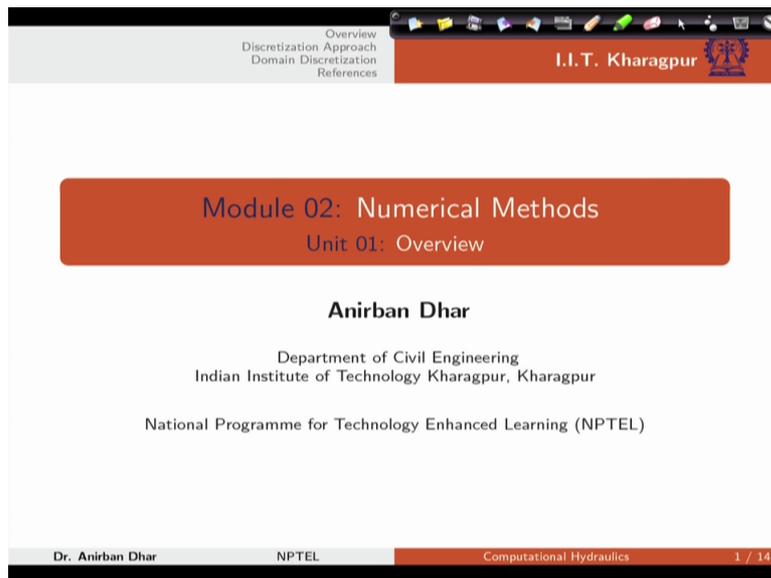**Computational Hydraulics**
ProfessorAnirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture5
Numerical Methods: Overview

Welcome to this lecture number 5 and from this lecture onwards we will start this module number 2 numerical methods.
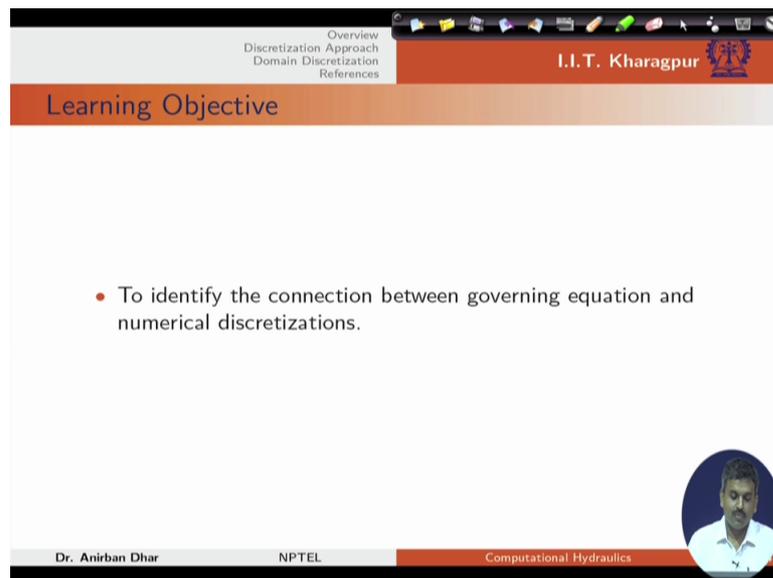
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This is unit number 1 overview. Learning objective for this unit at the end of this unit student will be able to identify the connection between governing equation and numerical discretization.

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So exact solution is not available for most of the complex problems. If we consider hydraulic system, close from solutions is not available for most of the cases.

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If we have complex boundary condition if we consider heterogeneity present in the system or if we consider hydraulic structures present in the channel network, different operating condition, recharge or discharge conditions. So we need to find out the numerical solution which is equivalent to our exact solution. But the problem is exact solution is continuous and that is defined for the whole system. But in numerical methods this defines for set of points or set of elements.

So this ordinary and partial differential equations are mostly defined for continuous domain. That means the value should be available for all points within the domain. And this continuous domain can be divided into parts or sub parts for discrete representation. So we can divide into equal parts or this size of parts can be different. And depending on the requirement we can change this division pattern and we can solve the problem.

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Numerical discretization defines the mathematical relation between parts or sub parts in terms of field variable. Let us say that in case of groundwater flow, groundwater flow the variable is h which is function of x and t. If we consider one dimensional space and time problem. So this domain can be divided into parts for solution. And this is the field variable.

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So for each of that part maybe we have one dimensional domain we can divide it into some zero to L is the length of the domain or one dimensional space domain. We can divide it into sub parts or parts.

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Now if weconsider this as first part so h1 associated with this part, h2 value is associated with this part, h3, h4 these values are associated with this part. So with this numerical discretization we can define the relationship between h1 h2, h2 h3, h3 h4, that means the connection between these values.

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So if we consider a problem definition to solution, so first we need to definethe problem for hydraulic system. If you consider the example of groundwater flow we need to know the hydraulic head variation within the system. If we want to know the velocity and depth in open channel flow network, then we have to get information aboutdepth and velocity. If we need information about the surface flow then we need to know the surface flow depth and velocity in co-ordinate directions.

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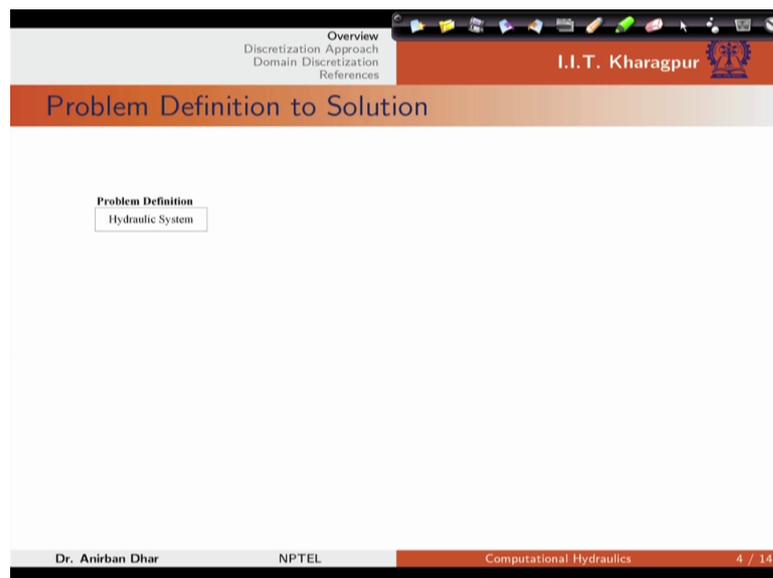
So with that information we can proceed and we can define our governing equations. That is basically mathematical conceptualization. Governing equation can be ODE or PDE with initial condition and boundary condition or only with boundary condition. In our previous module we have covered this definition of governing equation initial condition and boundary conditions.

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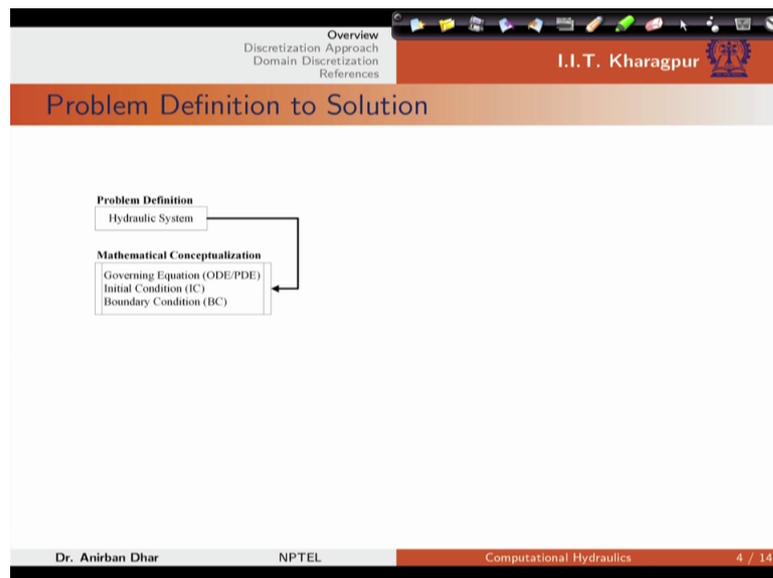
So with that definition we can proceed and we can divide the solution domain or spatial domain and temporal domain into 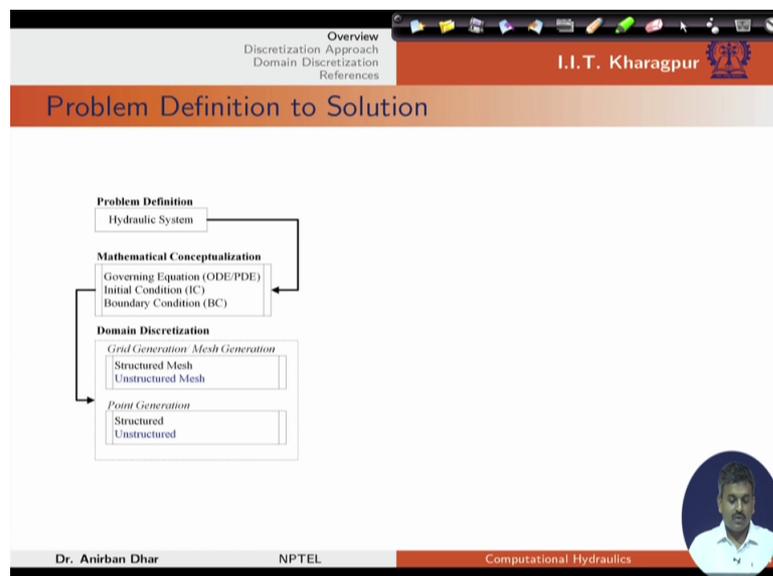parts.But domain discretization mainly it considers the discretization of the spatial domain. So within that spatial domain we can generate grid or generate points.They can be either structured or unstructured in nature. So for discrete points, we can get the variable or field variable value.

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So to define the relationship between field valuable values of sub parts or parts we need to do this numerical discretization and numerical discretization can be with Eulerian approach that is with finite difference approach or finite element or finite volume or spectral element or mesh free method.

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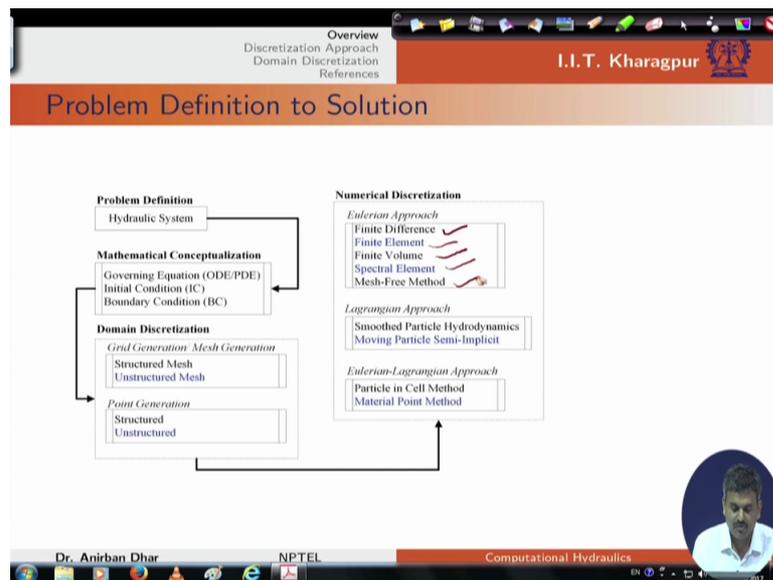
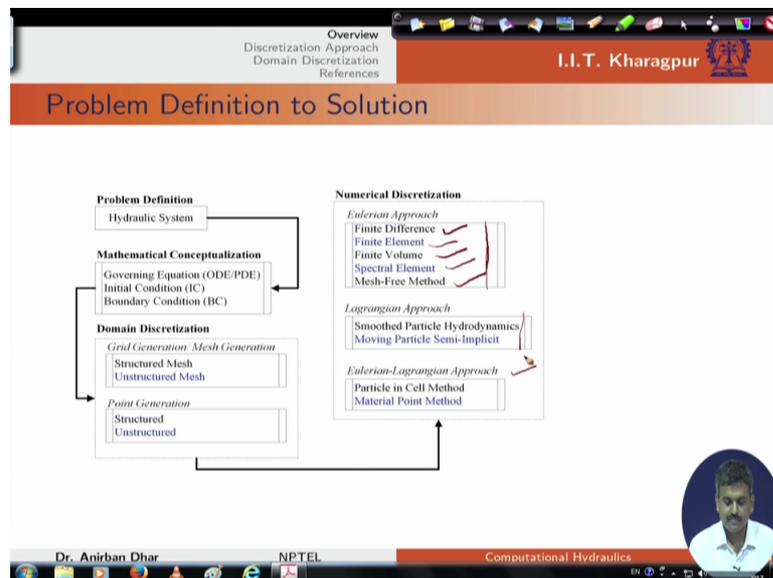We can use any of thesemethods for this numerical discretization or we can follow the Lagrangian approach like smoothed particle hydrodynamics, moving particle semi implicit method or combination of Eulerian-Lagrangian approach that is particle in cell method, material point method.

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So we can select let us say finite difference method from this Eulerian approach and we can perform this numerical discretization to relate thefield variables for sub parts.

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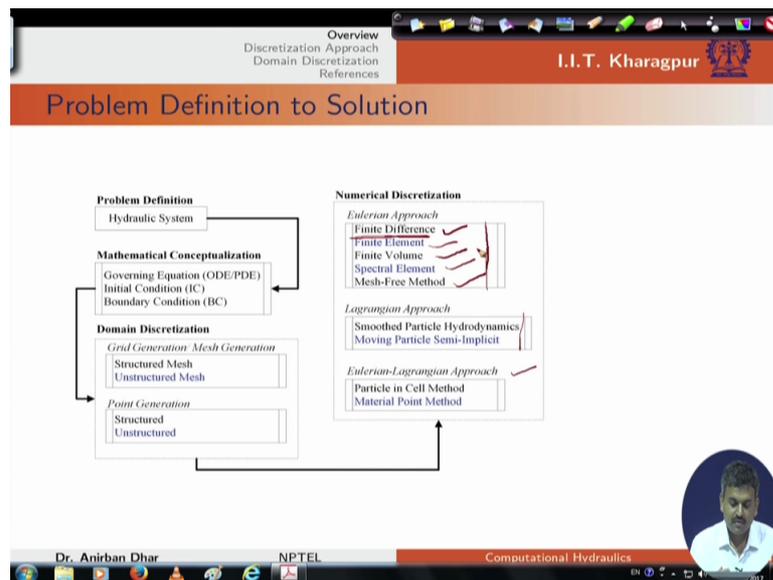
With this numerical discretization we can proceed and form the algebraic equations. These equations can be linear equations or nonlinear equations depending on the nature of differential equation.
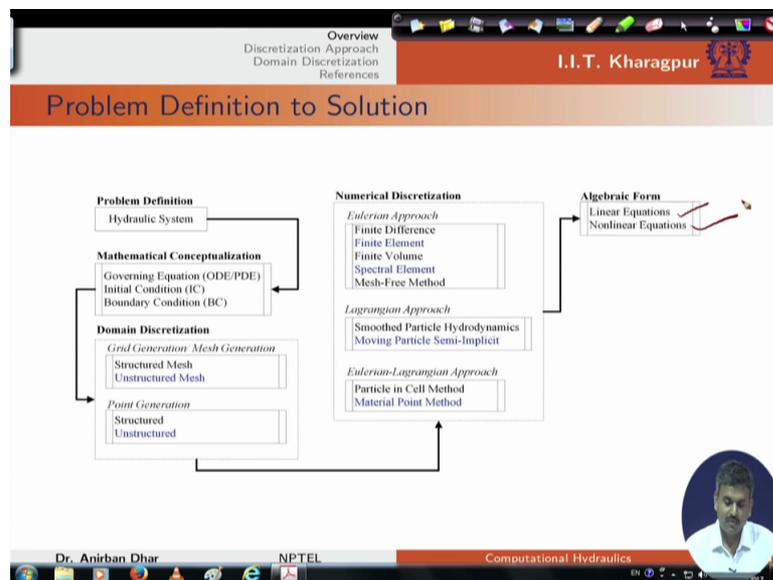
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So with this algebraic form we can proceed to the solution process. So solution process for linear and nonlinear problemswe should use a different approach. So linear solver if we consider either that can be solved with direct approach or iterative approach.If we consider the nonlinear equations then we can consider the iterative approach for solution.

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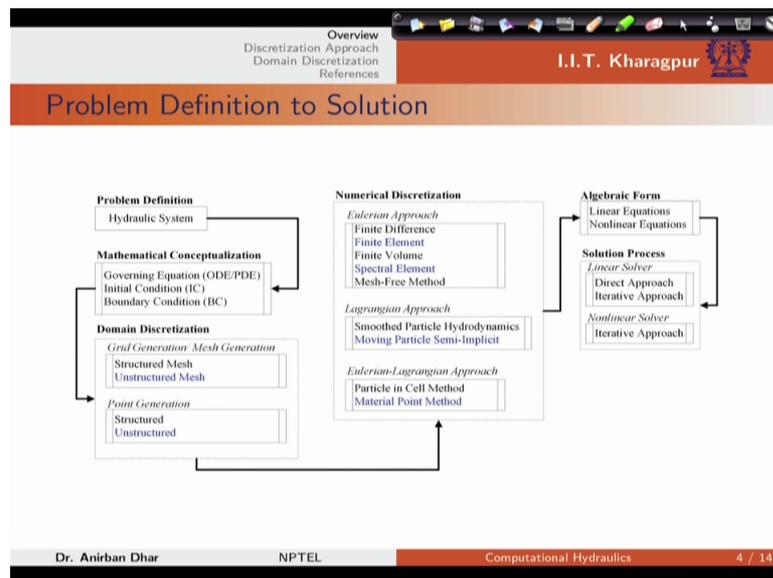
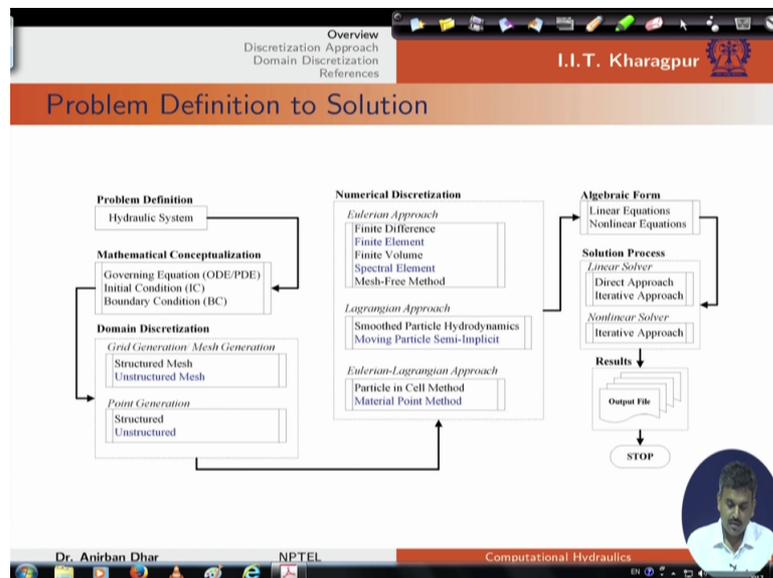With this solution we can find out the results in different output files and we can analyze that forthe relation between different variables and variation ofdependent variables with space and time. Ifthe results are up to satisfaction level then we can stop this process and we canuse those values for further analysis.

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Depending on length, time scale and corresponding methods we can consider these models. So one is classicalmolecular dynamics simulation. These models are at atomistic levels.

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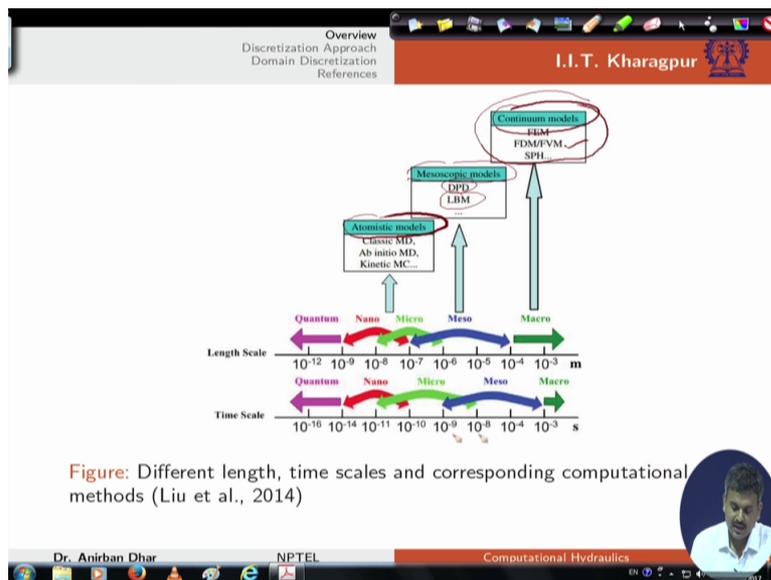Figure: Different length, time scales and corresponding computational methods (Liu et al., 2014)
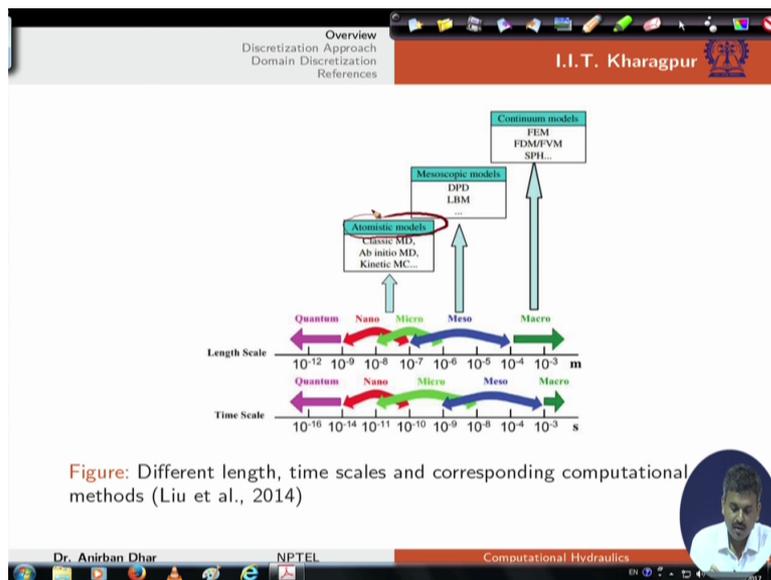
And mesoscopic models DPDor Lattice Boltzmann and this course will concentrate on this finite difference - finite volume method and mesh free method. This is at Continuum scale.

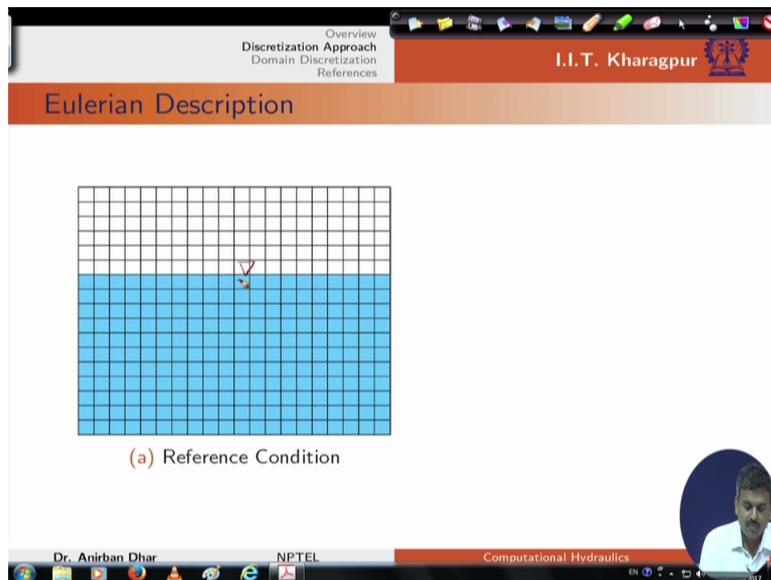(Refer Slide Time 12:22)

Figure: Different length, time scales and corresponding computational methods (Liu et al., 2014)
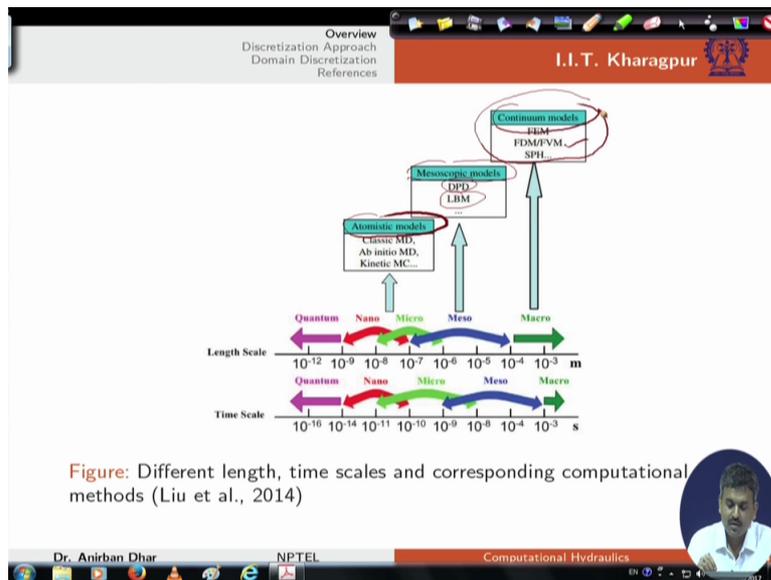
That means length scale is in the range of 10 to the power minus 3 meters and time scale is 10 to the power minus 3 seconds.
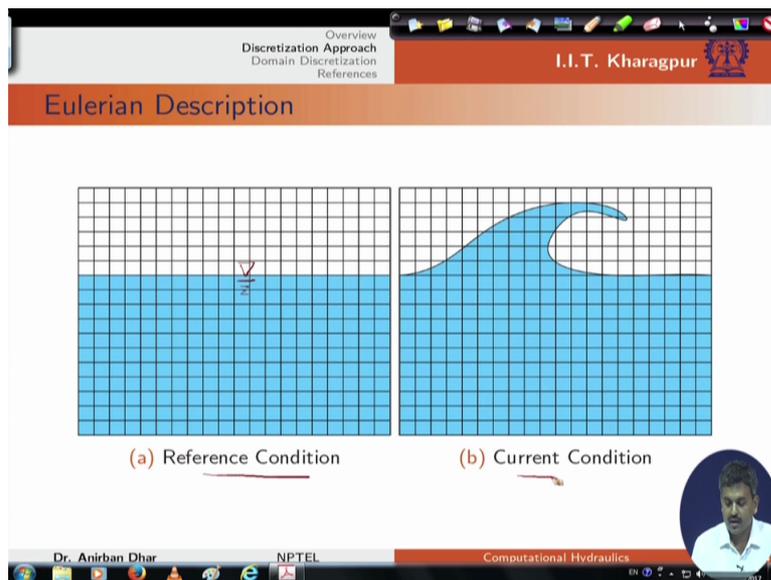
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So what is this Eulerian description? Let us say that we have one floor domain where water level up to this part.
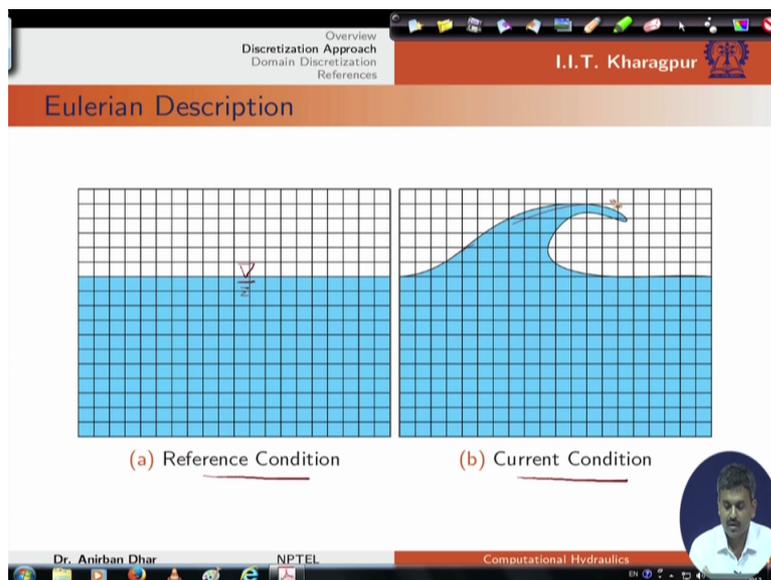
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We can divide this domain into grids and with this information if we proceed this is our reference condition this is our reference condition and this is the current condition.

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So obviously there will be changed in the water surface. So change we are tracking with the available grid. That means our grid is constant. That is not changing.
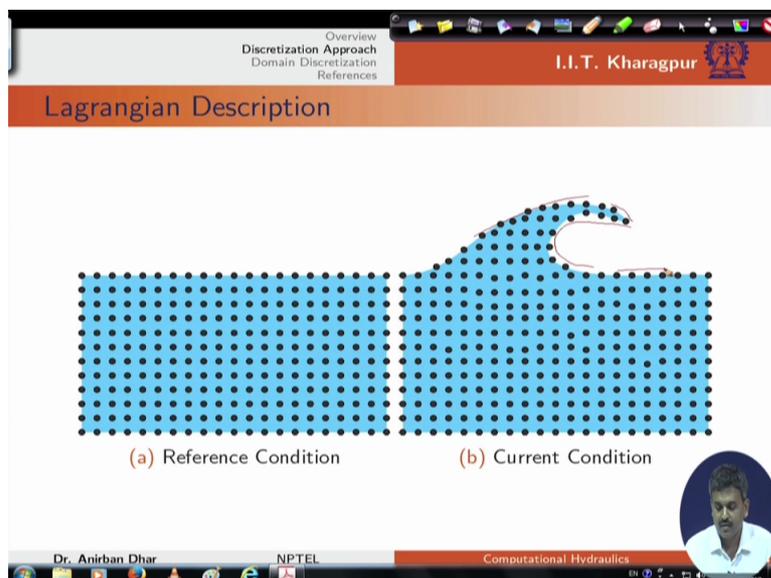
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In case of Lagrangian description, we divide the domaininto number of particles and we track these particles with time and important point is that these particles basically represent the volume.

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(a) Reference Condition
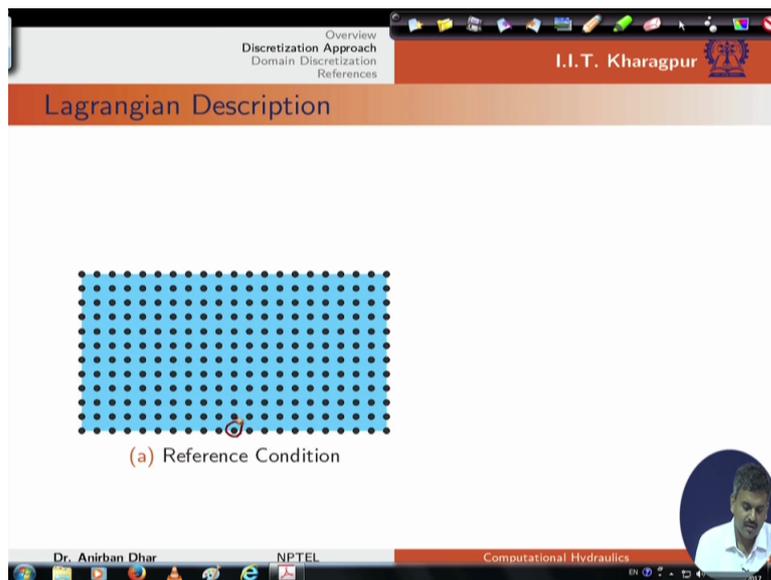
These are not individual orisolated particles. These volumes these particles represent elemental volume and for each particlewe can get information about the fill variance. So this is a reference condition. If we solve this problem so obviously we can see thatthe particle positions are changing and in terms of particles we can define our current position or current condition.

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(a) Reference Condition   (b) Current Condition

In Eulerian-Lagrangian descriptionwe consider both the things that is fix grid system and moving particle system. So this is our reference condition again.This is the water level.

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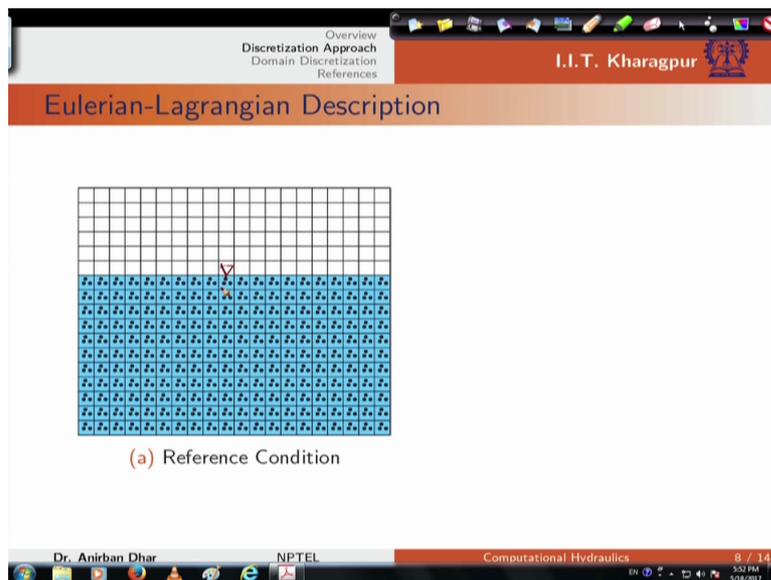
So with this information if we proceed then we can see that there will be movement of particles from grid to grid and with this information we can get the field variable values based on grid and we can move the particles based on the velocity calculatedwith the help of the values of support domain.

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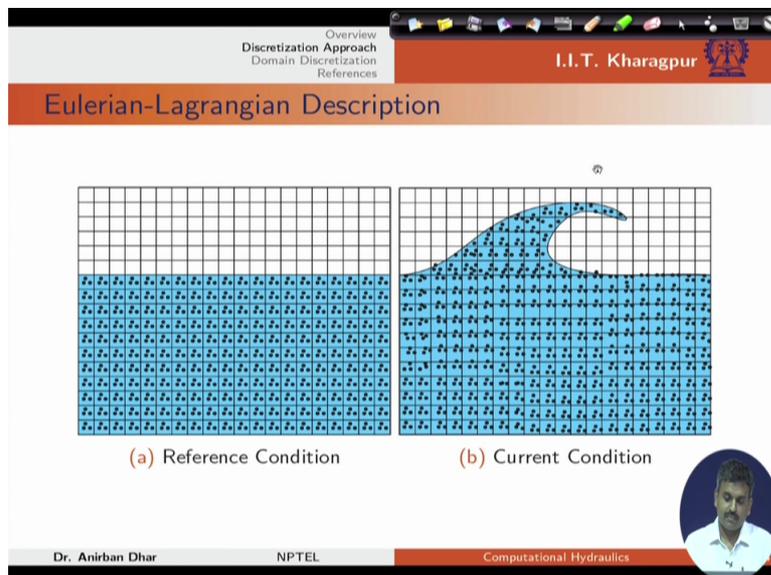
So if we consider the general differential equationwith the general function that is phi x, y, z, t.
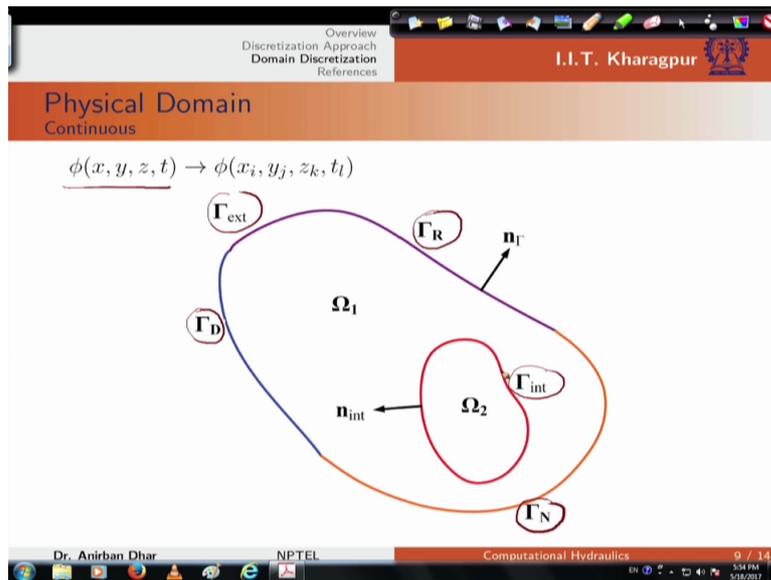
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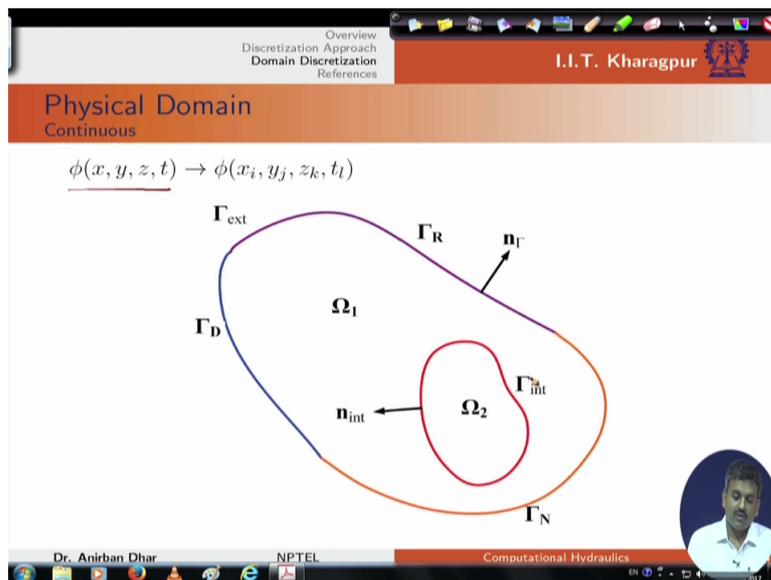
For continuous domain we should have close form solution. But through numerical discretization we are getting discrete values. So xi, yj, zk, tl these areactually corresponding to a particular space time point and for that space time point we can get the value of field variable.
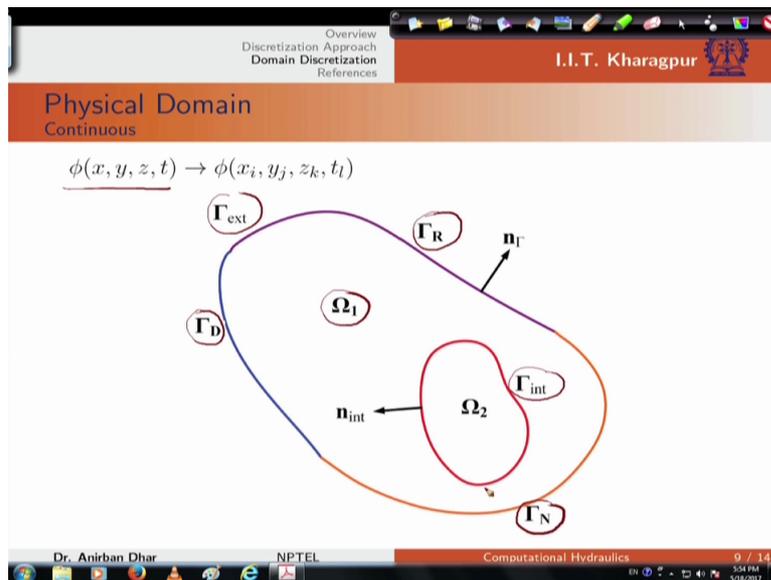
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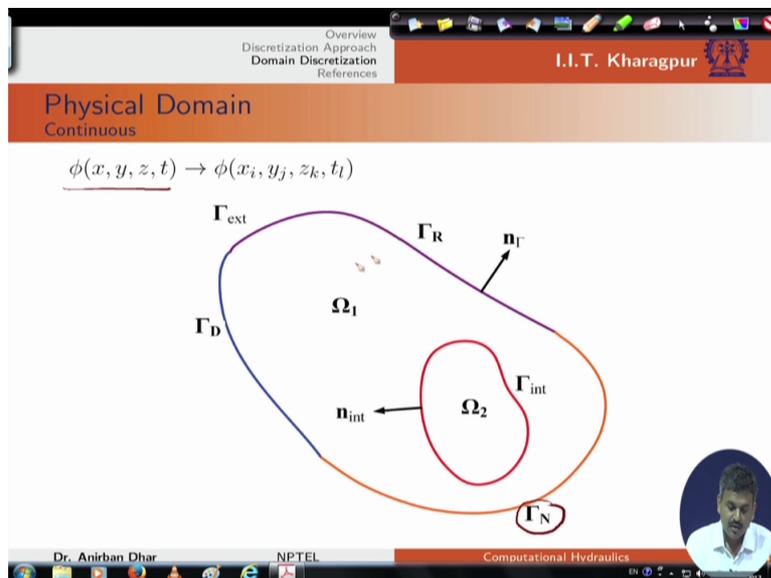
So if we divide this domain where gamma N represents Neumann boundary condition. Gamma D represent the Dirichlet boundary condition and gamma Rrepresents (ro) robin kind of boundary condition and in this case gamma exterior in general representing the external boundary condition, gamma interior is for internal boundary condition.
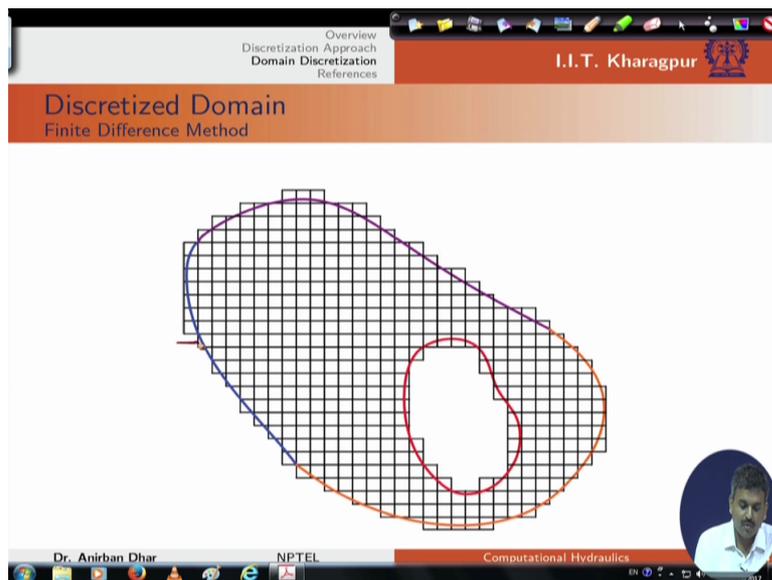
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So if we consider flow or other kind of problems within this omega 1 domain, excluding this omega 2 domain, then we need to divide this domain into parts or sub parts.
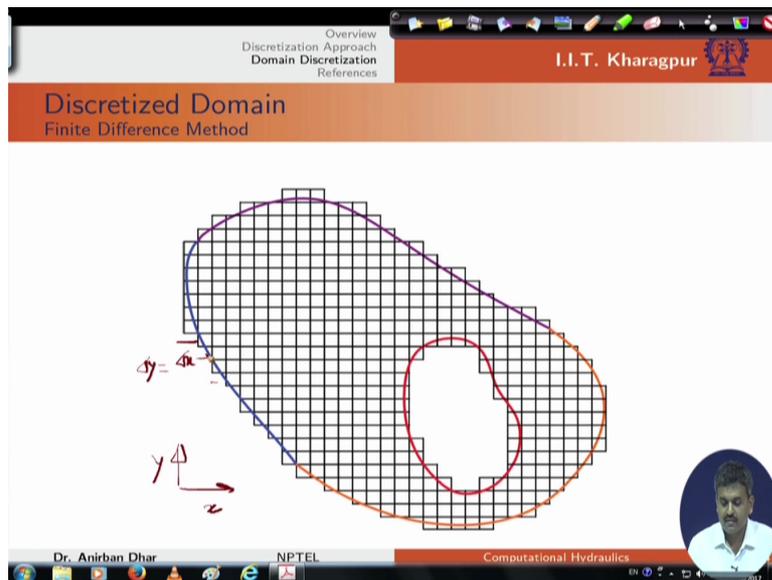
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So how to perform that division? So we can dividethis domain with rectangular grids and forthis grid you can see that a rectangular gridding is uniform in nature.
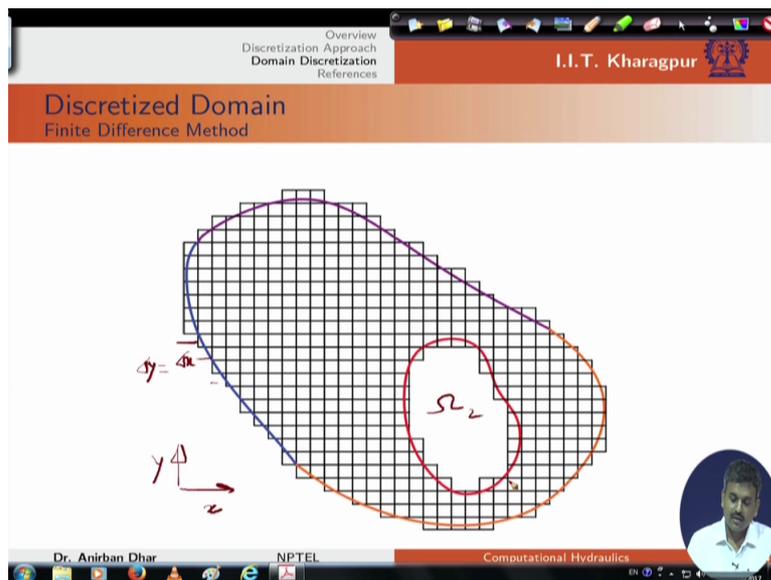
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So these are of equal size both. If we consider this as X direction and this as Y, so delta X equals to delta Y in this case.
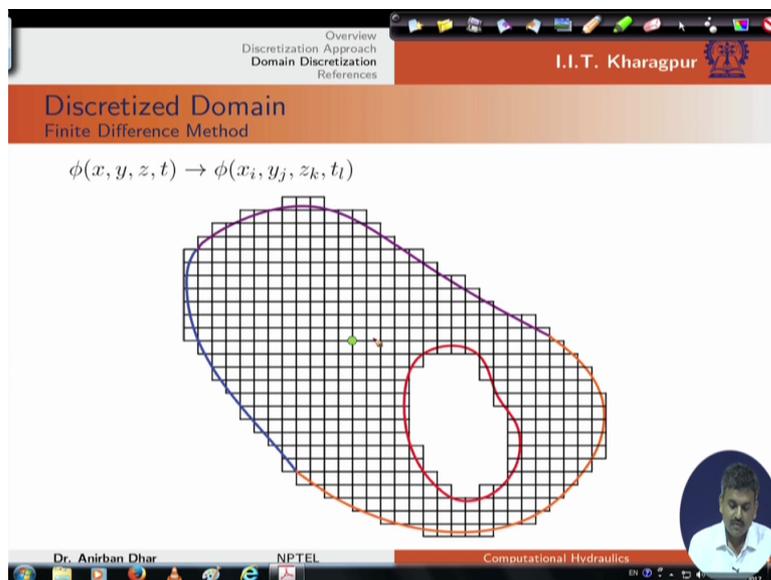
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So excluding this interior omega 2 domain, we can get the gridding.

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And if we consider single point value within this domain so we cannot choose any arbitrary point. So we need to considerthese intersection points for our analysis. So these are called as grid points and for these grid points only our field variables are defined.

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In this case this phi xi, yj zk, tl are although this is two dimensional problem we can see. So for this one it will be xi, yj and with time there will be variation. So with this approximation we can get the information about the field variable.

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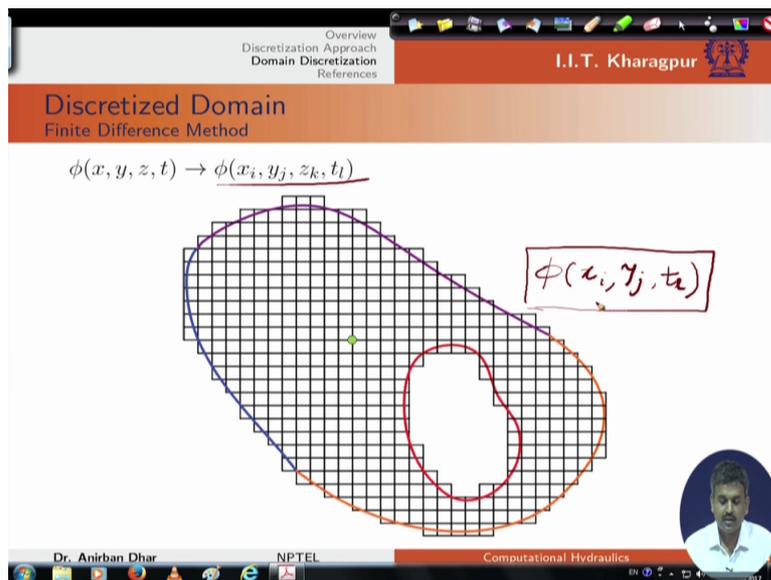
If we discretize using finite volume method, so we can use either structured or unstructured mesh.We can divide it into triangular parts. Againwe can get information for a particular point in the domain. But in this case finite volume is considered as representative volume or elemental volume.
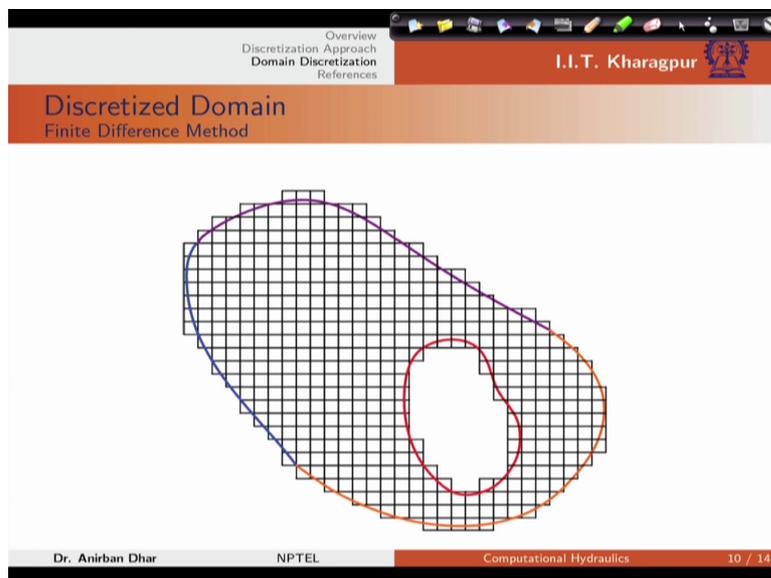
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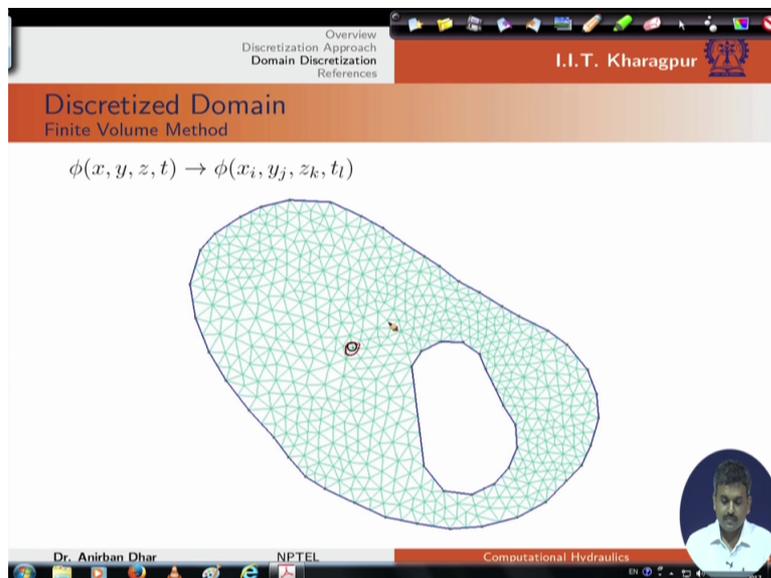
If we consider point generation method, so again we can dividethis domain in terms of points. With this points we can get the information about the field variables. Again if we consider the information about the field variable at any arbitrary point so we should choose that point within the domain and our numerical discretization approach should consider or should define the relationship between the field variable value at this point and surrounding points.
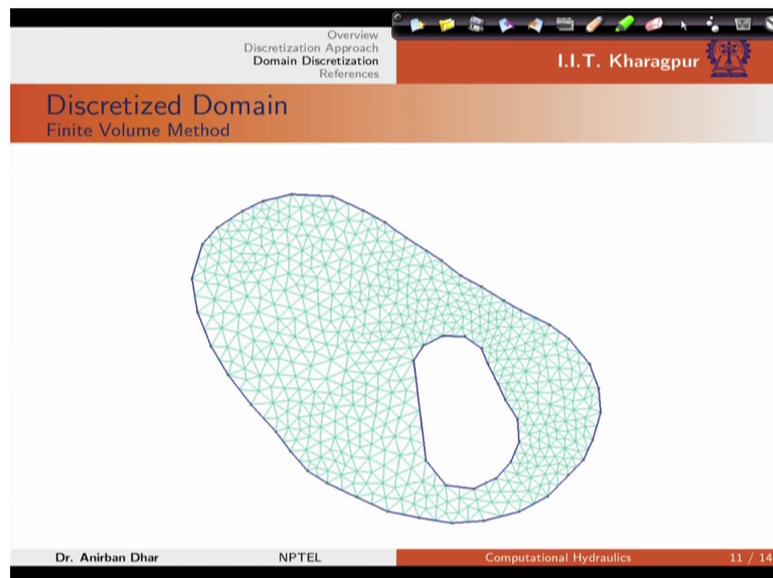
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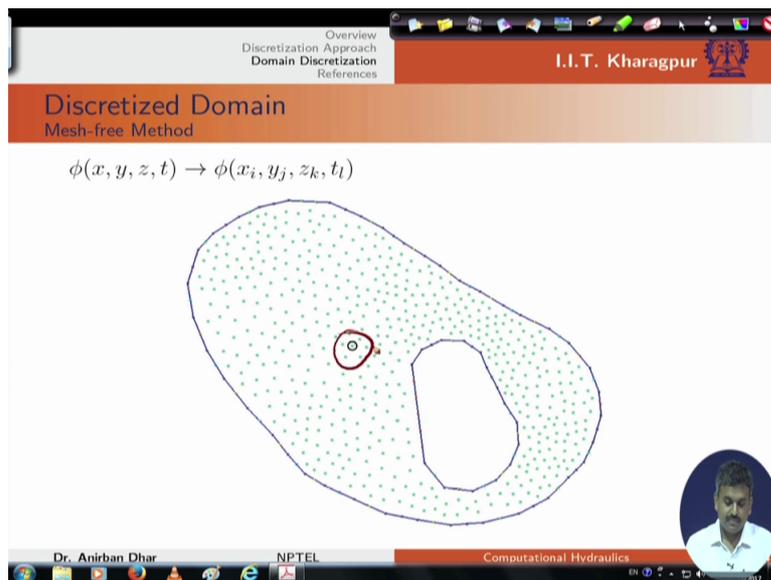
So with this information we can proceed and we can discretize our domain into different parts, either in terms of grids or meshes or in terms of points. So in generalif we consider, so we have finite difference method that is for grid based systems.
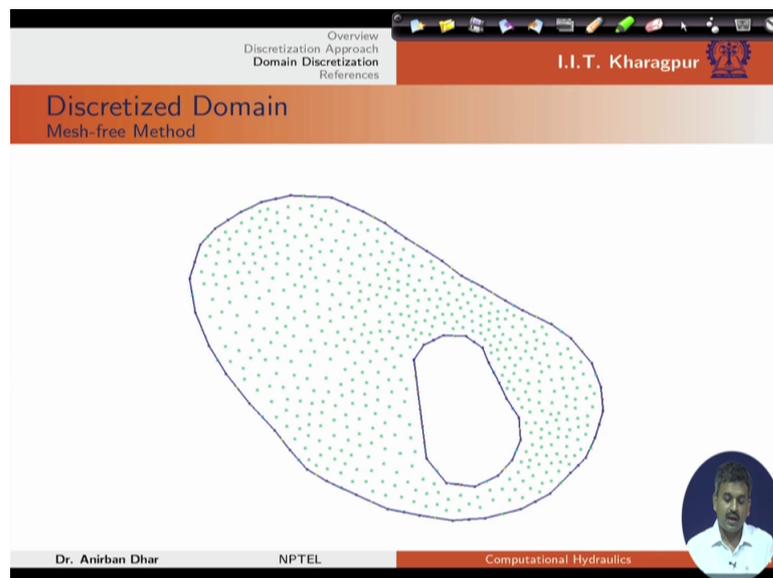
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If you have finite volume method either it can be grid based or this kind of unstructured meshing system.

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If we have mesh free methods, we should use points and for these points we can define the field variables.

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So with this information we can start the numerical discretization. Thank you.