Computational Hydraulics
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Lecture 39
Steady Channel Flow: Channel Network

Welcome to this lecture of the course computational hydraulics. We are in module 4, surface water hydraulics. And this is lecture number 4 of this module which is unit 4, steady channel flow and channel network. In our previous lecture class we have discussed the problems for single channel and channel in series.

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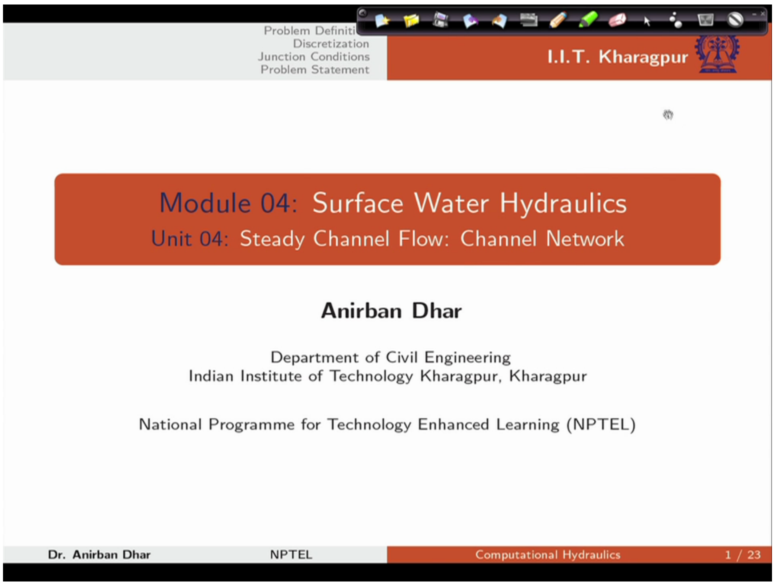

What is the learning objective for this particular lecture class? To solve steady channel flow for channel network problem using implicit method. Again we will be talking about the solution of nonlinear equation. This is not time implicit. This is implicit equation.

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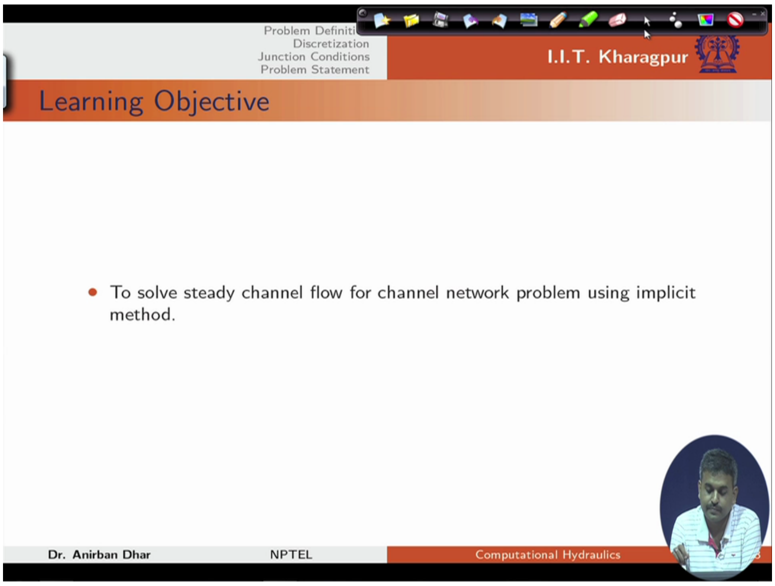

Problem definition to solution, in our previous lecture class we have talked about single channel. Let us say we have single channel and we have also solved the case where two channels are connected. So initial one was having milder slope, then we have comparatively steeper slope like this. And there was junction point and single channel. In this case the situation is that whatever flow is entering into the channel we are getting same amount of flow here.

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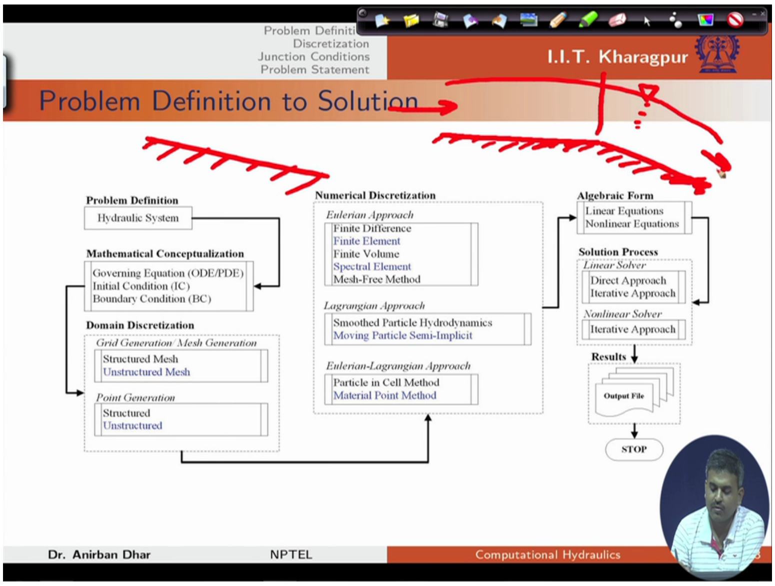

So in this case our given discharge we can solve this problem. And only variable is y or flow depth. But if we have multiple channels, let us say that if we have this kind of situation where flow is coming from here and there are two channels in the downstream main, so obviously if the flow here is Q, flow there in the one section is Q2 and another section is Q3. Obviously this Q1 should be equal to Q2 and Q3. But we do not know the exact value of Q2 and Q3.

It depends on the nature of the flow and the upstream and downstream conditions and junction conditions available within the channel network system.

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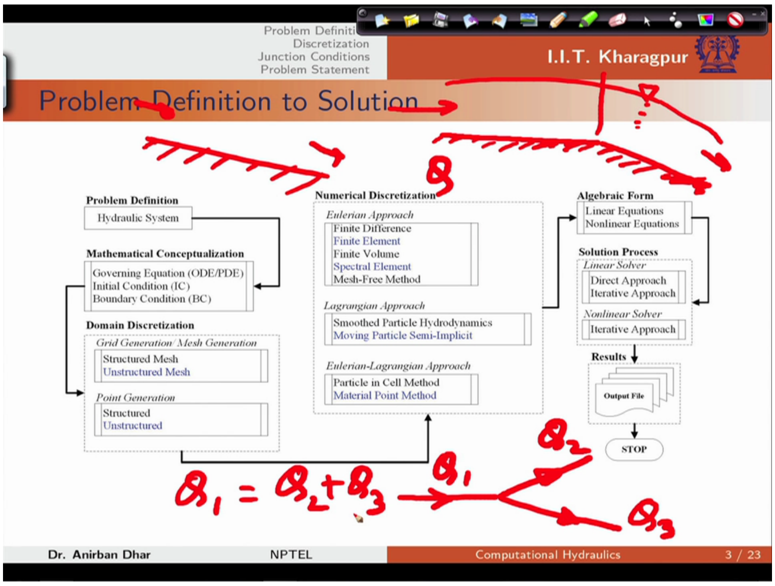

Problem definition, governing equation. Governing equation for channel flow can be written as boundary value problem. Again we will be using same state of governing equations. So continuity equation, momentum equation. But in our previous lecture class we have not utilised this governing equation during our solution process.

But in channel flow again at any junction there will be multiple channels involved in the flow process. So we cannot exactly calculate or estimate the discharge value at the junction without solving these equations.

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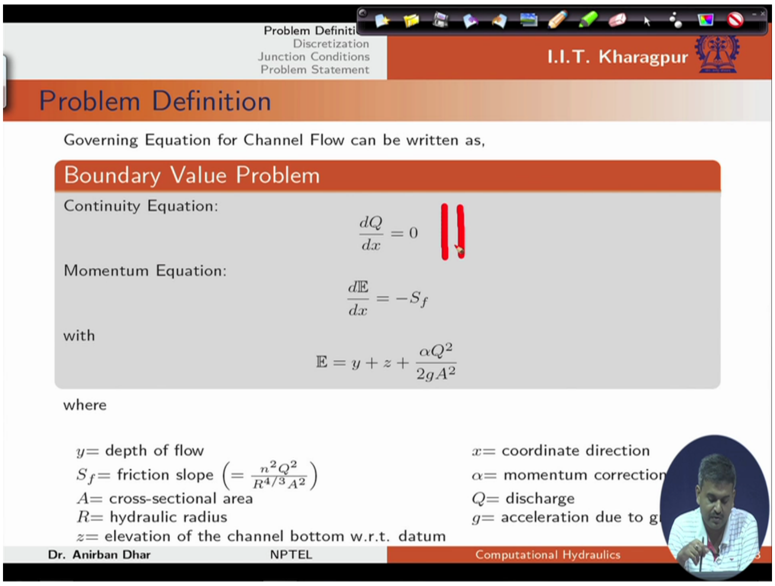

So what we will do, we will try to utilise the discretized form of both the equations and we will try to solve it simultaneously so that we will have the solution for both y that is the flow depth and discharge for any channel network. Interestingly in our last lecture class we have considered NL plus 1 number of variables. That means we have considered only y as flow variable. Now if we introduce this Q discharge as another variable then we need to solve 2 NL plus 1 number of equations.

That means we need equations which will be coming or discretized equation which will be coming from this continuity, momentum. But these equations are applicable for segments. So obviously we will get 2 NL number of equations here. And we need 2 NL plus 1 into 2. That means these many variables. So extra equations we have to add to get the solution.

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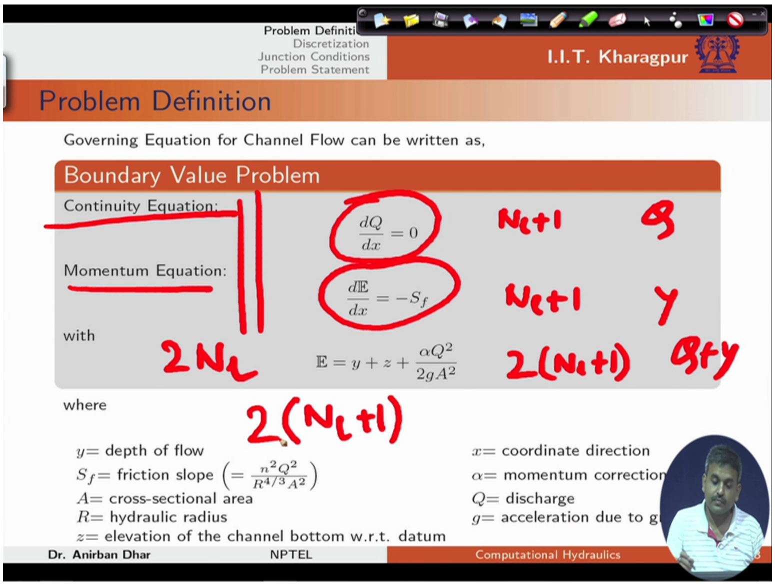

So for channel flow we have this general structure. We have our elevation head, flow depth, in this case velocity head or kinetic energy head. On the downstream end also we have our elevation head, flow depth, this kinetic energy head here and this is the friction slope which is the average one by considering the two friction slopes at two consecutive sections, i and i plus 1.

And i is the segment number. XL which is the channel section or del x. And it is associated with the channel reach. For a particular reach we will consider a single value of del x.

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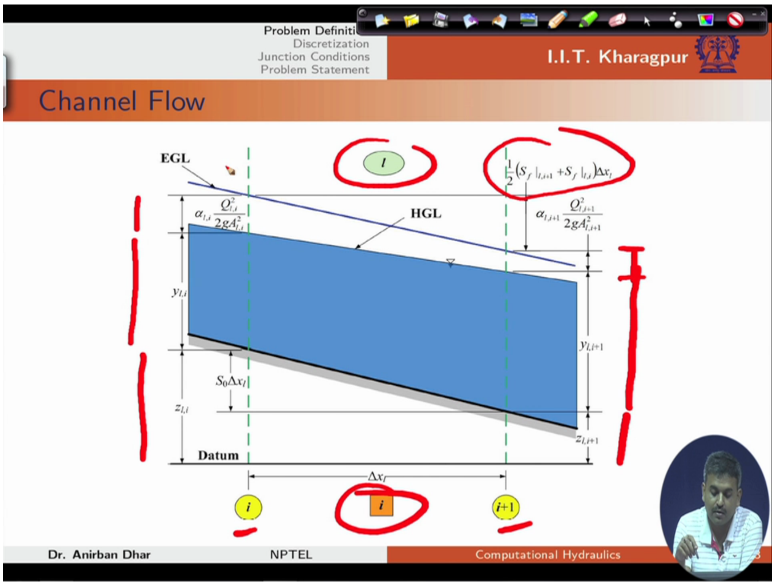

Now start of discretization we need to discretize the continuity equation for ith segment. If we discretize this equation for ith segment of Lth channel then basically we are getting this for ith segment. We have any arbitrary case.

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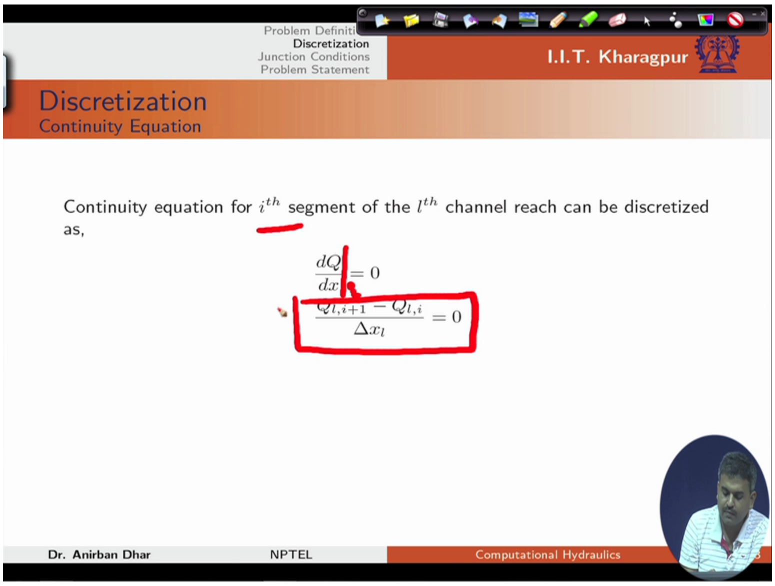

L is the index for channel number, i is the index for different sections within a channel reach. Now in simplified form for ith segment of the Lth channel we have this equation. Now we will not consider a constant value corresponding to this and we are not going to solve the same state of equations that we have solved in our previous case. We need to consider these two terms as variables. So we will not assign any value or we do not have any value for our system.

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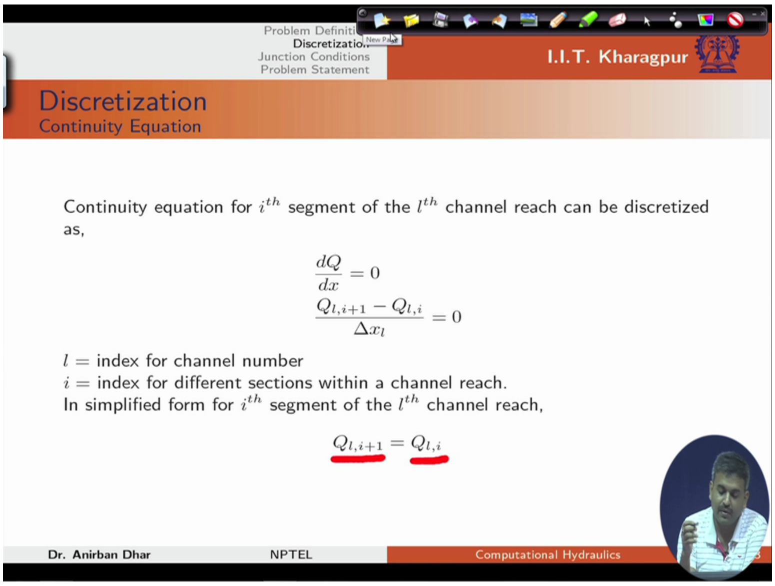

In functional form I can just transfer this value on the left hand side. I can write this continuity L i. This is valid for 1 to NL number of segments.

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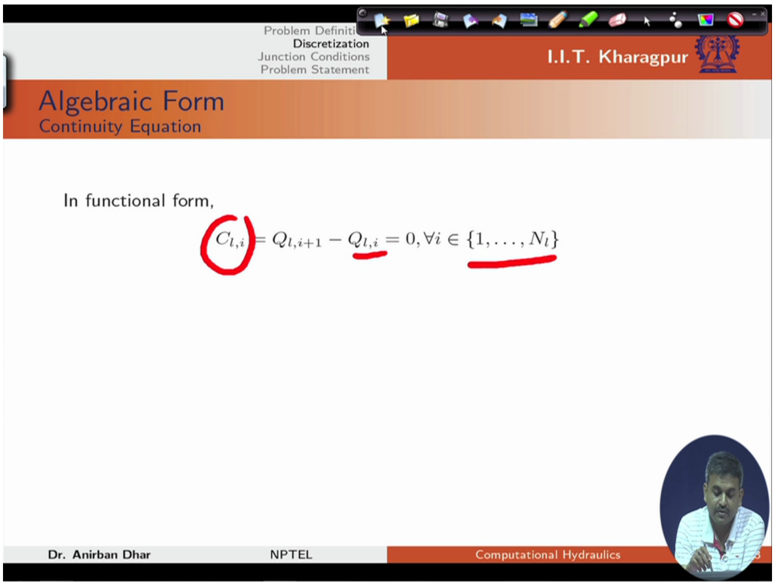

Now again we can use the Newton Raphson method and for Newton Raphson method although this equation is linear we need to calculate the coefficients or elements of our Jacobian matrix. Now we have in each case if we consider ith reach. Let us say this is ith reach and we have i and i plus 1. These two are sections.

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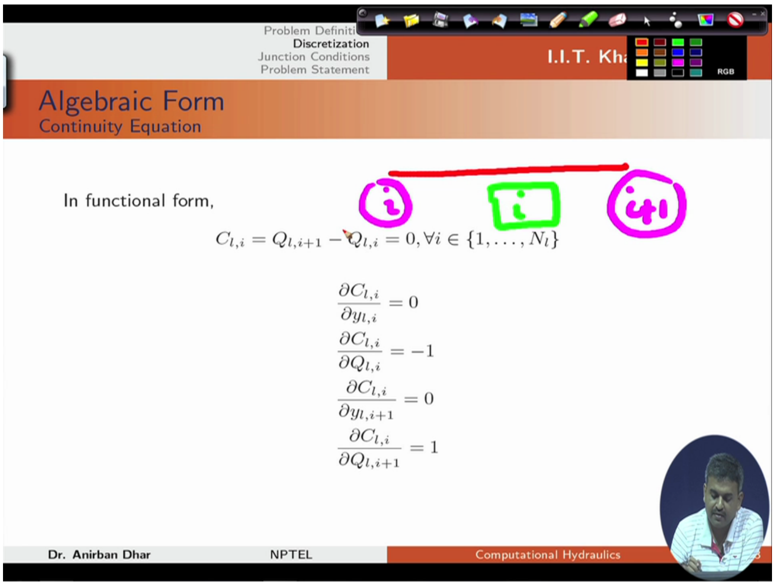

Now in this case for section i we have variable L i and Q L i. On the right hand side we have y L i plus 1, Q L i plus 1. So all total for any segment we will have four variables, two for depth and two for your discharge.

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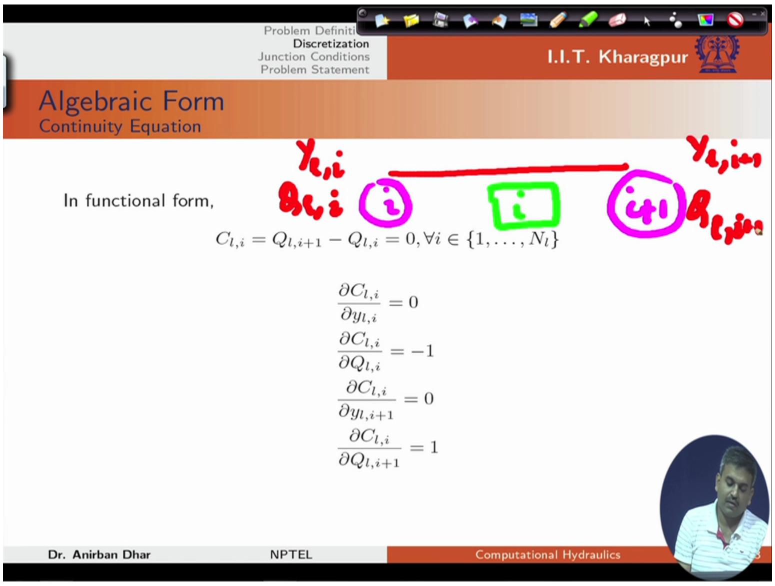

Now in this case if we have one equation so we should get derivative terms with respect to these variables. So if we have C L i equals to zero, this is our functional form. We should differentiate it with respect to Y L i, Q L i, Y L i, i plus 1, Q L i i plus 1. So first of all we do not have any y here. So this is zero.

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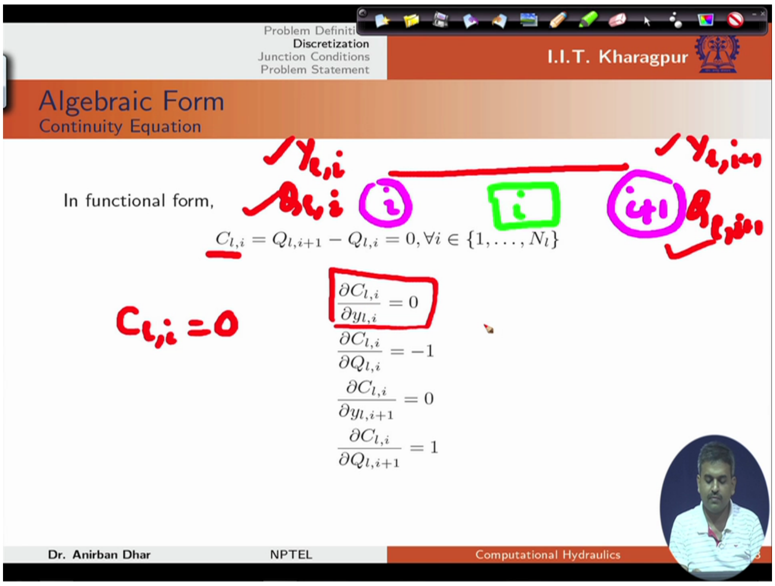

Q L i this is minus 1 because minus sign is there. Y L i plus 1 no y i plus 1 term, this is zero. This is Q C L i del Q L i plus 1, this is 1 again because it has got positive sign here. So we have determined our coefficients for the Jacobian matrix for our continuity equation in this case.

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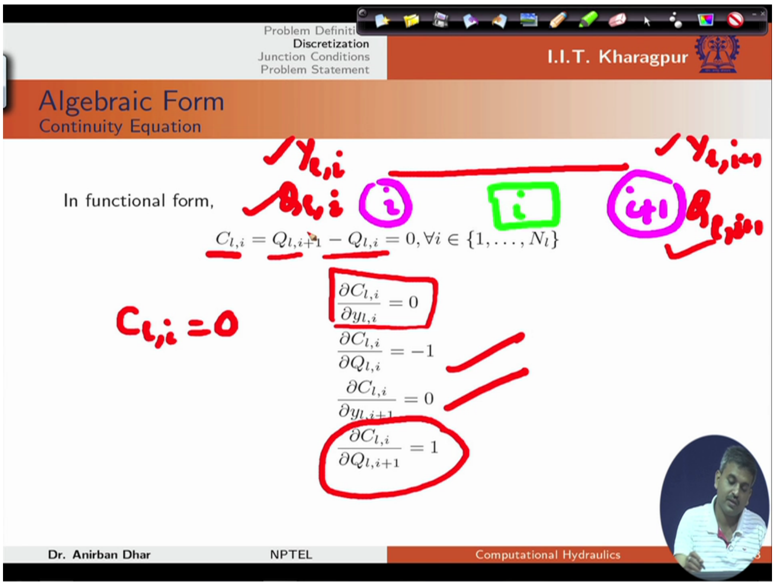

Now again we need the discretization of our momentum equation. Now to solve this momentum equation we can discretize it using forward difference L i plus 1 and E L i and we will take average of this friction slope in this case.

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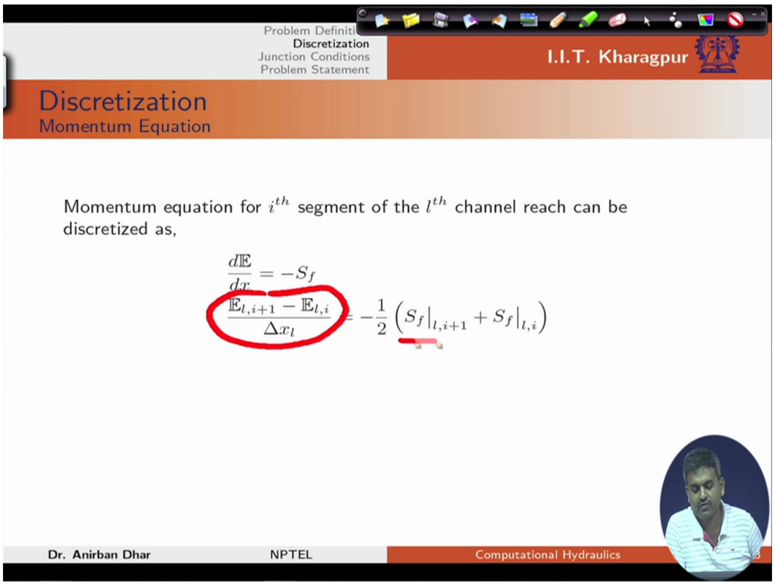

In expanded form we can write this y, z, alpha Q square, 2 gA square, L i plus 1, l i these are the things we will get. And on the right hand side we have averaged form of SF.

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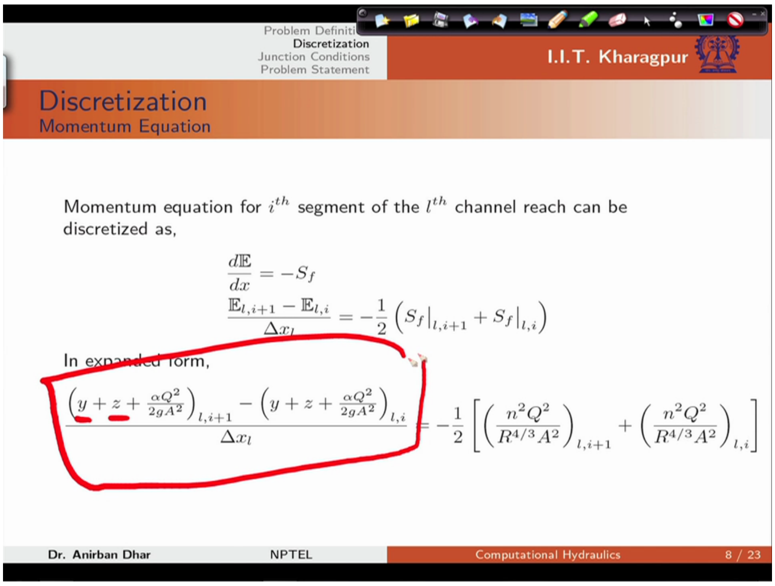

Now from here we can get the functional form. Again these are variables. Q is variable. Y L i plus 1 that is function of y L i plus 1. So these are variables. So direct variables is y and Q.

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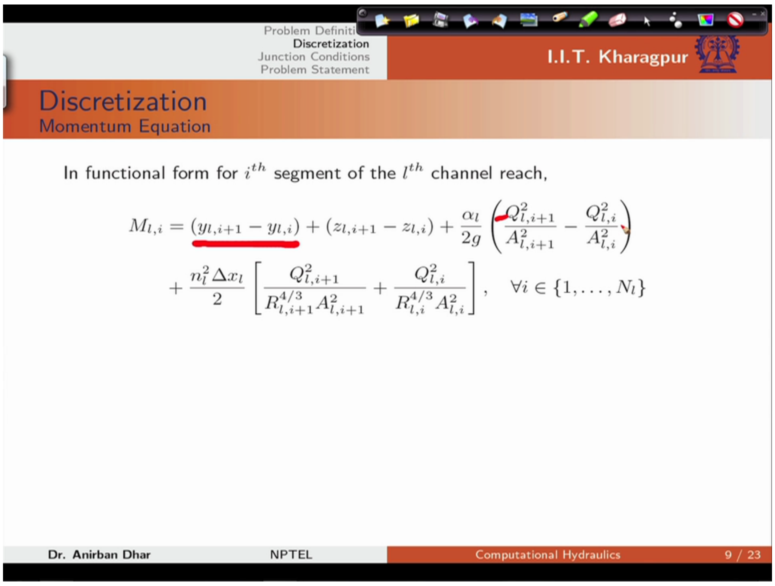

Now if we write this we need to consider the rivers flow situation. So if the flow is in forward direction we will consider plus Q. If it is in backward direction we will consider as negative Q. So if we use this square term here there will be loss of information. So instead of using this square we will try to utilise this Q L i plus 1, absolute value of Q L i plus 1. So obviously one term will get positive value. Another term if it is positive or negative that will be considered during calculation process. So similarly we can define the terms here.

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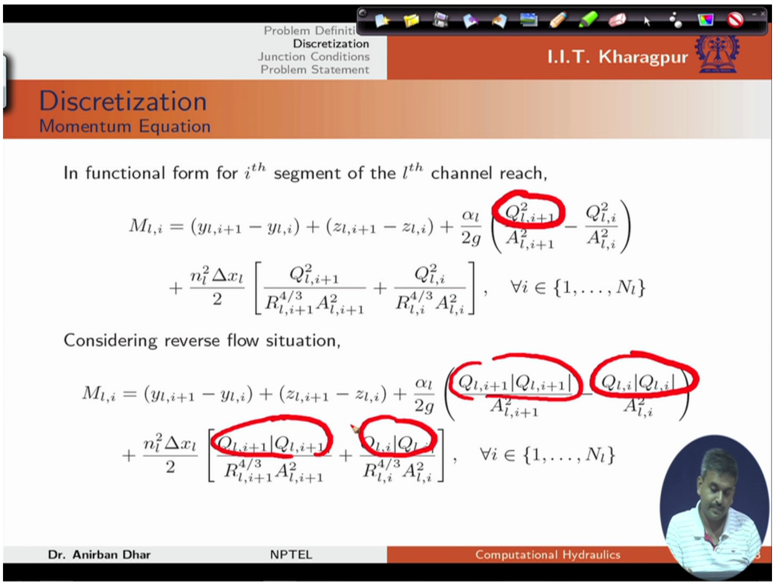

Now outside we have only alpha L divided by 2g, nL square del xL divided by 2. So in this case this term is not C1 and C2. So these are not C1 C2 because we do not have Q square term there.

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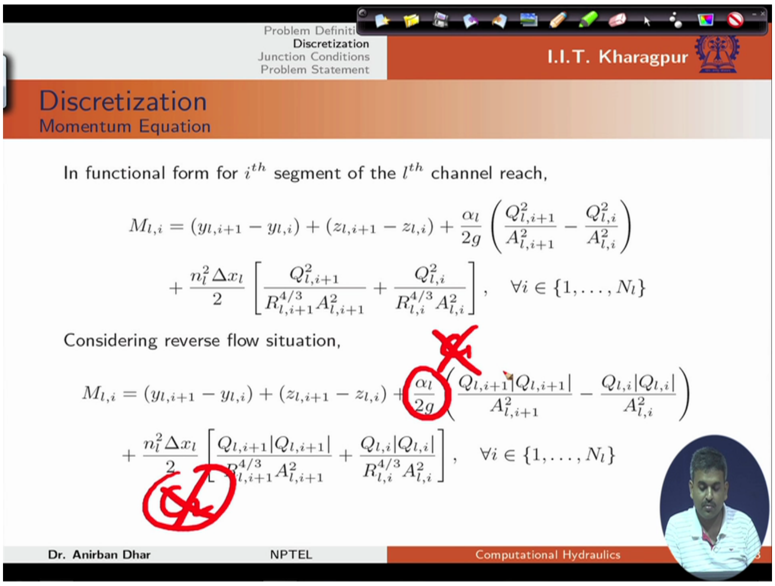

So we can define two new terms in the next slide. But in this case as I have already mentioned we have 2 NL non linear equations. So NL number of equations which are corresponding to C L i or continuity equation. And NL number of equation which are corresponding to momentum and this is corresponding to momentum equation. So all total we have 2 NL plus 1 number of unknowns.

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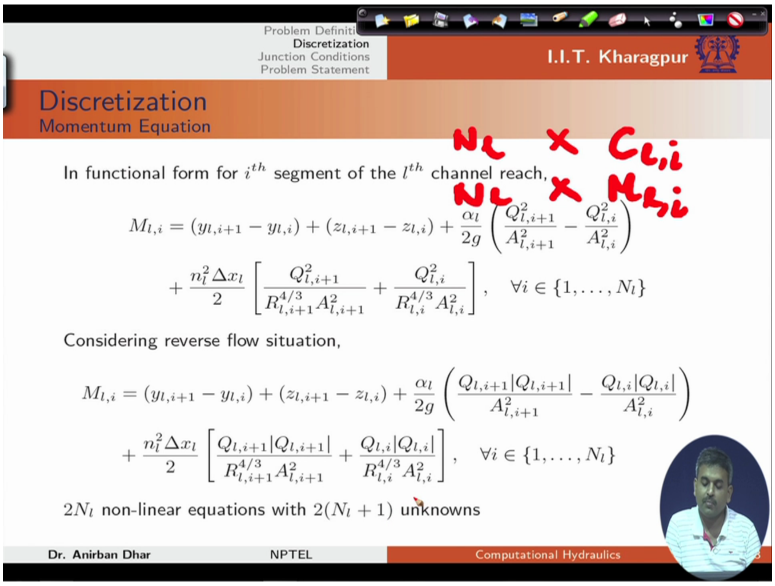

So in this case whenever we are going to (dis) take the derivative of this M, we should use this squared terms only because that absolute terms that is not differentiable.

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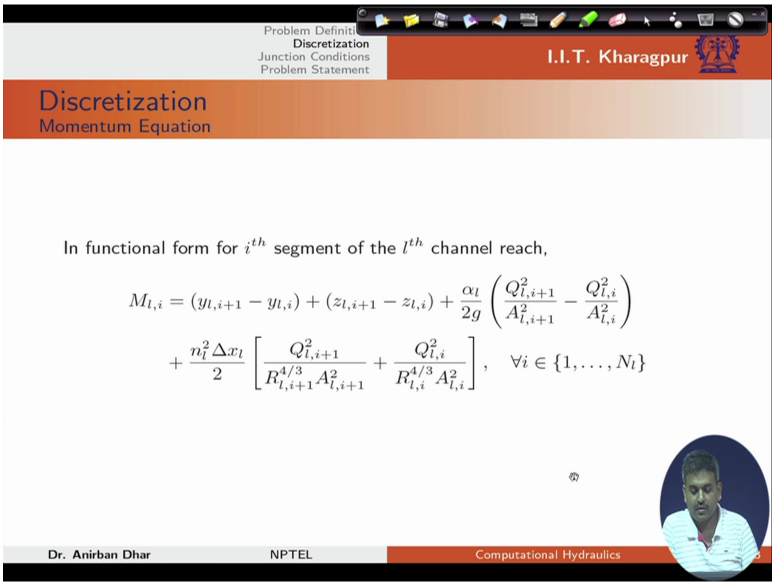

So if I write that the elements of the Jacobian matrix corresponding to momentum term so we have these expressions. These are similar to the expression that we have already got for our channel in series or single channel case. But we have two extra terms here because we need to consider two extra variables in this case. So we have this D1 and D2. D1, D2 these are constant terms and these are corresponding to Lth channel because alpha L, nL, delta xL, these are parameters which are function of or which are dependent on our channel reach.

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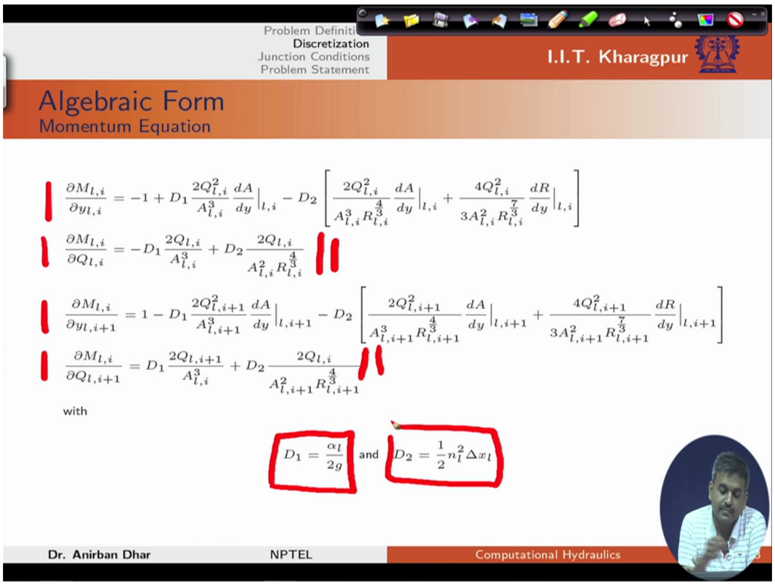

In general case we already know that dR by dy we can calculate like this where T is the top width, P is the weighted perimeter, R is hydraulic radius. So this is the change in hydraulic radius with respect to y. DA and dY, this is dA dy and dR dy. Both are functions of y.

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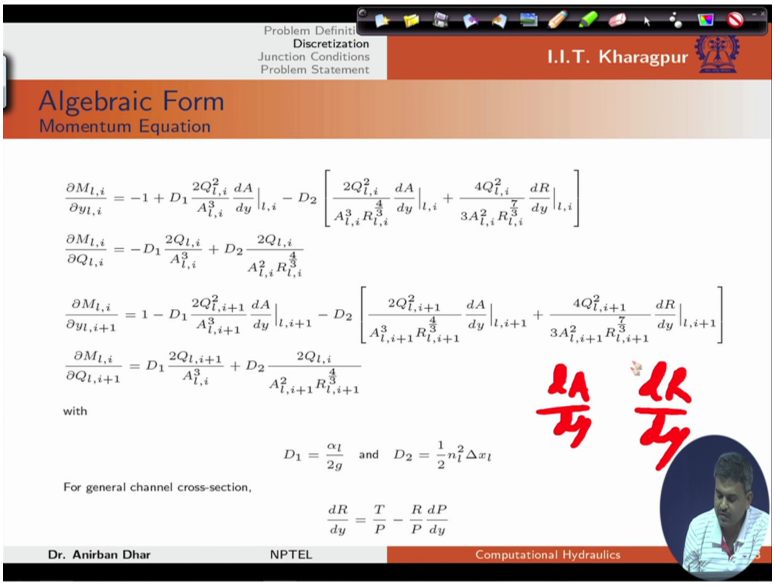

Boundary condition for subcritical flow again we will consider that on the downstream end we will have the specified flow condition. So this is the condition number one. Condition number one because we have considered NL equations in terms of C and n. And we have 2 NL plus 1 number of variables. So if we subtract this we need two more equations.

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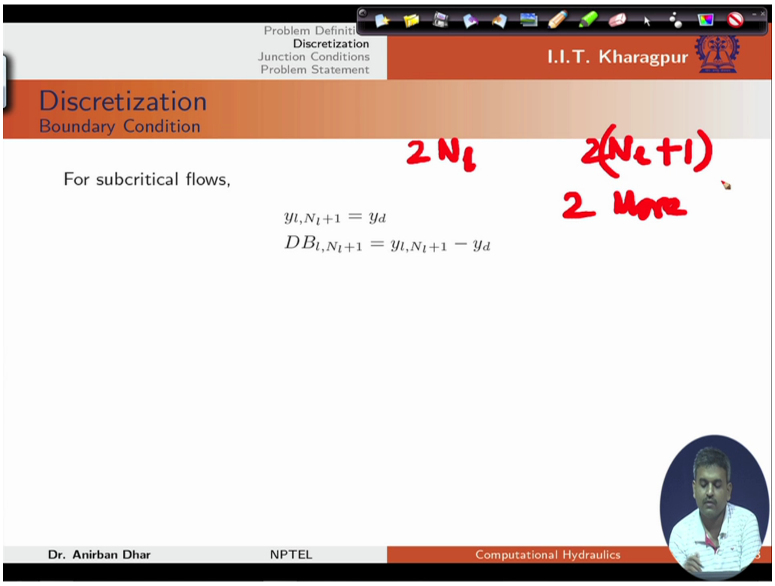

So what we can do, we can specify maybe discharge boundary condition at the upstream end. So at the upstream maybe which is the channel 1, we can specify this is for channel reach 1 and section 1 we can specify 1-1 equals to Q which is specified Q or Q upstream value. And that will be the second condition and we can solve this for our case.

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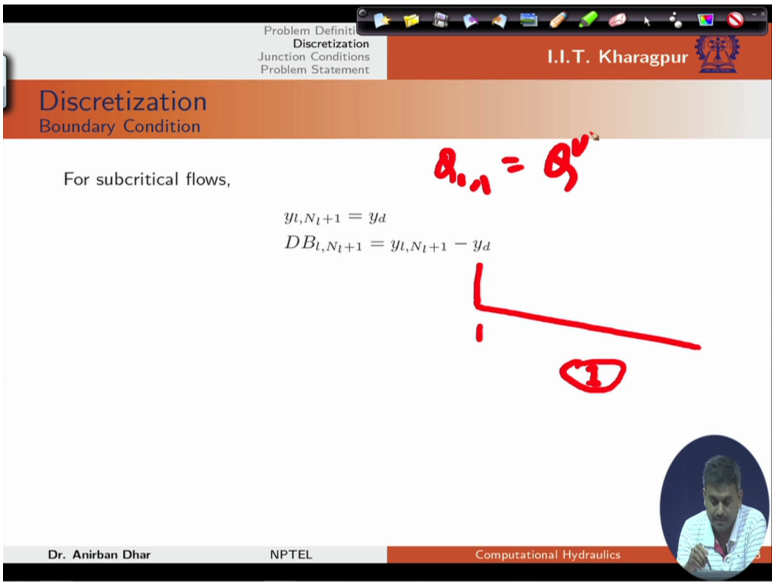

In this case obviously we have four terms. But these first two are zero because we do not have any yL NL or QL NL terms. So these are zero. Again we do not have any QL NL plus 1 term here, this is zero. So only term left is corresponding to yL NL plus 1.

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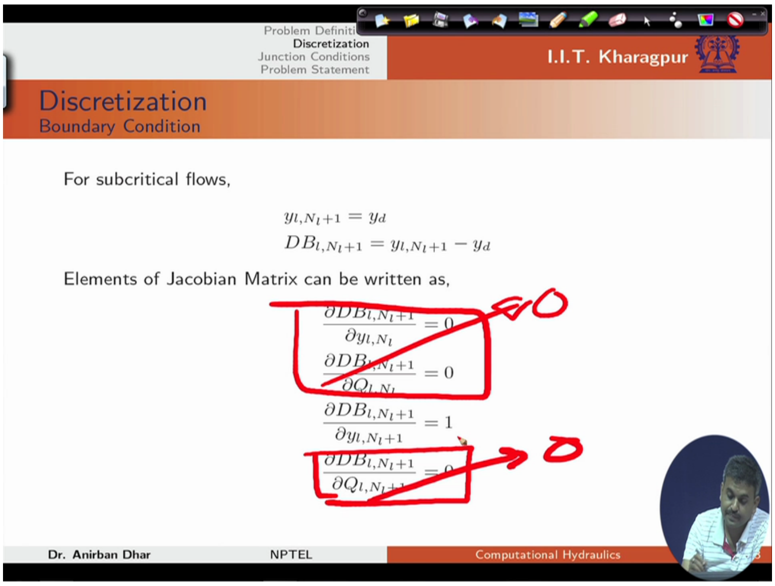

Now we have all those equations there. Now we can have other conditions within our system. If we have a channel network like we have discussed our single channel or channel in series, we can solve this problem. But we need to specify our channel junction conditions. So in this case like previous one we have channel number L, L plus 1, L plus 2, L plus 3.

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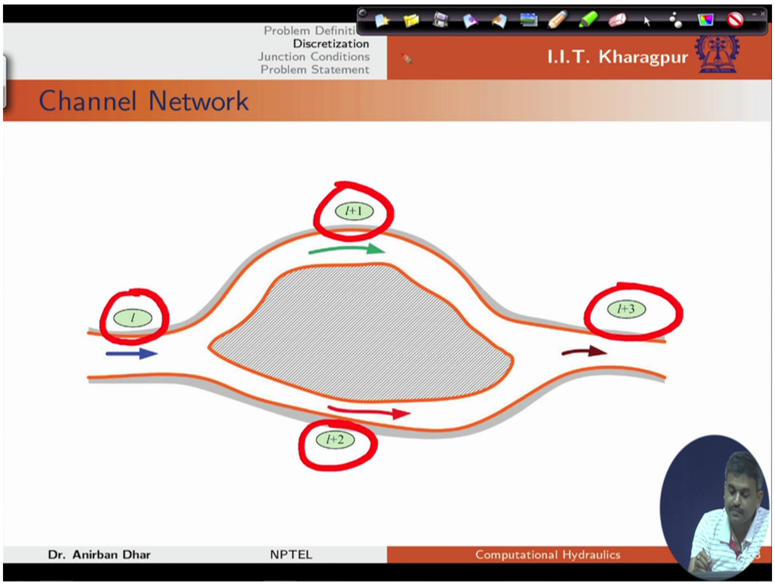

Now if we impose the channel conditions or the channel junctions conditions we need to consider those conditions between this blue one, green one, blue one, red one. So one condition is that the flow from left side should be equal to the summation of flow from the rightward or if we consider this JN or junction number 1 as general function, whatever inflow is coming to the system that should be equal to outflow.

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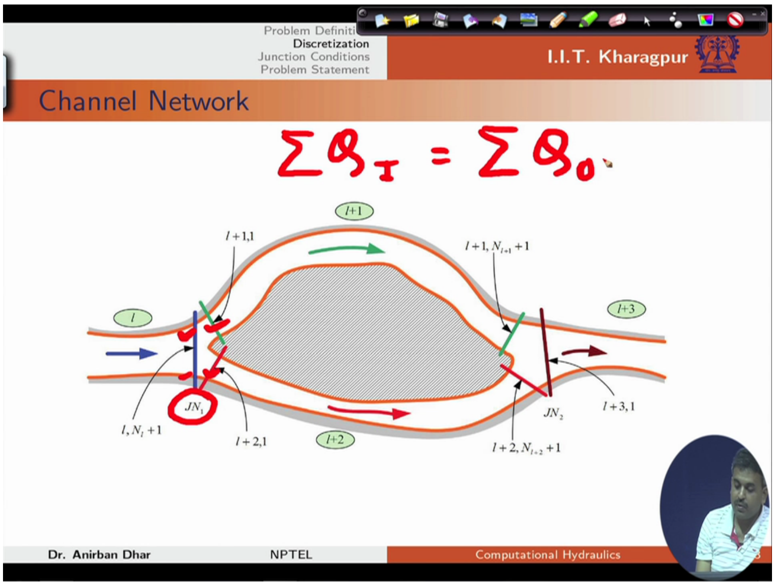

And in this case we can have multiple channel junctions. So in this case if we have another junction in the downstream portion we need to specify both continuity and energy conditions. Energy condition means we need to see the energy balance condition between this section and the section and these two sections.

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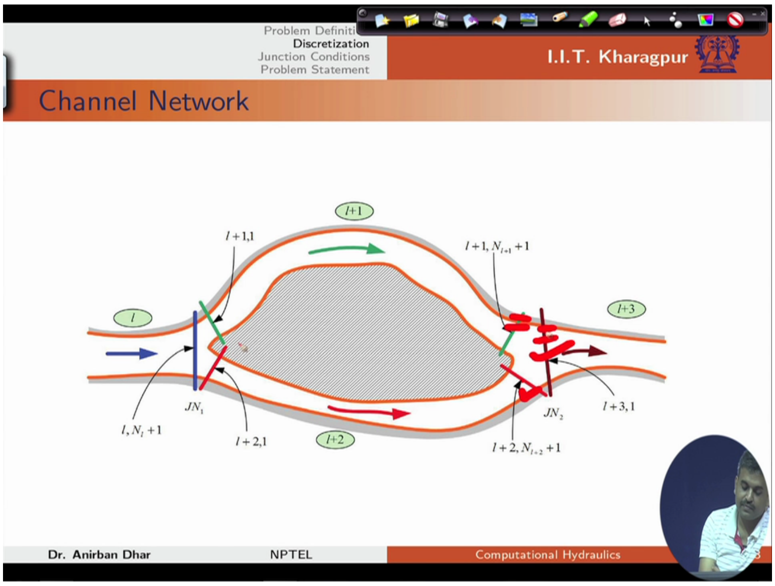

So obviously for any channel junction if we have this three case or three channel case we will have one continuity and two momentums. So all total three conditions because continuity is one because inflow and outflow values we need to consider. And two momentums or two energy condition. So in this case we need to consider energy between these two and again energy between this one and this one.

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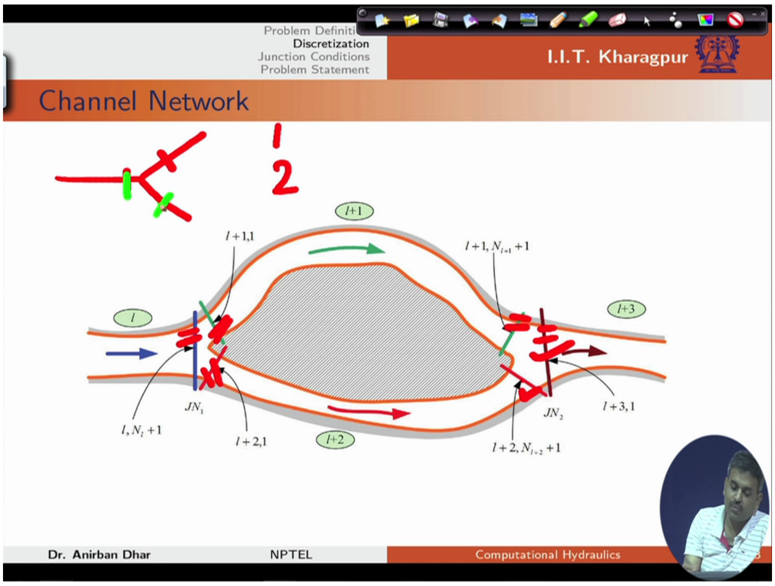

So all total for two junctions six conditions will be there. So this is a general thing, mass conservation. So channel discharge at inflow branch and channel discharge at the outflow branch considering all branches. And energy condition or neglect the losses. If we have different channel elevation then we should add elevation term. Otherwise we can directly consider the flow depth conditions.

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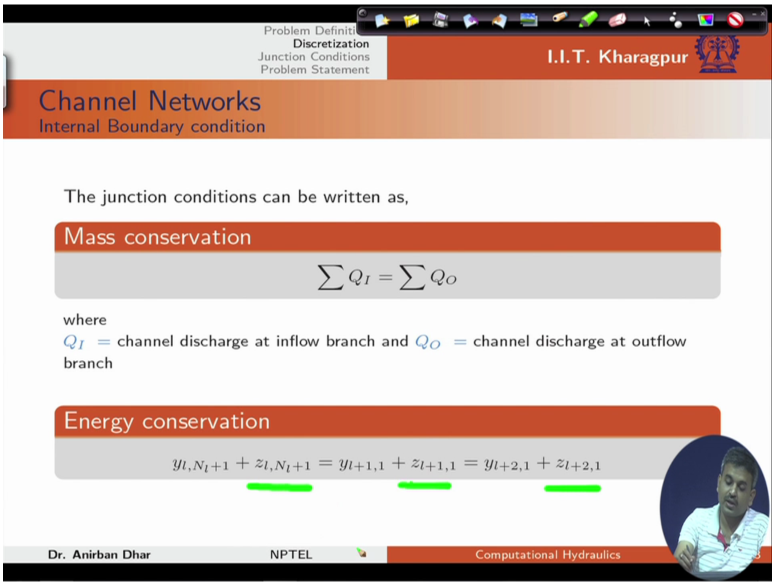

For junction let us say that we have junction 1. For junction 1 we have this continuity condition. Then we need to satisfy that energy condition there and energy condition we have two conditions. So out of this because in every case for any segment we need to find out variables or four number of variables or derivatives or Jacobian matrix terms for four number of variables.

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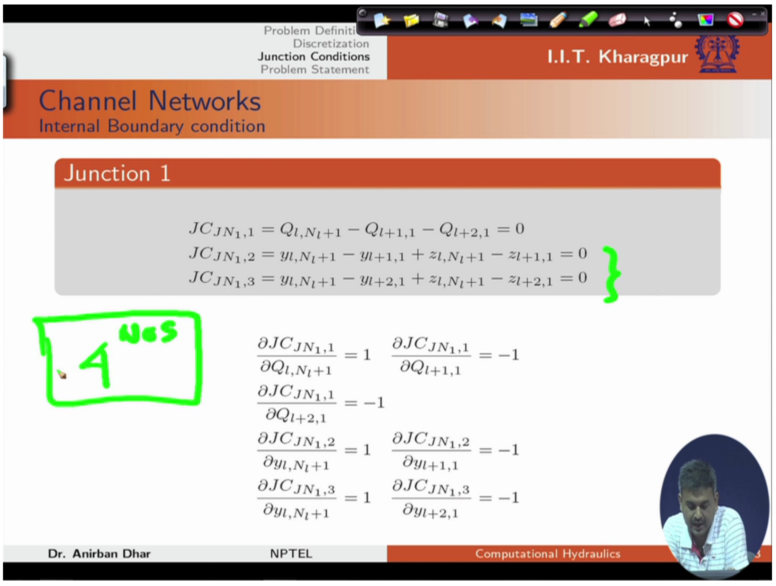

In this case from our continuity three entries will be there. So this is corresponding to NL plus 1 which is the end section of the channel L and the first section of channel L plus 1 and first section of channel L plus 2. This was the case.

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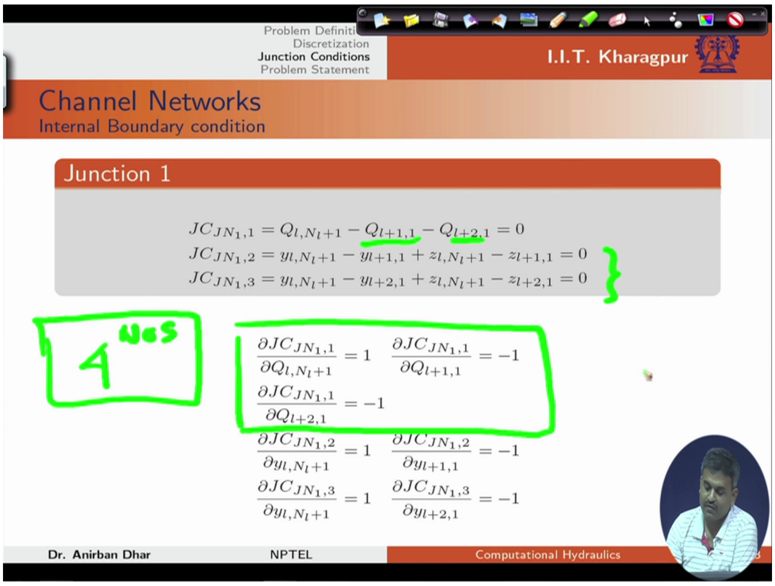

So this is NL plus 1 of Lth channel. This is first section of L plus 1 and first section of L plus 2. So from next two equations we have two variables. So 2, 2, here 3. For these three equations we will get total seven entries in the Jacobian matrix. So corresponding to first continuity equation we will get three entries. For next two we will get two entries each.

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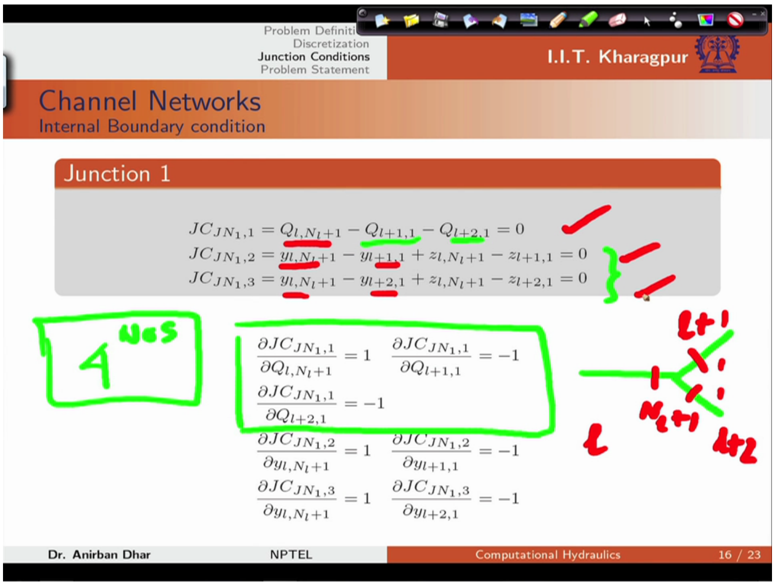

Similarly for junction 2 we can expand this and write the elements of the Jacobian matrix.

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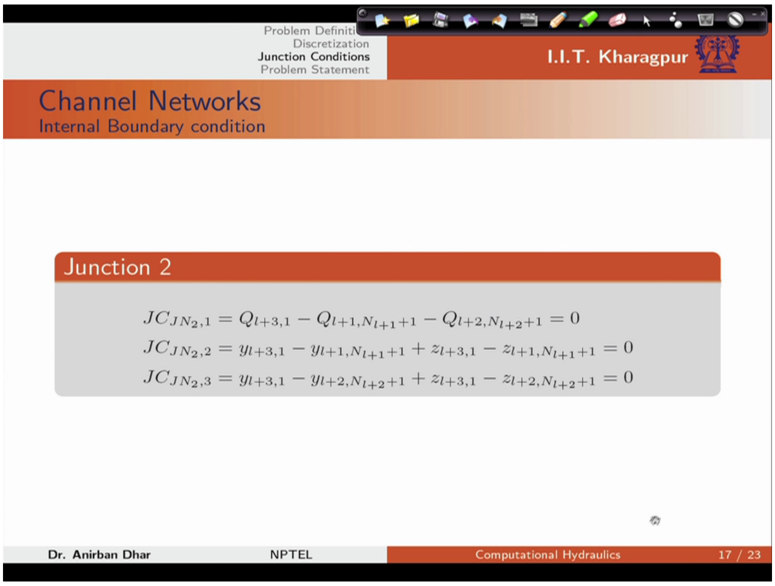

Now in subcritical flow situation we have NL number of equations for the segments and NL number of equation from the continuity and momentum. First one is from momentum, second one is from continuity. And if we have two boundary conditions, this is subcritical boundary condition at the downstream boundary and at upstream boundary if we have specified boundary condition then we can directly utilise that for the solution process.

Now with this conditions we have 2 NL plus 2. That means 2 NL plus 1 number of equations. Now we can solve the system and we can get the solution for the problem.

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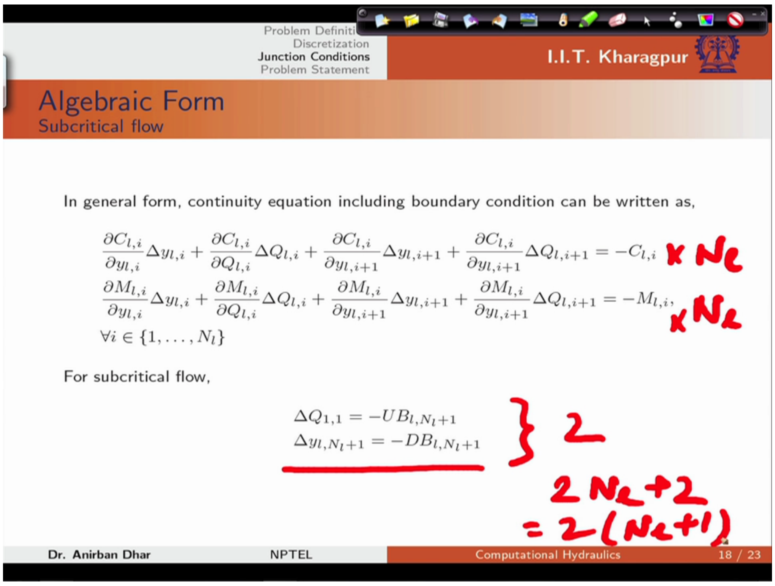

What is the structure? Structure is that for this matrix we have four entries. This blue one is corresponding to continuity, the red one is corresponding to momentum. So at each segment, so this is let us say channel reach. For each channel reach L we will have 1, 2 like that i, i plus 1. We have ith segment.

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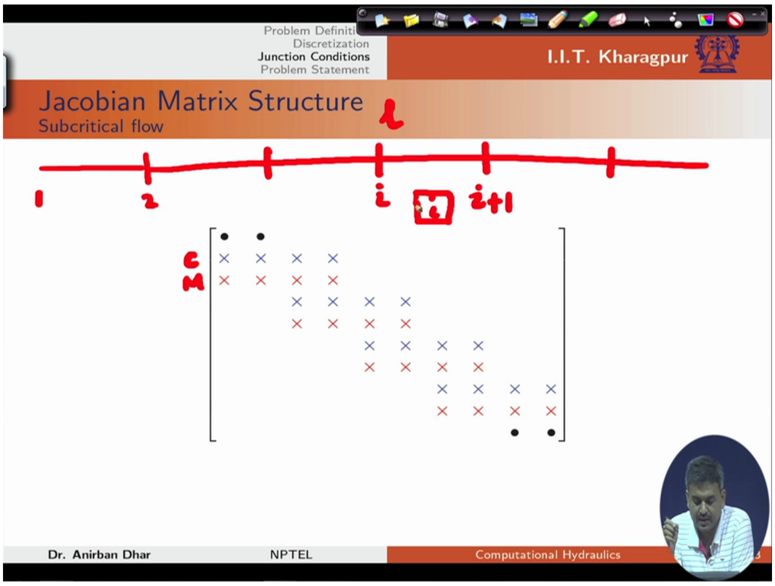

Now in this case first entry corresponding to this segment will be from continuity. This is momentum. Now for section i plus 1 or segment i plus 1 we have connections between i plus one and i plus 2. So obviously we need to consider the entries corresponding to i plus 1 if this is.

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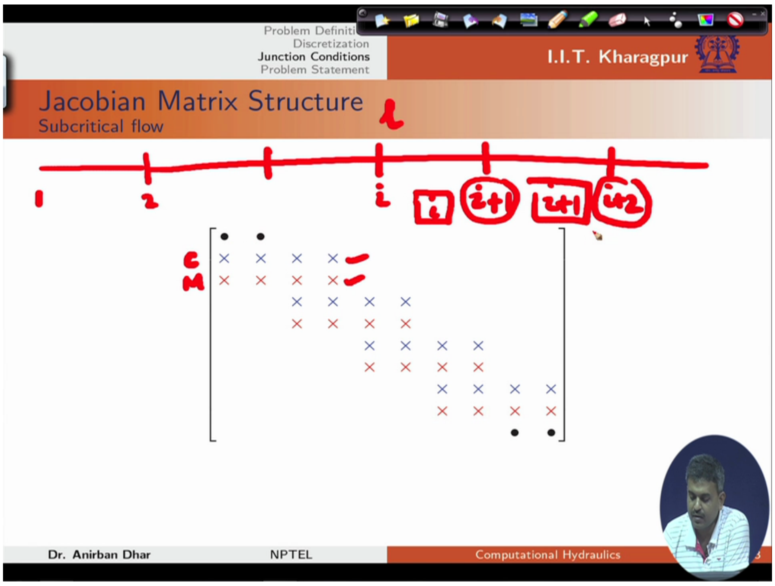

In general we will try to write this variable as y1, Q1, y2, Q2, y3, Q3, like that we will consider our variables. So if you multiply y1 with the first term, Q1 the second term, y2 with the third term and Q2 with the fourth term. So that is the case. Again for momentum we need to multiply the same variables or change in variables, del Q and del y.

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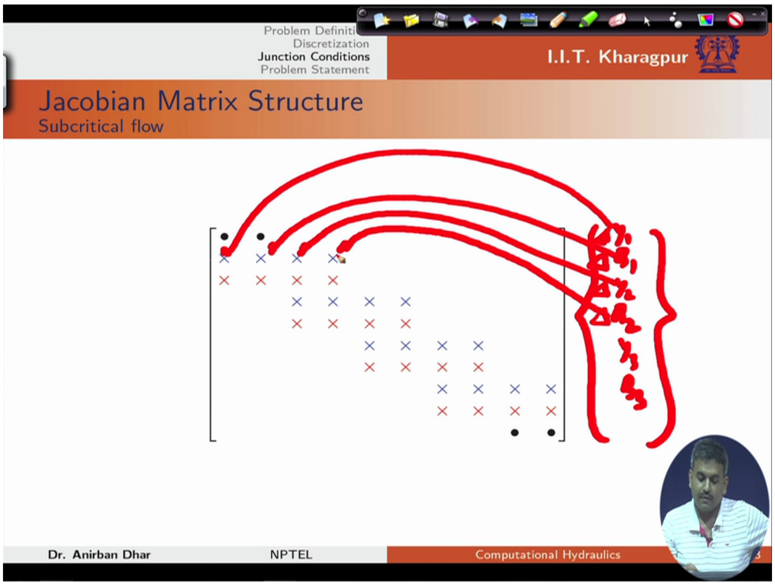

And for the next segment we need to shift our focus from these four entries to the next four entries. So in next four entries, these two entries will be common. So these two entries are common.

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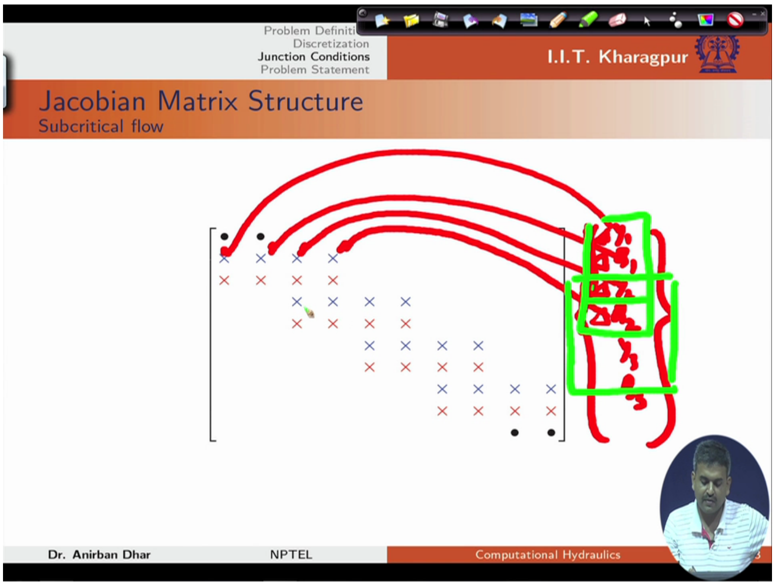

We will multiply it. So obviously in this case we will get the solution. These two are basically corresponding to boundary conditions and if required we can incorporate the junction conditions.

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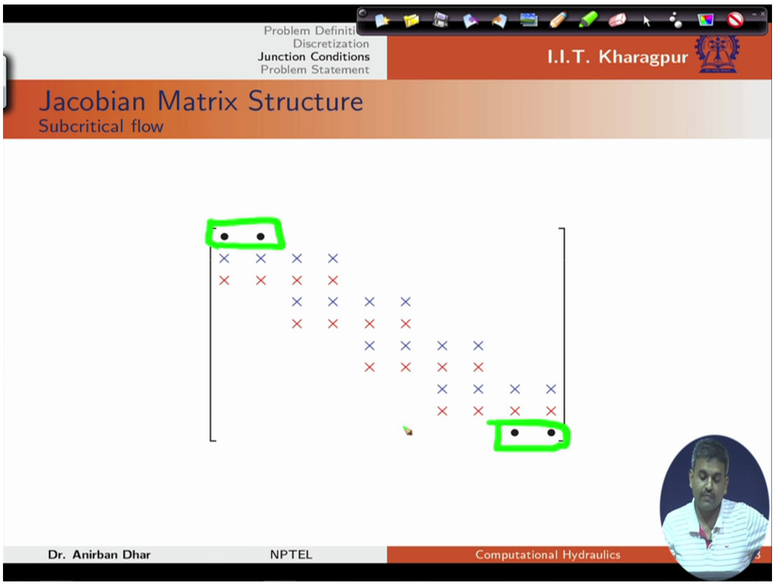

I have not shown those junctions conditions within this matrix structure. Now in this case we can again solve the problem that we have solved in our previous lecture class that is channels in series. We will consider two channels because in previous lecture class we have considered discharge as constant. But in this particular lecture class we are considering a separate continuity equation during solution process.

So if we are considering separate continuity equation for the solution process so we should get the same result that we have obtained using the single variable case. So let us say that the given information first is channel cross section type is rectangular, B is 15 metres, g is 9 point 81 metre per second square, S not 1 is point triple not 4, S not 2 this is S2 or bed slope for second channel.

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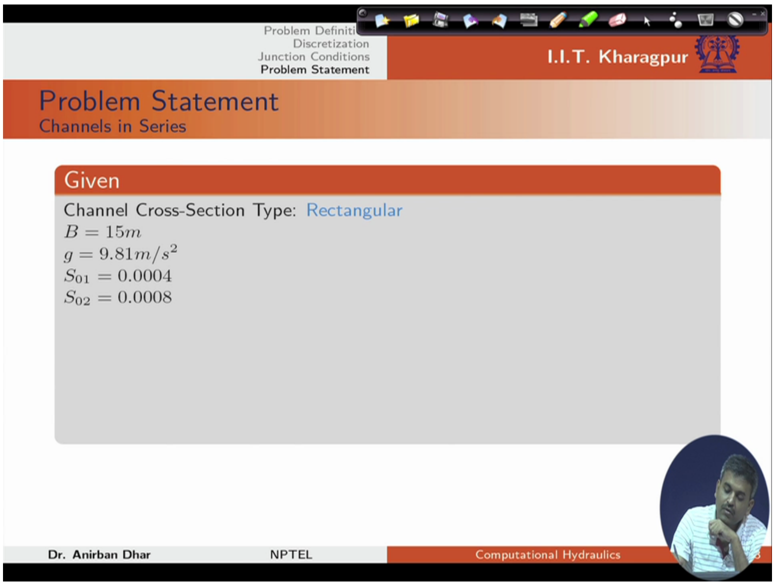

So obviously in this case we are considering milder channel in the upstream and comparatively steeper channel in the downstream section. Next is n1, n1 is point not 1, n2 is point not 15, Lx1 is hundred metres, Lx2 is hundred metres, Q is 20 metres cube per second and yd is point 6 metres. Required, estimate the flow that across the channels in series.

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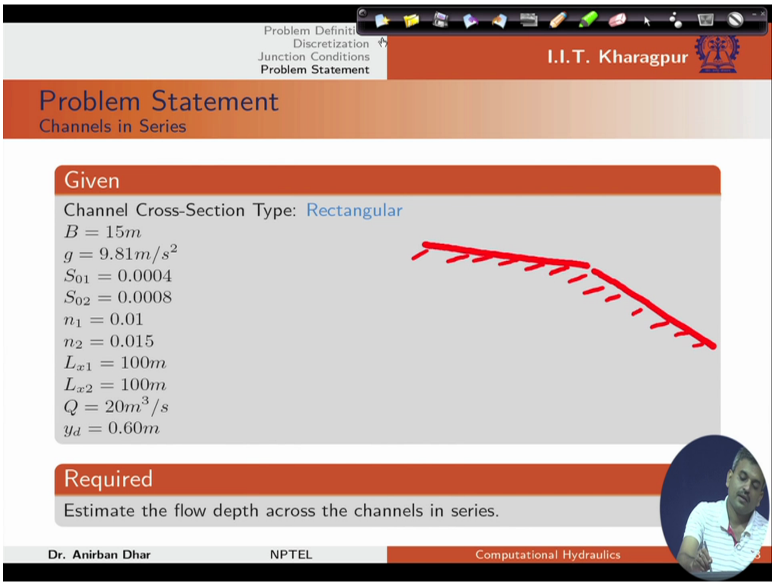

Now we are not changing any information in this one. Only thing is that we are utilising one extra equation during our solution process. Let us see how we can utilise that thing here. So in this case I have considered the same code and changed that code for this problem. So junc is the junction number, chL is I have two channel reaches, Qi is the 20 m cube per second, this is the discharge at upstream section. S not, this is for one and two point triple not 4 and point triple not 8, n is point not 1, point not 15.

B is point this is 15 metres. G is 9 point 81. Lx for both the channel length this is hundred metres. And yd in the downstream section we have point 6 metres. Mnode which is 101-101 and that I have considered here. And epsilon max, this is for iteration we need, 1 into 10 to the power minus 6. And global variable is B and g because Q is again changing here.

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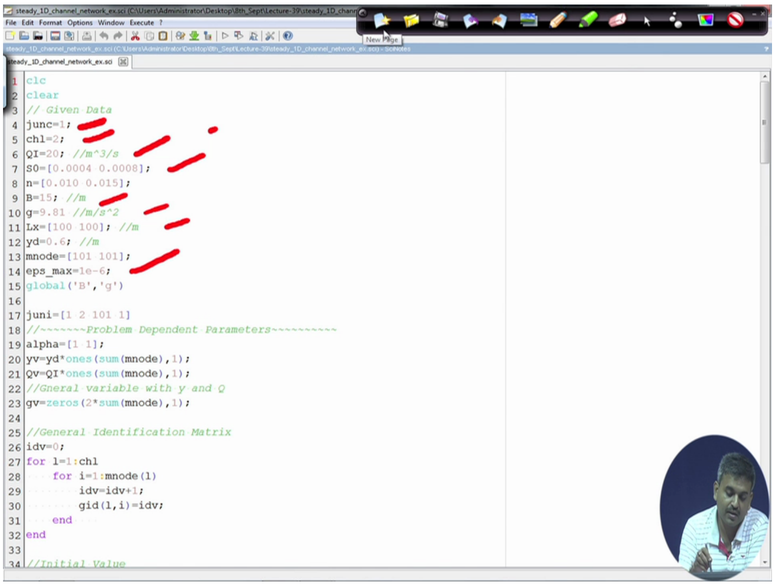

We are not considering constant discharge for this problem. So in this case I have defined alpha first. So alpha is 1 for both the channels. Now I have defined this yv. Yv is the variable and I should consider all the nodes and I should assign this initial value here, yd into ones sum mnode. Sum mnode that means 101 plus 101, 202 nodes this value will be assigned yv. Qv again I need to assign initial value for this one. This is Qi into ones sum mnode. That means 101 plus 101 total 202 nodes I have assigned this Qv value.

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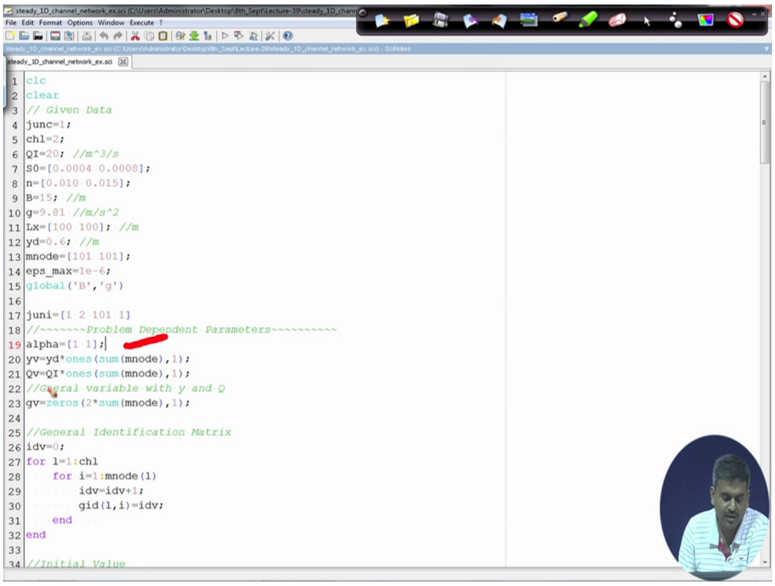

Now in this case because this channel reach is there. This is for L equals to 1, this is for L equals to 2. So starting from 1 to NL plus 1, again starting from 1 to this is N1 plus 1, this is N2 plus 1. So these are the sections in between.

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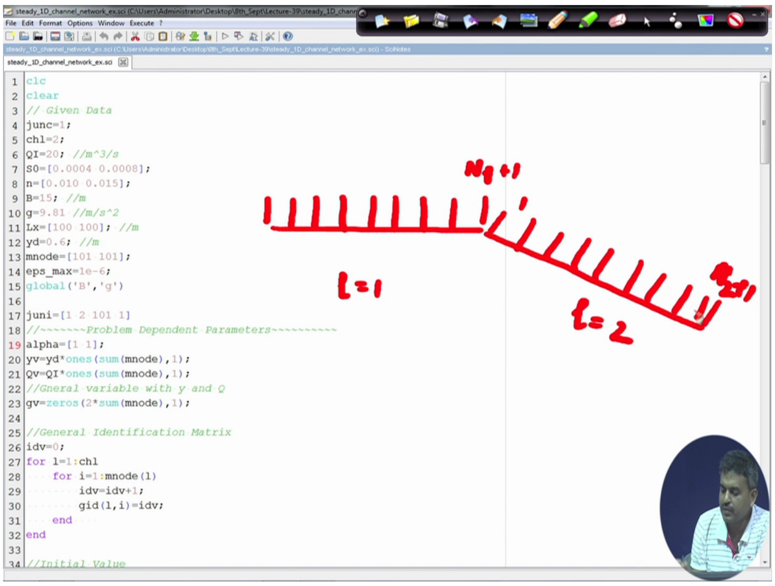

So in this case we have double index notation. That means L i. If I take L i that means it is representing the channel section in the Lth channel and for ith section, okay. So this is the local numbering. I need one global numbering for this one. What is that global numbering? I should use a global number that information I can utilise during the construction of Jacobian matrix. So that global number is gid or global id here, ok.

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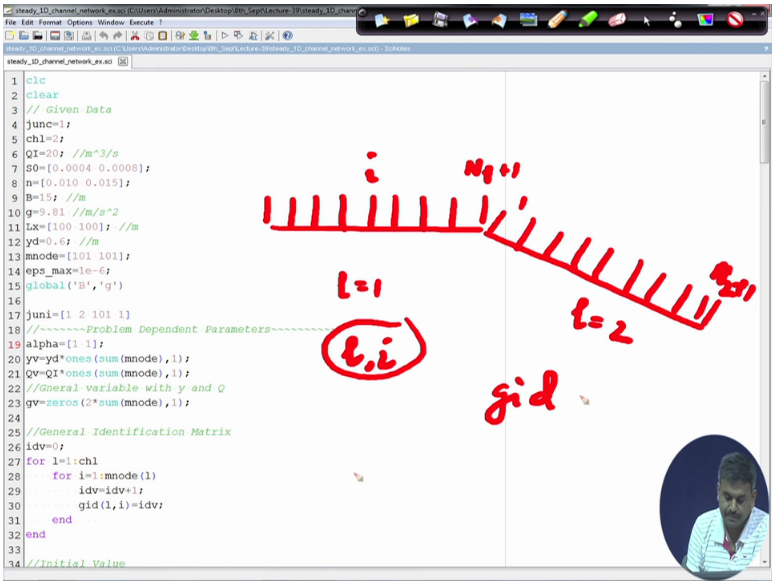

Now I will start from id value equals to zero. So starting from L is equal to 1 to CHL. That means for all channel reaches and from 1 to mnode. Mnode means 1 to NL plus 1. All nodes I should sign idv equals to idv plus 1. Whenever I am entering in this loop I am adding idv equals to idv plus 1. So gid equals to idv. That means one by one I have assigned serial number against each of those sections in different channel reaches. So I will have unique number for any channel section in a particular channel reach.

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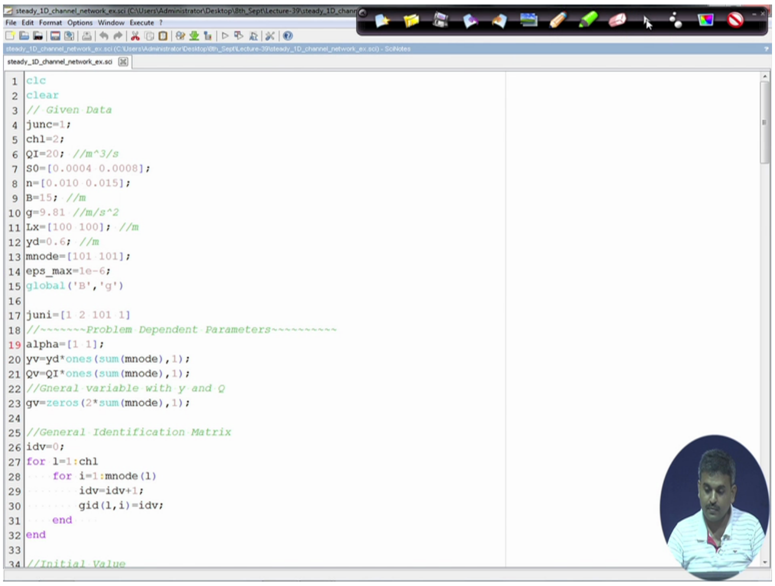

Now here after assigning this one so this is in this structure that if I have this structure which is L i. I should get one value. So for different values of L i, I will get gid. So let us say L1, i1

I am starting from 1. In our case if L equals to 2 and mnode equals to 101 that means for the second channel reach and 101 node my gid should be 202 because I have considered all the nodes or all the sections during this assignment process.

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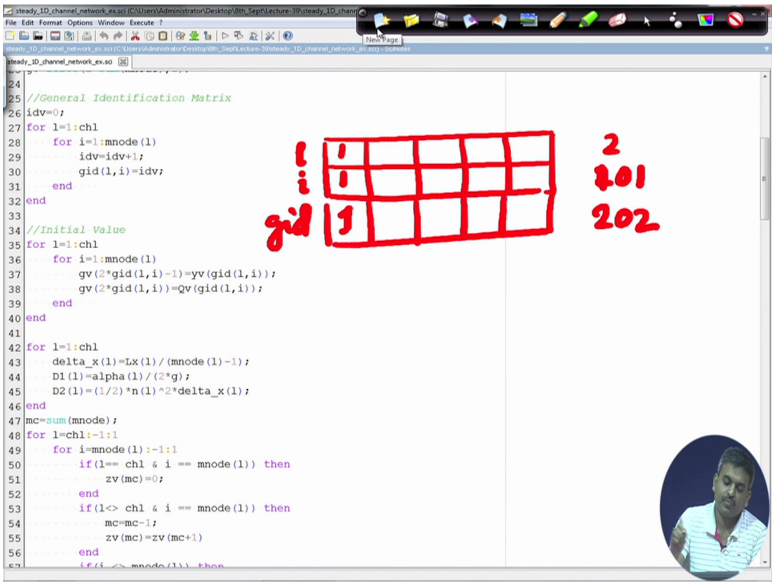

Now initial value, I have assigned initial value for individual variables yv Qv. But during my solution process I will have only one variable. Let us say that A phi equals to r equation in this case I have J into del phi because change in that variable is required. So this is my right hand side, something is there. But in this process this del phi it considers both y and my Q values. And they are arranged like this, y1 Q1, y2 Q2. And all are del values. So in a single calculation process I cannot use two different vectors y and Q.

So I need one global vector for this definition. So for this global vector definition what I am writing this 2 into gv, this gv is a general vector or general variable in this case. So general variable, what I am doing? I am assigning gid. So if my assignment of gid, this is ranging from 1 to 202.

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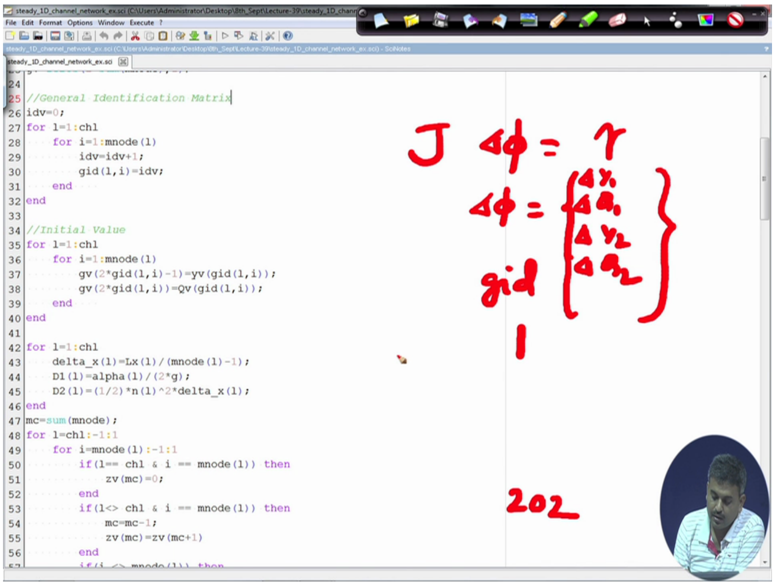

Now this one is valid for yv which is y value and Q value. But in case of gv which contains both y and q so I need 404 that means 202 into 2 number of variables. So for that one if I take any general gid for Lth segment and ith section and if I multiply it with 2, I should get the index corresponding to Q.

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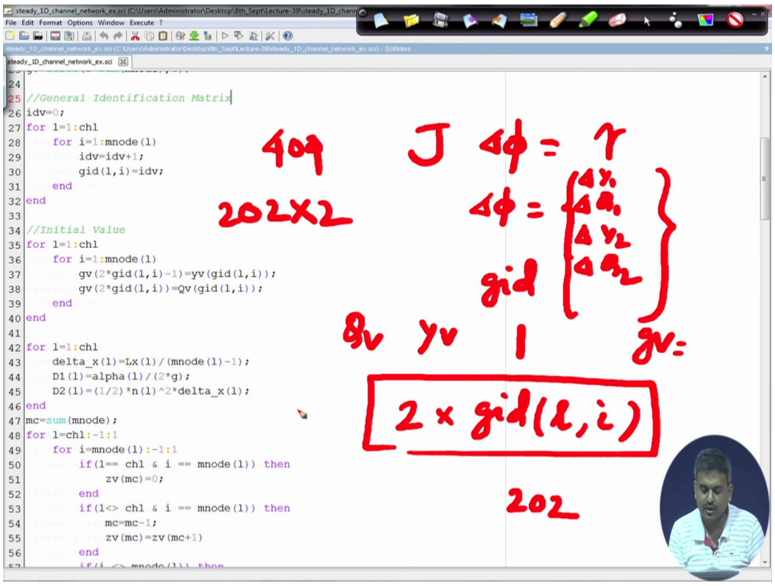

Because I am arranging the variable like this. Q1, y1, y2, Q2, y3, Q3, this is y4, Q4. So if my gid for L i for any arbitrary section or any arbitrary section in this case is 2. If gid equals to 2 that means gid into 2. That means 2 into gid is 4. That means I am considering the fourth term here. So what is my fourth term? Fourth term is Q2. So all even terms are discharge terms.

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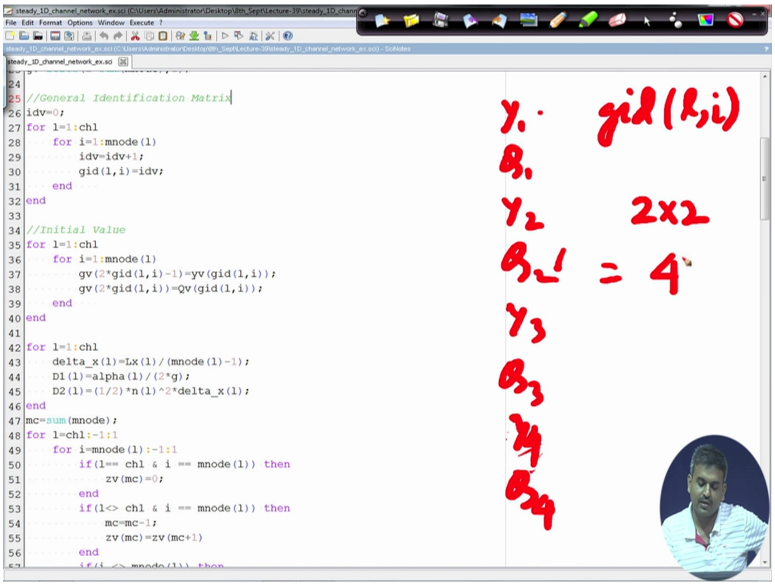

Now if I take 2 into gid L i minus 1, this is 3. Now 3 means this one. So all odd terms these are corresponding to flow depth values and even terms these are corresponding to discharge values.

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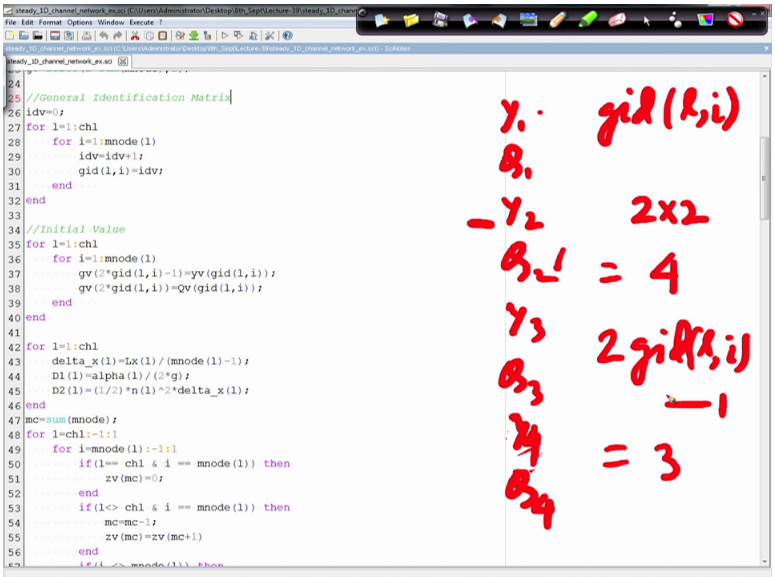

So with this assignment with this initialisation, because initial values I can transfer here. After that I can define the channel reach dependent parameters. Now channel reach dependent first parameter is del xL. So which is Lx divided by mnode L minus 1 and D1 D2. In this case the difference is C1 C2 compared to our previous lecture class is that we do not have any Q squared term here. So in this case again we can get the z values here.

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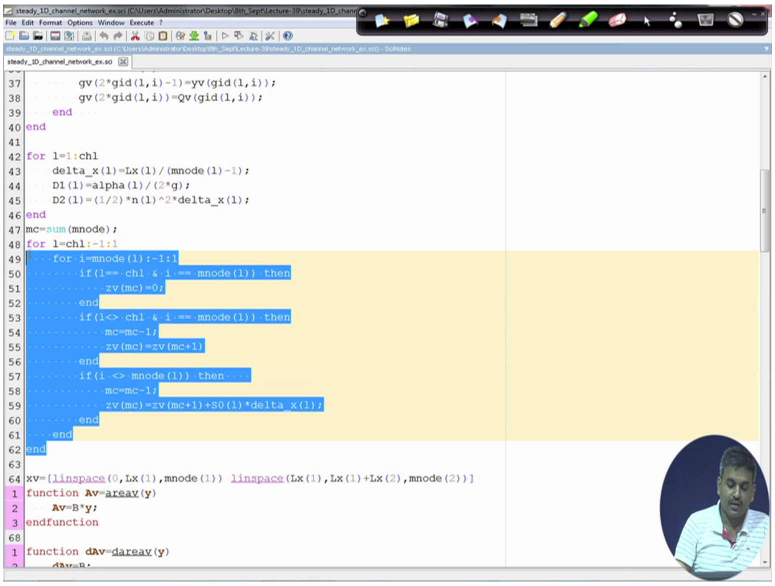

This is our area calculation. We are considering rectangular channel dA by dy, R, dR by dy calculation and this is mLi. I have not written that cLi because cLi the coefficients are either 1, minus 1 or zero. So I have directly assigned those values. So these are similar to our previous one or previous definition.

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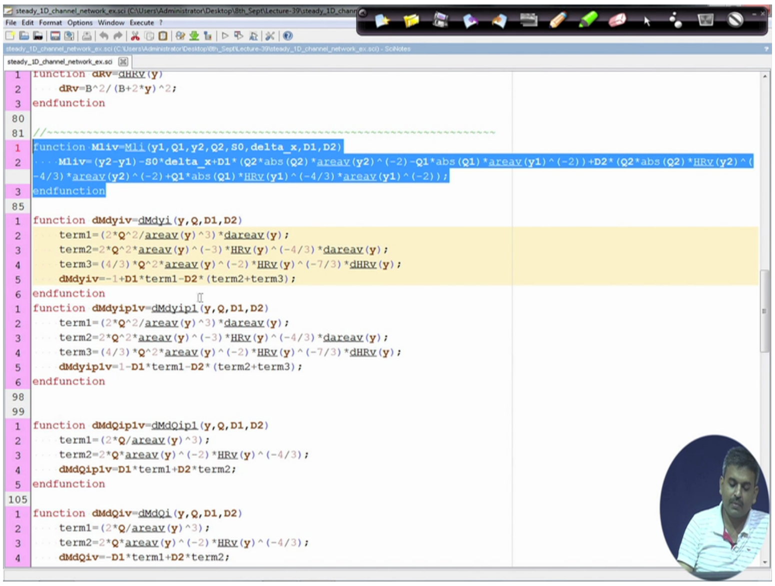

Now only difference is here that we need to transfer this D1 D2 values. We need to transfer del x, S not, y1, y2, y1, Q1, y2, Q2, y Q in this case also y Q. This is for M L i which is for ith segment of the Lth channel reach. This is del M by dy which is the coefficient for L i. This is dM by dy. This is L i, this is y i, this is L i, this is y i plus 1. So this is y i plus 1, this is y i, this is M and now we need dM L i, this is dQ i plus 1.

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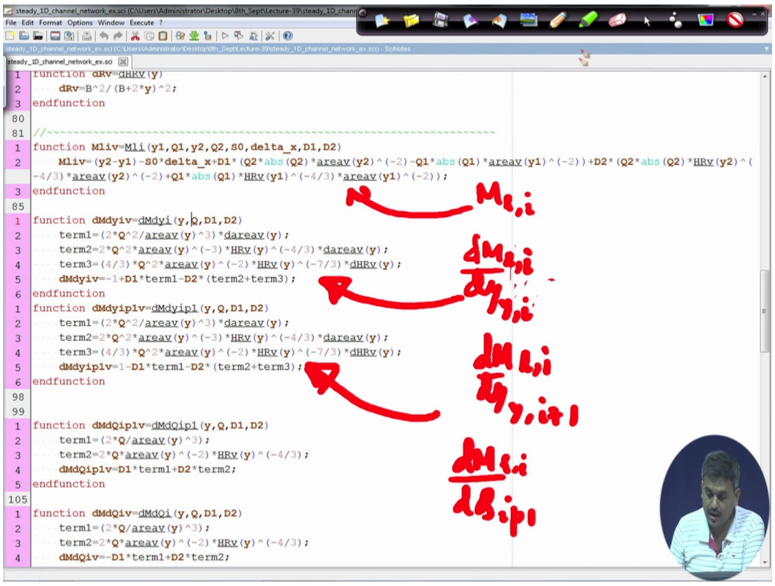

And the term which is dM by L i by dQ this is for L i. So this term is here.

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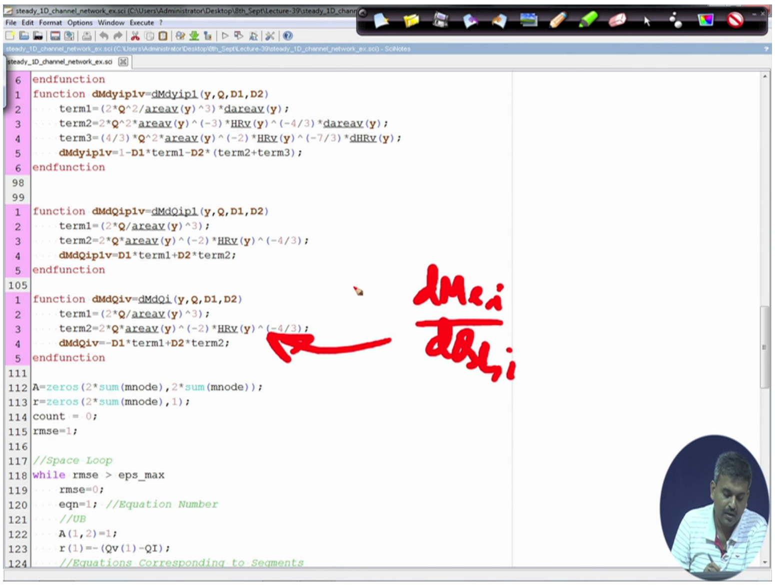

Now after defining these terms we can start defining our Jacobian matrix. Initialisation I can start, initialisation is with this A zeros 2 into sum mnode. That means if we have total 202 nodes, 101 plus 101. So obviously we have 404 variables. So 404 into 404 should be the size of A. And the size of R should be 404 into 1 in this case. This count is zero, rmse is 1.

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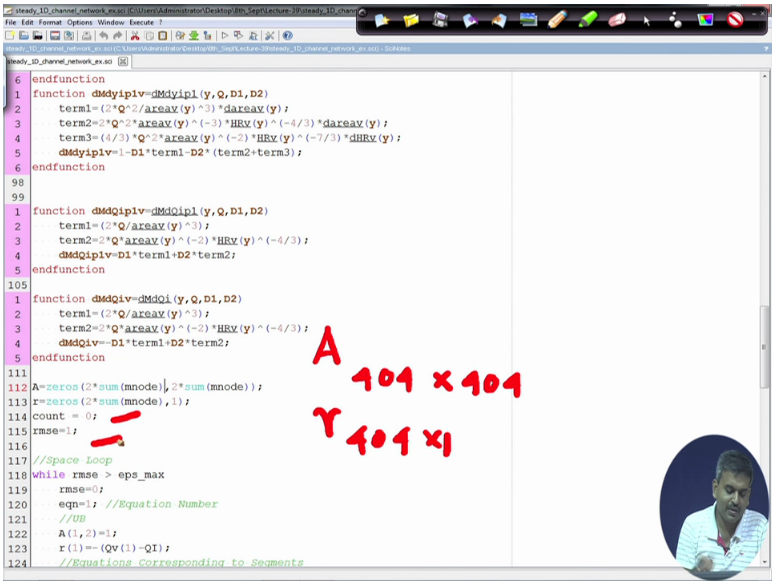

We have already initialised the gv or general variable there. Now in this case what is happening initially we have written this rmse and we are starting with equation number 1. So first equation is our upstream boundary condition because we will have entry for 1, 2. This is y1, Q1, y2, Q2. So if our Q1 value is specified which is Q directly we can say that this is Q1 minus Q. Some upstream location. And for that qv what is required? We need to differentiate it. If we differentiate it we will get only second term.

That means A 1-1 is zero. No entry for A 1-1. This is valid for section only. So we will have only two terms associated with it. But the first term A 1-1 is zero. But A 1-2, this is 1 and right hand side we need to write that upstream boundary minus. So if we write that upstream boundary minus this means Q1 minus Q upstream here. That is what I have written here.

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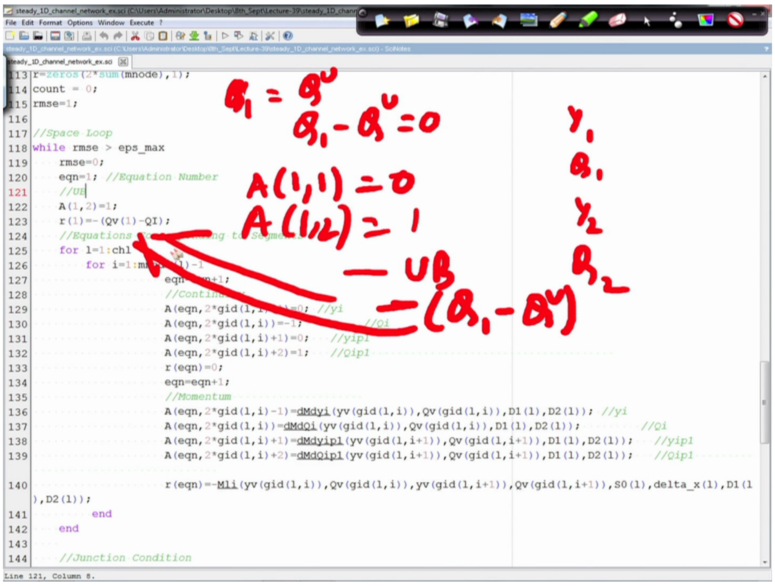

Now after that we can enter into this equations corresponding to segments. For equations corresponding to the segments we have L equals to 1 to channel number. Then i equals to 1 to mnode minus 1. Mnode minus 1 means that I am considering NL number of segments for each channel. So in this process I am considering 2 into NL number of equations. I have already considered 1 there. So I need one more extra. So that I can complete 2 NL plus 2 number of equations there.

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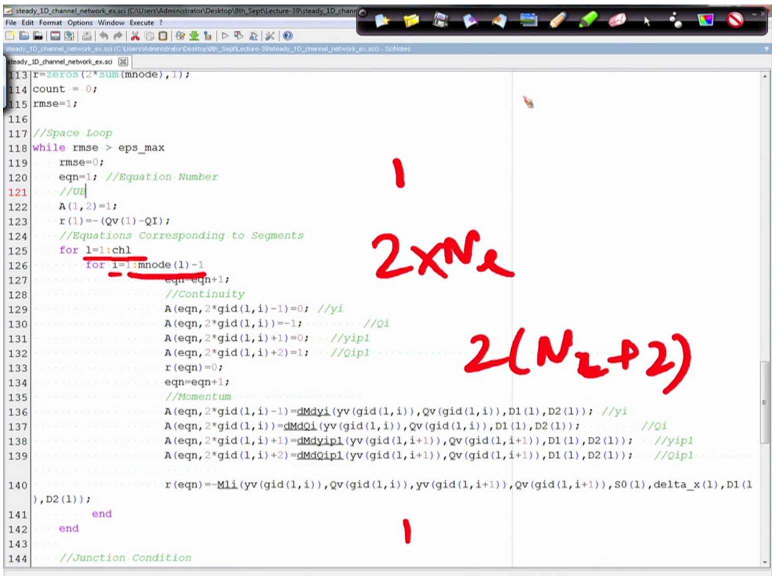

So A whenever we are entering into the system before this continuity equation I am increasing this equation number. Equation number equals to equation number plus 1. A eqn is

2 gid L i minus 1. That means whatever maybe this L i value, I will have one unique id or gid for my case. And in that case if I consider that 2 gid L i minus 1, that is the thing.

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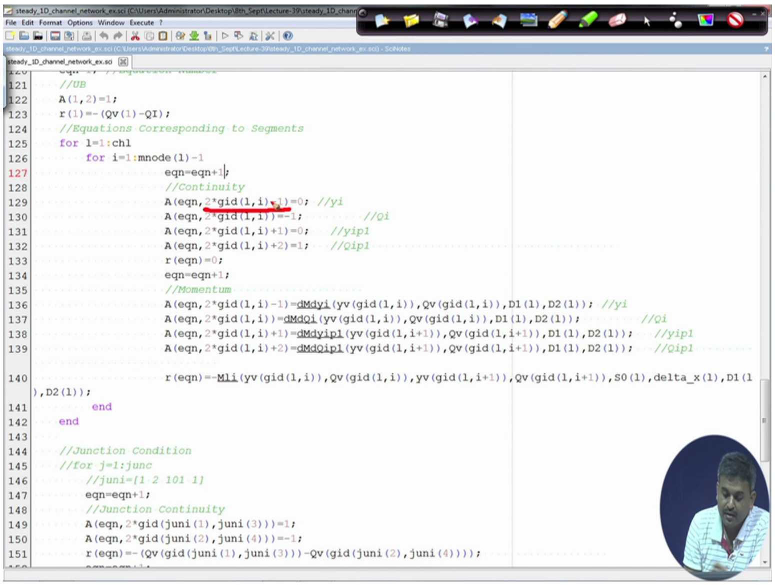

Now in this case another way I can represent it, what is that? That is nothing but we have this L plus 1. Now again I can consider that this is corresponding to our flow depth y i. This is corresponding to Q i, this is y i plus 1, this is Q i plus 1 because we are considering even numbers here. Even numbers because i plus 1, i, i plus 1 thing we are considering. This is gid L i plus 1. This is gid L i plus 1 2 into. But one minus 1 is here, minus 1 is here. So these are corresponding to y values.

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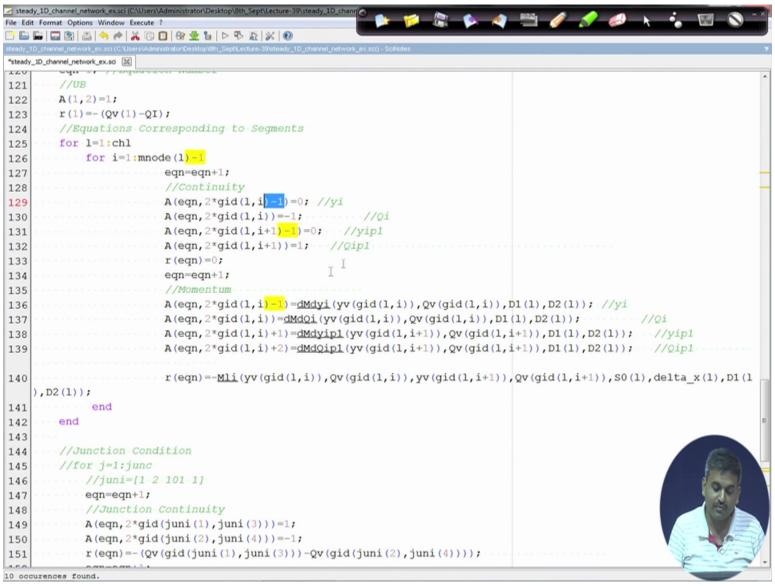

Now in this case again we can avoid this and we can write it in general terms. So this is the case where we have for any Jacobian matrix we will have equation number. Corresponding to that equation number we will have entries. So first entry that is corresponding to, 2 into gid L i minus 1. Next one is gid L i. That means this is corresponding to y, this is del i and this is del Q i, this is i.

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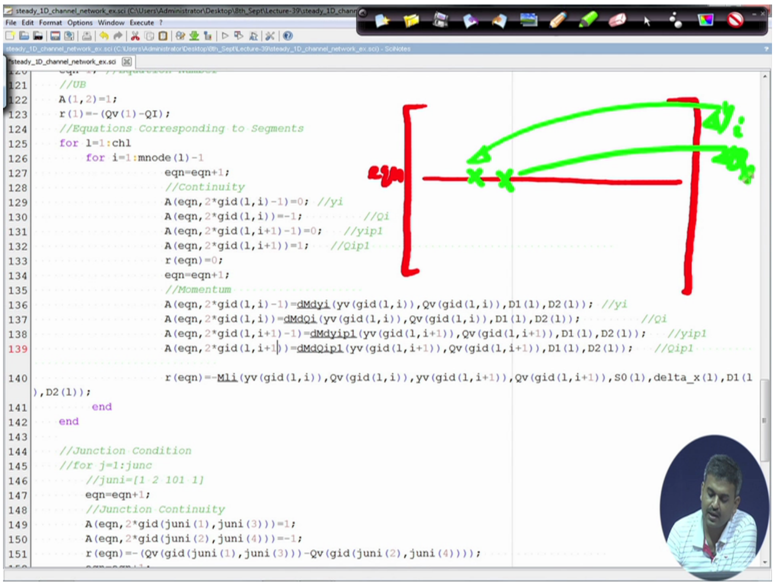

And this is corresponding to del y i plus 1 and the fourth one is corresponding to del y i plus 1.

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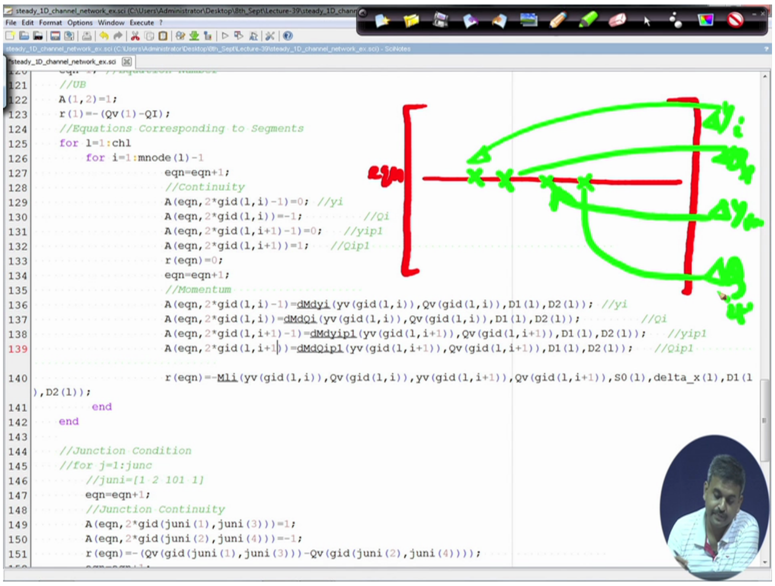

So in this process we have generated these equations. And the first equations are basically continuity equation. That is why I have not written the coefficient separately because we have 0, minus 1, 0, plus 1 terms here because we have Q I this is corresponding to L i plus 1 minus Q L i this is equal to zero. And we differentiate it with respect to these two variables, obviously plus 1 will be the coefficient of this one and minus 1 will be the coefficient of this one. So this is minus 1, this is plus 1 here.

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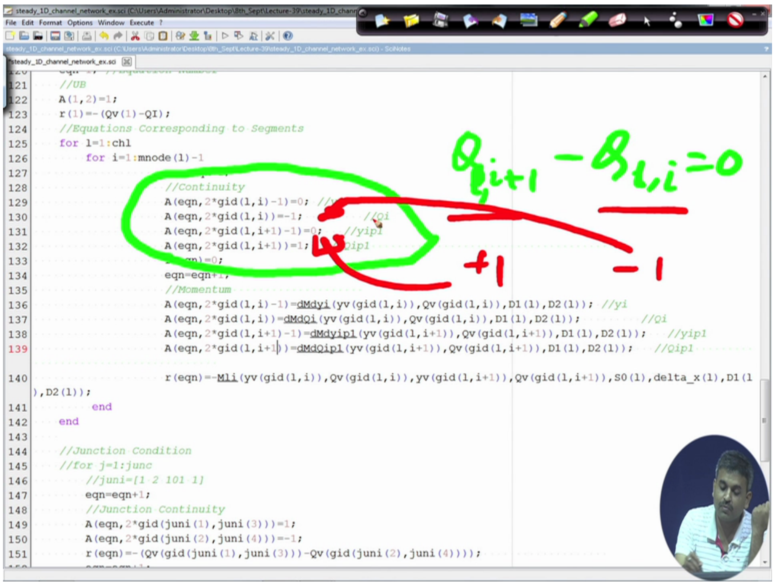

Now we have completed our continuity. Again after completing this continuity again we need to add this eqn. For eqn again we will be adding one momentum equation, For momentum equation we have to utilise this gid L i and gid L i plus 1. These two segments or these two sections for a particular segment.

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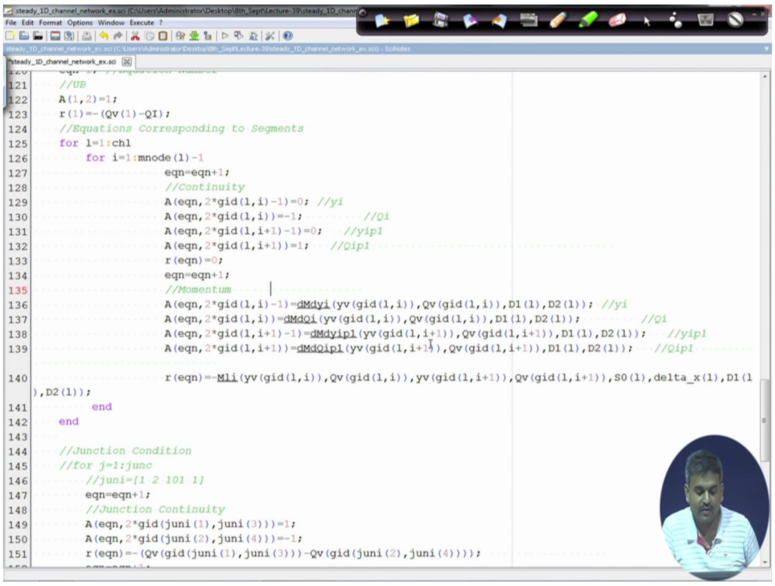

Now with this we can calculate our thing. Next thing is we need to specify certain junction conditions. Why this junction conditions are required? Because in this case we have only one condition that is I have 1 and 2. This is my 1 or channel 1 and 2. These are connected from 101 node to 1 in the second channel. So that is why junction information is 1 to 101, 1.

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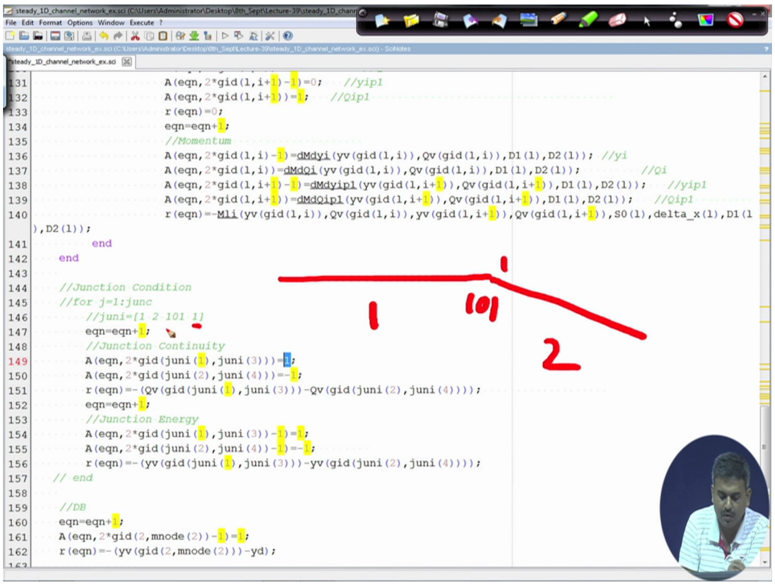

Now again I need to add one continuity equation here and one condition here. So if I add these conditions because the first term just junction information. I have junction information 1. This is the 1, the first channel. This is corresponding to L. Second one is i. So and third one is i junction condition for third. That means the location of the end section, this is 3. That means 101.

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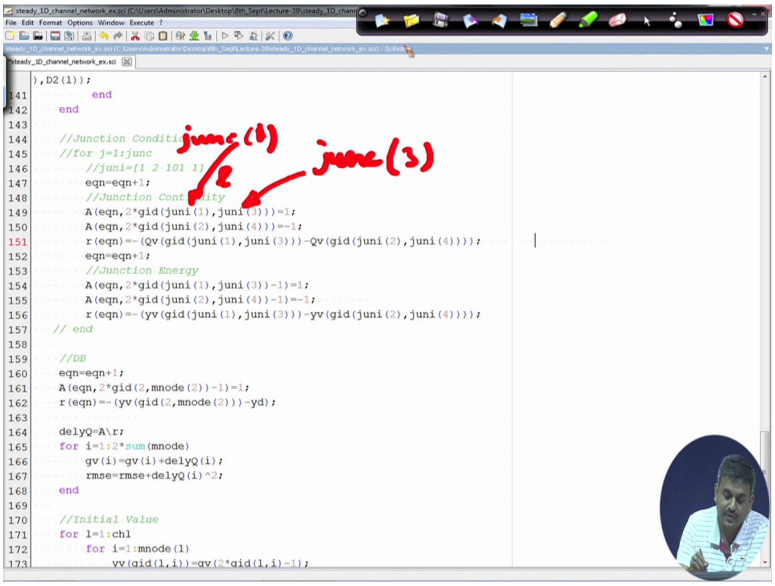

So with this information if I run this, so obviously we have these equations. So we have the condition for steady series or channel flow in series.

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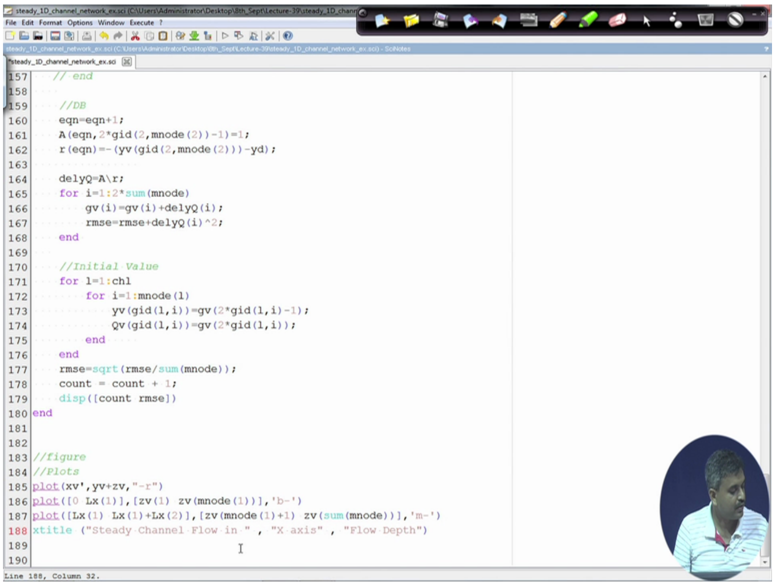

Now in this case we are getting one transition at this level.

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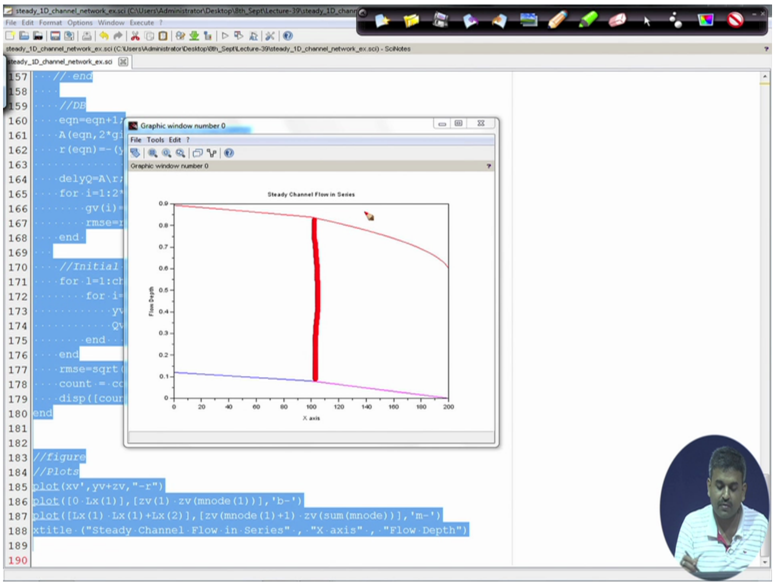

Again we need to transfer these values. All values we need to transfer to y and Qv values directly to or in this case this is update values because we need to update values after solution of del Q del y Q. Del y Q we need to add with gv values. Gv is assigned initial value. After every iteration this is getting updated.

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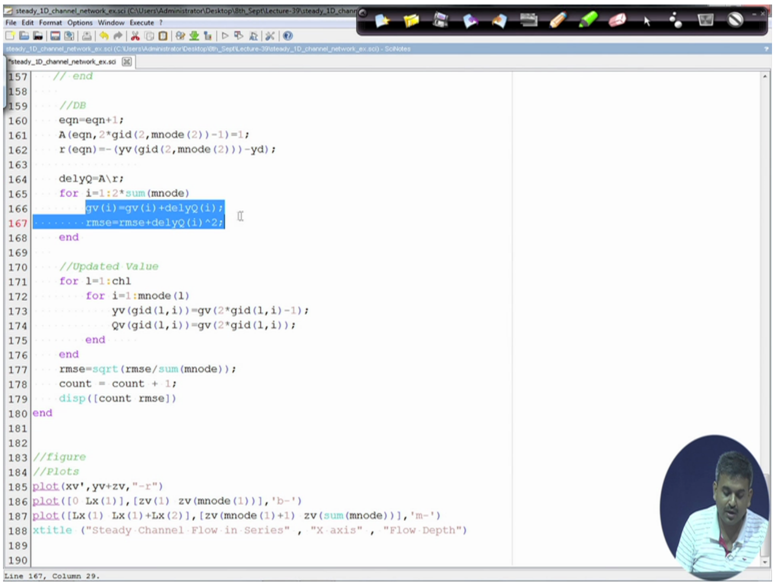

And after we are getting updated values we need to update y and v because during calculation we are using this y and v values. So in this case we are directly getting the solution. So obviously it is similar to our previous case. This is the channel transition in this case. So obviously colour is different.

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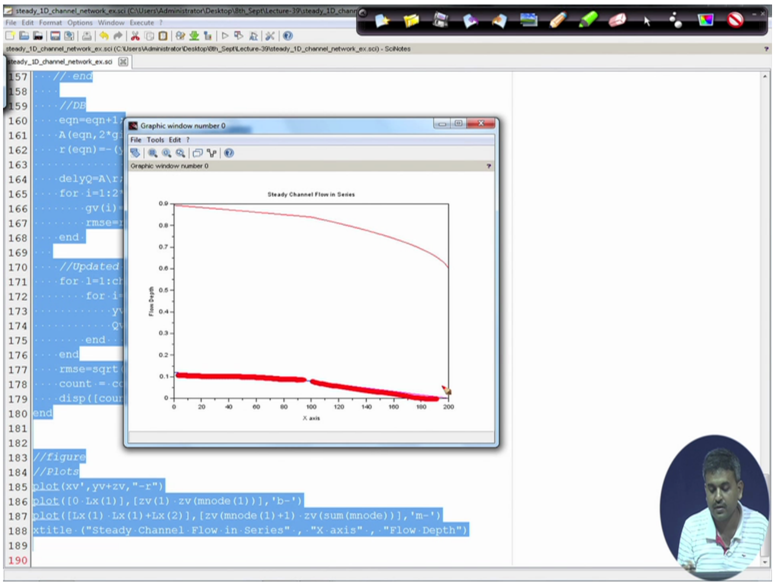

And we are getting one transition here.

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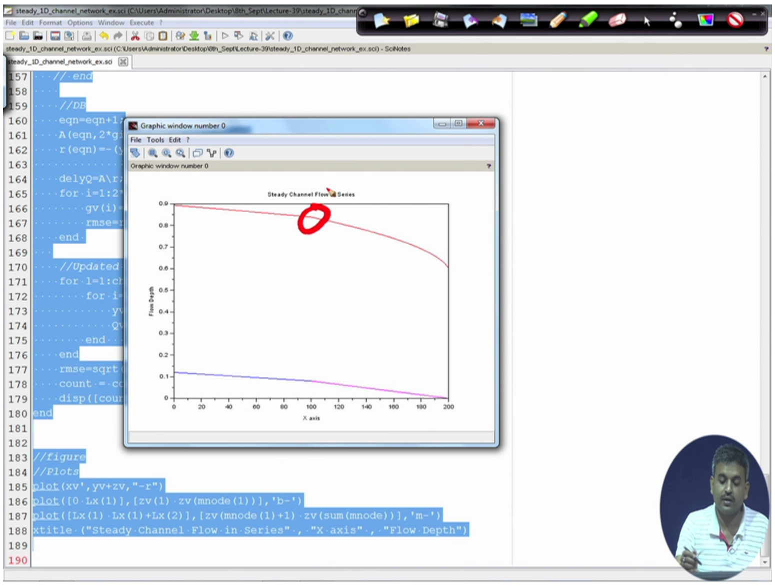

So with this approach we have solved both discharge and continuity equations for both the problems. Now we have all total junction conditions, boundary conditions and if we utilise those information we can directly get the solution out of this.

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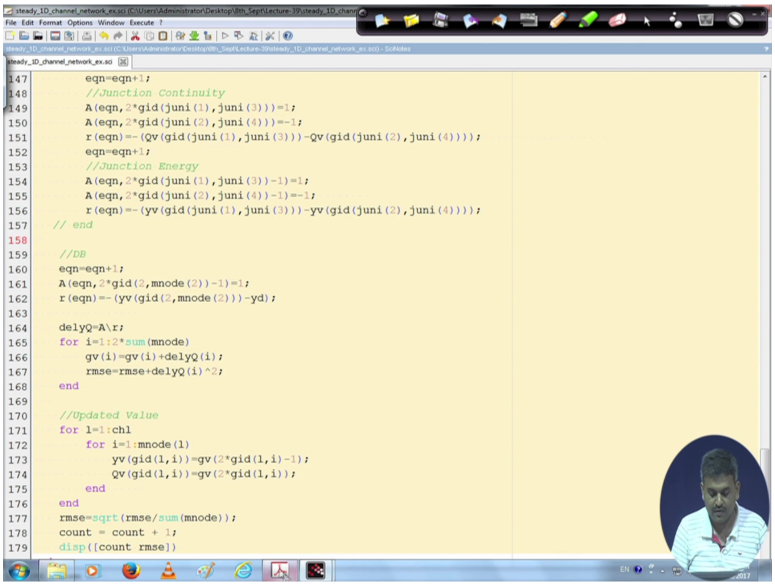

So this is the estimation of flow depth across the channel and we have solved this problem.

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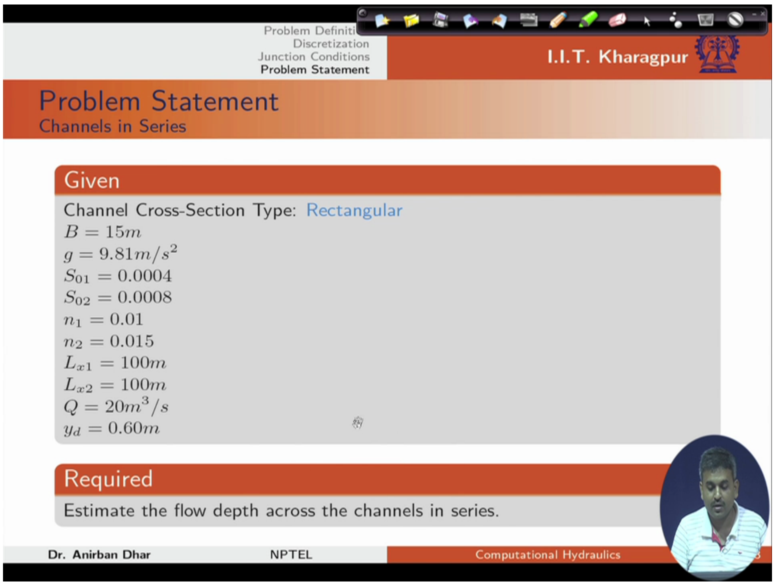

So this is with rectangular cross section we have solved. So code is steady 1D channel network sci. So you can utilise this code to check the solution process.

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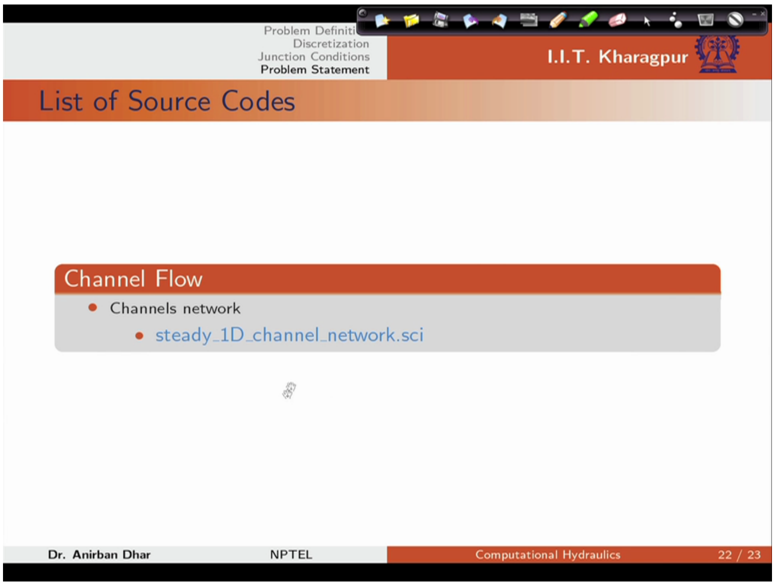

Thank you.