

**Structural Analysis 1**  
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**Lecture 11**  
**Analysis of Truss: Method of Sections (Contd.)**

Hello welcome to the last lecture of unit 2. You know in the previous in we introduced the concept of method of sections for determining member forces in statically determinate trusses. What we will do today is we will demonstrate method of sections through some examples. Okay. So now let us now consider this example. Okay.

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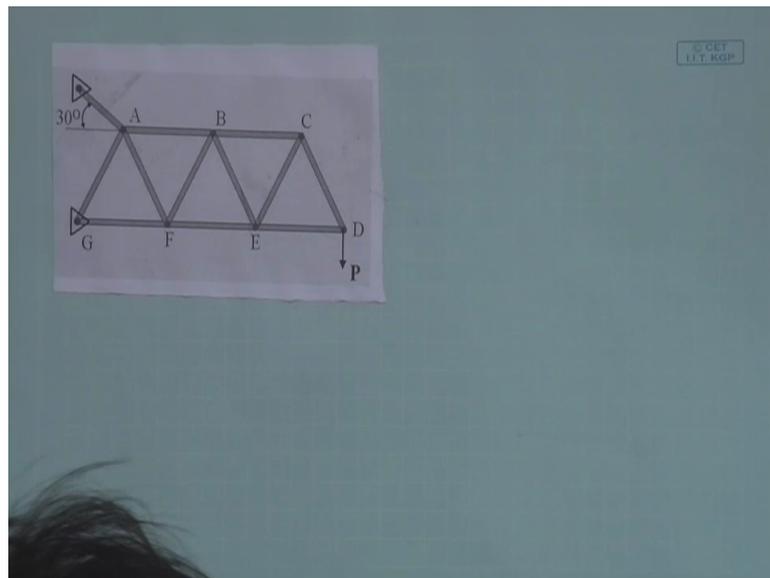
**Method of Sections: Example 2**

Determine forces in members AB, EF, BC, BE and BF. All triangles are equilateral triangle.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Example Courtesy: J. L. Meriam and L. G. Kraige, Engineering Mechanics Statics, John Wiley & Sons, 5<sup>th</sup> Edition

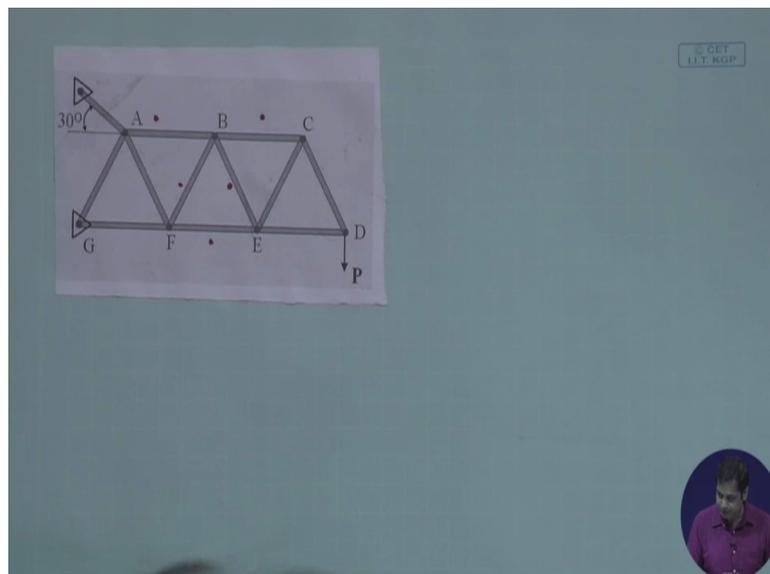
Now you need to find out the member forces in these members. Okay. Now let us see this. Now this is the problem. Okay. We need to find out forces in some of this members. Okay.

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But we will demonstrate the concept through some of the members, but other members also can be determined if they are following the same process. Okay. We need to find out the forces in member AB, this member. Then member EF, this member and BC, this member, BE, this member and BF, this member. These five members we need to determine the member forces, right?

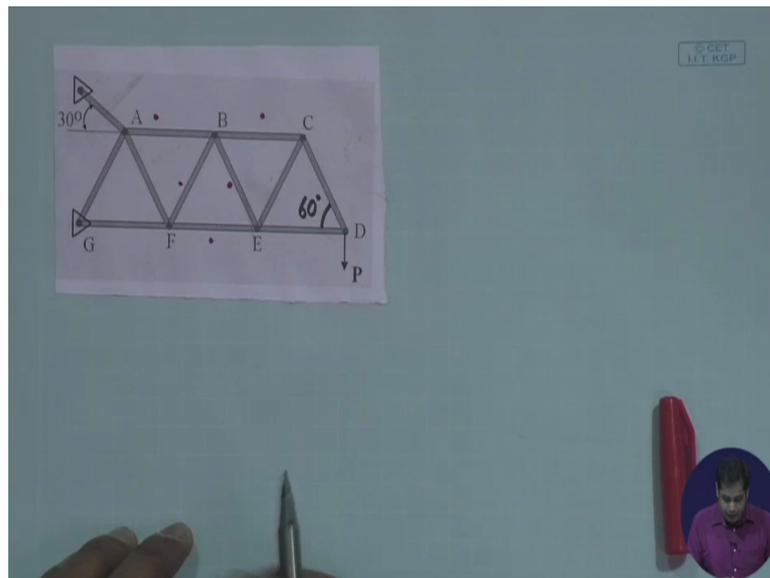
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Now let us first take and that we will be doing using method section. This angle is 60 degree. Okay.

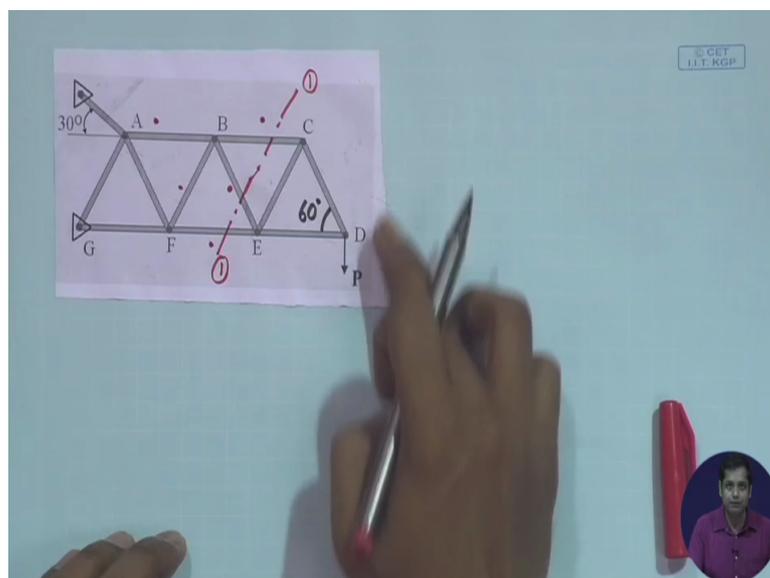


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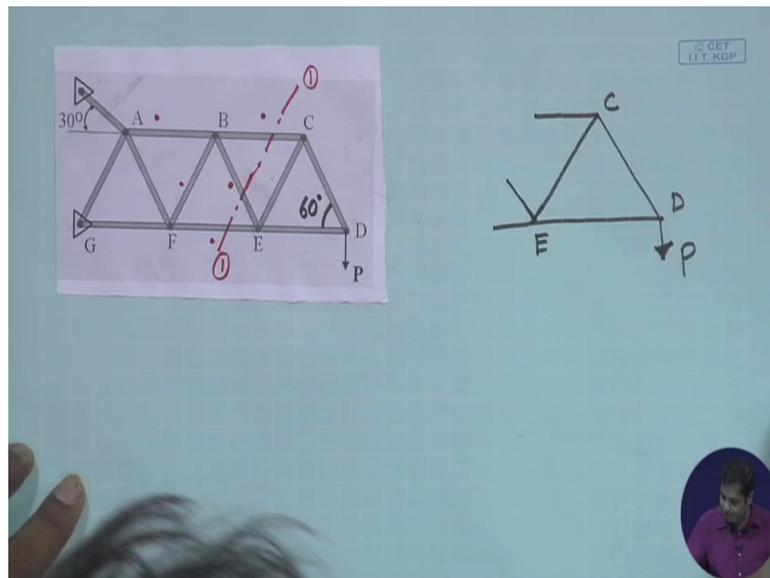
Let us first take a section here. Say this is section 1-1. Okay. Now take this part out from the entire structure and draw the free body diagram of this part. Okay.

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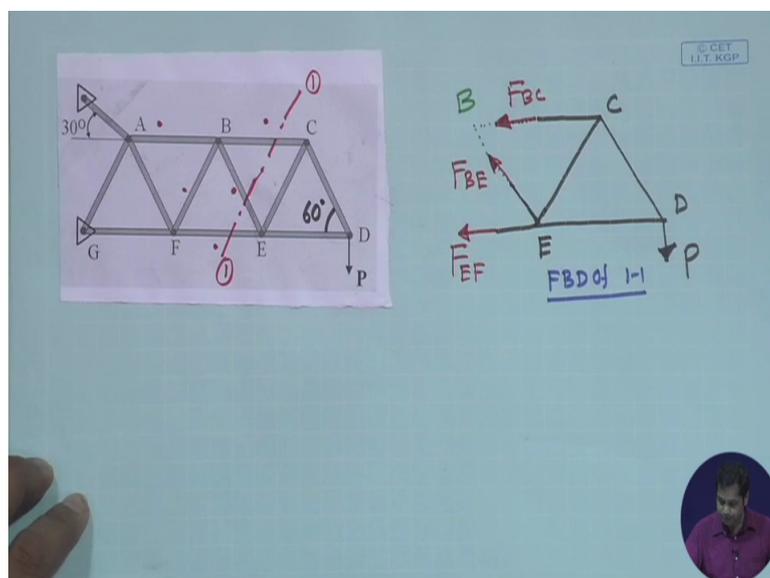
What will be the free body diagram of this part? Let us first draw this part, the free body diagram will be, say this is joint C, this is joint E, this is joint D, joint D we have a force externally applied force P and then here this is the member EF and similarly we have here the member EB. This is EB and this is EF.

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So this joint is actually B. Now draw the forces. The forces will be this force is in member EF so this is FEF. Now this is member BC so this is member FBC. Similarly we have a force in member BE and this is FBE. So this is free body diagram of section 1. This is FBD of section 1-1. Okay.

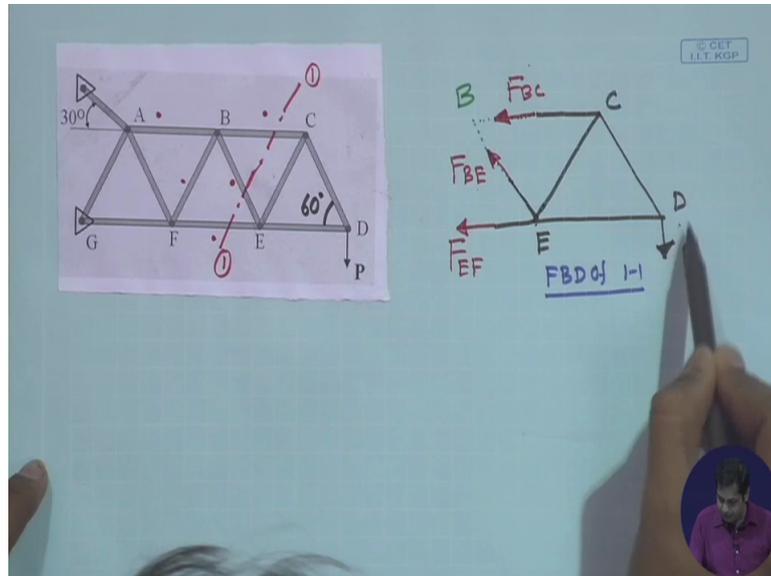
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Now next step is applied the equilibrium condition on this section on this free body diagram. Now if I apply the equilibrium condition first we need to determine member force BC. Now how many unknowns we have here? We have three unknowns. Now if we take say moment about E then what will happen? This B will not contribute because its line of action is passing

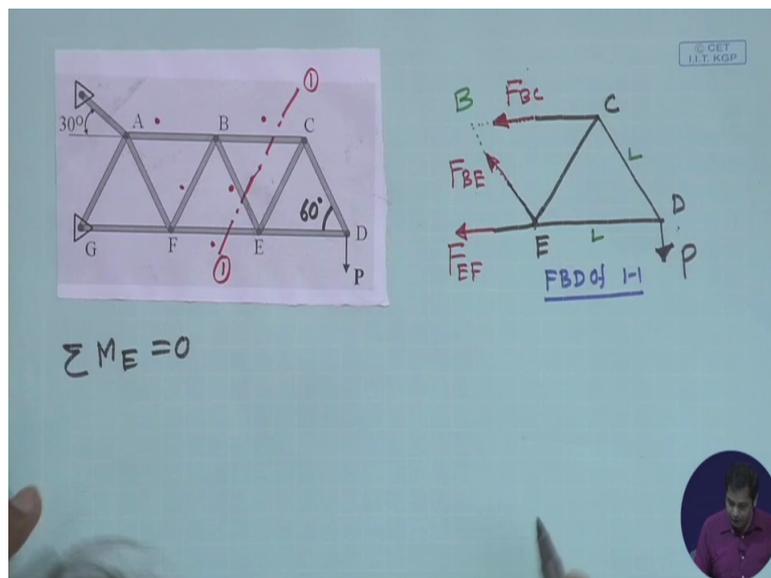
through E. EF will not contribute because its line of action is passing through E. So only force that will contribute is member BC and force P and this externally applied load P.

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So if we write that summation of E is equal to zero. The moment about at point E is equal to zero. So if we take moment FBC will produce anticlockwise moment and P will produce clockwise moment and all these distances are L, equilateral triangle. Okay.

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Now this gives us FBC is minus FBC, this distance is L, this is anti clockwise that is why it is negative, plus P into L which is clockwise. This is equal to zero. And this gives you FBC is

equal to P. So FBC is this. Okay. So we e already have determined FBC. FBC is determined, right Okay.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2}L + P \cdot L = 0$   
 $\Rightarrow F_{BC} = \frac{2}{\sqrt{3}}P$

Now what we next we need to determine is member force in BE. Now if we take summation of FY is equal to zero, so what are the forces that will contribute? FBC is in horizontal direction so it will not contribute, it is also, it will not contribute. P will contribute, it is in vertical direction and the vertical component of B will contribute. Now the vertical component of B is in upward direction and P is in downward direction, so this will be negative and this will be positive.

So vertical component of B will be,  $F_{BE} \cos 30$  degree minus P. P is downward direction. That is equal to zero. So this gives you  $F_{BE}$  is equal to  $2P$  by root 3. Okay. So force in member BC is this. Okay.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2}L + P \cdot L = 0$   
 $\Rightarrow F_{BC} = \frac{2}{\sqrt{3}}P$

$\sum F_y = 0$   
 $F_{BE} \cos 30^\circ - P = 0$   
 $\Rightarrow F_{BE} = \frac{2P}{\sqrt{3}}$

Now next is when you determine force in member EF. So BE is done and also you need to determine force in member EF. Force in member EF is if you take summation of FX is equal to zero and what are the forces we will contribute? This force, this force and the horizontal component of this force. And those forces are minus FBC, because this is this direction and as per our sign convention this is negative.

And then minus FEF, this is also negative as per our sign convention and then again minus FBE, this is also the horizontal component of this force will be this direction, therefore this is negative. That is equal to zero. Now from that we have determined BC, we have already determined BE, only unknown is EF and from this you can calculate EF is equal to minus root 3 P. Okay. So this is EF.

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$\sum F_y = 0$   
 $F_{BC} \times \frac{\sqrt{3}}{2} L + P \cdot L = 0$   
 $F_{BC} = \frac{2}{\sqrt{3}} P$

$\sum F_y = 0$   
 $F_{BE} \cos 30^\circ - P = 0$   
 $\Rightarrow F_{BE} = \frac{2P}{\sqrt{3}}$

$\sum F_x = 0$   
 $-F_{BC} - F_{EF} - F_{BE} \cos 60^\circ = 0$   
 $\Rightarrow F_{EF} = -\sqrt{3} P$

So just by considering one section we could determine the forces in members BC, forces in member BE and forces in member EF. Now remember if you have to determine this member forces using method of joints at least for this problem then what you had to do is first you had to consider this joint and calculate forces in member this and this.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2} L + P \cdot L = 0$   
 $\Rightarrow F_{BC} = \frac{2}{\sqrt{3}} P$

$\sum F_y = 0$   
 $F_{BE} \cos 30^\circ - P = 0$   
 $F_{BE} = \frac{2P}{\sqrt{3}}$

$\sum F_x = 0$   
 $-F_{BC} - F_{EF} - F_{BE} \cos 60^\circ = 0$   
 $\Rightarrow F_{EF} = -\sqrt{3} P$

Once you have this, consider this joint, then calculate force in member this and this and then consider this joint, calculate force member in this and this.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2} L = 0$   
 $\Rightarrow F_{BC} = \frac{2P}{\sqrt{3}}$

$\sum F_y = 0$   
 $F_{BE} \cos 30^\circ - P = 0$   
 $\Rightarrow F_{BE} = \frac{2P}{\sqrt{3}}$

$\sum F_x = 0$   
 $-F_{BC} - F_{EF} = 0$   
 $-F_{BE} \cos 60^\circ = 0$   
 $\Rightarrow F_{EF} = -\sqrt{3}P$

So we have to use free body diagram of 3 joints. But in this case just with one section we could determine the member forces. Now still we are not done. We need to determine the forces in other two members as well, this member and this member as well.

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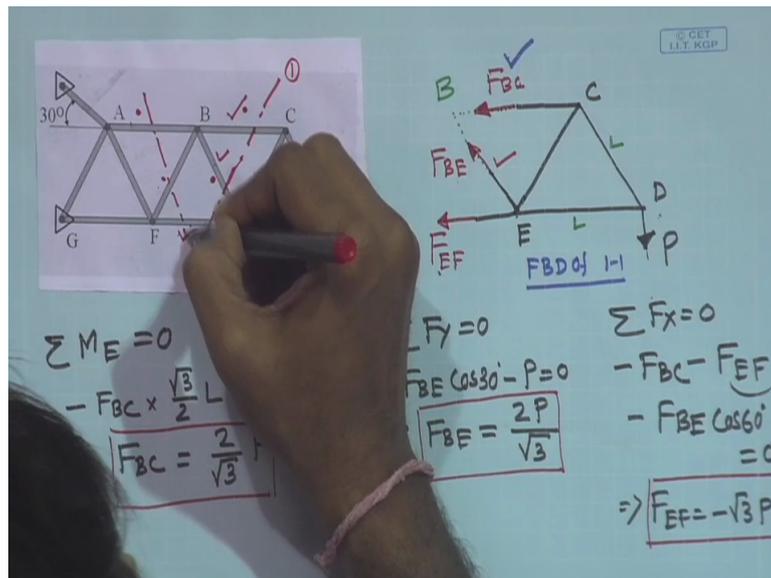
$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2} L = 0$   
 $\Rightarrow F_{BC} = \frac{2P}{\sqrt{3}}$

$\sum F_y = 0$   
 $F_{BE} \cos 30^\circ - P = 0$   
 $\Rightarrow F_{BE} = \frac{2P}{\sqrt{3}}$

$\sum F_x = 0$   
 $-F_{BC} - F_{EF} = 0$   
 $-F_{BE} \cos 60^\circ = 0$   
 $\Rightarrow F_{EF} = -\sqrt{3}P$

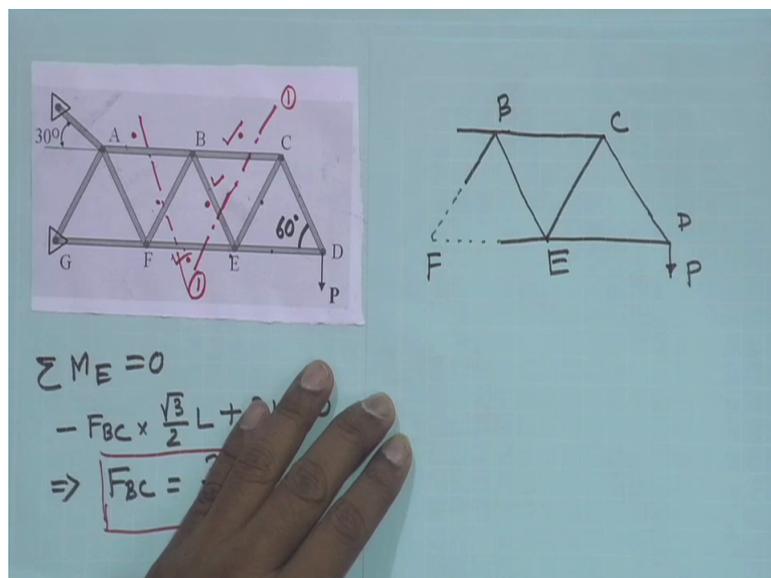
Now in order to do that let us take one more section and let us take another section, this section. Okay and let us draw the free body diagram of this section.

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Now draw the free body diagram. Free body diagram will be like this. This is triangle ECD, then another triangle, this is BCE and then AB, BF and then EF. And this is D which is force P, this is C and then this is E, B and if you extend the line of action this is point F, right?

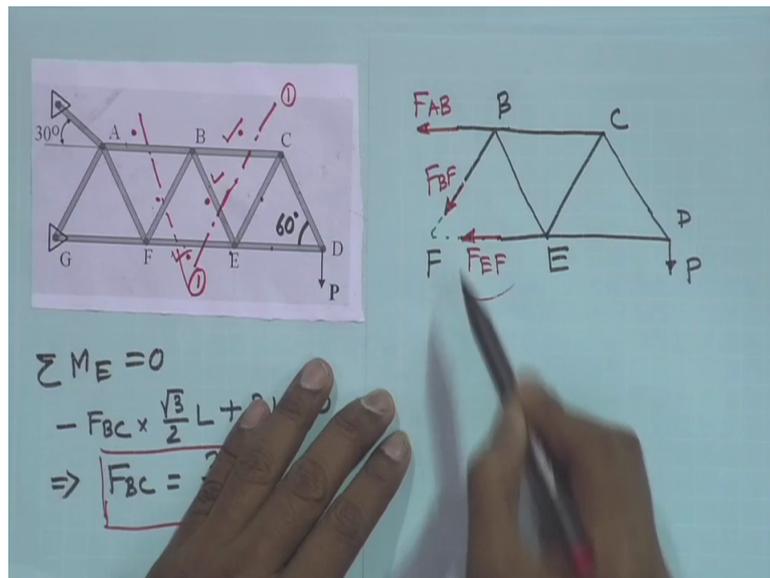
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Now what are the forces we have? This will be the force member AB then force in member BF, FBF and then here the force in member EF. What we need to determine? We need to determine the force in member FB and BF. This is already known because we have already computed this considering the other section.

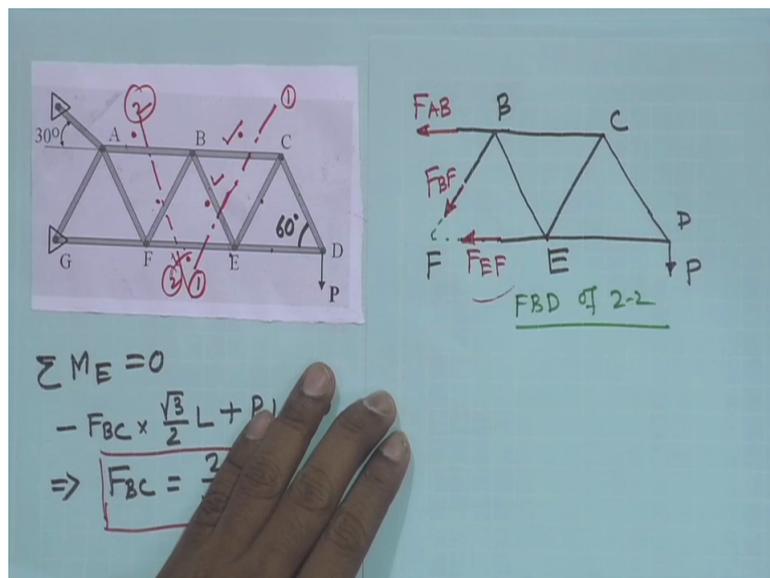


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So this is free body diagram of 2-2. Say this is section 2-2.

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Now next take summation of  $F_Y$  is equal to zero. The forces that will contribute this is this vertical component of this and this. And this component will be negative and  $P$  is also negative because they are downward. So this is minus  $F_{BF}$ , this is 30 degree.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2}L + P \cdot L = 0$   
 $\Rightarrow F_{BC} = \frac{2}{\sqrt{3}}P$

$\sum F_y = 0$   
 $-F_{BF} = 0$

Minus P is equal to zero. So FBF is equal to minus 2P by 3. Okay. Now then this is member FBF. Okay.

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$\sum M_E = 0$   
 $-F_{BC} \times \frac{\sqrt{3}}{2}L + P \cdot L = 0$   
 $\Rightarrow F_{BC} = \frac{2}{\sqrt{3}}P$

$\sum F_y = 0$   
 $-F_{BF} \cos 30^\circ - P = 0$   
 $\Rightarrow F_{BF} = -\frac{2P}{\sqrt{3}}$

Now next take, so this is determined, AB needs to be determined. Now if you take moment about F, it is not that you have to see if draw the point F does not belong to (diag) this part of the structure. But equilibrium when we say the summation of moment is equal to zero means summation of moment about any axis is equal to zero taking at any point is equal to zero, right?

So now take if you take moment about point F, the advantage will be because this force and this force will pass through this point, so they will not contribute. Only nonzero contribution will be from force P and from force AB.

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$$\sum F_y = 0$$

$$-F_{BF} \cos 30^\circ - P = 0$$

$$F_{BF} = -\frac{2P}{\sqrt{3}}$$

So if we take summation of moment about point F is equal to zero. So this length is L, these are L. So P will produce clockwise moment and F will (probu) produce anticlockwise moment that is equal to zero. And then finally we get FAB is equal to 4 root 3. So this is member force in AB.

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$$\sum M_F = 0$$

$$\Rightarrow P \times 2L - F_{AB} \cdot \frac{\sqrt{3}}{2} L = 0$$

$$\Rightarrow F_{AB} = \frac{4P}{\sqrt{3}}$$

$$\sum F_y = 0$$

$$-F_{BF} \cos 30^\circ - P = 0$$

$$F_{BF} = -\frac{2P}{\sqrt{3}}$$

So like this if you have to determine the forces in other members you can take other sections and continue this process to determine the forces in all the members. Okay. Now let us demonstrate one more example, let us see one more example. This is example 2 and the previous one was example 1.

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Method of Sections: Example 2

Determine forces in members IK and HK

40kN → A, 40kN → D, 40kN → G

Dimensions: 2.7 m, 2.4 m, 7.5 m, 3 @ 2.7 m

Example Courtesy: F. P. Beer, E. R. Johnston and E. L. Eisenberg, Vector Mechanics for Engineers, Statics, Mc Graw Hill, 8th Edition

What you need to determine here is, you need to determine forces in member IK and forces in member HK. These two, this member and this member you need to determine the forces using method of joints. Okay.

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Method of Sections: Example 2

Determine forces in members IK and HK

40kN → A, 40kN → D, 40kN → G

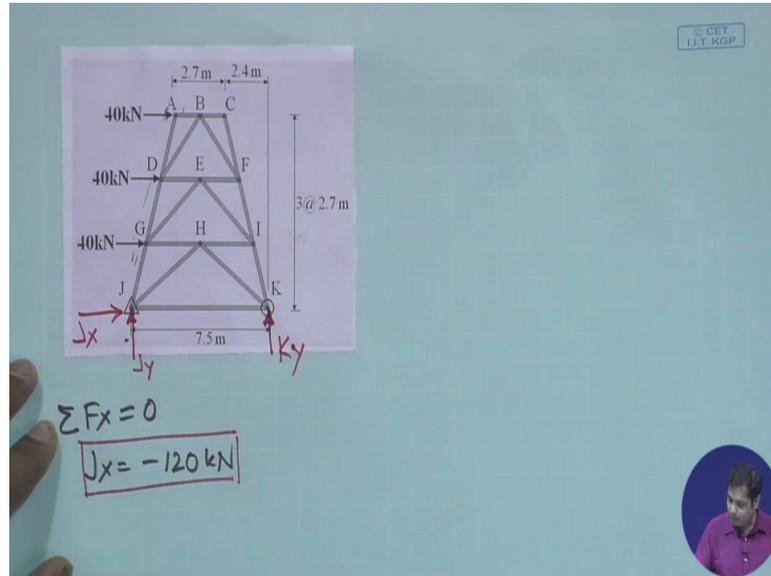
Dimensions: 2.7 m, 2.4 m, 7.5 m, 3 @ 2.7 m

Example Courtesy: F. P. Beer, E. R. Johnston and E. L. Eisenberg, Vector Mechanics for Engineers, Statics, Mc Graw Hill, 8th Edition



Now take summation of  $F_x$  is equal to zero. Only forces we have in  $J_x$  and this is externally applied forces, so that immediately gives you  $J_x$  is equal to minus 120 Newton. So this is  $J_x$ . Okay.

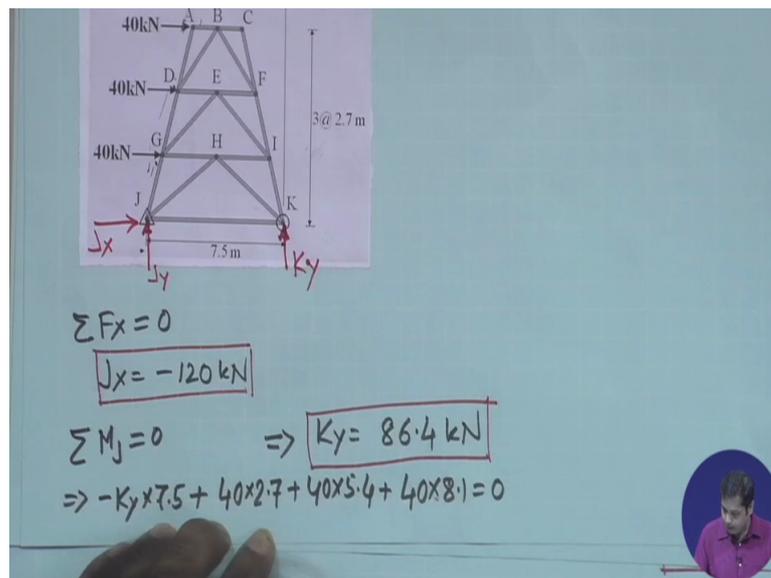
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Then if we take summation of moment about J is equal to zero, then what are the forces which will contribute to this moment? All these forces which will produce clockwise moment and then force  $K_y$  which will provide anticlockwise moment. Okay.

So this gives you minus  $K_y$  into 7 point 5 plus, this is in clockwise that is why it is minus and then 40 into 2 point 7 plus this, 40 into 5 point 4 plus another 40 into 8 point 1. That is equal to zero. Okay. And that will give you  $K_y$  is equal to 86 point 4 kilo Newton. Okay.

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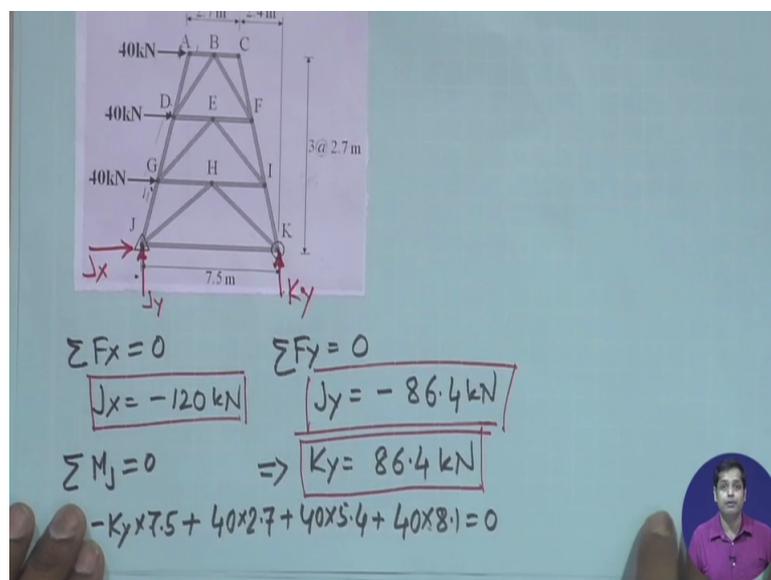
The diagram shows a truss structure with three horizontal levels of joints. The top level has joints A, B, and C; the middle level has D, E, and F; and the bottom level has G, H, and I. The base joints are J and K. Three horizontal forces of 40 kN are applied at joints A, D, and G. The truss is supported by a pin support at joint J and a roller support at joint K. The horizontal distance between J and K is 7.5 m. The vertical height of the truss is 8.1 m, divided into three equal segments of 2.7 m each. Reaction forces  $J_x$ ,  $J_y$ , and  $K_y$  are indicated at the supports.

Handwritten equations on the whiteboard:

$$\sum F_x = 0$$
$$J_x = -120 \text{ kN}$$
$$\sum M_J = 0 \Rightarrow K_y = 86.4 \text{ kN}$$
$$\Rightarrow -K_y \times 7.5 + 40 \times 2.7 + 40 \times 5.4 + 40 \times 8.1 = 0$$

And then summation of  $F_y$  is equal to zero so vertical forces only. Vertical forces are this and this and this gives you  $J_y$  is equal to 86 point 4 kilo Newton. Okay. So these are the member forces, right? Okay.

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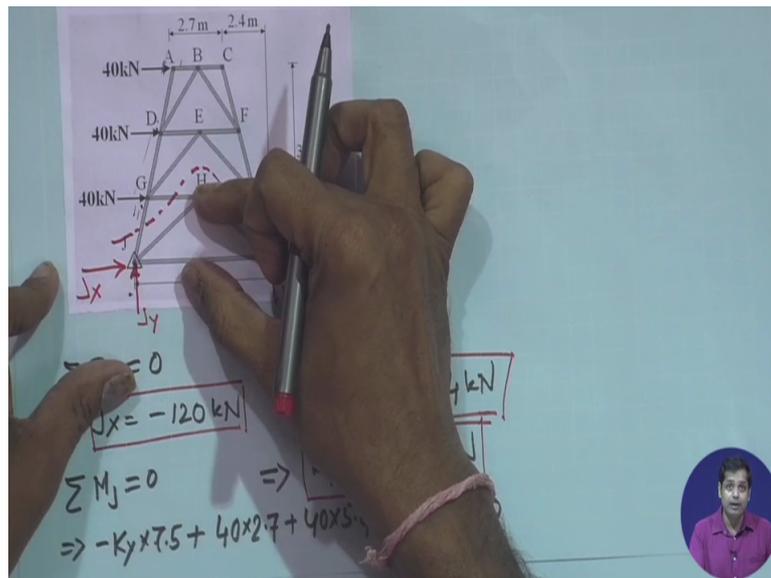
The diagram is identical to the one in the previous slide, showing a truss structure with horizontal forces and reaction forces.

Handwritten equations on the whiteboard:

$$\sum F_x = 0 \quad \sum F_y = 0$$
$$J_x = -120 \text{ kN} \quad J_y = -86.4 \text{ kN}$$
$$\sum M_J = 0 \Rightarrow K_y = 86.4 \text{ kN}$$
$$-K_y \times 7.5 + 40 \times 2.7 + 40 \times 5.4 + 40 \times 8.1 = 0$$

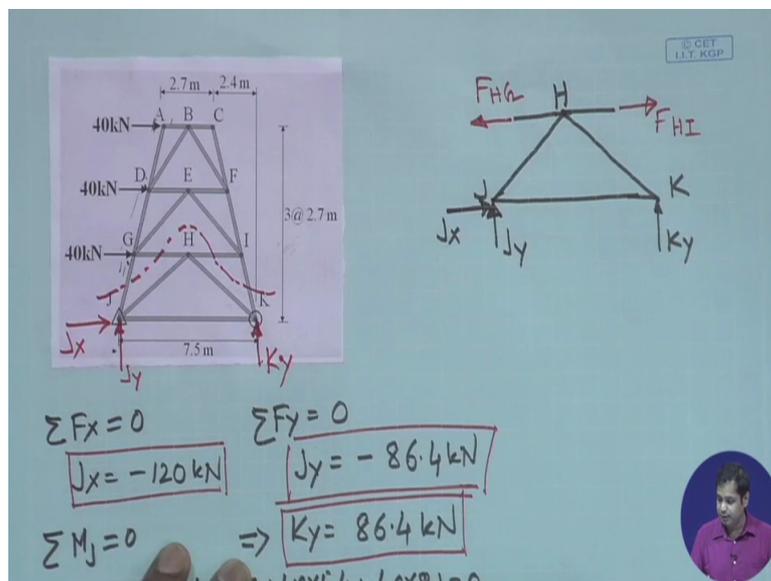
Now next let us take a section like this. Okay. And then take the free body diagram of this part. If we take the free body diagram of this part, this becomes.

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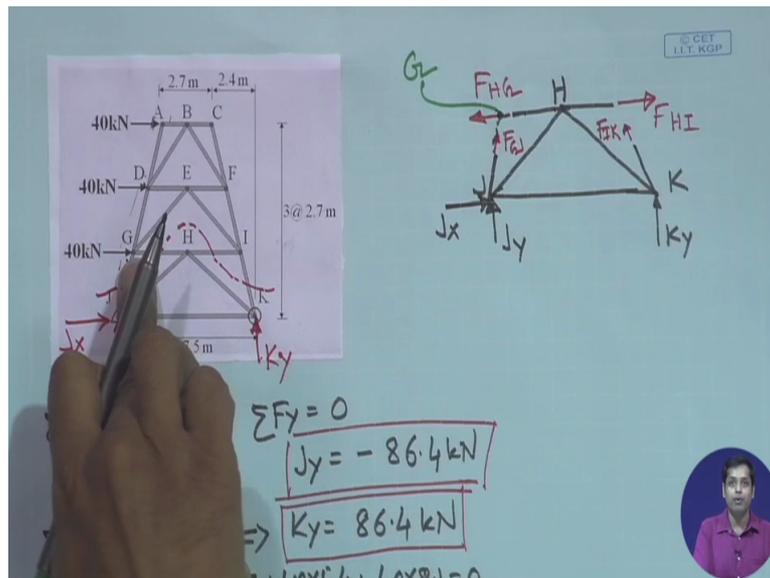
This is J, this is K and this is JY support reactions, this is JX, this is KY and this member is HI. This is H, so this member is HI, so this force will be F of HI and this force is F of HG. Okay.

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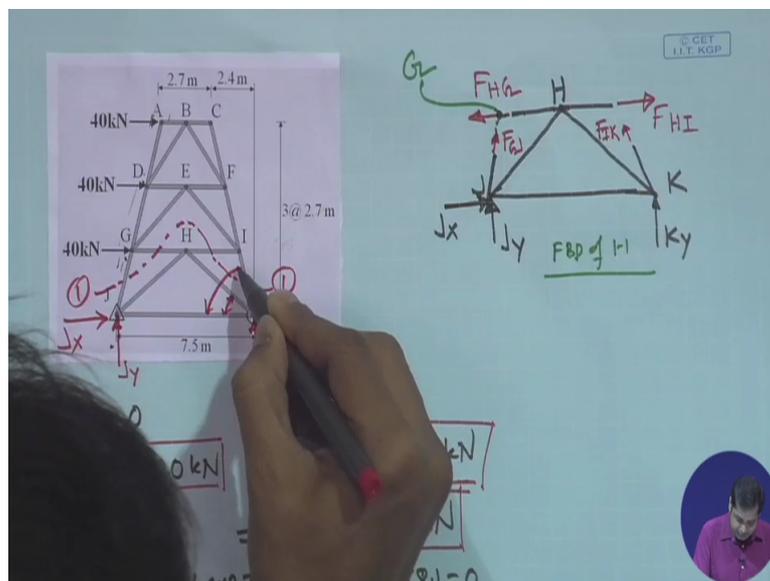
Now and one more, this and this. This is member IK so this is force IK and another is this force GJ. Okay. And their line of action, if we join together they will meet here and this point is point G. Okay.

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Now this is the free body diagram of FBD of section 1-1. So this section is 1-1, right? Now once we have the free body diagram let us apply the equilibrium condition on this free body diagram. But before that what we need is, we need this angle and also this angle. Okay.

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Now suppose this angle is theta 1 and this angle is theta 2. So all the dimensions are given. You can verify that theta 1 which is the smaller one, theta 1 comes around 35 point 75 and theta 2 is 73 point 5. Okay. That you can determine. Just apply simple trigonometry on this. Okay. Because all these dimensions are given. So this is theta 1 and theta 2.



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$\sum F_x = 0 \Rightarrow J_x = -120 \text{ kN}$   
 $\sum F_y = 0 \Rightarrow J_y = -86.4 \text{ kN}$   
 $\sum M_J = 0 \Rightarrow K_y = 86.4 \text{ kN}$

$\theta_1 = 35.75^\circ$   
 $\theta_2 = 73.5^\circ$

These are equal because once we take the summation of forces in particular direction we have to take the components. Okay. Now we need to determine force in member IK. Okay. Now how many unknown we have in this figure? We have these are the support reactions which are known. We have this is unknown, this is unknown, then this is unknown, this is unknown. Okay.

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$\sum F_x = 0 \Rightarrow J_x = -120 \text{ kN}$   
 $\sum F_y = 0 \Rightarrow J_y = -86.4 \text{ kN}$   
 $\sum M_J = 0 \Rightarrow K_y = 86.4 \text{ kN}$

$\theta_1 = 35.75^\circ$   
 $\theta_2 = 73.5^\circ$

Now if we take summation of  $F_x$  is equal to zero and summation of  $F_y$  is equal to zero then what happens? Then still all this unknowns will be there, right? So instead of that let us take summation of moment about point G is equal to zero. Now if we take summation of moment

about point G then what will happen? If the line of action of this force passing through point G, so it will not contribute, this will not contribute.

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$\sum F_x = 0$   
 $J_x = -120 \text{ kN}$

$\sum F_y = 0$   
 $J_y = -86.4 \text{ kN}$

$\sum M_j = 0$   
 $\Rightarrow K_y = 86.4 \text{ kN}$

$\Rightarrow -K_y \times 7.5 + 40 \times 2.7 + 40 \times 8.1 = 0$

Now line of action of FGJ that is also passing through G. So this will also not contribute. Line of action of HJ, that is also passing through point G. So this will also not contribute. So if we take moment about J, the only forces that will contribute to this moment are JX, then JY and then KY and this member force in member IK, this one. Okay.

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$\sum F_x = 0$   
 $J_x = -120 \text{ kN}$

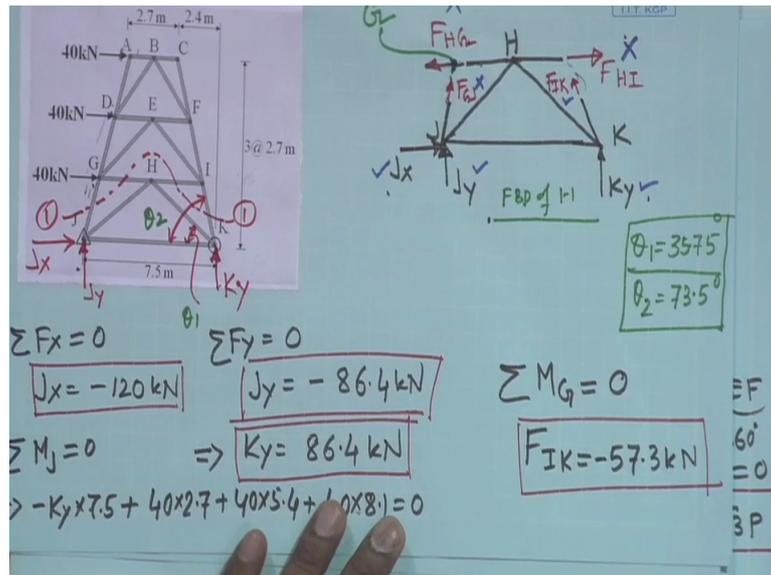
$\sum F_y = 0$   
 $J_y = -86.4 \text{ kN}$

$\sum M_j = 0$   
 $K_y = 86.4 \text{ kN}$

$\Rightarrow -K_y \times 7.5 + 40 \times 2.7 + 40 \times 8.1 = 0$

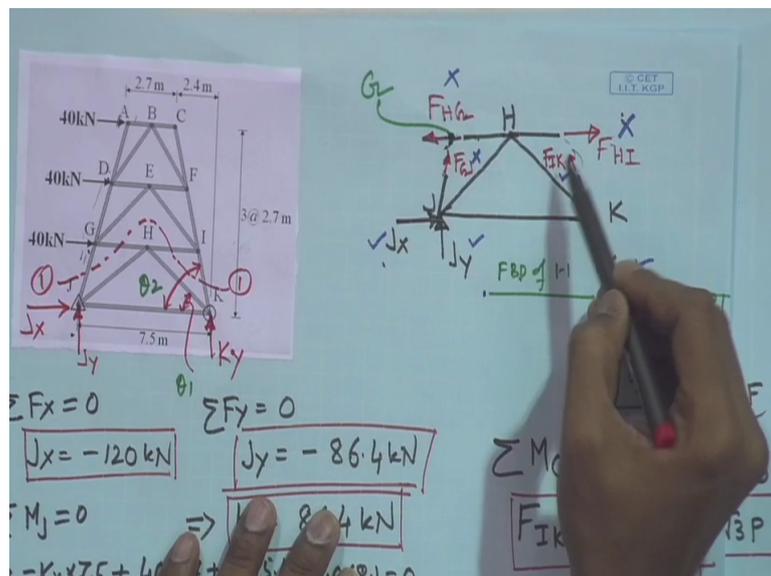
Now all these four forces only one unknown. We know JX and JY. Just now we have determined and we know KY also determined and we need to determine only IK. Now so next step would be summation of M with about G that is equal to zero. Okay. Now all the forces are determined, the dimensions are given. If you substitute that then what we get is FIK is equal to this. Okay.

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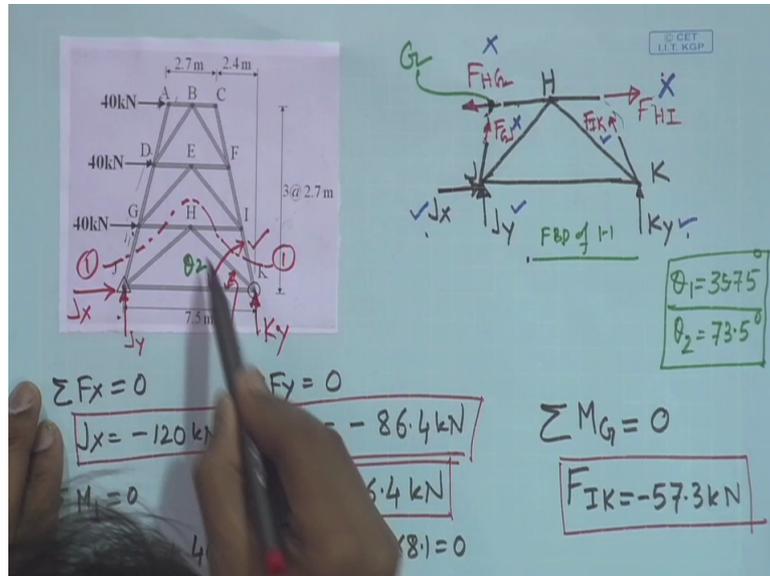
That exercise just substituting all these values and arriving at this value I leave it to you. But the important point here is you we have taken moment about G so that the contribution of all this forces becomes zero. The only contribution we get from this.

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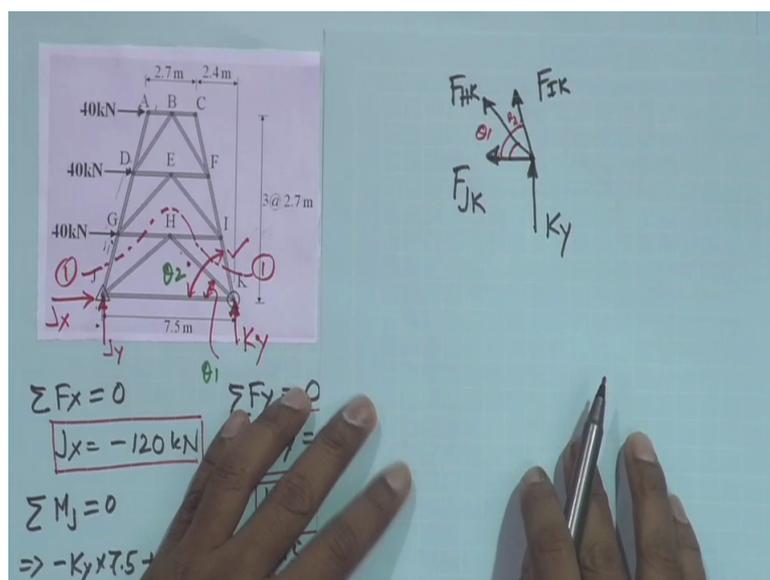
So now we have the force in member IK is determined. Okay. Now another force we need to determine is HK. We need to determine the force in member HK. This as well.

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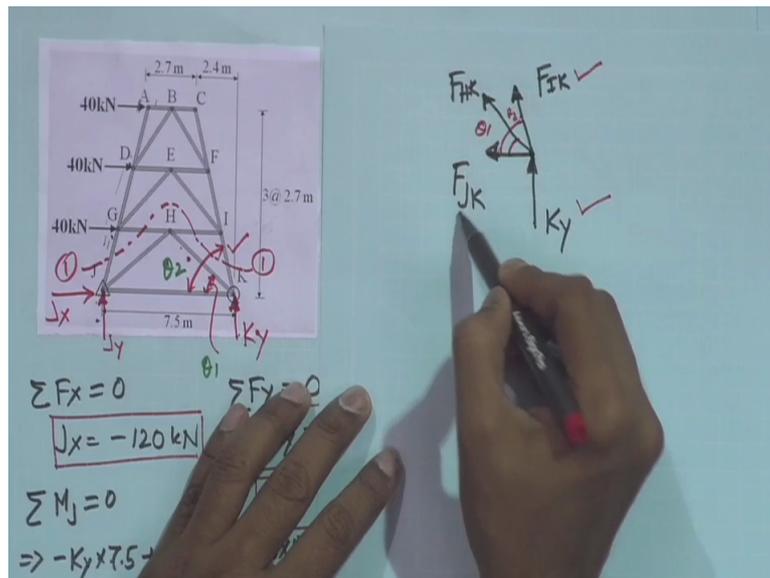
Now in order to do that let us take free body diagram of joint K. Now if we take free body diagram of joint K what are the forces we have in this free body diagram? This is joint K so first is vertical reaction which is  $K_y$  and then we have this  $F_{JK}$ . And then this is  $F_{HK}$  which is acting at an angle  $\theta_1$  and then  $F_{IK}$  which is acting at an angle  $\theta_2$ . Okay. This is  $\theta_2$  and this is  $\theta_1$ . Okay.

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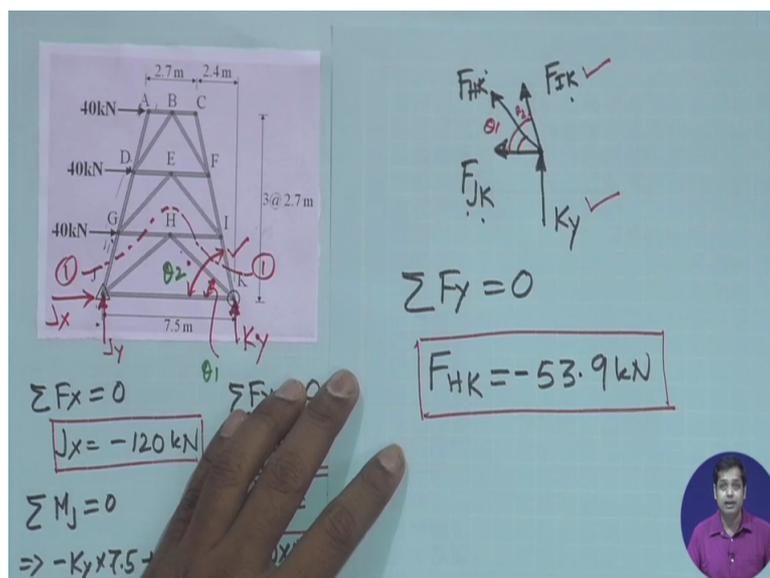
Then HIK just now we determined. This is also known. Only unknowns are this and this.

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Now the next step is we take the component of the forces in x direction or in y direction and take the summation in that direction, all the forces is equal to zero. So if we take summation of FY is equal to zero then what are the components we have? The KY, component of IK, component of HK, there will be no component of JK because it is horizontal direction and if you do that then finally, again I leave this part to you. HK is equal to 53 point 9 kilo Newton minus. So these are the forces in 2 members.

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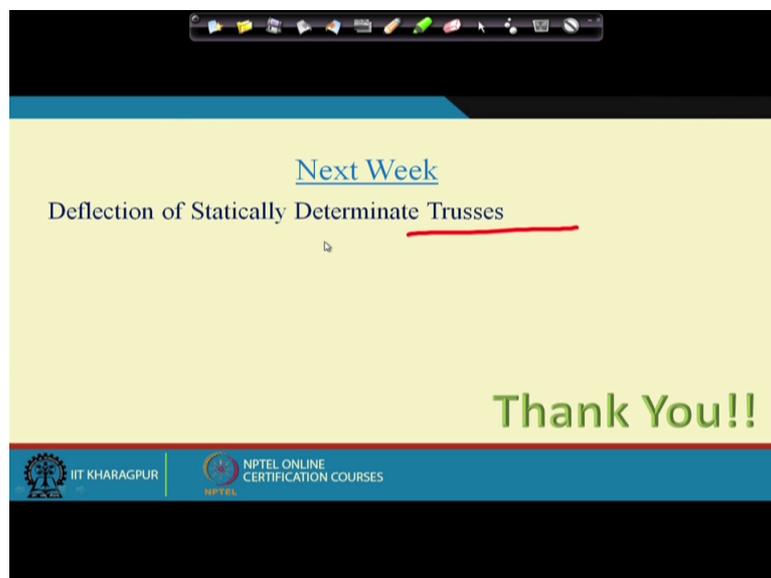
So previous example was where your application it was purely method of sections. All the member forces are determined using method of sections. But as I said in most economic way

if we apply wherever method of section or method of joints both together in the same problem, this is the second problem. Second example was a demonstration of that. Okay. That is what for today. So but if you take any structural analysis book there are many problems.

Some of them are very easy some of them are very tricky. What important is as a concept wise it is nothing new. You draw the free body diagram and apply the equilibrium conditions, right? What is important here then which free body diagram to consider that is important. Which joints to be considered? Which sections to be taken? And that sense, there is no guideline for that. That sense you have to generate in yourself and that sense will come as you attempt more and more problems. Okay.

So my suggestion is please go through the books and solve some of the exercise problems. This week we discussed how to determine the internal forces in statically determinate truss. Now we are interested in two kinds of response, internal forces and as well as deflection. Next week what we will do is we will see how to determine deflection in statically determinate trusses.

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With this thank you.