Structural Analysis 1
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Lecture 15
Analysis of Statically Determinate Structure: Method of Virtual Work (Contd.)

Hello everyone, welcome. What we are going to do today is we already discussed what is unit load method and how the unit load method can be applied to truss. Now we are going to demonstrate that through some examples. Okay. The first example that we will be doing is, take this example. It is a statically determinate truss and what we need to find out is deflection at joint C. And the axial rigidity is same for all members. So all the dimensions and loads are given. Okay. So let us start the analysis.

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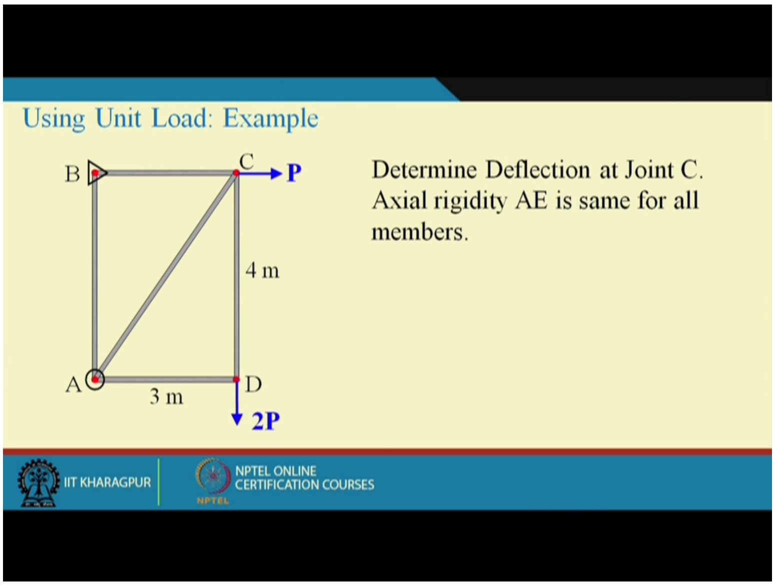

So the truss is like this. This is and then this is member A, this is member B, then C, then D. And then we have horizontal load of P at C and then a vertical load 2P at D. Okay. And this is roller support, this is hinge support and this is roller support here. Okay.

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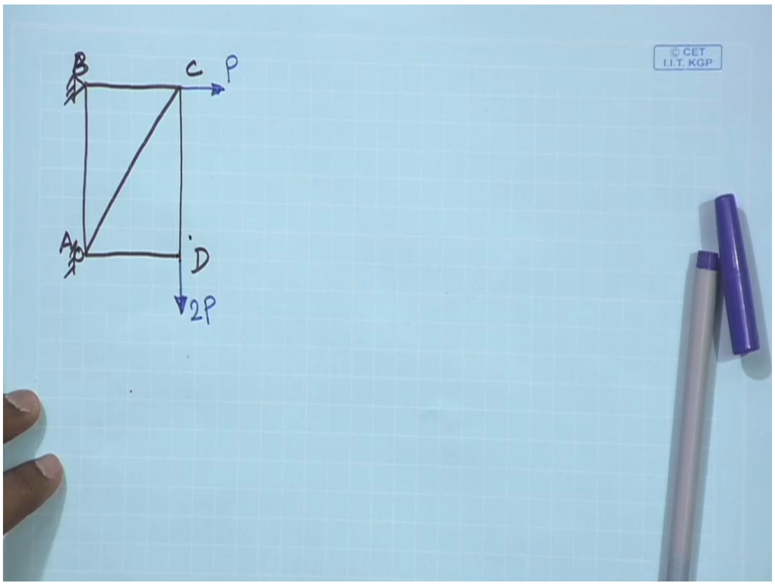

Now the first step in unit load method is first we need to analyze the structure as we have. We need to determine the forces in all the members. Then apply unit in the same structure. Remove all the loads and apply the unit load in the direction at the point where we want to find out the displacement and also the load should be in the direction in which we want to find out the displacement. And then with those two reasons we will get the final displacement. Okay.

Now let us first analyze this. Draw the free body diagram of this member, the entire structure. The free body diagram will be, we have this will be horizontal reaction at Ax, okay? And this will be the horizontal reaction Bx and then vertical reaction By.

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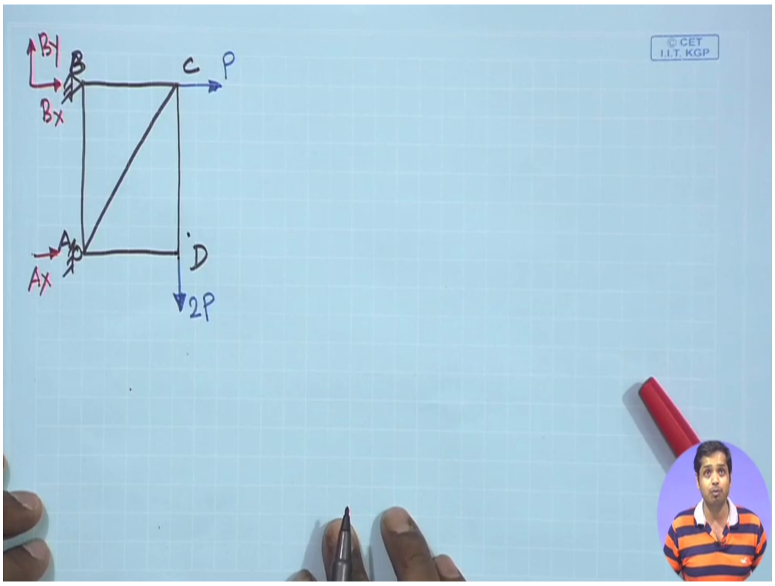

Now if we apply the equilibrium equation, let summation of Fy is equal to zero. The forces we have in Fy direction in 2P and By. We have By is equal to, this will give us By is equal to 2P, okay?

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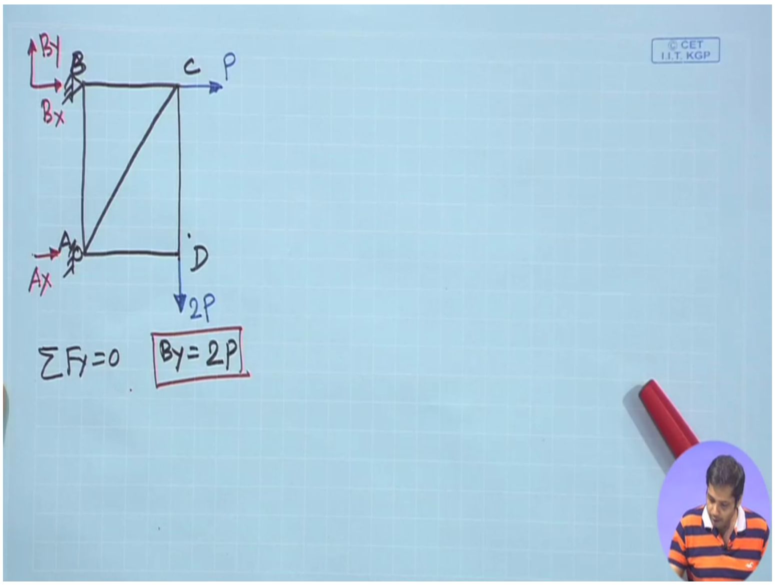

And then we can take summation of moment about A is equal to zero. Again please remember when you draw the free body diagram then the structure needs to be free from the support and that support needs to be represented by the forces. So (in) when you actually draw it if you are showing this reactions here so don't show this supports.

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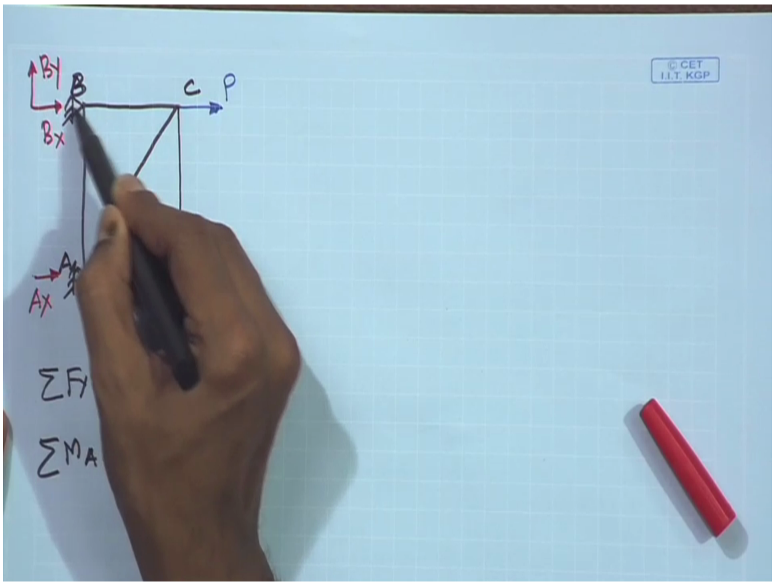

I am showing it because then I don't have to draw the same figure once again. Okay. Now let us summation of moment about A is equal to zero. Then the moment will be we have the contribution will be from 2P which is clockwise and then P which is clockwise and then contribution from Bx which is other clockwise and as per sign convention clockwise is positive.

So what we have is this will give us P into this distance 3 meters, this distance is 4 meters. So 2P into 3 plus P into 4 plus By into 4, this equals to zero. And this will give us By is equal to minus 5 by 2 P. So this is By. Okay.

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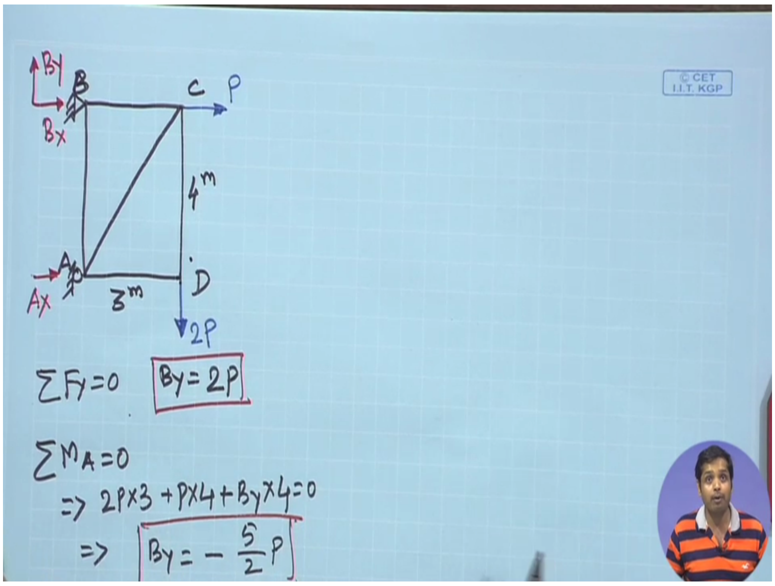

Now then next what we can do is we take summation of, we have used summation of Fy is equal to zero, moment about A is equal to zero, take another equilibrium condition. Summation of Fx is equal to zero and this will give us Ax is equal to the forces we have in x direction is only Bx P and Ax. Their sum will be zero and we will get Ax is equal to 3P by 2. So this is the reaction, all these reactions. Okay.

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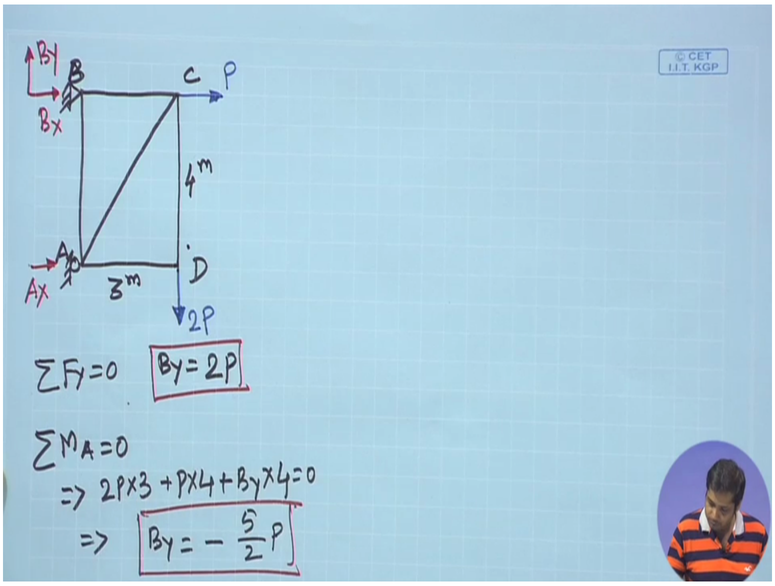

So once we have these reactions we need to find out what are the member forces? Now we have already discussed method of sections, method of joints. We can apply any method to find out the forces in these members. But let us try to find out in, before actually we use any method of section or method of section and do the calculation, just by (inspecti) inspection let us find out whether we can have some information about some of the members. Okay.

For instance if you take joint D then the forces will have the member forces in AD, member forces in CD which is in vertical direction and then in vertical direction 2P. So there is no horizontal force which can balance the horizontal force in member AD. So naturally the force in this member will be zero. So this is a zero force member. Member AD is a zero force member. Okay.

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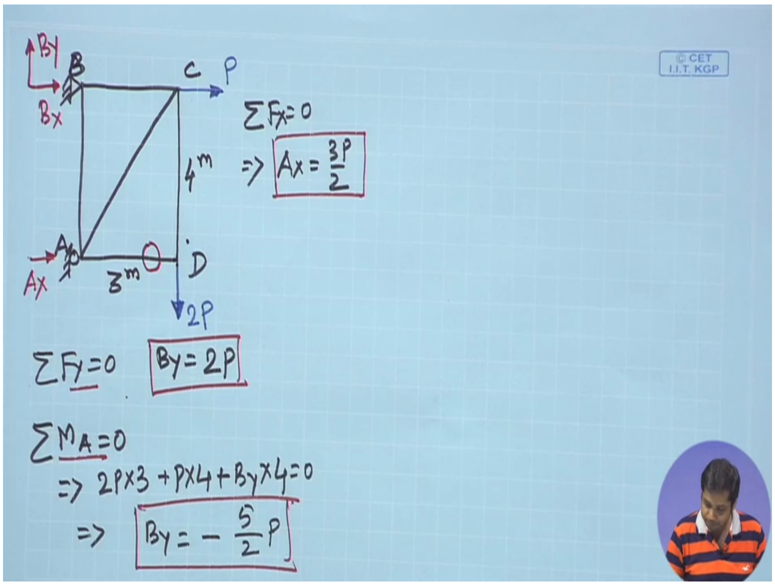

Now if the member AD is zero force member then if we take joint B then the free body diagram of joint D will be FBD of D. If we take then this will be the force in this direction which is 2P and then this force in member CD, FCD and naturally from this we can say that FCD is equal to 2P.

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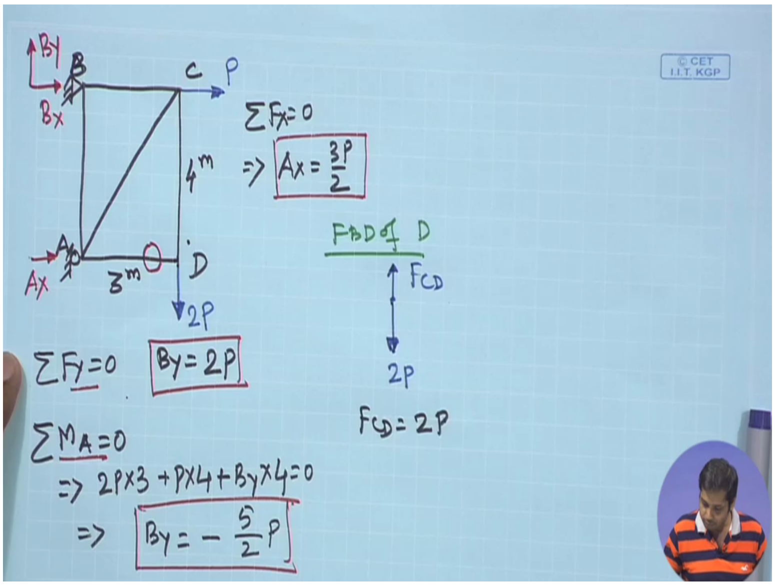

So for say FCD is equal to 2P, really we don’t have to do this. By just looking you can, we are actually applying the equilibrium condition but you don't have to write (equi) and see and take the component of forces in y direction and find out FCD is equal to 2P. Just by looking at we can say that FCD will be 2P.

So in this member, FCD is equal to 2P. Okay. So FCD is equal to 2P. Great. Okay. Let us write it here. So the force in this member is 2P. Okay.

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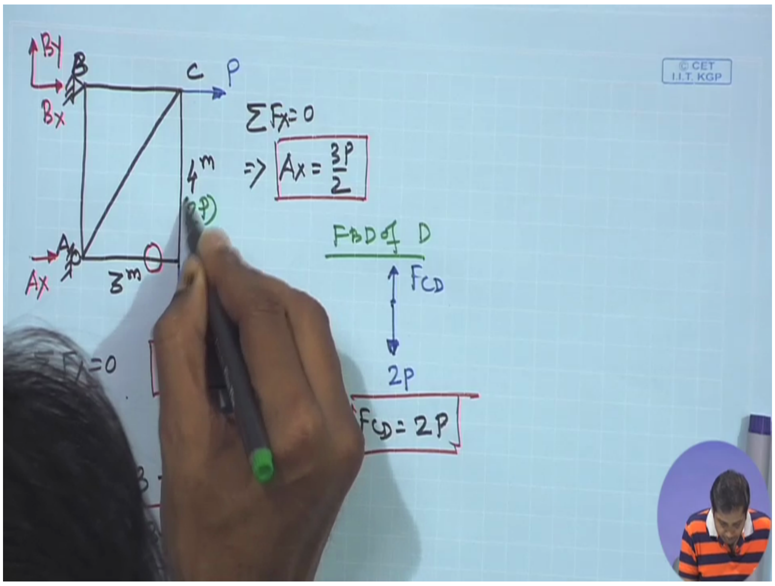

Now then once this is done, so this is your zero force member and then again we can take free body diagram of joint B. If we take free body diagram of joint B then what are the forces we have? FBD of joint B. Then we have member force BC, FBC. Then member force BA. If you remember that is our sign convention. We assume tension in member is positive FBA and then Bx and then By. Bx and By already we have determined.

Then we need to apply the equilibrium equation on this. Summation of Fx and summation of Fy is equal to zero. Then if we do that, summation of Fx is equal to zero, this will give us FBC is equal to minus Bx. Now Bx is equal to, already we have determined. This is Bx. If Bx is equal to minus 2 point 5, so FBC will be 2 point 5 P. So this is force in member BC.

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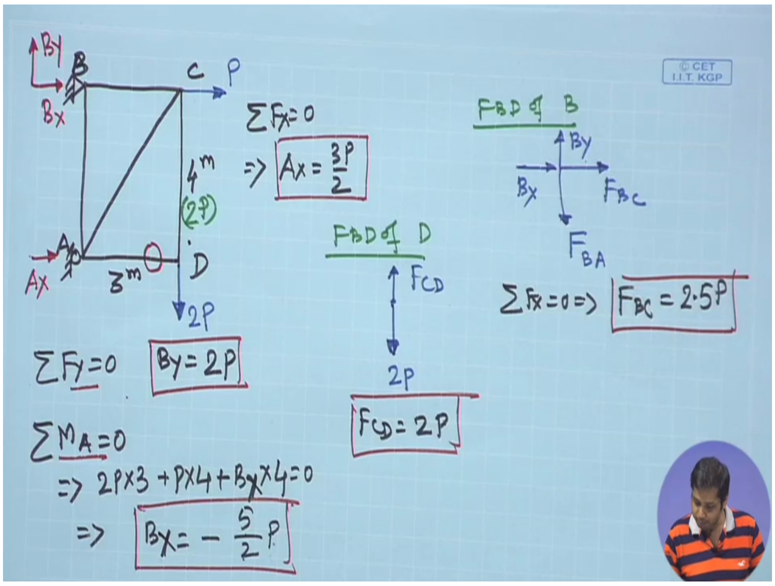

Similarly if we take summation of Fy is equal to zero, then this will give us FBA will be By. By is equal to 2P, already we have obtained. So this is 2P. So this force BC is 2 point 5 P and in BA it is 2P here.

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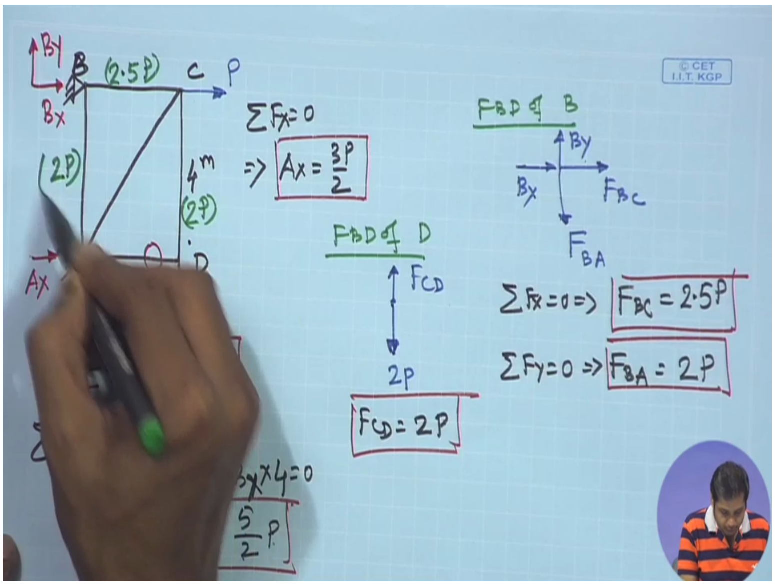

So only thing left is the force in member AC. That we can determine by taking (doi) free body diagram of either joint A or joint C. If you take free body diagram of joint A, the forces will be. So FBD of A we have there is the reaction force Ax and then FBC we have already determined and then this force is FCD. There is force FAD also but since FAD is a zero force member we are not showing it explicitly.

And Ax, the value of Ax is already obtained. Value of FBC is already obtained and we know this angle because this is 4 metre, this is 3 metre.

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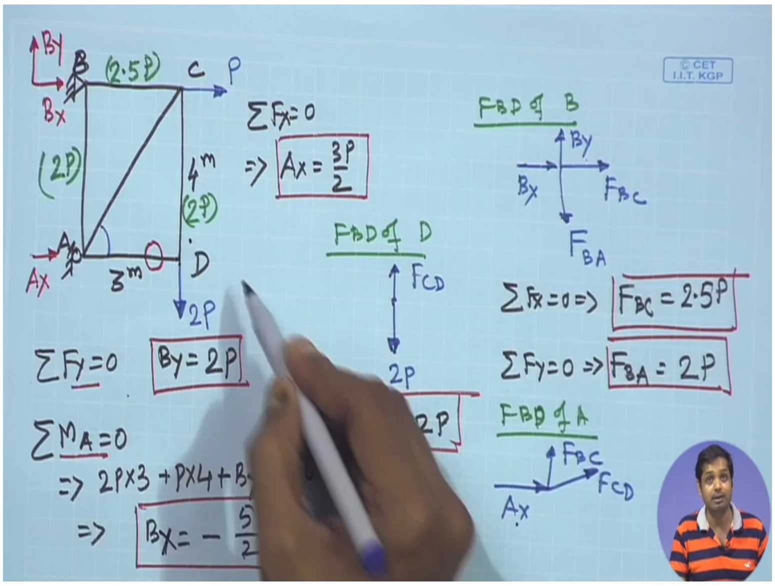

If we apply the equilibrium condition we get, this is FCA or AC. FCA we will get minus 2 point 5 P. So this is the reactions, okay?

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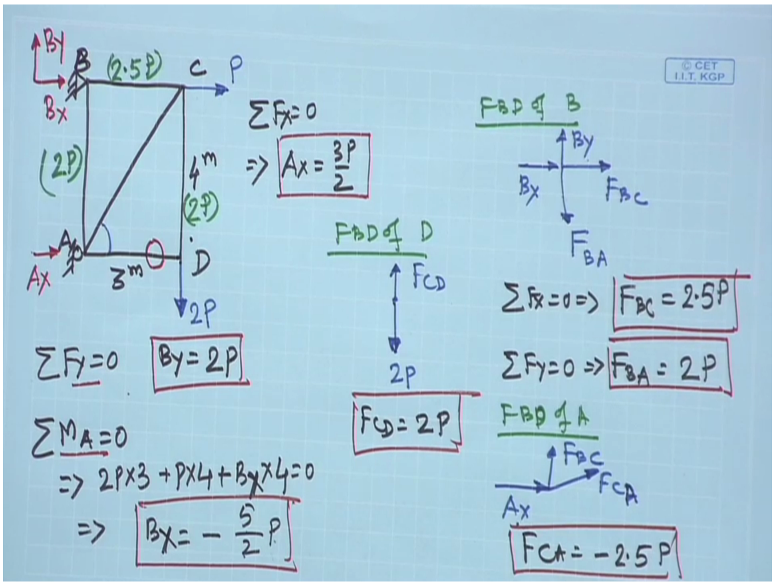

So now what we have? We have the forces in all the members because of the external load. Now next we what we need to do is unit load method, we take the same structure and apply unit load at the point where we want to find displacement and also the load has to be applied in the direction of the displacement. Okay.

Now so what you can do is we take the same structure once again and first we apply the unit load. So take the same structure. This is hinge, this is roller A, B, C, D and then we have diagonal angle AC.

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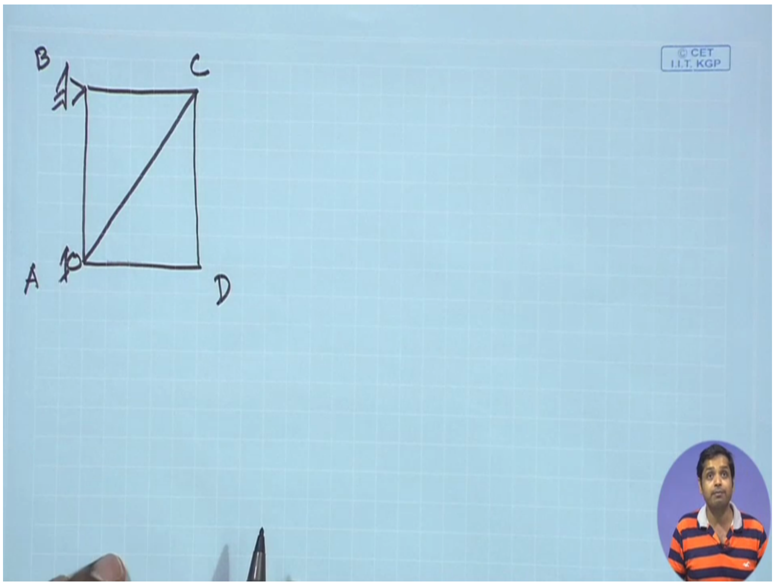

Now first let us find out the horizontal displacement of C. If we want to find out the horizontal displacement at C then this will be 1. Okay. We need to apply unit load in this direction. And now because of this unit load what are the forces in the members that we need to determine?

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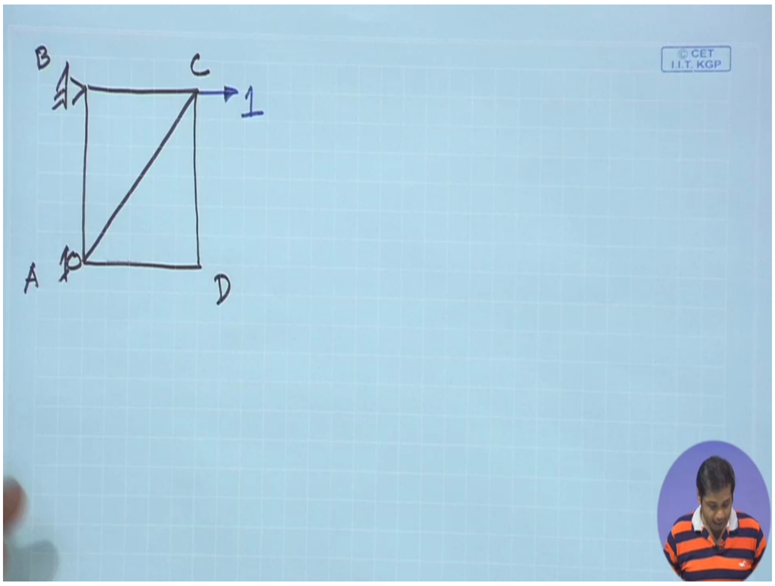

Again without doing any calculation just by looking at the structure we can find out the forces. For instance if we see the free body diagram of this joint, forces will be member force in AD and member force in CD which are horizontal and vertical respectively. There is no other horizontal or vertical load to balance them. So we can say that this force and this force they all are all zero force member. Okay.

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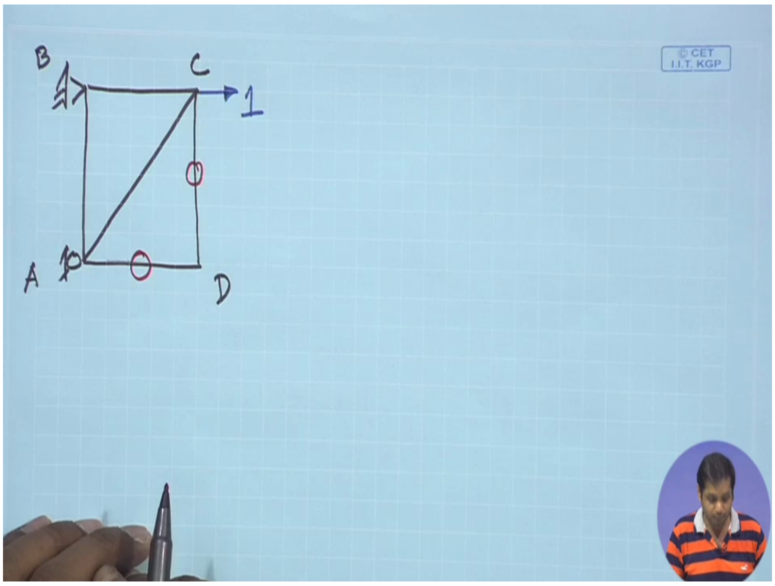

Now since you look at joint C, joint C have horizontal load of 1 Newton and then there will be horizontal load which is the member force in BC. And then member force in CA. Now member force in CA, the force in member CA has horizontal and vertical component but there is no other vertical load which can balance the vertical component of member CA. So therefore the force in member CA has to be equal to zero, right?

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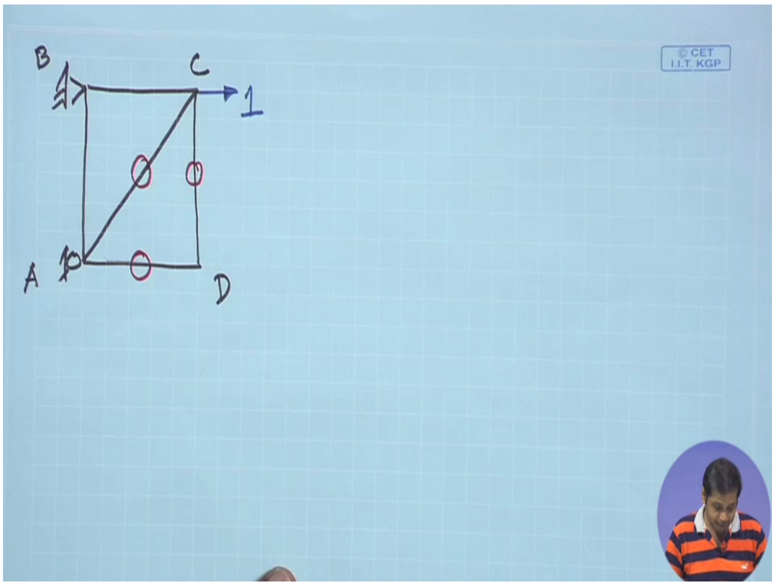

Now if the force in member CA is equal to zero, look at joint A. Reaction in joint A will be in horizontal direction because it is a roller support and then joint A only vertical force we will have the member force in BA. But force in this member and force in this member, they are zero.

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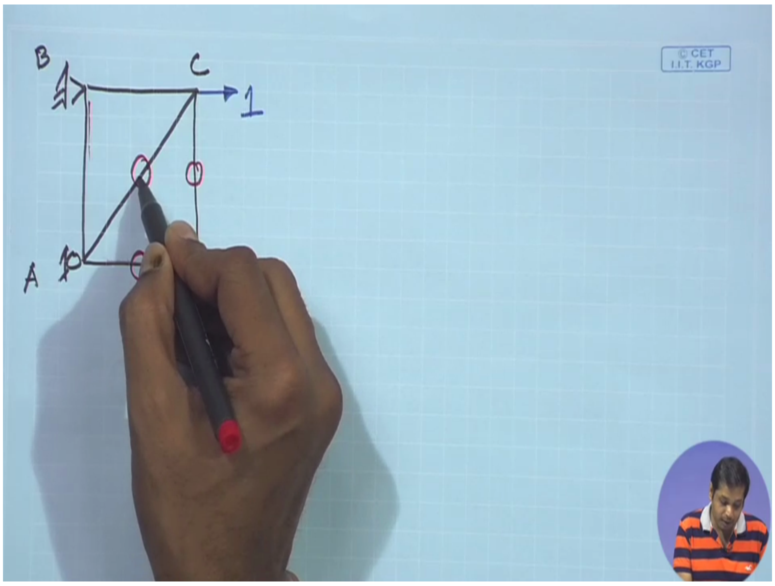

So there is no other vertical force which can balance the member force in AB. So naturally this also has to be a zero force member.

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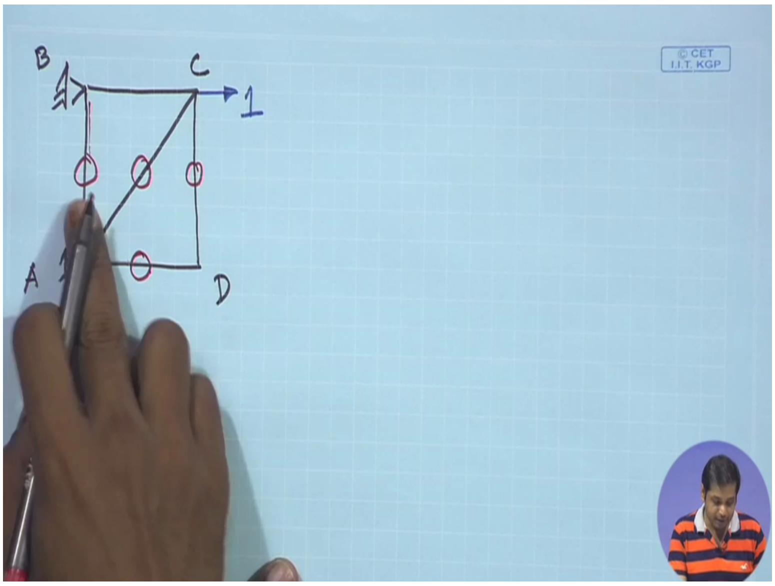

Now if it is zero force member, now look at joint C. If we look at joint C, then forces in member will be CB and all other forces as zero. And then what would be the force in member CB? Force in member CB will be 1. Okay.

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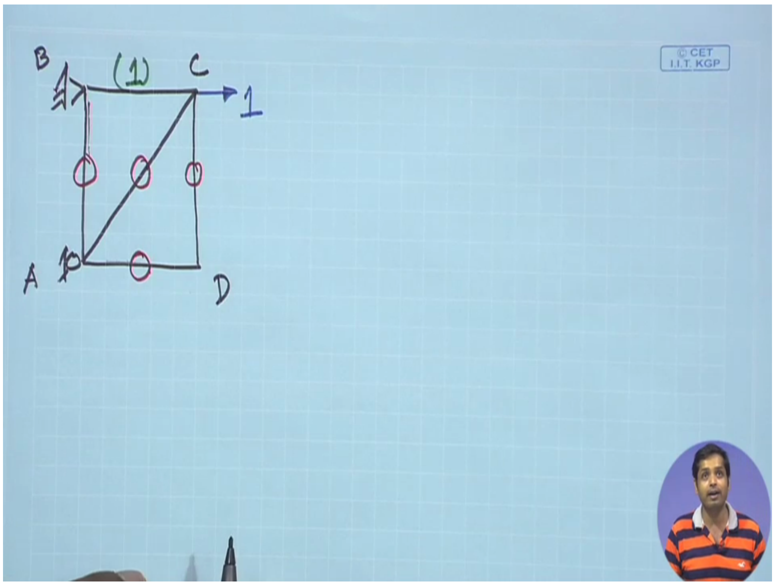

Now so you see without actually doing any calculation we should be able to find out, for such small structure if you can identify the zero force member by looking at the structure then our problem becomes easier. Now this will give you the forces in this. . Now this we require to determine force in horizontal direction.

Now we also need to find out force in vertical direction and for that take the same structure once again and this is roller, this is hinge and then this is A, B, C and D. Then we need to apply a force unit load in vertical direction. Okay.

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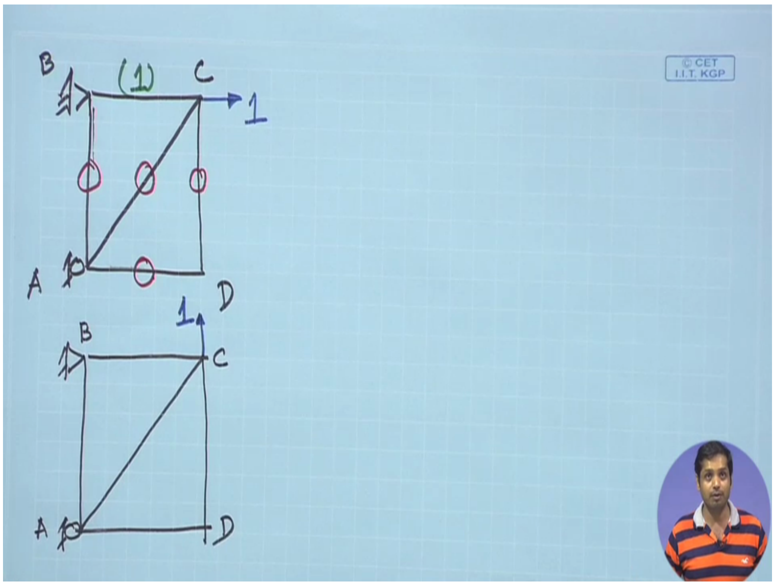

Now let us find out the forces in the member. For the same reason here the force in member CD and force in member AD will be zero. Okay. Now then again we need to find out, we can do the similar exercise. Any method of joints or method of section to determine the forces in other members.

And if you do that, the final forces in member will be, at this member will be 1 point 25. This will be minus 3 by 4 and this member force will be minus 1. Okay. We know how to determine this. Okay.

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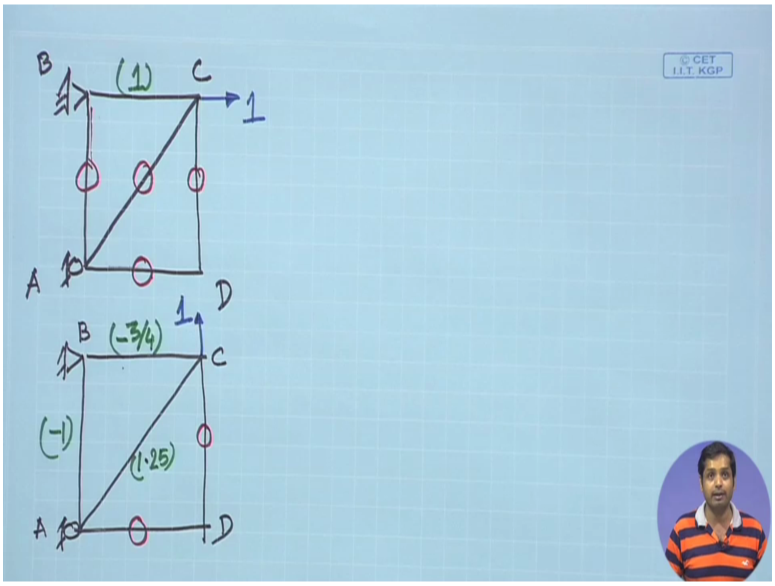

Now so this will give us horizontal displacement and this will give us vertical displacement. Now let us write everything in tabular form. So take a column for member. Okay. Then column for the forces in member due to the externally applied load. Then member in this force in here and then another column for this and then we need a column for length and then we need column for, okay let us write it.

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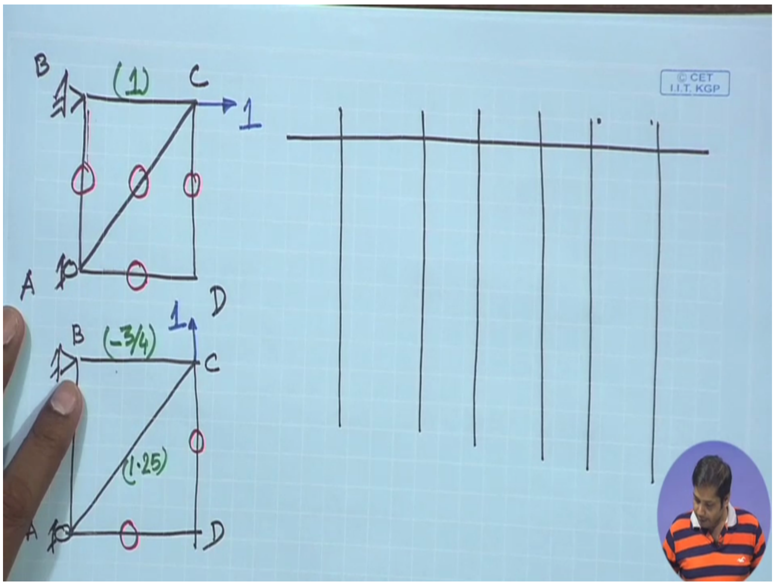

This is member. Okay. And this is N. What is N? N is the member forces. These are the member forces due to the external load on the structure. Okay.

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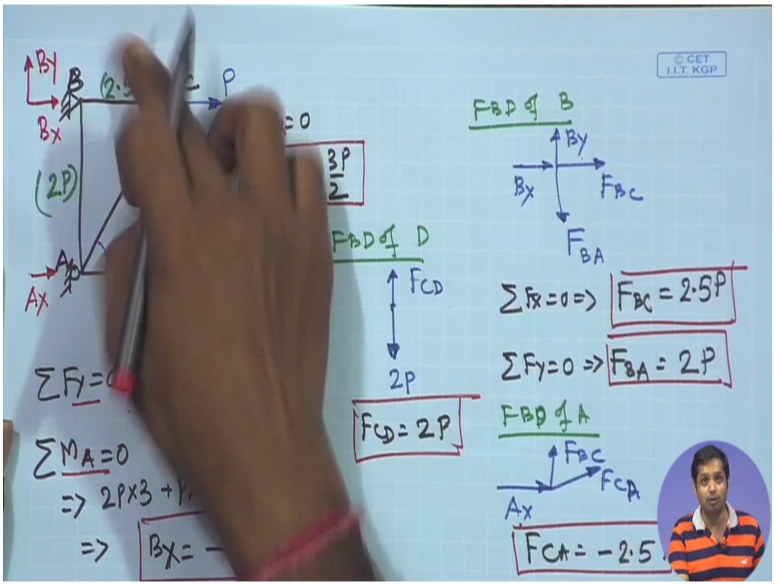

And then we have n1. N1 is what? N1 is the force in this direction due to the unit load at C in the horizontal direction.

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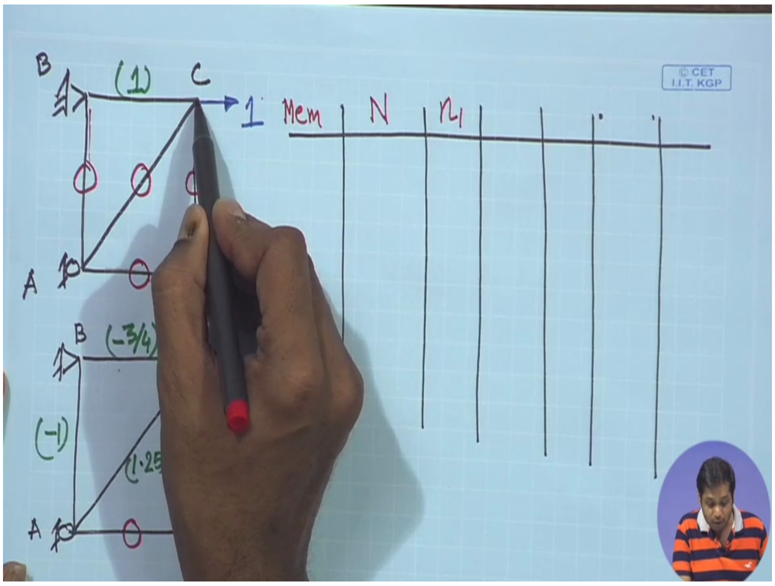

And then n2 which is the forces in this. N2 are the member forces due to the unit load at C in vertical direction. Okay.

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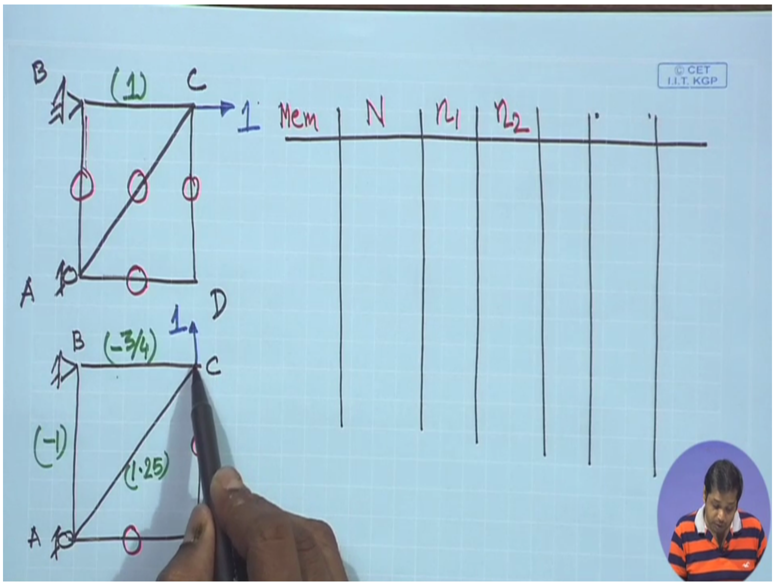

And then this is the length of each member. Okay. And then this is n1NL and this is n2NL. So this will give us this displacement and this will gives us this displacement. Okay.

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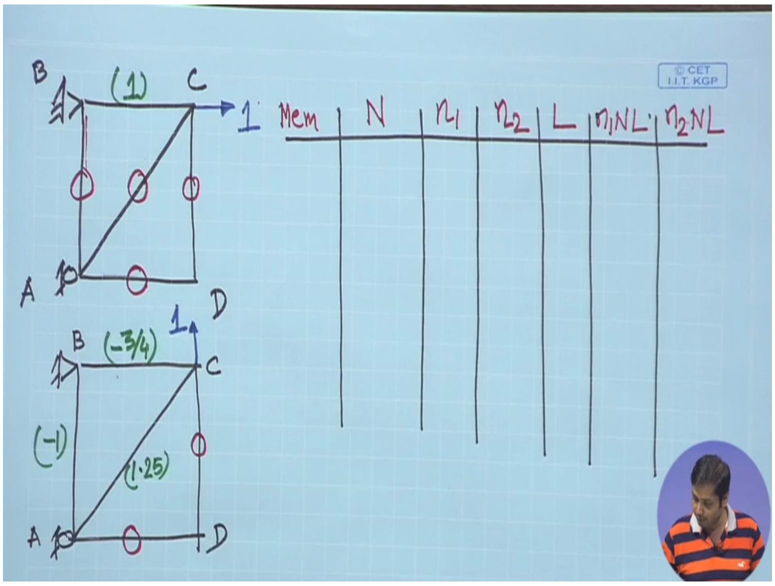

Now so let us write all the all the members. So we have total five members here. So 1, 2, 3, 4, and then 5.

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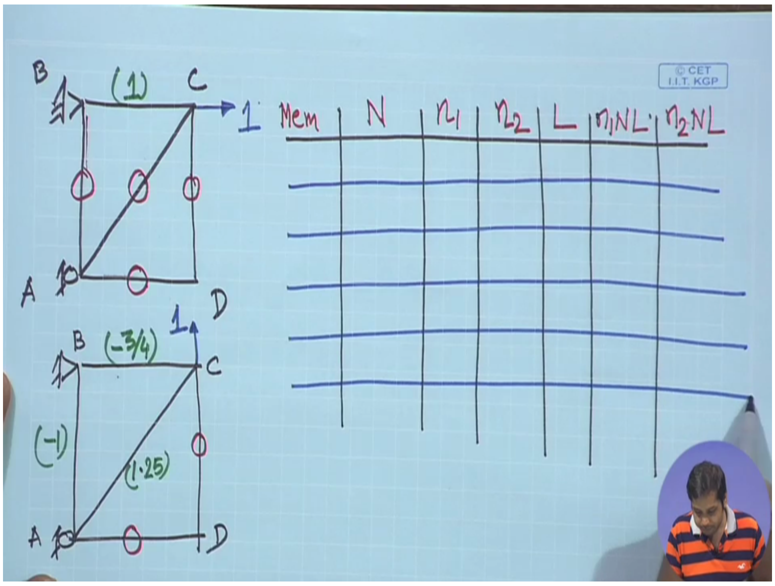

So this is member AB and then BC, CD and then AD and finally the diagonal member AC. Okay.

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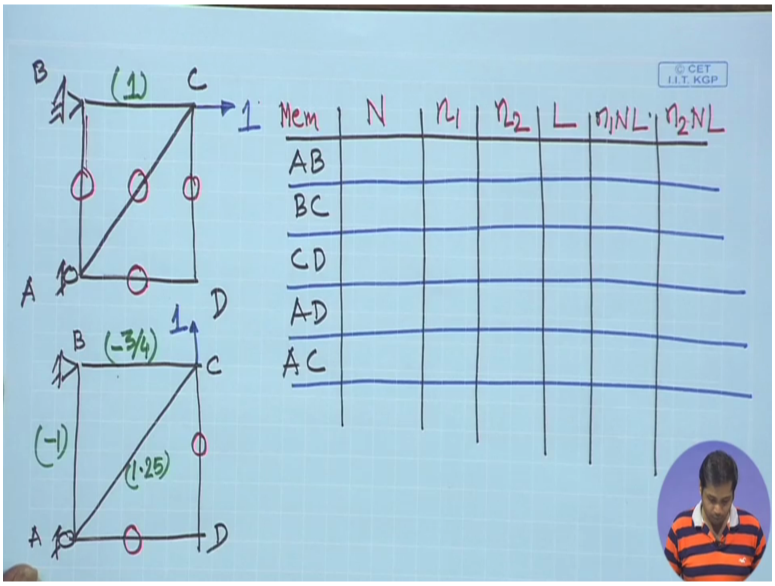

Now let us see what are the forces we have? The force in member if you can see this, the member forces in AB it is 2P. So this is 2P. And in BC it is 2 point 5 P. And then CD it is 2P. These members are zero force members. It is zero and then this member is minus 2 point 5, we have already determined it. So it is minus 2 point 5 P. So this value is minus 2 point 5 P.

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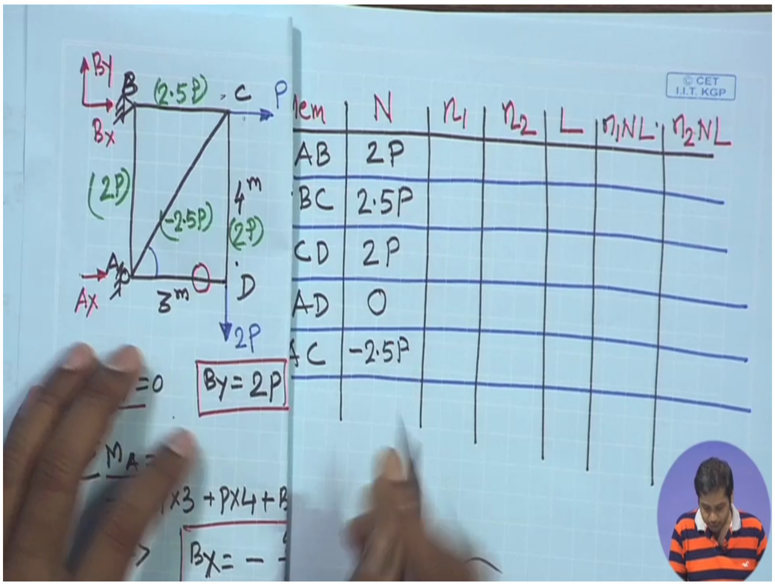

So these are the forces in the member due to the external load. Now what is n1? N1 is only, BC is one and all other are zero. So this is 0, this is 1, 0, 0, 0. And similarly n2, AD and CD are zero. So AD, this is zero and this is zero. And then member AB is minus 1 and then members BC is minus 3 by 4 and then finally member AC is 1 point 25.

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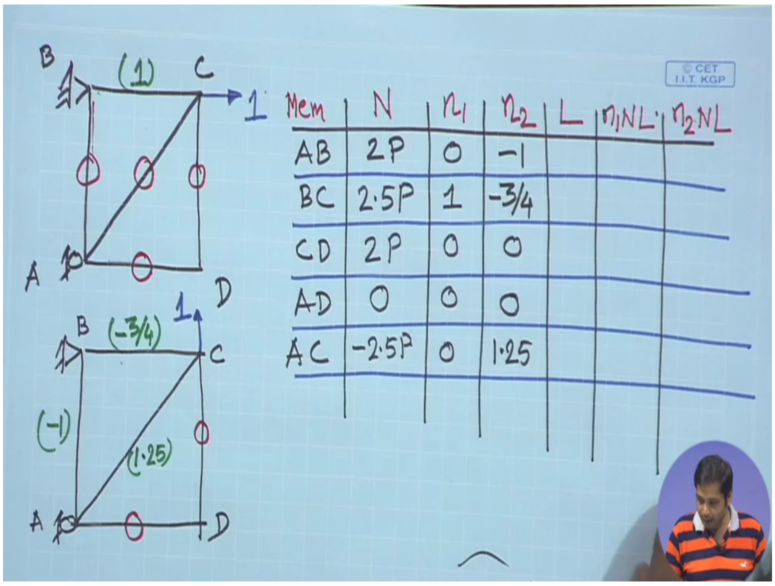

Lengths are this is 4 metre and then this is 3, then 4 metre, 3 metre and this length will be 5 metre. Okay.

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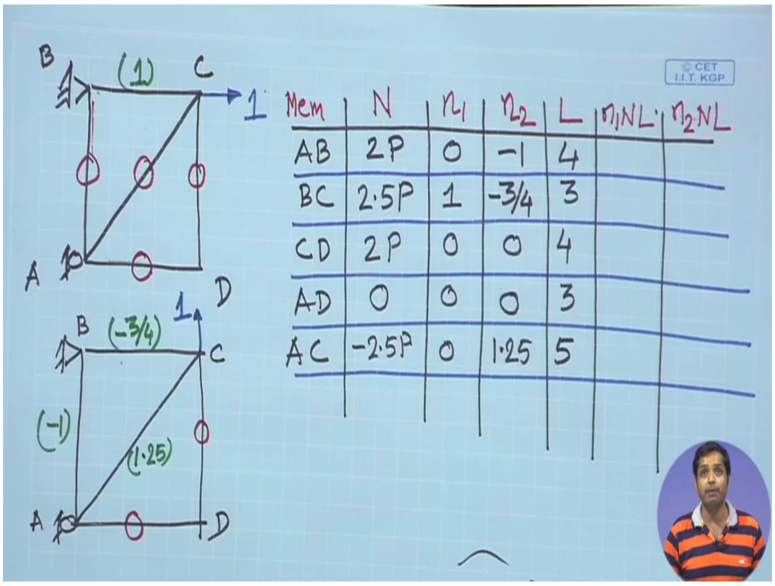

Then what is n1L? This all will be zero, this is zero, this is zero, this is zero and this will become 2 point 5 P into 1 into 3, 7 point 5 P. Okay. And similarly in this case minus capital N into n2 into L. So this become minus 8P and then this becomes 2 point 5 into minus 3 by 4 into 3. This is minus 5 point 625. And this becomes 0, this is 0, and this become minus 2 point 5 into 1 point 25 into 5. This is minus 15 point 625 P. Okay. These are all P.

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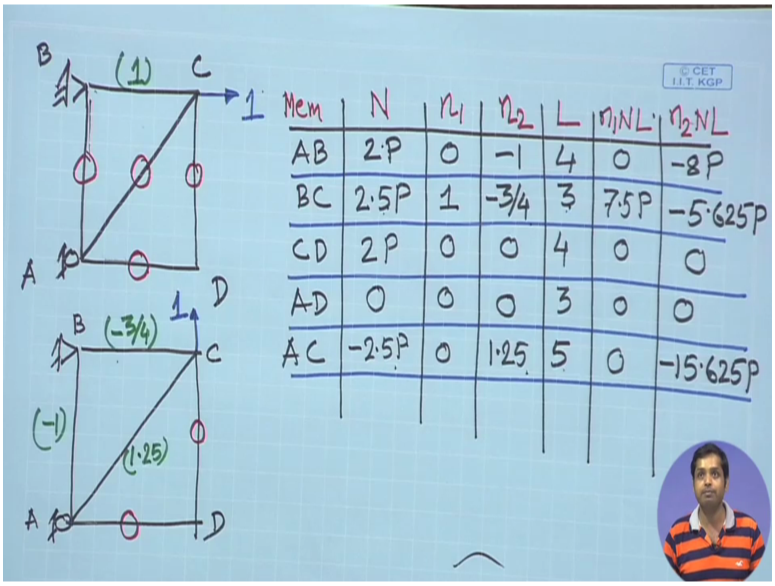

So this is P. Now then what is delta? If we remember, delta C1 will be means it is the displacement at C in, this is suppose direction 1 and this is direction 2. Okay.

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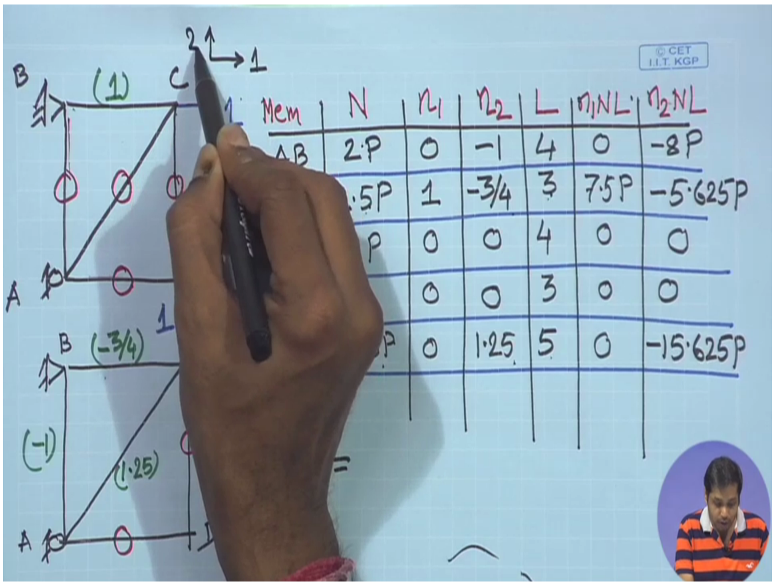

So delta C1 is the displacement at C in 1 direction. In direction 1 will be summation of, we know that it is summation of Nn1L divided by A. Okay. Now A is constant for all the members. This is actual rigidity and Nn1L, this is the summation of this column. So this gives us delta C1 is equal to 7 point 5 P by A. So this is the displacement at C in direction 1.

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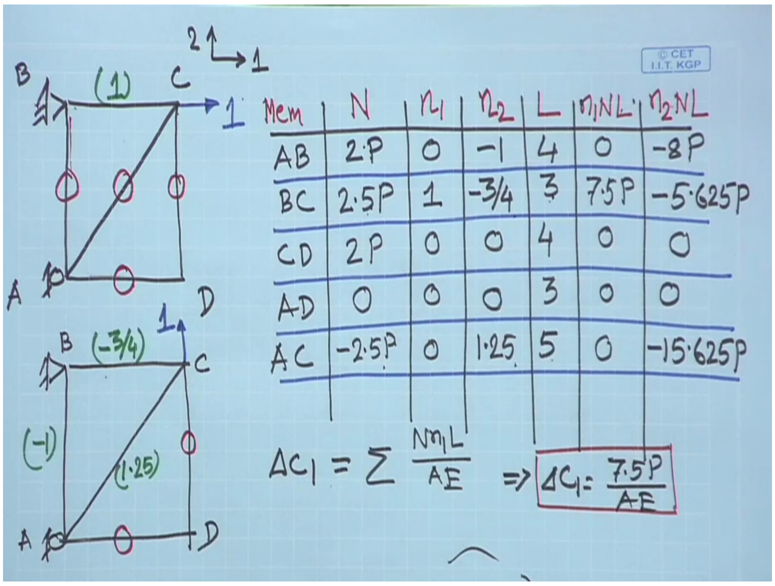

Similarly displacement at C in direction 2 will be summation of Nn2L by AE. Okay. Now Nn2L are already obtained. This is all the summation of the entire column. And if you sum them then this becomes minus. So this becomes delta C2 is equal to minus, if you sum them all, 29 point 25 by AE .

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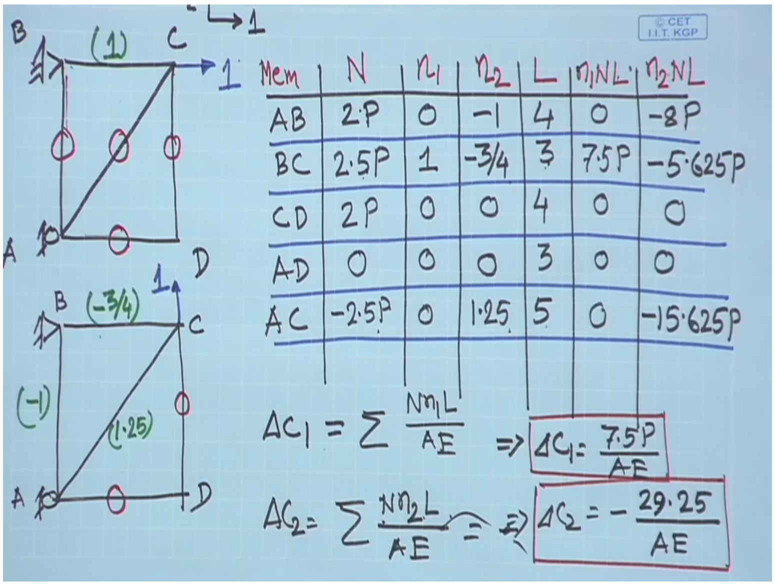

So what it says that delta point C, this is our positive direction 1 and positive direction 2.

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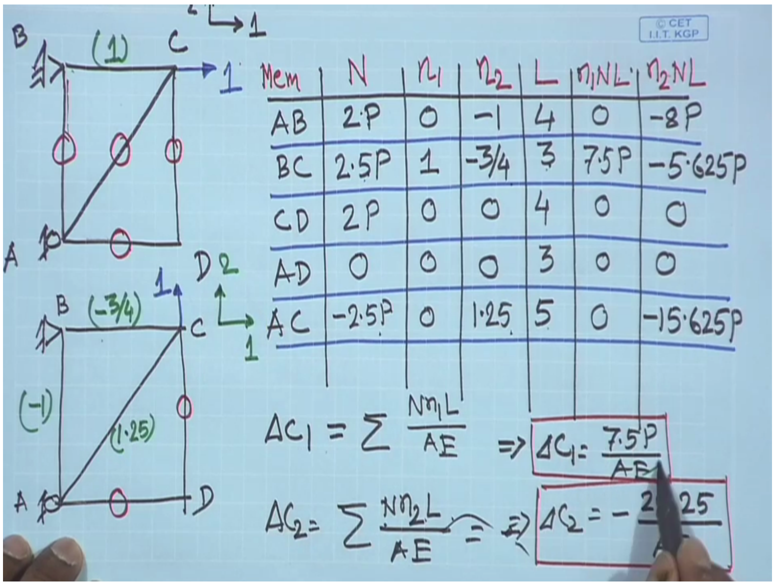

So this is positive it means that in direction 1 the displacement will be in the direction of unit load. And we (obt) applied unit load in this direction so displacement will be in this direction.

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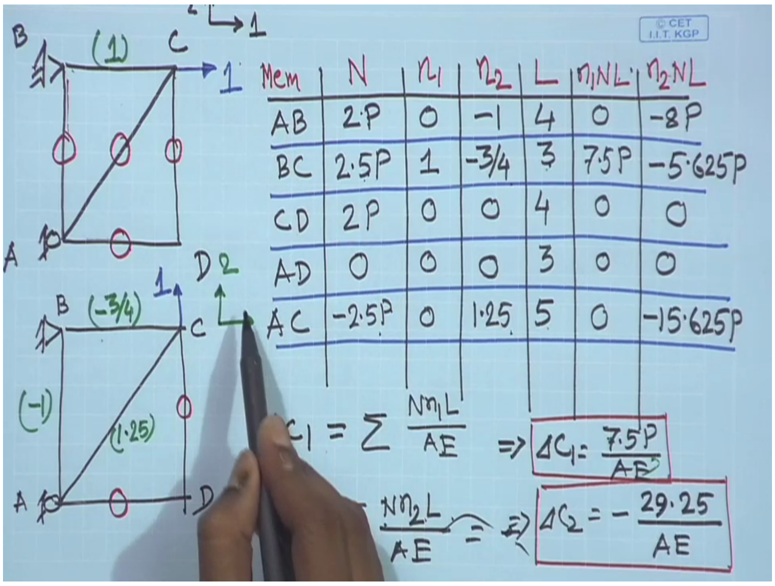

But delta C is negative means the displacement will be in the direction opposite to the unit load. Now we applied unit load in this upward direction.

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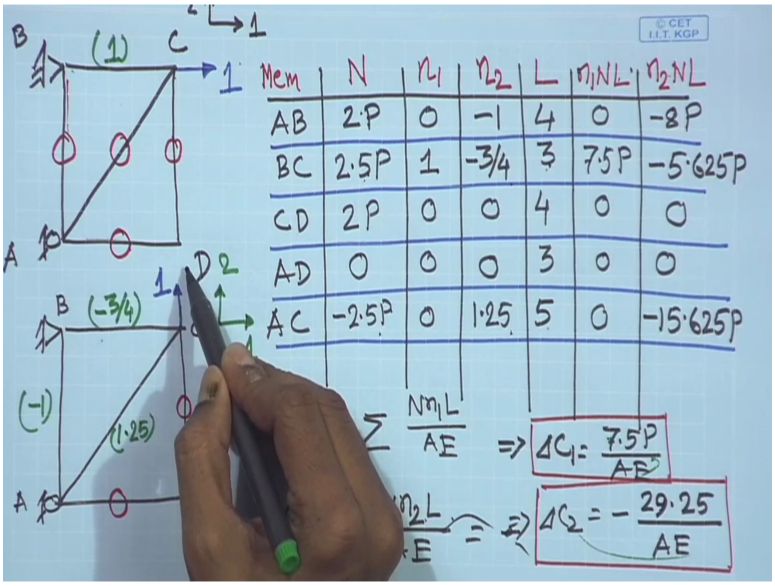

But since it is negative, displacement will be in the opposite direction. Opposite direction means displacement will be in this direction. Okay.

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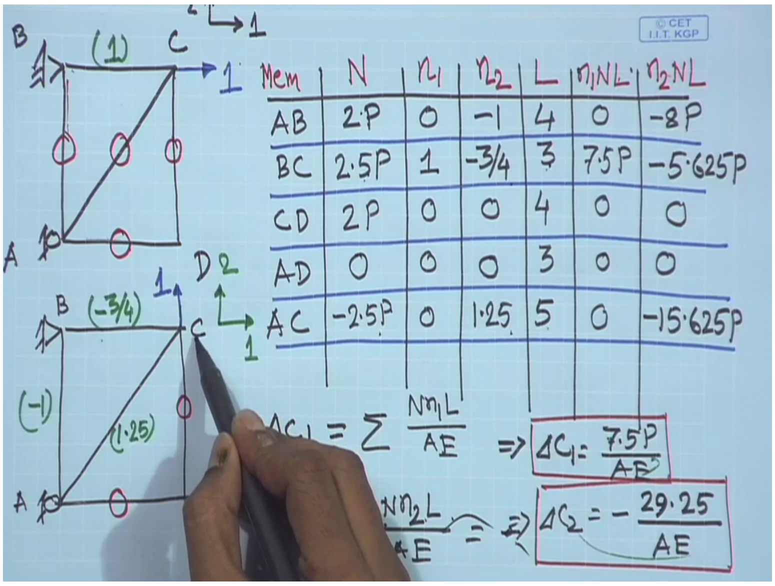

So now similar procedure you can you can apply to find out displacement at any point. Okay. N will remain same, this column, this column, this column, this column will remain same for the structure. Only thing is n1, n2. These forces in the members due to unit load will only change.

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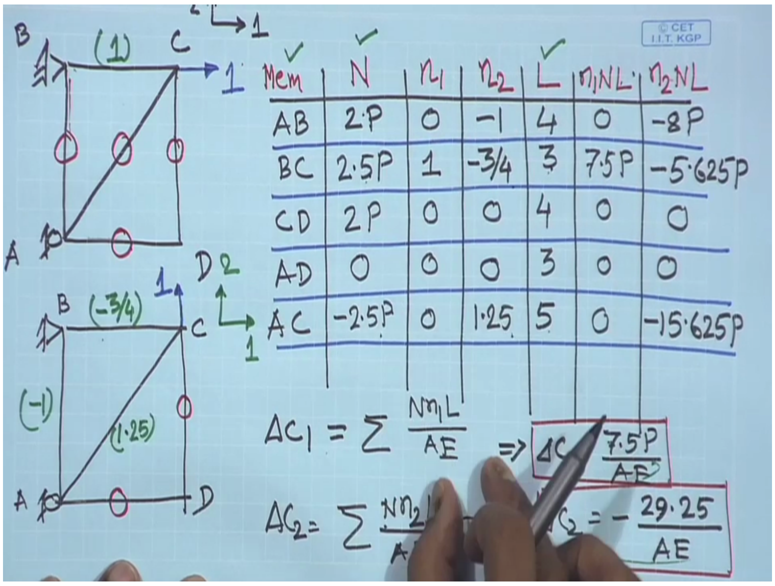

So wherever if you want to find out displacement at point D, similar way we need to apply unit load at point D in the direction in which we want to determine the displacements. Okay. So this is how we can apply unit load method to find out displacement at any particular joint in truss. Okay. Now the same truss if you want to see the (deform) deflected profile, you can see this is deflected profile of the truss.

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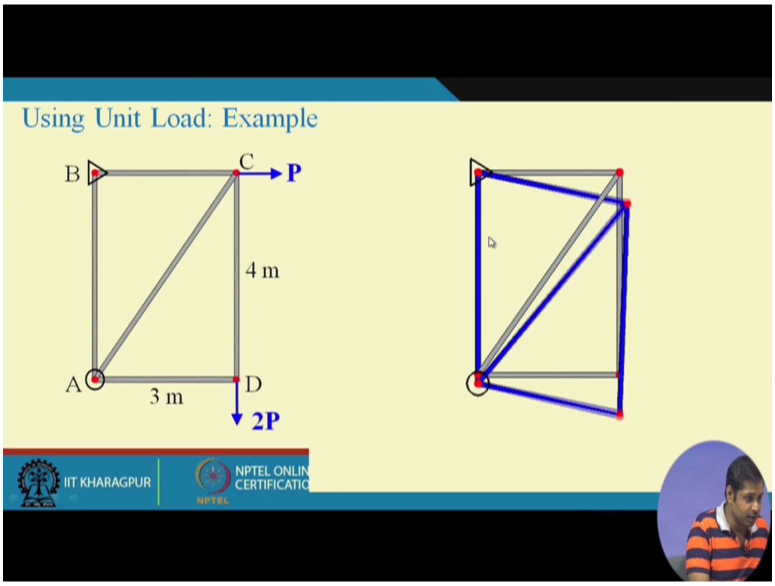

Now as I said this member as per our calculation, the displacement in member C, horizontal displacement in this direction and the vertical displacement will be downward direction and that is what it is happening here. And another thing is which is very obvious, you see the horizontal displacement is smaller as compared to the vertical displacement which you also have observed. Okay.

So this is how the member. Now and other thing you see the point B here, since it is a hinge support, it cannot move in any direction, either horizontal or vertical direction.

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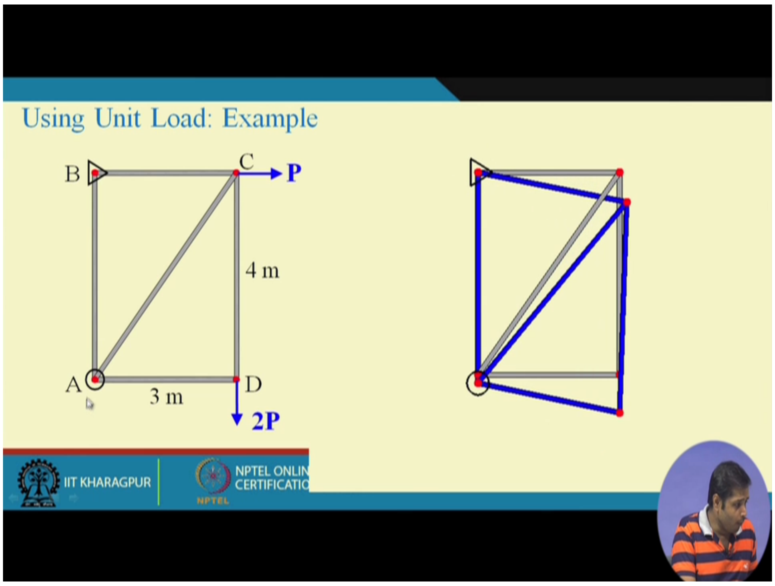

Whereas joint A is roller support. So this joint A cannot move in this direction because the moment in the transition in this direction is constraint. But joint A is free to move in vertical direction, that is what is happening here.

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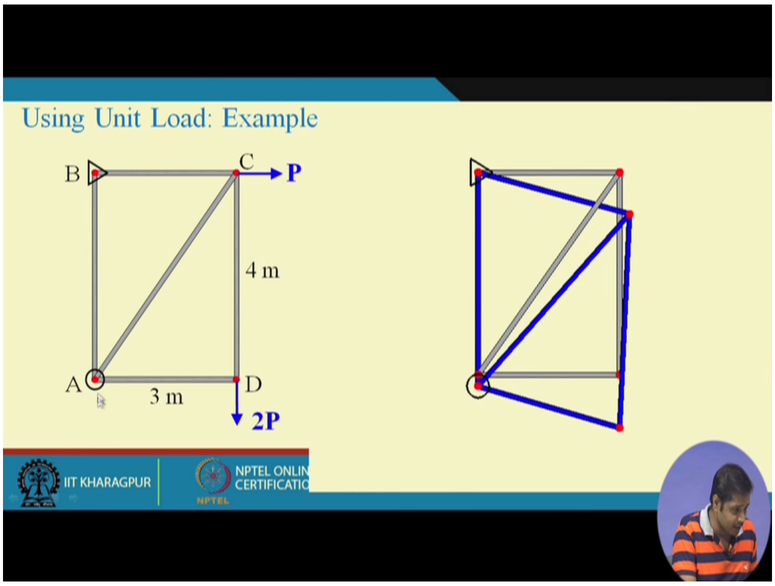

This is the animation that you can see it is a scaled deformation. So actual deformation is not that much in this structure but it is just to have a proper view of the deform shape it is scaled. Okay. So that is how we can apply unit load method in truss. We will discuss more on the principle of virtual work and its application to different structures. Okay. But as for today class is concerned we will stop here. See you in the next class. Thank you.