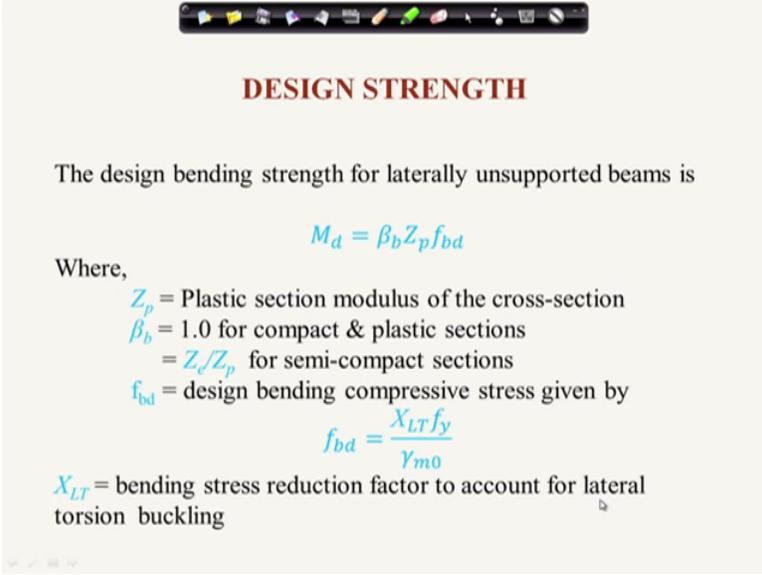


**Course on Design of Steel Structures**  
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**Module 10 Lecture 51**  
**Strength Calculation of Laterally Unsupported Beams**

Based on the last lecture, now we will go to one work out example and we will see how to calculate the strength design bending strength of a member which is laterally unsupported.

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**DESIGN STRENGTH**

The design bending strength for laterally unsupported beams is

$$M_d = \beta_b Z_p f_{bd}$$

Where,

- $Z_p$  = Plastic section modulus of the cross-section
- $\beta_b = 1.0$  for compact & plastic sections
- $= Z_e/Z_p$  for semi-compact sections
- $f_{bd}$  = design bending compressive stress given by

$$f_{bd} = \frac{X_{LT} f_y}{\gamma_{m0}}$$

$X_{LT}$  = bending stress reduction factor to account for lateral torsion buckling

So before going to that example, first I will show the glimpse of the lecture what we have discussed roughly that is first we have to find out the  $X_{LT}$  value, the bending stress reduction factor  $X_{LT}$ , if we can find out the value of  $X_{LT}$  bending reduction factor due to lateral torsional buckling then I can find out the design bending compressive stress and once I find the design bending compressive stress then I can find out the design bending strength. So to find out the finally the design bending strength of the member, I need to first find out the bending stress reduction factor to account for the lateral torsional buckling.

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$$X_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \leq 1.0$$

Where,  $\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$   
 $\alpha_{LT}$  = imperfection factor for lateral torsional buckling of beams  
= **0.21** for rolled steel sections  
= **0.49** for welded steel sections  
 $\lambda_{LT}$  = non-dimensional slenderness ratio given by,

$$\lambda_{LT} = \sqrt{\frac{\beta_b Z_p f_y}{M_{cr}}} \leq \sqrt{1.2 \frac{Z_e f_y}{M_{cr}}}$$
$$= \sqrt{\frac{f_y}{f_{cr,b}}}$$

So that XLT value, I can find out from this expression, which we have shown earlier and to find out the value of XLT we need to find out value of phi LT and where the phi LT depends on the imperfection factor for laterally torsional buckling that is alpha LT and it may be 0.21 for rolled section and 0.49 for welded steel sections and the lambda LT the non-dimensional slenderness ratio can be calculated from this formula where we need to know again the value of M<sub>cr</sub>, M<sub>cr</sub> is the elastic sorry, lateral torsional buckling moment.

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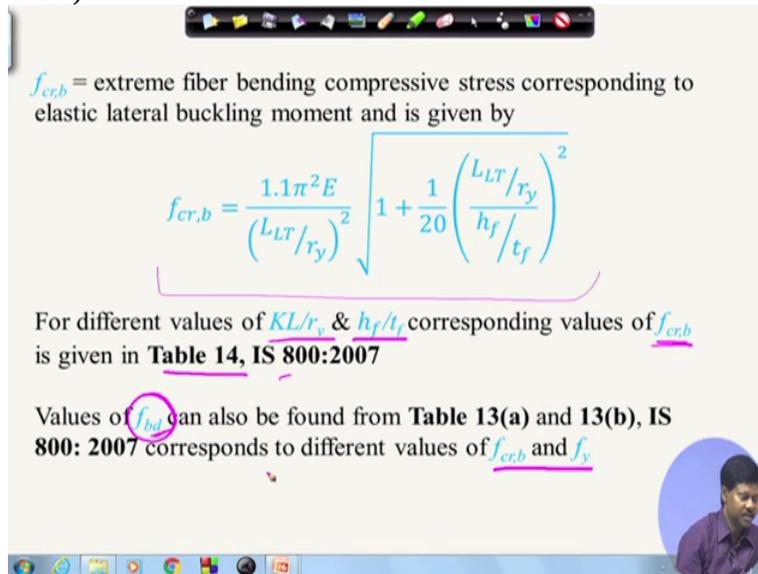
Where,  
 $M_{cr}$  = elastic lateral buckling moment (Cl. 8.2.2.1) is given by,

$$M_{cr} = \sqrt{\left\{ \left( \frac{\pi^2 E I_y}{(L_{LT})^2} \right) \left[ G I_t + \frac{\pi^2 E I_w}{(L_{LT})^2} \right] \right\}} = \beta_b Z_p f_{cr,b}$$

$I_t$  = torsional constant =  $\sum b_i t_i^3 / 3$  for open section  
 $I_w$  = warping constant  
 $I_y$  = moment of inertia about weaker axis  
 $r_y$  = radius of gyration about weaker axis  
 $L_{LT}$  = effective length for lateral torsional buckling (Clause 8.3)  
 $h_f$  = centre-to-centre distance between flanges  
 $t_f$  = thickness of flange  
G = shear modulus

So  $M_{cr}$  can be calculated from this that is the elastic lateral torsional buckling moment that can be found from this expression. So once we find the value of  $M_{cr}$  then I can find out finally the  $F_{bd}$  value and then the  $M_d$  value, right.

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$f_{cr,b}$  = extreme fiber bending compressive stress corresponding to elastic lateral buckling moment and is given by

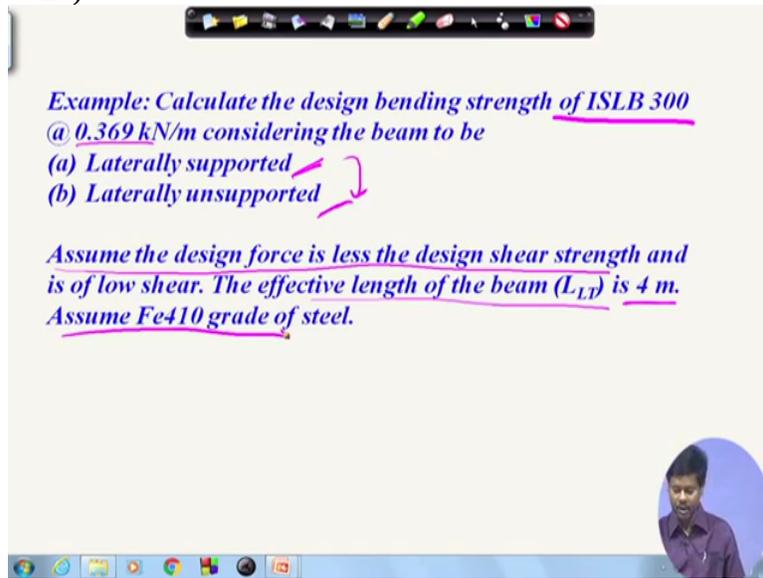
$$f_{cr,b} = \frac{1.1\pi^2 E}{(L_{LT}/r_y)^2} \sqrt{1 + \frac{1}{20} \left( \frac{L_{LT}/r_y}{h_f/t_f} \right)^2}$$

For different values of  $KL/r_y$  &  $h_f/t_f$  corresponding values of  $f_{cr,b}$  is given in **Table 14, IS 800:2007**

Values of  $f_{bd}$  can also be found from **Table 13(a) and 13(b), IS 800: 2007** corresponds to different values of  $f_{cr,b}$  and  $f_y$

$F_{cr,b}$  value also can be found from this formula otherwise as I told we can find out from table 14, in which we do not need to calculate in details through these expressions. So simply we can find out the value of  $Kl$  by  $r_y$  and  $h_f$  by  $t_f$  then we can find out the, from table 14, we can find out the value of  $F_{cr,b}$ . So once we find the value of  $F_{cr,b}$  we can find out  $F_{bd}$  correspondings to  $F_y$  and  $F_{cr,b}$ . So once we find  $F_{bd}$  value then we can find out the  $M_d$ , the design bending strength.

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*Example: Calculate the design bending strength of ISLB 300 @ 0.369 kN/m considering the beam to be*

- (a) Laterally supported*
- (b) Laterally unsupported*

*Assume the design force is less the design shear strength and is of low shear. The effective length of the beam ( $L_{LT}$ ) is 4 m. Assume Fe410 grade of steel.*

Now with this formula, now let us calculate the design bending strength of a section. So first we will see that to calculate the design bending strength of ISLB 300, okay. So if we use ISLB 300, 0.369 kilo newton per meter then let us find out the design bending strength considering laterally supported and laterally unsupported, laterally supported we have already done; however it is very easy, we see what is the bending strength is coming which is to be laterally unsupported and we will compare this with laterally unsupported (4:07), right we will try to find out how much reduction is coming for the same member.

So here to make the things easy we can assume the design force is less than the design shear strength and is of low shear. So we will calculate the things in a considering low shear and the effective length of the beam  $L_{LT}$  is given as 4 meter and we can use Fe410 grade of steel.

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ISLB-300

$D = 300$   
 $b_f = 150$   
 $t_w = 6.7$   
 $t_f = 9.4$

$r_x = 124$   
 $r_y = 28$   
 $Z_{px} = 554.32 \times 10^3$   
 $Z_{ex} = 488.9 \times 10^3$   
 $I_z = 7333 \times 10^4$   
 $I_y = 376 \times 10^4$

$d = D - 2(t_f + R_1) = 300 - 2(9.4 + 15) = 251.2 \text{ mm}$

$\alpha_{LT} = 0.21$

$f_y = 250$   
 $\gamma_{m0} = 1.10$

So with this data, now let us try to find out the design bending strength, okay. So the relevant properties of ISLB 300, we can find from sp6 (( ))(5:01) as D is equal to 300 Millimeter, width of flange is equal to 150 millimeter thickness of web is equal to 6.7 millimeter. Thickness of flange is equal to 9.4 millimeter and  $r_x$  value is given 124 millimeter, the radius of direction about x, radius direction about y is given 28 millimeter okay.

So all the values we are writing here which will be require. Then  $Z_{px}$  value we can find out 554.32 into 10 cube millimeter cube. This is plastic section modulus and elastic section modulus we can find out as 48.9 into 10 cube millimeter cube and also I can find out  $I_z$  as 7333 into 10 to the 4 millimeter to the 4,  $I_y$  as 376 into 10 to the 4 millimeter to the 4 and also I can find out the effective depth of the web  $d$  as  $D$  minus 2 into  $t_f$  plus  $R_1$ . So if we calculate these value we can find out effective depth as 251.2 millimeter, right and for steel rolled section, the imperfection factor for lateral torsional  $\alpha_{LT}$ , we can find as 0.21, because we are using steel rolled sections.

So for steel rolled section, the imperfection factor for lateral torsion is 0.21, right and as we are using Fe400 steel we can use  $f_y$  as 250 and  $\gamma_{m0}$  from table 5 we can find out 1.10, right. So relevant properties of the ISLB 300 is found.

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$\frac{b}{t_f} = \frac{150}{9.4} = 7.98 < 9.4$   
 $\frac{d}{t_w} = \frac{251.2}{6.7} = 37.49 < 67 \epsilon$

(a) Laterally supported

$M_d = \frac{\beta_b Z_p f_y}{\gamma_{m0}} = \frac{1 \times 554.32 \times 10^3 \times 250}{1.1} = 125.98$   
 $\leq 1.2 Z \epsilon \frac{f_y}{\gamma_{m0}} = \frac{1.2 \times 488.9 \times 250}{1.1} = 133.34$

$M_d = 125.98 \text{ kNm}$

Now on the basis of this we will try to classify first the section, whether it is plastic, compact or semi-compact. So for that we have to find out, first the ratio  $b$  by  $t_f$ ,  $b$  by  $t_f$  will be 150 by 2, because this is  $b_f$  if we see, so this is  $b_f$  is 150 and  $b$  will be from center to the extreme this is  $b$ . So  $b$  by the over on (7:58) portion  $b$  by  $t_f$  is the thickness of flange that is 9.4 is coming 7.98, which is less than 9.4, okay and  $d$  by  $t_w$  is coming 251.2 by  $t_w$  is 6.7, right 37.49 and this is less than 67 epsilon.

So basically shear buckling shape of web will not be required for this case we do not need to check shear buckling on web and also as it is low shear, so satisfying this we may not check the shear buckling, right. So now first we will consider laterally supported beam. So for laterally supported beam, as it is low shear, we can simply find out  $M_d$  as  $\beta_b$  into  $Z_p$  into  $F_y$  by  $\gamma_{m0}$ . So here  $\beta_b$  will be 1, because from this we can see that this is a plastic section, right.

So for plastic section  $\beta_b$  value is 1. So 1 into  $Z_p$  value was given earlier means from that table from the IS from the sp6 (9:52) no  $Z_p$  value is not given in sp6 (9:56) you can find out in IS 800-2007. So  $Z_p$  value 554.32 into 10 cube into  $F_y$  is 250 by 1.1. So 125.98 kilo newton meter and it should be in any case less than or equal to  $1.2 Z \epsilon F_y$  by  $\gamma_{m0}$ , okay. So this will become 1.2 into 4  $F_y$  88.9  $Z \epsilon$  the elastic section modulus we can find from sp6,

remember the elastic section modulus can be found from sp6, but the plastic section modulus we have to find from IS 800-2007. So into  $F_y$  you can make 250 by  $\gamma_{m0}$  is 1.1. So here it is coming 133.34 kilo newton. So that means the  $M_d$  value we can consider as 125.98 kilo newton. So design strength of the member when it is laterally supported is calculated as 125.95 kilo newton meter, right.

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$$M_{cr} = \sqrt{\left(\frac{\pi^2 E I_y}{(L_{LT})^2}\right) \left(G I_t + \frac{\pi^2 E I_w}{L_{LT}^2}\right)}$$

$$L_{LT} = 4000 \text{ mm.}$$

$$G = \frac{E}{2(1+\mu)} = \frac{2 \times 10^5}{2(1+0.3)} = 76.92 \times 10^3$$

$$I_t = \left[ \frac{b_i t_i^3}{3} \right] = \underbrace{2 \times \frac{150 \times 9.4^3}{3}}_{\text{flange}} + \underbrace{\frac{(300 - 2 \times 9.4) \times 67^3}{3}}_{\text{web}}$$

$$= 11.12 \times 10^4 \text{ mm}^4.$$

Now we will try to find out the design bending strength due to laterally unsupported. If the beam is laterally unsupported the let us see how to calculate the design bending strength and we will see also we will compare with the lateral supported one, how much reduction we have to get, okay. So for that laterally unsupported beam we can find out the first torsional buckling moment  $M_{cr}$  as this,  $\pi^2 E I_y$  by  $L_{LT}$  square into  $G I_t$  plus  $\pi^2 E I_w$   $L_{LT}$  square, right. So here we need to know before calculating this value we need to know, the value of the all parameters like the effective length due to lateral torsional buckling  $L_{LT}$  is 4000 millimeter, right. Now the shear modulus  $G$  we can find out as  $E$  by 2 into 1 plus  $\mu$ . So  $E$  is 2 into 10 to the 5 newton per millimeter square and 2 into 1 plus  $\mu$ ,  $\mu$  is the poisons ratio for steel so, we can consider as 0.3. So after calculating we can find out the shear modulus as 76.92 into 10 cube.

Similarly the torsional constant  $I_t$  we can find out as we know this will be for open section, it will be  $b_i t_i$  cube by 3, so it will be a 2 into 150 is the flange width into flange thickness is 9.4

cube by 3. So this is 1 for flange and for web it will be 300 minus 2 into 9.4 into  $t_f$  cube into 6.7  $t_w$ , okay thickness of web by 3. This is of flange, so the torsional constant for flange and for web has been calculated and is found as 11.12 into 10 to the 4 millimeter to the 4, right. So torsional constant we found.

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$$I_w = (1 - \beta_f) \beta_f I_y h_f^2$$

$$h_f = 300 - 9.4 (D - t_f) = 290.6 \text{ mm}$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5 \quad \underline{I_{fc} = I_{ft}}$$

$$I_w = (1 - 0.5) 0.5 \times 376 \times 10^4 \times 290.6^2 = \underline{7.94 \times 10^{10} \text{ mm}^6}$$

Now we will find out warping constant  $I_w$ , the warping constant we know that will be 1300 minus beta f into beta f  $I_y h_f$  square. Now here  $h_f$  is the center to center distance between flanges. So this will become 300 minus 9.4 means  $D$  minus  $t_f$  center to center distance between flanges. So this is becoming 290.6 millimeter. So  $h_f$  we can find out from this and beta f will become moment of inertia of compression flange by moment of inertia of compression flange plus moment of inertia of tension flange and since,  $I_{fc}$  and  $I_{ft}$  is same here, because this is an I section. So because of I section we can consider  $I_{fc}$  and  $I_{ft}$  as same. So this will become 0.5, right.

So beta f also we are putting as 0.5. Now the  $I_w$  value the warping constant we can find out. So this will be beta f is 0.5 into  $I_y$  we got 376 into 10 to the 4 then 290.6 is the  $h_f$ , so  $h_f$  square right. So  $I_w$  value can be found as 7.94 into 10 to the power 10 millimeter to 6, right. So warping constant  $I_w$  can be calculated as 7.94 into 10 to the 10.

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$$M_{cr} = \sqrt{\left( \frac{\pi^2 \times 2 \times 10^5}{(4000)^2} \right) \left( 76.92 \times 10^3 \times 11.22 \times 10^4 + \frac{4000^2}{4000} \right)}$$

$$= \underline{92.45 \text{ kN}}$$

$$M_{cr} = \frac{\pi^2 E I_y h_f}{2 L T^2} \left[ 1 + \frac{1}{20} \left( \frac{L T / r_y}{h_f / t_f} \right)^2 \right]^{0.5}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 376 \times 10^4 \times 290.6}{2 \times 4000^2} \left[ 1 + \frac{1}{20} \left( \frac{4000 / 28}{290.6 / 9.4} \right)^2 \right]^{0.5}$$

$$= \underline{96.92 \text{ kN m}}$$

Now all the values of the parameters are obtained, so we can find out the moment due to torsional buckling. So  $M_{cr}$  we can find out, so  $\pi^2$  into  $E$  into  $I_y$  that is  $376$  into  $10$  to the  $4$  by  $L T$  square, the length due to lateral torsional buckling that was  $4000$ , right  $4000$  square into  $G$  into  $I_t$ , so  $G$  value we can find out as  $76.92$  into  $10$  cube  $I_t$  value we found as  $11.22$  into  $10$  to the  $4$  plus  $\pi^2$   $E I_w$   $\pi^2$   $E I_w$ ,  $7.94$  into  $1100$  to the  $10$  by  $L T$  square  $4000$  square, right. So after calculation the  $M_{cr}$  value we could find  $92.45$  kilo newton.

So once we find the value of  $M_{cr}$  we can find the value of  $\lambda_{LT}$ , right. So  $M_{cr}$  value we can find from this or from the simplified equation also we can find out  $M_{cr}$  as  $\pi^2$   $E I_y h_f$  by  $2 L T$  square, I am calculating this to check how much difference is coming for how much difference in  $M_{cr}$  value as coming for these two expressions. So this is  $L T$  by  $r_y$  by  $h_f$  by  $t_f$  square to the  $0.5$ . So if we put this value we can see that is  $\pi^2$  into  $2$  into  $10$  to the  $5$  into  $376$  into to the  $4$   $h_f$  is  $290.6$  by  $2$  into  $L T$  is  $4000$  square into  $1$  plus  $1$  by  $20$  into  $L T$  by  $r_y$  by  $h_f$  is  $290.6$  by  $t_f$  is  $9.4$ , right. So  $M_{cr}$  value from this we can find out as  $96.92$  kilo newton meter, which is this, okay. So the lesson 1 is this means more or less which is coming closer.

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$$\lambda = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} = \sqrt{\frac{1 \times 554.32 \times 10^3 \times 250}{92.45 \times 10^6}}$$
$$= 1.22 > 0.4$$
$$\phi_{LT} = 0.5 \left[ 1 + 0.21 (\lambda_{LT} - 0.2) + \lambda_{LT}^2 \right]$$
$$= 1.35$$
$$X_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \leq 1.0$$

Now lambda LT value we can find out by putting the values what we have obtained earlier beta b Zpz Fy by Mcr. So for plastic section the beta b value will be 1 and Zpz value we found as 554.32 and Fy as 250 and Mcr value we have calculated we are taking the lesser one to get higher lambda LT so that the lesser Fbd value we obtain for 92.45 into 10 to the 6. So this value is coming 1.22 and it is more than 0.4. So we know if lambda value is more than 0.4 then effect of lateral torsional buckling has to be consider.

So non-dimensional slenderness ratio value we could calculate as 1.22, which is more than 0.4 and as it is more than 0.4 we have to consider the lateral torsional buckling effect, okay. So we can now find out phi LT, so this will be 0.5 into 110 plus alpha LT, alpha LT value will be 0.21 here into lambda LT. Lambda LT is 1.22 minus 0.2 plus lambda LT square, right. So phi LT value we are getting 1.35. Now the reduction factor due to lateral torsional buckling, the bending reduction factor XLT we can find. So we know XLT is phi LT plus square root of phi LT square minus lambda LT square, so and it has to be less than 1.

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Handwritten calculations on a blue background:

$$= \underline{0.52}$$
$$f_{bd} = \frac{X_{LT} f_y}{\gamma_{m0}} = \frac{0.52 \times 250}{1.1} = 118.2$$
$$M_d = 1 \times 554.32 \times 10^3 \times 118.2 \times 10^{-6}$$
$$M_d = 65.52 \text{ kNm} \quad \leftarrow \text{Unsupported}$$
$$M_d = 125.98 \text{ kNm} \quad \leftarrow \text{Supported}$$

A small circular inset image of a man is visible in the bottom right corner of the slide.

So now if I put the value of phi LT and lambda LT, I can find out the bending stress reduction factor XLT as this, 1 by 1 plus 1.35 plus root over 1.35 square minus 1.22 square. So this bending stress reduction factor we could find out as 0.52, right. So we could see that bending stress reduction factor is coming 0.52 means almost 50 percent reduction has to be done. So Md no no Fbd the design bending compressive stress can be found now XLT Fy by gamma m0. So now if we provide the value of bending stress reduction factor as 0.52 and Fy as 250 and gamma m0 as 1.1 then we can find out the value as 118.2, right 118.2 newton per millimeter square.

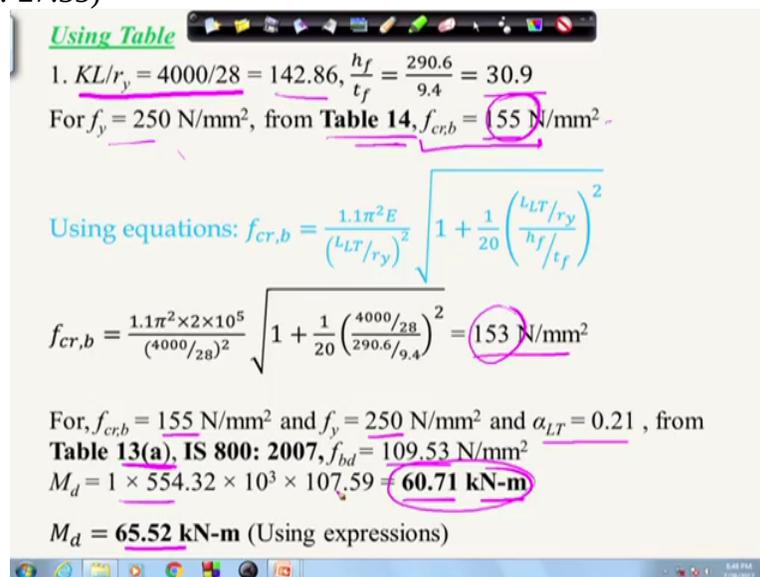
So now Md the design compressive design bending moment we can find out. This is beta b Zp into Fbd, so that is 1 into 554.32 into Fbd is 118.2 and to make it kilo newton meter, we can multiply 10 to the minus 6. So 65.52 kilo newton meter, right. So the design bending strength of the member when lateral torsional buckling is consider is 65.52 kilo newton meter and earlier we found Md as 125.98 kilo newton meter, right. So what we could see that if lateral torsional buckling is consider then the means if it is unsupported laterally unsupported then the bending strength is coming 65.52 and if it is supported then the design bending strength is coming 125.98 almost double, okay.

So we could see from this demonstration that the design bending strength of the member is decreasing to a certain extent if the lateral torsional buckling effect is consider that means when

the section is laterally unsupported its bending strength is reduced to certain extent and what will be the reduction that will depend on the section shape, the lateral torsional length of the lateral torsional LLT and the support condition. So depending on those aspects the moment due to lateral torsional buckling can be calculated and from that finally we can find out the reduction stress reduction factor for design bending stress and once, we find the design bending stress due to lateral torsional buckling then we can find out the design bending strength and from this example we have seen the design bending strength is quite reduced when the member is laterally unsupported with compared to laterally supported, right

In next class, we will discuss about the design procedure and we will go through one example to see how to find out a section size for a member when it is laterally unsupported with this I conclude today, okay.

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**Using Table**

1.  $KL/r_y = 4000/28 = 142.86$ ,  $\frac{h_f}{t_f} = \frac{290.6}{9.4} = 30.9$

For  $f_y = 250 \text{ N/mm}^2$ , from **Table 14**,  $f_{cr,b} = 155 \text{ N/mm}^2$

Using equations:  $f_{cr,b} = \frac{1.1\pi^2 E}{(L_{LT}/r_y)^2} \sqrt{1 + \frac{1}{20} \left(\frac{L_{LT}/r_y}{h_f/t_f}\right)^2}$

$f_{cr,b} = \frac{1.1\pi^2 \times 2 \times 10^5}{(4000/28)^2} \sqrt{1 + \frac{1}{20} \left(\frac{4000/28}{290.6/9.4}\right)^2} = 153 \text{ N/mm}^2$

For,  $f_{cr,b} = 155 \text{ N/mm}^2$  and  $f_y = 250 \text{ N/mm}^2$  and  $\alpha_{LT} = 0.21$ , from **Table 13(a), IS 800: 2007**,  $f_{bd} = 109.53 \text{ N/mm}^2$

$M_d = 1 \times 554.32 \times 10^3 \times 107.59 = 60.71 \text{ kN-m}$

$M_d = 65.52 \text{ kN-m}$  (Using expressions)

Now whatever we have discussed that means whatever we have calculated through expressions can also be done through this table. Now I am demonstrating using the table. Now we see if we know the Kl by ry, so for example here Kl by ry is 4000 by 28. So this is becoming 142.86 and hf by tf ratio that also we are getting 30.9 and corresponding to this we can use table 14 and for Fy 250 we can find out the value of Fcrb, the bending stress Fcrb due to lateral torsional buckling we found as 155 newton per millimeter square.

Now using equation if you see the  $F_{crb}$  value we found as 153 newton per millimeter square. So which is comparable, we are getting through interpolation in table 155 and by using expression we are getting as 150. So very quickly I can find out the value of  $F_{crb}$  through table 14, right. Now again if we use table 13 correspondings to  $F_{crb}$  value and  $F_y$  value then I can find out the design bending compressive stress  $F_{bd}$ . Now table 13, which table we will use table 13(a) or (b) depends on the section. So as we are using steel rolled section, so  $\alpha_{LT}$  we are using as 0.21, right and therefore, we are using table 13(a). So if we use table 13(a) corresponding to  $F_{crb}$  as 155 and  $F_y$  as 250 we get a  $F_{bd}$  value as 109.53.

So the  $M_d$  value we can find out as 60.71 kilo newton meter and using expression if you remember we obtain  $M_d$  value as 65.53 kilo newton meter. So in place of using those expressions we can simply use the table 13 and table 14 then we can find out the value of  $F_{bd}$ . So very quickly we can find out the value of  $m_d$  using table 13 and table 14, right. Okay.