Computational Hydraulics
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Lecture 37
Steady Channel Flow: Single/Series

Welcome to this lecture of course computational hydraulics. We are in module 4, surface water hydraulics. And this is unit number 3, steady channel flow, channels in series or single one.

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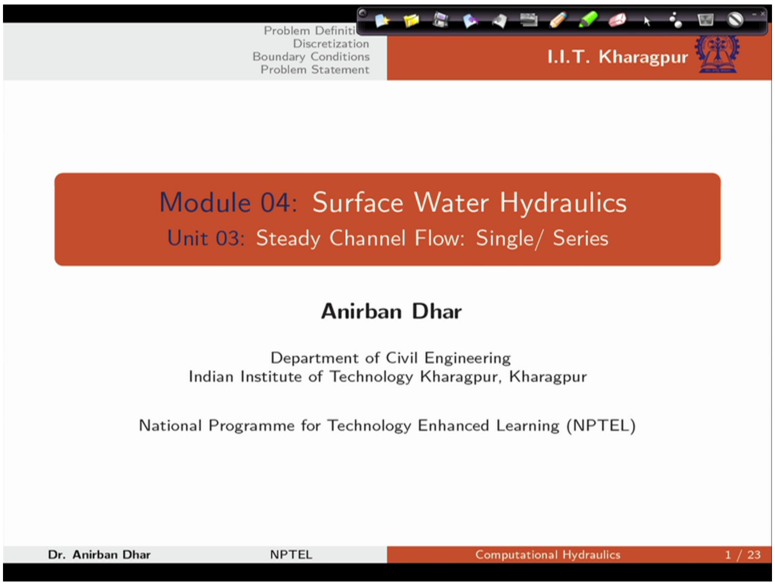

What is the learning objective of this particular unit? At the end of this unit students will be able to solve steady channel flow problem, single or in series using implicit method. Although this implicit approach is not related to time. We will be talking about steady channel flow. Essentially we will be talking about the solution of nonlinear discretized equations.

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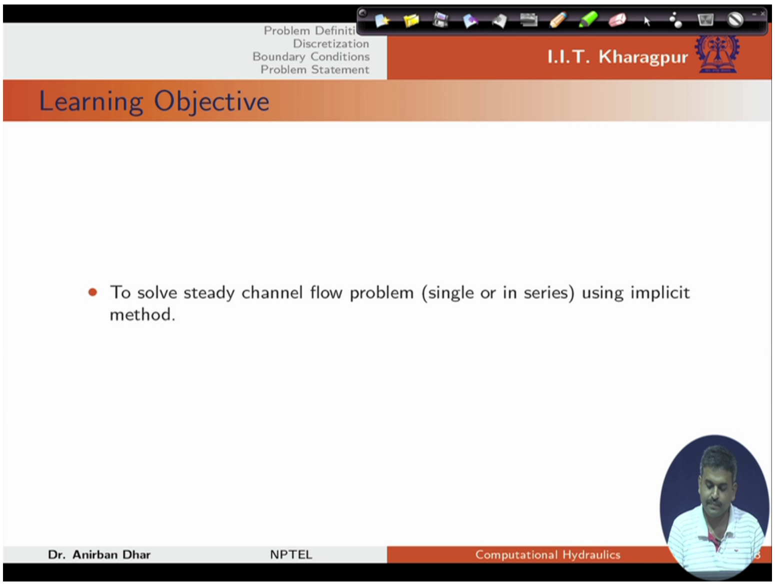

In our last two lectures we have discussed this gradually varied flow or GVF. In GVF we have seen that we need to solve this dy by dx S not minus SF 1 minus Fr square, this first order equation. And we have this initial condition x is equal to zero, we have y not. If we solve this one, we can get the variation of y with x. So from this one we can classify the type of GVF. But for classification further we need two main flow depths. One is yc which is critical depth and this corresponds to our critical depth line.

So if we consider this as channel, then flow is from left to right so obviously for this one if we have CDL or critical depth line here and normal depth line above then we have M profile conditions. And we can find out this NDL based on the value yn. Yn is nothing but this is normal depth. We can calculate this normal depth from our Mannings equation.

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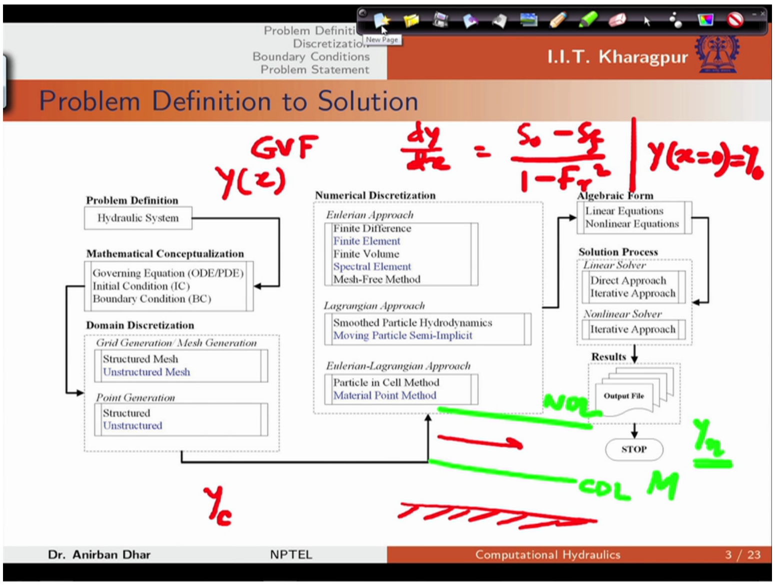

So in last two lecture classes we have discussed the solution process. One is implicit and explicit, explicit and implicit approach for GVF profile solution. And we have solved one initial value problem with one initial condition or one condition specified only at one end. And what is the value on the other end? That we do not know.

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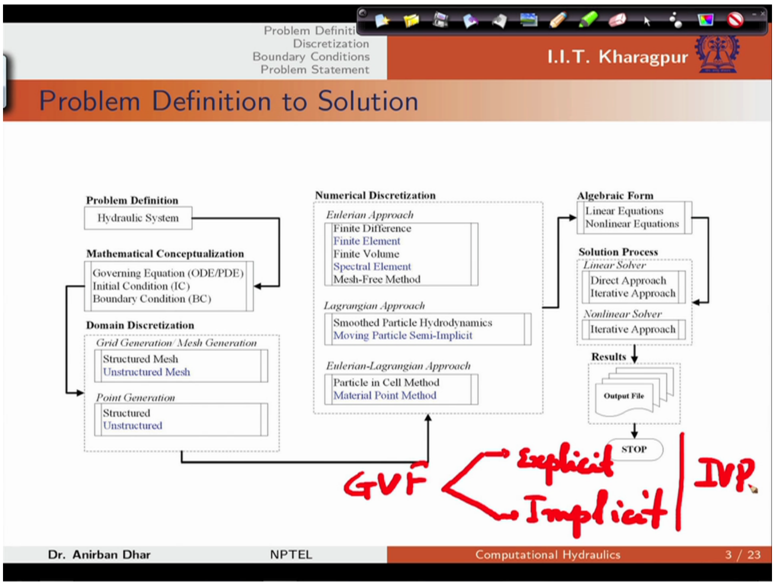

Now we will be talking about this continuity and momentum together. But we need to solve those two equations simultaneously. But for special cases we can omit one of the equations and continue with a single equation to get the solution. Let us say that we have a problem where we have a channel which is single channel. This is channel bed and this is the flow direction and obviously this is flow depth at any location.

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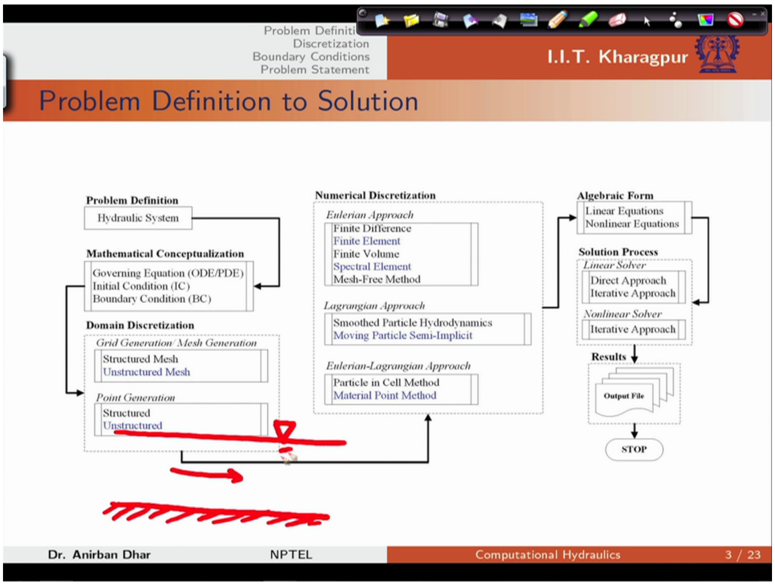

So this is single channel. Now we can have situation when we can have multiple channels but they are in series. They are directly connected. Let us say this is our first channel, this is second one. These are directly connected. If we see the top view of this one so we can see that either the cross section is not varying. There can be situations where there will be change in cross sections. But in this case we have considered that at this junction point, depth on left side and right side is equal.

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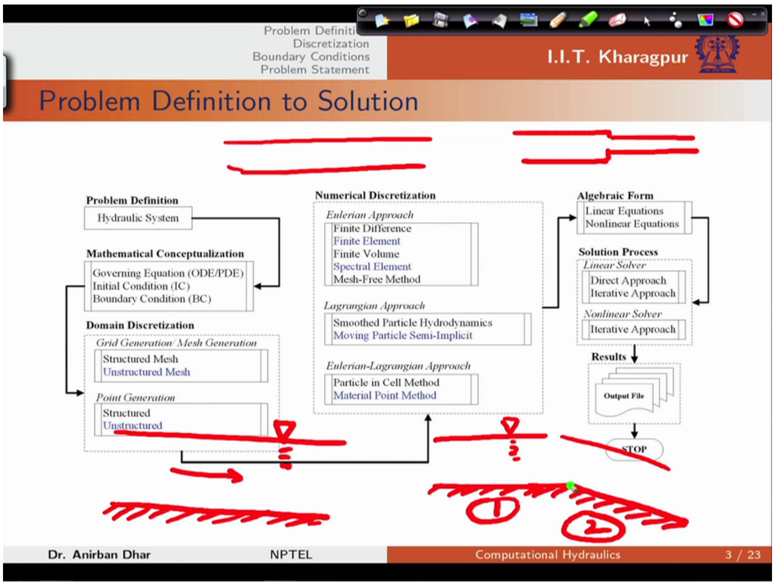

But we can have situations where if we have some hydraulic structures at this junction point then there will be two different conditions on the left and the right hand side. So let us see how we can solve this problem. Now first of all we need governing equation to solve this problem and we will try to solve this problem as BVP or boundary value problem and we will need these nonlinear equations or nonlinear equations will be generated as per our discretization process.

And finally we can solve this nonlinear equations using iterative approach for nonlinear equations.

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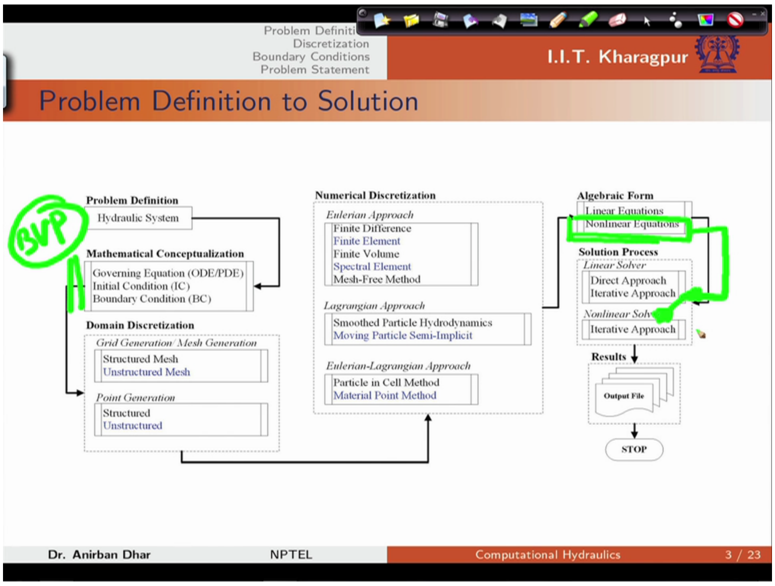

Now let us see what is the governing equation? Now this is boundary value problem. What is continuity equation? Continuity equation is dQ by dx is equal to zero. That means along the longitudinal direction there is no change in or the variation of Q is zero. So dQ dx. So in this case we will have another momentum equation dE by dx, this is minus SF. And E, this is the total energy in this case.

Y is a flow depth, z is the elevation head, alpha this is momentum correction factor, Q square divided by 2g A square. Essentially this is v square by 2g.

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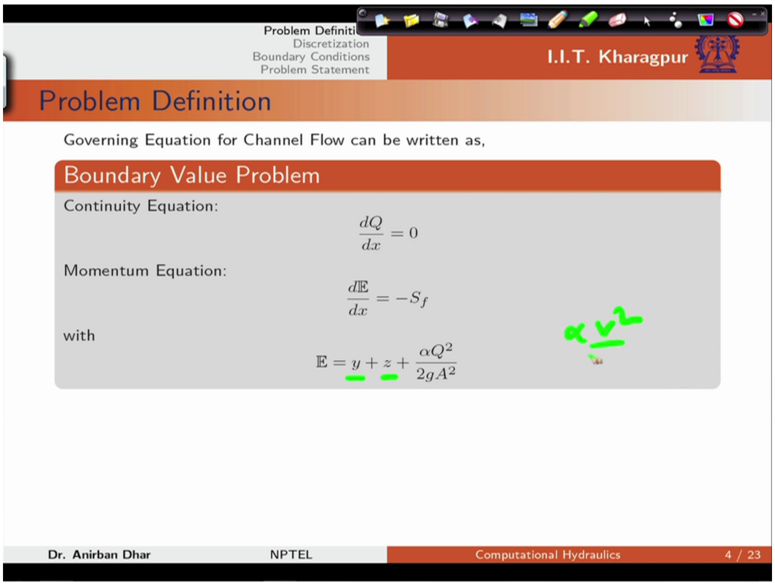

Now if you replace this v with Q by a we will get this term. So this is alpha Q square by 2g A square.

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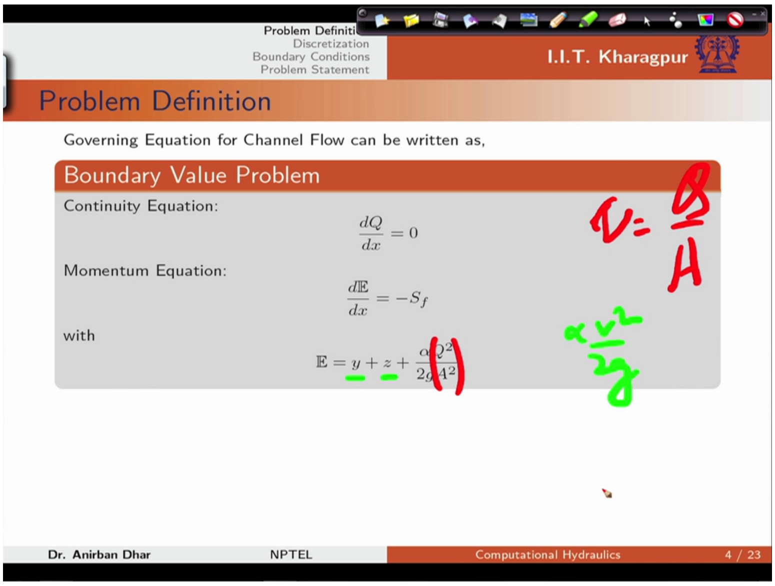

Now in this case these are the terms, y is flow depth. SF, this is friction slope. We can get this one from this expression where n is Mannings roughness. R is hydraulic radius, A is cross sectional area in this case and z, elevation of channel bottom with respect to datum. Alpha, this is momentum correction factor, Q is discharge, G is acceleration due to gravity.

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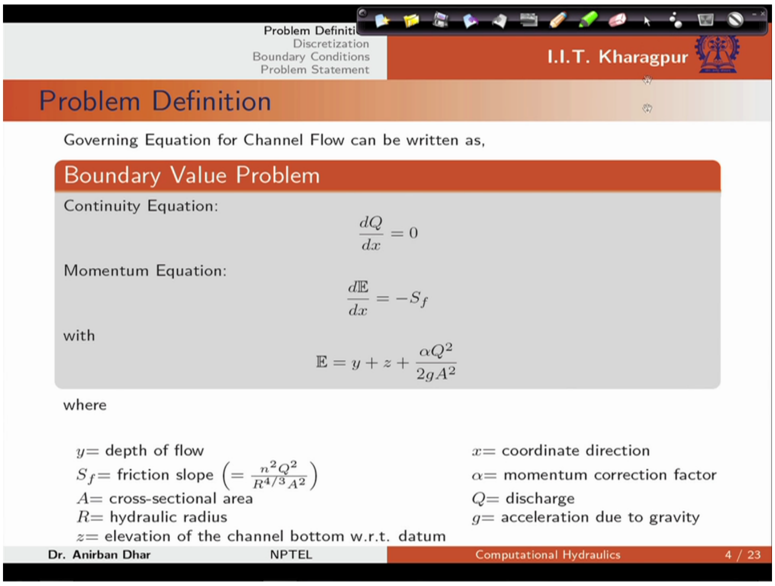

Now we need to define certain terms. First of all we will consider different channel reaches. So this is one channel reach that is under consideration. Now within this channel reach if we divide it in number of parts starting from 1 to NL plus 1. NL is the number of segments starting from 1 to NL here. So these are essentially number of segments. And this red dotted line, these are sections. So we have channel reach, segment, section.

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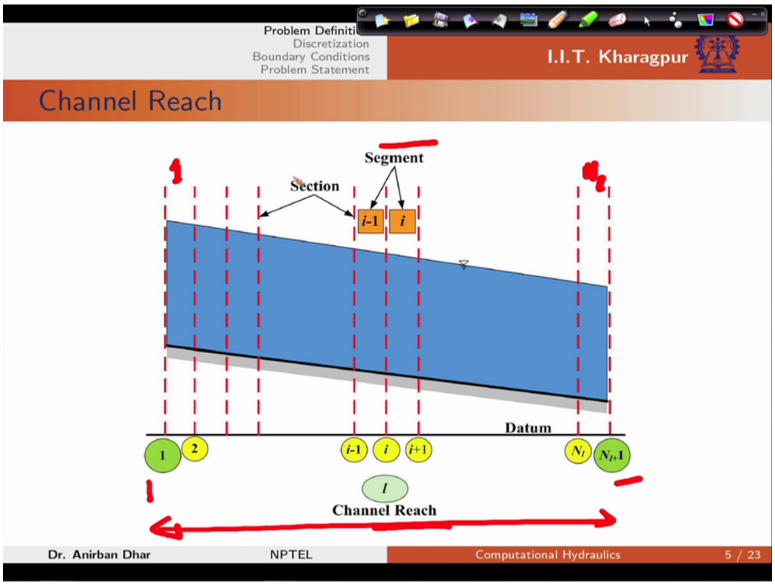

Now out of this all this total energy or E value is defined with respect to this datum. Now if you have a segment i that means it is connecting the sections i and i plus 1. So i this is section, i plus 1 is the section, i is a section and in this case i is our segment number.

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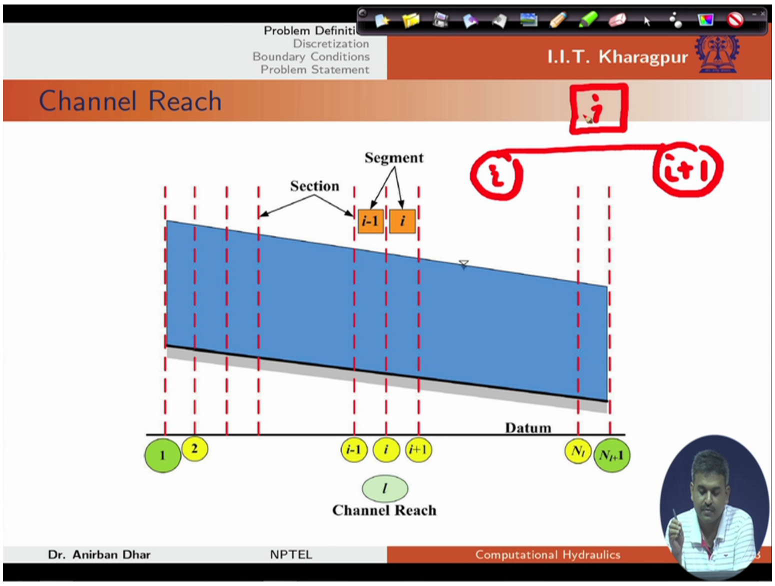

So this is a general notation that we will be utilising. And channel reach number we will denote it with this L, L is the channel reach number.

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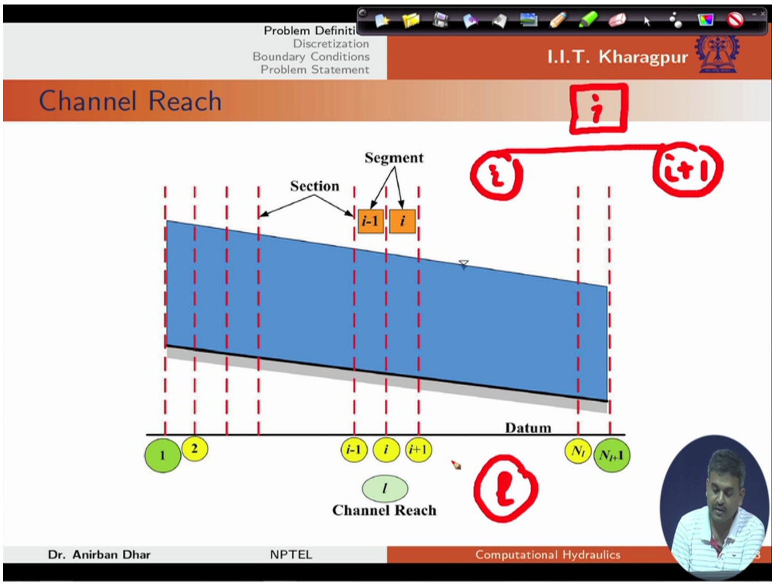

Now if we consider our general channel flow for the segment i with the sections i and i plus 1 then we can define each term on that energy expression or E. So first of all with respect to datum if we consider this section i, we have elevation head. So this is elevation head, then y L i this is our flow depth and on top of that we have kinetic energy head or this is alpha L i, Q L i square divided by 2g A L i square.

In this case this Q L i, this means that for Lth channel reach at ith section, what is the discharge? Similarly area L i, this means at Lth channel reach at ith section, what is area?

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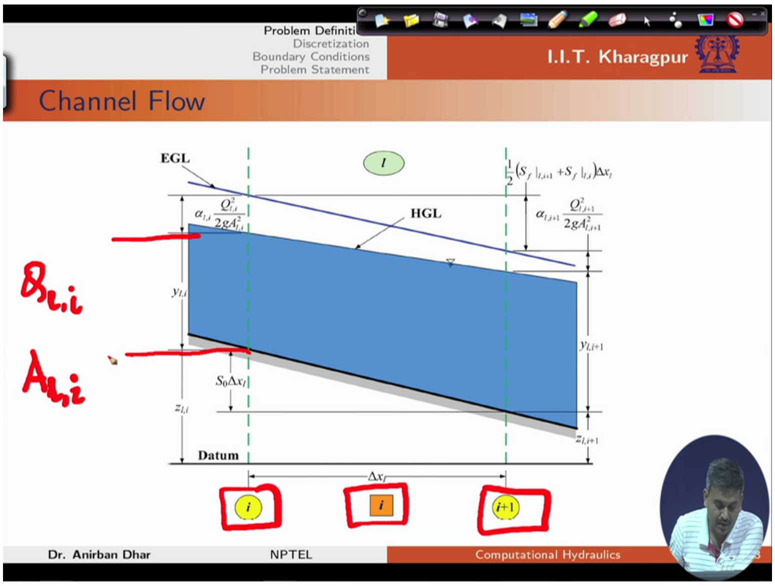

Similarly on the downstream end this ith i plus 1 section, we have this y L i plus 1, z L i plus 1 and this is again our thing. In this case we have alpha L i plus 1, this is Q L i plus 1 square divided by 2g A square. Again A should be at L i plus 1. So with this we have defined different variables or parameters for these two sections, i and i plus 1. Now this top surface is representing HGL or hydraulic grid line and this top one is representing energy grid line.

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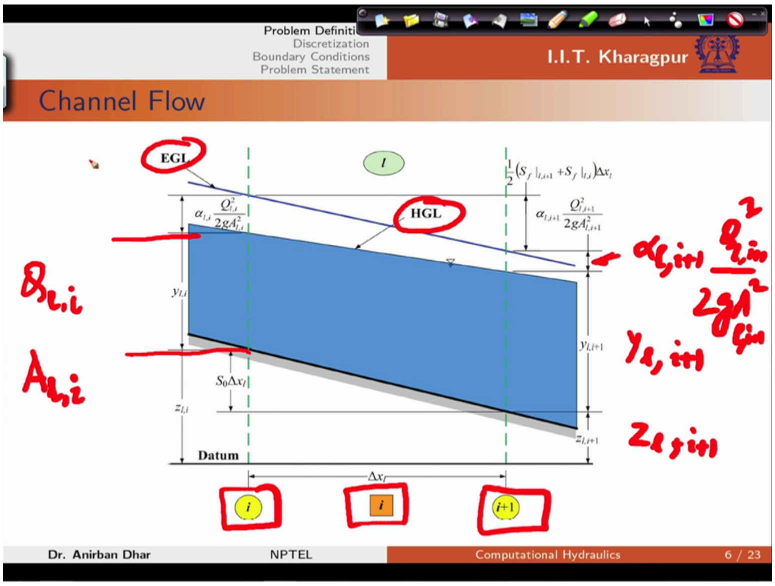

Now in this case we can see that there is difference between this point and this point. This is essentially the average of the friction slope multiplied by del xL. Del xL is the distance between these two sections. And in this case we will consider uniform del x for a particular channel reach.

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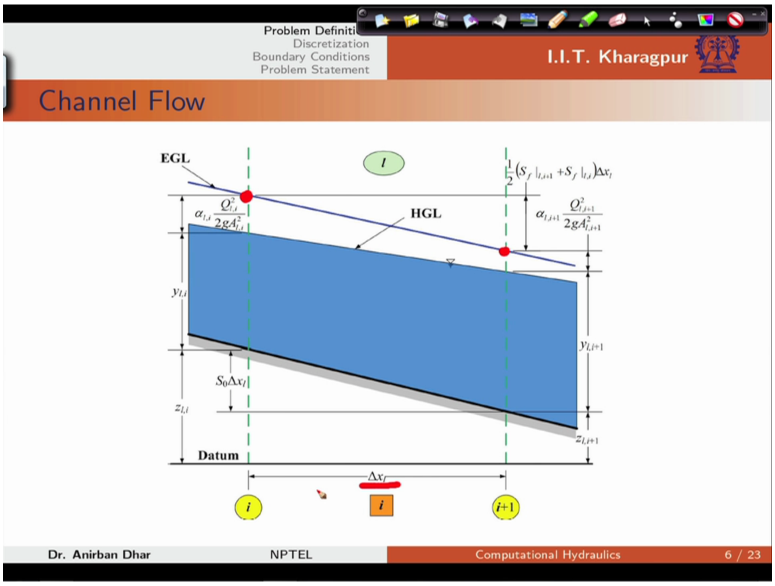

Now for discretization we will consider this continuity equation. And if we consider the ith segment of the Lth channel reach then this is dQ by dx. This dQ by dx essentially we are considering at ith segment only. Now for this case we can get this discretization, Q L i plus 1 minus Q L i divided by del xL, this is equal to zero.

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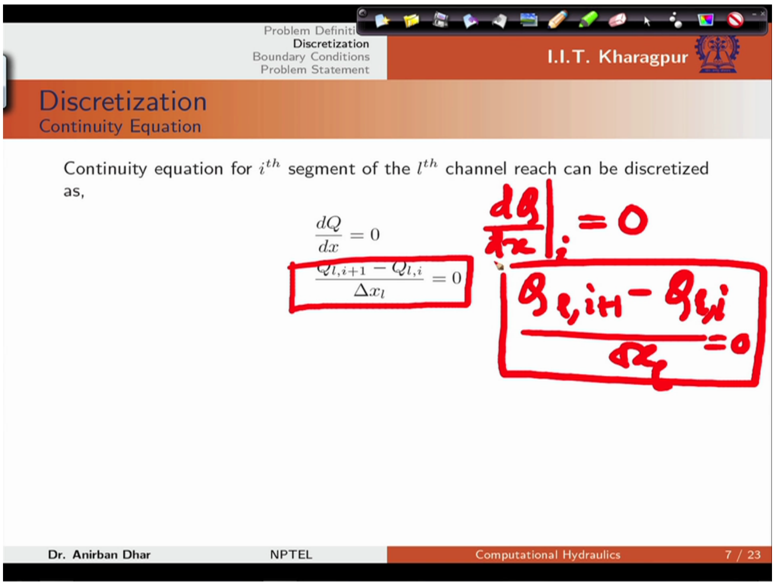

Now we can utilise this information during our solution process. Now L index for channel number and i index for different sections within a channel reach. In a simplified form for ith segment or the Lth channel we can write directly from this expression that Q L i plus 1 and L i, these two values are equal.

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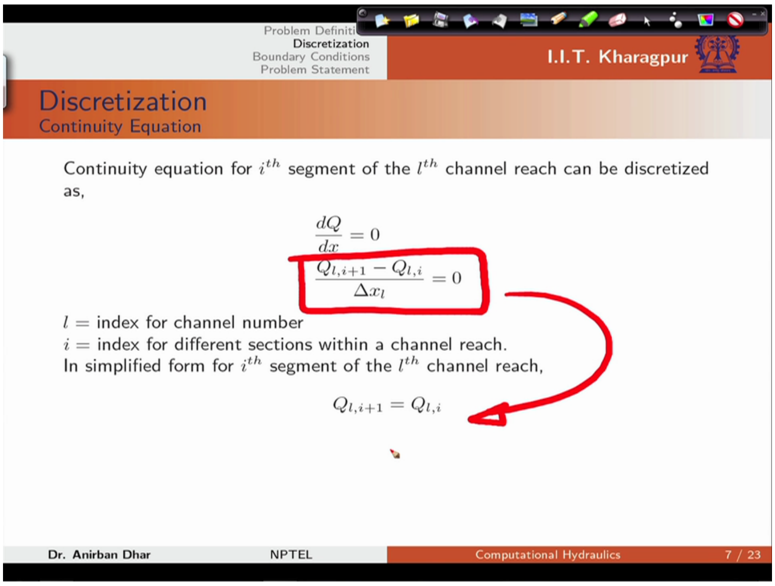

For single channel that means if you consider any channel L, for that channel we can have multiple sections. But if we consider the discharge Q L or inlet discharge for this channel L then this Q L 1 at section 1 to NL plus 1, NL plus 1 is the total number of sections for this Lth channel reach. So this is equal to Q L. So for all sections or discharge is equal and NL is a number of segments for this case. So in this case we are considering NL plus 1 number of sections.

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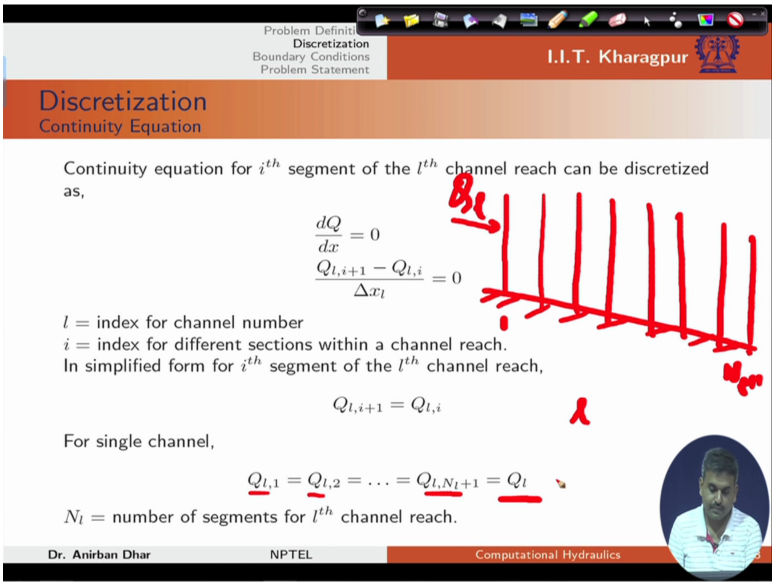

Now if we consider channel in series then also the situation will be same because if we consider two channel sections. Let us say this is our first one and this is the second one, 1 and 2. In this case we will have inflow discharge (shum) some Q1. And this case if we consider this discharge is Q2, so obviously in this case this Q1 should be Q2.

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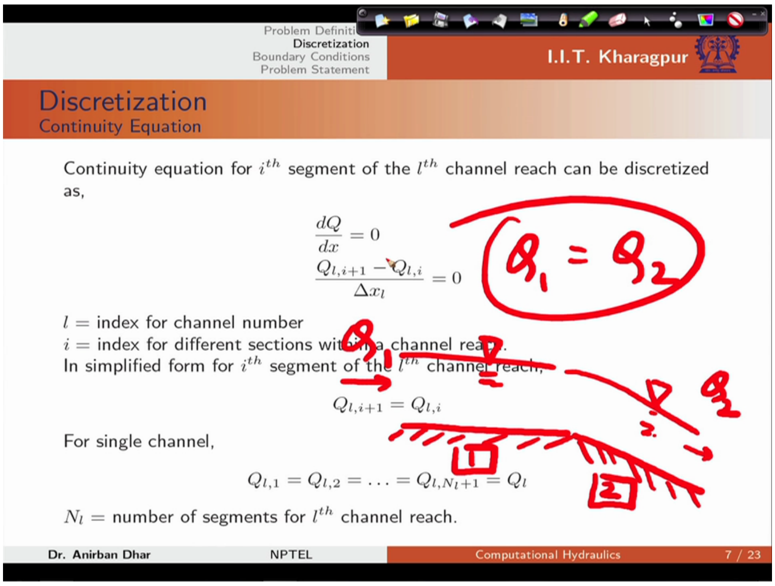

Now discretization of this momentum equation. Again we will consider the discretization for channel segment. So this is dE by dx at i, we will consider at that thing here. Now if we consider this discretization here, we can use simple our forward difference. So we are getting

E L i plus 1 minus E L i divided by del xL. L is the channel reach number. And minus we have SF in the right hand side.

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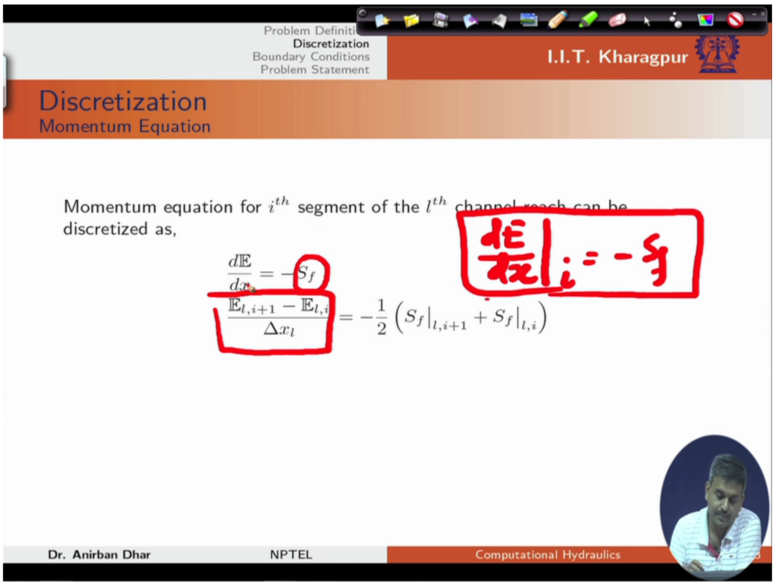

So we can get SF i value at channel section i and SF i plus 1 value at channel section i plus 1. So if we take average of this two we can represent the right hand side term.

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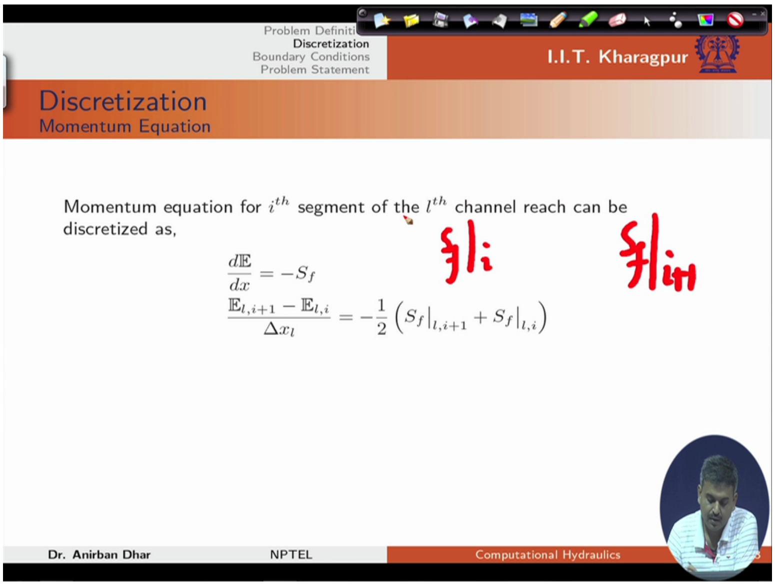

So further we can expand this. So expansion is done by putting E equals to y plus z alpha Q square 2g A square. Now in this case we can see that we have number of terms available here. But if we closely consider the parameters at different variables we can easily identify the unknown variable. So unknown variables in this case is y because for single channel or channel in series we have constant discharge condition.

And discharge is also not varying with time. So we will have a constant value of discharge for all channels and those values are equal.

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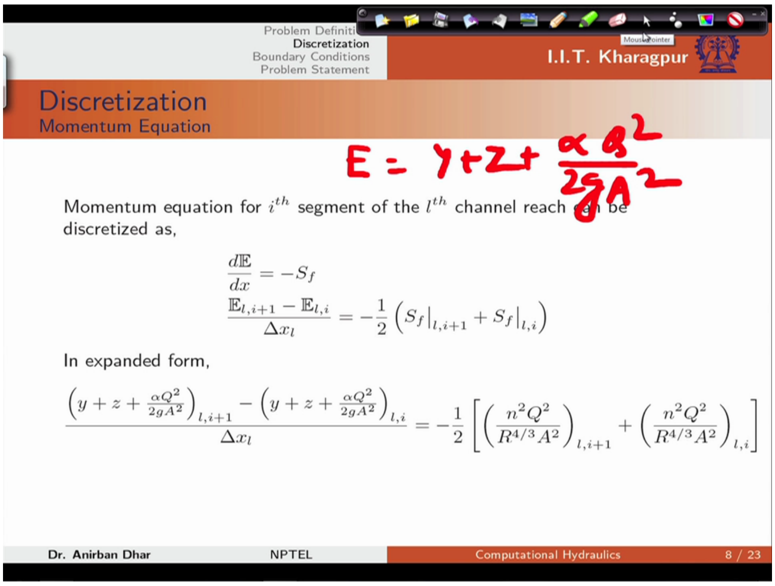

So in a particular problem z is known, Q is known, A which is again function of y in this case also y unknown, z is known. Only unknown is y and A is the function of y only. On the right hand side we have another value of this hydraulic radius which is again function of y. And we have discharge value constant n which is Mannings roughness coefficient that is also constant.

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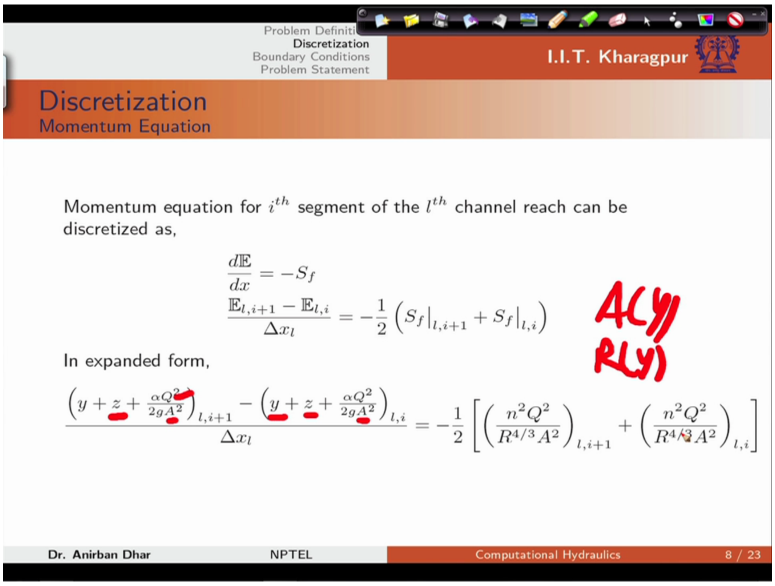

So in functional form this is again nonlinear equation. So we can write it in functional form for solution because we need to utilise Newton Raphson technique. For Newton Raphson technique first step is that we need to represent it in terms of some F phi where phi is a vector equals to zero. So this expression we need to generate.

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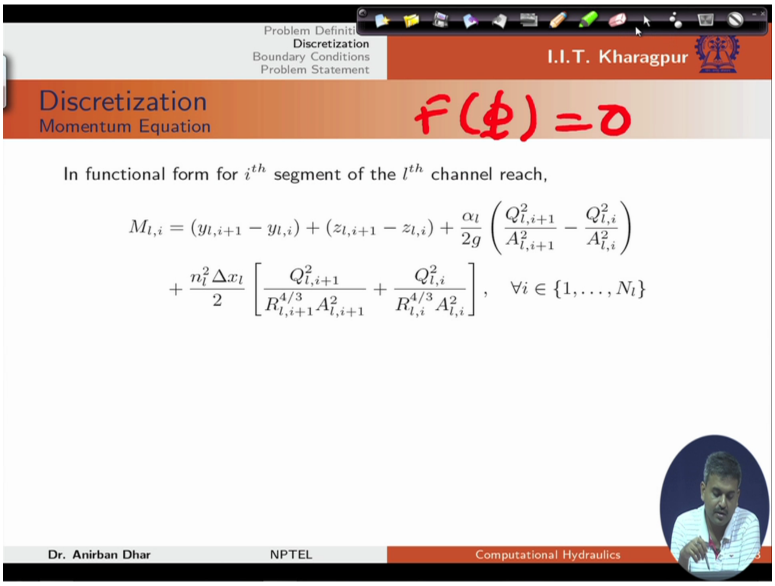

So for any segment we will get this M L i or momentum L i function and this should be equal to zero. And this thing is valid for 1 to NL number of segments. We have NL plus 1 number of sections but NL number of segments. So these equations will be there for NL number of segments.

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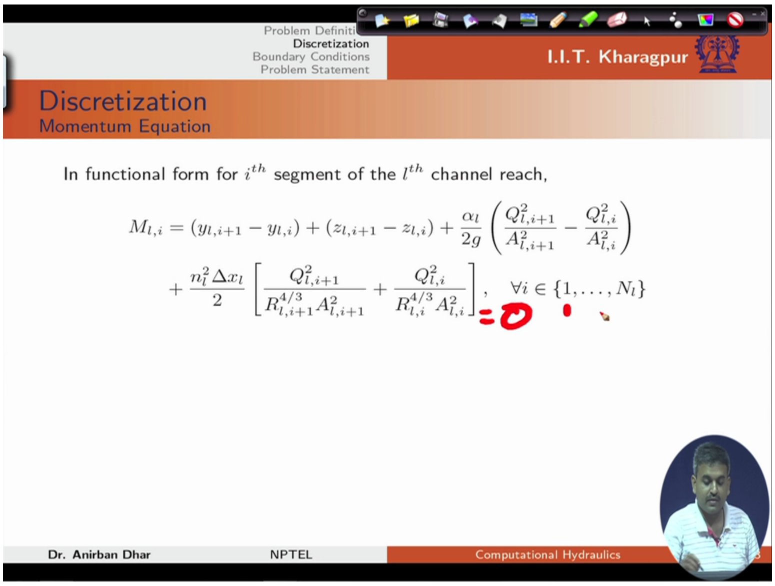

Now in reduced form because we know that for single channel or channel in series we have constant value of Q L. So we can take out this Q L square out of this right hand term. Now we can define two parameters because these are channel reach dependent parameters for our case. And R and A, these are functions of y. This is again a constant term for two consecutive channel sections in this case.

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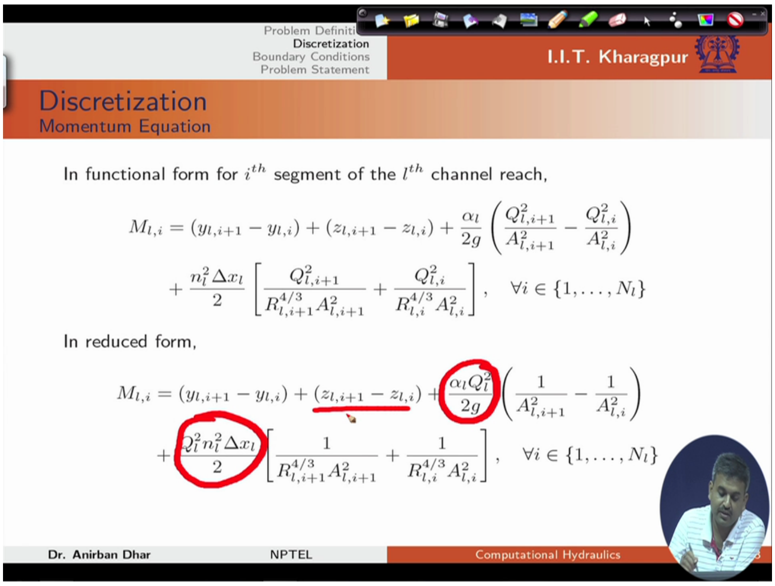

Now we have NL nonlinear equations with NL plus 1 unknown flow depths. In this case if we consider a single channel then we are starting from 1 and going up to NL plus 1. But in this case we are generating M L i equations for segment 1, segment 2, segment 3 up to NL. So we will have in NL number of nonlinear equations. Like M L i equals to zero. But we have unknown flow depths for all nodes. So these nodes 1, 2, 3 up to NL plus 1. Now we need one extra equation to solve this particular problem.

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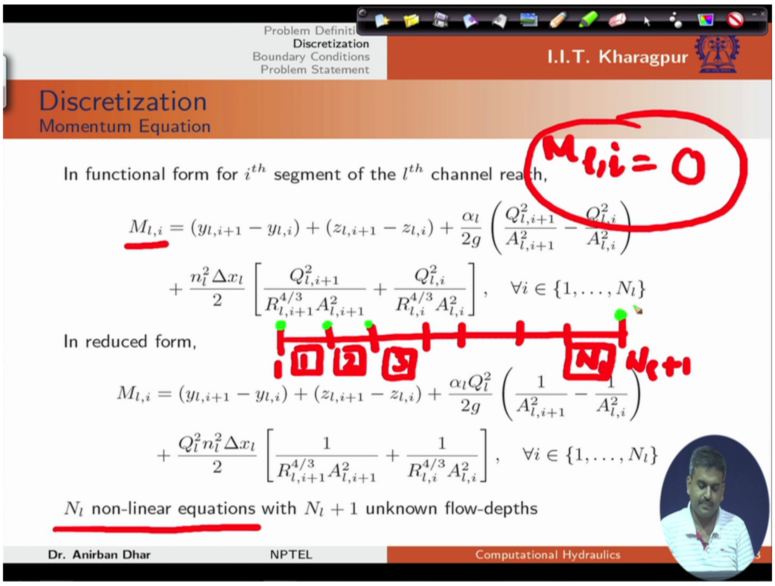

So one condition that we can utilise for subcritical flow that is at the downstream end we have specified depth conditions. So DB L that means downstream boundary condition for Lth channel reach at NL plus l segment, this is equal to is equal to y L nN L plus 1 minus yd. Yd is specified value at downstream end.

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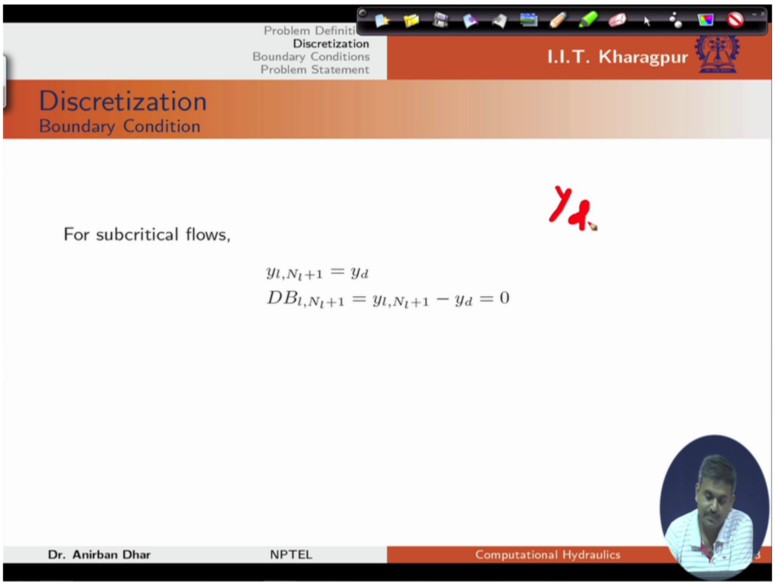

But if you have supercritical flow condition this will be y L 1 yu. That means at the starting of the channel or at the first section itself our depth is defined. Then we will have this upstream boundary condition. So upstream boundary L NL plus 1 because we already have

NL number of equation. This is NL plus 1 number equation in this case. So y L 1 equals to yu, this is for (up) downstream boundary, this is for upstream boundary condition.

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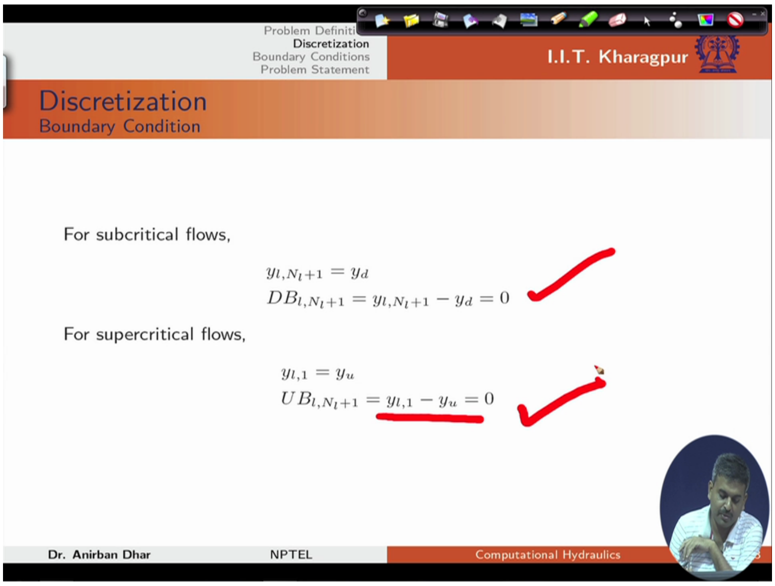

Now we already have our discretized momentum equation. In this case for each of those equations we have two unknown variables. One is y L i plus 1 another one is y L i. So for solution process using Newton Raphson we need define the coefficients or coefficient matrix which is again a Jacobian matrix. Now in this case by assuming this term as C1 and the second term as C2, we can reduce the size of this equation. At least in compact form we can represent this.

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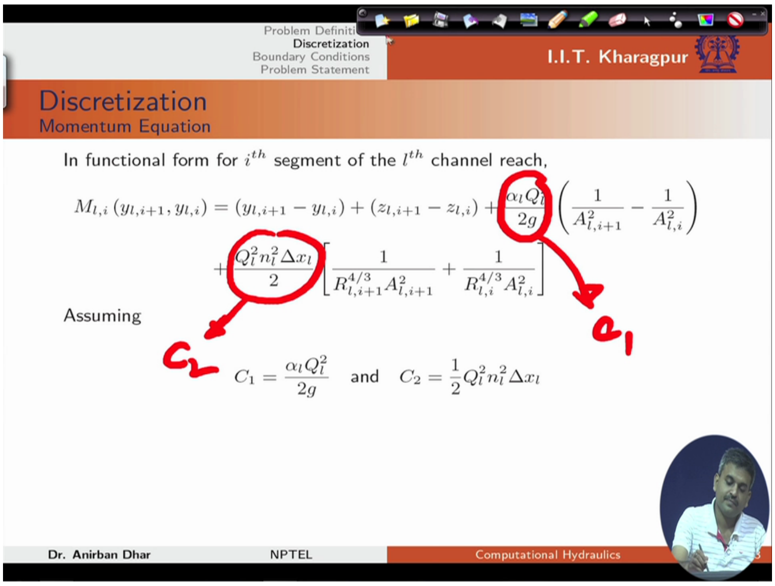

Now with this compact representation now we have all the variables on this right hand side of the equation. And R A, these are variables of y L i plus 1 and y L i. Essentially in this case R L i plus 1 this is function of y L i plus 1. Again area L i plus 1 is function of y L i plus 1. And this hydraulic radius at L i, this is function of L i. A L i is function of A L i. Now we have these terms in our equations.

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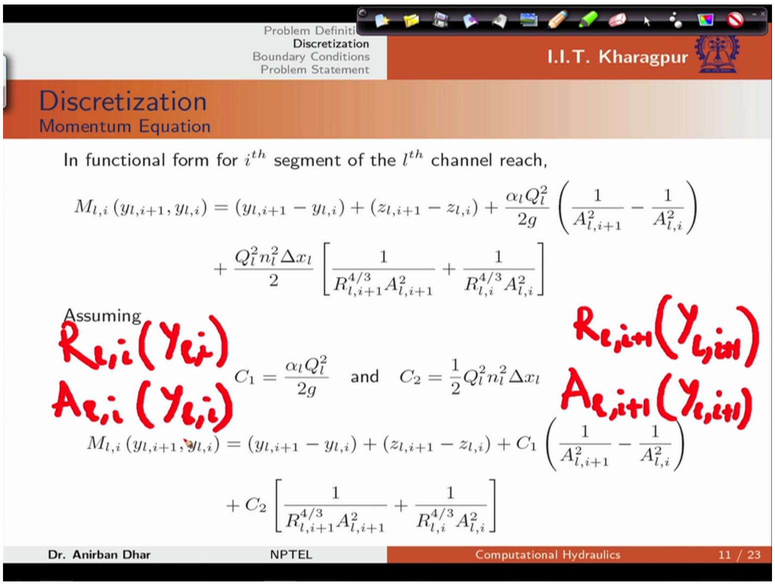

If we take derivative to get the coefficients or elements of Jacobian matrix then from our momentum equation we will get two terms because we have two variables. So one is corresponding to y L i another one is corresponding to this y L i plus 1. And C1 C2 these terms we have already defined.

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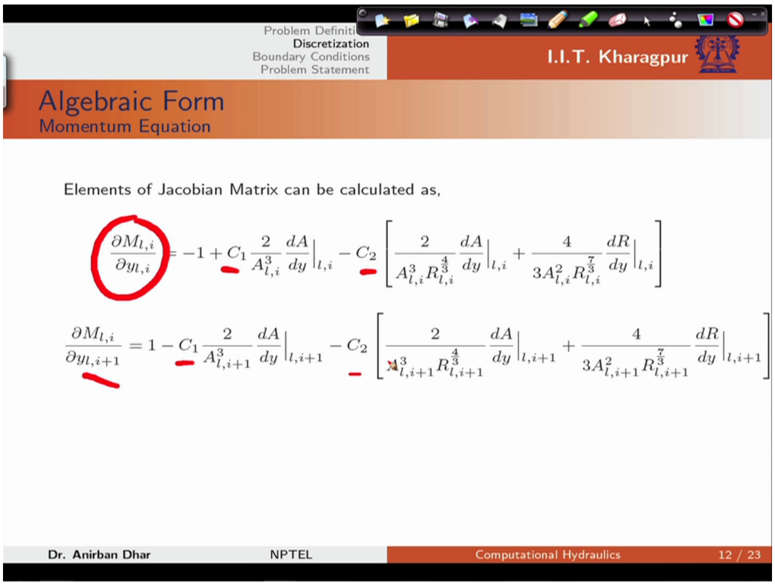

Now in this case for any general prismatic cross section we need to find out what is this dA by dy? And it should be evaluated at L plus 1. Similarly dR dy L i index. It is dA by dy at L i. Similarly for the second term we will have dA by dy at L i plus 1 and dR by dy at L i plus 1.

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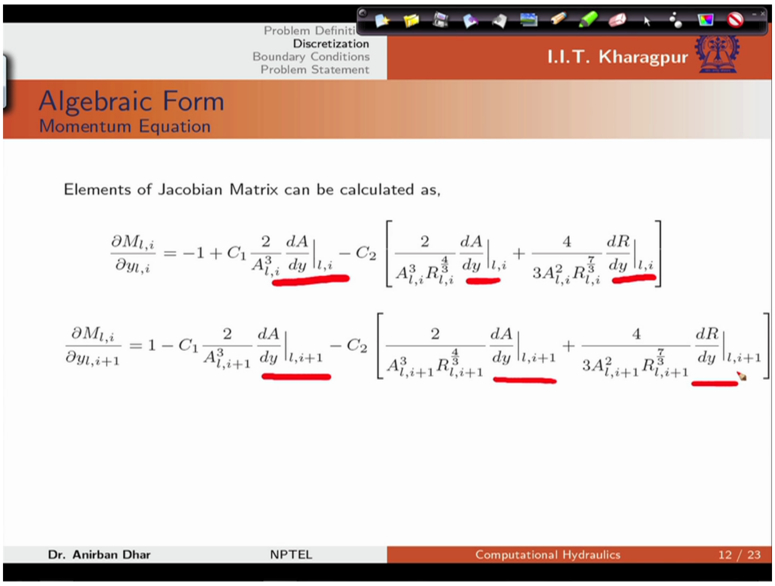

For general channel cross section this dR by dy, this is T by P. T is the top width of the channel. P is the weighted perimeter. R is the hydraulic radius divided by again weighted perimeter dP by dy in this case.

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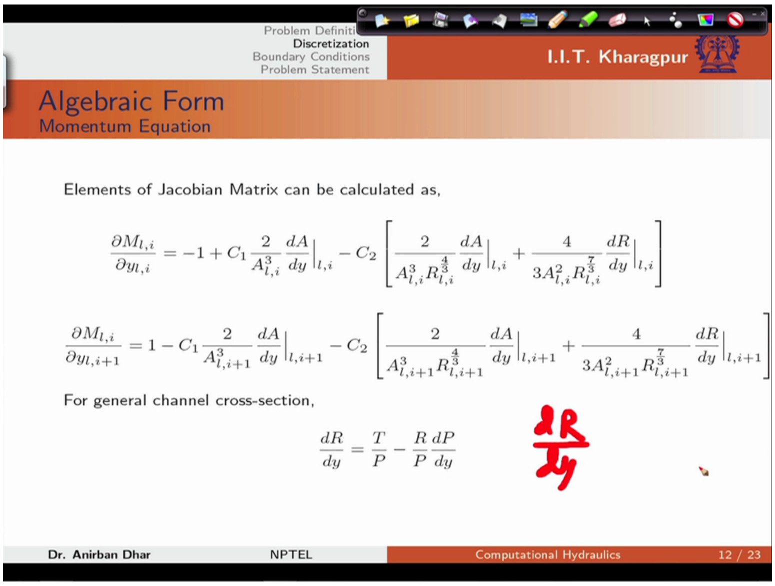

Now for subcritical flow again in case of subcritical flow we have seen that the condition is y L NL plus 1, this should be equal to yd. If I transfer this yd on the left hand side, this is zero. And this is our downstream boundary condition or DB L NL plus 1 equal. Now if I take derivative with respect to first y L NL, so I should get zero here and in this case I am getting 1 because this is only y L NL plus 1.

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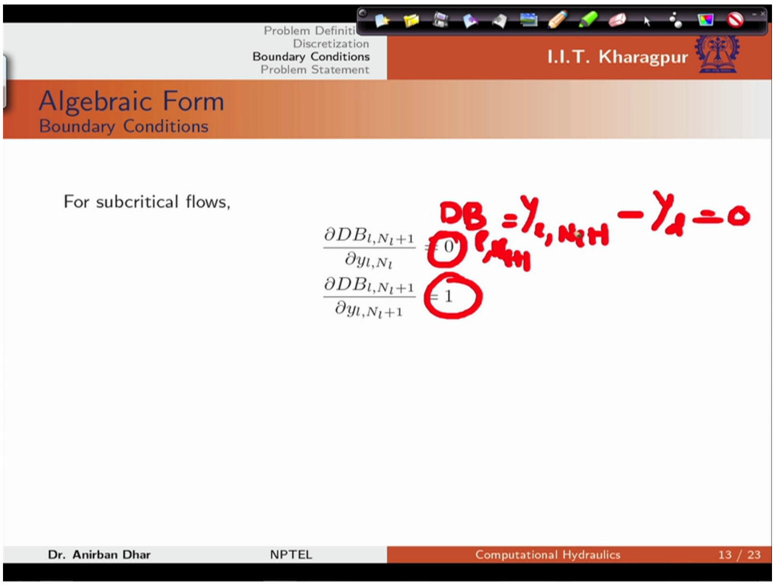

Now for supercritical flow condition, similarly if we have this y L 1 equal to yu and we can transfer it on the left hand side upstream boundary L NL plus 1 equal. Now in this case if we take derivative with respect to y L 1, so obviously we are getting 1 here and for second term we are getting zero.

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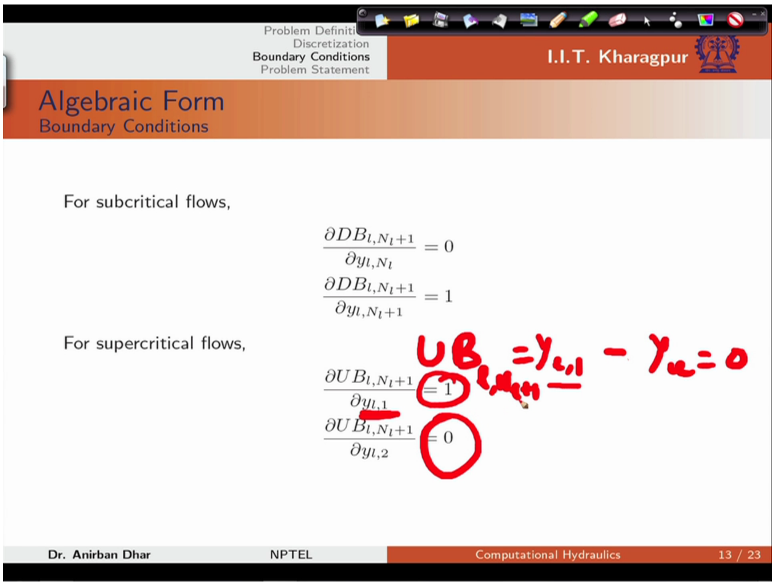

Now in general form the governing equation including boundary condition can be written as, this coefficient is there. The coefficients which we have calculated for our Jacobian matrix and this delta L i and delta L i plus 1, these are the increment values. Now in this case this del M L i, this is again second term in the Jacobian matrix. And these are the equations.

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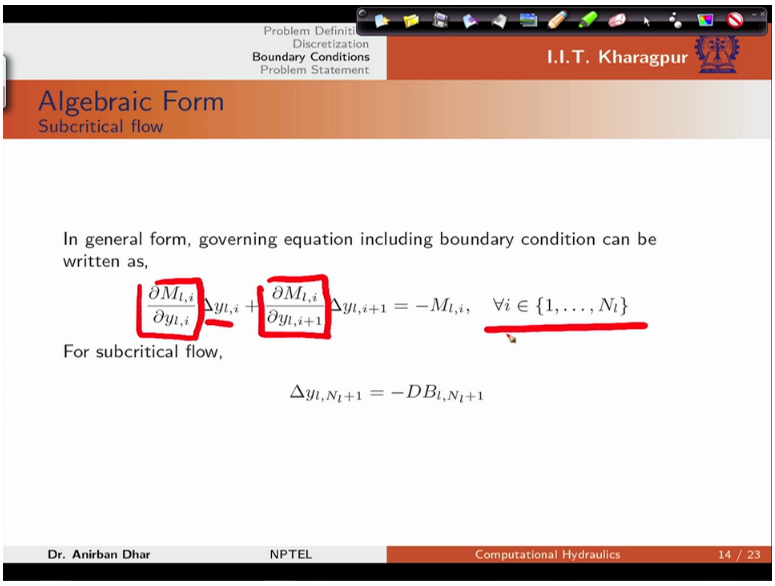

So starting from 1 to NL segment we have this equation. This is the thing that we can directly solve but we need another boundary condition here that is subcritical flow condition. In this case coefficient is 1. That is why I have not written there.

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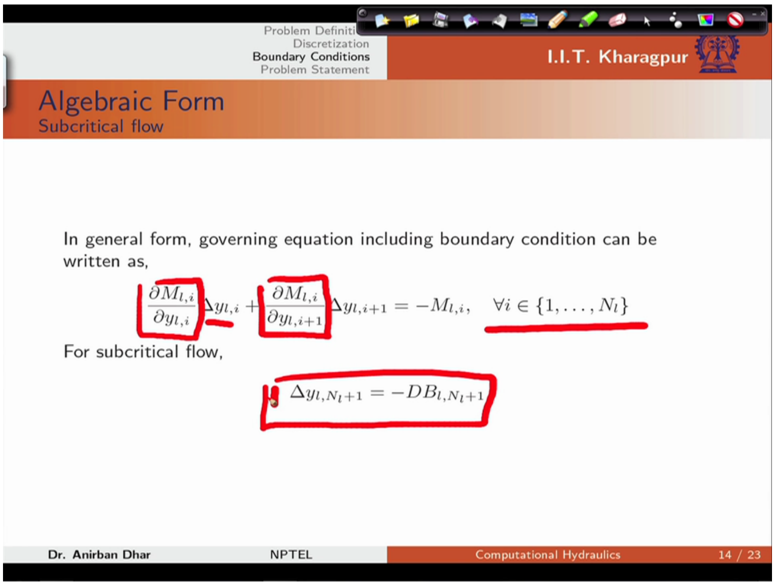

Now after getting the solution through iteration, at every iteration we need to update the values. So update thing we can do like this, y L that means y L vector at Pth iteration, this should be calculated at y L P minus 1. And this minus sign is already there.

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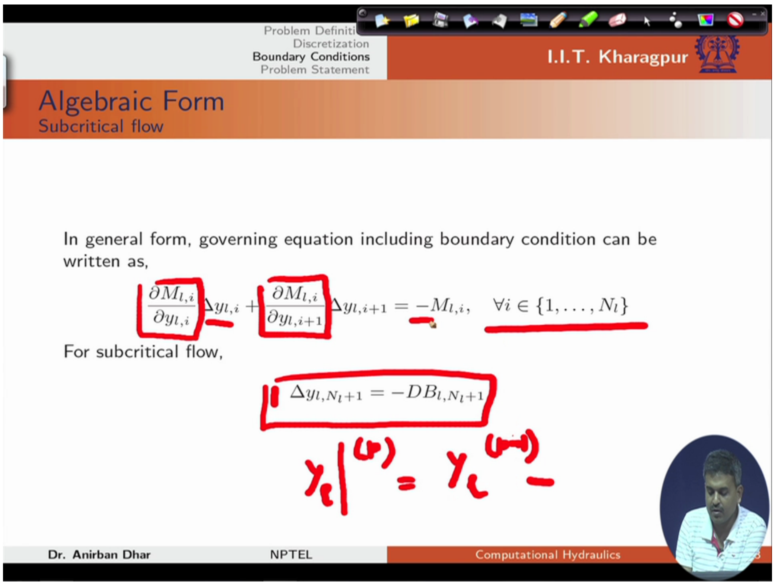

So if we get some solution out of this so we should add this directly. So this is again based on the values at P minus 1. So after getting the solution for this del values here, we can add that value with the guess value or previous iteration value and we can get the updated value. And we can iterate it until and unless we get the convergence for Newton Raphson method. And finally we will get the solution.

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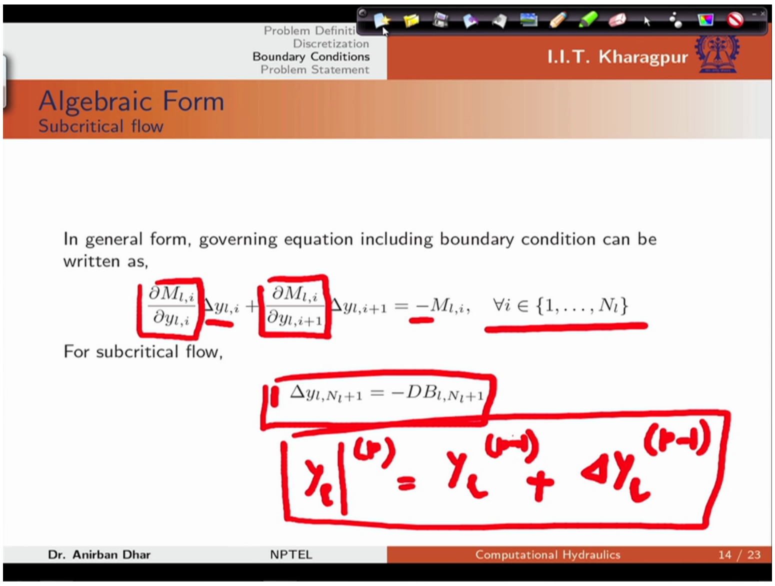

So what is the matrix structure? We have i, i plus 1 in this case. So we have one diagonal term and another one which is banded one and the green one is that. And last dot is for the boundary condition that we will be utilising for our case.

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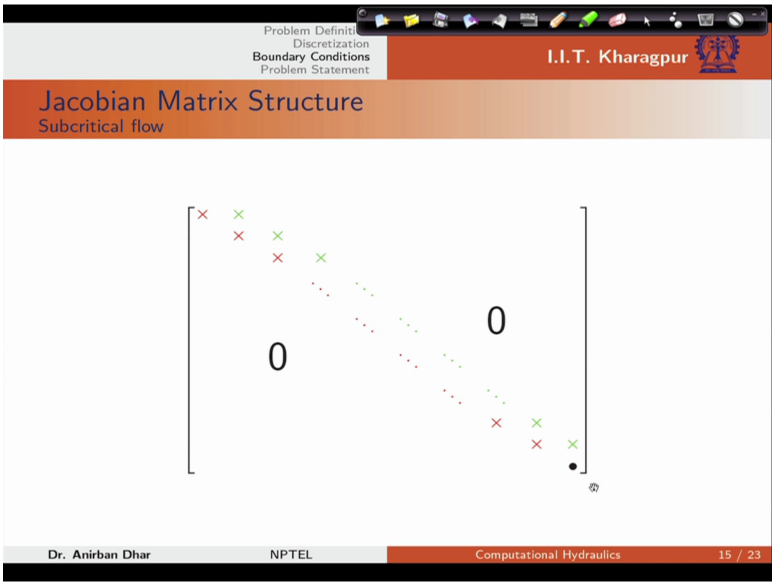

Now for supercritical flow condition the situation is same. But in this case we have first condition available here or condition which is available at the first section.

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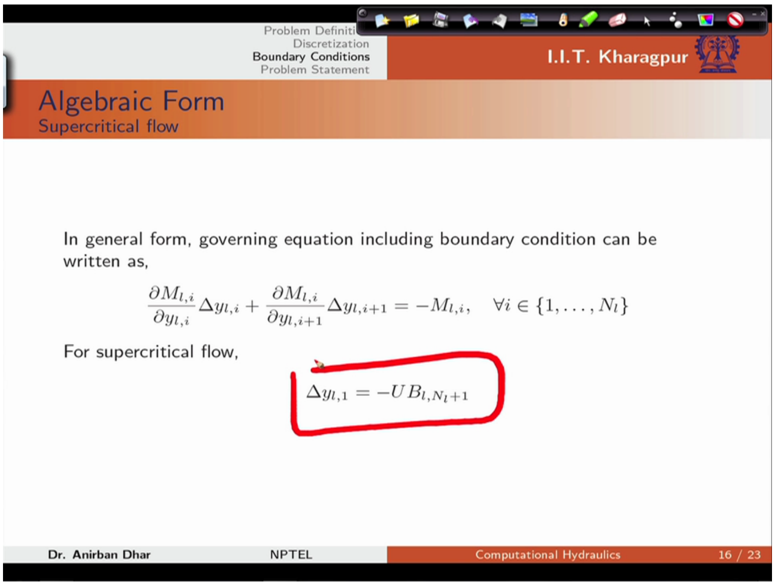

Now the structure is like this. In supercritical we have this structure of the matrix.

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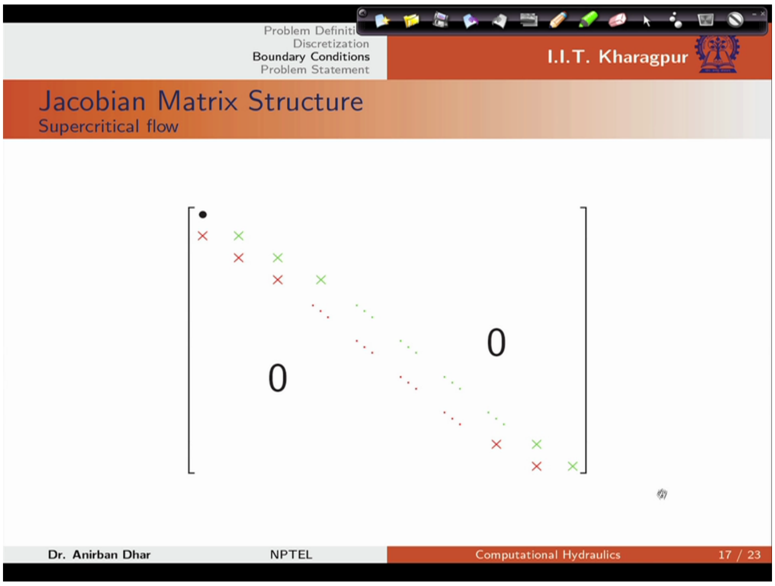

Now if we have channel in series, obviously our discharge will be same for both the cases and we need to satisfy the energy condition at junction point. Now if we have two z values available for this section then for these two channels at this section so we can consider that z values are different. Otherwise if both the points are the same points, we can neglect this elevation thing and we can just equate the flow depth here.

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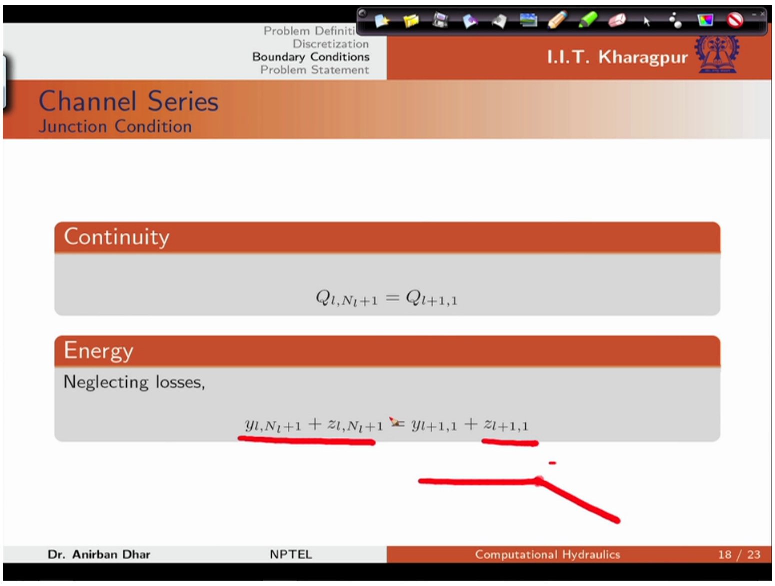

Now let us consider a problem where we have a given cross section type rectangular, B which is a width of the rectangular section that is 15 metres. G is 9 point 8 metre per second square. S not, which is point 0001. Mannings roughness, point 015. Lx is 200 metres. Q is 20 metre cube per second and yd which is depth at the downstream boundary is point 6 metres. Now required is, estimate the flow depth across the channel reach.

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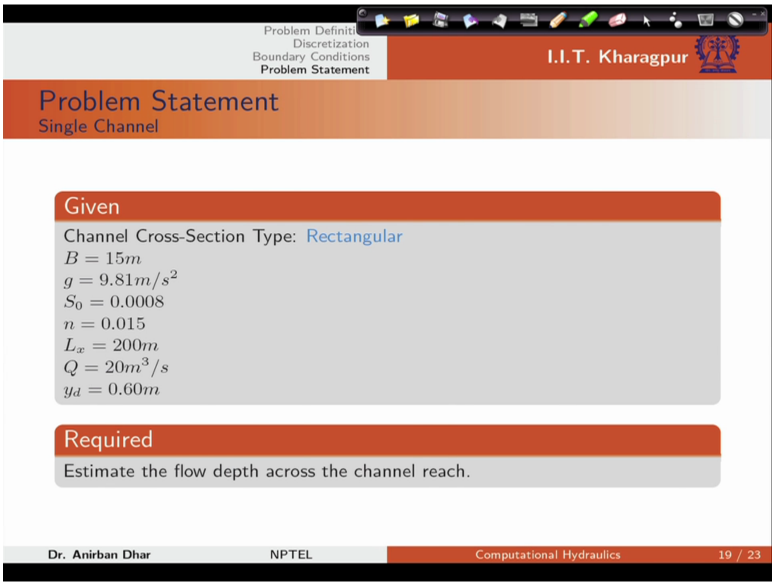

Now this problem is similar to our problem that we have solved in GVF case. In that case the critical depth was around point 567. Now what is the downstream boundary condition here? This is point 6. Now it is just above the critical depth. And with this condition if we simulate using this boundary value problem so we should get similar profile in this problem also. So if we consider this one, for rectangular channel this dR by dy, this is B square by B plus 2y square. And top width is B. Now we can utilise this information here.

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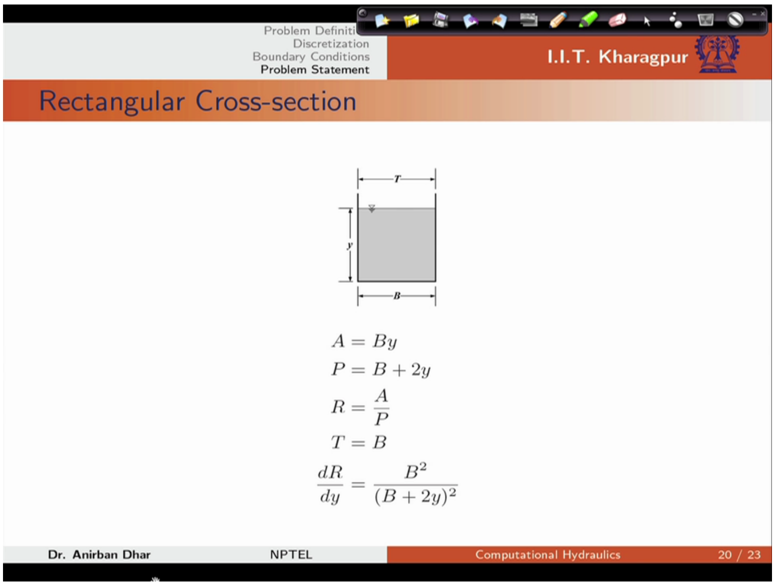

Now if we take our scilab. Again I have written one code so that we can simulate this channel flow condition. So first is 1D channel. This is single channel. Now this information like Q equals to 20, S not equals to point. These are given information.

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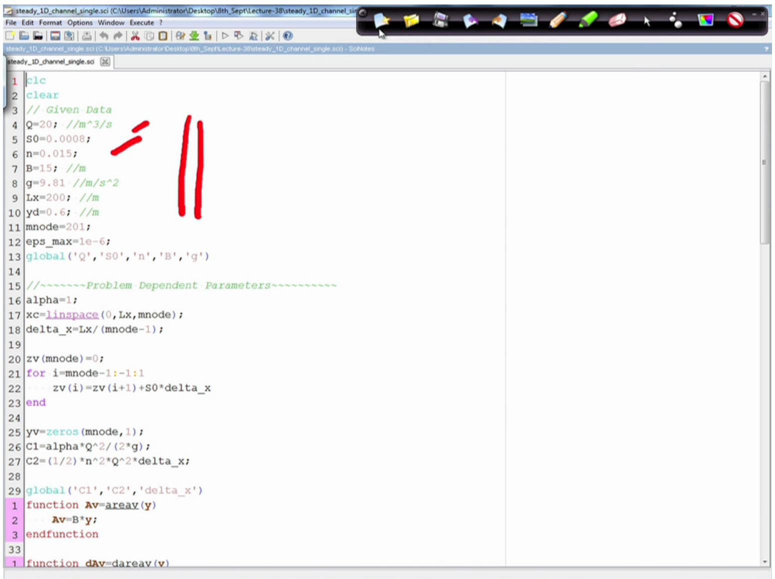

I have considered mnode equals to 201 like our previous problem that we have discussed in previous lecture class. Then epsilon max, this is 1 into 10 to the power minus 6. This is required for Newton Raphson method. Global we have constant discharge value S not, n, B, g. These values are available. So we can solve this.

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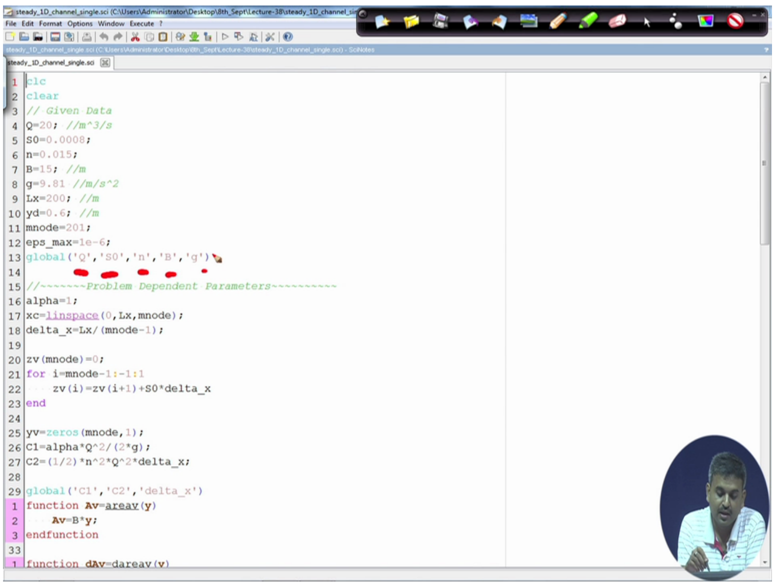

Now what are these problem dependent parameters? Problem dependent alpha, we need alpha. For single channel I have considered alpha equals to 1. Xc which is linspace between zero to Lx and mnode that means mnode number of nodes which is 201. This del x is Lx divided by mnode minus 1. That means it should be divided by the number of segments. Now z values, now z value is zero. I have considered at the n section from datum. And from there we can just add S not into delta x to get the z value for other sections.

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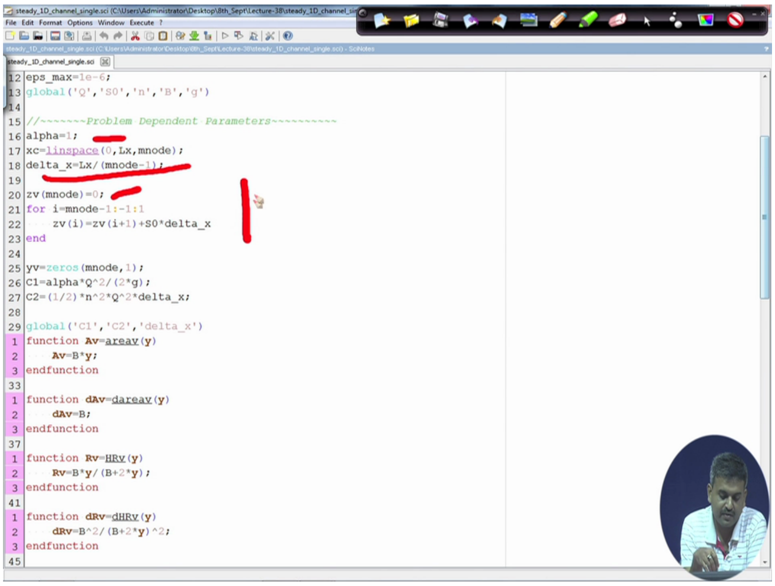

Now yv which is a variable, this is defined for mnodes. That means NL plus 1 number of nodes.

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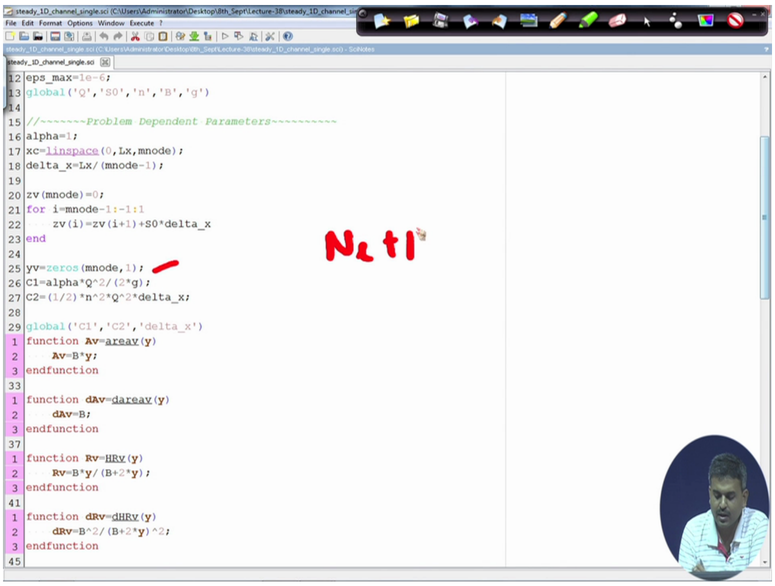

Now in this case we can define this C1 and C2. C1 and C2 these are constant parameters. Now interesting to see that for rectangular channel, area is By. DA by dy, this is B. R which is By divided by B plus 2y. And dR by dy we can directly calculate using this one.

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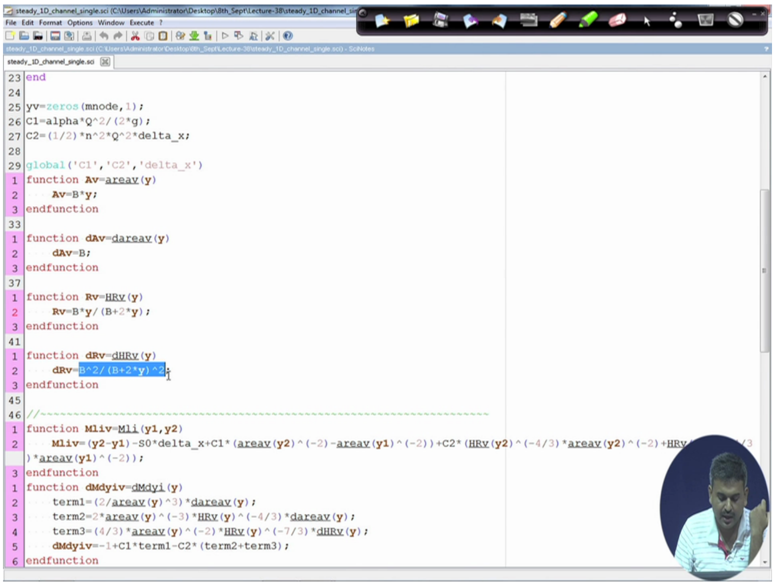

Now we can define the equations for each segment. For each segment we need to calculate this M L i. This value is required for the right hand side for the solution process.

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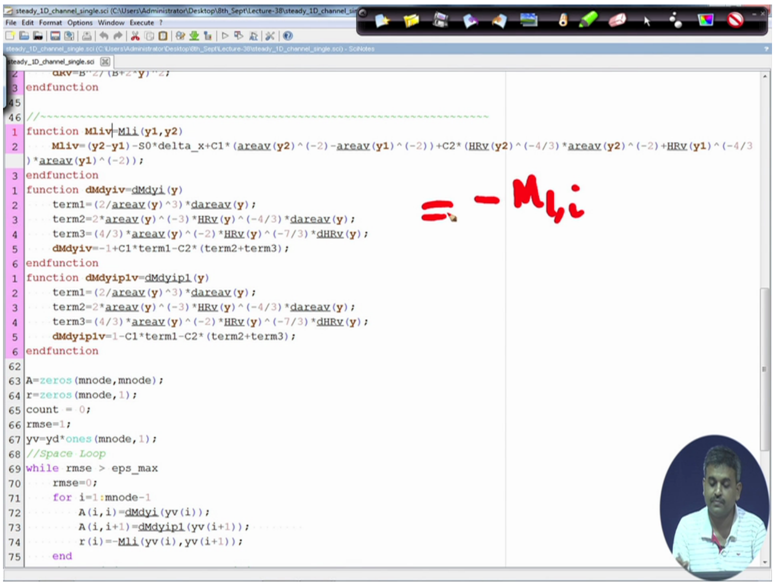

And this is dM by dy L i. And this is the dM by dy L i plus 1. So these two values are required for our calculation.

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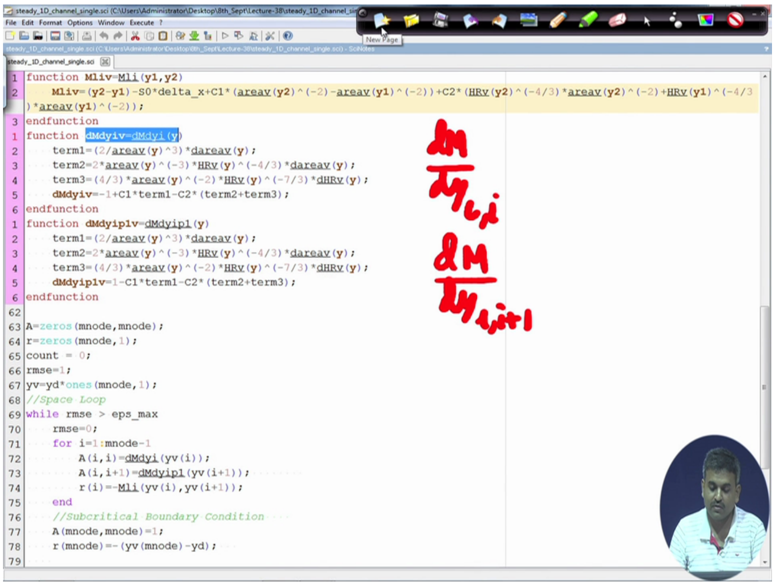

Now we can start with our Jacobian matrix. This is zeros mnode, mnode. And r right hand side we have zeros mnode 1. Now with the count I am starting zero rmse is equal to 1. And guess value I am starting from the downstream end value which is yd for all values or all sections.

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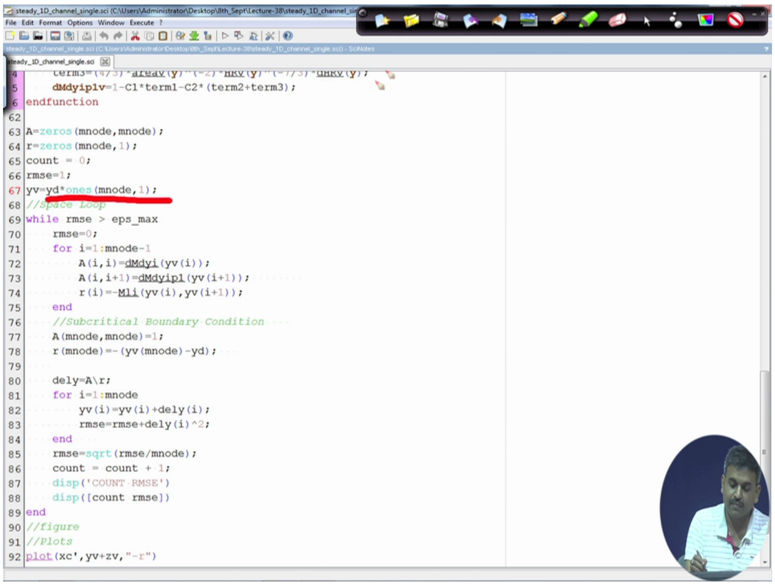

Now this rmse is greater than epsilon max. Then only we should iterate. So after entering into this loop I have used this rmse equals to zero.

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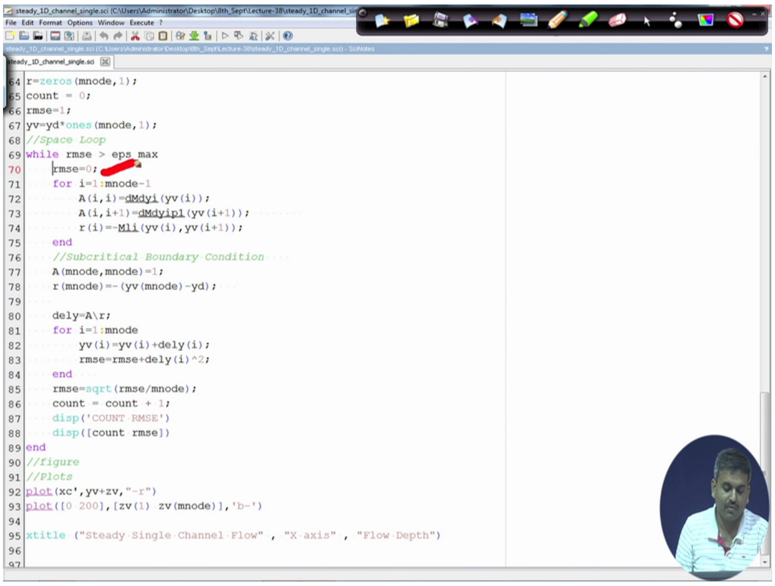

Now from there we are getting this i equals to mnode minus 1 and A i i which is our diagonal term, this is dm by dy i. And A i i plus 1, this is dm by d i i plus 1 and r i which is minus M L i. I have generated these equations and I need the boundary condition.

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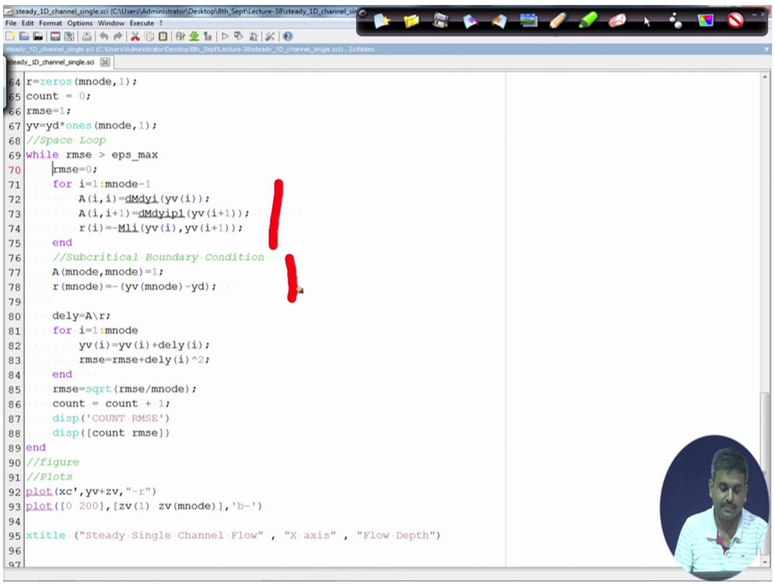

Now finally I need to solve this delta y. After solving this delta y using A backslash, this is internal function of scilab. I am utilising it for del y calculation.

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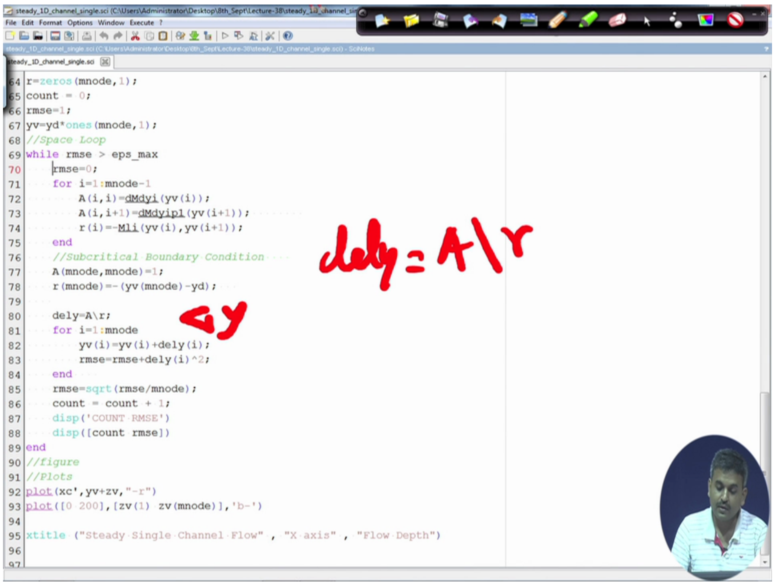

After getting this we need to update this yv and simultaneously we can also calculate this rmse value and finally we can get this rmse value and i count here.

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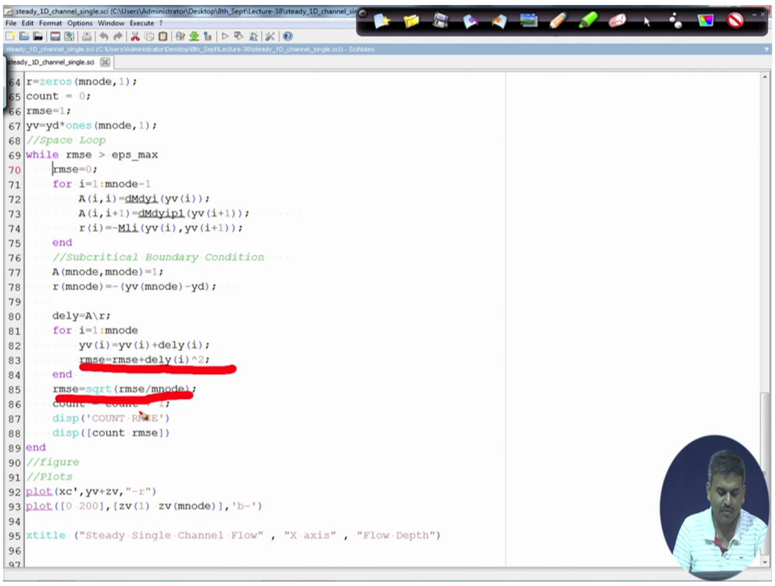

So now if I use this xc which is the x coordinate, yv and yz the zv. Zv are the z values for each (loca) section. We can generate the GVF curve here. And interesting part here is that this is like bed profile. And this is point 6.

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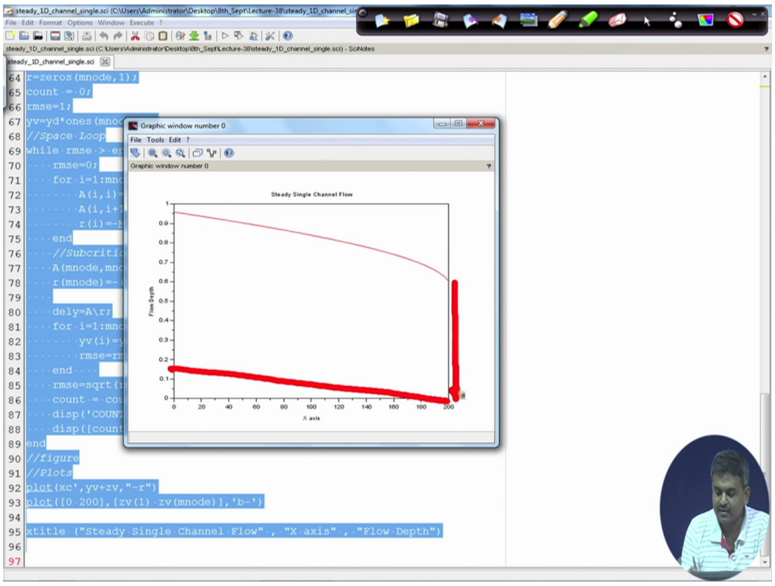

And again we are getting some value here. This is the value that we are getting out of this. And in this case upstream section we have value which is more than point 95.

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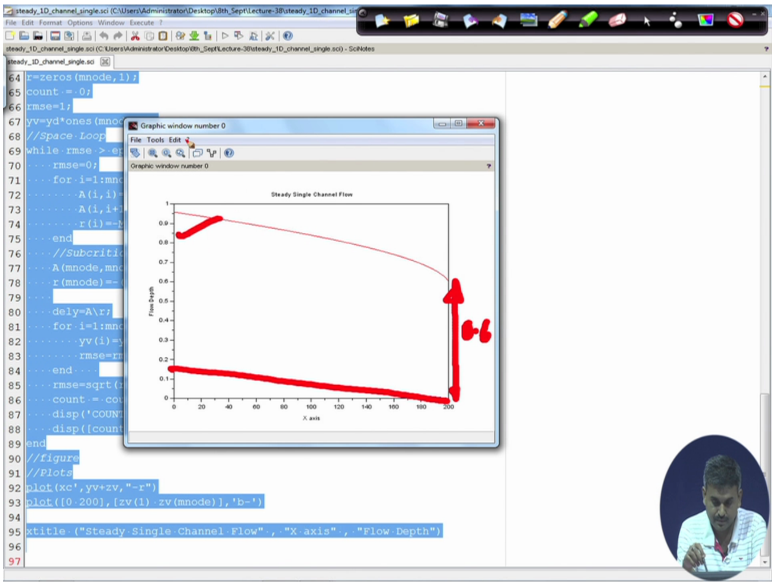

Now if you see our channel in series we have rectangular channel. And let us say that if I divide the same 200 metres length into two parts, 100 metres and 100 metres with point not 4 to point not not 8, we can get the profile there.

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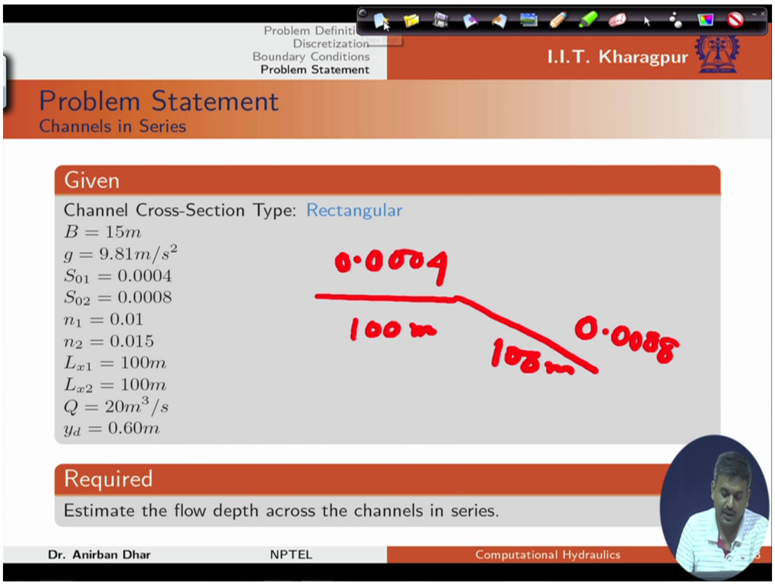

And both the cases n1 n2 values are this and L1 is 100, L2 is 100. Discharge is same because we need to utilise that. And yd, this is in the downstream point 6.

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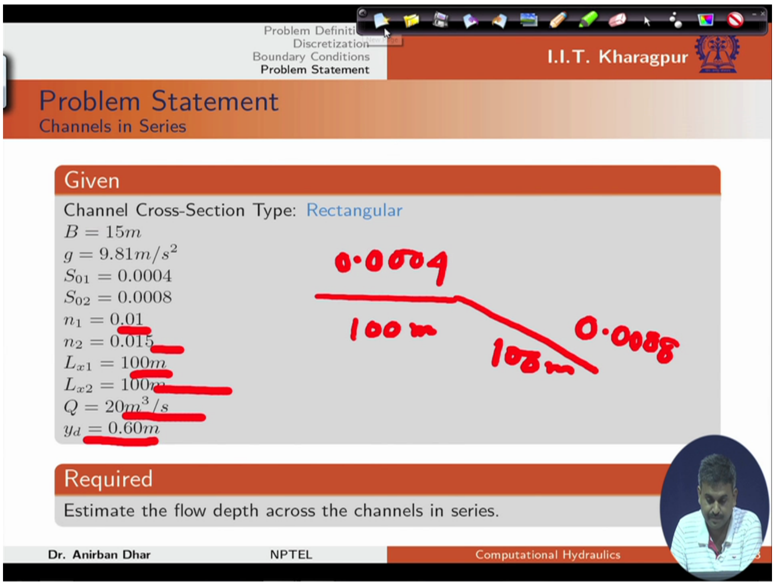

Now what is there? If we see this channel in series, now in channel in series we have this discharge value, channel CHL, we have two channel reaches, S not we have two S not values, n we have two n1 values. Lx, we have to Lx values here. Mnode because we need to divide both the parts separately so we have mnode for channel 1, 101. Mnode for channel 2, again that is 101. Now epsilon max, 1 into 10 to the power minus 6.

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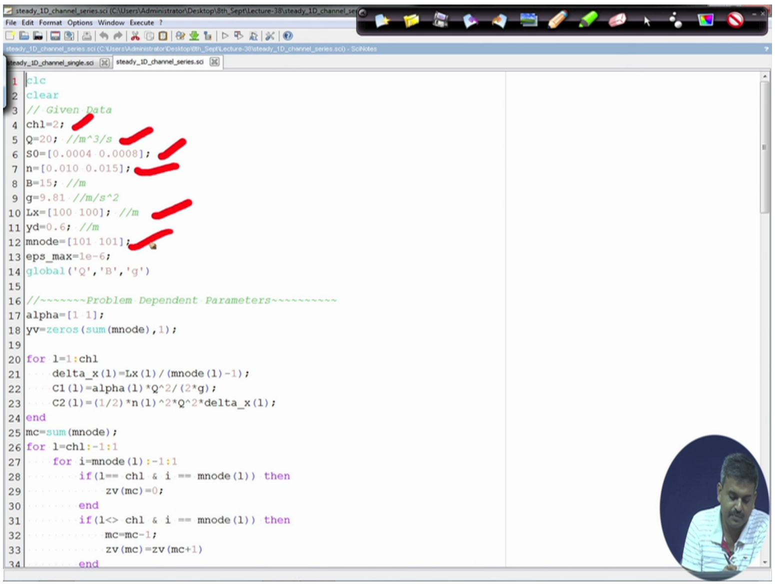

In this case S not, n, these are varying. That is why I have not kept it here as global variable. So Q, B and g, these three are global parameters here.

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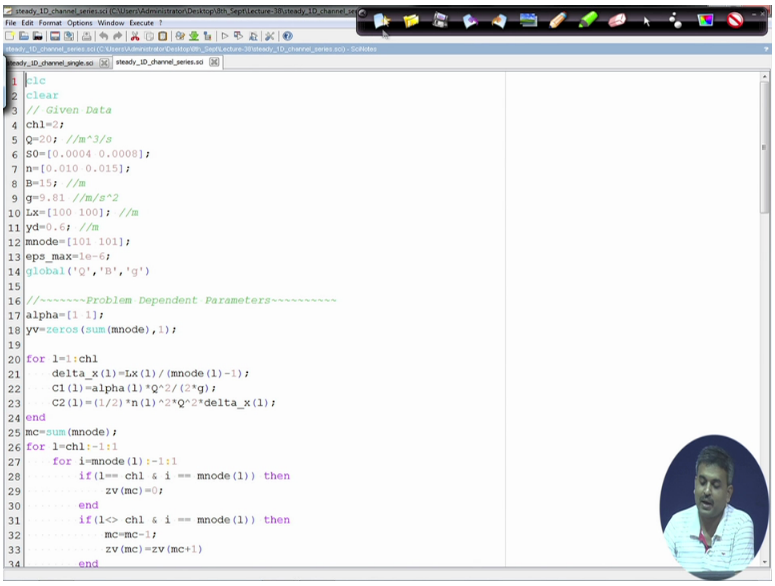

Now alpha I have considered 1 and 1 for both the channels. And yv, this is sum node. That is mnode 1 plus mnode 2. We have total 202 variables because we have 101 nodes each for each of these channel reaches.

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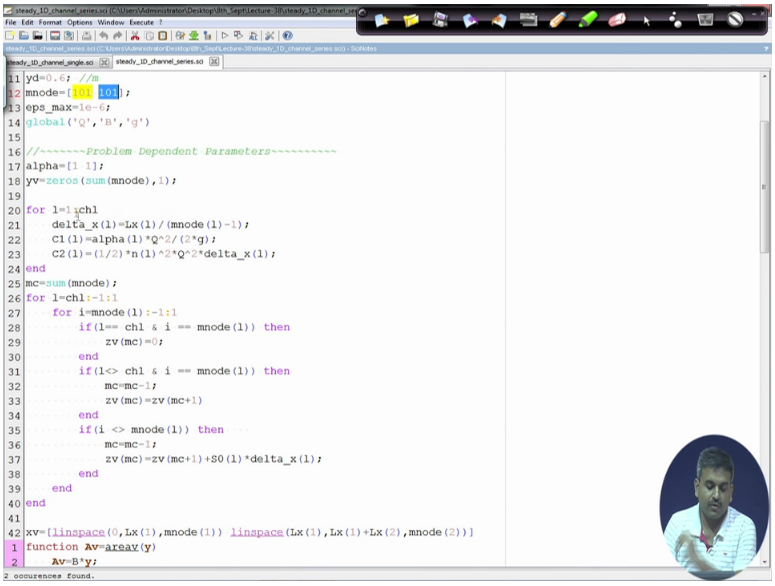

So in this case I can define this delta x. Delta x is again varying for L. L is corresponding to a particular channel. C1 is also varying with L. C2 is also varying with L. Now we can define this mc which is the total number of nodes there and we need those many flow depth values. In this case I can calculate the z values starting from zero value at the downstream end portion.

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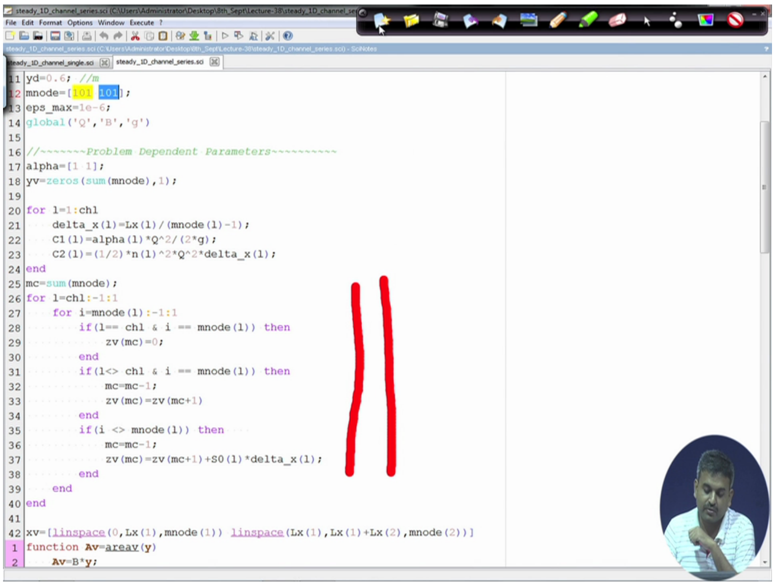

So if L equals to CHL which is a downstream channel, we can calculate the zv value here. So in this case we can calculate that zv from there and these derivatives are same like our previous one.

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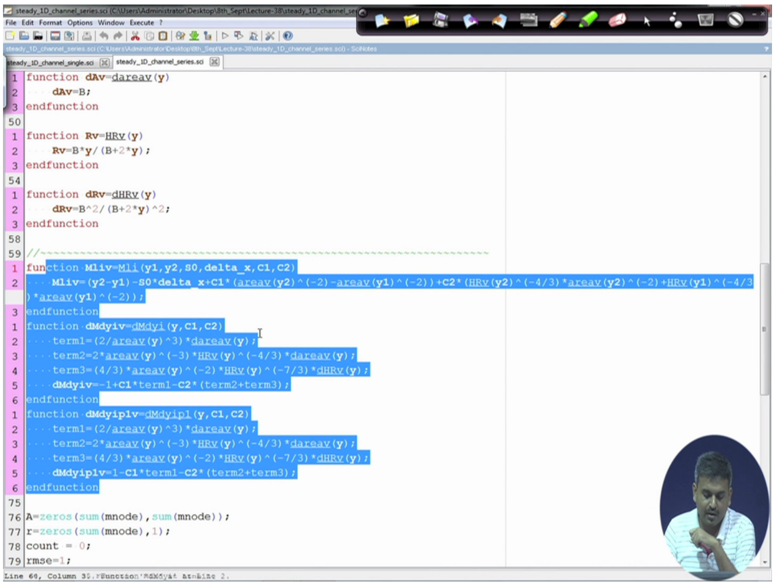

But we need to transfer the C1 C2 and del x, S not values here because these are not constant value now.

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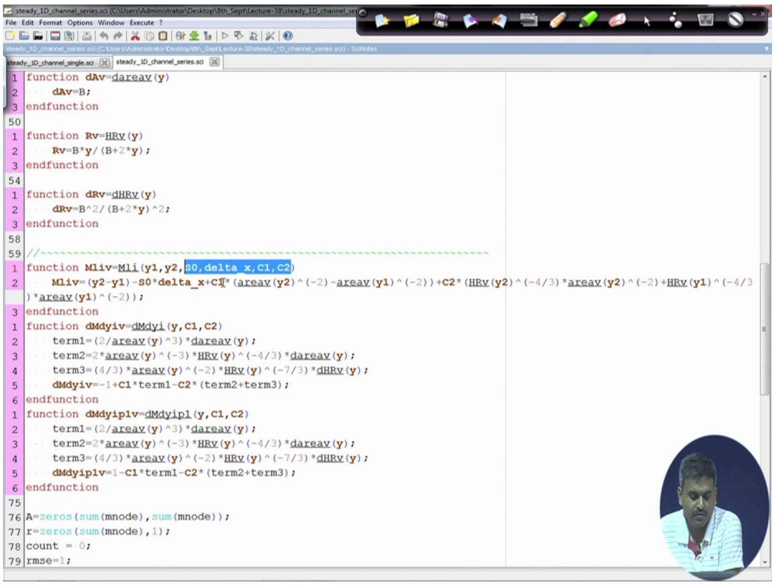

Again y C1 C2 these are required here. Again A sum mnode into sum mnode here, r zeros sum mnode. That means if we have two channels, 1 and 2, we have mnode 1 and mnode 2 then we can add this to those many variables. We need to determine because we need flow depth for the channel length.

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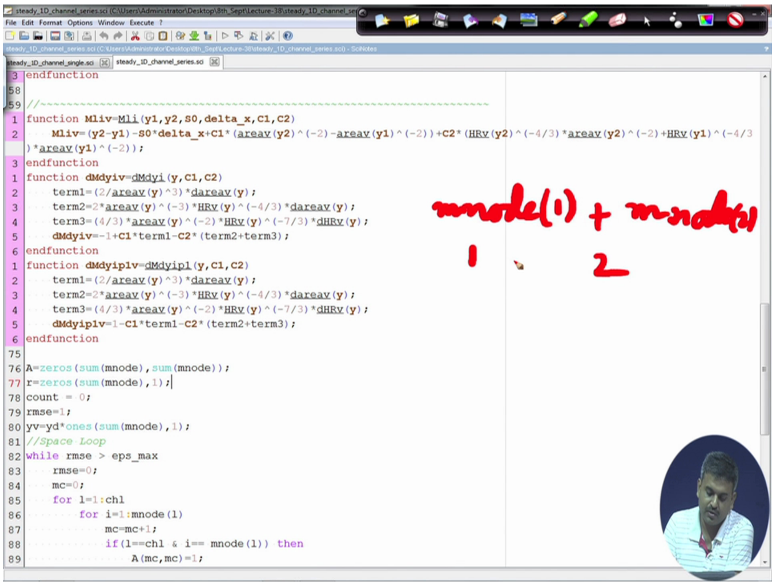

So in this case again we will consider yd which is the initial guess for our Newton Raphson. In this case first thing what we will do is that mc equals to zero. This is one counter here and with this counter we will create or add the values here. Mc, this is starting for channel L 1 to CHL, i 1 to mnode, mc equals to mc plus 1. after entering into this loop we will add this mc or increase this mc. And this is CHL i mnode.

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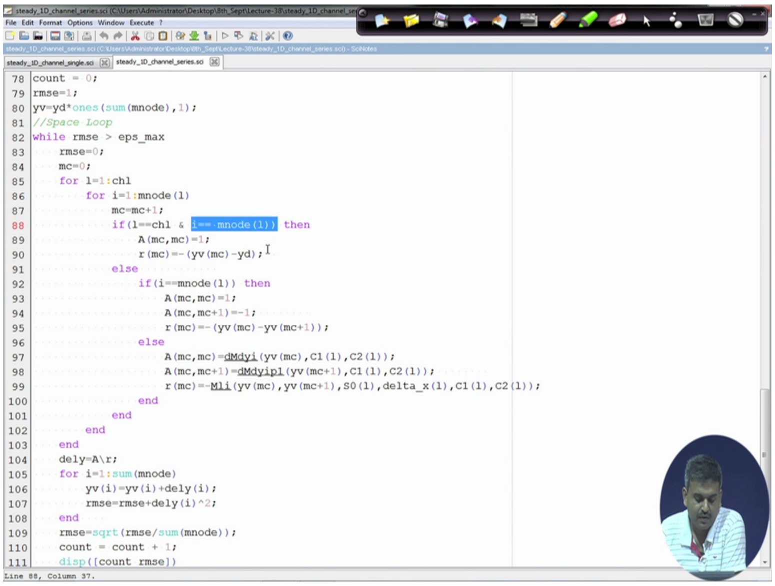

So for this one we need this condition which is the specified depth there. And in this case we have one junction node. For that junction node we need to satisfy the depth continuity because we are not considering variation in bed. So A mc, mc, this is for any general element we are considering.

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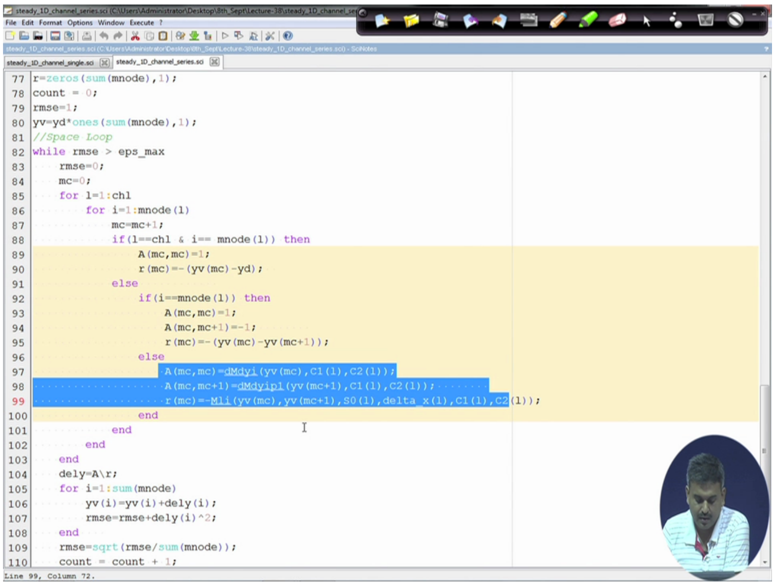

Now delta y we can calculate from Ar. This is from sum mnode. And yv again we can update and calculate the rmse values and finally we can get the solution from this iteration process.

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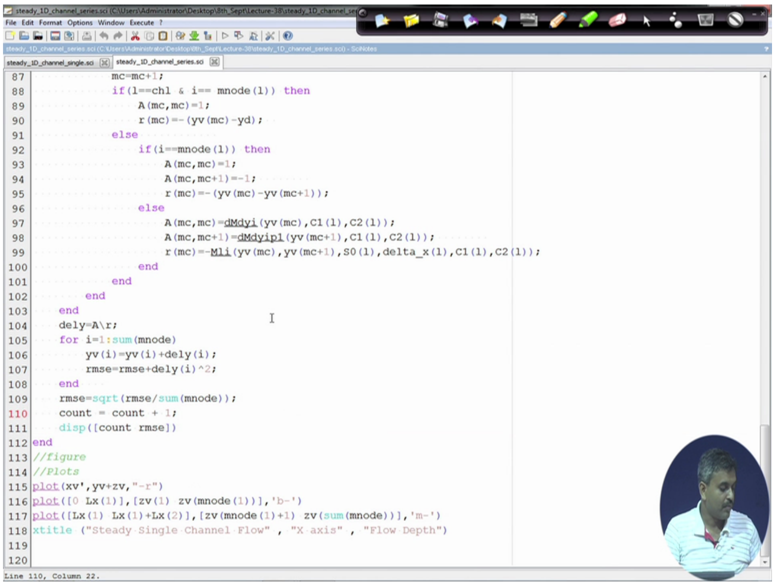

So if we see this value again this value is somewhat near point 9 in the upstream section. Downstream it is point 6 exactly. And interestingly in this case slope is varying. So initially it was milder, now it is a mild slope. And obviously we can see one change in profile at this junction point.

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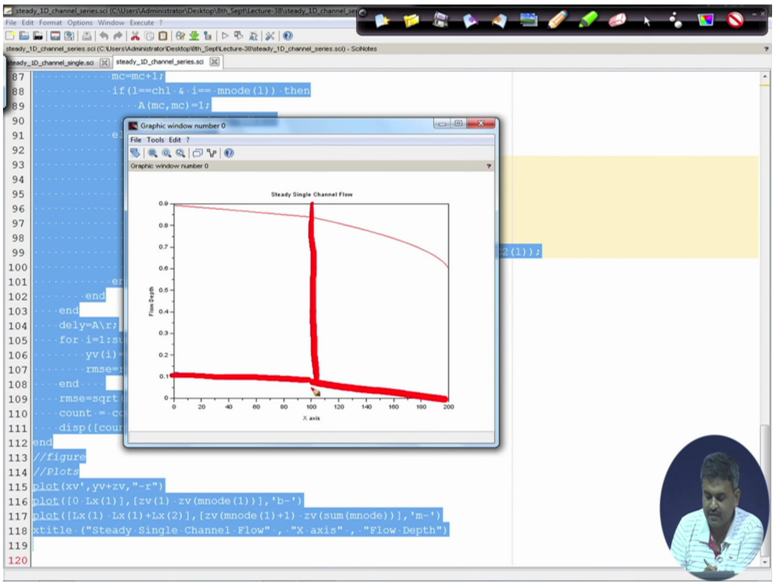

But continuity is there in the junction point. So we have calculated or estimated the flow depths for different sections. Now we have a framework in place where we can utilise the constant discharge values. And maybe upstream and downstream boundary condition depending on the flow situation we can get the corresponding flow profile there.

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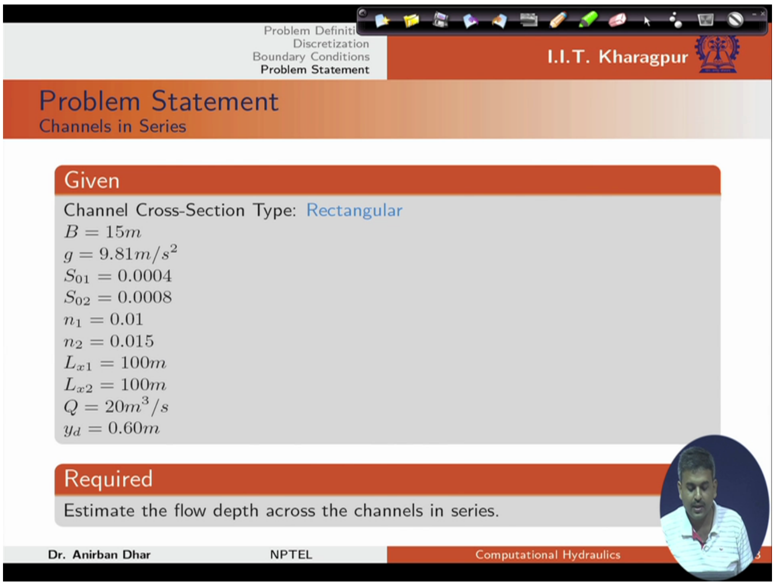

So we have used these two codes. One is single channel, steady 1D channel single. And channels in series, steady 1D channel in series. And we have got the full profile using these two codes.

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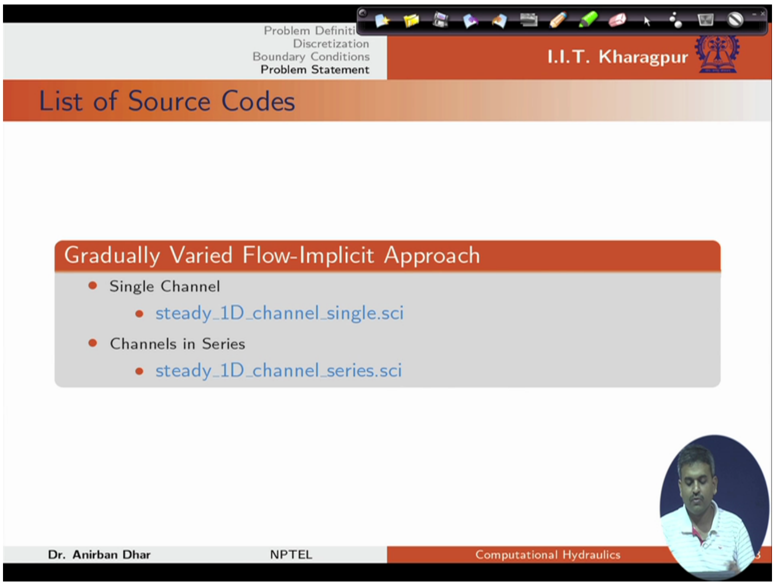

In the next lecture we will be talking about channel either in parallel or they will have more number of junction points or multiple channels connected there. Thank you.