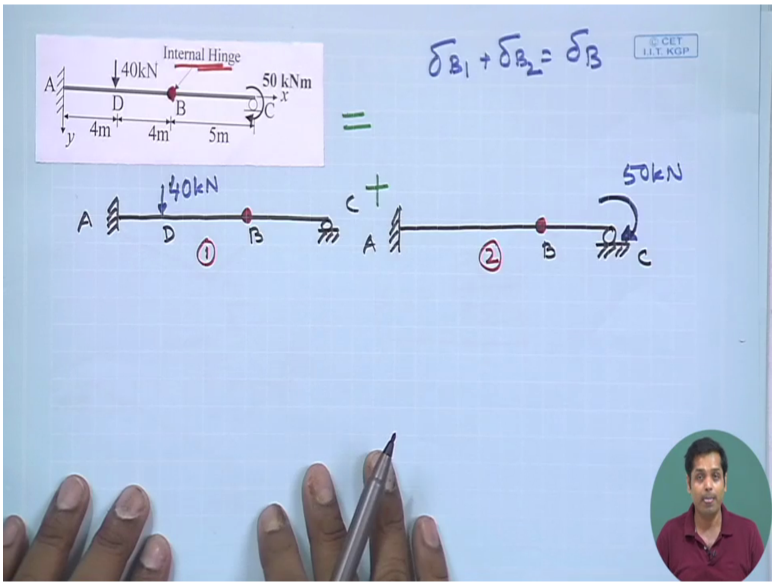

Now apply the conjugate beam method. Before we apply the conjugate first thing is we need to find out what is the bending moment diagram. So I know you can do the calculation to find out the bending moment diagram. Let us just by looking at the beam get some idea about the bending moment diagram, okay. Now so first draw this beam, okay. Let us first start with this. This is the beam, okay.

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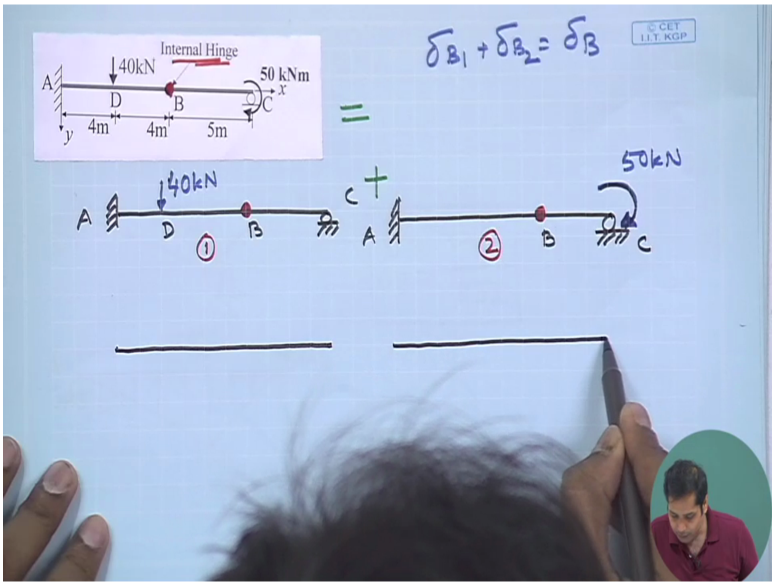

Now let us see what is the bending moment diagram for this case? At this point the bending moment is 50, okay.

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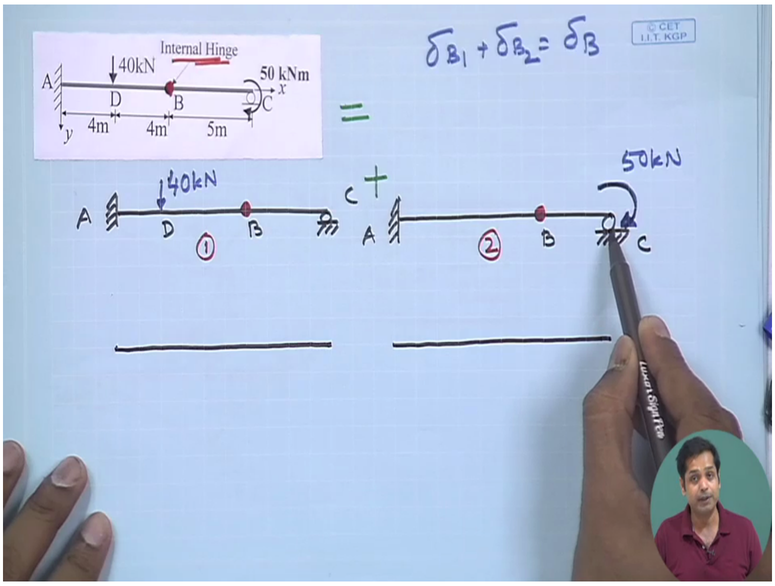

And as per our sign convention this is hogging moment. So at point C bending moment will be this. This is minus 50 or minus means it is downwards, okay.

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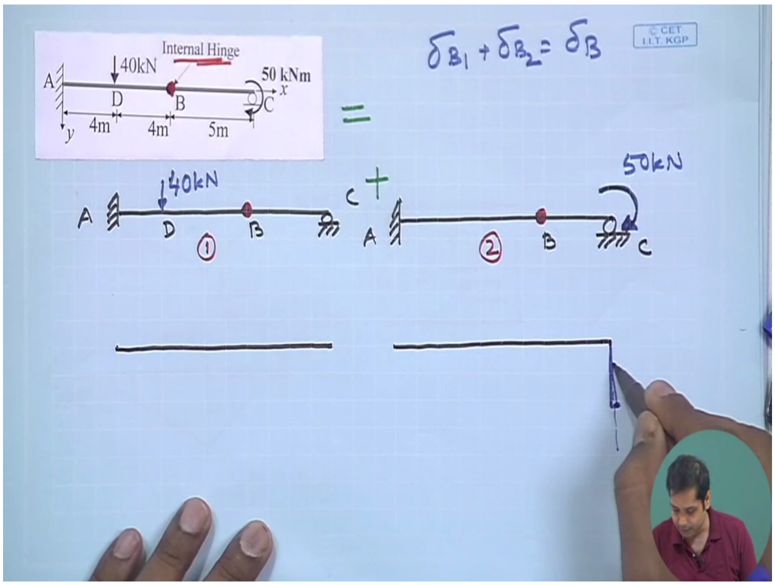

Now you see between B and C there is no other externally applied load here, okay.

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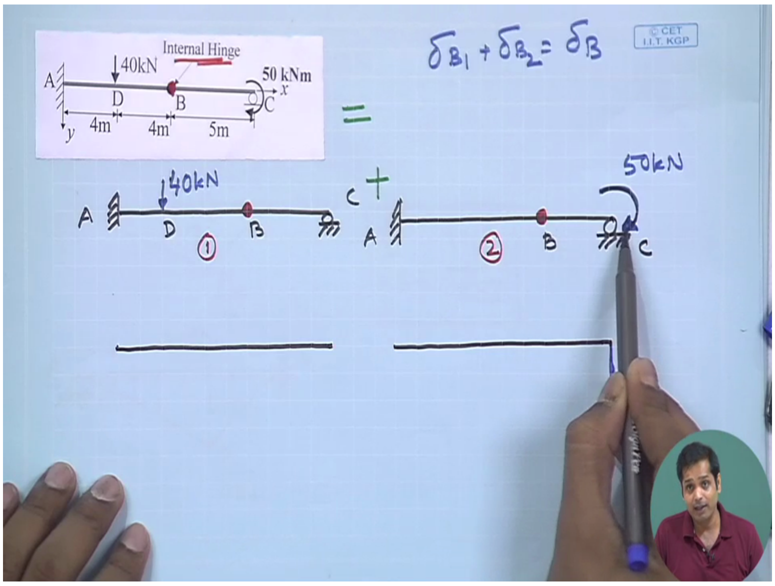

And also we know that bending moment at B will be zero because it is hinge, okay. So then if you take any section then at any section the bending moment (fa) what are the forces that will cause variation of bending moment between B and C? This is the initial moment at B and then you have a support reaction and then support reaction will decrease this moment and eventually at point B your moment will be zero.

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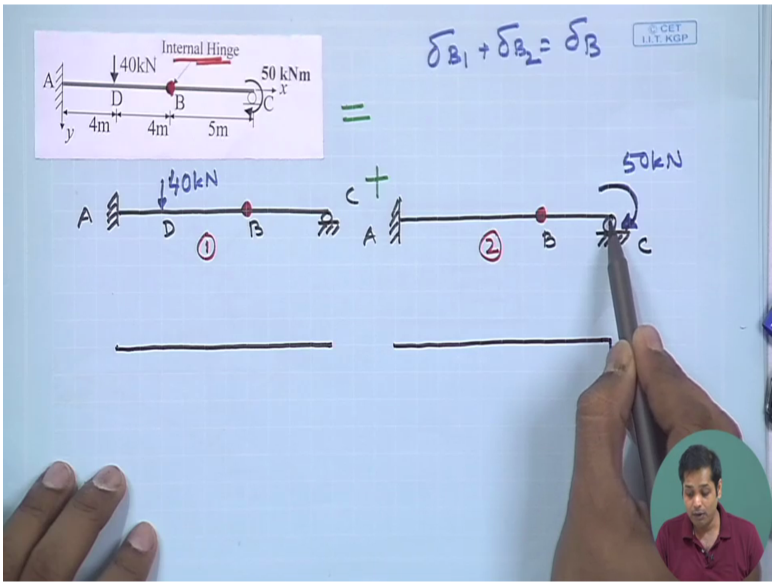

So up to point B the moment is zero, okay. Now again between A to B there is no other load, okay. And there is no externally applied moment also so the slope of the moment will be continuous. There is no additional external other load here.

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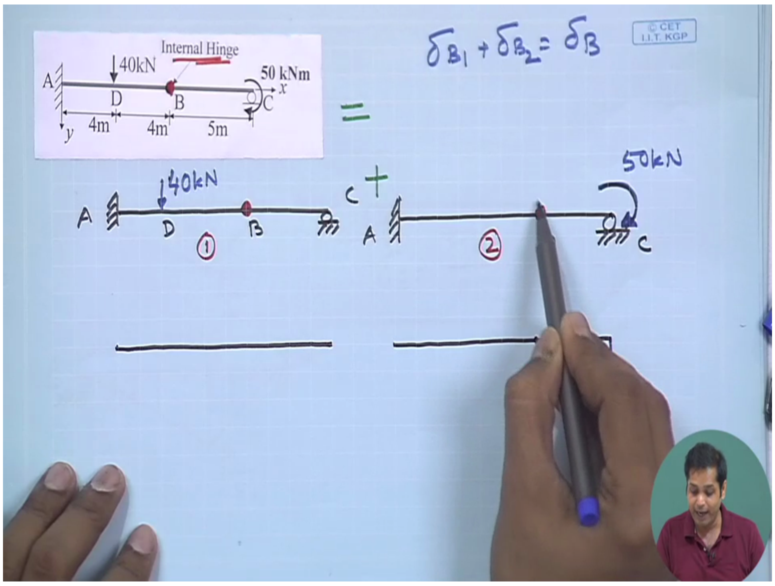

So what will be the bending moment diagram? Bending moment diagram will be something like this, okay.

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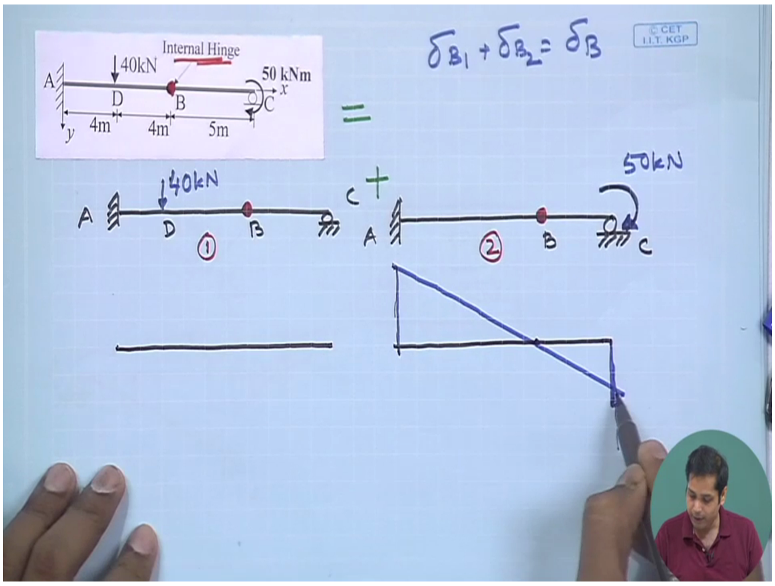

This will be 50 and I am not writing the calculation this will be 80, okay.

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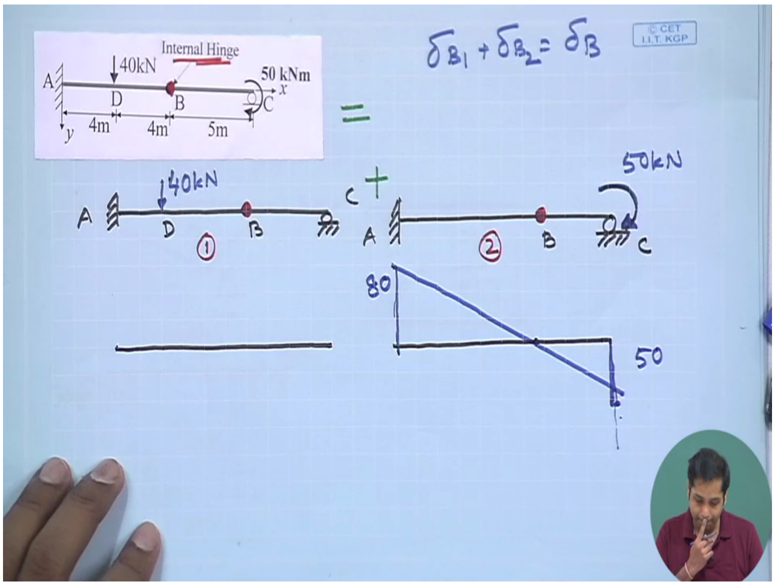

Now this you can calculate actually this length is 50 so anyway. This length is 5 metres, this length is 8 metres.

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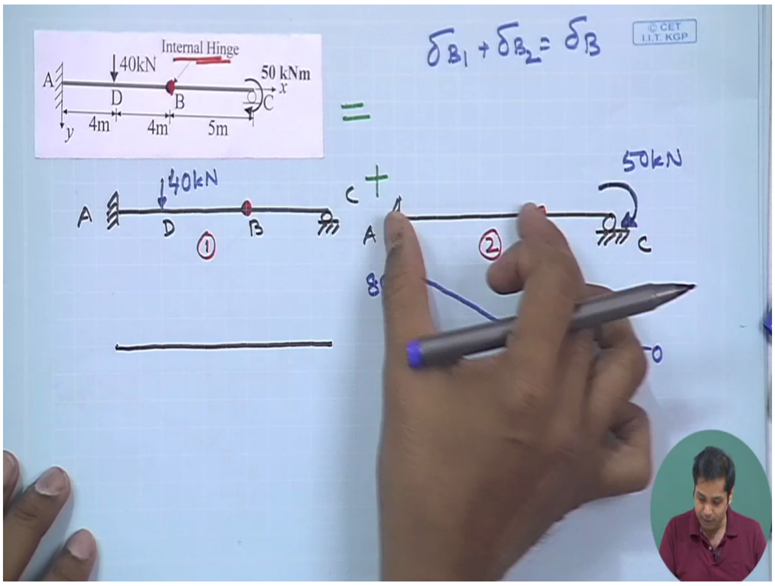

So this is the bending moment diagram for this. Now let us find out what is the bending moment diagram for this. Now if you take BC separately you see there is no other external load here. So what will happen, the reaction at C will be zero.

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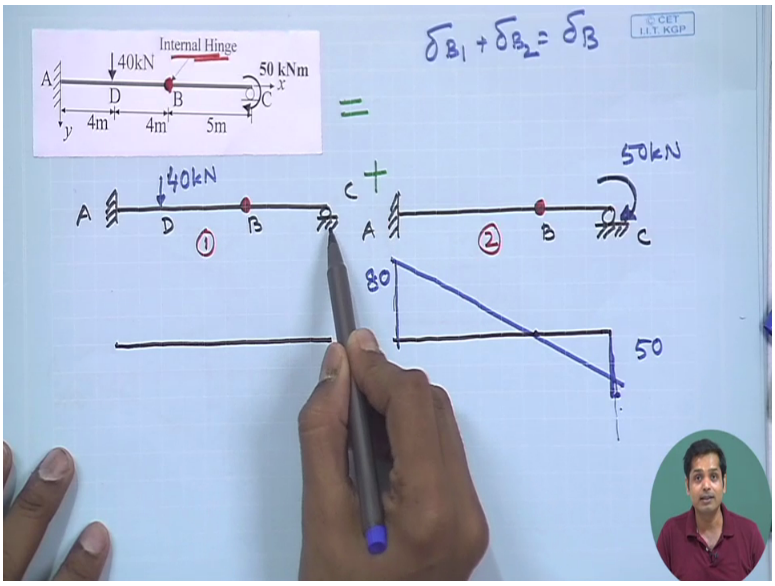

Now reaction at C will be zero means there is no bending moment in BC. Even between BD also there is no other load. Reaction at B is zero, C is zero and up to C and D there is no other load. So naturally between C and D there will be no load. And you have load 40 kilo Newton at D and between A and D there is no other load so the bending moment at any section will be caused due to this load. And the bending moment will be this load multiplied by this distance.

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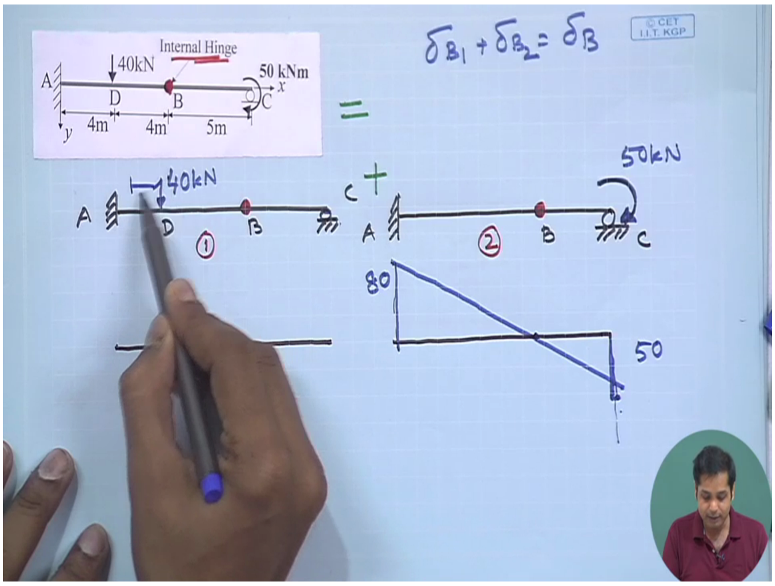

So bending moment will be linearly varying between A to D. And what would be the sign of the bending moment? This beam will deflect like this.

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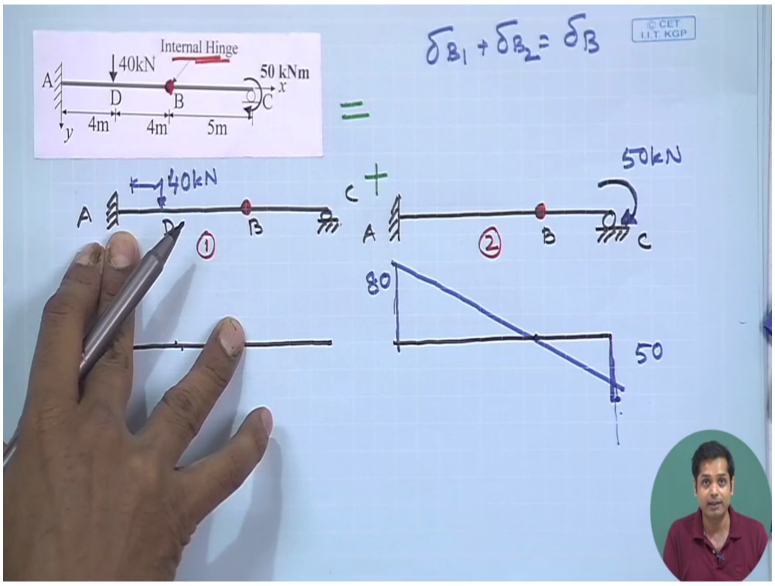

So bending moment will be hogging bending moment. So this is bending moment diagram between D and this length is 40 kilo Newton and this length is 4 metre.

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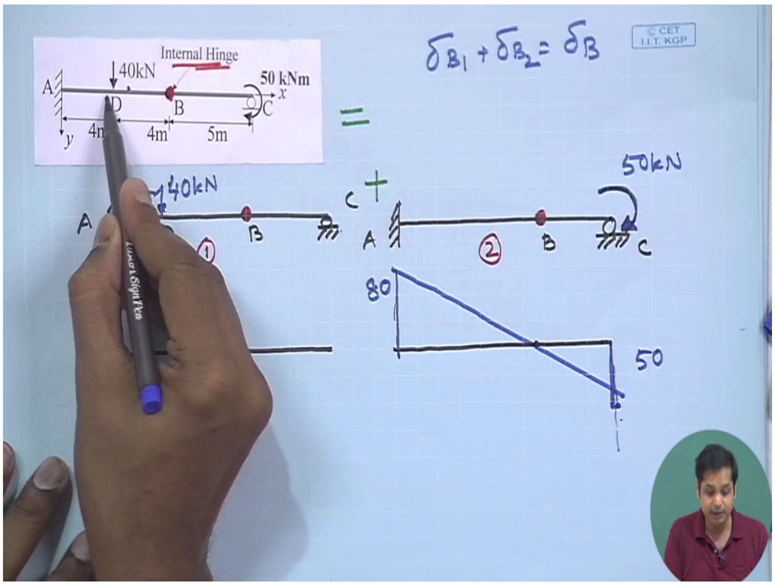

So this bending moment will be 160 kilo Newton metre, okay. So this is bending moment diagram for this part, this is bending moment diagram for this part.

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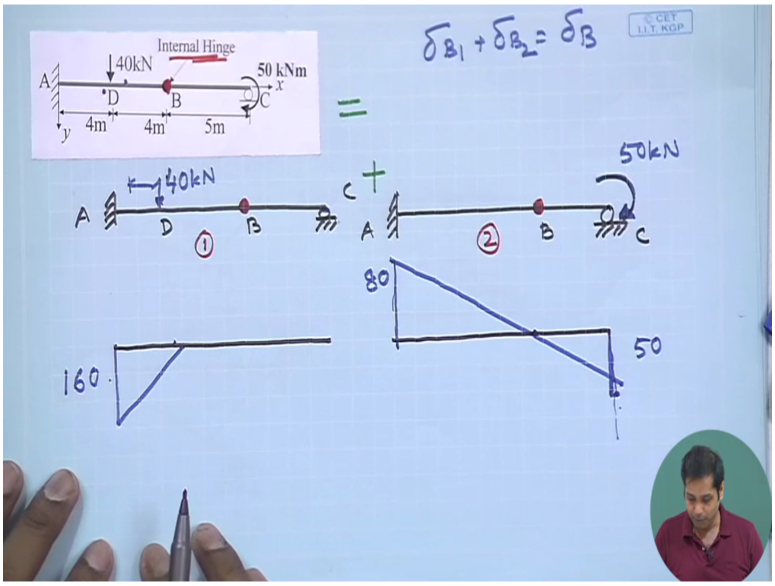

Now let us find out the conjugate beam. The first step is the loading on the conjugate beam will be the curvature diagram of the real beam. So loading on the conjugate beam will be, this becomes 80 by EI and this becomes 50 by EI, okay.

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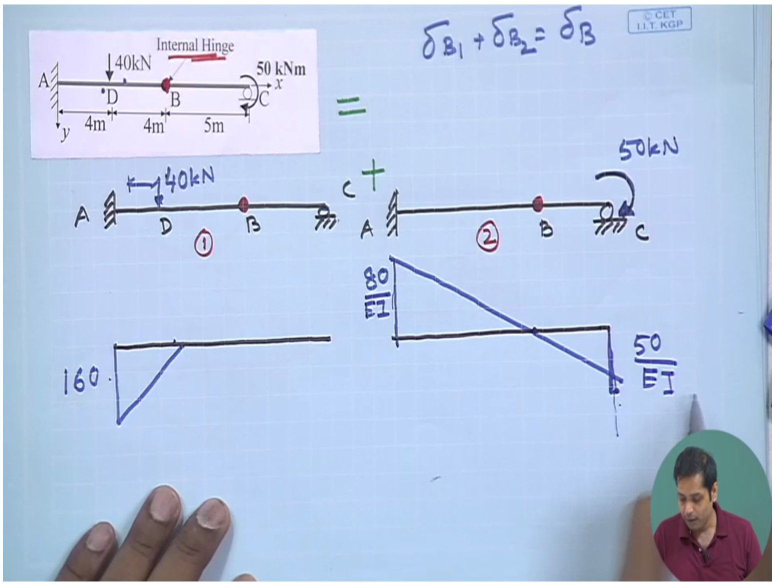

This is positive so this will act downward and this will act upward, right?

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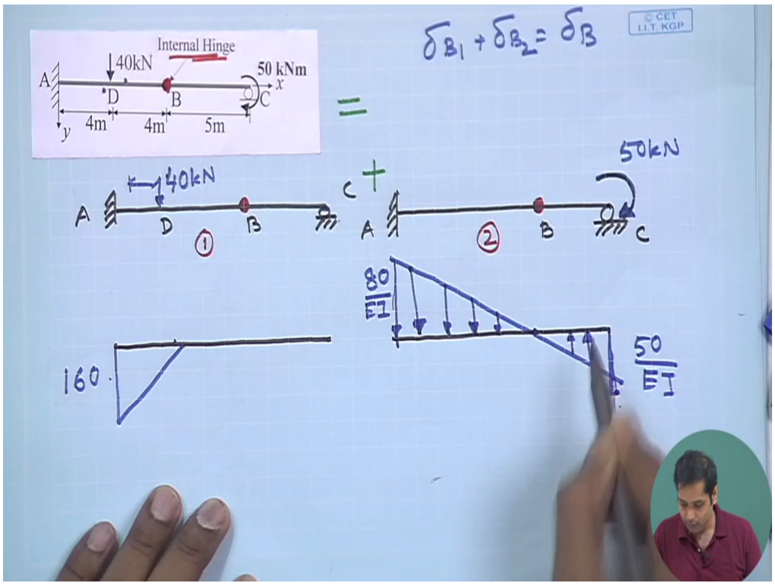

And what would be the support condition? Again find out the support condition from here. If you see the roller support remain as roller and then if you have internal hinge then internal hinge becomes internal support and the fixed support becomes free.

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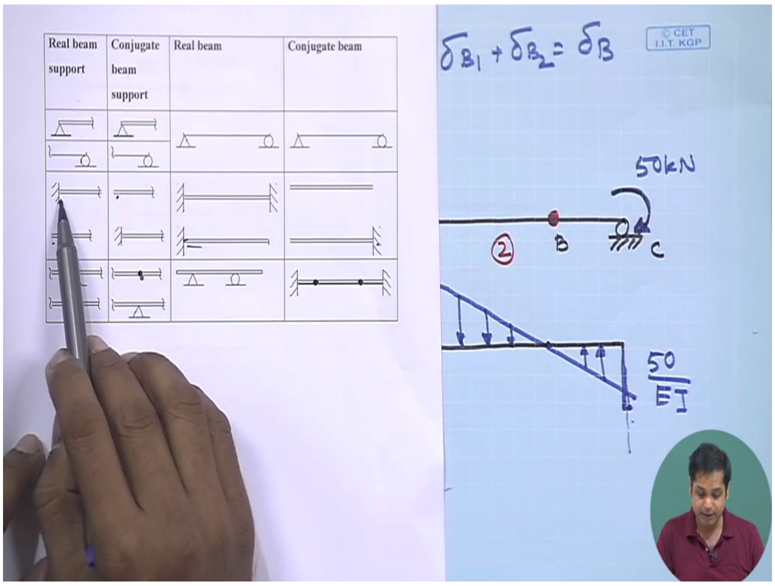

So the support condition will be this internal hinge becomes roller.

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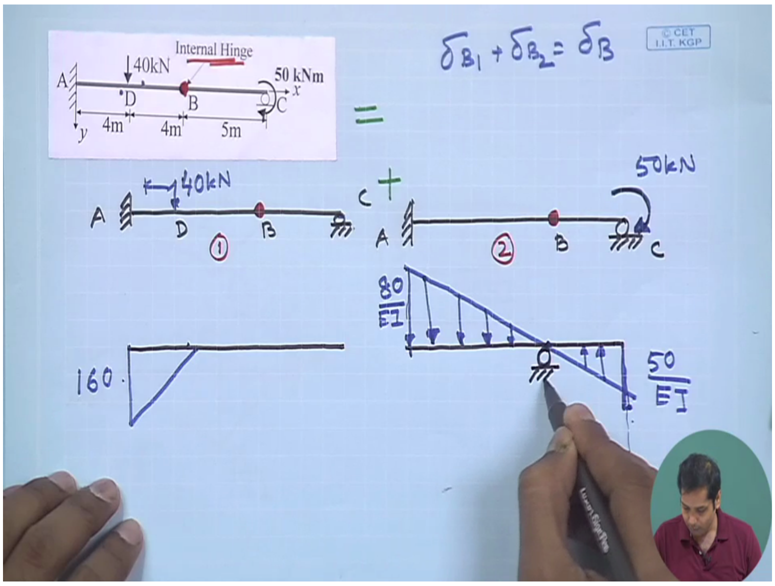

This is fixed so this will be free and this remain as roller, okay.

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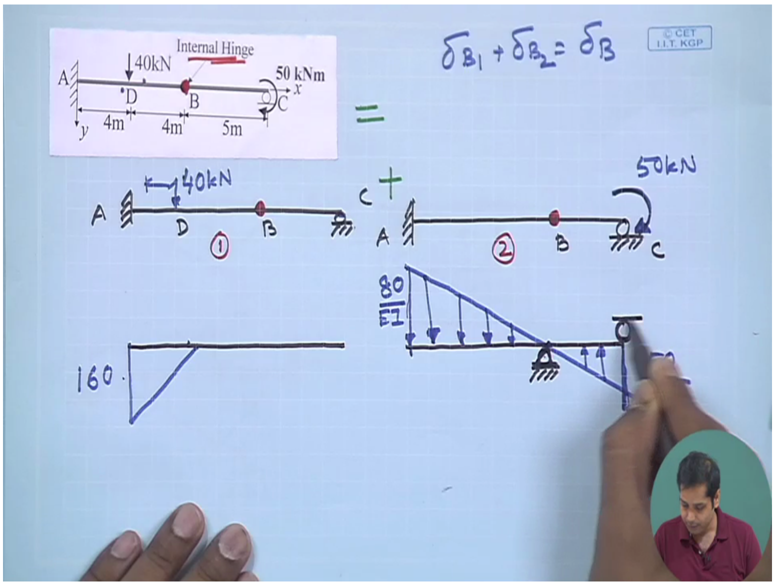

This is the support condition, okay. Now let us see what is the support condition for this? The support condition for this will be again same. So this is roller and this internal hinge become external intermediate hinge support and its fixed end becomes free.

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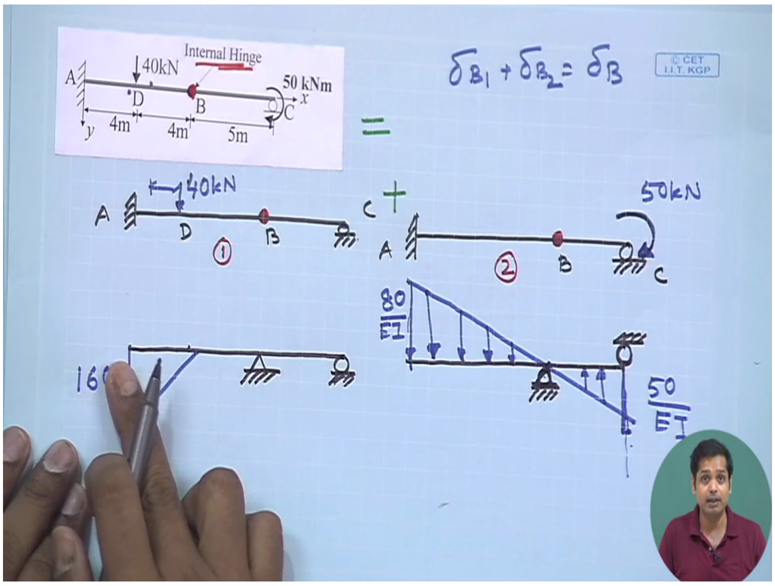

And what would be the loading? This loading will be upward direction, okay.

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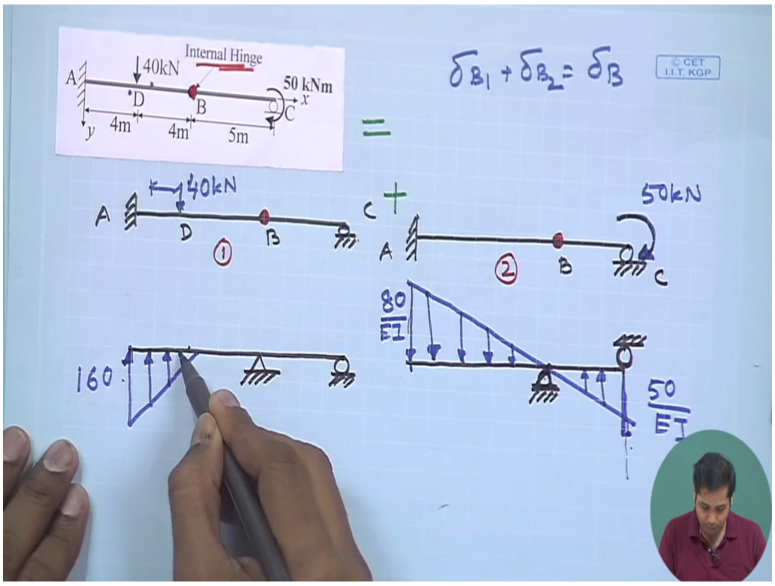

Now what we need to find out? We need to find out what is delta B. So we to find out what is delta B1 from here and then what is the delta B2 from here.

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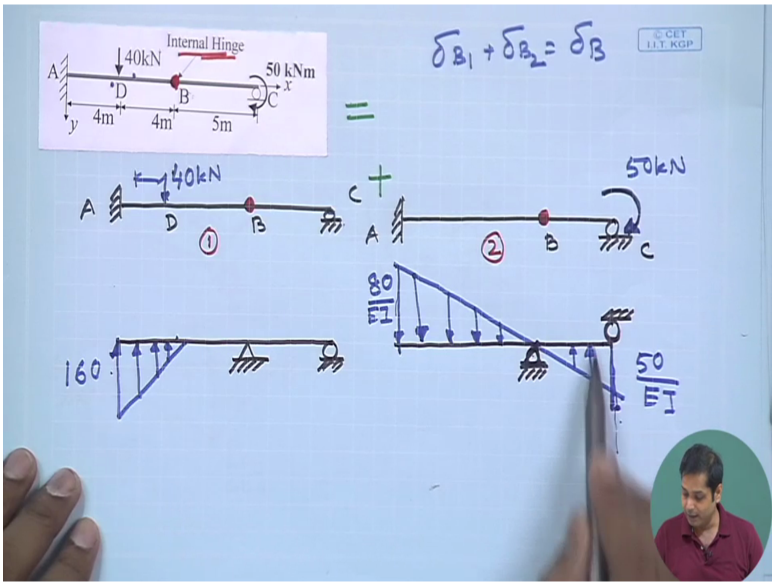

So now this is A, this is B, this is C, A, B and C, okay. Now we need to find out what is MB here. And from this also we need to find out what is MB here, okay.

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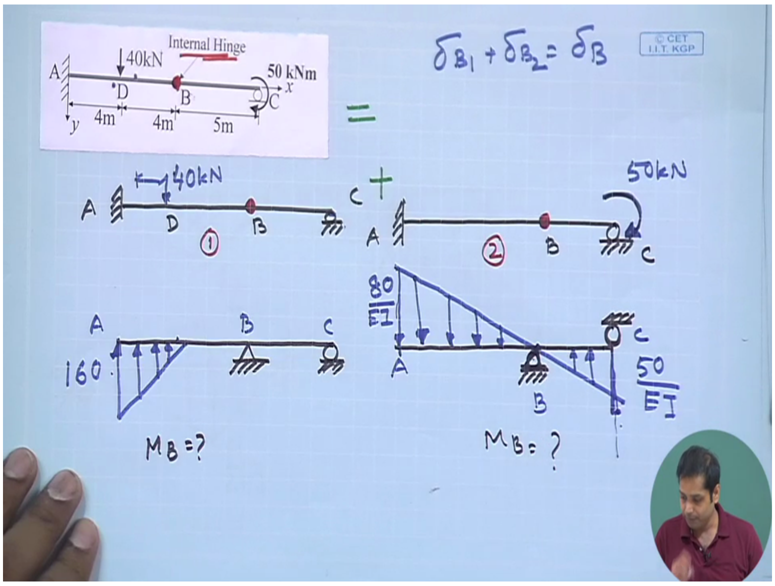

Now what is MB? Now this MB will give us delta B1 and this gives us delta B2, okay.

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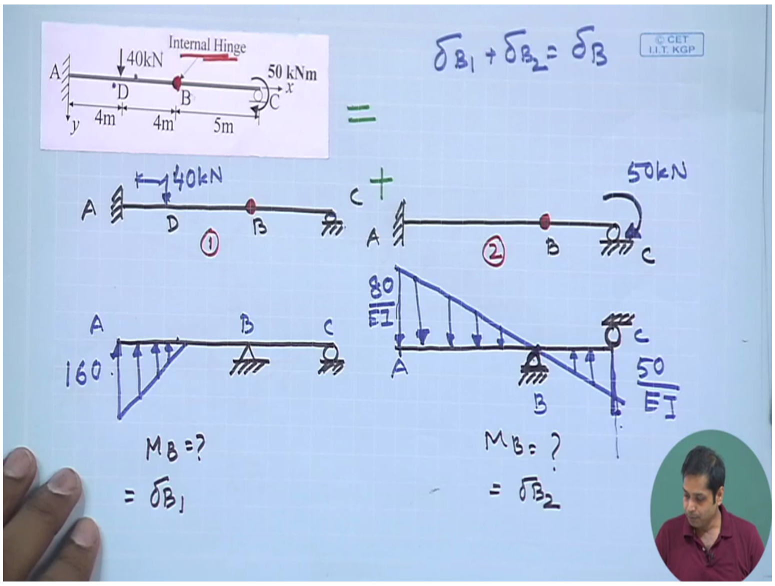

Now then add delta B1 and delta B2 we will get delta B. And I will just give you the final result. Let us give you the final result so you can find out what is delta B1 and what is delta B2. Delta B becomes 427 by EI.

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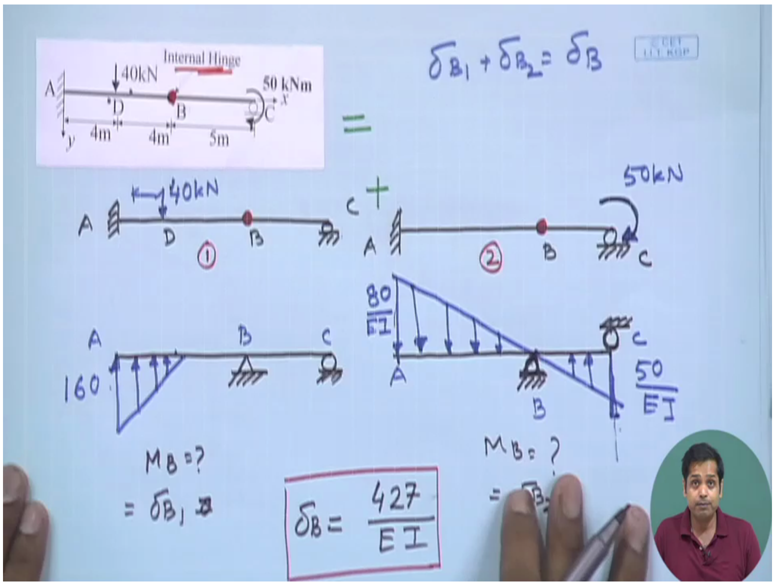

So now what is the advantage of dividing (divi) the problem into two? In both the cases the bending moment becomes very simpler, okay. Without that also we could determine what is delta B but in that case bending moment diagram would have been this plus this and then the variation in the same length will be more. That is why probably this approach the calculation becomes easier, okay.

So please find out what is the delta B1 and what is delta B2 and then check whether you are getting this results or not. In addition to that you also find out what is the slope at theta B, okay. Now you see since it is hinge support, the hinge support does not give you any rotational constraint. So not necessarily that your slope is continuous at point B.

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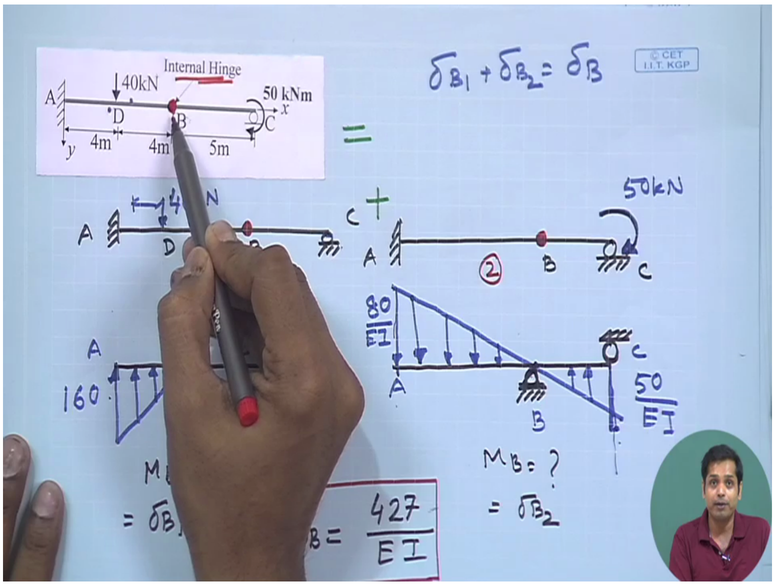

For instance if you see the animation, this is the deflected shape of this beam. Now you see this point is the hinge point and how they behave and if you see the deflected profile, deflection is continuous up to this part. The slope is continuous up to this part.

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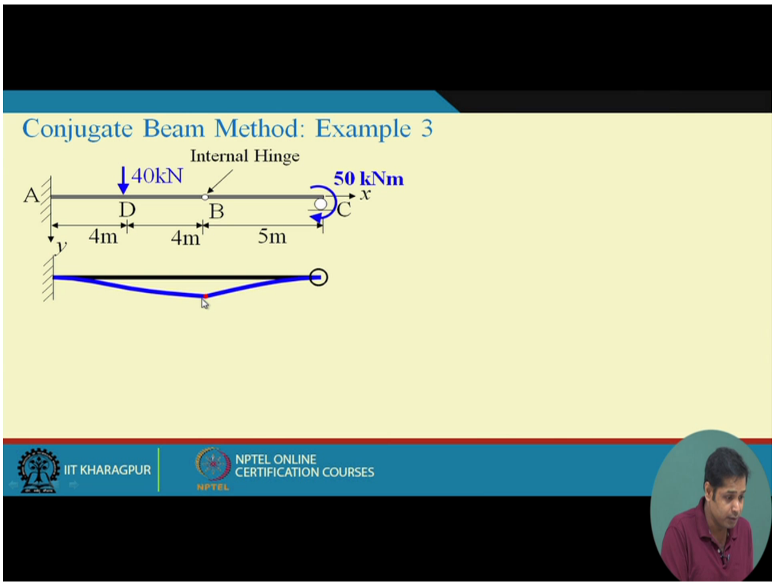

For this part BC slope is also separately continuous but at point B the slope is not continuous. So if you calculate for slope at B for part AB and slope at B for part BC, those two slopes will not be same.

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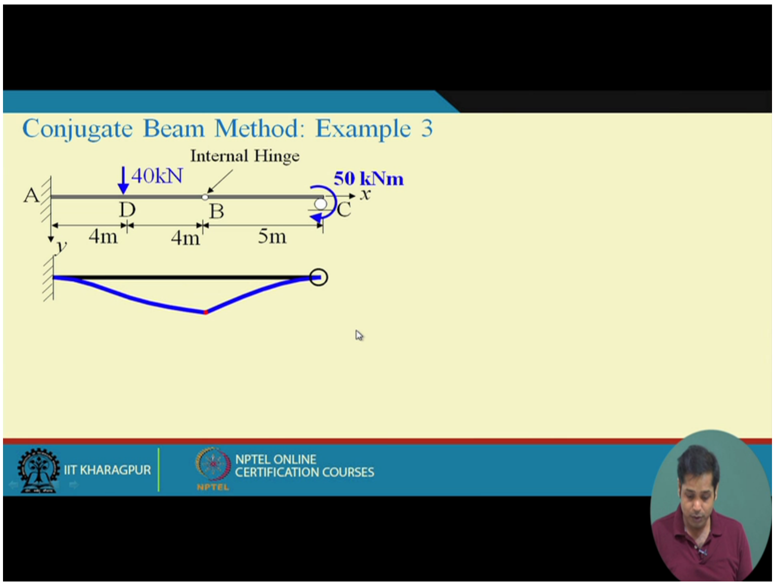

So you find out what are the slopes at same point B due to part BA and due to part BC. And the slope will be if you get the shear force here and shear force here.

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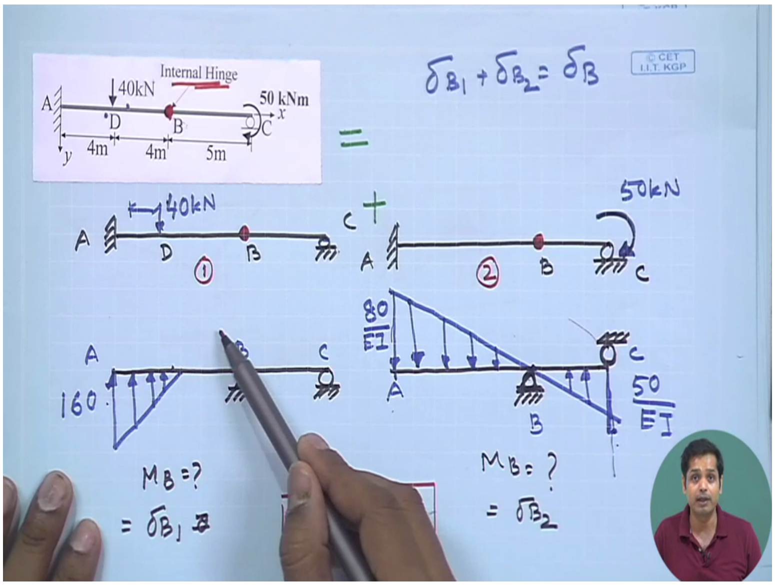

You add this to shear forces and you will get the slope at point B, okay. So these were some demonstrations of conjugate beam theory. So there are again many examples given in the book. Please do some examples and make yourself comfortable with the concept. So next class what we will do is we will briefly review the internal forces in statically determinate frames, okay. See you in the next class. Thank you.