

Computational Hydraulics
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Lecture 39
Steady Channel Flow: Channel Network

Welcome to this lecture of the course computational hydraulics. We are in module 4, surface water hydraulics. And this is lecture number 4 of this module which is unit 4, steady channel flow and channel network. In our previous lecture class we have discussed the problems for single channel and channel in series.

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The image shows a presentation slide with a white background and a red header. The header contains the text "Problem Definition", "Discretization", "Junction Conditions", and "Problem Statement" on the left, and "I.I.T. Kharagpur" with the institute's logo on the right. The main content area features a red box with the text "Module 04: Surface Water Hydraulics" and "Unit 04: Steady Channel Flow: Channel Network". Below this, the name "Anirban Dhar" is displayed, followed by "Department of Civil Engineering" and "Indian Institute of Technology Kharagpur, Kharagpur". At the bottom, it says "National Programme for Technology Enhanced Learning (NPTEL)". The footer of the slide includes "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "1 / 23".

What is the learning objective for this particular lecture class? To solve steady channel flow for channel network problem using implicit method. Again we will be talking about the solution of nonlinear equation. This is not time implicit. This is implicit equation.

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Problem Definition
Discretization
Junction Conditions
Problem Statement

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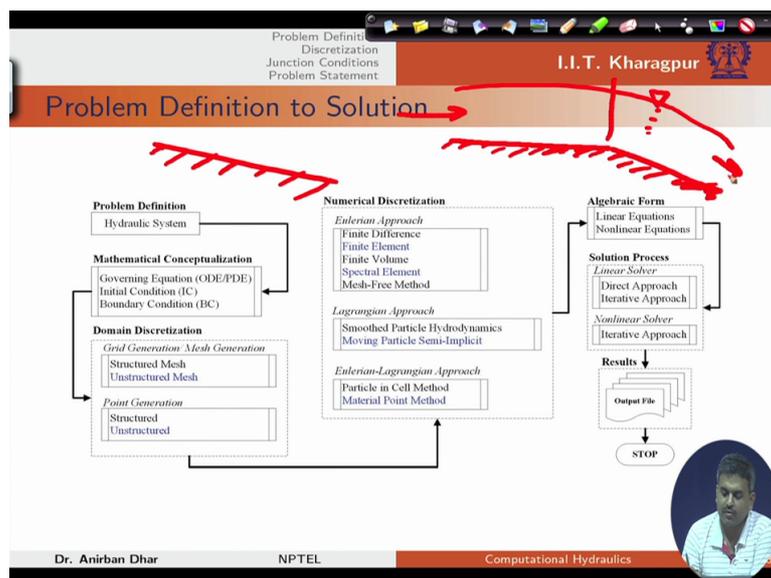
Learning Objective

- To solve steady channel flow for channel network problem using implicit method.

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Problem definition to solution, in our previous lecture class we have talked about single channel. Let us say we have single channel and we have also solved the case where two channels are connected. So initial one was having milder slope, then we have comparatively steeper slope like this. And there was junction point and single channel. In this case the situation is that whatever flow is entering into the channel we are getting same amount of flow here.

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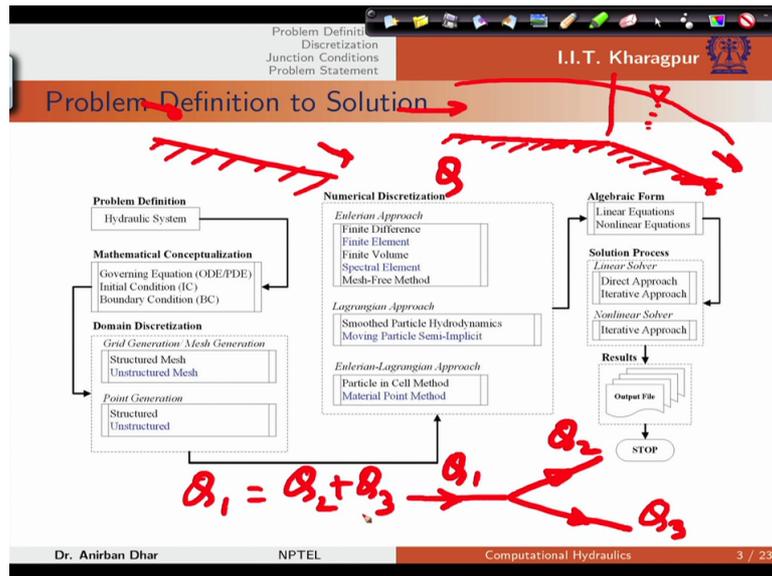


So in this case our given discharge we can solve this problem. And only variable is y or flow depth. But if we have multiple channels, let us say that if we have this kind of situation where

flow is coming from here and there are two channels in the downstream main, so obviously if the flow here is Q , flow there in the one section is Q_2 and another section is Q_3 . Obviously this Q_1 should be equal to Q_2 and Q_3 . But we do not know the exact value of Q_2 and Q_3 .

It depends on the nature of the flow and the upstream and downstream conditions and junction conditions available within the channel network system.

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Problem definition, governing equation. Governing equation for channel flow can be written as boundary value problem. Again we will be using same state of governing equations. So continuity equation, momentum equation. But in our previous lecture class we have not utilised this governing equation during our solution process.

But in channel flow again at any junction there will be multiple channels involved in the flow process. So we cannot exactly calculate or estimate the discharge value at the junction without solving these equations.

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Problem Definition
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Problem Definition

Governing Equation for Channel Flow can be written as,

Boundary Value Problem

Continuity Equation: $\frac{dQ}{dx} = 0$

Momentum Equation: $\frac{dE}{dx} = -S_f$

with

$$E = y + z + \frac{\alpha Q^2}{2gA^2}$$

where

y = depth of flow	x = coordinate direction
S_f = friction slope $(= \frac{n^2 Q^2}{R^4/3 A^2})$	α = momentum correction
A = cross-sectional area	Q = discharge
R = hydraulic radius	g = acceleration due to g
z = elevation of the channel bottom w.r.t. datum	

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So what we will do, we will try to utilise the discretized form of both the equations and we will try to solve it simultaneously so that we will have the solution for both y that is the flow depth and discharge for any channel network. Interestingly in our last lecture class we have considered NL plus 1 number of variables. That means we have considered only y as flow variable. Now if we introduce this Q discharge as another variable then we need to solve $2NL$ plus 1 number of equations.

That means we need equations which will be coming or discretized equation which will be coming from this continuity, momentum. But these equations are applicable for segments. So obviously we will get $2NL$ number of equations here. And we need $2NL$ plus 1 into 2. That means these many variables. So extra equations we have to add to get the solution.

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Problem Definition

Governing Equation for Channel Flow can be written as,

Boundary Value Problem

Continuity Equation: $\frac{dQ}{dx} = 0$ N_{t+1} Q

Momentum Equation: $\frac{dE}{dx} = -S_f$ N_{t+1} γ

with $2N_L$ $E = y + z + \frac{\alpha Q^2}{2gA^2}$ $2(N_{t+1})$ $Q\gamma y$

where

y = depth of flow x = coordinate direction
 S_f = friction slope ($= \frac{\tau^2 Q^2}{R^4/3 A^2}$) α = momentum correction
 A = cross-sectional area Q = discharge
 R = hydraulic radius g = acceleration due to g
 z = elevation of the channel bottom w.r.t. datum

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So for channel flow we have this general structure. We have our elevation head, flow depth, in this case velocity head or kinetic energy head. On the downstream end also we have our elevation head, flow depth, this kinetic energy head here and this is the friction slope which is the average one by considering the two friction slopes at two consecutive sections, i and i plus 1.

And i is the segment number. Δx which is the channel section or Δx . And it is associated with the channel reach. For a particular reach we will consider a single value of Δx .

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Channel Flow

The diagram illustrates the relationship between the Energy Grade Line (EGL) and the Hydraulic Grade Line (HGL) for a channel reach of length Δx . The EGL is shown as a dashed line, and the HGL is shown as a solid line. The channel bottom is represented by a solid line. The water surface profile is shown as a dashed line. The diagram includes various parameters such as Q^2 , α , g , A , y , z , S_0 , and S_f . Handwritten red annotations highlight the friction slope term $\frac{1}{2}(S_{f,i+1} + S_{f,i})\Delta x$ and the segment number i .

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Now start of discretization we need to discretize the continuity equation for i th segment. If we discretize this equation for i th segment of L th channel then basically we are getting this for i th segment. We have any arbitrary case.

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Problem Definition
Discretization
Junction Conditions
Problem Statement

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Discretization

Continuity Equation

Continuity equation for i^{th} segment of the l^{th} channel reach can be discretized as,

$$\frac{dQ}{dx} = 0$$
$$\frac{Q_{l,i+1} - Q_{l,i}}{\Delta x_l} = 0$$

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L is the index for channel number, i is the index for different sections within a channel reach. Now in simplified form for i th segment of the L th channel we have this equation. Now we will not consider a constant value corresponding to this and we are not going to solve the same state of equations that we have solved in our previous case. We need to consider these two terms as variables. So we will not assign any value or we do not have any value for our system.

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Discretization

Continuity Equation

Continuity equation for i^{th} segment of the l^{th} channel reach can be discretized as,

$$\frac{dQ}{dx} = 0$$

$$\frac{Q_{l,i+1} - Q_{l,i}}{\Delta x_l} = 0$$

l = index for channel number
 i = index for different sections within a channel reach.
 In simplified form for i^{th} segment of the l^{th} channel reach,

$$\underline{Q_{l,i+1}} = \underline{Q_{l,i}}$$


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In functional form I can just transfer this value on the left hand side. I can write this continuity L i. This is valid for 1 to NL number of segments.

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Algebraic Form

Continuity Equation

In functional form,

$$\underline{C_{l,i}} = \underline{Q_{l,i+1}} - \underline{Q_{l,i}} = 0, \forall i \in \{1, \dots, N_l\}$$


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Now again we can use the Newton Raphson method and for Newton Raphson method although this equation is linear we need to calculate the coefficients or elements of our Jacobian matrix. Now we have in each case if we consider ith reach. Let us say this is ith reach and we have i and i plus 1. These two are sections.

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Algebraic Form

Continuity Equation

In functional form,

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \dots, N_l\}$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

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Now in this case for section i we have variable L_i and Q_{L_i} . On the right hand side we have y_{L_i+1} , Q_{L_i+1} . So all total for any segment we will have four variables, two for depth and two for your discharge.

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Algebraic Form

Continuity Equation

In functional form,

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \dots, N_l\}$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

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Now in this case if we have one equation so we should get derivative terms with respect to these variables. So if we have $C_{L_i} = 0$, this is our functional form. We should differentiate it with respect to y_{L_i} , Q_{L_i} , y_{L_i+1} , Q_{L_i+1} . So first of all we do not have any y here. So this is zero.

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Algebraic Form

Continuity Equation

In functional form, $C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \dots, N_l\}$

$C_{l,i} = 0$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

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Q L i this is minus 1 because minus sign is there. Y L i plus 1 no y i plus 1 term, this is zero. This is Q C L i del Q L i plus 1, this is 1 again because it has got positive sign here. So we have determined our coefficients for the Jacobian matrix for our continuity equation in this case.

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Algebraic Form

Continuity Equation

In functional form, $C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \dots, N_l\}$

$C_{l,i} = 0$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

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Now again we need the discretization of our momentum equation. Now to solve this momentum equation we can discretize it using forward difference L i plus 1 and E L i and we will take average of this friction slope in this case.

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Discretization

Momentum Equation

Momentum equation for i^{th} segment of the l^{th} channel reach can be discretized as,

$$\frac{dE}{dx} = -S_f$$

$$\frac{E_{l,i+1} - E_{l,i}}{\Delta x_l} = -\frac{1}{2} (S_f|_{l,i+1} + S_f|_{l,i})$$


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In expanded form we can write this $y, z, \alpha Q^2, 2gA^2, L_{i+1}, L_i$ these are the things we will get. And on the right hand side we have averaged form of SF.

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Discretization

Momentum Equation

Momentum equation for i^{th} segment of the l^{th} channel reach can be discretized as,

$$\frac{dE}{dx} = -S_f$$

$$\frac{E_{l,i+1} - E_{l,i}}{\Delta x_l} = -\frac{1}{2} (S_f|_{l,i+1} + S_f|_{l,i})$$

In expanded form,

$$\frac{\left(y + z + \frac{\alpha Q^2}{2gA^2} \right)_{l,i+1} - \left(y + z + \frac{\alpha Q^2}{2gA^2} \right)_{l,i}}{\Delta x_l} = -\frac{1}{2} \left[\left(\frac{n^2 Q^2}{R^{4/3} A^2} \right)_{l,i+1} + \left(\frac{n^2 Q^2}{R^{4/3} A^2} \right)_{l,i} \right]$$

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Now from here we can get the functional form. Again these are variables. Q is variable. $Y L_{i+1}$ that is function of $y L_{i+1}$. So these are variables. So direct variables is y and Q .

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Discretization

Momentum Equation

In functional form for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

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Now if we write this we need to consider the rivers flow situation. So if the flow is in forward direction we will consider plus Q. If it is in backward direction we will consider as negative Q. So if we use this square term here there will be loss of information. So instead of using this square we will try to utilise this Q L i plus 1, absolute value of Q L i plus 1. So obviously one term will get positive value. Another term if it is positive or negative that will be considered during calculation process. So similarly we can define the terms here.

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Discretization

Momentum Equation

In functional form for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

Considering reverse flow situation,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^2} - \frac{Q_{l,i}|Q_{l,i}|}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}|Q_{l,i+1}|}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}|Q_{l,i}|}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$


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Now outside we have only alpha L divided by 2g, nL square del xL divided by 2. So in this case this term is not C1 and C2. So these are not C1 C2 because we do not have Q square term there.

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Discretization

Momentum Equation

In functional form for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

Considering reverse flow situation,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^2} - \frac{Q_{l,i}|Q_{l,i}|}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}|Q_{l,i+1}|}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}|Q_{l,i}|}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

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So we can define two new terms in the next slide. But in this case as I have already mentioned we have 2 NL non linear equations. So NL number of equations which are corresponding to C L i or continuity equation. And NL number of equation which are corresponding to momentum and this is corresponding to momentum equation. So all total we have 2 NL plus 1 number of unknowns.

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Discretization

Momentum Equation

In functional form for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

Considering reverse flow situation,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^2} - \frac{Q_{l,i}|Q_{l,i}|}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}|Q_{l,i+1}|}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}|Q_{l,i}|}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

2 N_l non-linear equations with 2(N_l + 1) unknowns

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So in this case whenever we are going to (dis) take the derivative of this M, we should use this squared terms only because that absolute terms that is not differentiable.

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Discretization

Momentum Equation

In functional form for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g} \left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2} \right) + \frac{n_l^2 \Delta x_l}{2} \left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2} \right], \quad \forall i \in \{1, \dots, N_l\}$$

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So if I write that the elements of the Jacobian matrix corresponding to momentum term so we have these expressions. These are similar to the expression that we have already got for our channel in series or single channel case. But we have two extra terms here because we need to consider two extra variables in this case. So we have this D1 and D2. D1, D2 these are constant terms and these are corresponding to Lth channel because alpha L, nL, delta xL, these are parameters which are function of or which are dependent on our channel reach.

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Algebraic Form

Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}^2}{A_{l,i}^3 R_{l,i}^{4/3}} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}^2}{3A_{l,i}^2 R_{l,i}^{5/3}} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2Q_{l,i}}{A_{l,i}^2 R_{l,i}^{4/3}}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}^2}{A_{l,i+1}^3 R_{l,i+1}^{4/3}} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}^2}{3A_{l,i+1}^2 R_{l,i+1}^{5/3}} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2Q_{l,i+1}}{A_{l,i+1}^2 R_{l,i+1}^{4/3}}$$

with

$$D_1 = \frac{\alpha_l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

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In general case we already know that dR by dy we can calculate like this where T is the top width, P is the weighted perimeter, R is hydraulic radius. So this is the change in hydraulic radius with respect to y . DA and dY , this is dA dy and dR dy . Both are functions of y .

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Algebraic Form

Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}^2}{A_{l,i}^3 R_{l,i}^3} \frac{dA}{dy} \Big|_{l,i} + \frac{4Q_{l,i}^2}{3A_{l,i}^2 R_{l,i}^3} \frac{dR}{dy} \Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2Q_{l,i}}{A_{l,i}^2 R_{l,i}^3}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}^2}{A_{l,i+1}^3 R_{l,i+1}^3} \frac{dA}{dy} \Big|_{l,i+1} + \frac{4Q_{l,i+1}^2}{3A_{l,i+1}^2 R_{l,i+1}^3} \frac{dR}{dy} \Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2Q_{l,i+1}}{A_{l,i+1}^2 R_{l,i+1}^3}$$

with

$$D_1 = \frac{\alpha l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

For general channel cross-section,

$$\frac{dR}{dy} = \frac{T}{P} - \frac{R}{P} \frac{dP}{dy}$$

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Boundary condition for subcritical flow again we will consider that on the downstream end we will have the specified flow condition. So this is the condition number one. Condition number one because we have considered NL equations in terms of C and n . And we have 2 NL plus 1 number of variables. So if we subtract this we need two more equations.

(Refer Slide Time: 19:31)

Problem Definition
Discretization
Junction Conditions
Problem Statement

I.I.T. Kharagpur

Discretization

Boundary Condition

For subcritical flows,

$$y_{l,N_t+1} = y_d$$

$$DB_{l,N_t+1} = y_{l,N_t+1} - y_d$$

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So what we can do, we can specify maybe discharge boundary condition at the upstream end. So at the upstream maybe which is the channel 1, we can specify this is for channel reach 1 and section 1 we can specify 1-1 equals to Q which is specified Q or Q upstream value. And that will be the second condition and we can solve this for our case.

(Refer Slide Time: 20:18)

Problem Definition
Discretization
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Problem Statement

I.I.T. Kharagpur

Discretization Boundary Condition

For subcritical flows,

$$y_{l,N_t+1} = y_d$$

$$DB_{l,N_t+1} = y_{l,N_t+1} - y_d$$

Handwritten red notes: $a_{1,1} = Q^u$

Diagram: A horizontal line representing a channel reach. A vertical dashed line on the left is labeled '1-1' in red, indicating a discharge boundary condition.

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In this case obviously we have four terms. But these first two are zero because we do not have any y_L NL or Q_L NL terms. So these are zero. Again we do not have any Q_L NL plus 1 term here, this is zero. So only term left is corresponding to y_L NL plus 1.

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Problem Definition
Discretization
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Problem Statement

I.I.T. Kharagpur

Discretization Boundary Condition

For subcritical flows,

$$y_{l,N_t+1} = y_d$$

$$DB_{l,N_t+1} = y_{l,N_t+1} - y_d$$

Elements of Jacobian Matrix can be written as,

$$\frac{\partial DB_{l,N_t+1}}{\partial y_{l,N_t}} = 0$$

$$\frac{\partial DB_{l,N_t+1}}{\partial Q_{l,N_t}} = 0$$

$$\frac{\partial DB_{l,N_t+1}}{\partial y_{l,N_t+1}} = 1$$

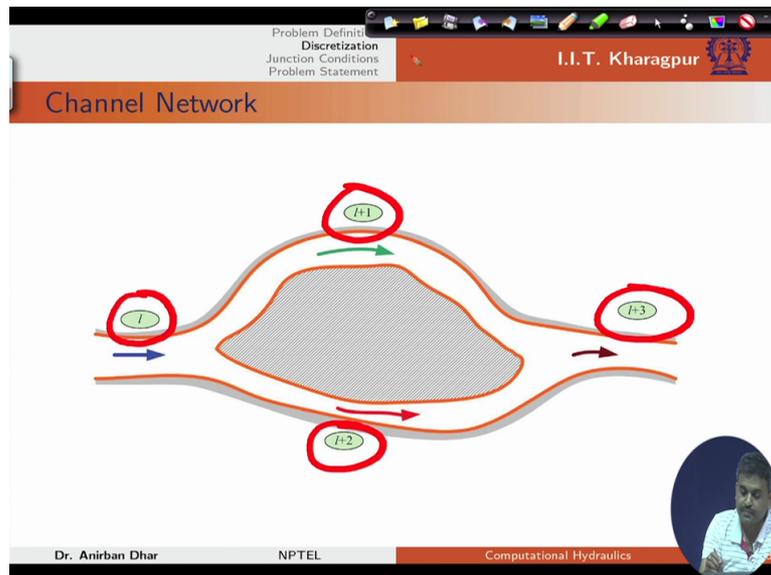
$$\frac{\partial DB_{l,N_t+1}}{\partial Q_{l,N_t+1}} = 0$$

Handwritten red annotations: The first two equations are boxed and labeled '0'. The last equation is boxed and labeled '0' with an arrow pointing to the right.

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Now we have all those equations there. Now we can have other conditions within our system. If we have a channel network like we have discussed our single channel or channel in series, we can solve this problem. But we need to specify our channel junction conditions. So in this case like previous one we have channel number L , $L + 1$, $L + 2$, $L + 3$.

(Refer Slide Time: 21:54)



Now if we impose the channel conditions or the channel junctions conditions we need to consider those conditions between this blue one, green one, blue one, red one. So one condition is that the flow from left side should be equal to the summation of flow from the rightward or if we consider this JN or junction number 1 as general function, whatever inflow is coming to the system that should be equal to outflow.

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Problem Definition
Discretization
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Channel Network

$$\sum Q_T = \sum Q_0$$

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And in this case we can have multiple channel junctions. So in this case if we have another junction in the downstream portion we need to specify both continuity and energy conditions. Energy condition means we need to see the energy balance condition between this section and the section and these two sections.

(Refer Slide Time: 23:25)

Problem Definition
Discretization
Junction Conditions
Problem Statement

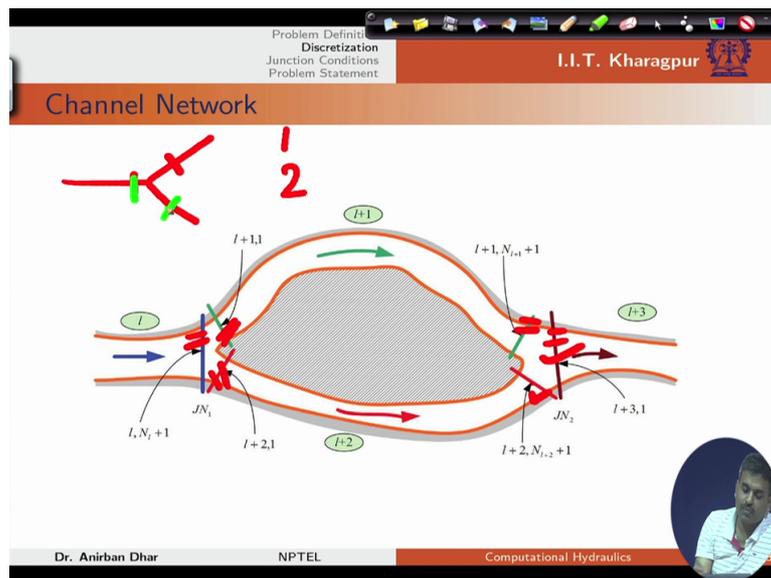
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Channel Network

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So obviously for any channel junction if we have this three case or three channel case we will have one continuity and two momentums. So all total three conditions because continuity is one because inflow and outflow values we need to consider. And two momentums or two energy condition. So in this case we need to consider energy between these two and again energy between this one and this one.

(Refer Slide Time: 24:14)



So all total for two junctions six conditions will be there. So this is a general thing, mass conservation. So channel discharge at inflow branch and channel discharge at the outflow branch considering all branches. And energy condition or neglect the losses. If we have different channel elevation then we should add elevation term. Otherwise we can directly consider the flow depth conditions.

(Refer Slide Time: 24:54)

Problem Definition
Discretization
Junction Conditions
Problem Statement

I.I.T. Kharagpur

Channel Networks

Internal Boundary condition

The junction conditions can be written as,

Mass conservation

$$\sum Q_I = \sum Q_O$$

where
 Q_I = channel discharge at inflow branch and Q_O = channel discharge at outflow branch

Energy conservation

$$y_{l, N_l+1} + z_{l, N_l+1} = y_{l+1, 1} + z_{l+1, 1} = y_{l+2, 1} + z_{l+2, 1}$$

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For junction let us say that we have junction 1. For junction 1 we have this continuity condition. Then we need to satisfy that energy condition there and energy condition we have two conditions. So out of this because in every case for any segment we need to find out variables or four number of variables or derivatives or Jacobian matrix terms for four number of variables.

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Problem Definition
Discretization
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Problem Statement

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Channel Networks

Internal Boundary condition

Junction 1

$$JC_{JN_1,1} = Q_{l, N_l+1} - Q_{l+1, 1} - Q_{l+2, 1} = 0$$

$$JC_{JN_1,2} = y_{l, N_l+1} - y_{l+1, 1} + z_{l, N_l+1} - z_{l+1, 1} = 0$$

$$JC_{JN_1,3} = y_{l, N_l+1} - y_{l+2, 1} + z_{l, N_l+1} - z_{l+2, 1} = 0$$

4 nos

$$\frac{\partial JC_{JN_1,1}}{\partial Q_{l, N_l+1}} = 1 \quad \frac{\partial JC_{JN_1,1}}{\partial Q_{l+1, 1}} = -1$$

$$\frac{\partial JC_{JN_1,1}}{\partial Q_{l+2, 1}} = -1$$

$$\frac{\partial JC_{JN_1,2}}{\partial y_{l, N_l+1}} = 1 \quad \frac{\partial JC_{JN_1,2}}{\partial y_{l+1, 1}} = -1$$

$$\frac{\partial JC_{JN_1,3}}{\partial y_{l, N_l+1}} = 1 \quad \frac{\partial JC_{JN_1,3}}{\partial y_{l+2, 1}} = -1$$

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In this case from our continuity three entries will be there. So this is corresponding to NL plus 1 which is the end section of the channel L and the first section of channel L plus 1 and first section of channel L plus 2. This was the case.

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Problem Definition
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I.I.T. Kharagpur

Channel Networks

Internal Boundary condition

Junction 1

$$JC_{JN_{1,1}} = Q_{l,N_l+1} - Q_{l+1,1} - Q_{l+2,1} = 0$$

$$JC_{JN_{1,2}} = y_{l,N_l+1} - y_{l+1,1} + z_{l,N_l+1} - z_{l+1,1} = 0$$

$$JC_{JN_{1,3}} = y_{l,N_l+1} - y_{l+2,1} + z_{l,N_l+1} - z_{l+2,1} = 0$$

4 Nos

$$\frac{\partial JC_{JN_{1,1}}}{\partial Q_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,1}}}{\partial Q_{l+1,1}} = -1$$

$$\frac{\partial JC_{JN_{1,1}}}{\partial Q_{l+2,1}} = -1$$

$$\frac{\partial JC_{JN_{1,2}}}{\partial y_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,2}}}{\partial y_{l+1,1}} = -1$$

$$\frac{\partial JC_{JN_{1,3}}}{\partial y_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,3}}}{\partial y_{l+2,1}} = -1$$

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So this is NL plus 1 of Lth channel. This is first section of L plus 1 and first section of L plus 2. So from next two equations we have two variables. So 2, 2, here 3. For these three equations we will get total seven entries in the Jacobian matrix. So corresponding to first continuity equation we will get three entries. For next two we will get two entries each.

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Problem Definition
Discretization
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I.I.T. Kharagpur

Channel Networks

Internal Boundary condition

Junction 1

$$JC_{JN_{1,1}} = Q_{l,N_l+1} - Q_{l+1,1} - Q_{l+2,1} = 0$$

$$JC_{JN_{1,2}} = y_{l,N_l+1} - y_{l+1,1} + z_{l,N_l+1} - z_{l+1,1} = 0$$

$$JC_{JN_{1,3}} = y_{l,N_l+1} - y_{l+2,1} + z_{l,N_l+1} - z_{l+2,1} = 0$$

4 Nos

$$\frac{\partial JC_{JN_{1,1}}}{\partial Q_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,1}}}{\partial Q_{l+1,1}} = -1$$

$$\frac{\partial JC_{JN_{1,1}}}{\partial Q_{l+2,1}} = -1$$

$$\frac{\partial JC_{JN_{1,2}}}{\partial y_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,2}}}{\partial y_{l+1,1}} = -1$$

$$\frac{\partial JC_{JN_{1,3}}}{\partial y_{l,N_l+1}} = 1 \quad \frac{\partial JC_{JN_{1,3}}}{\partial y_{l+2,1}} = -1$$

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Similarly for junction 2 we can expand this and write the elements of the Jacobian matrix.

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Problem Definition
Discretization
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Channel Networks

Internal Boundary condition

Junction 2

$$JC_{JN_2,1} = Q_{l+3,1} - Q_{l+1,N_{l+1}+1} - Q_{l+2,N_{l+2}+1} = 0$$

$$JC_{JN_2,2} = y_{l+3,1} - y_{l+1,N_{l+1}+1} + z_{l+3,1} - z_{l+1,N_{l+1}+1} = 0$$

$$JC_{JN_2,3} = y_{l+3,1} - y_{l+2,N_{l+2}+1} + z_{l+3,1} - z_{l+2,N_{l+2}+1} = 0$$

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Now in subcritical flow situation we have NL number of equations for the segments and NL number of equation from the continuity and momentum. First one is from momentum, second one is from continuity. And if we have two boundary conditions, this is subcritical boundary condition at the downstream boundary and at upstream boundary if we have specified boundary condition then we can directly utilise that for the solution process.

Now with this conditions we have 2 NL plus 2. That means 2 NL plus 1 number of equations. Now we can solve the system and we can get the solution for the problem.

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Problem Definition
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I.I.T. Kharagpur

Algebraic Form

Subcritical flow

In general form, continuity equation including boundary condition can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -C_{l,i} \quad \times N_L$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} \Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}} \Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}} \Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}} \Delta Q_{l,i+1} = -M_{l,i} \quad \times N_L$$

$\forall i \in \{1, \dots, N_L\}$

For subcritical flow,

$$\Delta Q_{1,1} = -UB_{l,N_{l+1}}$$

$$\Delta y_{l,N_{l+1}} = -DB_{l,N_{l+1}}$$

} 2
2 N_L + 2
= 2(N_L + 1)

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What is the structure? Structure is that for this matrix we have four entries. This blue one is corresponding to continuity, the red one is corresponding to momentum. So at each segment, so this is let us say channel reach. For each channel reach L we will have 1, 2 like that i, i plus 1. We have i th segment.

(Refer Slide Time: 29:33)

The slide shows a horizontal line representing a channel reach with nodes labeled 1, 2, ..., i , $i+1$. Below the line is a matrix structure. The matrix is partitioned into two parts: a blue part labeled 'C' (continuity) and a red part labeled 'M' (momentum). The blue part has non-zero entries (marked with 'x') at the diagonal and between adjacent nodes. The red part has non-zero entries (marked with 'x') at the diagonal and between adjacent nodes, but with a different pattern than the blue part. A small inset photo of a man is visible in the bottom right corner.

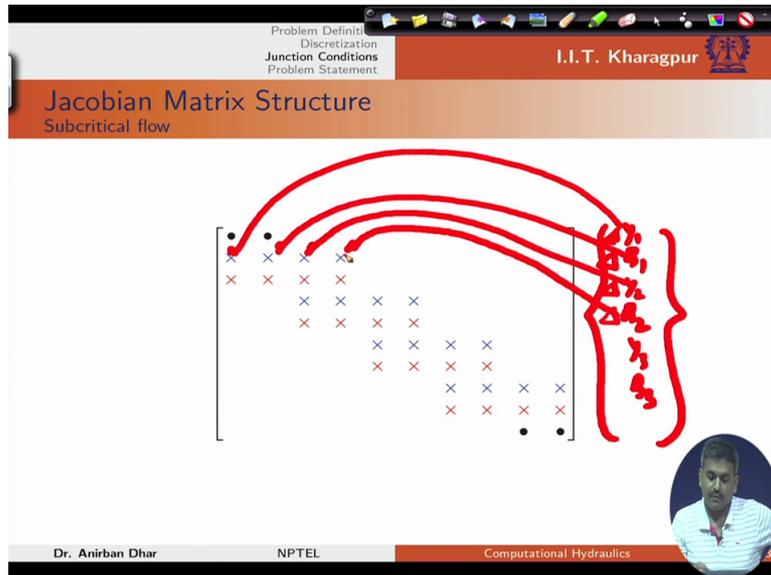
Now in this case first entry corresponding to this segment will be from continuity. This is momentum. Now for section i plus 1 or segment i plus 1 we have connections between i plus one and i plus 2. So obviously we need to consider the entries corresponding to i plus 1 if this is.

(Refer Slide Time: 30:11)

The slide shows a horizontal line representing a channel reach with nodes labeled 1, 2, ..., i , $i+1$, $i+2$. Below the line is a matrix structure. The matrix is partitioned into two parts: a blue part labeled 'C' (continuity) and a red part labeled 'M' (momentum). The blue part has non-zero entries (marked with 'x') at the diagonal and between adjacent nodes. The red part has non-zero entries (marked with 'x') at the diagonal and between adjacent nodes, but with a different pattern than the blue part. A small inset photo of a man is visible in the bottom right corner.

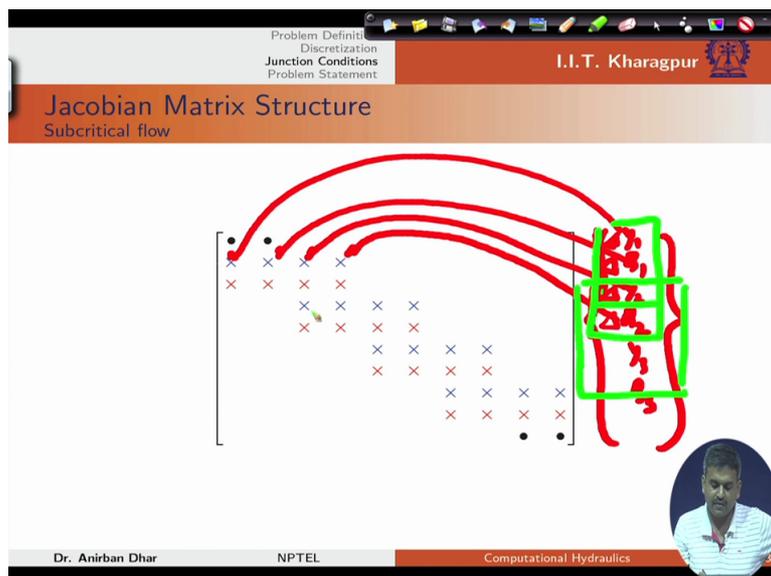
In general we will try to write this variable as $y_1, Q_1, y_2, Q_2, y_3, Q_3$, like that we will consider our variables. So if you multiply y_1 with the first term, Q_1 the second term, y_2 with the third term and Q_2 with the fourth term. So that is the case. Again for momentum we need to multiply the same variables or change in variables, ΔQ and Δy .

(Refer Slide Time: 31:13)



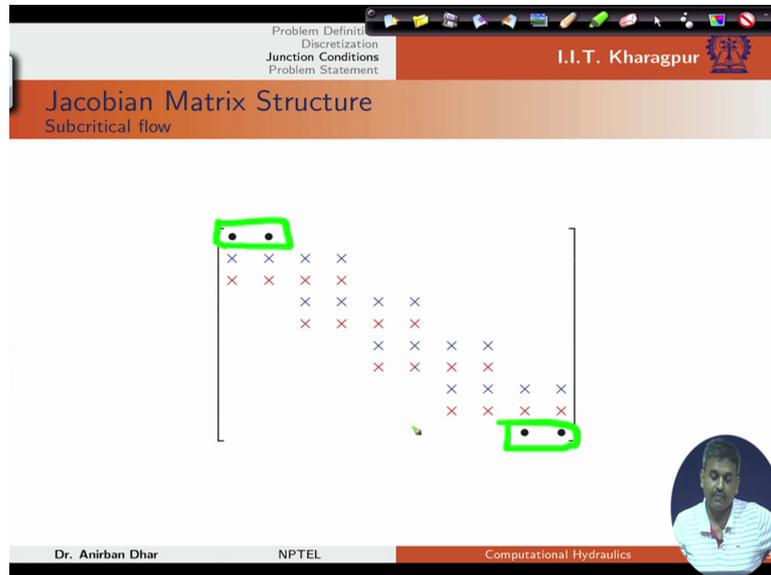
And for the next segment we need to shift our focus from these four entries to the next four entries. So in next four entries, these two entries will be common. So these two entries are common.

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We will multiply it. So obviously in this case we will get the solution. These two are basically corresponding to boundary conditions and if required we can incorporate the junction conditions.

(Refer Slide Time: 31:54)



I have not shown those junctions conditions within this matrix structure. Now in this case we can again solve the problem that we have solved in our previous lecture class that is channels in series. We will consider two channels because in previous lecture class we have considered discharge as constant. But in this particular lecture class we are considering a separate continuity equation during solution process.

So if we are considering separate continuity equation for the solution process so we should get the same result that we have obtained using the single variable case. So let us say that the given information first is channel cross section type is rectangular, B is 15 metres, g is 9 point 81 metre per second square, S not 1 is point triple not 4, S not 2 this is S_2 or bed slope for second channel.

(Refer Slide Time: 33:43)

Problem Definition
Discretization
Junction Conditions
Problem Statement

I.I.T. Kharagpur

Problem Statement

Channels in Series

Given

Channel Cross-Section Type: Rectangular

$B = 15m$
 $g = 9.81m/s^2$
 $S_{01} = 0.0004$
 $S_{02} = 0.0008$

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So obviously in this case we are considering milder channel in the upstream and comparatively steeper channel in the downstream section. Next is n_1 , n_1 is point not 1, n_2 is point not 15, L_{x1} is hundred metres, L_{x2} is hundred metres, Q is 20 metres cube per second and y_d is point 6 metres. Required, estimate the flow that across the channels in series.

(Refer Slide Time: 34:27)

Problem Definition
Discretization
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Problem Statement

I.I.T. Kharagpur

Problem Statement

Channels in Series

Given

Channel Cross-Section Type: Rectangular

$B = 15m$
 $g = 9.81m/s^2$
 $S_{01} = 0.0004$
 $S_{02} = 0.0008$
 $n_1 = 0.01$
 $n_2 = 0.015$
 $L_{x1} = 100m$
 $L_{x2} = 100m$
 $Q = 20m^3/s$
 $y_d = 0.60m$

Required

Estimate the flow depth across the channels in series.

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Now we are not changing any information in this one. Only thing is that we are utilising one extra equation during our solution process. Let us see how we can utilise that thing here. So in this case I have considered the same code and changed that code for this problem. So $junc$ is the junction number, chL is I have two channel reaches, Q_i is the 20 m cube per second,

this is the discharge at upstream section. S not, this is for one and two point triple not 4 and point triple not 8, n is point not 1, point not 15.

B is point this is 15 metres. G is 9 point 81. Lx for both the channel length this is hundred metres. And yd in the downstream section we have point 6 metres. Mnode which is 101-101 and that I have considered here. And epsilon max, this is for iteration we need, 1 into 10 to the power minus 6. And global variable is B and g because Q is again changing here.

(Refer Slide Time: 36:34)

```

1 clc
2 clear
3 // Given Data
4 junc=1;
5 chl=2;
6 Ql=20; //m^3/s
7 S0=[0.0004 0.0008];
8 n=[0.010 0.015];
9 B=15; //m
10 g=9.81 //m/s^2
11 Lx=[100 100]; //m
12 yd=0.6; //m
13 mnode=[101 101];
14 eps_max=1e-6;
15 global('B','g')
16
17 juni=[1 2 101 1]
18 //-----Problem Dependent Parameters-----
19 alpha=[1 1];
20 yv=yd*ones(sum(mnode),1);
21 Qv=Ql*ones(sum(mnode),1);
22 //General variable with y and Q
23 qv=zeros(2*sum(mnode),1);
24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
  
```

We are not considering constant discharge for this problem. So in this case I have defined alpha first. So alpha is 1 for both the channels. Now I have defined this yv. Yv is the variable and I should consider all the nodes and I should assign this initial value here, yd into ones sum mnode. Sum mnode that means 101 plus 101, 202 nodes this value will be assigned yv. Qv again I need to assign initial value for this one. This is Qi into ones sum mnode. That means 101 plus 101 total 202 nodes I have assigned this Qv value.

(Refer Slide Time: 37:34)

```

1 clear
2 clear
3 // Given Data
4 junc=1;
5 chl=2;
6 QI=20; //m^3/s
7 SO=[0.0004 0.0008];
8 n=[0.010 0.015];
9 B=15; //m
10 g=9.81 //m/s^2
11 Lx=[100 100]; //m
12 yd=0.6; //m
13 mnode=[101 101];
14 eps_max=1e-6;
15 global('B','g')
16
17 juni=[1 2 101 1]
18 //-----Problem Dependent Parameters-----
19 alpha=[1 1];
20 yv=yd*ones(sum(mnode),1);
21 Qv=QI*ones(sum(mnode),1);
22 //General variable with y and Q
23 gv=zeros(2*sum(mnode),1);
24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value

```

Now in this case because this channel reach is there. This is for L equals to 1, this is for L equals to 2. So starting from 1 to NL plus 1, again starting from 1 to this is N1 plus 1, this is N2 plus 1. So these are the sections in between.

(Refer Slide Time: 38:19)

```

1 clear
2 clear
3 // Given Data
4 junc=1;
5 chl=2;
6 QI=20; //m^3/s
7 SO=[0.0004 0.0008];
8 n=[0.010 0.015];
9 B=15; //m
10 g=9.81 //m/s^2
11 Lx=[100 100]; //m
12 yd=0.6; //m
13 mnode=[101 101];
14 eps_max=1e-6;
15 global('B','g')
16
17 juni=[1 2 101 1]
18 //-----Problem Dependent Parameters-----
19 alpha=[1 1];
20 yv=yd*ones(sum(mnode),1);
21 Qv=QI*ones(sum(mnode),1);
22 //General variable with y and Q
23 gv=zeros(2*sum(mnode),1);
24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value

```

So in this case we have double index notation. That means L i. If I take L i that means it is representing the channel section in the Lth channel and for ith section, okay. So this is the local numbering. I need one global numbering for this one. What is that global numbering? I should use a global number that information I can utilise during the construction of Jacobian matrix. So that global number is gid or global id here, ok.

(Refer Slide Time: 39:14)

The image shows a MATLAB script window with the following code:

```
1 clear
2 clear
3 // Given Data
4 junc=1;
5 chl=2;
6 QI=20; //m^3/s
7 SO=[0.0004 0.0008];
8 n=[0.010 0.015];
9 B=15; //m
10 g=9.81 //m/s^2
11 Lx=[100 100]; //m
12 yd=0.6; //m
13 mnode=[101 101];
14 eps_max=1e-6;
15 global('B','g')
16
17 juni=[1 2 101 1]
18 //-----Problem Dependent Parameters-----
19 alpha=[1 1];
20 yy=yd*ones(sum(mnode),1);
21 Qv=QI*ones(sum(mnode),1);
22 //General variable with y and Q
23 gv=zeros(2*sum(mnode),1);
24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
```

Handwritten red annotations include a diagram of a channel reach with two sections of length L_1 and L_2 . The first section is labeled $L=1$ and the second $L=2$. A circled L, i is written below the first section. The variable gid is written in red. A small circular inset shows a man's face.

Now I will start from id value equals to zero. So starting from L is equal to 1 to CHL. That means for all channel reaches and from 1 to mnode. Mnode means 1 to NL plus 1. All nodes I should sign idv equals to idv plus 1. Whenever I am entering in this loop I am adding idv equals to idv plus 1. So gid equals to idv. That means one by one I have assigned serial number against each of those sections in different channel reaches. So I will have unique number for any channel section in a particular channel reach.

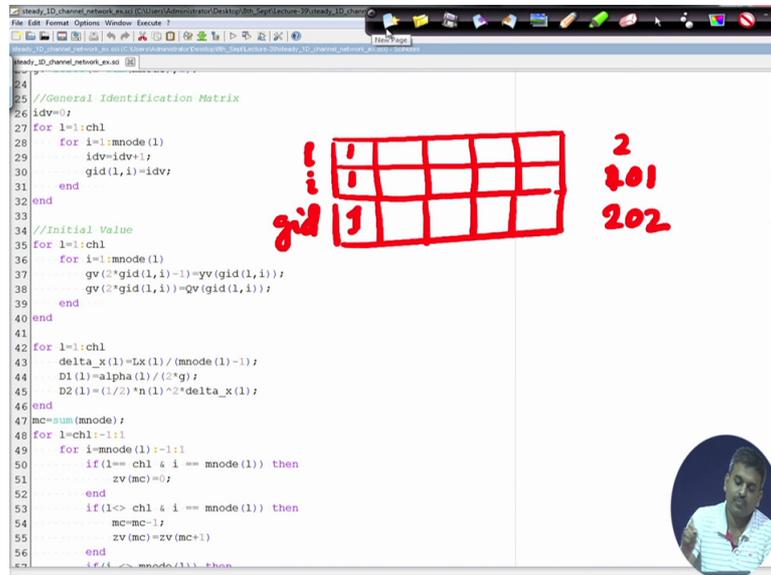
(Refer Slide Time: 40:13)

The image shows the same MATLAB script window as above, but without the handwritten annotations. A small circular inset shows a man's face.

Now here after assigning this one so this is in this structure that if I have this structure which is L i. I should get one value. So for different values of L i, I will get gid. So let us say L1, i1

I am starting from 1. In our case if L equals to 2 and mnode equals to 101 that means for the second channel reach and 101 node my gid should be 202 because I have considered all the nodes or all the sections during this assignment process.

(Refer Slide Time: 41:28)



Now initial value, I have assigned initial value for individual variables yv Qv. But during my solution process I will have only one variable. Let us say that A phi equals to r equation in this case I have J into del phi because change in that variable is required. So this is my right hand side, something is there. But in this process this del phi it considers both y and my Q values. And they are arranged like this, y1 Q1, y2 Q2. And all are del values. So in a single calculation process I cannot use two different vectors y and Q.

So I need one global vector for this definition. So for this global vector definition what I am writing this 2 into gv, this gv is a general vector or general variable in this case. So general variable, what I am doing? I am assigning gid. So if my assignment of gid, this is ranging from 1 to 202.

(Refer Slide Time: 43:30)

```

24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
35 for l=1:chl
36     for i=1:mnode(l)
37         gv(2*gid(l,i)-1)=yv(gid(l,i));
38         gv(2*gid(l,i))=Qv(gid(l,i));
39     end
40 end
41
42 for l=1:chl
43     delta_x(l)=Lx(l)/(mnode(l)-1);
44     D1(l)=alpha(l)/(2*g);
45     D2(l)=(1/2)*n(l)^2*delta_x(l);
46 end
47 mc=sum(mnode);
48 for l=chl:-1:1
49     for i=mnode(l)-1:1
50         if (l== chl & i == mnode(l)) then
51             zv(mc)=0;
52         end
53         if (l<> chl & i == mnode(l)) then
54             mc=mc-1;
55             zv(mc)=zv(mc+1);
56         end
57     end
58 end

```

Handwritten notes in red:

- $J \Delta\phi = \gamma$
- $\Delta\phi = \begin{Bmatrix} \Delta q_1 \\ \Delta y_2 \\ \Delta q_2 \end{Bmatrix}$
- gid with a subscript l
- 202

Now this one is valid for yv which is y value and Q value. But in case of gv which contains both y and q so I need 404 that means 202 into 2 number of variables. So for that one if I take any general gid for Lth segment and ith section and if I multiply it with 2, I should get the index corresponding to Q.

(Refer Slide Time: 44:27)

```

24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
35 for l=1:chl
36     for i=1:mnode(l)
37         gv(2*gid(l,i)-1)=yv(gid(l,i));
38         gv(2*gid(l,i))=Qv(gid(l,i));
39     end
40 end
41
42 for l=1:chl
43     delta_x(l)=Lx(l)/(mnode(l)-1);
44     D1(l)=alpha(l)/(2*g);
45     D2(l)=(1/2)*n(l)^2*delta_x(l);
46 end
47 mc=sum(mnode);
48 for l=chl:-1:1
49     for i=mnode(l)-1:1
50         if (l== chl & i == mnode(l)) then
51             zv(mc)=0;
52         end
53         if (l<> chl & i == mnode(l)) then
54             mc=mc-1;
55             zv(mc)=zv(mc+1);
56         end
57     end
58 end

```

Handwritten notes in red:

- 404
- 202×2
- $J \Delta\phi = \gamma$
- $\Delta\phi = \begin{Bmatrix} \Delta q_1 \\ \Delta y_2 \\ \Delta q_2 \end{Bmatrix}$
- gid with a subscript l
- $2 \times gid(l,i)$ in a red box
- 202

Because I am arranging the variable like this. Q1, y1, y2, Q2, y3, Q3, this is y4, Q4. So if my gid for L i for any arbitrary section or any arbitrary section in this case is 2. If gid equals to 2 that means gid into 2. That means 2 into gid is 4. That means I am considering the fourth term here. So what is my fourth term? Fourth term is Q2. So all even terms are discharge terms.

(Refer Slide Time: 45:30)

```

24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
35 for l=1:chl
36     for i=1:mnode(l)
37         gv(2*gid(l,i)-1)=yv(gid(l,i));
38         gv(2*gid(l,i))=Qv(gid(l,i));
39     end
40 end
41
42 for l=1:chl
43     delta_x(l)=Lx(l)/(mnode(l)-1);
44     D1(l)=alpha(l)/(2*g);
45     D2(l)=(1/2)*n(l)^2*delta_x(l);
46 end
47 mc=sum(mnode);
48 for l=chl:-1:1
49     for i=mnode(l):-1:1
50         if(l==chl & i==mnode(l)) then
51             zv(mc)=0;
52         end
53         if(l<>chl & i==mnode(l)) then
54             mc=mc-1;
55             zv(mc)=zv(mc+1);
56         end
57         if(l<>mnode(l)) then

```

$y_1 \cdot \text{gid}(L,i)$
 B_1
 y_2 2x2
 $B_2! = 4$
 y_3
 B_3
 y_4
 B_4

Now if I take 2 into gid L i minus 1, this is 3. Now 3 means this one. So all odd terms these are corresponding to flow depth values and even terms these are corresponding to discharge values.

(Refer Slide Time: 45:55)

```

24
25 //General Identification Matrix
26 idv=0;
27 for l=1:chl
28     for i=1:mnode(l)
29         idv=idv+1;
30         gid(l,i)=idv;
31     end
32 end
33
34 //Initial Value
35 for l=1:chl
36     for i=1:mnode(l)
37         gv(2*gid(l,i)-1)=yv(gid(l,i));
38         gv(2*gid(l,i))=Qv(gid(l,i));
39     end
40 end
41
42 for l=1:chl
43     delta_x(l)=Lx(l)/(mnode(l)-1);
44     D1(l)=alpha(l)/(2*g);
45     D2(l)=(1/2)*n(l)^2*delta_x(l);
46 end
47 mc=sum(mnode);
48 for l=chl:-1:1
49     for i=mnode(l):-1:1
50         if(l==chl & i==mnode(l)) then
51             zv(mc)=0;
52         end
53         if(l<>chl & i==mnode(l)) then
54             mc=mc-1;
55             zv(mc)=zv(mc+1);
56         end
57         if(l<>mnode(l)) then

```

$y_1 \cdot \text{gid}(L,i)$
 B_1
 $-y_2$ 2x2
 $B_2! = 4$
 y_3 2gid(L,i)
 B_3
 y_4
 $= 3$

So with this assignment with this initialisation, because initial values I can transfer here. After that I can define the channel reach dependent parameters. Now channel reach dependent first parameter is del xL. So which is Lx divided by mnode L minus 1 and D1 D2. In this case the difference is C1 C2 compared to our previous lecture class is that we do not have any Q squared term here. So in this case again we can get the z values here.

(Refer Slide Time: 46:46)

```

37     gv(2*gid(1,i)-1)=yv(gid(1,i));
38     gv(2*gid(1,i))=Qv(gid(1,i));
39 end
40 end
41
42 for l=1:chl
43     delta_x(1)=Lx(1)/(mnode(1)-1);
44     D1(1)=alpha(1)/(2*g);
45     D2(1)=(1/2)*n(1)^2*delta_x(1);
46 end
47 mc=sum(mnode);
48 for l=chl:-1:1
49     for i=mnode(1):-1:1
50         if (l==chl & i == mnode(1)) then
51             zv(mc)=0;
52         end
53         if (l<>chl & i == mnode(1)) then
54             mc=mc-1;
55             zv(mc)=zv(mc+1);
56         end
57         if (l<>mnode(1)) then
58             mc=mc-1;
59             zv(mc)=zv(mc+1)+s0(1)*delta_x(1);
60         end
61     end
62 end;
63
64 xv=[linspace(0,Lx(1),mnode(1)) linspace(Lx(1),Lx(1)+Lx(2),mnode(2))]
65 function Av=areav(y)
66     Av=B*y;
67 endfunction
68 function dAv=dareav(y)
69     dAv=B;

```

This is our area calculation. We are considering rectangular channel dA by dy , R , dR by dy calculation and this is mLi . I have not written that cLi because cLi the coefficients are either 1, minus 1 or zero. So I have directly assigned those values. So these are similar to our previous one or previous definition.

(Refer Slide Time: 47:27)

```

1 function dRv=dHRV(y)
2     dRv=B^2/(B+2*y)^2;
3 endfunction
80
81 //-----
1 function Mli=Mli(y1,Q1,y2,Q2,s0,delta_x,D1,D2)
2     Mli=(y2-y1)-s0*delta_x*D1*(Q2*abs(Q2)*areav(y2)^(-2)-Q1*abs(Q1)*areav(y1)^(-2))+D2*(Q2*abs(Q2)*HRV(y2)^(-4/3)*areav(y2)^(-2)+Q1*abs(Q1)*HRV(y1)^(-4/3)*areav(y1)^(-2));
3 endfunction
85
1 function dMdyiv=dMdyi(y,Q,D1,D2)
2     term1=(2*Q^2/areav(y)^3)*dareav(y);
3     term2=2*Q^2*areav(y)^(-3)*HRV(y)^(-4/3)*dareav(y);
4     term3=(4/3)*Q^2*areav(y)^(-2)*HRV(y)^(-7/3)*dHRV(y);
5     dMdyiv=-D1*term1-D2*(term2+term3);
6 endfunction
1 function dMdyipiv=dMdyipiv(y,Q,D1,D2)
2     term1=(2*Q^2/areav(y)^3)*dareav(y);
3     term2=2*Q^2*areav(y)^(-3)*HRV(y)^(-4/3)*dareav(y);
4     term3=(4/3)*Q^2*areav(y)^(-2)*HRV(y)^(-7/3)*dHRV(y);
5     dMdyipiv=-D1*term1-D2*(term2+term3);
6 endfunction
98
1 function dMdyipiv=dMdyipiv(y,Q,D1,D2)
2     term1=(2*Q/areav(y)^3);
3     term2=2*Q*areav(y)^(-2)*HRV(y)^(-4/3);
4     dMdyipiv=-D1*term1-D2*term2;
5 endfunction
105
1 function dMdyiv=dMdyiv(y,Q,D1,D2)
2     term1=(2*Q/areav(y)^3);
3     term2=2*Q*areav(y)^(-2)*HRV(y)^(-4/3);
4     dMdyiv=-D1*term1-D2*term2;

```

Now only difference is here that we need to transfer this $D1$ $D2$ values. We need to transfer $del x$, S not, $y1$, $y2$, $y1$, $Q1$, $y2$, $Q2$, y Q in this case also y Q . This is for $M L i$ which is for i th segment of the L th channel reach. This is $del M$ by dy which is the coefficient for $L i$. This

is dM by dy . This is L_i , this is y_i , this is L_i , this is y_i plus 1. So this is y_i plus 1, this is y_i , this is M and now we need dM L_i , this is dQ i plus 1.

(Refer Slide Time: 49:15)

```

1 function drv=dRv(y)
2   drv=B^2/(B+2*y)^2;
3 endfunction
80
81 //-----
1 function Mli=Mli(y1,Q1,y2,Q2,s0,delta_x,D1,D2)
2   Mli=(y2-y1)-s0*delta_x*D1*(Q2*abs(Q2)*areav(y2)^(-2)-Q1*abs(Q1)*areav(y1)^(-2))+D2*(Q2*abs(Q2)*HRv(y2)^(-4/3)*areav(y2)^(-2)+Q1*abs(Q1)*HRv(y1)^(-4/3)*areav(y1)^(-2));
3 endfunction
85
1 function dMdyiv=dMdyi(y,Q,D1,D2)
2   term1=(2*Q^2/areav(y)^3)*dareav(y);
3   term2=2*Q^2*areav(y)^(-3)*HRv(y)^(-4/3)*dareav(y);
4   term3=(4/3)*Q^2*areav(y)^(-2)*HRv(y)^(-7/3)*dHRv(y);
5   dMdyiv=-D1*term1-D2*(term2+term3);
6 endfunction
1 function dMdyiplv=dMdyipl(y,Q,D1,D2)
2   term1=(2*Q^2/areav(y)^3)*dareav(y);
3   term2=2*Q^2*areav(y)^(-3)*HRv(y)^(-4/3)*dareav(y);
4   term3=(4/3)*Q^2*areav(y)^(-2)*HRv(y)^(-7/3)*dHRv(y);
5   dMdyiplv=-D1*term1-D2*(term2+term3);
6 endfunction
98
99
1 function dMdqiplv=dMdqipl(y,Q,D1,D2)
2   term1=(2*Q/areav(y)^3);
3   term2=2*Q*areav(y)^(-2)*HRv(y)^(-4/3);
4   dMdqiplv=-D1*term1-D2*term2;
5 endfunction
105
1 function dMdqiv=dMdqiv(y,Q,D1,D2)
2   term1=(2*Q/areav(y)^3);
3   term2=2*Q*areav(y)^(-2)*HRv(y)^(-4/3);
4   dMdqiv=-D1*term1-D2*term2;

```

And the term which is dM by L_i by dQ this is for L_i . So this term is here.

(Refer Slide Time: 49:34)

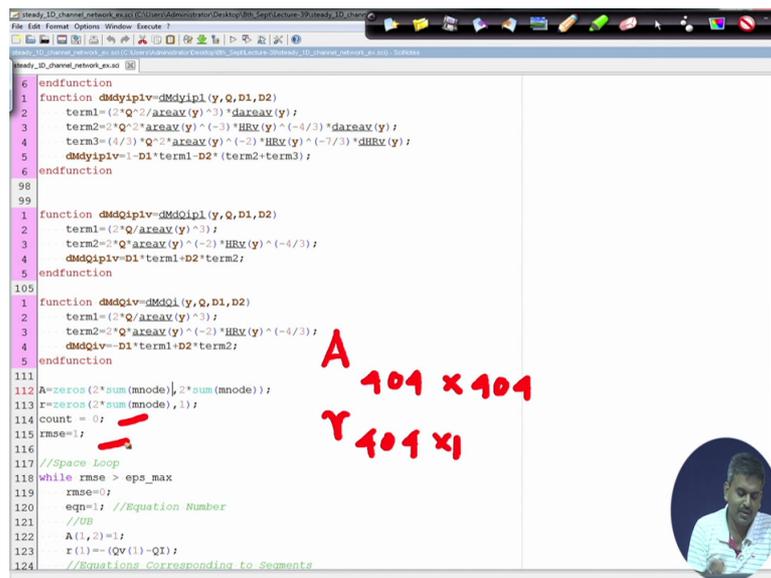
```

6 endfunction
1 function dMdyiplv=dMdyipl(y,Q,D1,D2)
2   term1=(2*Q^2/areav(y)^3)*dareav(y);
3   term2=2*Q^2*areav(y)^(-3)*HRv(y)^(-4/3)*dareav(y);
4   term3=(4/3)*Q^2*areav(y)^(-2)*HRv(y)^(-7/3)*dHRv(y);
5   dMdyiplv=-D1*term1-D2*(term2+term3);
6 endfunction
98
99
1 function dMdqiplv=dMdqipl(y,Q,D1,D2)
2   term1=(2*Q/areav(y)^3);
3   term2=2*Q*areav(y)^(-2)*HRv(y)^(-4/3);
4   dMdqiplv=-D1*term1-D2*term2;
5 endfunction
105
1 function dMdqiv=dMdqiv(y,Q,D1,D2)
2   term1=(2*Q/areav(y)^3);
3   term2=2*Q*areav(y)^(-2)*HRv(y)^(-4/3);
4   dMdqiv=-D1*term1-D2*term2;
5 endfunction
111
112 A=zeros(2*sum(mnode),2*sum(mnode));
113 r=zeros(2*sum(mnode),1);
114 count = 0;
115 rmse=1;
116
117 //Sparse Loop
118 while rmse > eps_max
119   rmse=0;
120   eqn=1; //Equation Number
121   //UB
122   A(1,2)=1;
123   r(1)=-(Qv(1)-Q1);
124   //Equations Corresponding to Segments

```

Now after defining these terms we can start defining our Jacobian matrix. Initialisation I can start, initialisation is with this A zeros 2 into sum mnode. That means if we have total 202 nodes, 101 plus 101. So obviously we have 404 variables. So 404 into 404 should be the size of A. And the size of R should be 404 into 1 in this case. This count is zero, rmse is 1.

(Refer Slide Time: 50:33)



```
6 endfunction
1 function dMdyipiv=dMdyipl(y,Q,D1,D2)
2 term1=(2*Q^2/arsav(y)^3)*darsav(y);
3 term2=2*Q^2*arsav(y)^(-3)*HRV(y)^(-4/3)*darsav(y);
4 term3=(4/3)*Q^2*arsav(y)^(-2)*HRV(y)^(-7/3)*dHRV(y);
5 dMdyipiv=1-D1*term1-D2*(term2+term3);
6 endfunction
98
99
1 function dMdqipiv=dMdqipl(y,Q,D1,D2)
2 term1=(2*Q/arsav(y)^3);
3 term2=2*Q*arsav(y)^(-2)*HRV(y)^(-4/3);
4 dMdqipiv=D1*term1+D2*term2;
5 endfunction
105
1 function dMdqiv=dMdqiv(y,Q,D1,D2)
2 term1=(2*Q/arsav(y)^3);
3 term2=2*Q*arsav(y)^(-2)*HRV(y)^(-4/3);
4 dMdqiv=D1*term1+D2*term2;
5 endfunction
111
112 A=zeros(2*sum(mnode),2*sum(mnode));
113 r=zeros(2*sum(mnode),1);
114 count = 0;
115 rmse=1;
116
117 //Space Loop
118 while rmse > eps_max
119     rmse=0;
120     eqn=1; //Equation Number
121     //for
122     A(1,2)=1;
123     r(1)=(Qv(1)-Q1);
124     //Equations corresponding to segments
```

We have already initialised the gv or general variable there. Now in this case what is happening initially we have written this $rmse$ and we are starting with equation number 1. So first equation is our upstream boundary condition because we will have entry for 1, 2. This is y_1, Q_1, y_2, Q_2 . So if our Q_1 value is specified which is Q directly we can say that this is Q_1 minus Q . Some upstream location. And for that qv what is required? We need to differentiate it. If we differentiate it we will get only second term.

That means A_{1-1} is zero. No entry for A_{1-1} . This is valid for section only. So we will have only two terms associated with it. But the first term A_{1-1} is zero. But A_{1-2} , this is 1 and right hand side we need to write that upstream boundary minus. So if we write that upstream boundary minus this means Q_1 minus Q upstream here. That is what I have written here.

(Refer Slide Time: 52:33)

```

113 r = zeros(2 * sum(mnode), 1);
114 count = 0;
115 rmse = 1;
116
117 //Space Loop
118 while rmse > eps_max
119     rmse = 0;
120     eqn = 1; //Equation Number
121     //u1
122     A(1,2) = 1;
123     r(1) = -(Qv(1) - Q1);
124     //Equations Corresponding to Segments
125     for l = 1:chl
126         for i = 1:mnode
127             //Continuity
128             A(eqn, 2 * gid(1, i) - 1) = 0; //y1
129             A(eqn, 2 * gid(1, i)) = -1; //q1
130             A(eqn, 2 * gid(1, i) + 1) = 0; //yip1
131             A(eqn, 2 * gid(1, i) + 2) = 1; //qip1
132             r(eqn) = 0;
133             eqn = eqn + 1;
134             //Momentum
135             A(eqn, 2 * gid(1, i) - 1) = dmdy1(yv(gid(1, i)), Qv(gid(1, i)), D1(1), D2(1)); //y1
136             A(eqn, 2 * gid(1, i)) = dmdq1(yv(gid(1, i)), Qv(gid(1, i)), D1(1), D2(1)); //q1
137             A(eqn, 2 * gid(1, i) + 1) = dmdyip1(yv(gid(1, i + 1)), Qv(gid(1, i + 1)), D1(1), D2(1)); //yip1
138             A(eqn, 2 * gid(1, i) + 2) = dmdqip1(yv(gid(1, i + 1)), Qv(gid(1, i + 1)), D1(1), D2(1)); //qip1
139             r(eqn) = -M1i(yv(gid(1, i)), Qv(gid(1, i)), yv(gid(1, i + 1)), Qv(gid(1, i + 1)), S0(1), delta_x(1), D1(1), D2(1));
140         end
141     end
142 end
143 //Junction Condition
144

```

Handwritten notes in red:

- $a_1 = a^u$
- $a_1 - a^u = 0$
- $A(1,1) = 0$
- $A(1,2) = 1$
- $r(1) = -(Qv(1) - Q1)$
- $-UB = (a_1, -a_2)$
- y_1, a_1, y_2, a_2

Now after that we can enter into this equations corresponding to segments. For equations corresponding to the segments we have L equals to 1 to channel number. Then i equals to 1 to mnode minus 1. Mnode minus 1 means that I am considering NL number of segments for each channel. So in this process I am considering 2 into NL number of equations. I have already considered 1 there. So I need one more extra. So that I can complete 2 NL plus 2 number of equations there.

(Refer Slide Time: 53:27)

```

113 r = zeros(2 * sum(mnode), 1);
114 count = 0;
115 rmse = 1;
116
117 //Space Loop
118 while rmse > eps_max
119     rmse = 0;
120     eqn = 1; //Equation Number
121     //u1
122     A(1,2) = 1;
123     r(1) = -(Qv(1) - Q1);
124     //Equations Corresponding to Segments
125     for l = 1:chl
126         for i = mnode(1) - 1
127             eqn = eqn + 1;
128             //Continuity
129             A(eqn, 2 * gid(1, i) - 1) = 0; //y1
130             A(eqn, 2 * gid(1, i)) = -1; //q1
131             A(eqn, 2 * gid(1, i) + 1) = 0; //yip1
132             A(eqn, 2 * gid(1, i) + 2) = 1; //qip1
133             r(eqn) = 0;
134             eqn = eqn + 1;
135             //Momentum
136             A(eqn, 2 * gid(1, i) - 1) = dmdy1(yv(gid(1, i)), Qv(gid(1, i)), D1(1), D2(1)); //y1
137             A(eqn, 2 * gid(1, i)) = dmdq1(yv(gid(1, i)), Qv(gid(1, i)), D1(1), D2(1)); //q1
138             A(eqn, 2 * gid(1, i) + 1) = dmdyip1(yv(gid(1, i + 1)), Qv(gid(1, i + 1)), D1(1), D2(1)); //yip1
139             A(eqn, 2 * gid(1, i) + 2) = dmdqip1(yv(gid(1, i + 1)), Qv(gid(1, i + 1)), D1(1), D2(1)); //qip1
140             r(eqn) = -M1i(yv(gid(1, i)), Qv(gid(1, i)), yv(gid(1, i + 1)), Qv(gid(1, i + 1)), S0(1), delta_x(1), D1(1), D2(1));
141         end
142     end
143 //Junction Condition
144

```

Handwritten notes in red:

- 1
- $2 * N_2$
- $2 * (N_2 + 2)$

So A whenever we are entering into the system before this continuity equation I am increasing this equation number. Equation number equals to equation number plus 1. A eqn is

2 gid L i minus 1. That means whatever maybe this L i value, I will have one unique id or gid for my case. And in that case if I consider that 2 gid L i minus 1, that is the thing.

(Refer Slide Time: 54:20)

```

121 //UB
122 A(1,2)=1;
123 r(1)=- (Qv(1)-Q1);
124 //Equations Corresponding to Segments
125 for l=1:chl
126     for i=1:mnode(l)-1
127         eqn=eqn+1;
128         //Continuity
129         A(eqn,2*gid(l,i)-1)=0; //yi
130         A(eqn,2*gid(l,i))=1; //oi
131         A(eqn,2*gid(l,i)+1)=0; //yip1
132         A(eqn,2*gid(l,i)+2)=1; //oip1
133         r(eqn)=0;
134         eqn=eqn+1;
135         //Momentum
136         A(eqn,2*gid(l,i)-1)=dmdy1(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //yi
137         A(eqn,2*gid(l,i))=dmdo1(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //oi
138         A(eqn,2*gid(l,i)+1)=dmdyip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //yip1
139         A(eqn,2*gid(l,i)+2)=dmdoip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //oip1
140         r(eqn)=-M1i(yv(gid(l,i)),Qv(gid(l,i)),yv(gid(l,i+1)),Qv(gid(l,i+1)),S0(1),delta_x(1),D1(1),D2(1));
141     end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni=[1 2 101 1]
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=- (Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)))));

```

Now in this case another way I can represent it, what is that? That is nothing but we have this L plus 1. Now again I can consider that this is corresponding to our flow depth y i. This is corresponding to Q i, this is y i plus 1, this is Q i plus 1 because we are considering even numbers here. Even numbers because i plus 1, i, i plus 1 thing we are considering. This is gid L i plus 1. This is gid L i plus 1 2 into. But one minus 1 is here, minus 1 is here. So these are corresponding to y values.

(Refer Slide Time: 55:33)

```

121 //UB
122 A(1,2)=1;
123 r(1)=- (Qv(1)-Q1);
124 //Equations Corresponding to Segments
125 for l=1:chl
126     for i=1:mnode(l)-1
127         eqn=eqn+1;
128         //Continuity
129         A(eqn,2*gid(l,i)-1)=0; //yi
130         A(eqn,2*gid(l,i))=1; //oi
131         A(eqn,2*gid(l,i)+1)=0; //yip1
132         A(eqn,2*gid(l,i+1))=1; //oip1
133         r(eqn)=0;
134         eqn=eqn+1;
135         //Momentum
136         A(eqn,2*gid(l,i)-1)=dmdy1(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //yi
137         A(eqn,2*gid(l,i))=dmdo1(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //oi
138         A(eqn,2*gid(l,i)+1)=dmdyip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //yip1
139         A(eqn,2*gid(l,i)+2)=dmdoip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //oip1
140         r(eqn)=-M1i(yv(gid(l,i)),Qv(gid(l,i)),yv(gid(l,i+1)),Qv(gid(l,i+1)),S0(1),delta_x(1),D1(1),D2(1));
141     end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni=[1 2 101 1]
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=- (Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)))));

```

Now in this case again we can avoid this and we can write it in general terms. So this is the case where we have for any Jacobian matrix we will have equation number. Corresponding to that equation number we will have entries. So first entry that is corresponding to, 2 into gid L i minus 1. Next one is gid L i. That means this is corresponding to y, this is del i and this is del Q i, this is i.

(Refer Slide Time: 56:39)

```

121 //UB
122 A(1,2)=1;
123 r(1)=-(Qv(1)-Q1);
124 //Equations Corresponding to Segments
125 for l=1:chl
126     for i=1:mnode(l)-1
127         eqn=eqn+1;
128         //Continuity
129         A(eqn,2*gid(l,i)-1)=0; //yi
130         A(eqn,2*gid(l,i))=-1; //qi
131         A(eqn,2*gid(l,i+1)-1)=0; //yip1
132         A(eqn,2*gid(l,i+1))=1; //qip1
133         r(eqn)=0;
134         eqn=eqn+1;
135         //Momentum
136         A(eqn,2*gid(l,i)-1)=dmdyi(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //yi
137         A(eqn,2*gid(l,i))=dmdqi(yv(gid(l,i)),Qv(gid(l,i)),D1(1),D2(1)); //qi
138         A(eqn,2*gid(l,i+1)-1)=dmdyip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //yip1
139         A(eqn,2*gid(l,i+1))=dmdqip1(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(1),D2(1)); //qip1
140         r(eqn)=-M1i(yv(gid(l,i)),Qv(gid(l,i)),yv(gid(l,i+1)),Qv(gid(l,i+1)),S0(1),delta_x(1),D1(1),D2(1));
141     end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni=[1 2 101 1]
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=(Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)))));

```

And this is corresponding to del y i plus 1 and the fourth one is corresponding to del y i plus 1.

(Refer Slide Time: 56:54)

So in this process we have generated these equations. And the first equations are basically continuity equation. That is why I have not written the coefficient separately because we have 0, minus 1, 0, plus 1 terms here because we have Q_i I this is corresponding to L_i plus 1 minus Q_{L_i} this is equal to zero. And we differentiate it with respect to these two variables, obviously plus 1 will be the coefficient of this one and minus 1 will be the coefficient of this one. So this is minus 1, this is plus 1 here.

(Refer Slide Time: 57:46)

```

121 //UB
122 A(1,2)=1;
123 r(1)=-Qv(1)-QI;
124 //Equations Corresponding to Segments
125 for l=1:chl
126     for i=l:mnode(l)-1
127         .....
128         //Continuity
129         A(eqn,2*gid(l,i)-1)=0; //Qv
130         A(eqn,2*gid(l,i))=-1; //QI
131         A(eqn,2*gid(l,i+1)-1)=0; //Qvpl
132         A(eqn,2*gid(l,i+1))=1; //QIpl
133         .....
134         eqn=eqn+1;
135         //Momentum
136         A(eqn,2*gid(l,i)-1)=dMdyi(yv(gid(l,i)),Qv(gid(l,i)),D1(l),D2(l)); //yi
137         A(eqn,2*gid(l,i))=dMdyi(yv(gid(l,i)),Qv(gid(l,i)),D1(l),D2(l)); //QI
138         A(eqn,2*gid(l,i+1)-1)=dMdyipl(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(l),D2(l)); //yipl
139         A(eqn,2*gid(l,i+1))=dMdyipl(yv(gid(l,i+1)),Qv(gid(l,i+1)),D1(l),D2(l)); //QIpl
140         r(eqn)=-Mli(yv(gid(l,i)),Qv(gid(l,i)),yv(gid(l,i+1)),Qv(gid(l,i+1)),S0(l),delta_x(l),D1(l),D2(l));
141     end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni={1 2 101 1}
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=-Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)));

```

Handwritten green text: $Q_{i+1} - Q_{L,i} = 0$

Red annotations: A green circle highlights the continuity section (lines 128-132). Red arrows point from the handwritten equation to the coefficients +1 and -1 in the code.

Now we have completed our continuity. Again after completing this continuity again we need to add this eqn. For eqn again we will be adding one momentum equation, For momentum equation we have to utilise this gid L_i and gid L_i plus 1. These two segments or these two sections for a particular segment.

(Refer Slide Time: 58:21)

```

121 //ID
122 A(1,2)=1;
123 r(1)=-Qv(1)-QI;
124 //Equations Corresponding to Segments
125 for l=1:chl
126     for i=1:mnode(l)-1
127         eqn=eqn+1;
128         //Continuity
129         A(eqn,2*gid(1,i)-1)=0; //yi
130         A(eqn,2*gid(1,i))=-1; //Qi
131         A(eqn,2*gid(1,i+1)-1)=0; //yip1
132         A(eqn,2*gid(1,i+1))=1; //qip1
133         r(eqn)=0;
134         eqn=eqn+1;
135         //Momentum
136         A(eqn,2*gid(1,i)-1)=dmdyi(yv(gid(1,i)),Qv(gid(1,i)),D1(1),D2(1)); //yi
137         A(eqn,2*gid(1,i))=dmdqi(yv(gid(1,i)),Qv(gid(1,i)),D1(1),D2(1)); //Qi
138         A(eqn,2*gid(1,i+1)-1)=dmdyip1(yv(gid(1,i+1)),Qv(gid(1,i+1)),D1(1),D2(1)); //yip1
139         A(eqn,2*gid(1,i+1))=dmdqip1(yv(gid(1,i+1)),Qv(gid(1,i+1)),D1(1),D2(1)); //qip1
140         r(eqn)=-Mli(yv(gid(1,i)),Qv(gid(1,i)),yv(gid(1,i+1)),Qv(gid(1,i+1)),S0(1),delta_x(1),D1(1),D2(1));
141     end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni=[1 2 101 1]
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=-Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)));

```

Now with this we can calculate our thing. Next thing is we need to specify certain junction conditions. Why this junction conditions are required? Because in this case we have only one condition that is I have 1 and 2. This is my 1 or channel 1 and 2. These are connected from 101 node to 1 in the second channel. So that is why junction information is 1 to 101, 1.

(Refer Slide Time: 59:14)

```

131 A(eqn,2*gid(1,i+1)-1)=0; //yip1
132 A(eqn,2*gid(1,i+1))=1; //qip1
133 r(eqn)=0;
134 eqn=eqn+1;
135 //Momentum
136 A(eqn,2*gid(1,i)-1)=dmdyi(yv(gid(1,i)),Qv(gid(1,i)),D1(1),D2(1)); //yi
137 A(eqn,2*gid(1,i))=dmdqi(yv(gid(1,i)),Qv(gid(1,i)),D1(1),D2(1)); //Qi
138 A(eqn,2*gid(1,i+1)-1)=dmdyip1(yv(gid(1,i+1)),Qv(gid(1,i+1)),D1(1),D2(1)); //yip1
139 A(eqn,2*gid(1,i+1))=dmdqip1(yv(gid(1,i+1)),Qv(gid(1,i+1)),D1(1),D2(1)); //qip1
140 r(eqn)=-Mli(yv(gid(1,i)),Qv(gid(1,i)),yv(gid(1,i+1)),Qv(gid(1,i+1)),S0(1),delta_x(1),D1(1),D2(1));
141 end
142 end
143
144 //Junction Condition
145 //for j=1:junc
146 //juni=[1 2 101 1]
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))=1;
150 A(eqn,2*gid(juni(2),juni(4)))=-1;
151 r(eqn)=-Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)));
152 eqn=eqn+1;
153 //Junction Energy
154 A(eqn,2*gid(juni(1),juni(3)))=-1;
155 A(eqn,2*gid(juni(2),juni(4)))=-1;
156 r(eqn)=-yv(gid(juni(1),juni(3)))-yv(gid(juni(2),juni(4)));
157 // end
158
159 //DB
160 eqn=eqn+1;
161 A(eqn,2*gid(2,mnode(2))=1;
162 r(eqn)=-yv(gid(2,mnode(2)))-yd;

```

Now again I need to add one continuity equation here and one condition here. So if I add these conditions because the first term just junction information. I have junction information 1. This is the 1, the first channel. This is corresponding to L. Second one is i. So and third one

is i junction condition for third. That means the location of the end section, this is 3. That means 101.

(Refer Slide Time: 01:00:14)

```

141     ), D2(1));
142     end
143     //Junction Condition
144     //for j=1:junc
145     //juni={1 2 101 1}
146     eqn=eqn+1;
147     //Junction Continuity
148     A(eqn, 2*gid(juni(1), juni(3)))=1;
149     A(eqn, 2*gid(juni(2), juni(4)))=-1;
150     r(eqn)=- (Qv(gid(juni(1), juni(3)))-Qv(gid(juni(2), juni(4)))));
151     eqn=eqn+1;
152     //Junction Energy
153     A(eqn, 2*gid(juni(1), juni(3))-1)=1;
154     A(eqn, 2*gid(juni(2), juni(4))-1)=-1;
155     r(eqn)=- (yv(gid(juni(1), juni(3)))-yv(gid(juni(2), juni(4)))));
156     // end
157     //DB
158     eqn=eqn+1;
159     A(eqn, 2*gid(2, mnmode(2))-1)=1;
160     r(eqn)=- (yv(gid(2, mnmode(2)))-yd);
161     delyQ=A*r;
162     for i=1:2*sum(mnmode)
163         gv(i)=gv(i)+delyQ(i);
164         rmse=rmse+delyQ(i)^2;
165     end
166     //Initial Value
167     for l=1:chl
168         for i=1:mnmode(1)
169             yv(gid(1,i))=gv(2*gid(1,i)-1);
170             Qv(gid(1,i))=gv(2*gid(1,i));
171         end
172     end
173     rmse=sqrt(rmse/sum(mnmode));
174     count = count + 1;
175     disp([count rmse])
176 end
177 //Figure
178 //Plots
179 plot(xv', yv+zv, '-r')
180 plot([0 Lx(1)], [zv(1) zv(mnmode(1))], 'b-')
181 plot([Lx(1) Lx(2)], [zv(mnmode(1)+1) zv(sum(mnmode))], 'm-')
182 title ("Steady Channel Flow in ", "X axis", "Flow Depth")
183
184
185
186
187
188
189
190

```

So with this information if I run this, so obviously we have these equations. So we have the condition for steady series or channel flow in series.

(Refer Slide Time: 01:00:58)

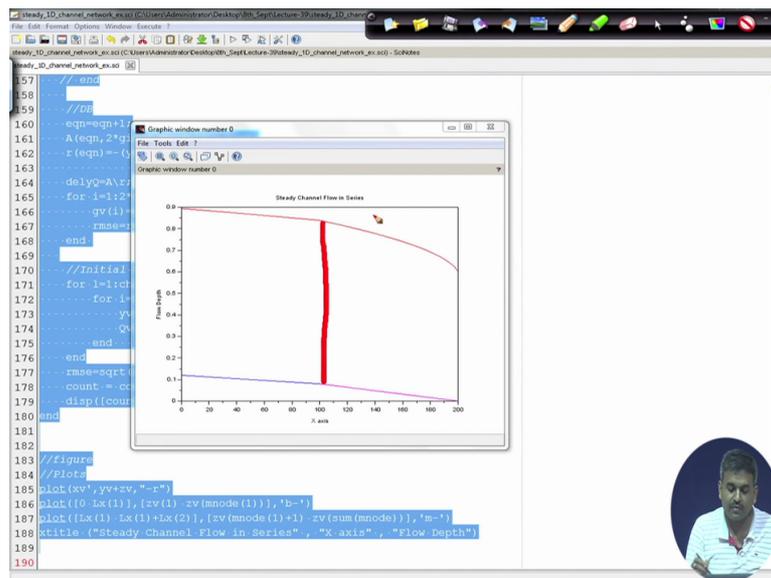
```

157     // end
158     //DB
159     eqn=eqn+1;
160     A(eqn, 2*gid(2, mnmode(2))-1)=1;
161     r(eqn)=- (yv(gid(2, mnmode(2)))-yd);
162     delyQ=A*r;
163     for i=1:2*sum(mnmode)
164         gv(i)=gv(i)+delyQ(i);
165         rmse=rmse+delyQ(i)^2;
166     end
167     //Initial Value
168     for l=1:chl
169         for i=1:mnmode(1)
170             yv(gid(1,i))=gv(2*gid(1,i)-1);
171             Qv(gid(1,i))=gv(2*gid(1,i));
172         end
173     end
174     rmse=sqrt(rmse/sum(mnmode));
175     count = count + 1;
176     disp([count rmse])
177 end
178 //Figure
179 //Plots
180 plot(xv', yv+zv, '-r')
181 plot([0 Lx(1)], [zv(1) zv(mnmode(1))], 'b-')
182 plot([Lx(1) Lx(2)], [zv(mnmode(1)+1) zv(sum(mnmode))], 'm-')
183 title ("Steady Channel Flow in ", "X axis", "Flow Depth")
184
185
186
187
188
189
190

```

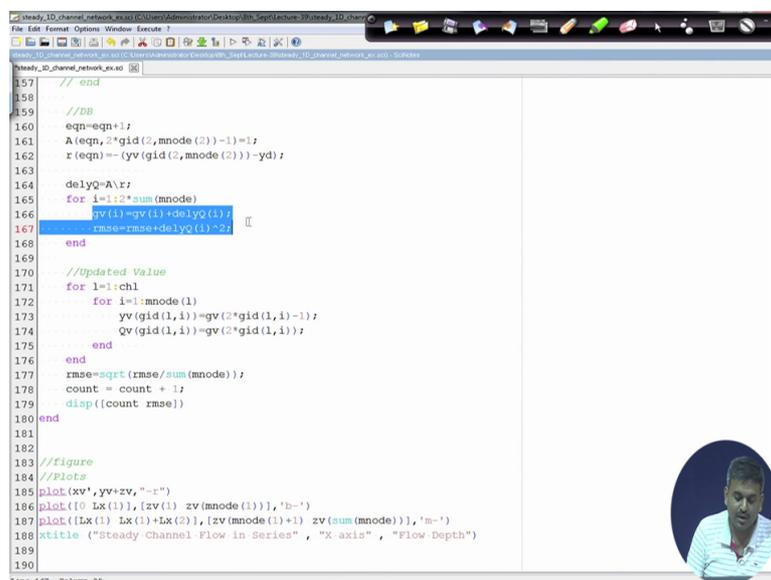
Now in this case we are getting one transition at this level.

(Refer Slide Time: 01:01:12)



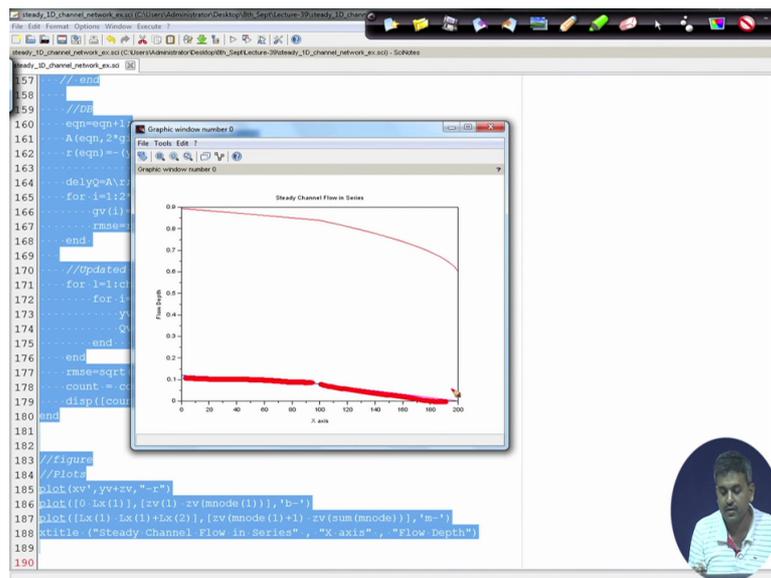
Again we need to transfer these values. All values we need to transfer to y and Qv values directly to or in this case this is update values because we need to update values after solution of $\frac{\partial Q}{\partial y}$. $\frac{\partial Q}{\partial y}$ we need to add with gv values. Gv is assigned initial value. After every iteration this is getting updated.

(Refer Slide Time: 01:02:00)



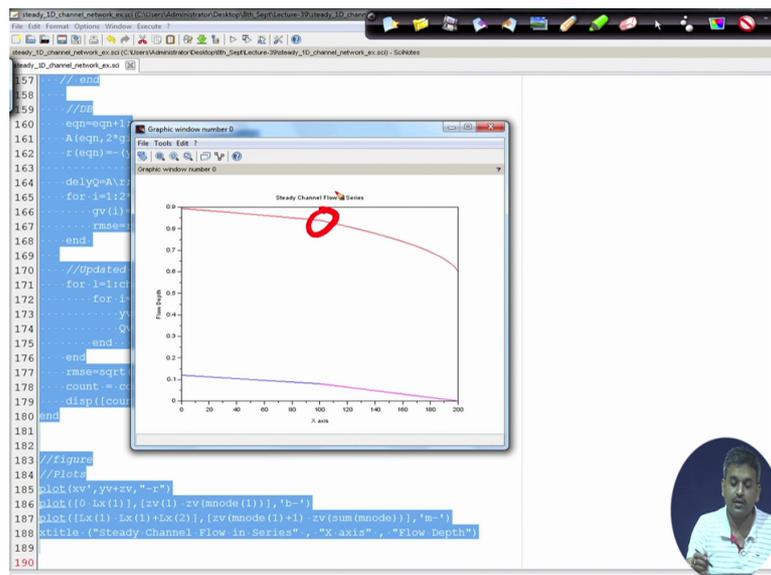
And after we are getting updated values we need to update y and v because during calculation we are using this y and v values. So in this case we are directly getting the solution. So obviously it is similar to our previous case. This is the channel transition in this case. So obviously colour is different.

(Refer Slide Time: 01:02:41)



And we are getting one transition here.

(Refer Slide Time: 01:02:45)



So with this approach we have solved both discharge and continuity equations for both the problems. Now we have all total junction conditions, boundary conditions and if we utilise those information we can directly get the solution out of this.

(Refer Slide Time: 01:03:32)

```
147 eqn=eqn+1;
148 //Junction Continuity
149 A(eqn,2*gid(juni(1),juni(3)))-1;
150 A(eqn,2*gid(juni(2),juni(4)))-1;
151 r(eqn)=-(Qv(gid(juni(1),juni(3)))-Qv(gid(juni(2),juni(4)))));
152 eqn=eqn+1;
153 //Junction Energy
154 A(eqn,2*gid(juni(1),juni(3))-1)-1;
155 A(eqn,2*gid(juni(2),juni(4))-1)-1;
156 r(eqn)=-(yv(gid(juni(1),juni(3)))-yv(gid(juni(2),juni(4)))));
157 // end
158
159 //DB
160 eqn=eqn+1;
161 A(eqn,2*gid(2,mnode(2))-1)-1;
162 r(eqn)=-(yv(gid(2,mnode(2)))-yd);
163
164 delyQ=A*r;
165 for i=1:2*sum(mnode)
166 gv(i)=gv(i)+delyQ(i);
167 rmse=rmse+delyQ(i)^2;
168 end
169
170 //Updated Value
171 for l=1:chl
172 for i=1:mnode(l)
173 yv(gid(l,i))=gv(2*gid(l,i))-1;
174 Qv(gid(l,i))=gv(2*gid(l,i));
175 end
176 end
177 rmse=sqrt(rmse/sum(mnode));
178 count = count + 1;
179 disp([count rmse])
```

So this is the estimation of flow depth across the channel and we have solved this problem.

(Refer Slide Time: 01:03:42)

Problem Definition
Discretization
Junction Conditions
Problem Statement

I.I.T. Kharagpur

Problem Statement

Channels in Series

Given

Channel Cross-Section Type: Rectangular
 $B = 15m$
 $g = 9.81m/s^2$
 $S_{01} = 0.0004$
 $S_{02} = 0.0008$
 $n_1 = 0.01$
 $n_2 = 0.015$
 $L_{x1} = 100m$
 $L_{x2} = 100m$
 $Q = 20m^3/s$
 $y_d = 0.60m$

Required

Estimate the flow depth across the channels in series.

Dr. Anirban Dhar NPTEL Computational Hydraulics

So this is with rectangular cross section we have solved. So code is steady 1D channel network sci. So you can utilise this code to check the solution process.

(Refer Slide Time: 01:04:00)

The image is a screenshot of a presentation slide. At the top, there is a navigation bar with the following text: "Problem Definition", "Discretization", "Junction Conditions", and "Problem Statement". To the right of this bar is the I.I.T. Kharagpur logo and name. The main title of the slide is "List of Source Codes". Below this, there is a section titled "Channel Flow" which contains a sub-section "Channels network" and a list item "steady_1D_channel_network.sci". At the bottom of the slide, there is a footer with the text: "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "22 / 23".

Thank you.