

Course on Design of Steel Structures
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Lecture 13

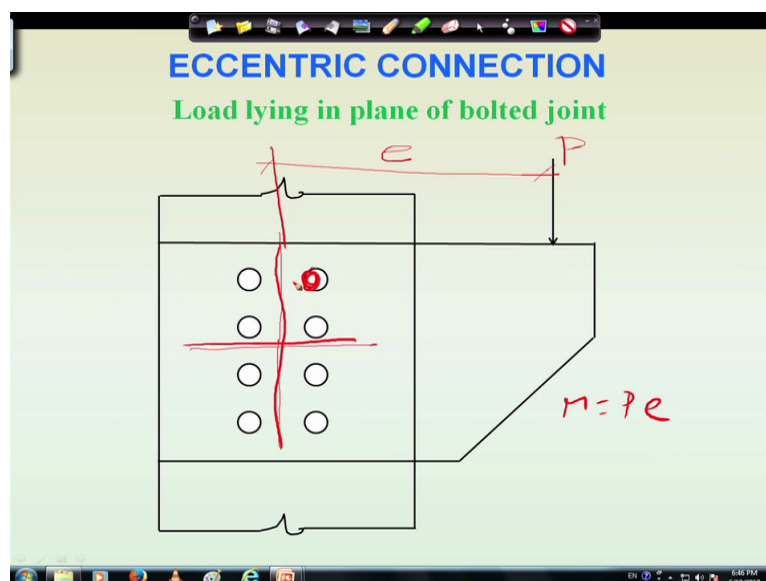
Module 3

Eccentric Connection (Load Lying in Plane of Bolted Joint)

Hello today I am going to discuss about eccentric connections in last few lectures we have discussed the different types of connections but mainly those are concentric. Now in case of eccentric connections cg of the connection and cg of the load will be in different position so if it does not coincide then eccentricity will develop and because of eccentricity extra moment will come into picture and because of the moment extra stress will come.

So therefore we have to design the joint taking consideration of the direct load as well as due to eccentricity say in case of eccentric joint we have different type of joint like when load is lying in the plane of joint it will be one type of eccentricity again that can be designed by weld connection as well as by bolt connection and similarly when load is lying in the perpendicular to the plane of joint then another type of eccentricity come into picture means another type of load reaction will come and that also can be designed using bolt joint and using weld joint. So basically four type of joints will be considered for design.

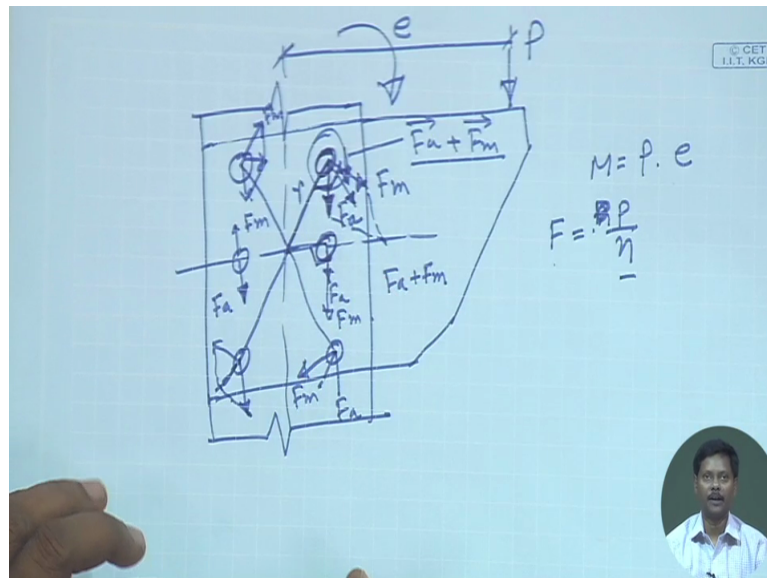
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Now I will discuss today mainly on the bolted type connection where load is lying in the plane of joint. Say for example if we see this this is a typical example of load lying in the

plane of bolted joint, so the cg of the bolted joint is this and the load is acting say here say we can load P and attend eccentricity e then the moment additional moment will come into picture that is P into e right so cg of the bolt joint is here so we have to calculate the load coming into the bolt at each bolt what are the load coming we have to calculate then we have to find out the bolt strength and we have to see whether this is safe or not.

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If we draw this figure we will see say a column is carrying some eccentric load which is coming from beam or say gusset plate, so this is column say the eccentricity of bolt joint is this now so load is acting here. Now P is the load so the eccentricity will be e. So the additional moment due to eccentricity will be P into e now what will happen to the joint in each joint will see that in each joint there will be a load which will act in this direction means if n number of joints are there then load in each joint I can I can write say P by N assuming that the loads are distributed equally to each joint.

So if we consider that every bolt is taking equal load every bolt is sharing equal load then load at each bolt will be F is equal to P by n this is this is the load which is coming due to direct load now another load will come because of moment, moment developing so this moment because of this moment so this is direct load and another load will come or stress will come in this direction means perpendicular to this axis.

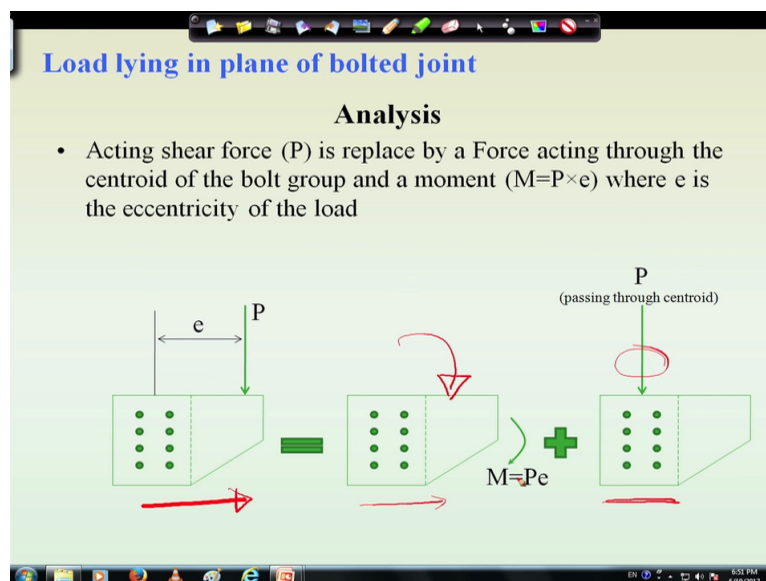
So this is the radial distance r we can say this is radial distance so for this bolt there are two types of forces will be acting one is called F_a due to direct load and another is called F_m due to moment moment and axial or direct load so we have to find out what is the resultant of this

load and these resultant load will be the load acting on the bolt. Similarly for this case will see F_a will be this and also F_m will be also in this direction because it acts 90 degree to the radial distance so here it will be F_a plus F_m directly it will be addition but in this case it will be F_a vector plus F_m vector that means we have to find out the resultant of that.

Similarly for this case will see this will act F_m will act in this way and F_a will act in this way so we have to find out the resultant and we have to analyse for each bolt and we have to see which one is the most critical one here you see if we take the resultant we will see that it will be much less, it will be less resultant will be less. In this direction in this case F_a will be this and F_m will be this, so the resultant will be F_a minus F_m similarly for this case we can find out F_m in this direction and F_a in this direction therefore the resultant will be this F_r will be in this direction.

So for each bolt the exacted force will be different we have to calculate the force coming on each bolt and we have to see which one is the critical so case to case we have to consider and we have to find out the critical one.

(Refer Slide Time: 6:44)



So if we start with the load lying in the plane of bolted joint then we can see that so if we come to the case of load lying in the plane of bolted joint then we can see that this case can be made equivalent to this M is equal to Pe plus this one this direct load, so we are dividing into two part one is moment due to eccentricity another is direct centroid load.

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Load lying in plane of bolted joint

If, Direct load $= P$, Bending moment, $M = Pe$
Therefore, Direct axial shear, $F_a = P/n$

Where
 $n \rightarrow$ No of bolts in the group
 $F_a \rightarrow$ Force on each bolt due to axial load

Direct Force + **Moment** = **Combined direct Force & moment**

So this two will be added and we have to find out the equivalent load. So this is what I have discussed that if we can find out the direct force that will be F_a and this F_a will be simply P by n if P is the load then P by n will be the F_a and similarly moment we can find out moment will be P into e where P is the axial load and e is eccentricity so if we add this direct force and moment we can find out the combination of this. The things whatever I have drawn there that again I have shown here that if F_m is this and F_a is this its resultant will be in this direction we can find out.

Similarly here resultant will be the simply adding and here we can see that this will be again adding so out of this six bolts in this particular case will see the extreme case will come either this one or this one because this bolt and this bolt have similar magnitude because the theta will be same here so resultant force will be same because distance radial distance from here to this bolt and to this bolt is same.

That is why resultant force will become same for this case and this case however in this case it will be simply adding but in this case we will see it will be reduced, this F_r value will be less and here it will be F_a minus F_m so it will be much more less than this and similarly this is will be equal to this one so it will be also less so what we can see that we have to check only two bolts means if we can check only these two bolts then we will be able to understand which one is means what is the critical bolt because this bolt and this bolt are same so we do not need to check one means if we check the extreme one that is sufficient and also we have to check this one then we have to find out the critical one.

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Load lying in plane of bolted joint

If, Direct load $= P$, Bending moment, $M = Pe$
 Therefore, Direct axial shear, $F_a = P/n$

Where
 $n \rightarrow$ No of bolts in the group
 $F_a \rightarrow$ Force on each bolt due to axial load

Direct Force **Moment** **Combined direct Force & moment**

Shear due to moment $F_m \propto r$

$$F_m = k \cdot r \quad k = \frac{F_m}{r}$$

$$M = F_m \cdot r = P \cdot k \cdot r \cdot r = k \cdot r^2$$

$$M_t = \sum F_m \cdot r = \sum k \cdot r \cdot r = k \sum r^2$$

$$M_t = k \sum r^2 = \frac{F_m}{r} \sum r^2$$

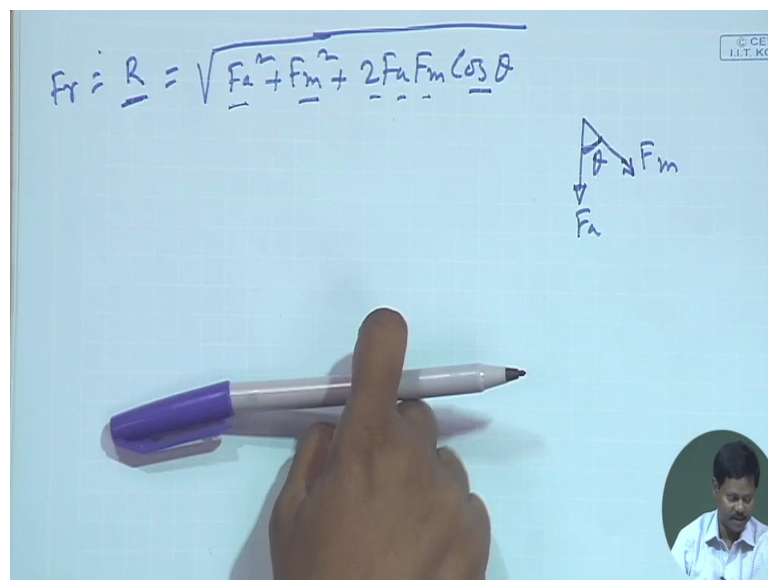
$$\text{P.e. } r^2 = \frac{M_t \cdot r}{\sum r^2} \quad F_a = \frac{P}{n}$$

So from this we can find out the shear due to moment we can find out shear due to moment that will be F_m that will be varying with r because in this case if we see that the moment will be more if the distance is more. So it is varying with its distance so F_m will be more because its distance is more but F_m will be less in this direction in this bolt, so it varies with the distance so F_m we can write as F_m varies r where r is the radial distance of bolts from the cg bolt. So what we can do so F_m we can write k into r where k is the proportionality constant k we can write as a proportionality constant. So F_m in each bolt is coming k into r where r is the radial distance from the bolt to the cg distance of the bolt group, so the moment on that particular bolt I can write moment will be F_m into r so F_m into r means kr into r that means kr square right.

So total moment it is individual moment so total moment say M_t I can write total moment will be F_m into r that means $k r$ into r that means k into summation r square right again from here I can write k is equal to F_m by r so M_t I can write that F_m by r means summation $k r$ square that means F_m by r , this is k into summation r square right. So from this I can write F_m that force due to moment will be M_t into r by summation r square that means P into e into r by summation r square.

So this is how the force on a particular bolt can be calculated from this formula where e is the eccentricity and r is the radial distance of bolt to the cg group of the bolt, so F_m the force on the on a particular bolt due to moment can be calculated this and F_a we know that is basically P by n where n is the number of bolt, so if n is the number of bolt then F_a I can find out as a P by n so this is how we can find out the value of F_a and F_m .

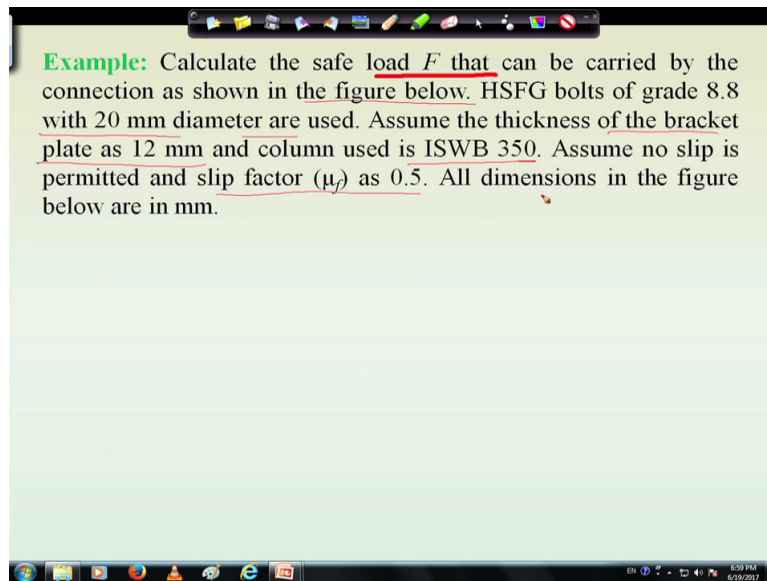
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Now the resultant force, so we can find out resultant force that will be F_a square plus F_m square plus $2 F_a F_m$ into $\cos \theta$ right, θ will be the if this is F_m and if this is F_a then θ is the angle between F_m and F_a . So I can find out R or I can write F_r resultant force at a particular bolt I can find out as a F_a square plus F_m square plus 2 into $F_a F_m \cos \theta$ right, now this will be clear if we go through one example.

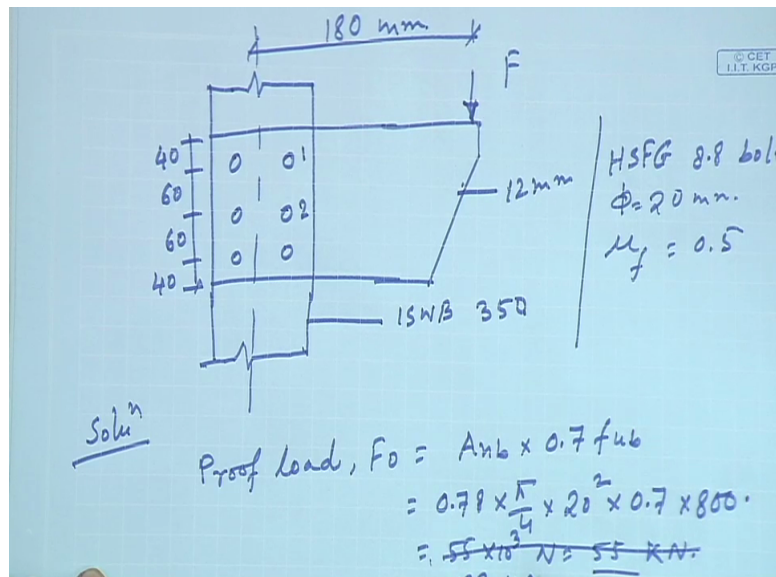
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Example: Calculate the safe load F that can be carried by the connection as shown in the figure below. HSFG bolts of grade 8.8 with 20 mm diameter are used. Assume the thickness of the bracket plate as 12 mm and column used is ISWB 350. Assume no slip is permitted and slip factor (μ_f) as 0.5. All dimensions in the figure below are in mm.



Now let us go through one example right, so this example let us go through. Say calculate the safe load F that can be carried by the connection as shown in the figure below I will show the figure. HSFG bolts of 8.8 grade with a 20 mm diameter are used. Assume the thickness of bracket as 12 mm and column used is ISWB 350. Assume no slip is permitted and slip factor μ_f has been given that is μ_f as 0.5. So all dimensions in figure are given in millimetre.

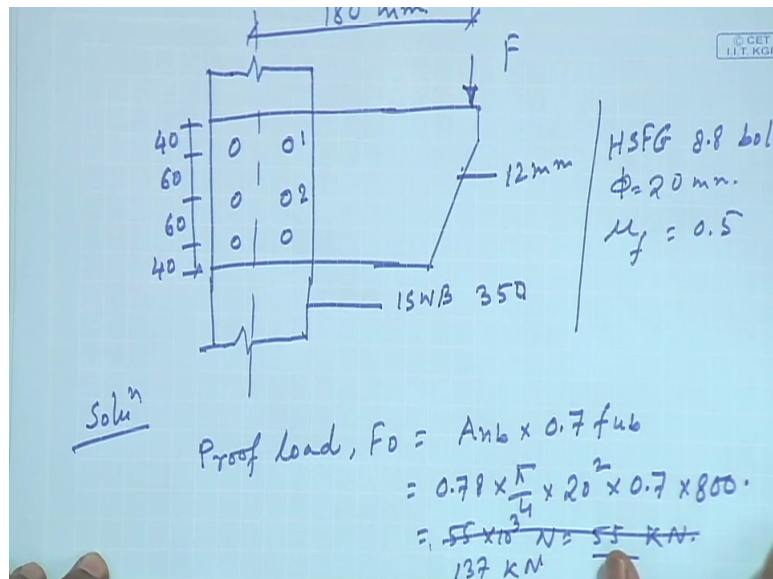
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HSFG 8.8 bolts
 $\phi = 20 \text{ mm}$
 $\mu_f = 0.5$

Soln

Proof Load, $F_0 = A_n b \times 0.7 f_{ub}$
 $= 0.78 \times \frac{\pi}{4} \times 20^2 \times 0.7 \times 800$
 $= 55 \times 10^3 \text{ N} = 55 \text{ kN}$



So if we try to solve this problem then it will be clear to us say in this problem the figure is given like this say it is a ISWB 350, is connected with a bracket and it has a force F this is F and it is situated at a distance of 180 millimetre. Now bolts are placed in this way six bolts are placed with a pitch distance of 60 mm and edge distance of 40 mm right. Say this is bolt number 1, this is bolt number 2 and this bracket thickness is 12 mm and this is ISWB 350 section.

Now here we are using HSFC bolt 8.8 grade bolt right and diameter of the bolt we are considering as 20 mm and μ_f was given as 0.5 mm. So if we want to solve this problem what will do will first try to find out proof load because this is a high strength friction bolt so due to friction we have to find out the load carrying capacity. So proof load we can find out F_0 as $A_n b$ into 0.7 f_{ub} right.

So $A_n b$ we can find out as 0.78 into pi by 4 into d square, d is 20 mm into 0.7 into f_{ub} is 800 in this case right. So this will be 55 into 10 cube newton or 55 kilonewton, this is one. So proof load we can find out as 55 sorry this is coming 137 kilonewton, this is coming 137 kilo newton.

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(1) Slip resistance of bolt, $V_{dsf} = \mu_f n_e k_n \frac{F_0}{\gamma_{mf}}$
 $= 0.5 \times 1 \times 1 \times \frac{137}{1.25} = \underline{\underline{55 \text{ kN}}}$

(2) strength of bearing, $V_{dpb} = 2.5 k_b d \cdot t \cdot \frac{f_u}{\gamma_{mb}}$
 $k_b = \text{least of } \left(\frac{e}{3d_0} = \frac{40}{3 \times 22} = 0.606 \right.$
 $\left. \frac{p}{3d_0} - 0.25 = \frac{60}{3 \times 22} - 0.25 = 0.6 \right.$
 $\left. \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975 \right)$

$= 0.5 \times 1 \times 1 \times \frac{137}{1.25} = \underline{\underline{55 \text{ kN}}}$

(2) strength of bearing, $V_{dpb} = 2.5 k_b d \cdot t \cdot \frac{f_u}{\gamma_{mb}}$
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 $k_b = 0.606$

Now we have to find out the slip resistance so slip resistance of bolt can be find as V_{dsf} is equal to $\mu_f n_e k_n$ into F_0 by γ_{mf} so this if we put the value μ_f is considered as 0.5 and if we assume this as 1 and F_0 we have calculated as 137 kilonewton and γ_f is 1.25 considering shop bolt, so sorry γ_{mf} is 1.25 so if we consider then it will be 55 kilonewton.

So V_{dsf} value is becoming 55 kilo newton and strength of bolt in bearing because it it may fail due to bearing also so we have to find out the strength of bearing V_{dpb} that we can find out as $2.5 k_b d t f_u$ by γ_{mb} right. Now we have to find out the constant k_b which will be least of e by $3d_0$ that means e in this case we have consider 40 mm and $3d_0$ d_0 will be 22 so this is coming 0.606 and P by $3d_0$ minus 0.25 so that is coming P is considered 60 into by

3 into 22 minus 0.25 this is 0.66 and f_{ub} by f_u is 400 by 410 that is 0.975 and 1. So list of these four 0.606, 0.66, 0.975 and 1 so k_b value we can consider k_b value we can consider as 0.606 so now we can find out the value of V_{dpb} .

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$$V_{dpb} = 2.5 \times 0.606 \times 20 \times 11.4 \times \frac{410}{1.25} \times 10^{-5}$$

$$= 113 \text{ kN.}$$

Bolt value = least (55 & 113) = 55 kN.

$$r_1 = \sqrt{50^2 + 60^2} = 78.1 \text{ mm}$$

$$r_2 = 50 \text{ mm}$$

$$\sum r^2 = 4r_1^2 + 2r_2^2 = 4 \times 78.1^2 + 2 \times 50^2 = 29398 \text{ mm}^2$$

So V_{dpb} value if we calculate will get V_{dpb} as 2.5, k_b is 0.606, d is 20, t is 11.4. This is 11.4 because the thickness of flange of ISWB 350 is 11.4 which is less than 12 so lesser of the two thickness that is lesser of the bracket plate and the column flange, column flange thickness is 11.4 and bracket thickness is 12 so list of these two we have taken as 11.4.

Now if we put this value we can get as 113 kilonewton. So bolt value we can find out as least of bearing and shearing, shearing is 55 and bearing is 113 so it will be 55 kilonewton right. So bolt value we could get now we have to find out what are the bolt reaction is coming due to external moment, external force and its moment. So here r_1 , r_1 we can find out r_1 will be r_1 means if we see here this is one bolt, this is another bolt so if we consider this r_1 r_1 will be this distance square plus distance square and its square root.

So this is 50 square plus 60 square of square root of this so this is 78.1 millimetre right 78.1 millimetre because this is 50 and this we have consider 60 right and r_2 r_2 will be this one r_2 will be simply 50 mm, so summation of square means summation of all r square will be $4r_1$ square plus $2r_2$ square so we can find out the value of this summation of r square sorry $4r_1$ square plus $2r_2$ square. So 2 into 50 square so this is becoming 29398 millimetre square right.

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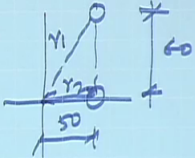
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$$r_1 = \sqrt{50^2 + 60^2} = 78.1 \text{ mm}$$

$$r_2 = 50 \text{ mm}$$

$$\sum r^2 = 4r_1^2 + 2r_2^2 = 4 \times 78.1^2 + 2 \times 50^2 = 29398 \text{ mm}^2$$

$$F_a = \frac{P}{6}$$


So now the force due to direct load in each bolt will be F_a is equal to P by 6 right.

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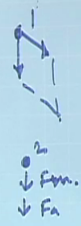
$$F_{m1} = \frac{P \times 180}{29398} \times 78.1 = 0.478 P$$

$$F_{m2} = \frac{P \times 180}{29398} \times 50 = 0.306 P$$

$$\theta_1 = \tan^{-1} \frac{60}{50} = 50.19^\circ$$

$$\theta_2 = 0.$$

$$F_{r1} = \sqrt{F_m^2 + F_a^2 + 2F_m F_a \cos \theta}$$

$$= \sqrt{(0.478 P)^2 + \left(\frac{P}{6}\right)^2 + 2 \cdot 0.478 P \cdot \frac{P}{6} \cdot \cos 50.19^\circ}$$


$$F_{m1} = \frac{P \times 180}{29398} = 0.478P$$

$$F_{m2} = \frac{P \times 180}{29398} \times 50 = 0.306P$$

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$$\theta_2 = 0.$$

$$F_{r1} = \sqrt{F_{m1}^2 + F_a^2 + 2F_{m1}F_a \cos \theta}$$

$$= \sqrt{(0.478P)^2 + \left(\frac{P}{6}\right)^2 + 2 \cdot 0.478P \cdot \frac{P}{6} \cdot \cos 50.19}$$

$$= 0.599P$$

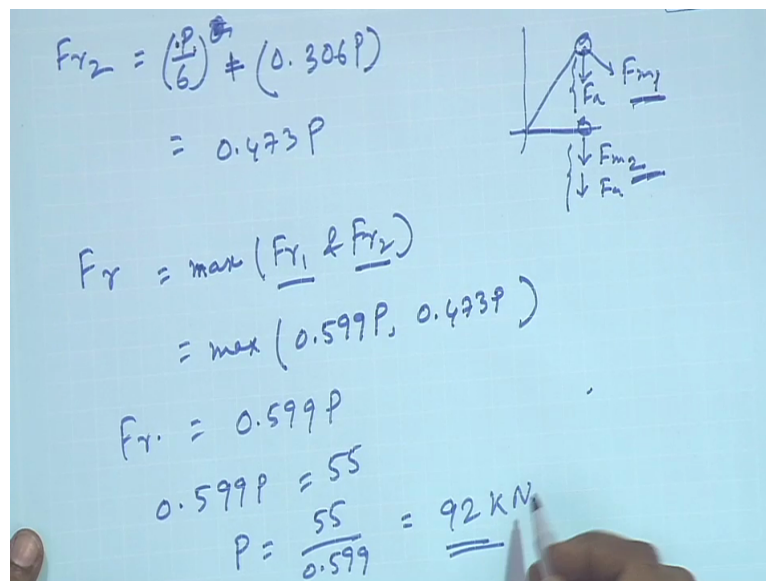
$F_a = \frac{P}{6}$

And force due to moment F_{m1} will be P into e , e is 180 this is moment by summation r square that is 29398 into r r means 78.1. This is F_{m1} means bolt number 1 which we had marked at the extreme end so that is coming $0.478P$. Similarly F_{m2} we can find out as P into 180 by that means m by summation r square into r 29398 into in this case r will be 50 so this will be $0.306P$.

So here the bolt is this so we are providing in this direction one and in this direction another one so we have to find out the this is bolt number 1 and in this case for bolt 2 this will be F_m and this is F_a so we have to add it right, so here θ_1 will be $\tan^{-1} 60$ by 50 right so it is 50.19 degree and θ_2 will be 0. So F_{r1} means resultant force on bolt 1 I can find out as F_m square plus F_a square plus $2F_mF_a \cos \theta$ right so if we put these values will get $0.478P$ square plus P by 6 square plus 2 into F_m is $0.478P$ into P by 6 into $\cos \theta$, $\cos 50.19$ right.

So after calculation we can get this as $0.599P$ right $0.599P$ right here let me tell you that F_a we got assuming that all bolts are sharing equal loads F_a will got P by 6 because number of bolt bolt are 6 so load will be shared by shared as P by 6.

(Refer Slide Time: 26:24)



The image shows handwritten calculations on a blue background. At the top left, the formula for F_{r2} is written as $F_{r2} = \left(\frac{1}{6}\right)P + (0.306P)$, which simplifies to $= 0.473P$. To the right of this is a diagram of a bolted joint. It shows a horizontal line representing a bolt. Above the line, a vertical arrow points down labeled F_m , and a horizontal arrow points right labeled F_{r1} . Below the line, a vertical arrow points down labeled F_{m2} , and a horizontal arrow points right labeled F_{r2} . Below the calculations, the formula for F_r is written as $F_r = \max(F_{r1}, F_{r2})$, which simplifies to $= \max(0.599P, 0.473P)$. Then, $F_r = 0.599P$ is written. Below that, the equation $0.599P = 55$ is written, followed by the calculation $P = \frac{55}{0.599} = 92 \text{ kN}$, where the final result is underlined.

$$F_{r2} = \left(\frac{1}{6}\right)P + (0.306P)$$
$$= 0.473P$$
$$F_r = \max(F_{r1}, F_{r2})$$
$$= \max(0.599P, 0.473P)$$
$$F_r = 0.599P$$
$$0.599P = 55$$
$$P = \frac{55}{0.599} = \underline{92 \text{ kN}}$$

So now again F_{r2} because we do not know whether in bolt 1 and bolt 2 will be maximum we do not know because here it is adding F_m and F_r is adding but F_m say 2 and here it is adding here (())(26:44) for means in the resultant it will be F_{m1} and F_m .

This F_{m1} and F_{m2} is not same, this is more and this is less again here directly we can add but here we have to get resultant so without calculation we will not able to see means we will not be able to predict which one will be more so we have to find out F_{r2} also. So F_{r2} we can find out in this way that F_{r2} we can find out that 1 by 6 1 by 6 P , P by 6 whole square plus P by 6 plus 0.306 we got P right.

So if we add these two will get $0.473P$, so F_r value will be the maximum of means maximum force will be maximum of these two F_{r1} and F_{r2} so this will be if we calculate we have seen that after calculation these values are basically F_{r1} was $0.599P$ and that is $0.473P$, so this will be $0.599P$ F_r right. So now we can find out we can equate with the bolt value bolt value we have obtained as 55 kilonewton.

So $0.599P$ is equal to 55 we can equate and then we can find out P as 55 by 0.599 and this will become 92 kilonewton that means to withstand the load on this arrangement of bolt we need means we can apply maximum value of P as 92 kilonewton, 92 kilonewton at a distance of 170 mm, I think 170 no 180 mm at a distance of 180 mm we can provide this. So this is how if we know the distribution of bolt and the loading distribution then either we can find out the load considering the maximum capacity of bolt due to shearing and bearing or we can apply certain load and we can check whether the existing bolt in what way it is distributed are

safe or not means if we provide certain load then we can check what is the load coming on these bolt or in the critical bolt what is the load coming and that load if it is more than the bolt strength then it is unsafe otherwise the bolt is safe.

So this is how in other way we can do either of the way we can do, either we can find out the critical load or we can find out whether the section is mentioned the connection group bolt connection group is safe or not. So in this way the calculation can be made when the eccentric connection in a sense that load is lying in the plane of connection if it is then I can find out the section whether it is adequate or not section means bolt group whatever it is being distributed is adequate or not, thank you very much.