Structural Analysis 1
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Lecture 13
Analysis of Statically Determinate Structures - Method of Virtual Work (Contd.)

Hello everyone. We were supposed to startprinciple of virtual forces today.But we discussed principle of virtual displacement as originally proposed by Bernoulli. That was the original version of principle of virtual work. And then we discussed then if you want to apply principle of virtual work in structural mechanics problem, that we are interested in, where the body is the deformable, we need to move beyond that.

We need to generalize the concept. And in that generalization we need to include the concept of virtual forces. And then which eventually gives us the principle of virtual forces. And then deformable body, since we need to consider both the external and internal work. But before we do that, let us first demonstrate principle of virtual displacement through some examples.

How the principle of virtual displacement can be usedto analyze the, at least some part of the analysisof statically determinate structures? Now, and then the principle of virtual forces will be discussed in the next lecture, means lecture 14.

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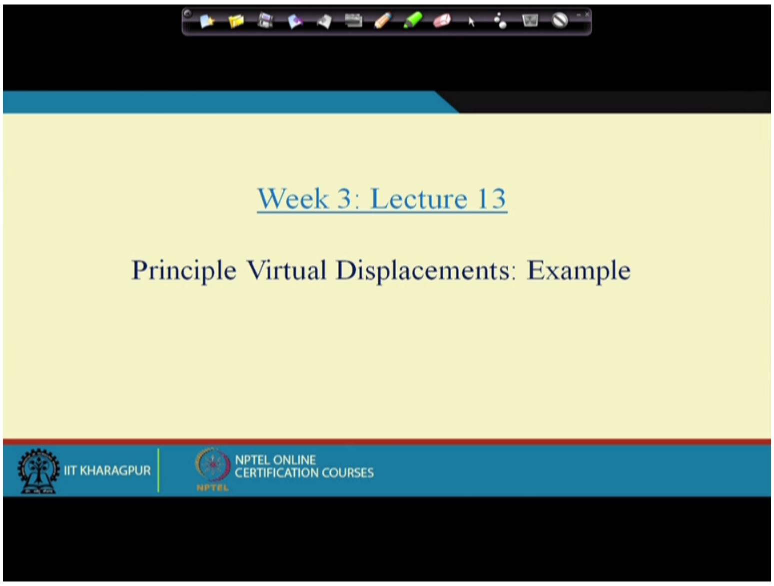

So today what you will do is, we will discuss, what is principle of virtual displacement? And we will see some example of that. And principle of virtual displacement, if you rememberit says that if a rigid body is an equilibrium then if it undergoes some small virtual displacement then the total work by theexternal forces acting on the bodywhich moving to the virtual displacement is zero. Now let ussee the first example.

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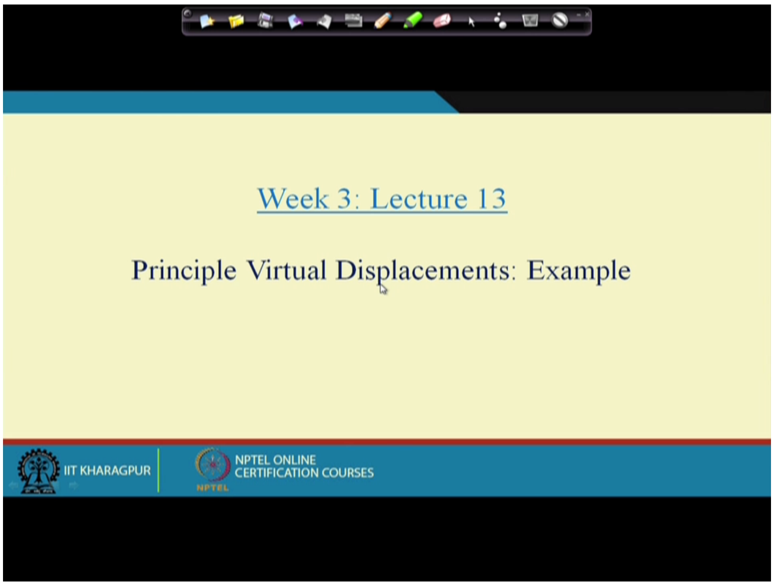

Suppose this examples we have already probably used for drawing bending moment and shear force diagram. And when we try to draw bending moment and shear force diagram defining the internal forces, we had to determine the support reactions and therewe determine support reactions using equilibriumconditions.

Now what we do is, we will try to find out this equilibrium equation using the principle of virtual displacement. This example is, it is a beam simply supported beam with overhang. Let us let us draw it, let us take some more space. This is roller, this ishinge support.

And then we have another roller support here. This is A, this is say B and this is point C. And this distance is L and this distance is L by 2. And it is subjected to force P.

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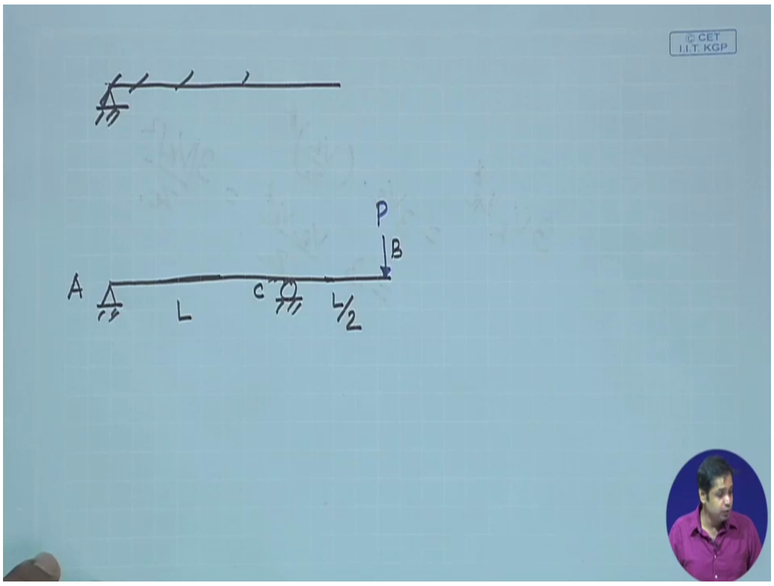

What we need to find out? We need to find out what is thesupport reactions? Now since we are applying now the principle of virtual displacement which we have already discussed for rigid bodies. So we assume that the member ABis rigid body. But as I said, along with the internal forces we are also interested in the formation of the body.

Then in order to find the deformation,weneed to move beyond theassumption of the rigid body. That we willbe doing in the principle of virtual forces and in the subsequent lecture. But for today when we apply the principle of virtual displacement, we assume this is a rigid body. Because we are not interested to find out the displacement. We are all interested to find out the reactions. Now the reactions will be what?

Reactions will be, we can see that the horizontal reaction at A is zero because there is no horizontal force. So there will be a reaction at Ay which is Ay and there will be reaction here at By, right? This is Cy.

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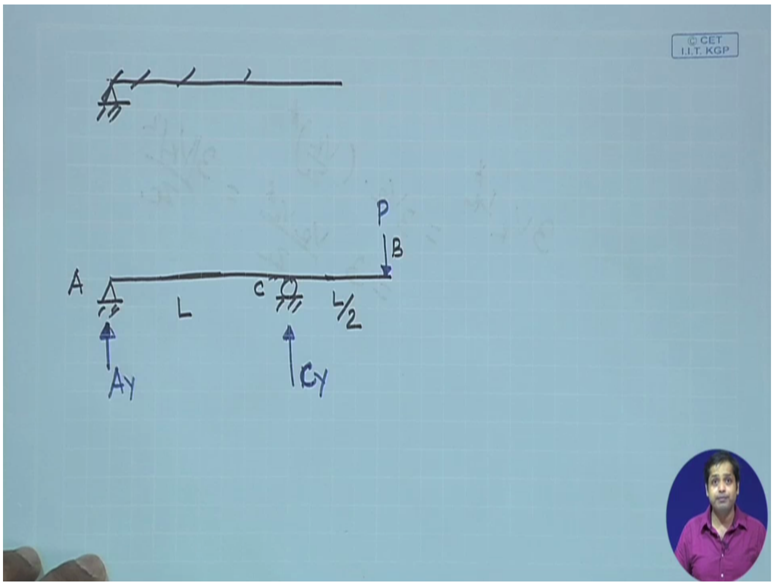

Now, so what are the force field we have? The force field we have is reaction Ay, Cy. When we draw these reaction we can remove this support conditions. So this is the force field.

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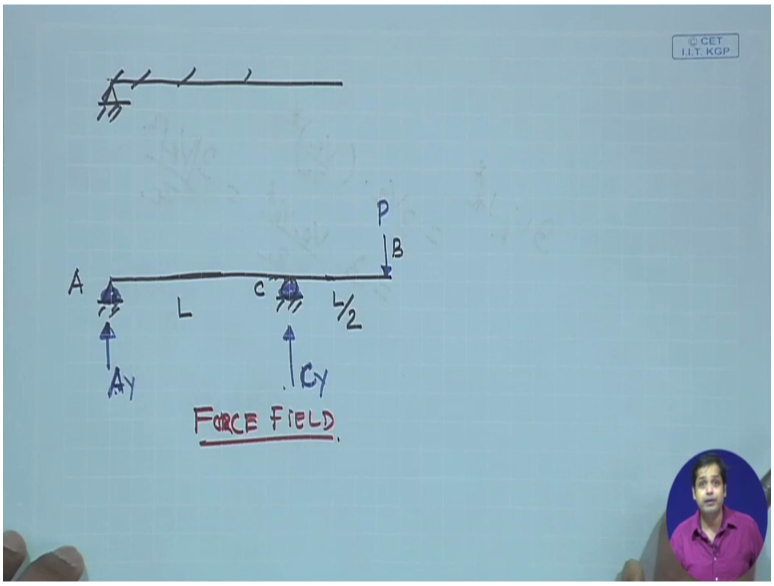

Now, only to construct the displacement field. Now what we have to do is, we have to apply virtual displacement in this body. Now let us give some virtual displacement. Let us do it here itself. First let us give a virtual displacement like this.

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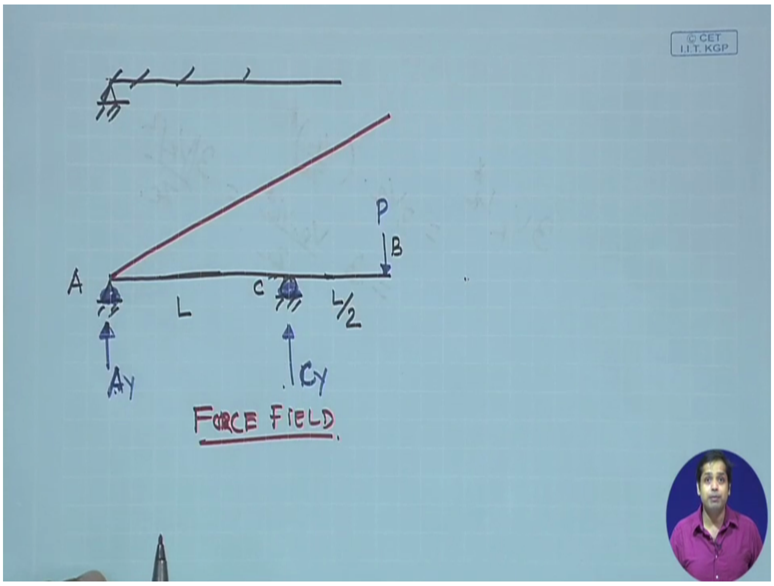

So virtual displacement not necessary that they have to satisfy the constraint. The only thing the virtual displacement needs to satisfy the internalcompatibility. And in this casesince it is rigid body, now if we give rotation like this,then it will be a rigid rotation.

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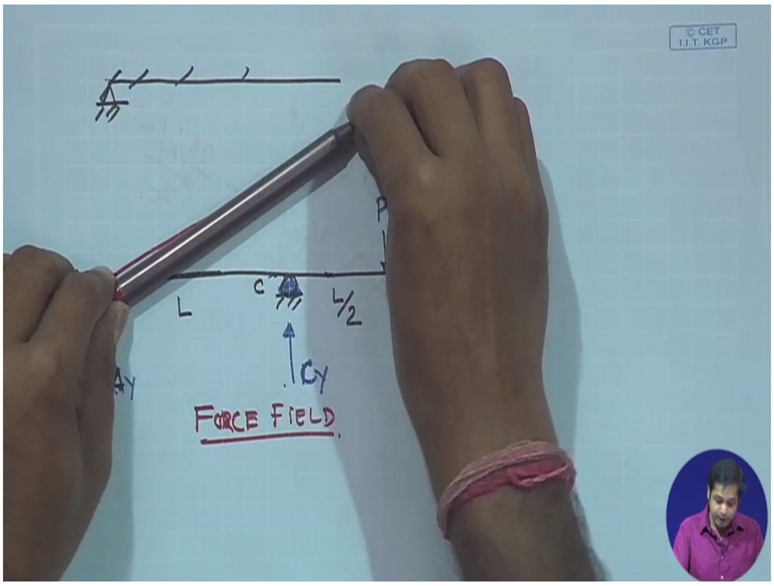

And suppose this distance is delta. Now you see (dis) this is displacement field. Now, what the principle of virtual displacement says that, if we have a virtual displacement, these are the actual force acting on the body, then the work done or the product of the (displace)actual force and their conjugate displacement, this conjugate to this force.

Multiply the product of the force and the conjugate displacement, the summation of that which is a virtual work that is zero.So what is the virtual work done? The virtual work Wisequal to. Now you see at this point there is no displacement in the direction of Ay. So Ay will not contribute anything in this work. Now if this is delta, then this distance will be 2 by 3 delta. This length is L by 2, this length is L.

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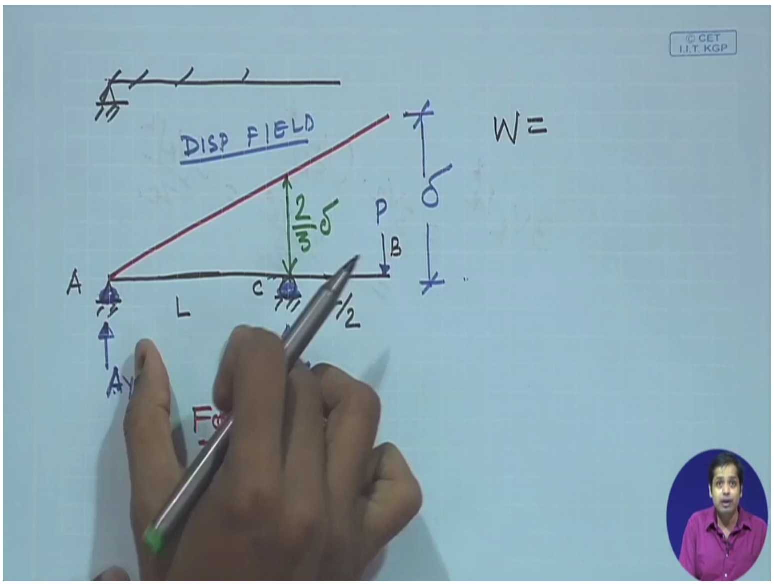

So then what would be the product of Cy into this? It is the virtual work by this force.

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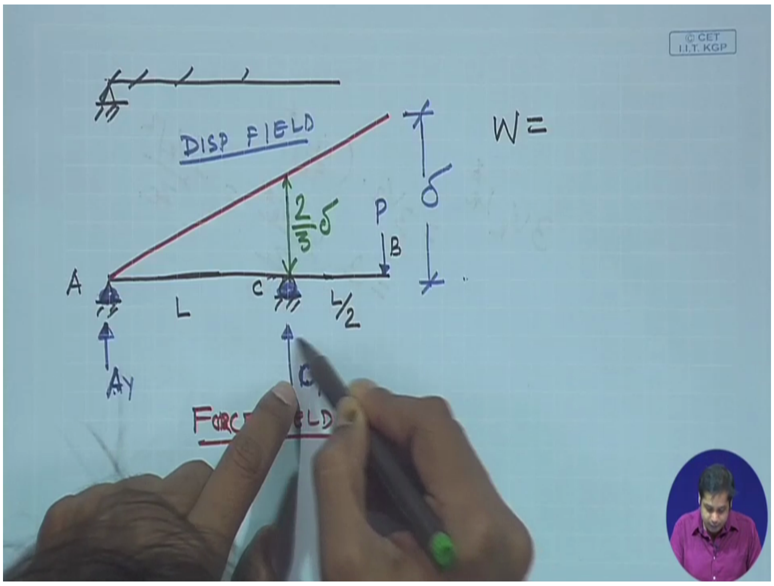

And then what will be the virtual work by this force? Virtual work by this is Pinto delta.

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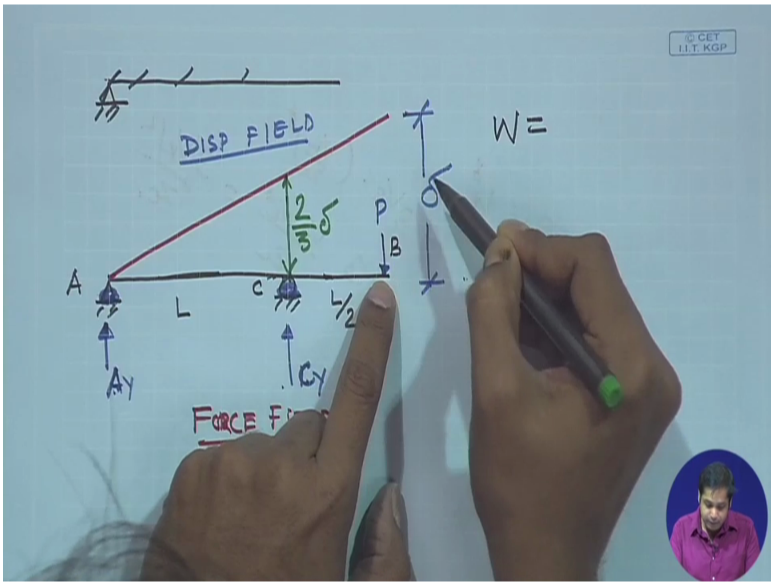

Now this displacement and the force, they are in the same direction. So this is positive and if it is delta. Now force is in downward direction, it is in upward direction, so this is negative. So now then what will be the contribution? Total force will be that 2 by 3 into delta into Cy and then minus P into delta. This is the total virtual work and virtual displacement says that this is equal to zero. So this gives us Cy is equal to 3 by 2 into P. By just equilibrium equation you can verify what is the value of Cy.

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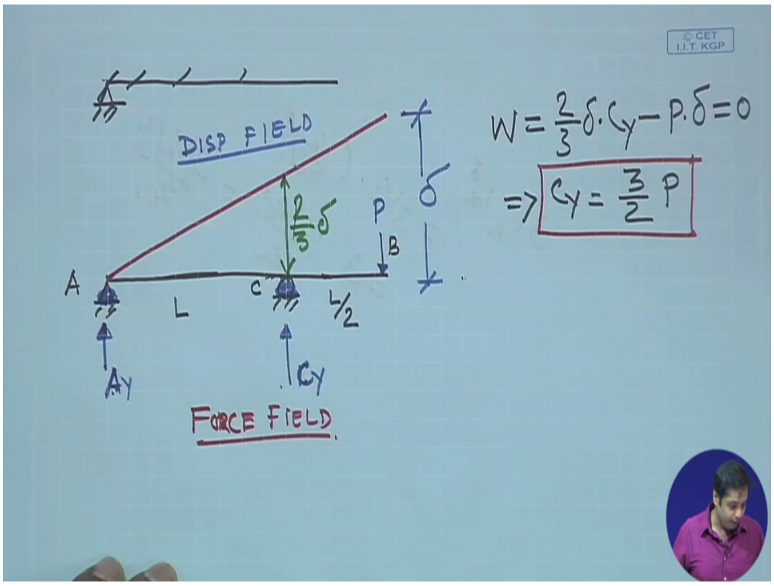

Now onceCy is obtained, we need to find out Ay. We won't be using equilibrium equation, right? We will be only using this principle. Now let us find out reaction at A. Again draw this beamsimply supported at A and then roller support at C. This is A, B, C. We havethis force here P. Now what we do, give a displacement like this. Now this length is L, this length is L by 2 and if this distance is delta then this distance becomes delta by 2.

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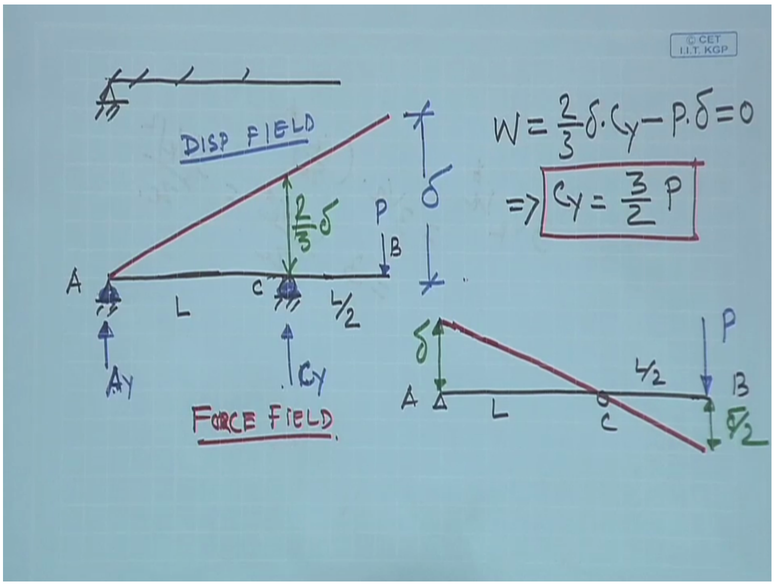

Now in the force system, we have force Ay here and then Cy here. Now what is the virtual work? Virtual work W is equal to. Now Cy there it is rotated above Cy, so there is no displacement at Cy. So this virtual contribution from Cy will be zero. Contribution from Ay will be, Ay into delta and then contribution from Py will be, plus P into delta by 2. Now P and delta, they are in the same direction, it is positive. Ay and delta is in same direction, this is positive.

This is equal to zero. Now if this is equal to zero, then this gives us Ay is equal to, we have minus P by 2. So support reactions are Ay is equal to minus P by 2 and Cy is equal to 3 by 2 P. And in order to get this support reaction, we have not used equilibrium equation.

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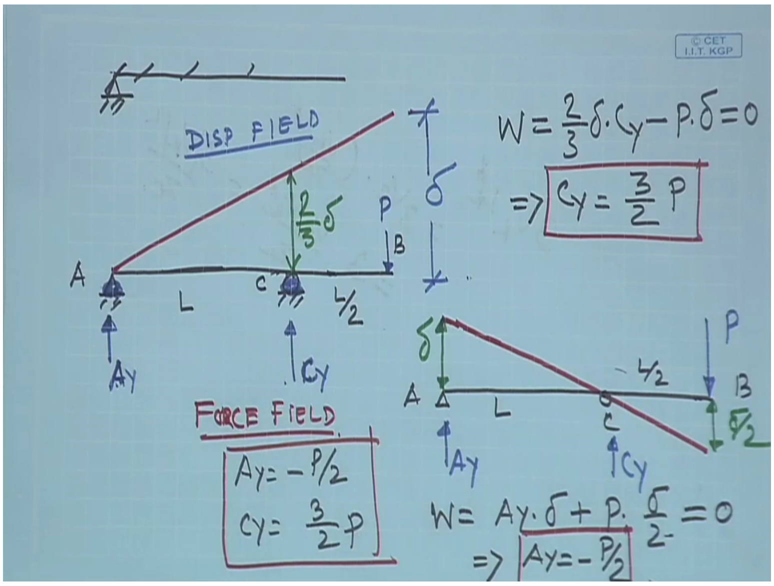

What we have used? The principle of virtual displacement which somewhere is essentially a different form of equilibrium equation, right? That we have seen. Because this is valid if the object is in equilibrium, right? So this was the first example. Now take one more example. This example.

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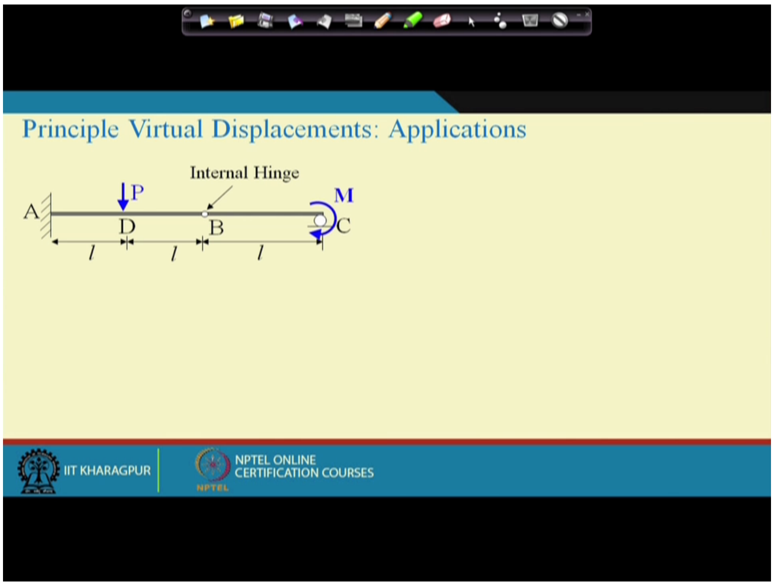

This is againstatically determinate structure.Because of this hinge, it is statically determinate structure. Internal hinge, if we do not have hinge, it is a propped cantilever which is statically indeterminate. Now again here we have to find out what is the reactions? Now how many reactions we have? We have vertical reaction at C and then vertical horizontal reactions at A and moment at A. Again draw this.

This is a point A. And then we have an internal hinge at point B. And then this is point C. This distance is 2L and then in addition to that we have force P here and then a moment at this is M. Then this distance is L here. This distance is also L, this distance is also L.

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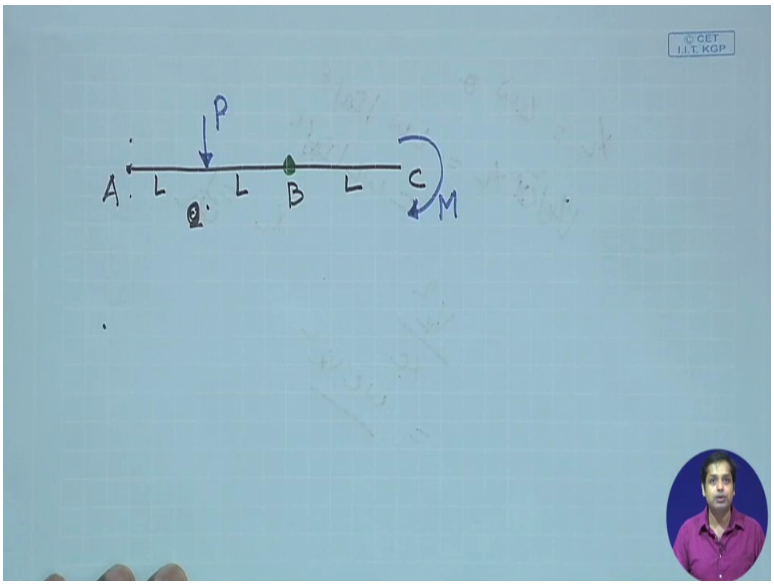

Now what are the forces we have? The forces we have is hereCywhich is the roller support. Then here also we have Ay and then MA. We assume there is no horizontal force so Ax is equal to zero. So it is not shown here.

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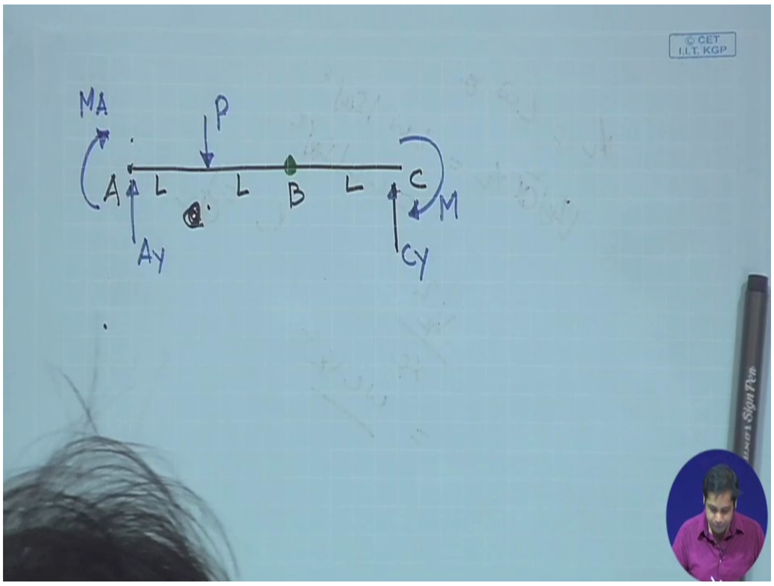

Now there are three unknown here. One is MA, Ay and Cy. Now we need to give the virtual displacement in some way that this unknown can be determined. Now first give virtual displacement like this. And again the virtual displacement should satisfy the internal compatibility. It is not that we have to satisfy the constraint as well. Now the virtual displacement here is something like this.

Means point A to B there is no displacement and since point B we have hinge here, so this part, the part BC can rotate like this. And this satisfy this compatibility displacement. So actually your displacement is this.

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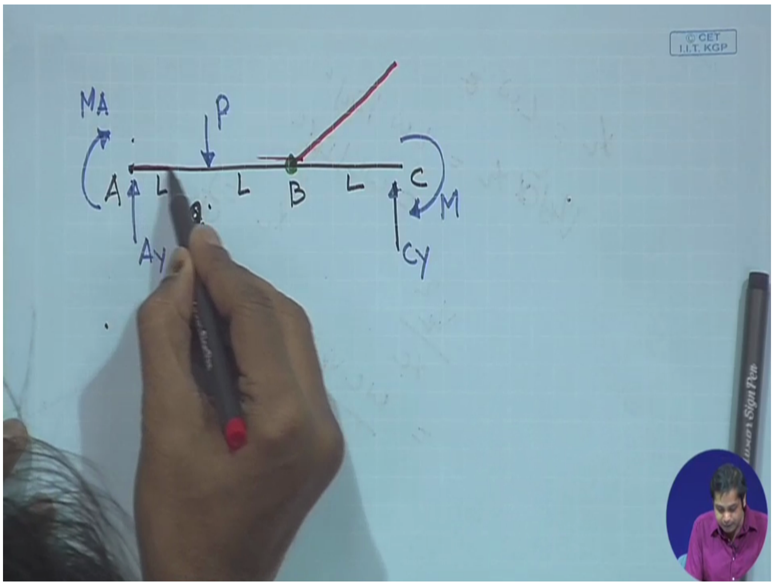

So this displacement is compatible, right? They satisfy the compatibility. Now suppose this is delta. Now if this is delta and this angle is theta,then delta you can write is equal to theta into L, right? This delta is equal to theta into L.

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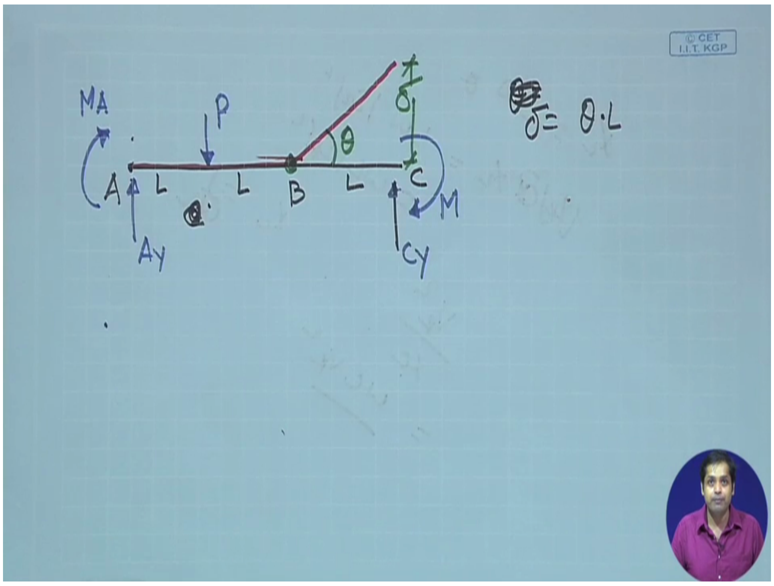

Now let's write the virtual work. The virtual work W will be, what are the contributions we have? See at point A there is norotation, so the contribution of Ay will be zero, contribution from My will be zero. At C there is a translation of delta which is in the direction of Cy. So Cy will contribute and that contribution will be Cy into delta. And then we have moment.

And then (co)associated displacement for the moment will be theta. And that will be minus M into theta. So that is equal to zero, right? Now delta is equal to theta into L. If we substitute delta is equal to theta into L, what we will get is Cy is equal to M by L.

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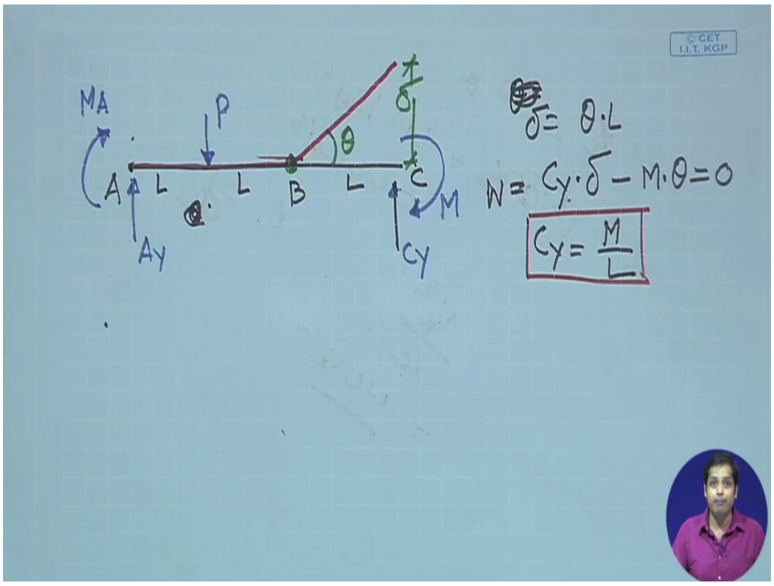

So this we obtained from this virtual work. And if we give this virtual displacement, then other unknowns cannot be determined. Now let us find out other unknowns. Let us find out what is the moment at A. So again take the same problem once again and give a virtual displacement like this.

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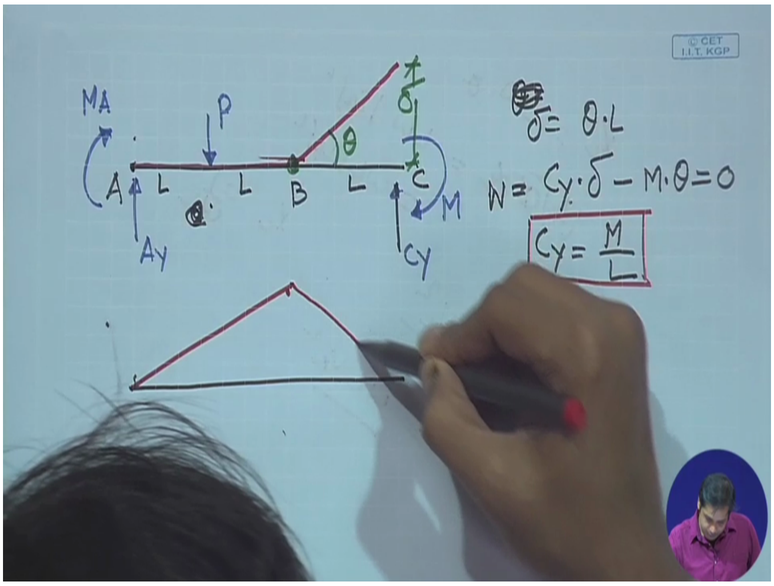

Since at B there is no rotational constraint. So it can take the form. Virtual displacement is still compatible. Andif angle is theta, this angle will become 2 theta because of the length given here. And then this distancebecome 2 theta into L.

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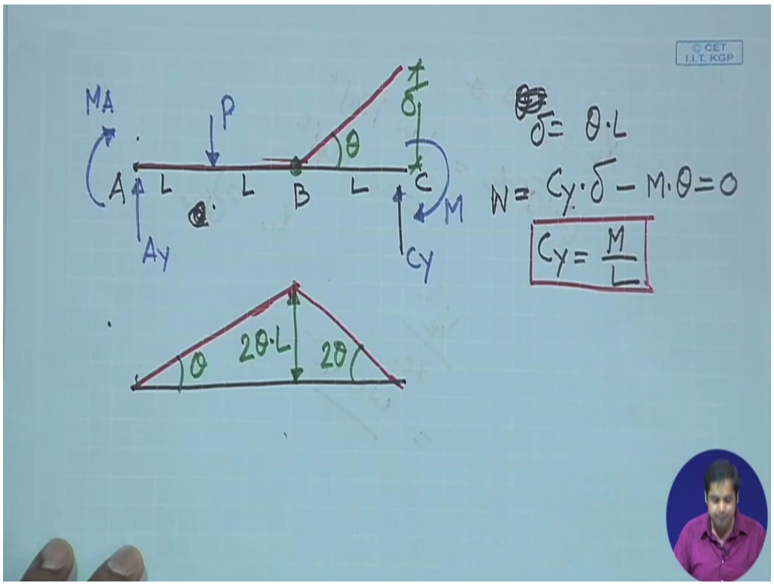

And then we have moment. This isthe moment we have. And then another moment here we have. And thentheforce here.

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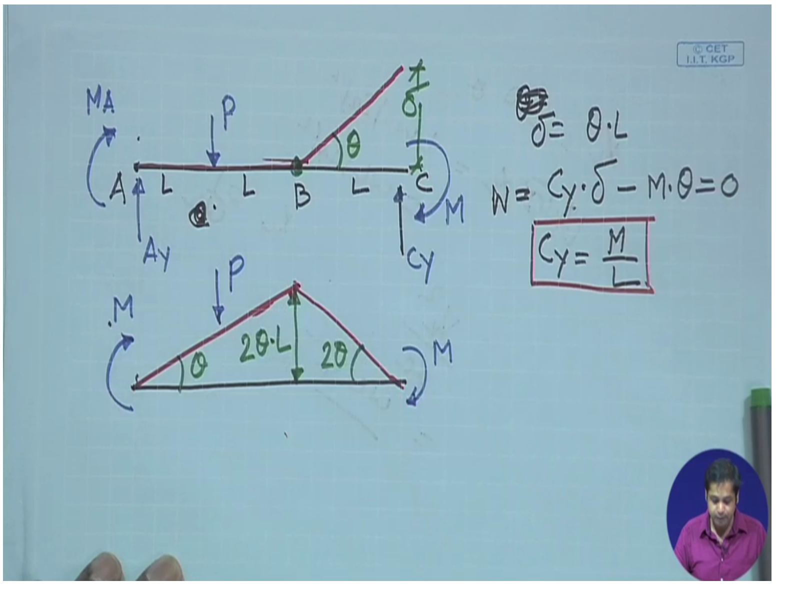

Now there is a force Ay as well. Now because of this virtual displacement Ay will not contribute. M will contribute, this M will contribute. This is MA. MA will contribute, this M will contribute and P will contribute. And if we take the virtual work total, virtual work W will be minus MA into theta.

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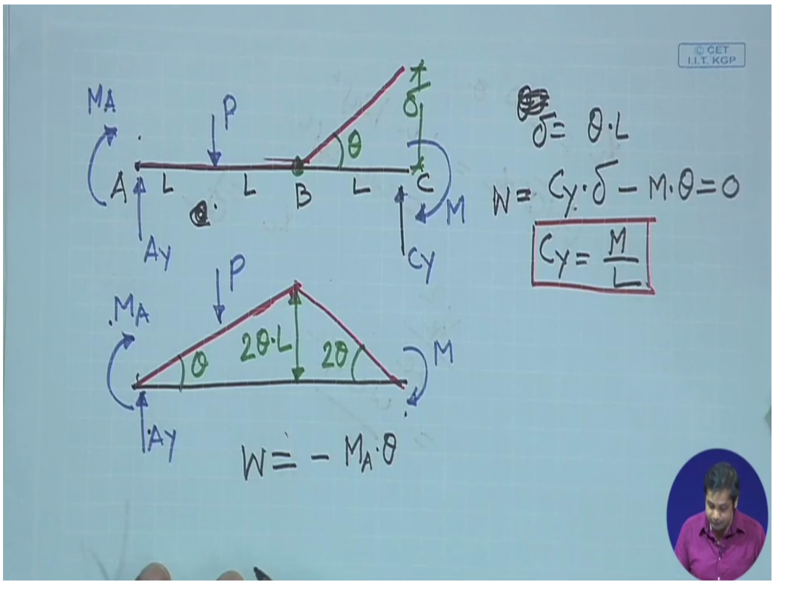

That is how we defined theta. And then contribution from this is plus. That is how we define that. Take this as minus and then minus P into,this P is downward, this is upward, minus P into,if this is to 2 theta L, this distance will be theta into L.

So P into theta into L and then plus this will be 2 theta into M. That is equal to zero.Now in this case that is equal to zero. So this gives us MA is equal to 2M minus PL.

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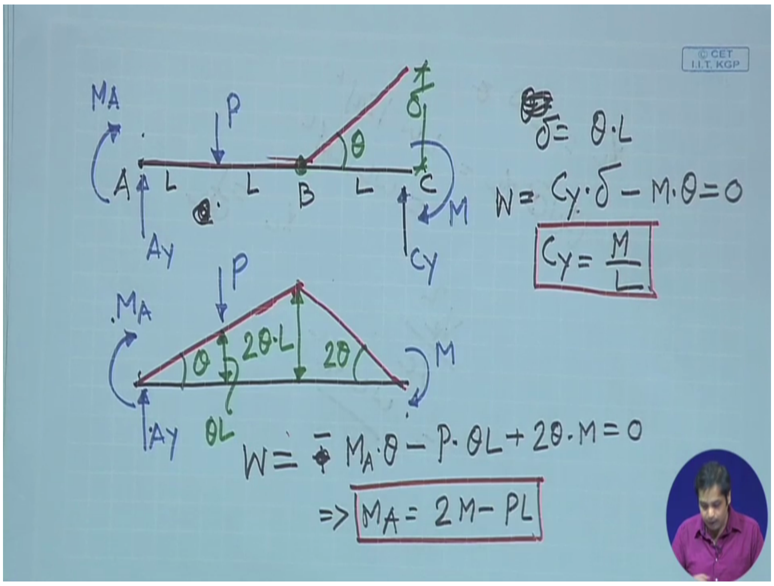

So this is the moment at A. Again we obtained this moment not by an equilibrium condition, by applying the principle of virtual displacement. Nowwe need to find out what is Ay? So we have to give the virtual displacement in such a way that we get the contribution from Ay. Now let us give the virtual displacement like this. Again this is the internal hinge. This is A and then this is C, this is B and we have the forces like this.

This is MA. When you sum then please make sure what sign you take and be consistent with the signs. Let's give a virtual displacement like this and then this is hinge. This is delta. The reason why this virtual displacement is given because you see there is no rotation between AB.

So the contribution from MA will be zero and there is no displacement at point C, so contribution from Cy will be zero. Here we get only contribution from Ay. So this is a virtual displacement.

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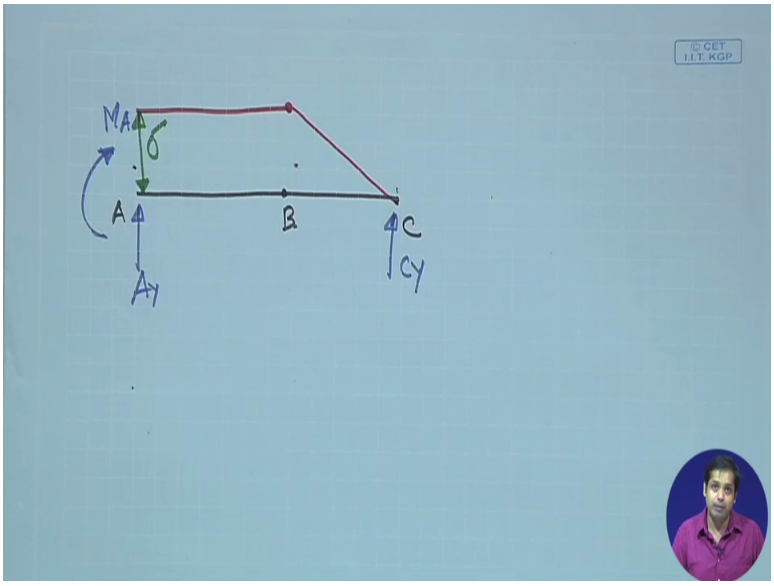

Again it not necessarily need to satisfy the constraints. Only thing it should satisfy the internal compatibility of this member.

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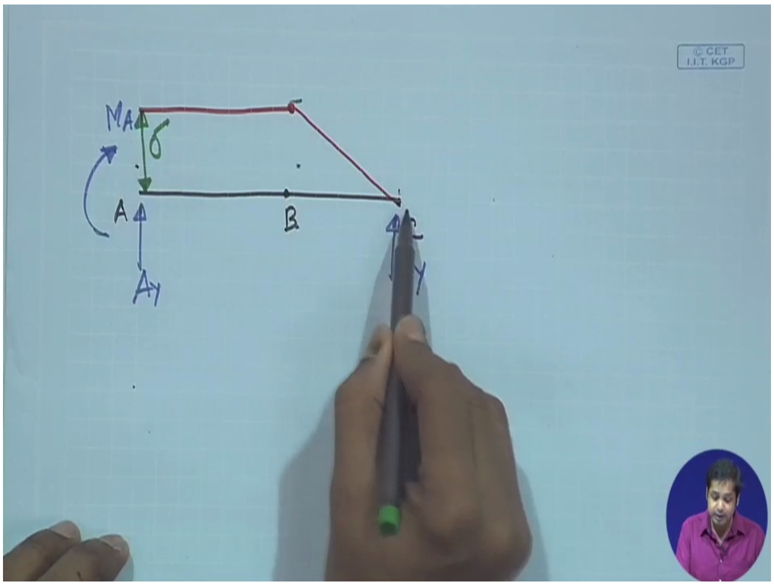

Now if this is theta, then we have another force here, this is P and then we have another moment here which is M.

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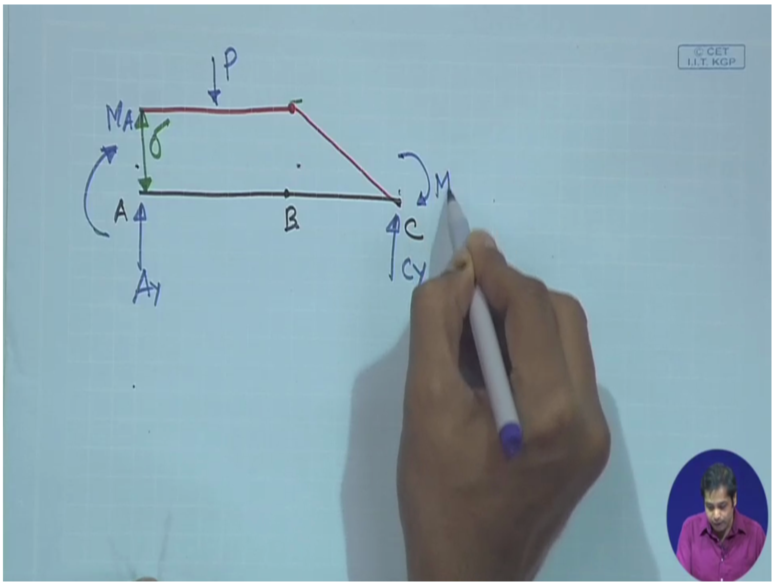

So delta will be is equal to, if this angle is theta and this distance is L and this distance is 2L, this distance become theta into L.So delta is equal to theta into L.

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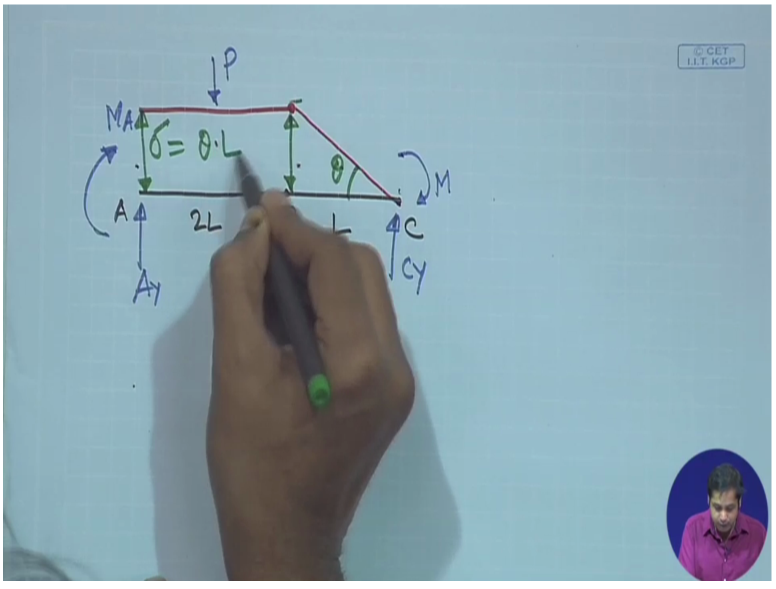

So write virtual work. So virtual work will be, Ay will contribute. So Ay into delta is equal to theta into L and then M will contribute this plus M into theta and then P will contribute. Contribution from P will be minus P into theta into L. Delta is equal to theta. So that is equal to zero and Ay becomes P minus Mby L. You verify with using equilibrium equation, you get this value.

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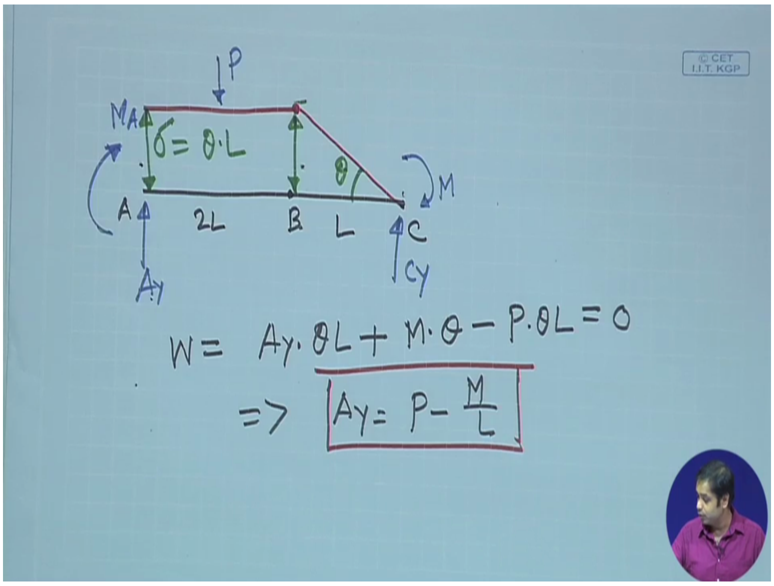

So let us quicklysee. Now you see here,please make sure the sign. The reason sign is because of the action of the slope. Because of the action itload, by intuition, you say that it can cause deflection like this. So this and this moment and this theta,they have the same sense of direction, right? So that's why this is positive.

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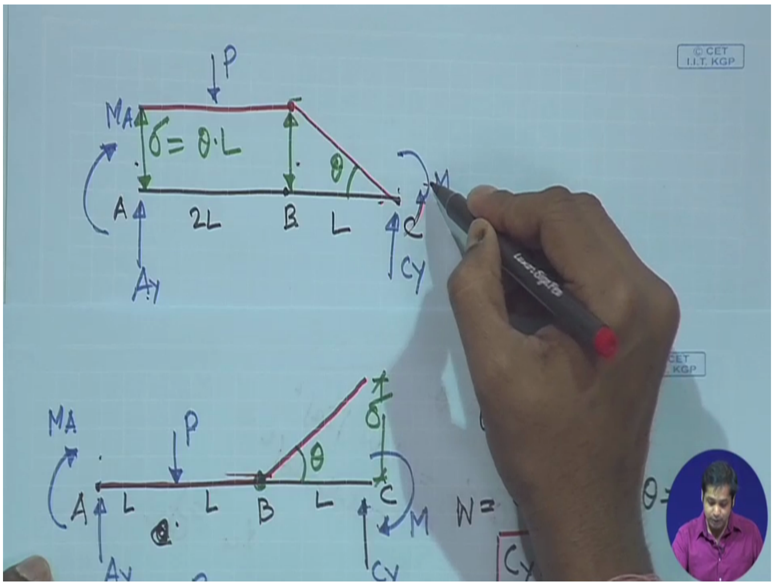

But in this case this M and this theta, they have opposite sense. Direction is in opposite sense. That's why it was negative, right?

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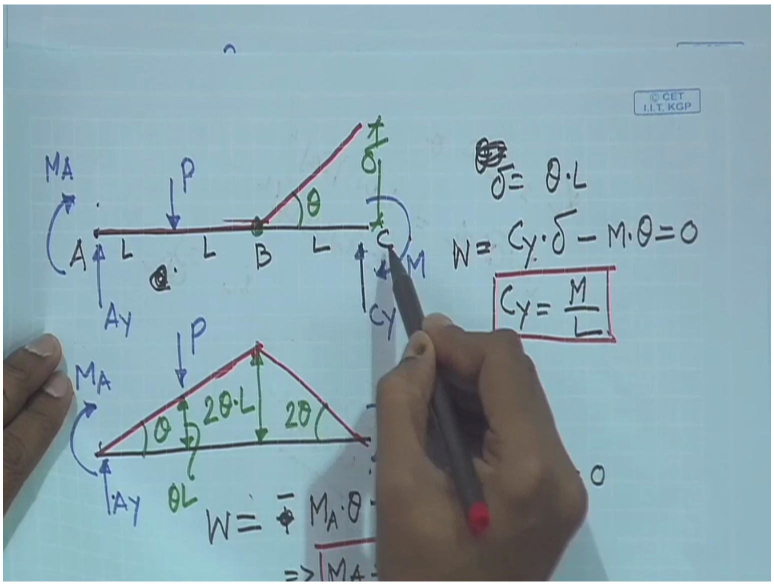

Similarlythis MA and this theta. This MA (wo) would have caused deflection like this. But now we have assume virtual displacement like this. So this MA and theta in terms of directional sense, they are not the same. That's why it is negative.

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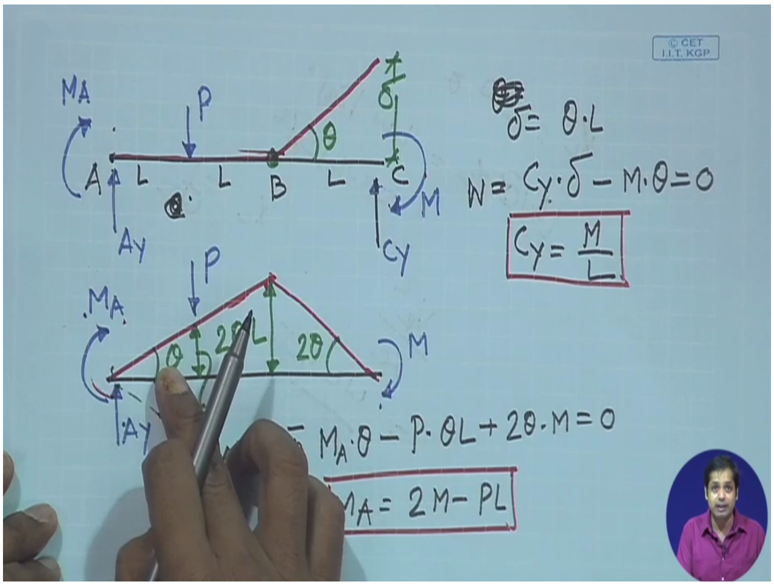

So you take the sign properly. So what you do is,you apply the equilibrium condition and verify whether the support reactions that we have obtained from principle of virtual displacement right or not. Now what we have actually used here? This is fine,this W is equal to zero. Now in addition to this W, all the contribution that in this expression we have, those contribution we have obtained from simple geometry, right? For instance, if this is theta, this has to be 2 theta L, this has to be 2 theta.

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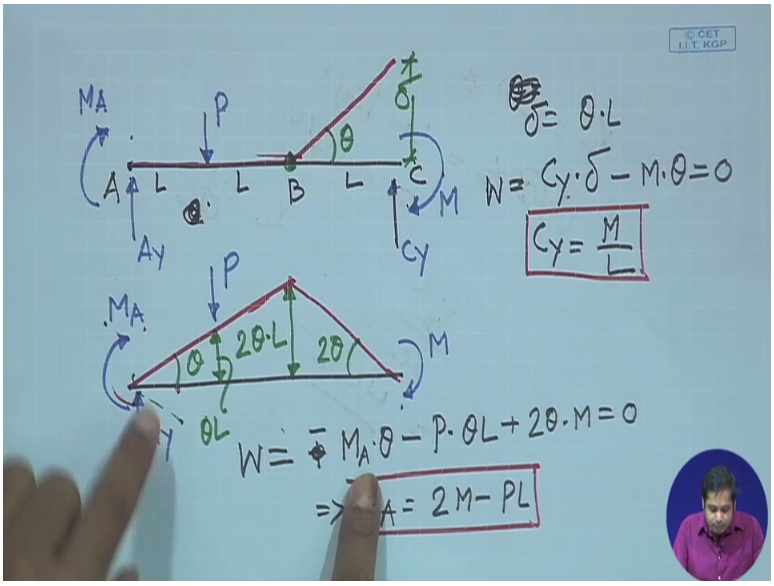

These values that we obtained, thatis simple geometry, right? Similarly in this problem as well. If this is delta then this should be 2 by 3 into delta. That also we have obtained by using simple geometry.

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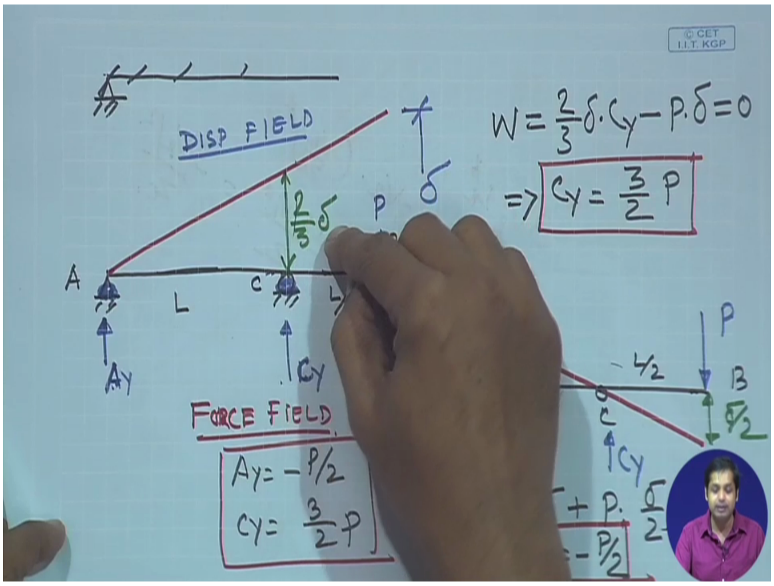

Means what we have done here is. Now these are the two problems that we have just now attempted.

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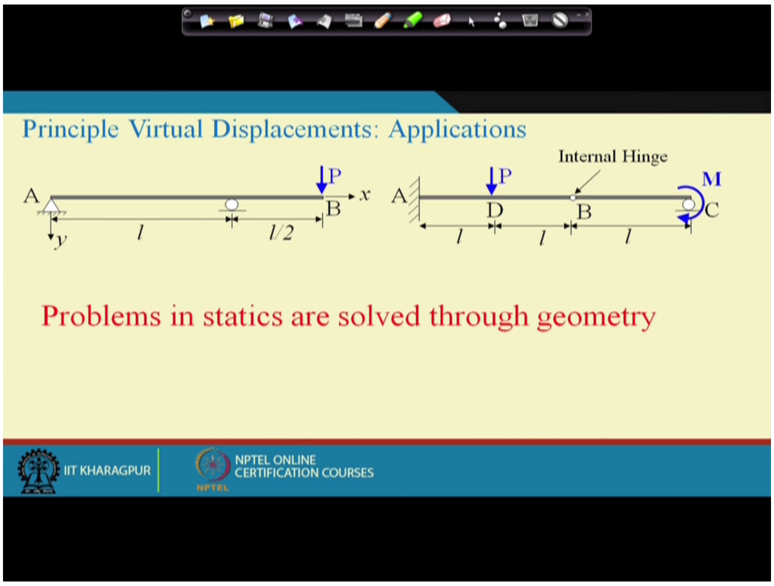

Now finding reactions, suppose a problem is given. We need to find out the reactions. This is essentially a problem in statics, right? Where if it is determinate structure then we need to draw the free body diagram, that's what we learnt as of now.If we forget what you are doing right now, at least till last week what we learnt is, if we draw the free body diagram of the object or the entire object, then apply the equilibrium conditions on the free body diagram and get the reactions.

So this is essentially problem in statics, right? What we have done here is, we solved the problem in statics but not using the equation of statics. What we have used is, principle of virtual displacement. But the virtual work in that equation, each term you have obtained from simple geometry, right? So essentially what we have done is, the principle of statics are solved through geometry.

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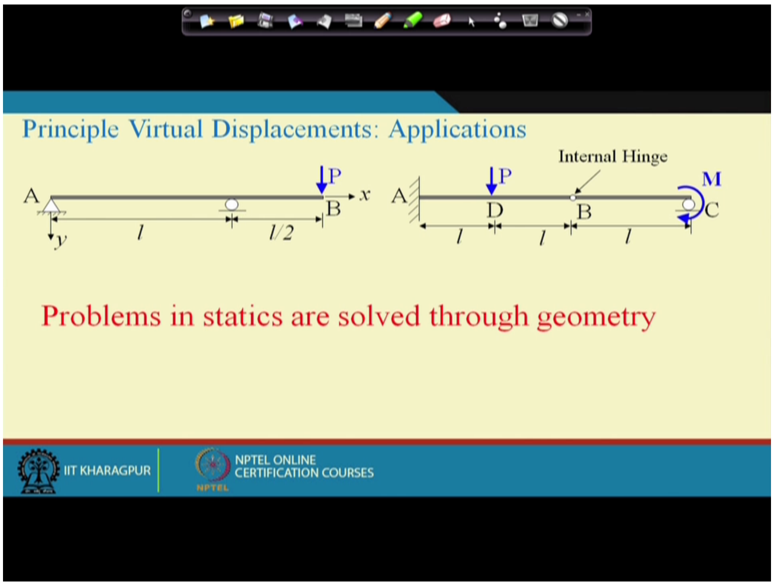

And that's what principle of virtual displacement allows you to do so. Now when we talk about principle of virtual forces then you will see, the virtual force is just the opposite where the problem of geometry will be solved using the concept of statics. But that we will discuss in detail. So we will stop here today.Whatwe have learnt so far is, we learnt what is the concept of virtual work? What is virtual displacement?

Then, what is the concept of virtual work? And then principle of virtual displacement, and how or what is the principle of virtual displacement? What is the principle of virtual work for a rigid body? And then how that principle of virtual displacement can be used to determinesupport reactions for a rigid bodies? But this understanding is not enough to solve the problems that we have toin structural mechanics.

Therefore we need to go beyond. We need to extend this principle, we need to generalize this principle and that we will be doing in the next class. Okay then. I stop here today. I will see you in the next class. Thank you.