Computational Hydraulics
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Lecture 35
Unsteady Flow In Unconfined Aquifer Using FVM

Welcome to this lecture number 35 of the course computational hydraulics. We are in module 3 groundwater hydraulics. This is unit number 5, unsteady flow in unconfined aquifer using finite volume method.

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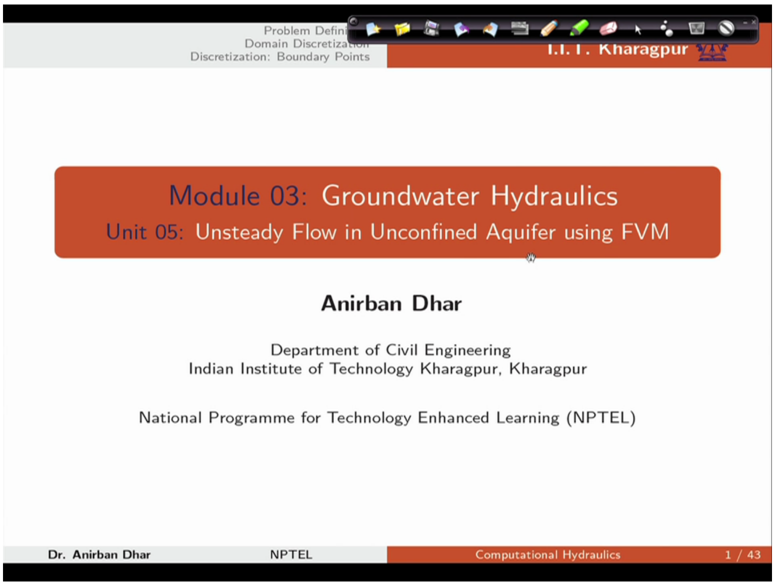

In our last lecture class we have discretized this unsteady flow equation for confined aquifers using finite volume method. So what is the learning objective for this particular lecture? At the end of this particular unit students will be able to solve unsteady two dimensional groundwater flow in unconfined aquifer using finite volume method.

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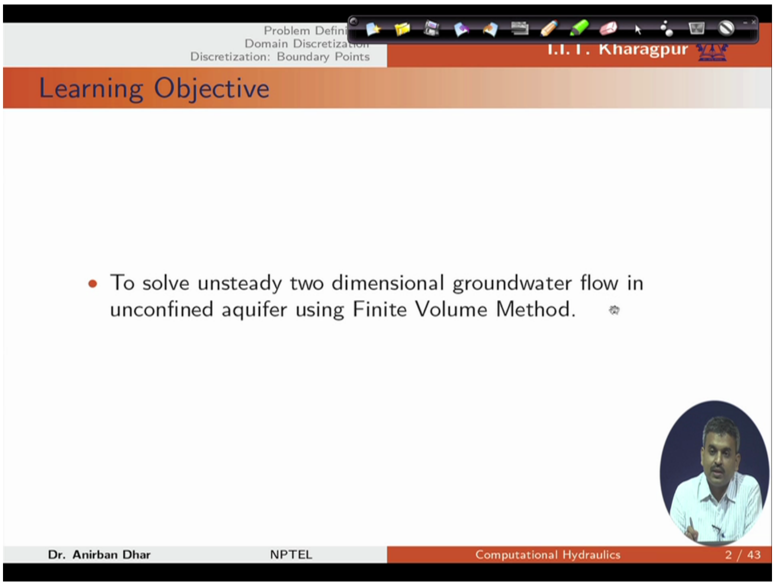

Problem definition to solution, we will be utilising h xyt because in unconfined aquifer flow, the flow we will consider only in horizontal direction. So in this case again we will be utilising this problem definition, mathematical conceptualization, domain discretization, numerical discretization, obviously in this case we will be utilising finite volume method. But the main difference with the confined aquifer flow is the nonlinear equations.

In case of this unconfined aquifer flow we will be utilising nonlinear equations and now solution approach will be different compared to our linear equation that we have utilised for confined aquifer flow.

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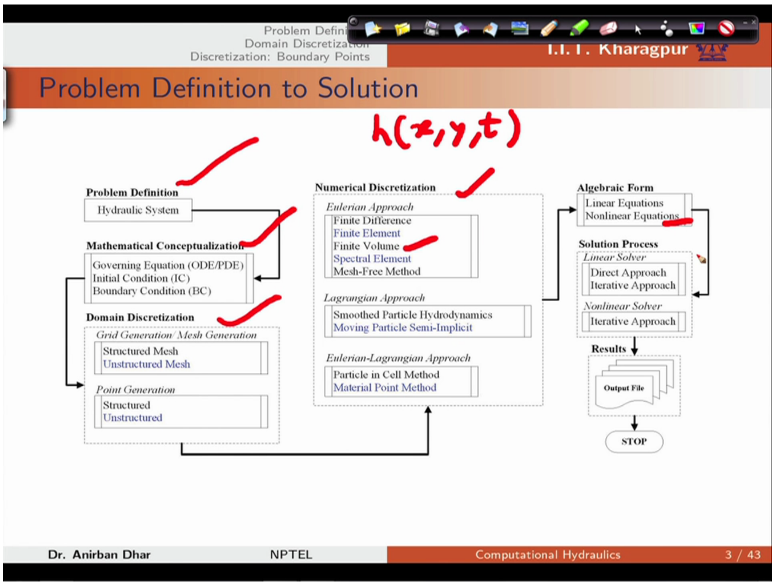

So problem definition, I am not going to the details of the problem definition. Now in this case again our system is homogeneous isotropic system. Boundary conditions are same like our previous confined aquifer flow. But only difference is that we are considering it as unconfined aquifer flow. In confined aquifer flow we need to define storativity but in unconfined aquifer we need to define specifically, obviously transmissivity is required.

We will define the hydraulic conductivity in terms of Kx, Ky. In isotropic system Kx will be able to Ky in this case.

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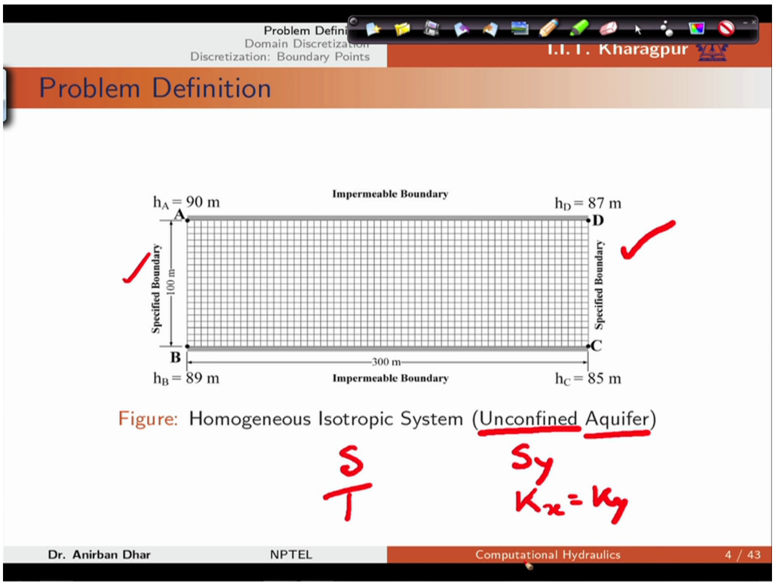

Let us define our two dimensional problem. In our confined aquifer problem we have utilised this equation del h by del t and del 2 h by del x 2 plus del 2 h by del y 2. Now in this case we have extra term this h. And because of this h, the equation is nonlinear in nature. Now let us consider that Sy specifically is point 25 and K is 20 metres per day. Obviously in this case Kx equals to Ky equals to 20 metres per day.

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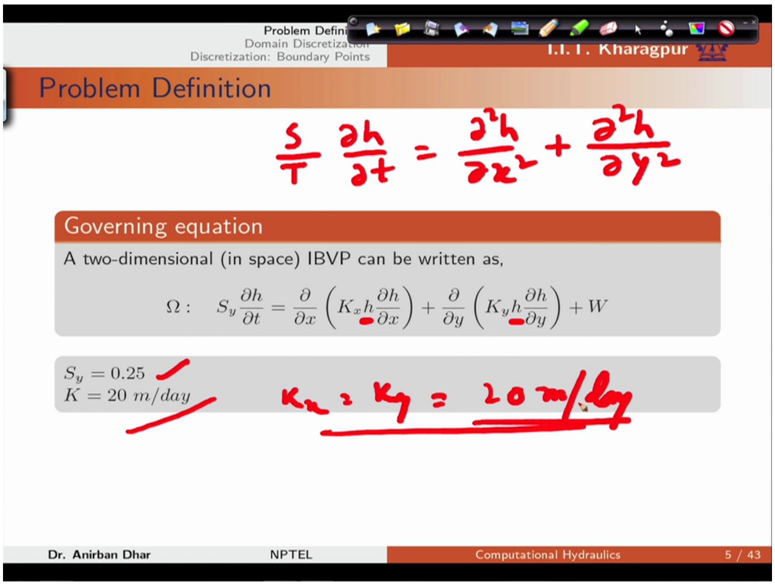

Now we need to see how we can solve this using our finite volume approach. Initial condition, again the conditions are same like over finite difference approach or finite volume approach that we have utilised for our confined aquifer system.

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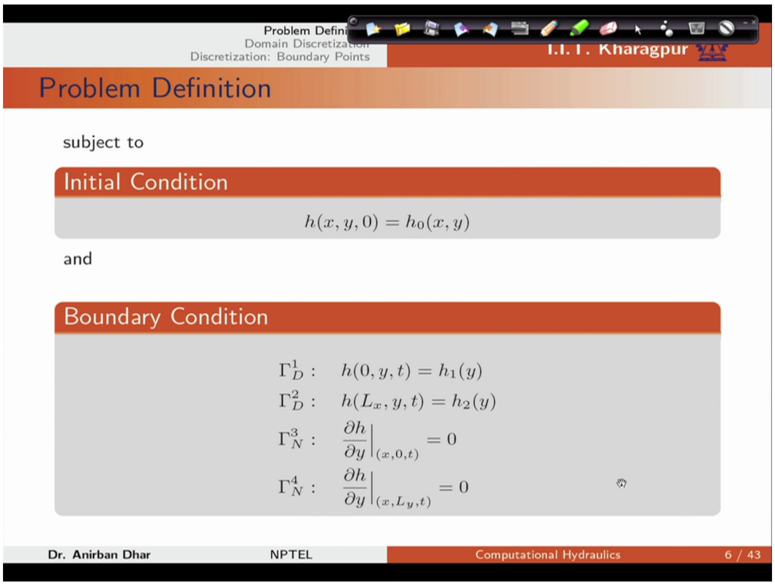

Domain discretization, again we will have interior nodes. So again 1 for interior node or I should say interior cell plus boundary cell we will have 4. And corner cell we will have 4 condition. So all total we need to consider 9 conditions for this problem. Obviously this is Lx and Ly. This is y direction, this is x direction.

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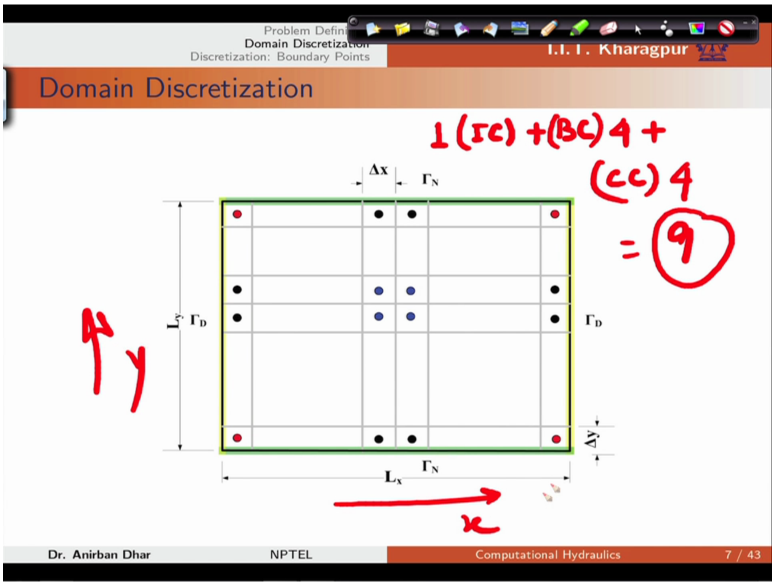

Now in finite volume method the governing equation is integrated over the element in space and time interval to form the discretized governing equation. Now in this case we have space and time derivatives. This F vector is fx fy. Obviously F vector if you consider, this is nothing but fi plus fy j where i and j, these are two unit vectors corresponding x and y directions.

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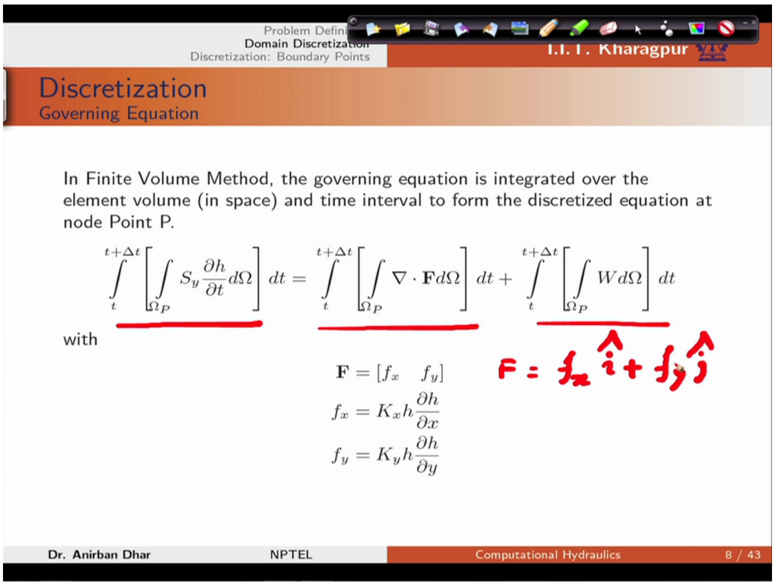

So this fx is this quantity. So initially this was the case where Kx, h, del h by del x plus this is y Ky. Initially let us consider that our Kx Ky are different and those are varying with x. So we will formulate the general problem and during solution we will consider Kx and Ky, those are having equal values. So this is nothing but fx plus fy. So we can write this as divergence of F.

Now if we write it in terms of divergence of F again we can utilise Gauss divergence theorem for the first term on the right hand side and we can use the integration concept for other terms to get the final discretization.

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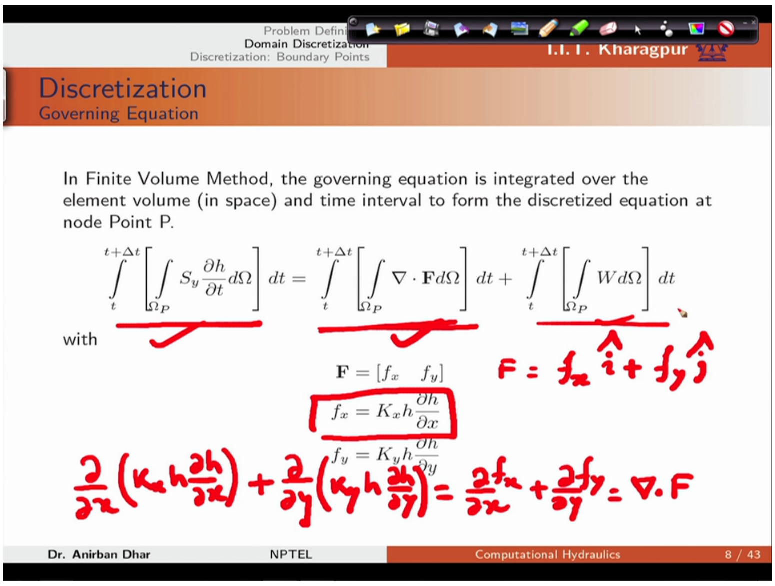

Now the first term on the left hand side. This is Sy del h by del t. If we consider that no variation of h within this omega P volume, we are considering that omega P is the volume in this case. So within this omega P there is the variation of h. Now we are considering constant value for this one.

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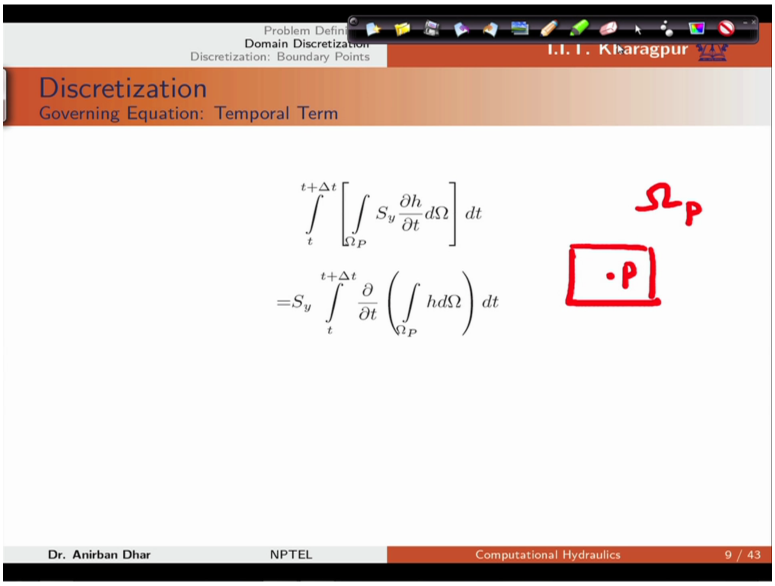

Now with this consideration if we further write this so this will be nothing but hP into del omega P which is a volume for this cell. Now we need to integrate. So obviously if we integrate it over time period or time interval will get something.

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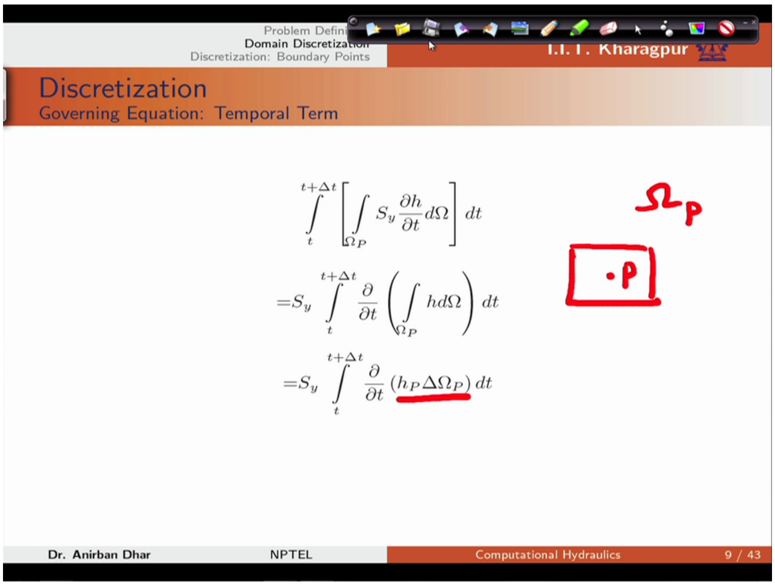

So what is that? So we will get this hP L plus 1 to L because L, this corresponds to t and L plus one, this corresponds to t plus delta t. Now if we use this finally we will get that our del omega P. This is nothing but del x del y and unit value on the other direction. We are considering 2D volume only in this case. So that is why this del x into del y.

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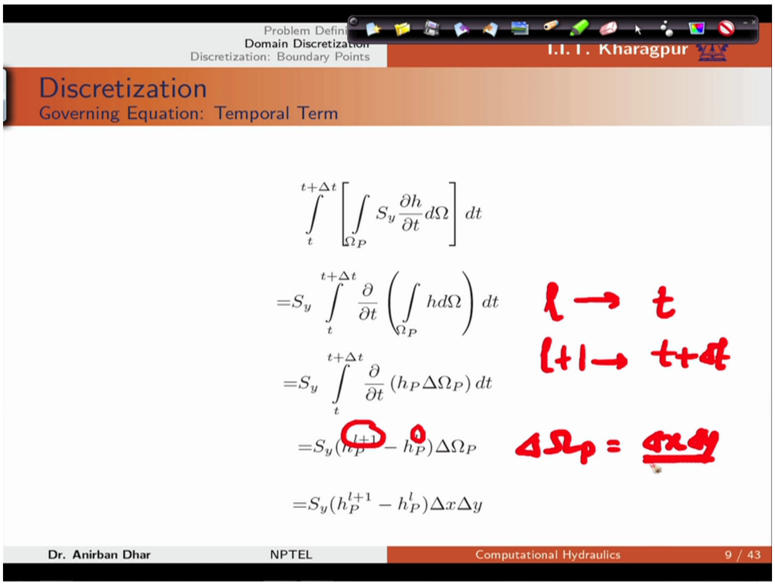

Now let us consider the first component on the right hand side. So this is the divergence form. Again divergence form of this one. As we have discussed that area is always outward positive. If area is outward positive then this is nothing but del y i. This is minus del y i. This is del x j, obviously negative sign will be there. On the upper side this will be del x j and this is Pth cell.

Now for the cell if we consider that F dot dA from over Gauss divergence theorem, so F dot dA, this component for all faces that means we need to consider this east, north, west and south faces here.

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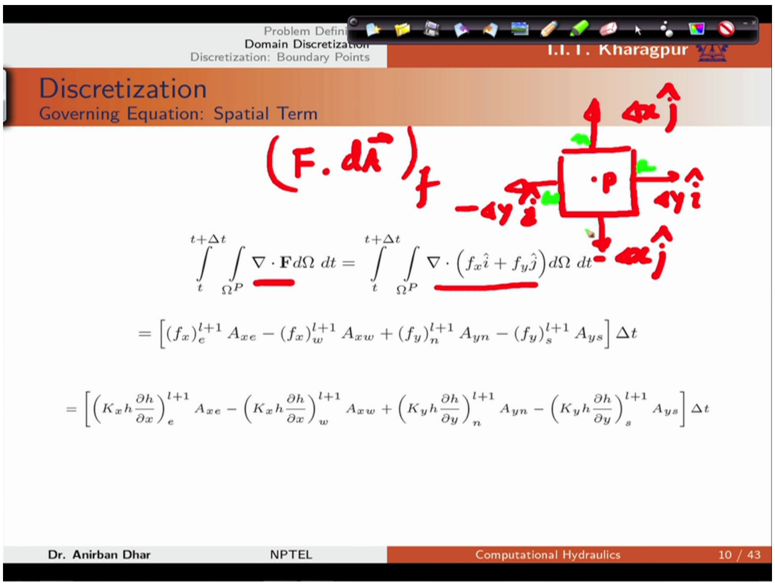

So for all faces if we consider, this quantity dot area, for eastern face obviously this will be positive. In this case A xe, this is nothing but del y. This is A xw, this is again del y because negative sign is already there so we should not consider this thing. So Ay north, again del x. Ay south again this is del x in this case. So we have discretized the second term in terms of these derivatives. East, west, north, south and del t will be there.

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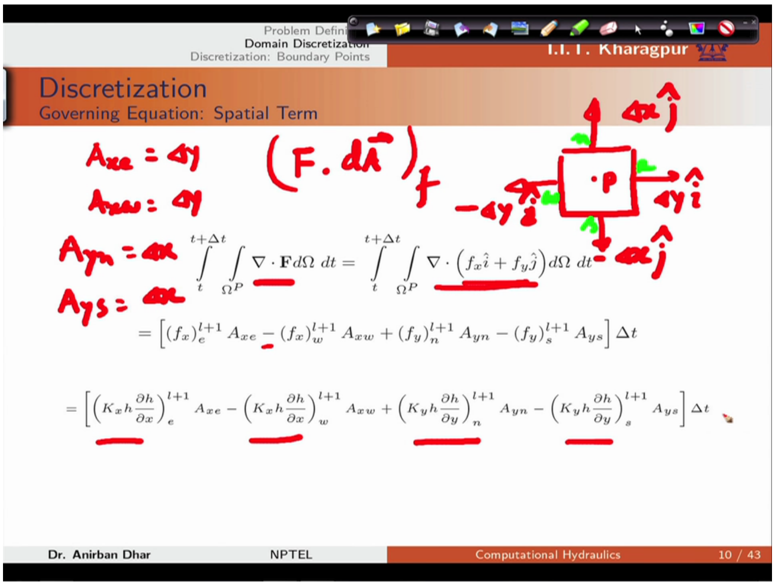

Now if we consider the uniform grid system as I have discussed. So this will be the case and source term, if we consider the source term obviously this is valid for cell value. W is valid for cell value. So obviously del x, del y, del t, this should be multiplied here.

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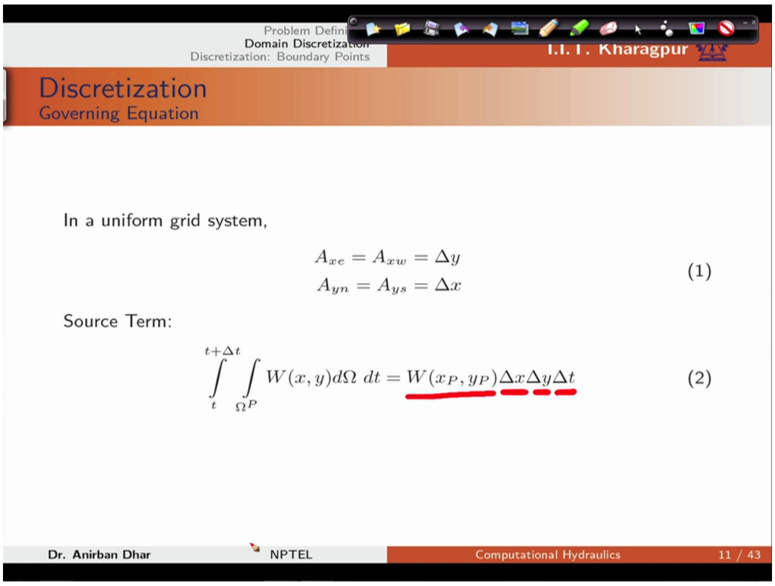

Now in compact form if we combine all terms we can get this. On the left hand side this is the thing and on the right hand side del y will be multiplied with eastern and western face. Del x will be multiplied here and del x, del y, del t on this side.

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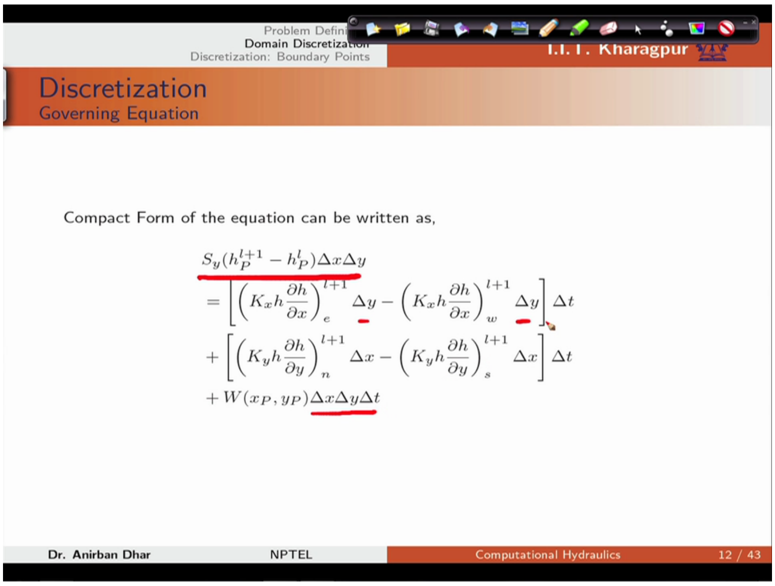

So if we see the case for interior cells, now interior cell east face, this is important. East face we are already know that del h by del x at east face we can write it in terms of hE L plus 1 minus hE L plus 1. This comes from the concept of implicit scheme because we are using L plus 1 index for time. And spatial derivative we are (usili) utilising the concept of finite difference.

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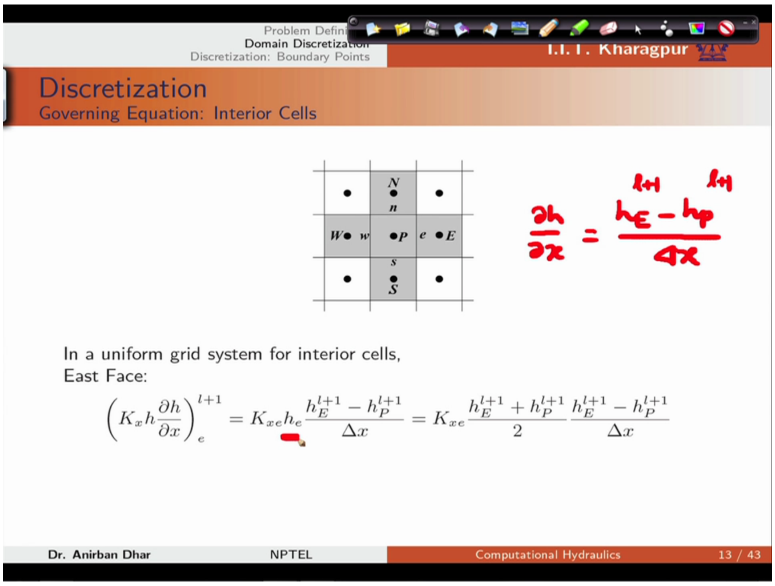

Now this is hE. What is this hE? HE is the h value at the face. Now how to calculate this h value at the face? So what is the readymade approximation? Readymade approximation is that if we can consider this hE plus hP by 2. That means the average value between this P and E cell that can be considered as our face value for the east face. So this is our face value and this is a derivative value multiplies by K xe. That means hydraulic conductivity at the face.

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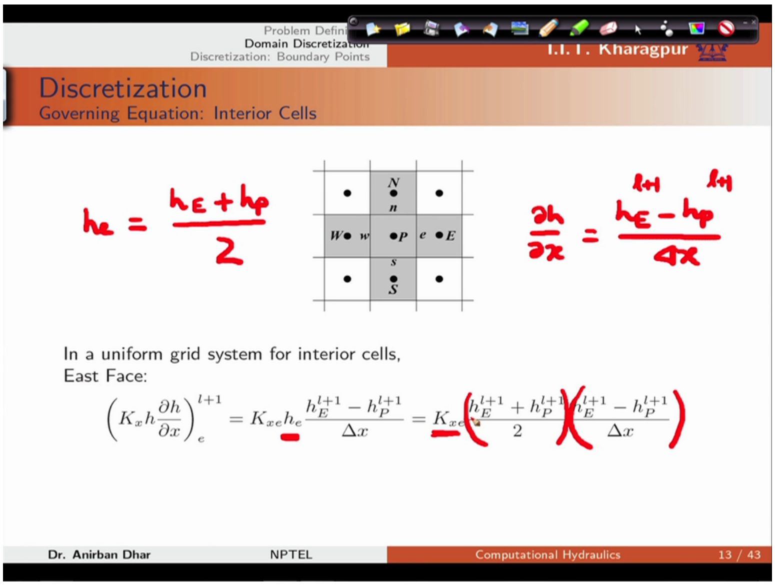

Now we are considering that we have homogeneous isotropic system. But in this case we are considering generalized formulation. So we will simply use Kx instead of this K xe.

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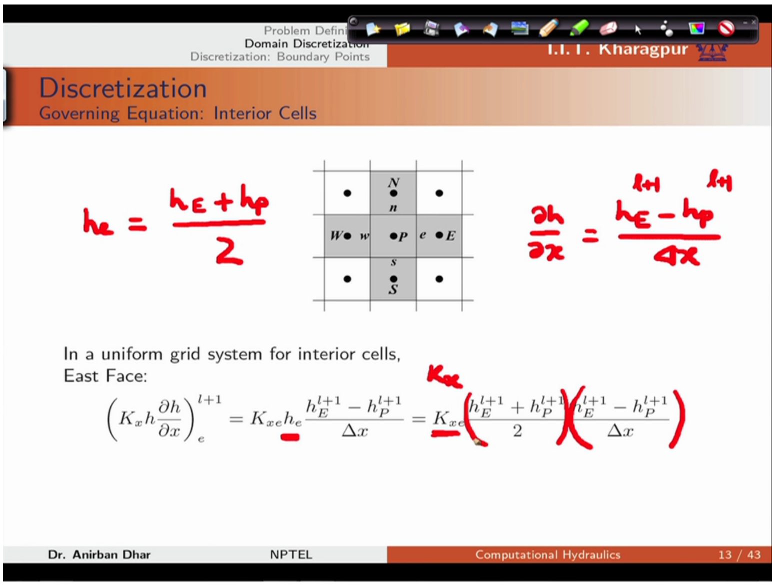

Now let us see what is the situation for west face? West face is again similar. We will consider hW as hP L plus 1 plus hW L plus 1 divided by 2. And this derivative we already know that del h by del x at western face hP minus hW divided by delta x.

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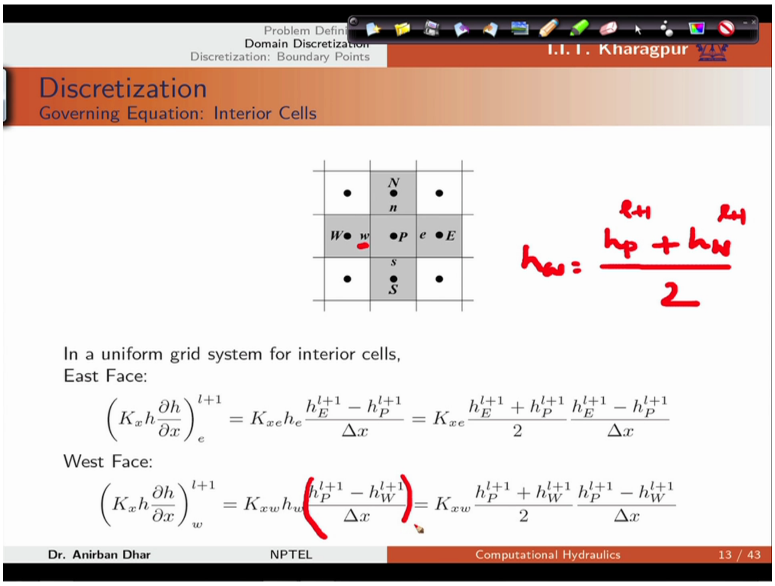

So this is the average value, this is the derivative. Now we can write the east face and west face like this.

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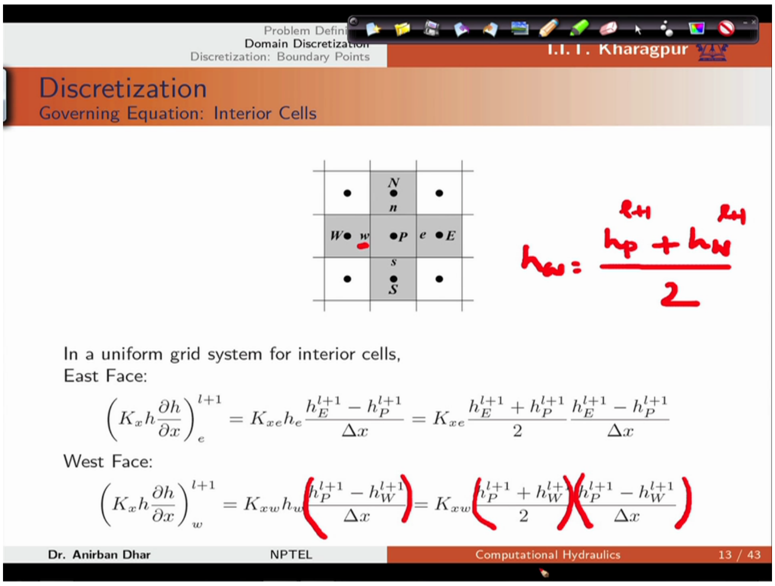

Now if you have northern face, northern face again we can discretize or approximate this hN in terms of hN hP and the derivative as hN minus hP by del y.

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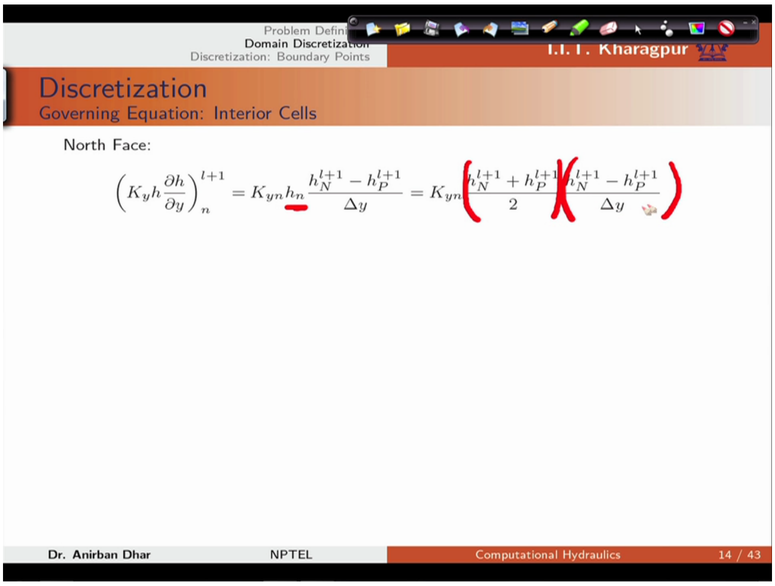

Now if we have southern face, for southern face we have hS, hS is nothing but hP and hS average and hP minus hS divided by del y.

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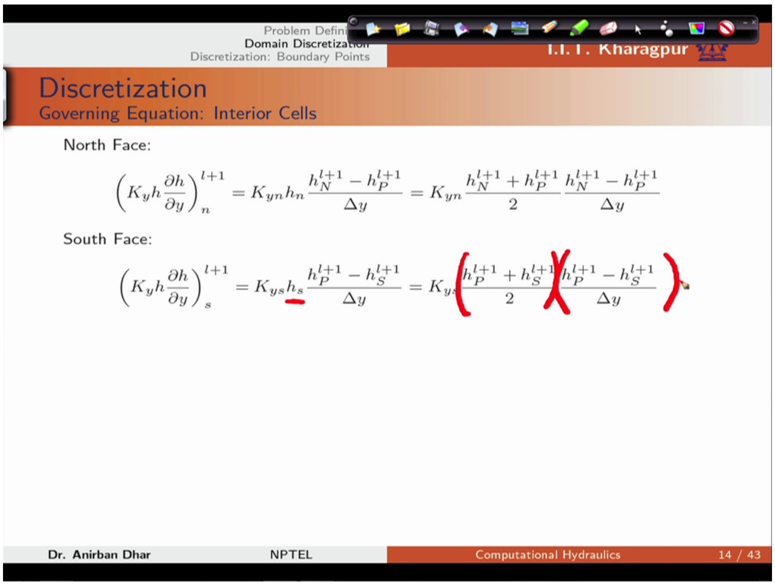

So with this we can further simplify our compact form of the equation. On the right hand side interestingly we are getting these square terms, hE minus hP square, hP minus hW square, hN, hP, hP, hS, all square terms. So this is one non linear equation.

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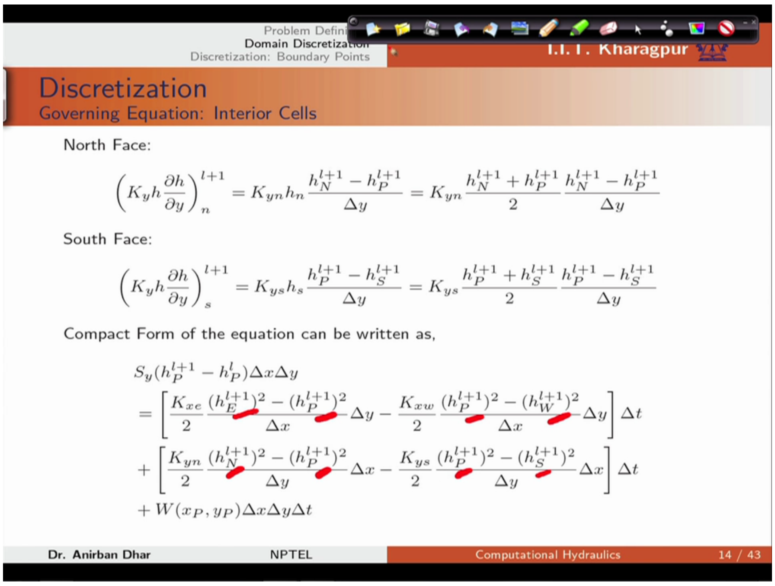

Now in compact form what we can do, we can transfer this Sy on the right hand side and del t we can include here. So del x square obviously in this case we will get this parameter kind of thing. Multiplied by 1 by del x square. Also Kx del t by 2 Sy, this should be multiplied by 1 by del x square.

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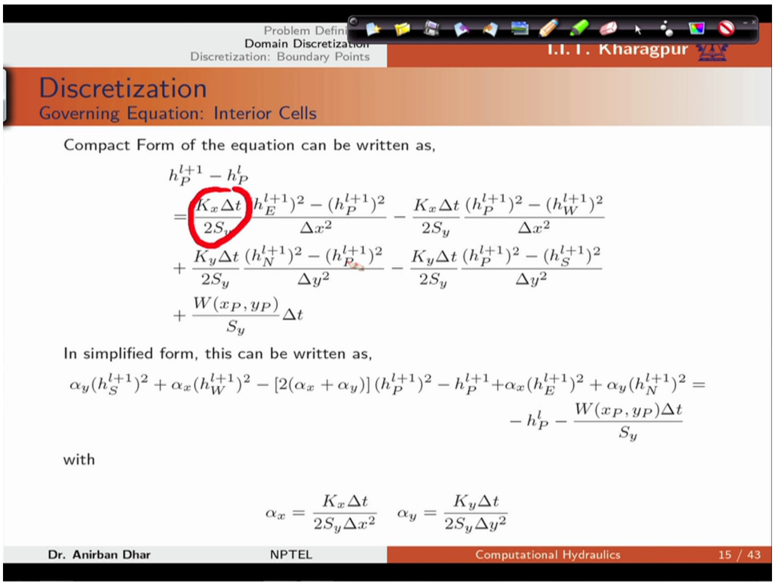

Now we can simplify it. So if we further simplify this thing with the approximation that Kx del t by 2 Sy del x square, Ky del t 2 Sy into del y square we can write it in approximate form.

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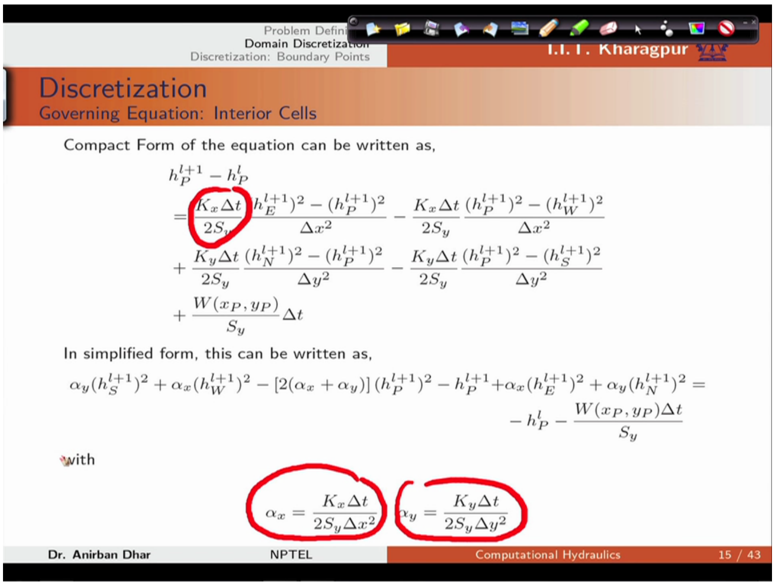

So now approximate form is this alpha y into hS square plus alpha x into hW square minus 2 alpha x alpha y. These are coming from here because coefficients two for alpha x, two for alpha y.

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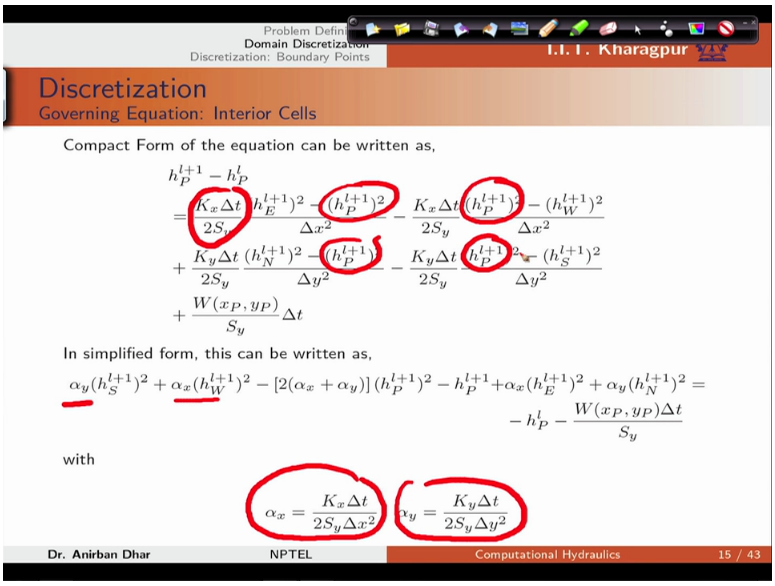

And one which is coming from this side. This is minus hP L plus 1. This is not square term. Remember that this is not a square term here. But this hP is a square term.

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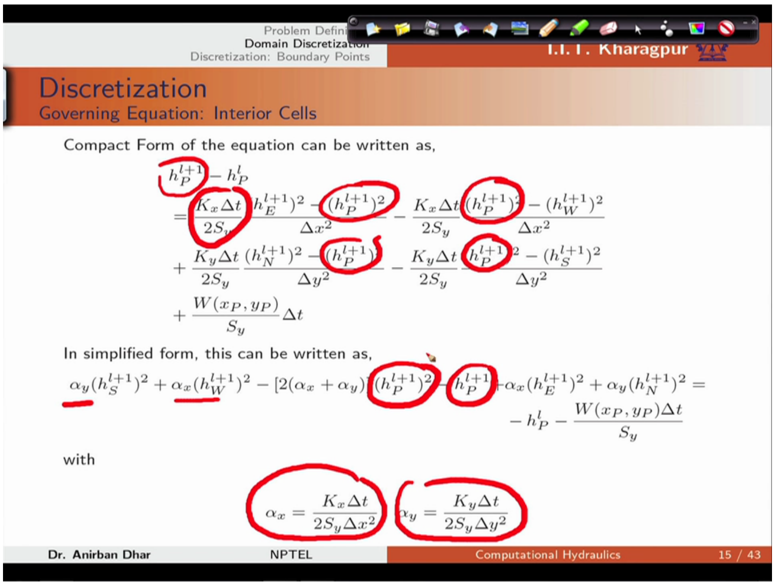

Now another term will be there alpha x hE square, alpha y hN square. So in this side we have all known quantities. Now this is a discretized form for interior points.

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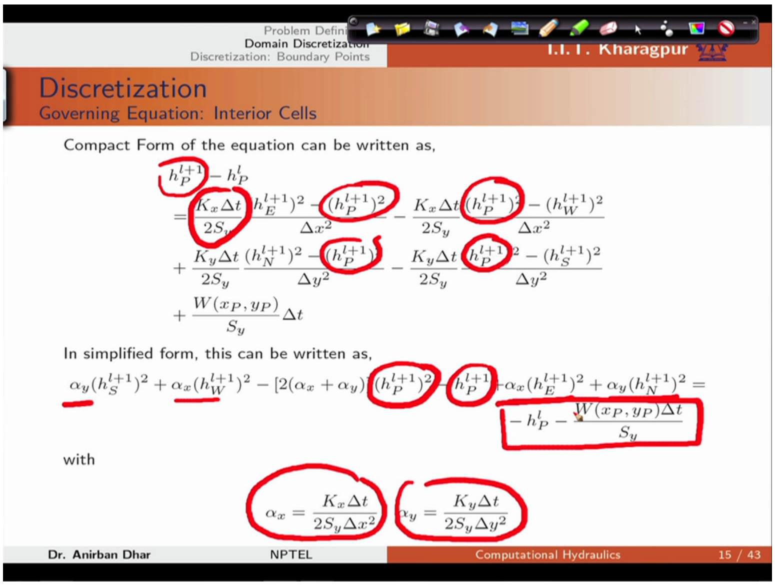

Now if we have non linear equation, obviously we need to solve it using some nonlinear technique and one of such technique is Newton Raphson. In Newton Raphson we need to define the Fm or our function which is function of h L plus 1, all unknown quantities. So I have written all left hand side terms here and transferred this right hand side to the left hand side and equated this one with zeros. So this is my functional form or function form.

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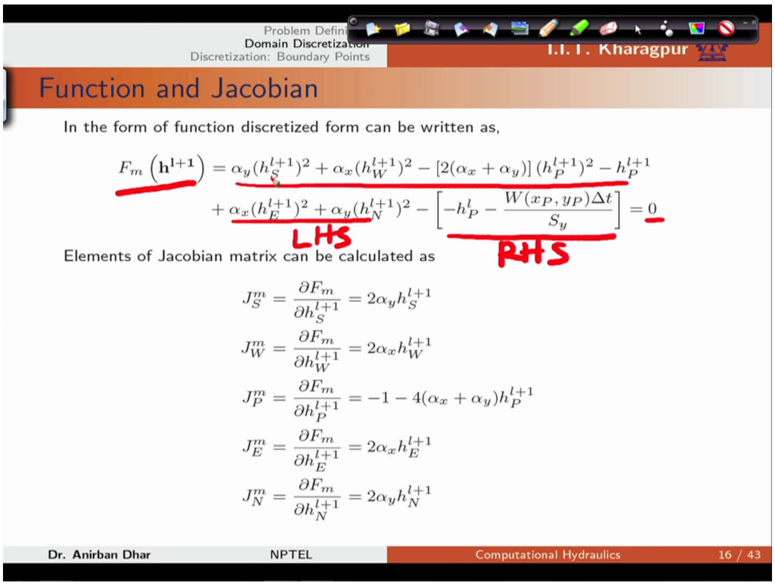

Now I need to calculate the derivative term or elements of Jacobian matrix. So to calculate the element of Jacobian matrix so I need to take derivative of Fm with respect to hS which is 2 alpha y hS. It is directly coming from here.

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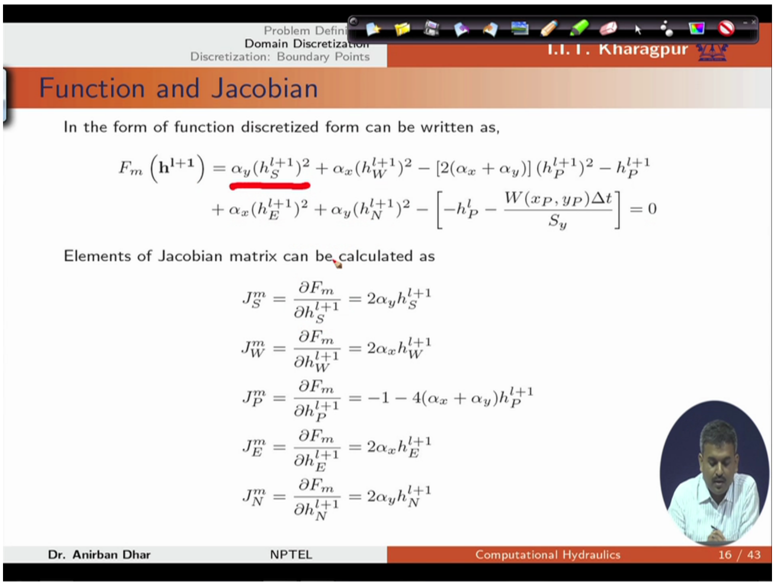

Now next one is hW which is coming from here. This is 2 alpha y hW L plus 1.

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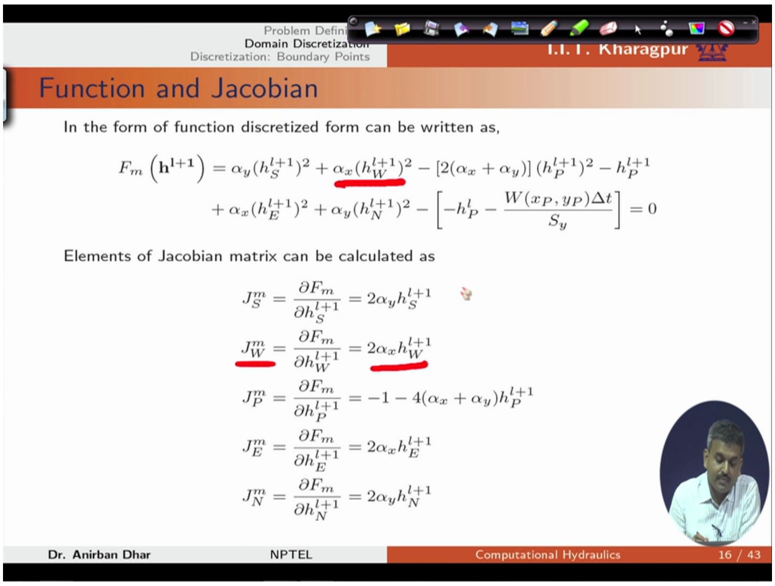

Then comes this hP which is again minus 1. Minus 1 means because in this case there is no coefficient available. You can simply take derivative here. We will get minus 1. Minus 4 because 2 is already, the square term, this is 2. So 4 alpha x alpha y into hP. Then hE on the eastern side we have 2 alpha x into hE, 2 alpha y into hN. This is there.

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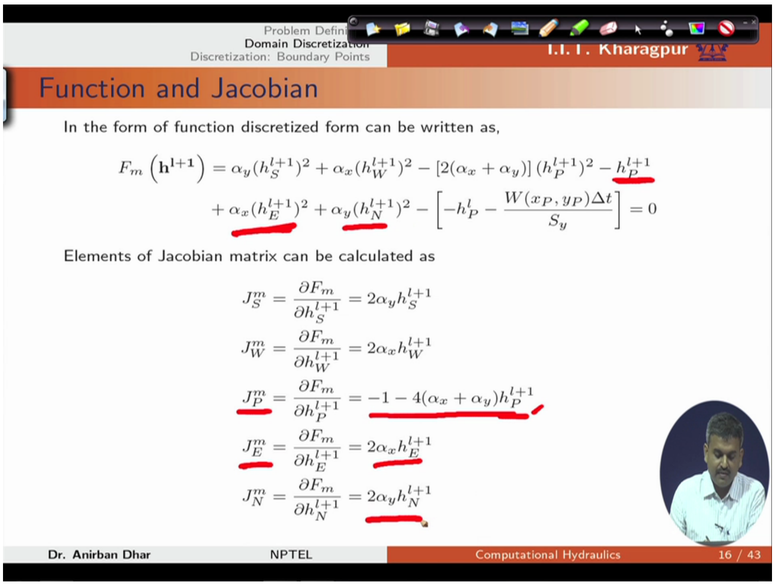

And this is our right hand side term. Obviously this is a constant term for function calculation because we are considering this unknown at L plus 1 level.

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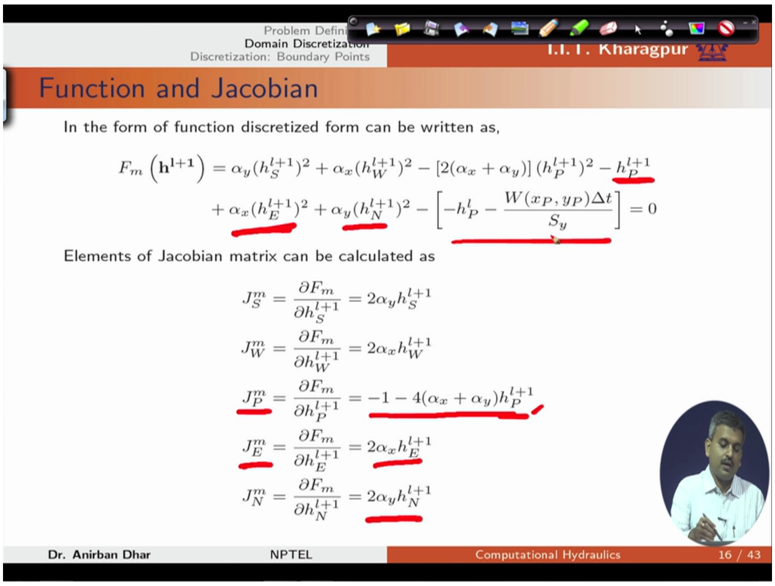

So with this if you proceed then we can construct one row of the Jacobian matrix. Now for all cells that means if you have M cells, Mc and N cells in y direction. This is in x, this is in y direction. For M Nc into Mc Nc this will be the size of Jacobian matrix. And we will have penta diagonal structure of the Jacobian matrix because you will have P term, W term, E term, N term and S term. So obviously no other term will be associated with the central point or point P.

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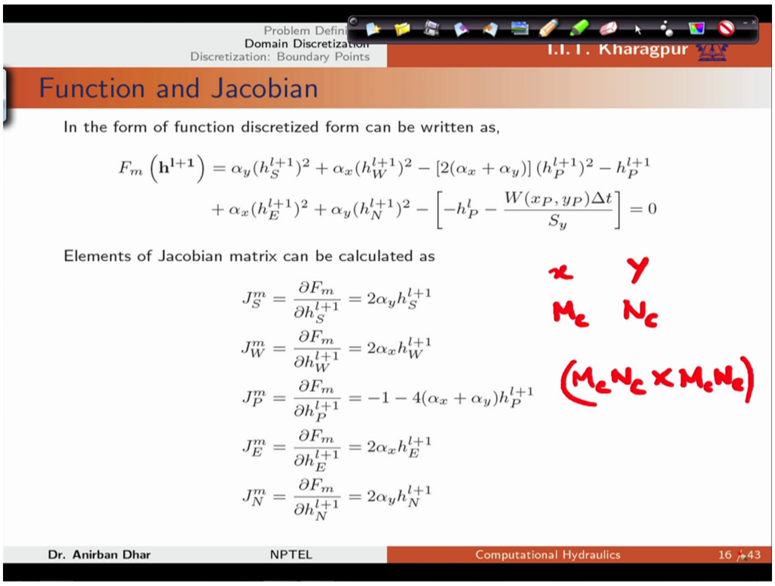

So obviously we will get this P. This is for our other ones on the right hand side which is eastern one, this is western and this is our northern, this is southern. So like this we will get the terms for interior points terms for interior points.

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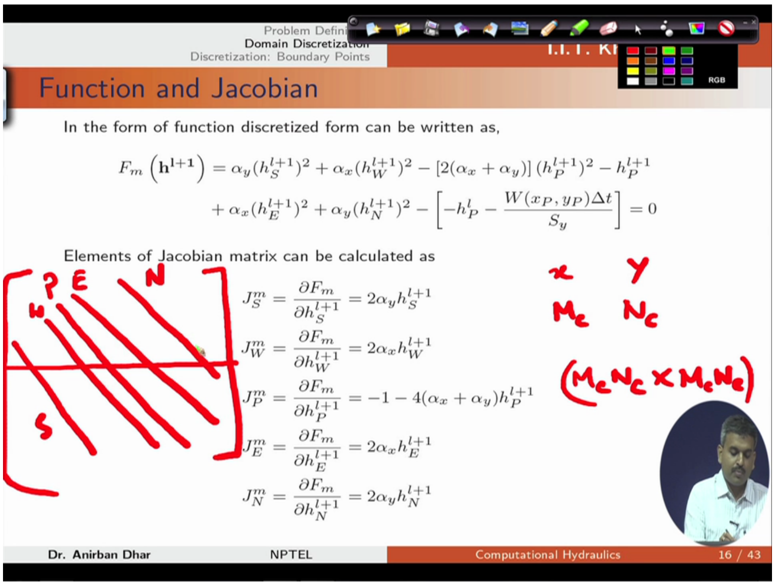

These points you will get from these equations. Obviously this will be JS, so S means this one. This is W, W is this one, P is this one, E is this one and N is this one. So for all interior nodes we need to calculate this.

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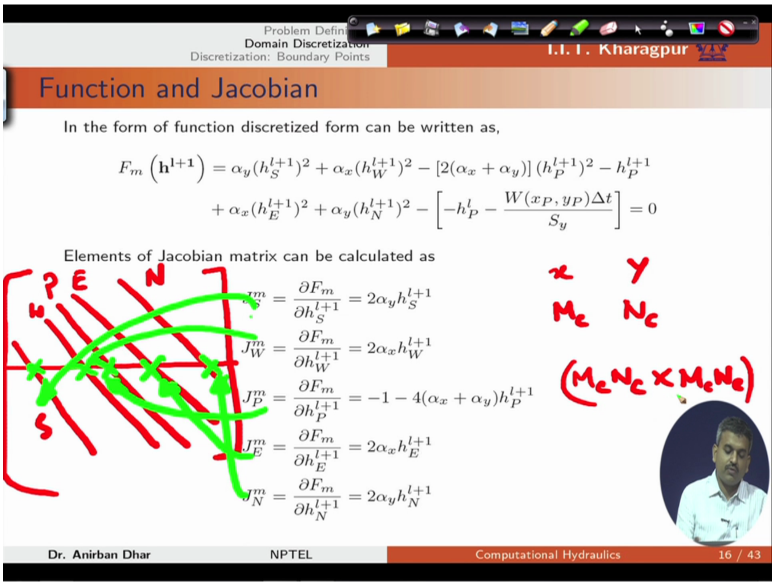

But this is a node number including the boundary nodes. For boundary nodes we need spatial treatment.

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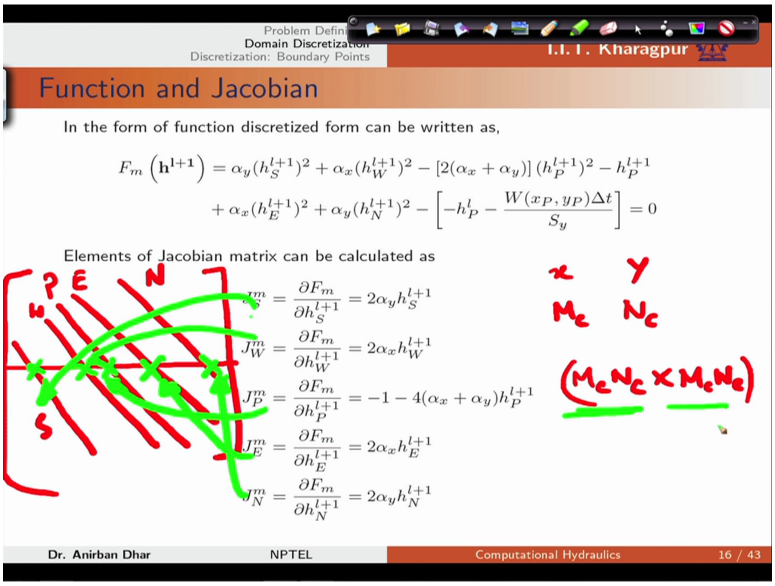

So let us see how we can utilise the concept for boundary nodes. Now we have left boundary, left boundary this discretization is exactly same as we have utilised for confined aquifer system because the derivative term will be same. But we need to multiply hW or value on the right hand side. This is our original equation which is in discretized form.

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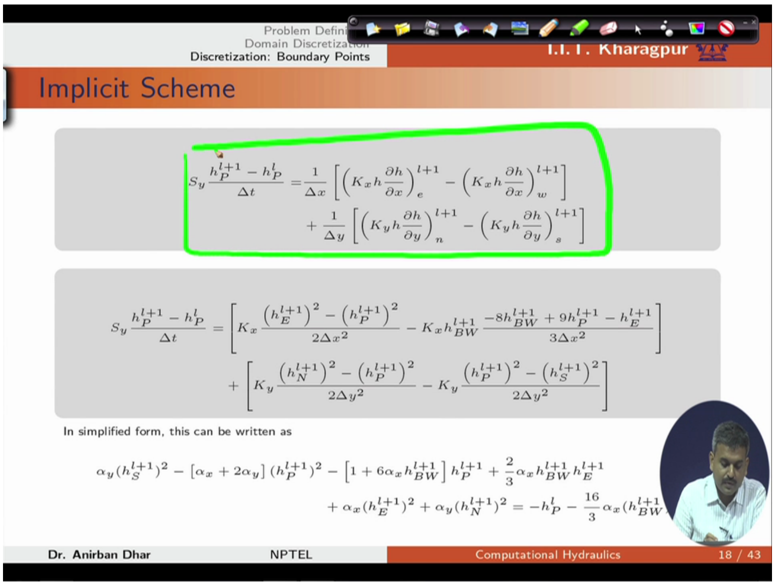

On the right hand side if we consider eastern face we can directly use the form that we have utilised for interior nodes. For northern face again we can use the same thing. For the southern face also we can use that node.

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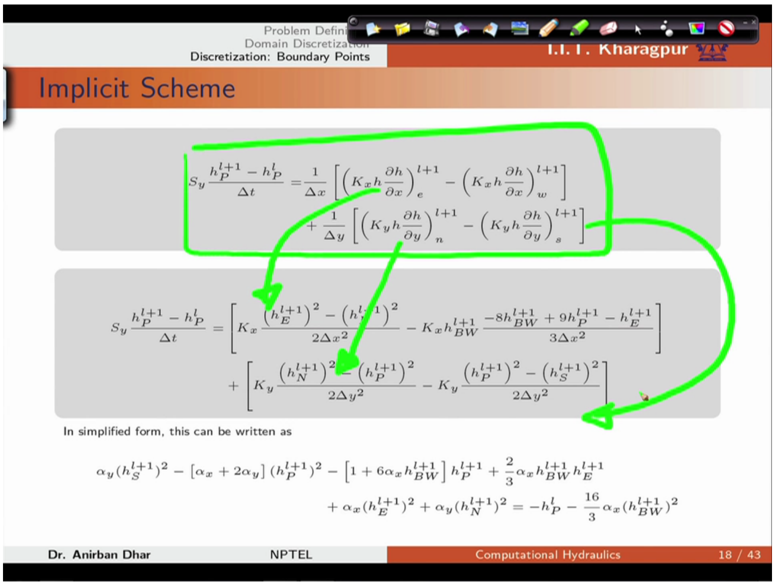

But for the western face we have something different. This is applicable for derivative then we need to multiply h BW which is the height at the face. Height at the face is equal to the specified value for this case. So if we have specified value then we can directly use that because that is a known value.

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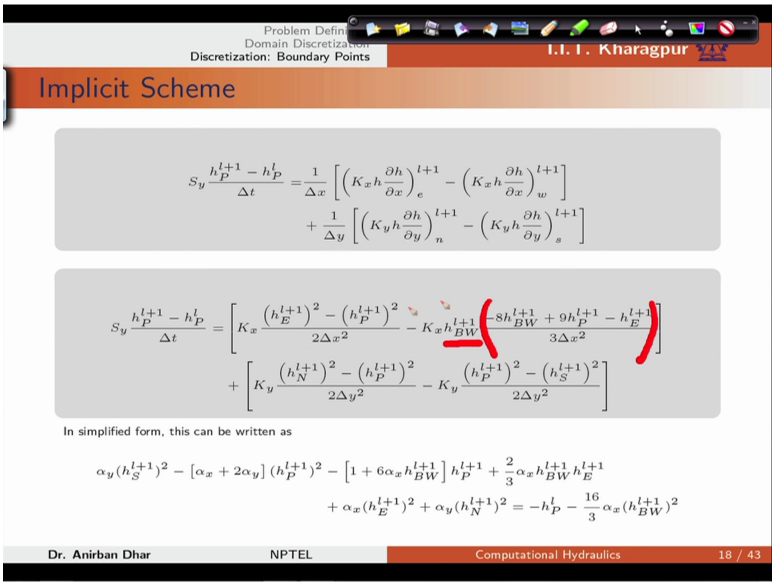

So if you simplify this again we will get terms with hS, hP, hP, hE, hE and hN. In this case we do not have any hW term because for hW term we do not have W cell in case of left boundary. And the right hand side we have this 16 by 3 alpha x BW. This is the first term.

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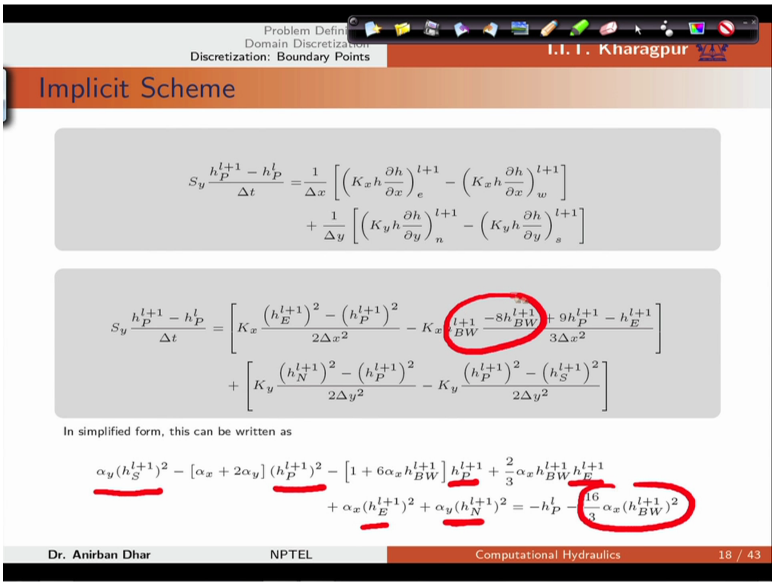

We need to transfer this on the right hand side. And again our approximation is based on this parameter alpha y equals to del t. This is nothing but alpha x equals to del t into Kx and this is Sy 2 into delta x square. This is alpha y equals 2 Ky del t divided by 2 Sy del y square.

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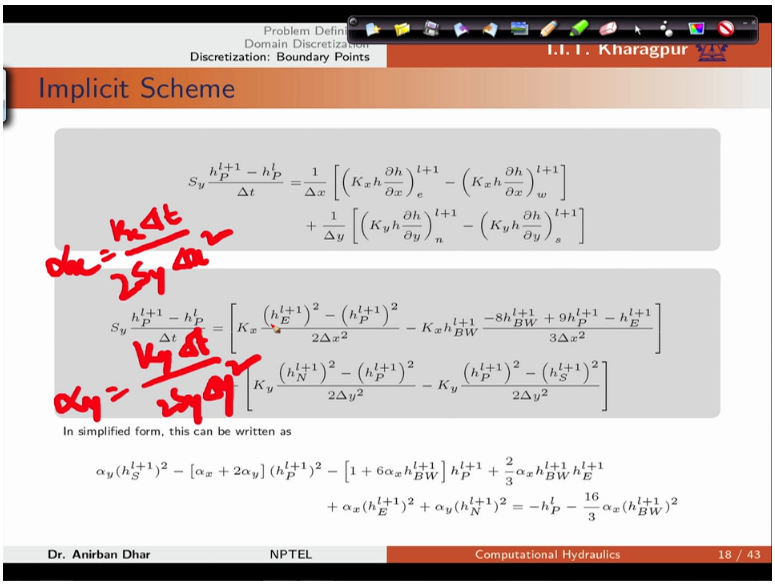

So further if we discretize this or if you write it in terms of functional format or a function format then we need to transfer the terms on the right hand side to the left hand side so that we can write it as equal to zero.

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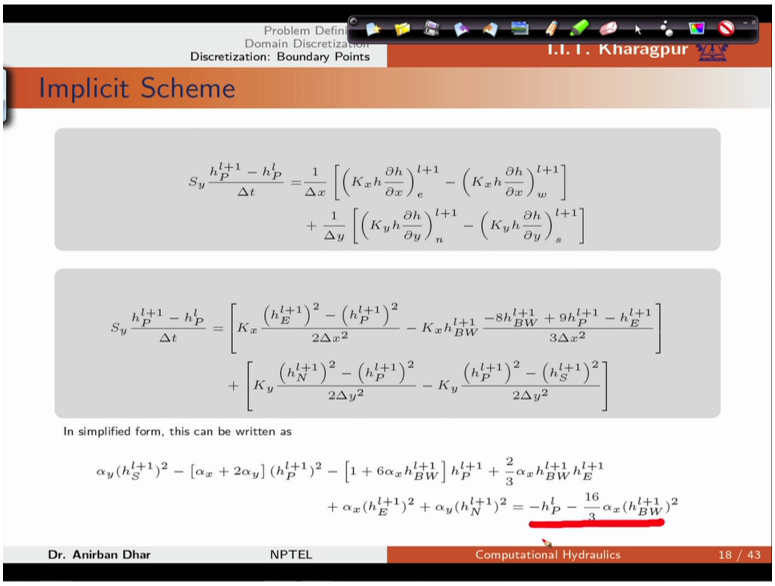

On the left hand side interestingly we have 2 alpha y which is coming from these two.

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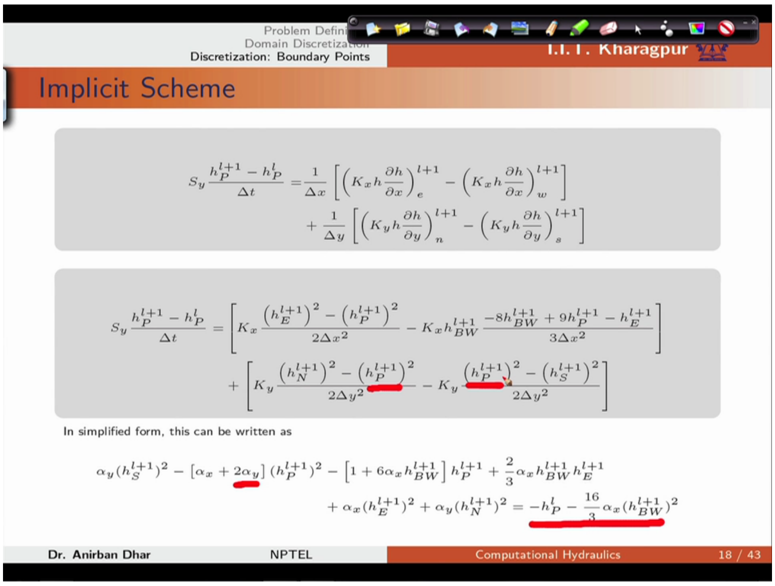

But only one alpha x which is coming from here.

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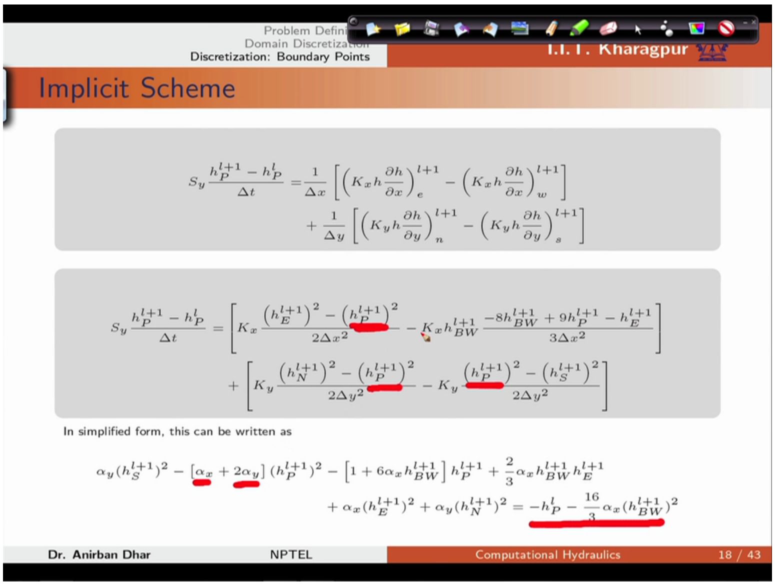

And the coefficient this 6 alpha x, this is coming from here. 3 into 3, this is 3 and from that 3 because we have considered this alpha x equals to this one. We need to consider this. Now this is 2 into 3 that is 6 here, 6 alpha x that is coming here.

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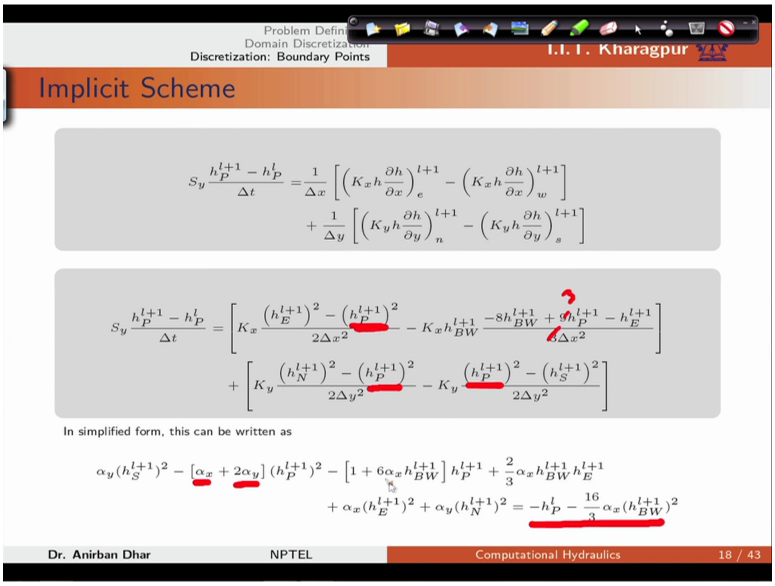

Now on the right hand side we have this alpha x. Whatever is there on the left hand side and this is from the right hand side directly coming.

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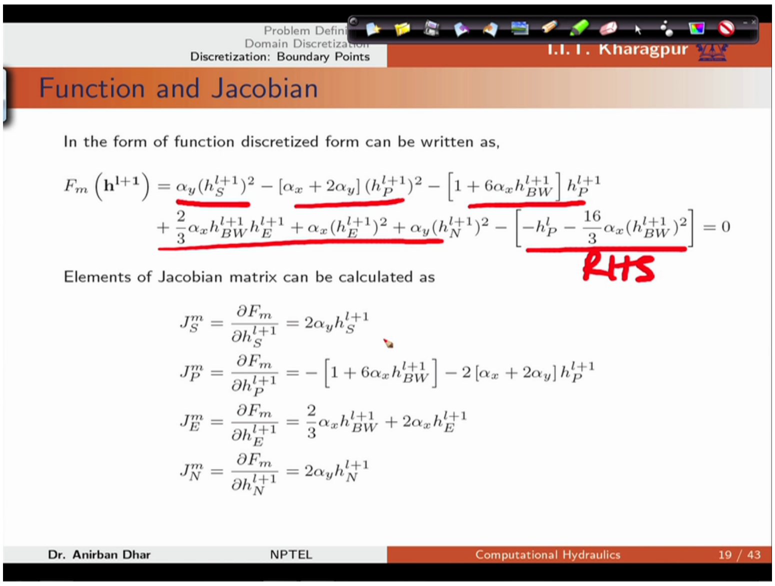

Now we have this derivative hS which is corresponding to this one. Then we have hP or JP. JP is minus 2 alpha x and alpha x plus 2 alpha y. This is 1 plus alpha x h BW. And JE again this is coming as 2 3rd alpha x h BW. And hE again there is another term with square. And there will be northern term or northern boundary.

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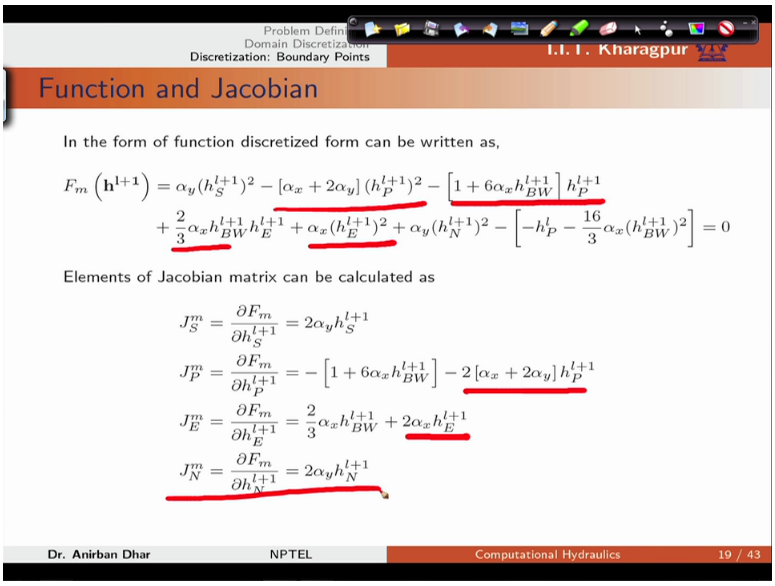

So in this case obviously we have JW and this is equal to zero. So we do not have this term in our Jacobian matrix.

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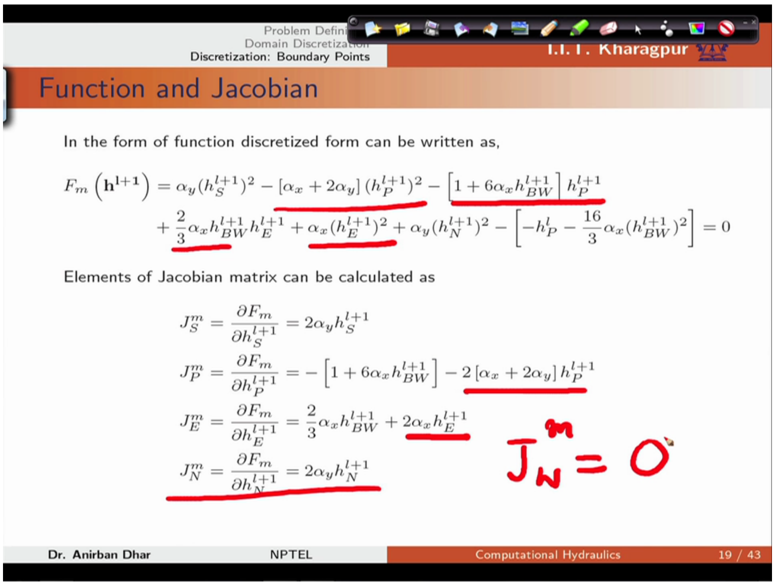

Now if we consider our right boundary, again the discretizations are exactly same as per our confined aquifer system.

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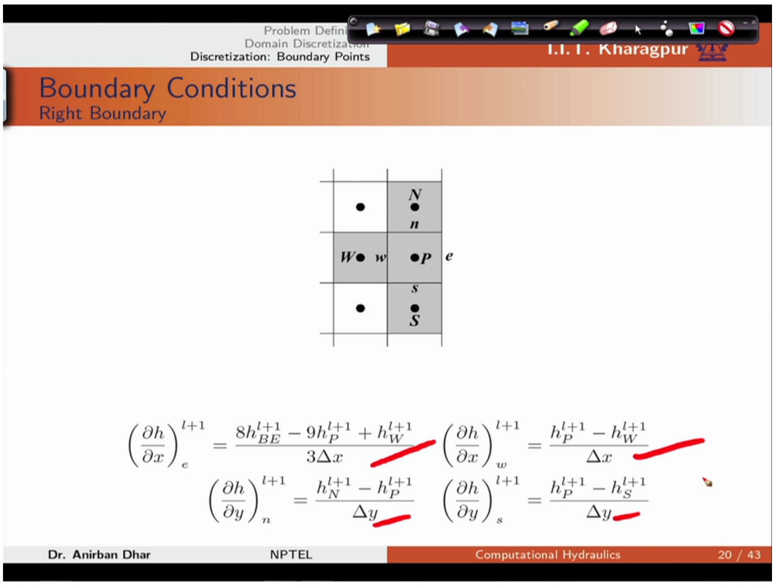

But in the next page where we have utilised the equations, the equations are different. So for eastern face this is the change. This is the only change required because other terms these are exactly coming from the interior node discretizations.

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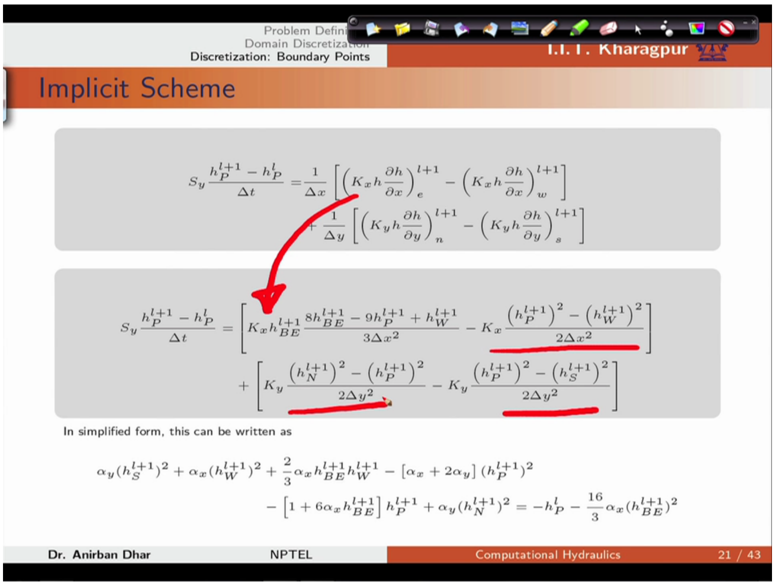

So in this case this BE term which is again specified thing is coming here on the right hand boundary. This 3 term this is coming here as 6 alpha x.

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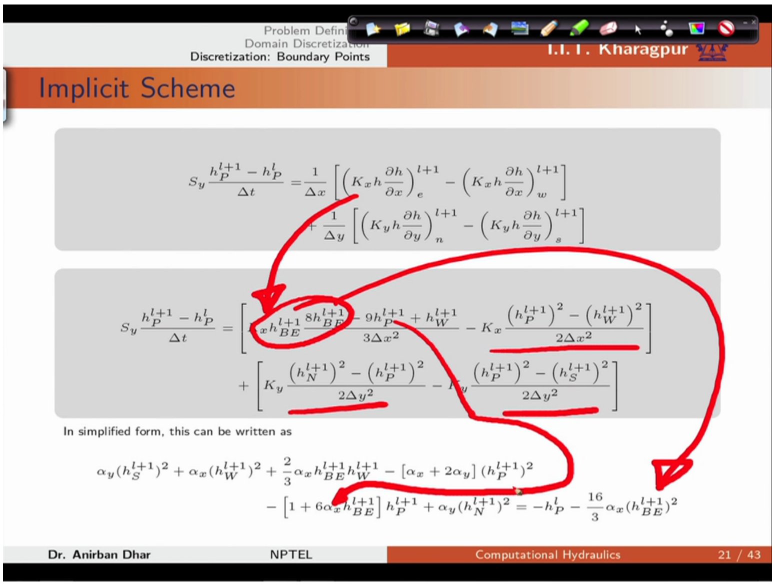

So we have this thing for our case where we are getting all terms except this hE because there should be no hE here. Only hP, hW, hP, hW, hS, hP, hN and all known values are there on the right hand side.

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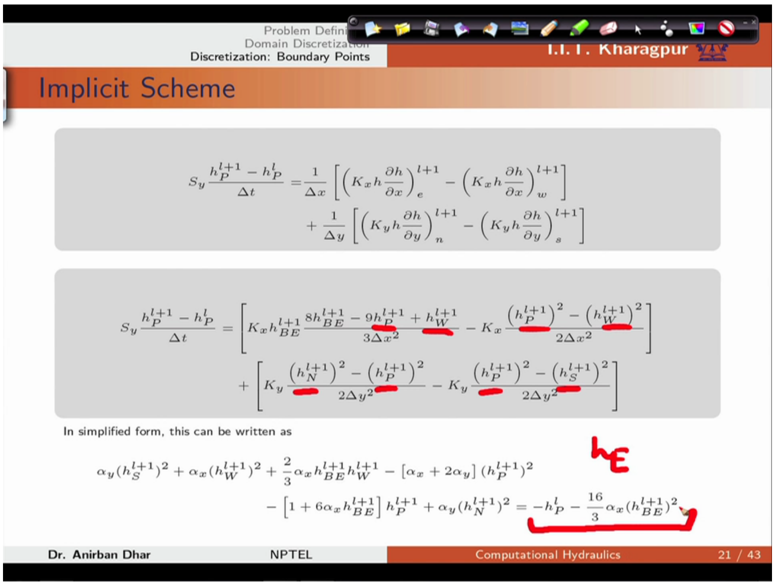

So alpha y, alpha x, this is coefficient of hS, hW and extra emphasis on hW here. And 2 alpha y, this is coming from this one, this one. But only one alpha x, this is coming from here.

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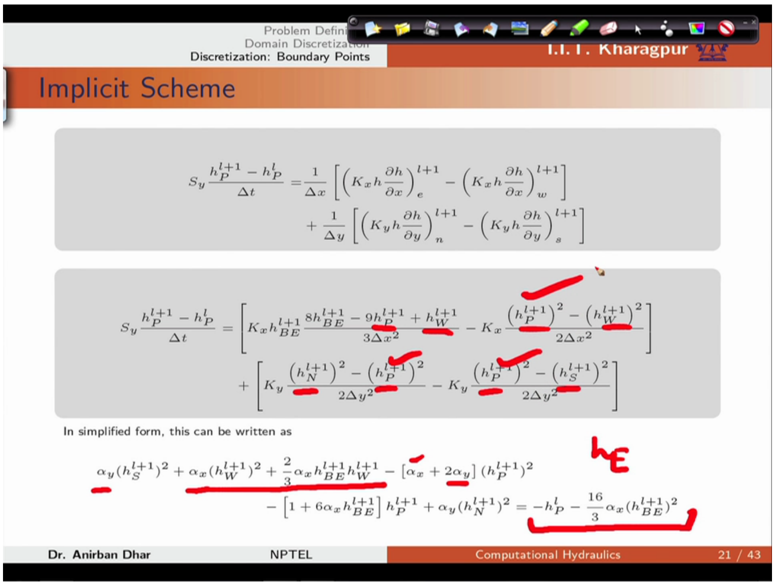

Now this is a functional form. This is left hand side, this is our right hand side.

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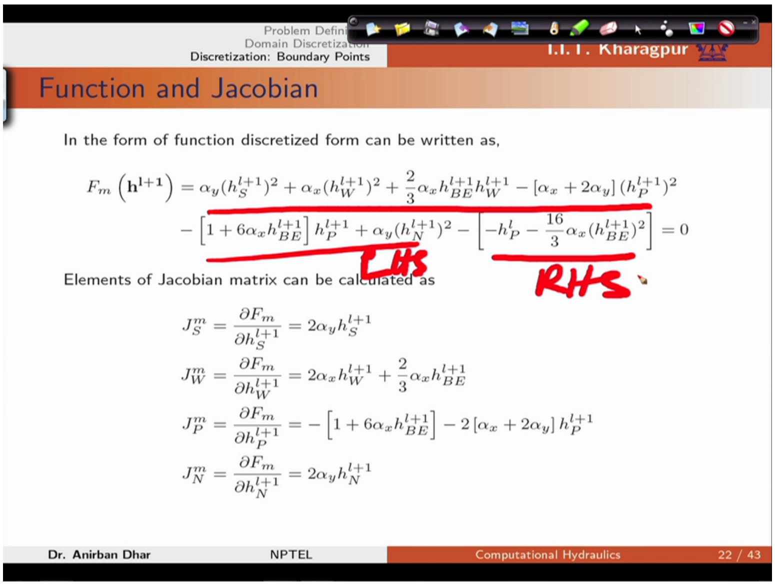

Now if you take individual elements of the Jacobian matrix we will get like this. This is 2 alpha y hS. JW, this is 2 alpha x, again 2 3rd h BE, this will be there. HP which is exactly same, 2 alpha x plus 2 alpha y hP. And finally we have hN but no hE or this case this is zero. We should remember this. This should be zero in this case because we are considering right boundary.

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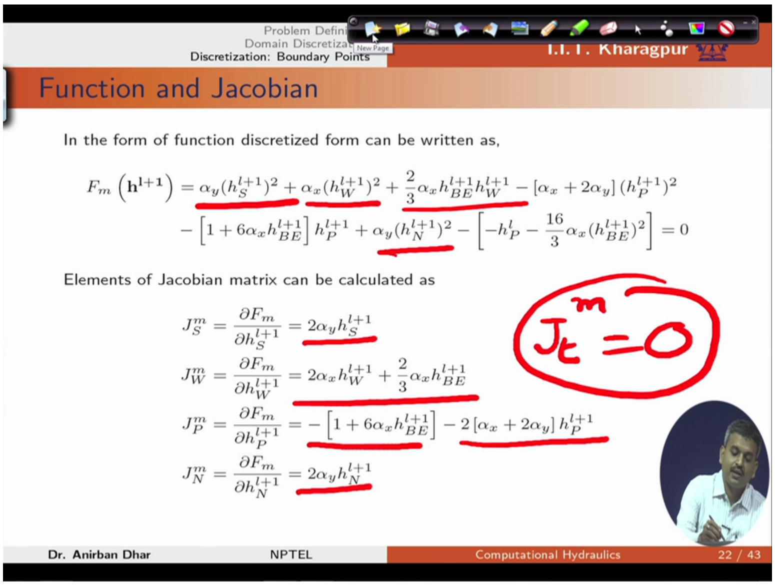

Now if we consider our top boundary. Top boundary again this point is important because we need to apply the zero condition on the derivative.

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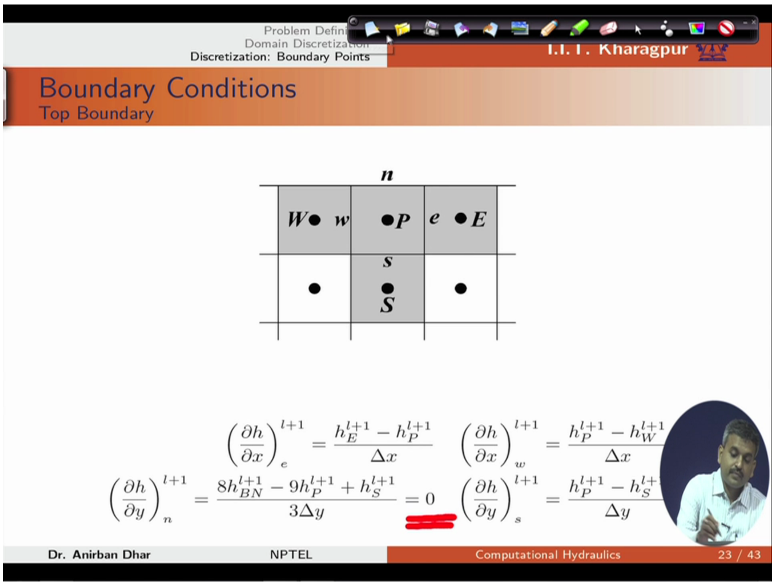

And if we apply this condition so obviously in this case on the northern boundary this is zero. And as we are considering other derivatives similar to interior points, these are exactly same like interior points and we will get like this.

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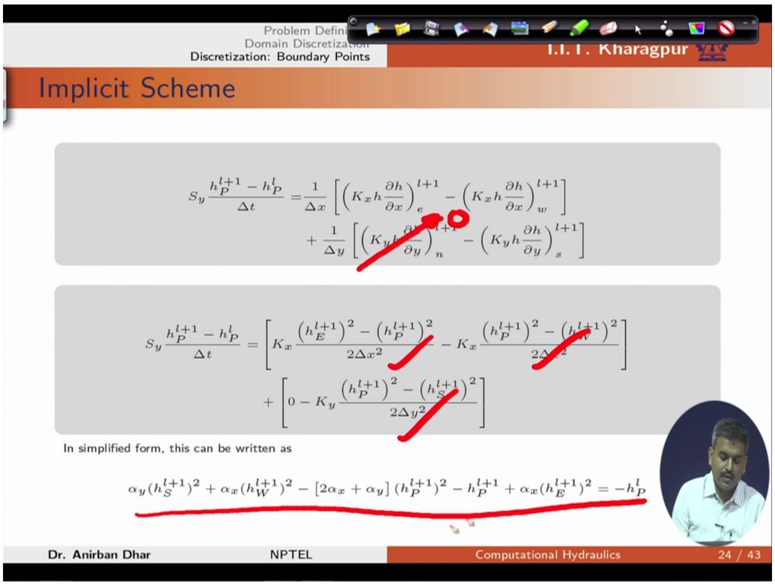

So obviously 2 alpha x because two P terms are there and 1 hP L plus 1 which is coming from here. And hP L which is there on the right hand side which is a known quantity.

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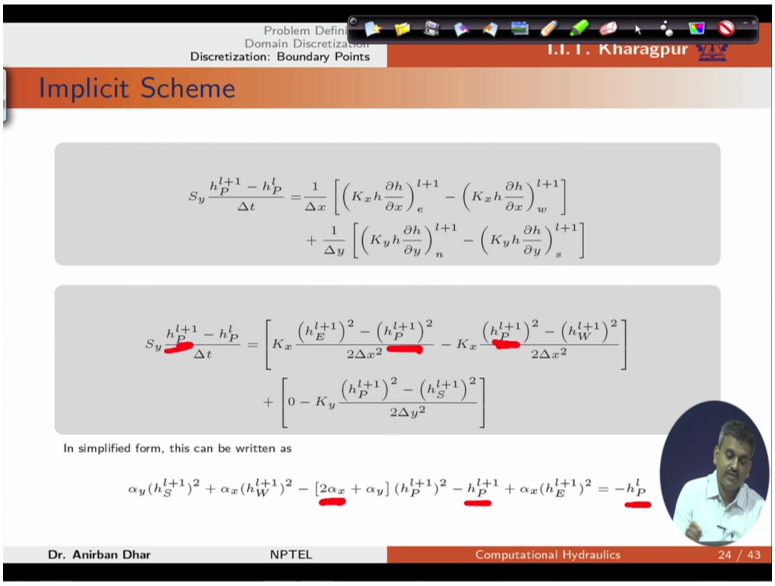

Now if I write it in function form, this is our left hand side, this is right hand side.

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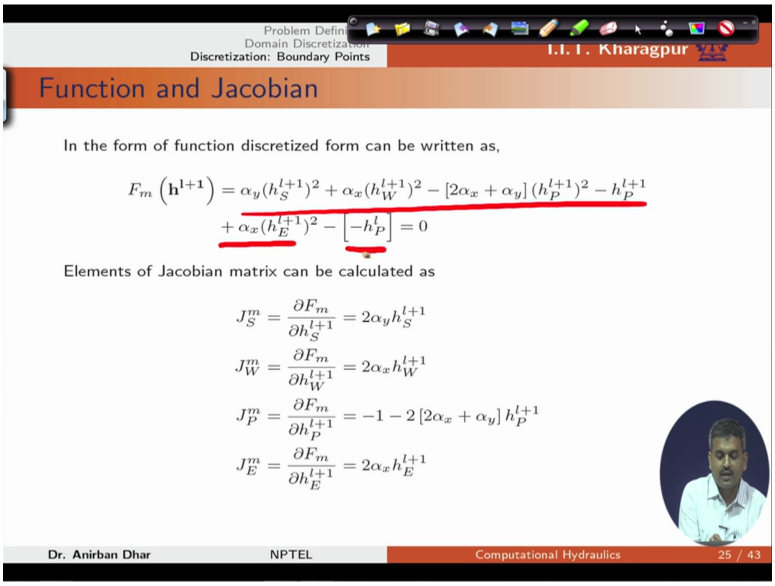

Now derivative for hS, hW, hS, hW. Then hP, for hP we have minus 1 then 2 alpha x plus alpha y into hP. This is there. And hE, obviously there should be no JNm term. This is zero in the Jacobian matrix.

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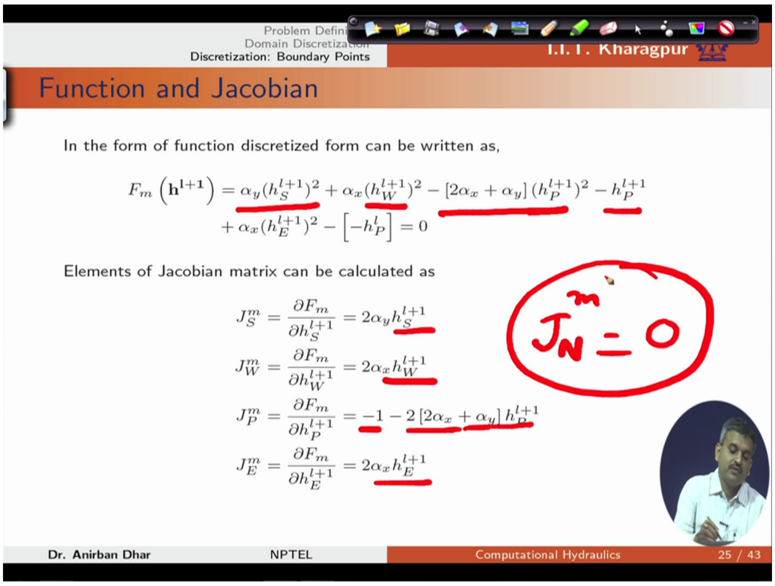

If we have bottom boundary, again for bottom boundary we need to apply this zero condition. Other terms are same like our interior points.

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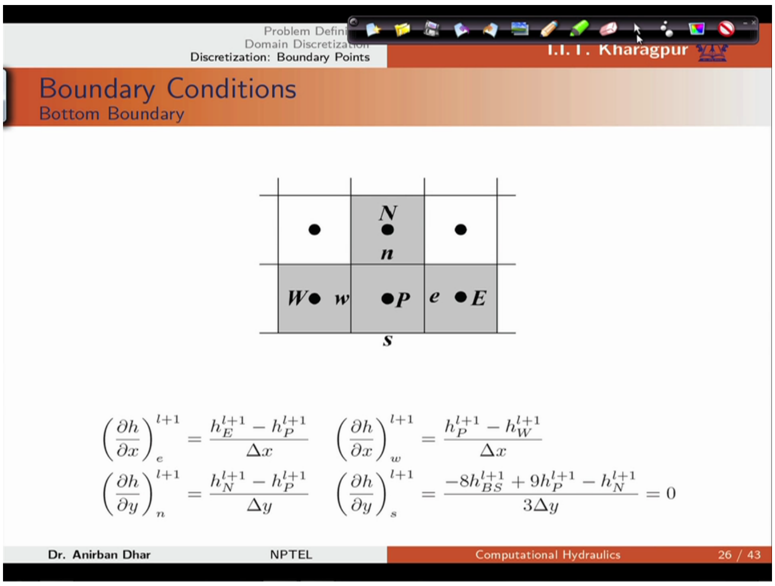

Now if we apply the same thing here so this is zero. For other faces we have these forms. Now we have 2 alpha x coming from here and alpha y only one term here and again hP L plus 1 which is coming here and alpha x E, N, W. But no hS L plus 1 term in this case.

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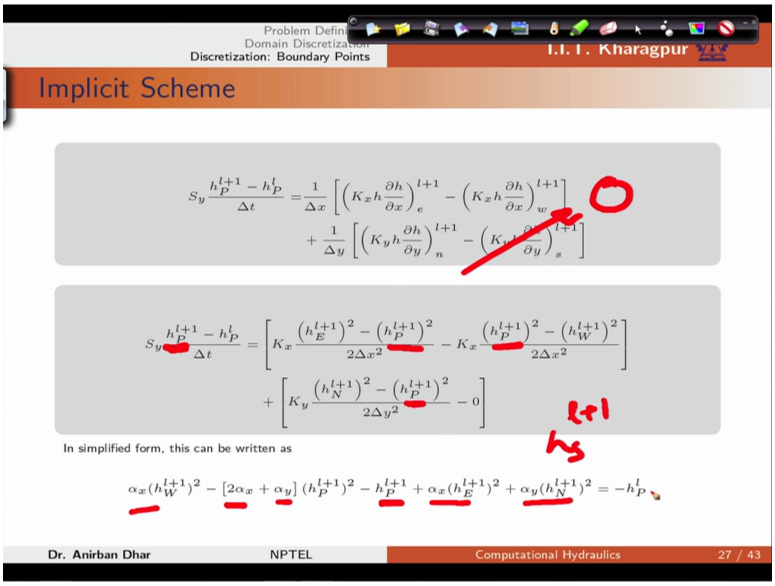

So we have left hand side, right hand side and W obviously this is 2 alpha x into hW L plus 1. Now JP is minus 1, this is 2 into 2 alpha x alpha y hP L plus 1. JE is 2 alpha x directly coming from here. JN, this is 2 alpha y hN L plus 1 from this one.

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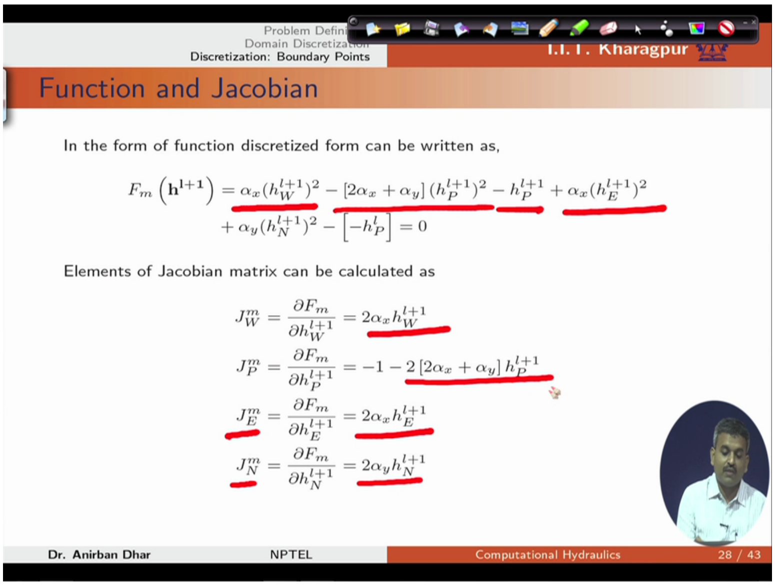

Now we need to consider this northeast or northwest corner. Northwest corner, in northern side we have impermeable boundary. Obviously we need to put the zero condition and the western boundary we need to put this Dirichlet or specified boundary condition.

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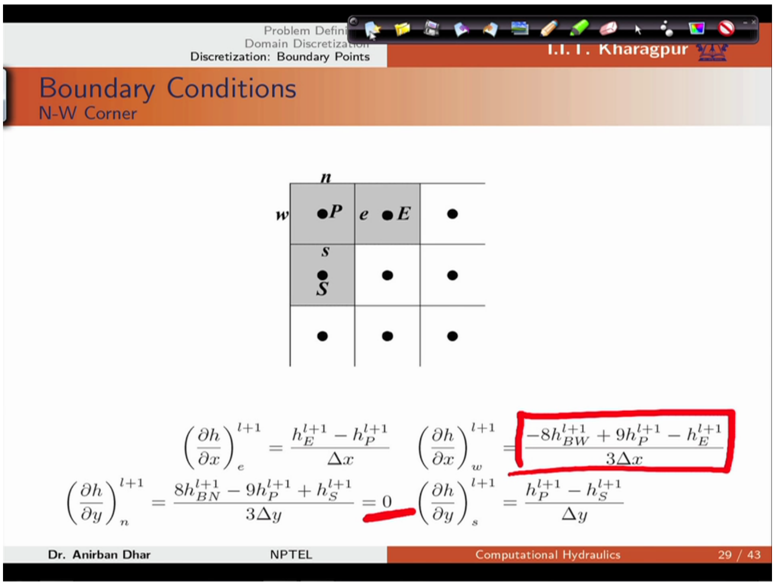

So if we apply that in this case interestingly we do not have any W term. Only E, P and P, S. That means no northern term or hN L plus 1 or hW L plus 1 here.

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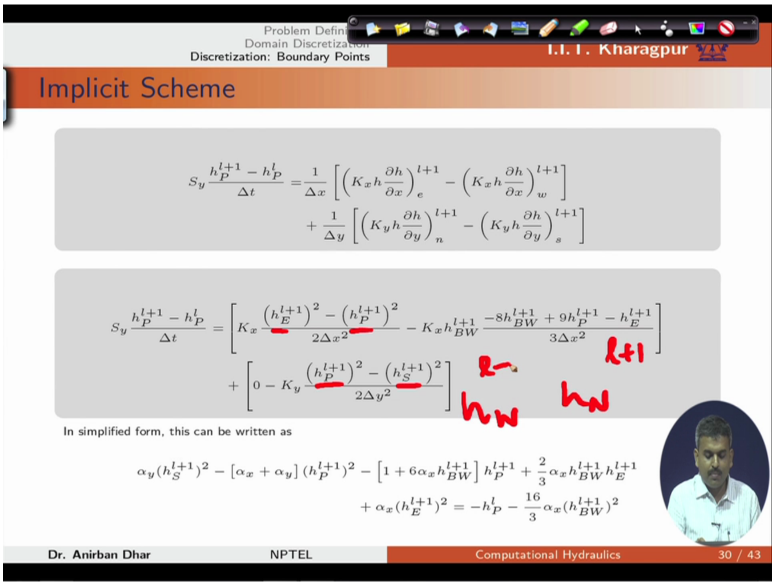

So only one here alpha x is coming here. From here we are getting one alpha y.

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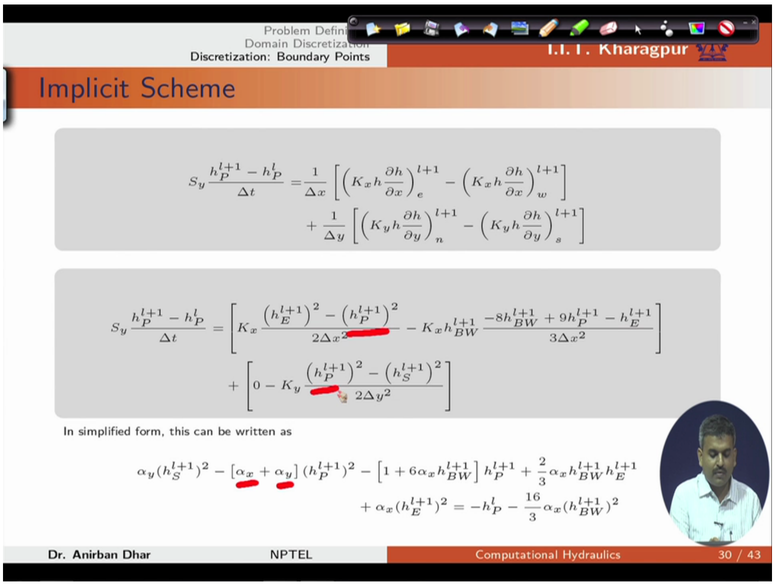

Again from here we are getting this (alp) 6 alpha x h BW and we need to transfer on the right hand side.

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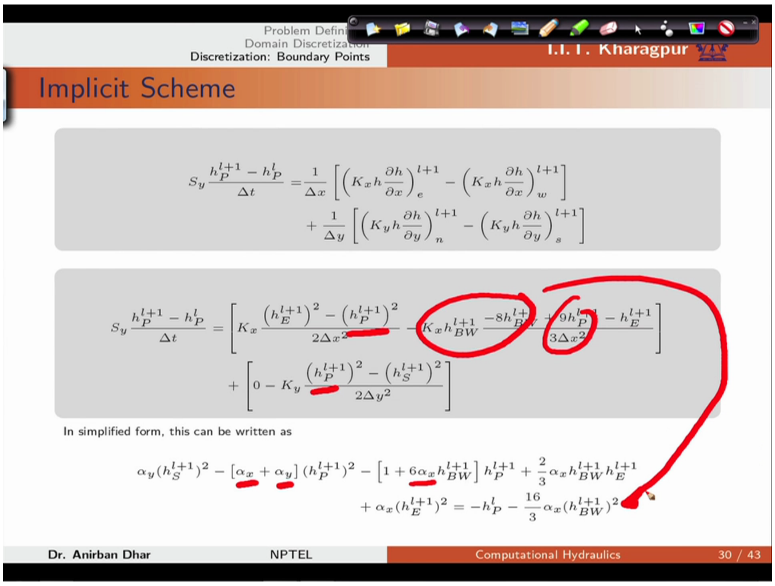

In this case this alpha y hS square, this term is there.

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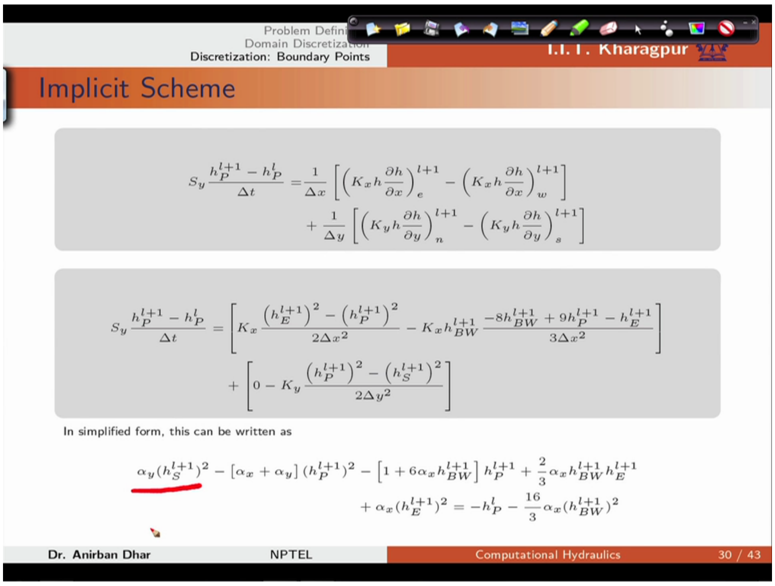

Now similarly we can write it in function format. So we have hS. This is JS. JS is 2 alpha y hS L plus 1. P is this one, 2 alpha x plus alpha y. This is 2 by 3 alpha x h BW L plus 1, 2 alpha x hE L plus 1.

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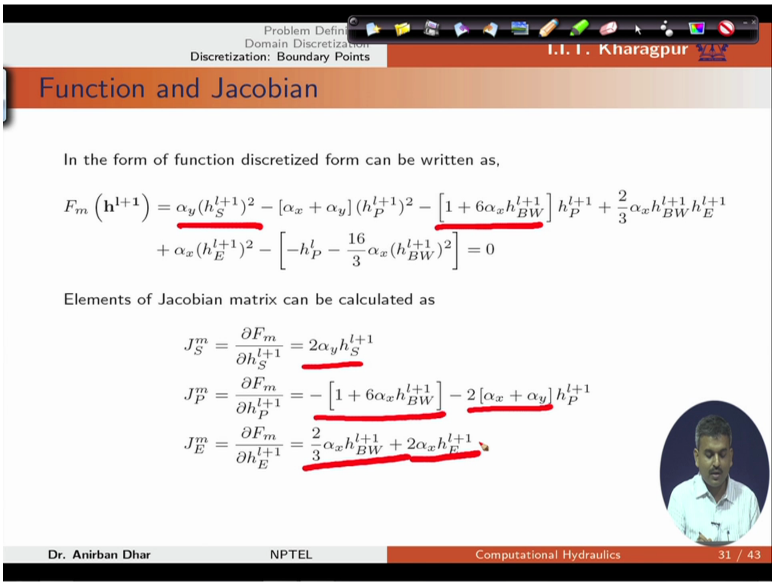

So we do not have JWm, this is zero and JNm, this is zero. So in Jacobian matrix we will not find these two terms.

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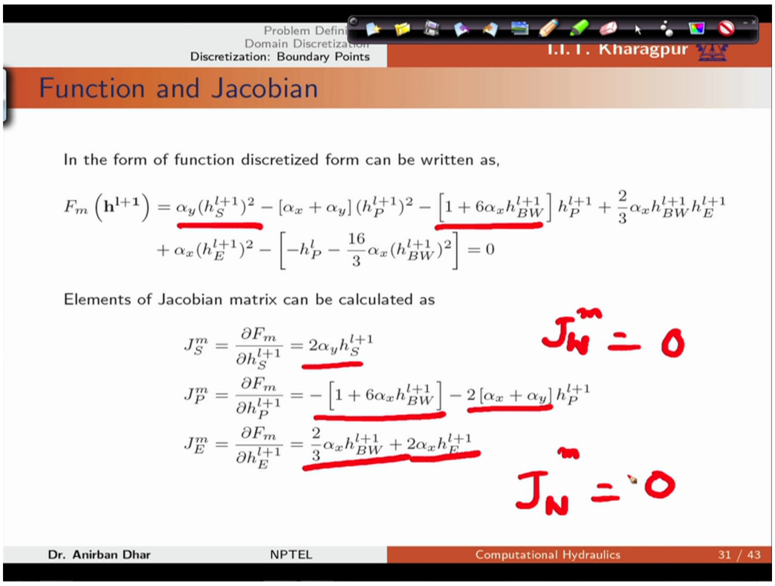

Now if you consider our northeast term, if we have northeast, on the northern side we have zero and eastern side we need to specify the specified boundary condition.

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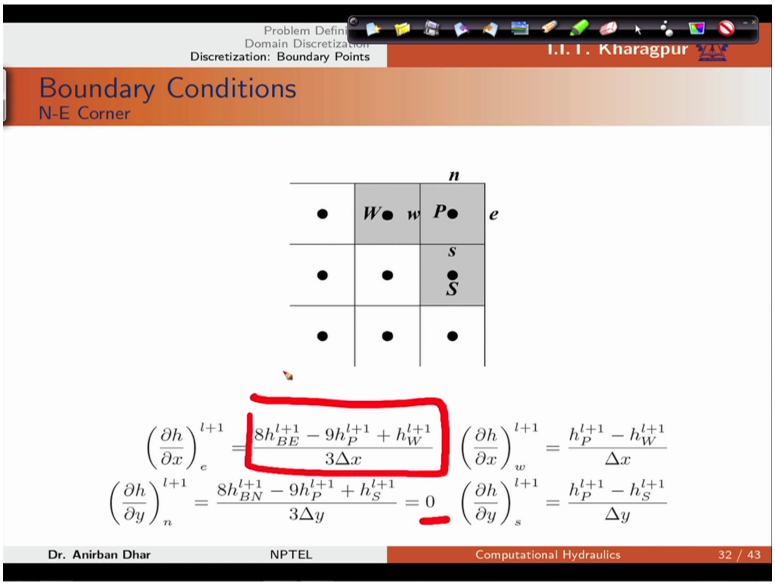

Now this is exactly same as our specified boundary condition. And northern side we need to put zero condition. And this is the condition that we have utilised from the specified boundary condition.

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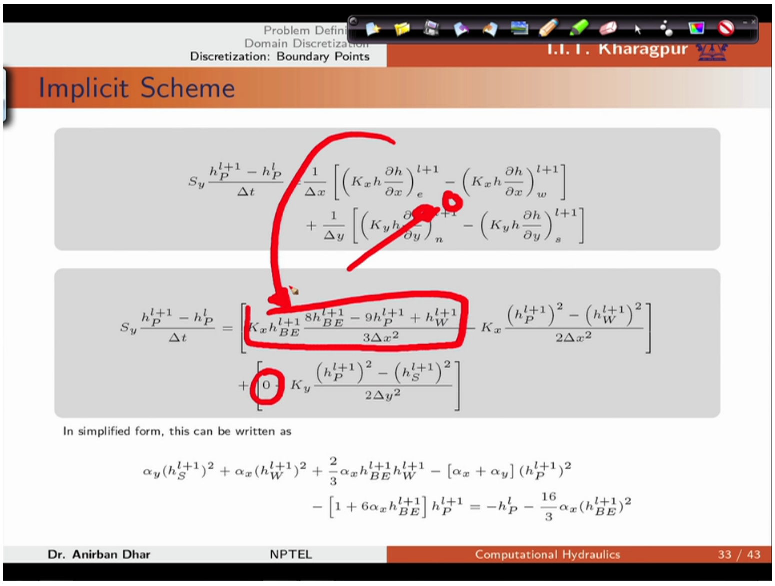

Now we will get hS, hW, hW, hP, hP. That means no N L plus 1 or E L plus 1 term here.

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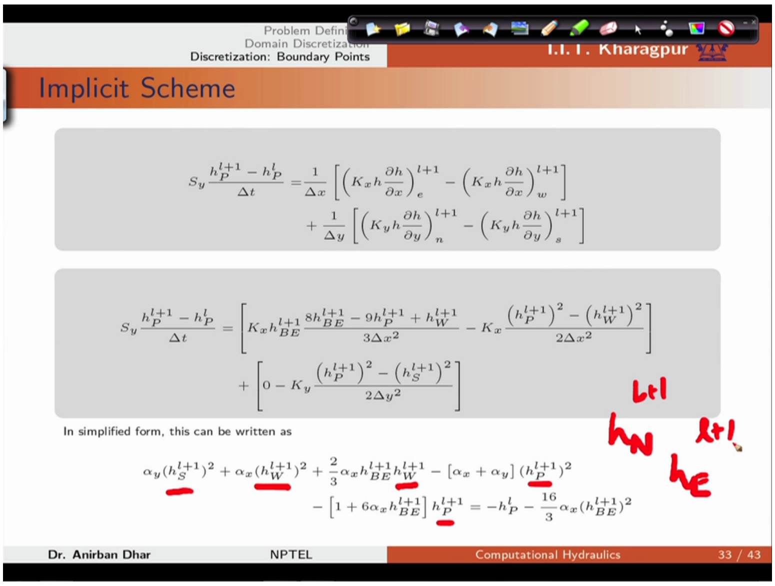

So if we consider our function format again 2 alpha y hS. This is coming as western. So from western side 2 alpha x hW L plus 1 and 2 3rd alpha x hBE L plus 1, this is there. This is JP and this is 1 plus 6 alpha x BE. This is directly coming from here. And this is 2 alpha x plus alpha y. This is the term that is coming from here.

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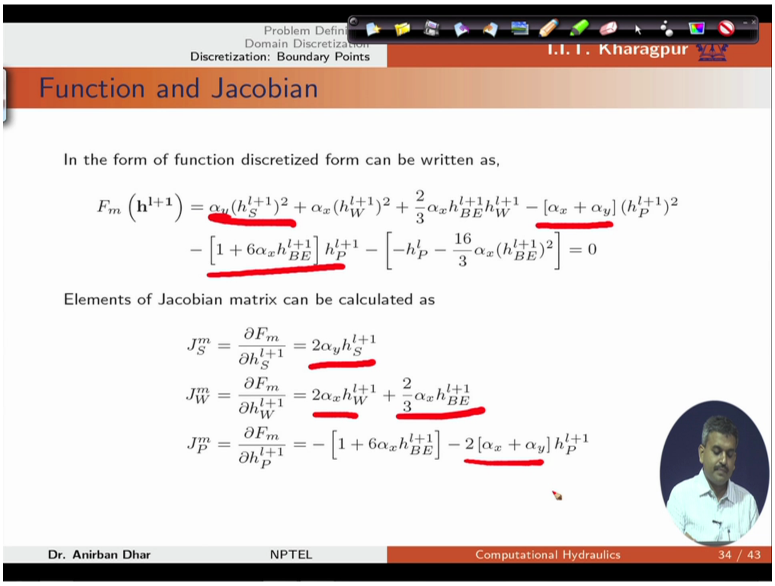

Now we do not have any JEm. That means this is zero. Then JNm, this is again zero. These two terms are zero terms.

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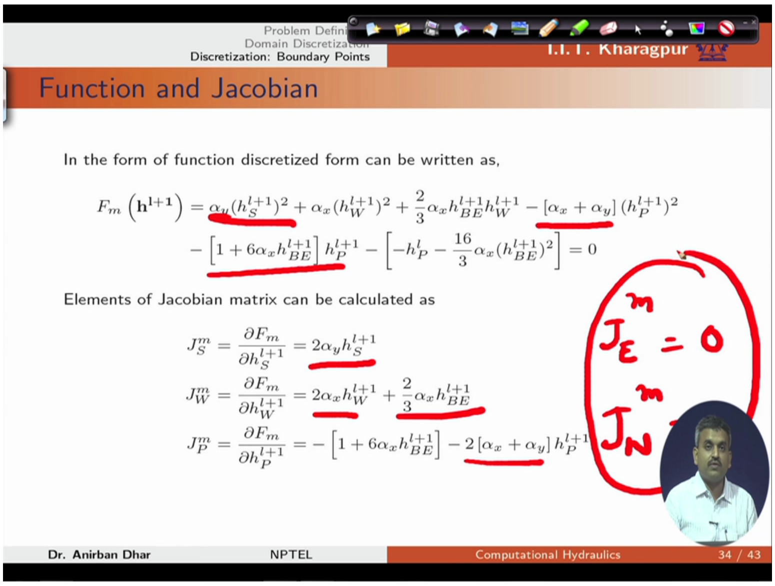

Now the southeast corner. On the southern side this will be impermeable. So obviously this will be zero. On the eastern side we have specified boundary condition.

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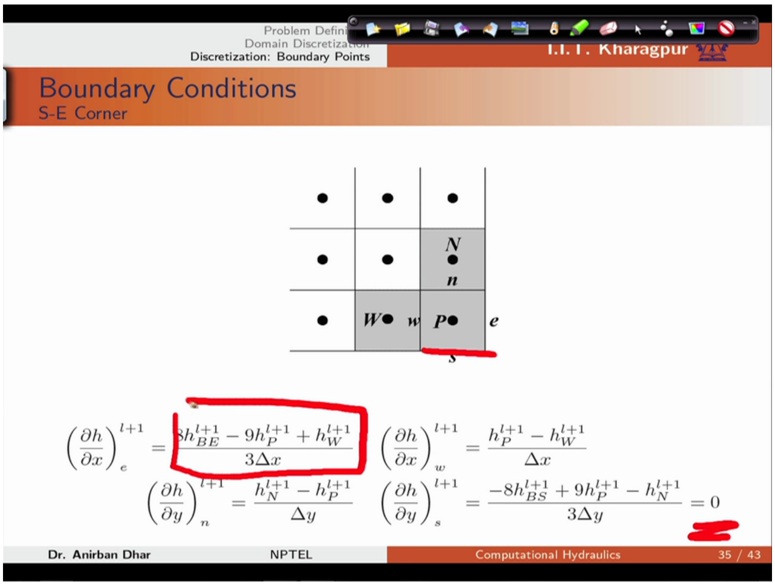

So for specified boundary condition this is zero on the southern side. On the eastern side we have a specified condition which is directly coming from here. So we will have western term or hW term, hP, hP, hN. So no hS and no hE term here.

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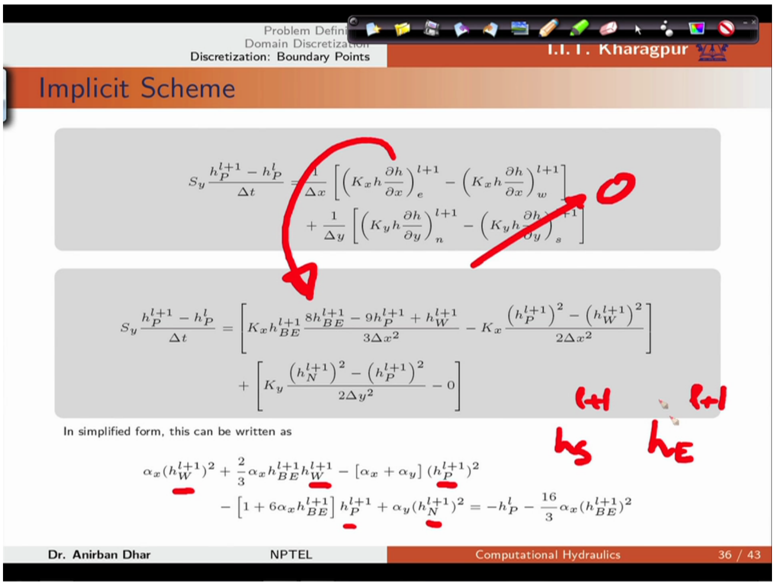

Again we can get equivalent function form. So from this JW, this is directly coming as 2 alpha x hW L plus 1, 2 3rd alpha x BE. JP, this is coming as 2 into alpha x hP L plus 1 and this term is there 1 plus 6 alpha x BE. And on the northern side we have 2 alpha y, this hN L plus 1. And this is our right hand side which is the known quantity. Now we do not have any JSm that is zero and we do not have any JEm zero. So these two terms are zero terms.

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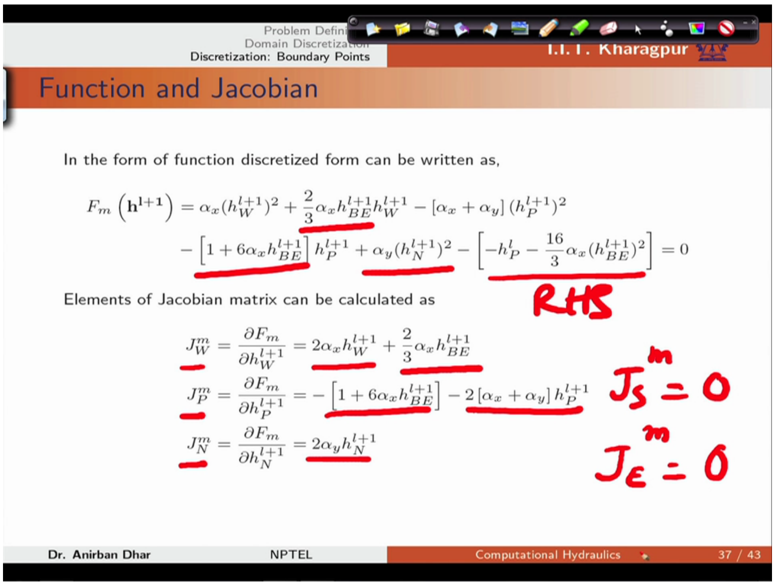

Now if you consider southwest corner, the approach is similar. We need to put zero condition here.

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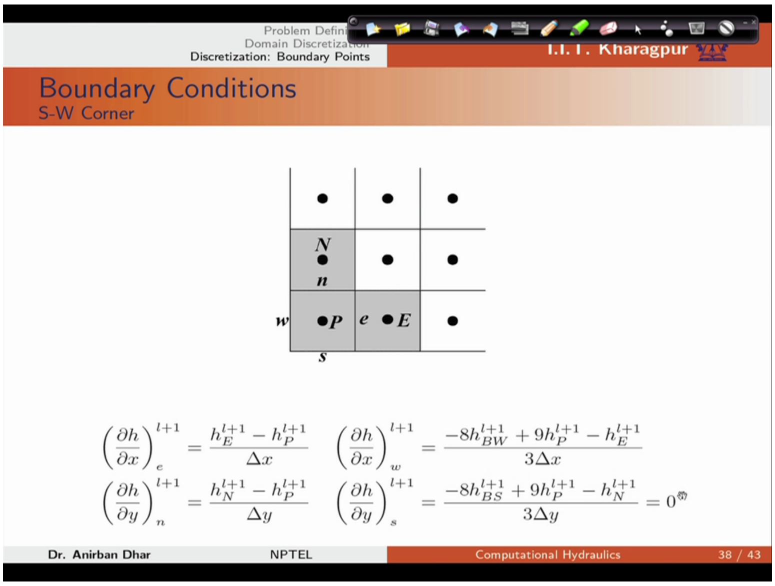

For southern boundary this is zero and for western case we have transferred these values and derivatives.

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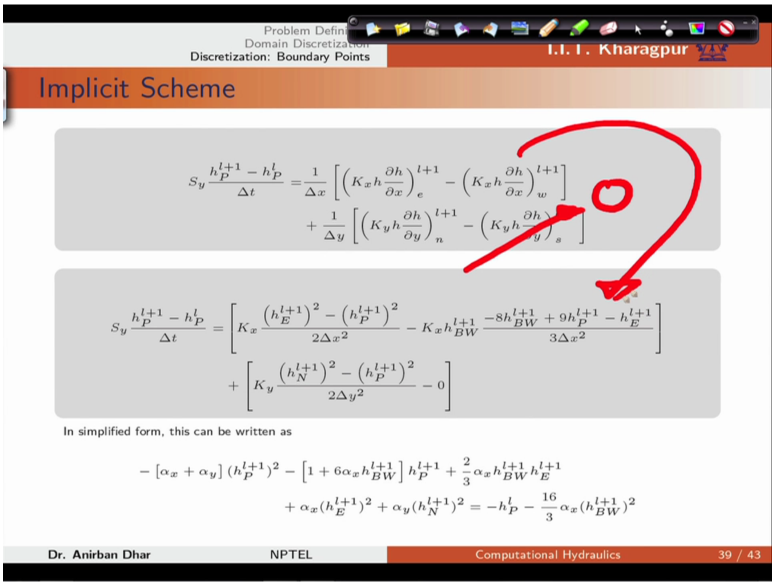

So for western boundary we do not have any hS L plus 1 or hW L plus 1 term. So directly we can get this discretized form.

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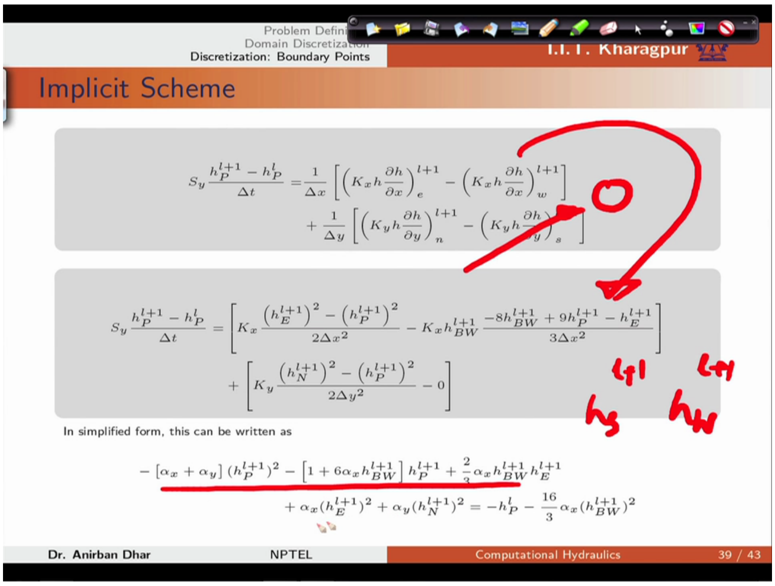

And this is the function in the form of function discretized form can be written. This is our form. Again JP, JP is coming from here, this is 2 alpha x, this is there. This is 2 3rd BW hW, 2 alpha x hWE, 2 alpha x hE coming from here and 2 alpha y hN L plus 1, this is directly coming. So we do not have HSm equals to zero and HWm, this is equal to zero. So this is the condition for our Jacobian matrix.

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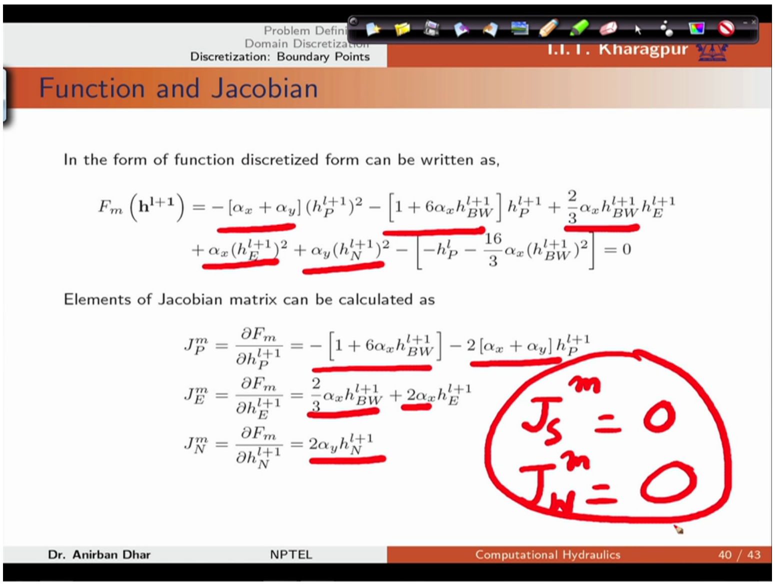

So we have defined all four corners or corner cells and all boundary cells here. Now we need to see how we can solve this problem. In general form the governing equation including the boundary conditions we can write like this. So this is nothing but JS into del hS L plus 1, this is JW, this is JP, JE, JN. So this is same. On the right hand side we have this minus Fm.

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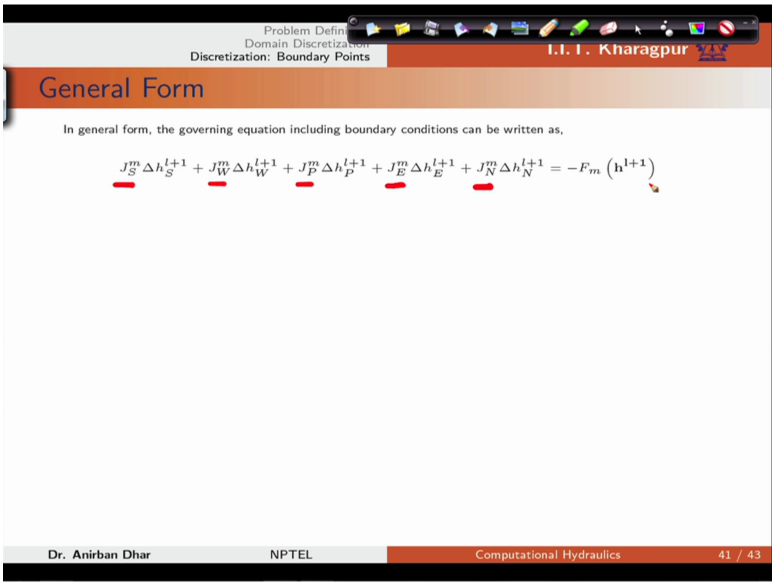

Now in this case we can solve this through iterative form. This hS can be expanded like this, hW can be expanded like this, hP, hE, hN. And on the right hand side Fm, this can be expanded in terms of hS, hW, hP, hE hN. So I have changed the colour for hS hW in this case.

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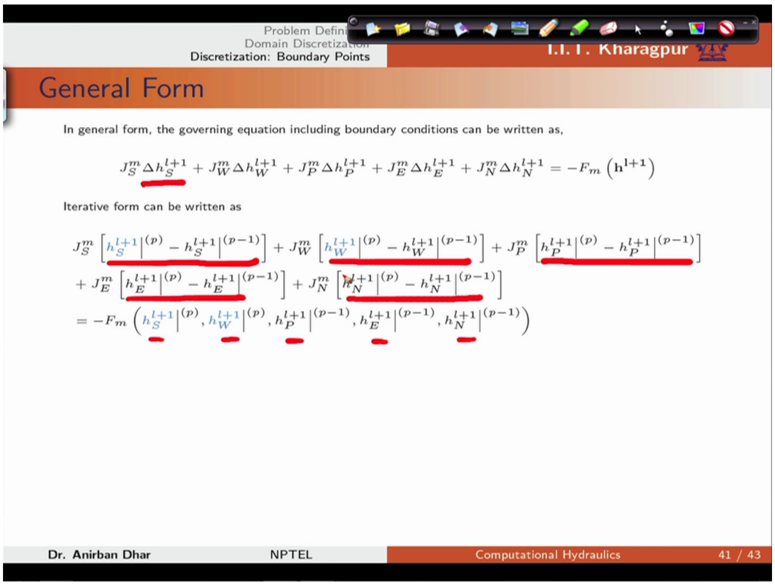

Why? Because if we consider our problem, then we have central cell which is our cell P. This is northern, this is southern cell, this is western, this is eastern. Now during calculation process we will start discretizing our domain into number of cells and we will start our calculation from this point.

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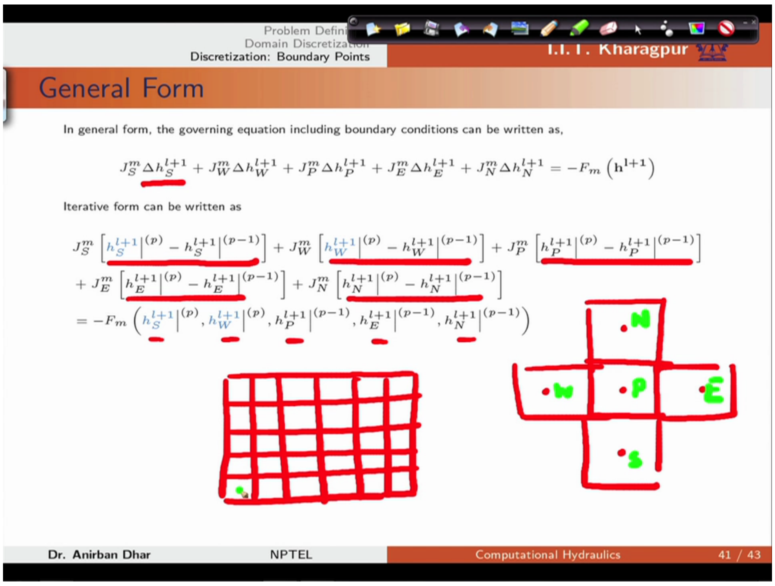

So from this point we will move towards this. Again we will start from this, again move like this. So in this case what is happening, first we are solving S then moving to this direction and we are coming back here and going to this W cell.

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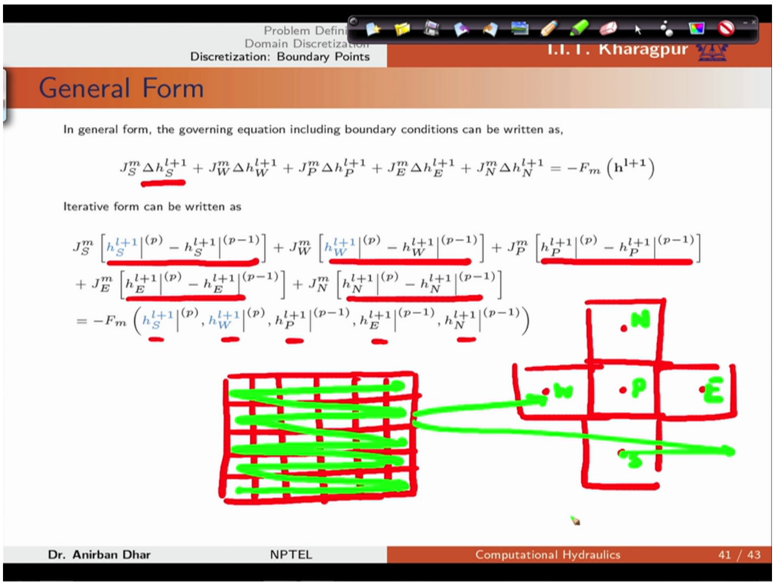

So for calculation of P, the value of S cell and W cell, these two are already updated values available from the calculation process. So I have mark these values as hS p, hW L plus 1 at pth iteration. This is again pth iteration, this is again at pth iteration. We have updated values available.

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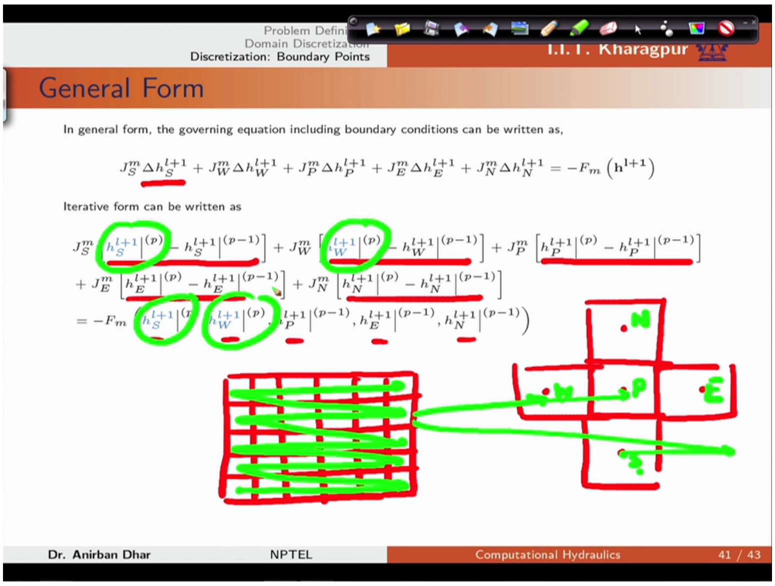

Now in this case further I can just simplify this and write it in final iterative form like this. So I can consider this value and transfer all other terms on the right hand side. So I will write this residual. So in this case residual excludes the central term. So we have JS, JW, JE, JN. Obviously in this case it is a pth iteration value itself available.

So I have written in terms of pth. And this is P minus 1 level values are available. That is why I have written in terms of P minus 1. So I can calculate this residual and divide it by JPm that is directly coming from here.

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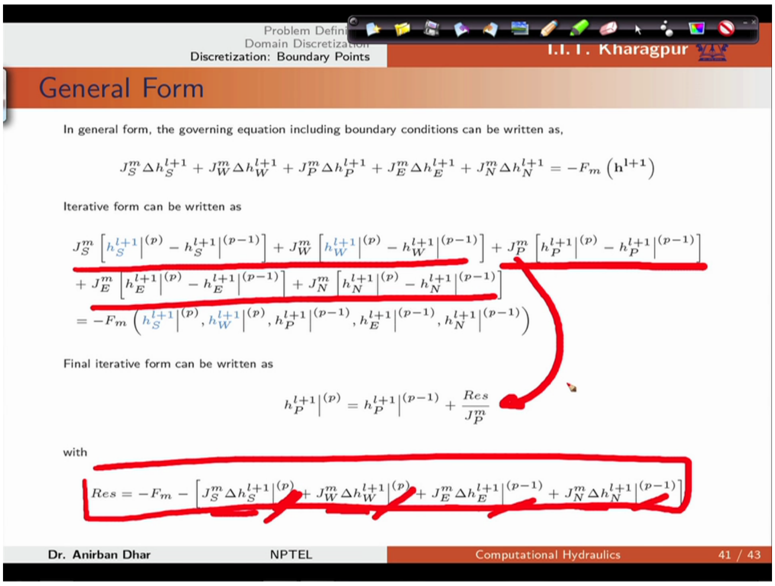

So this is the iterative form. Now what I can do? I can start the iteration and get the converged value of hP L plus 1. After getting convergence through the iterative format I can transfer this value to hP L. Then this will become my new old time level value which is calculated from the future time level and I can iterate for the time.

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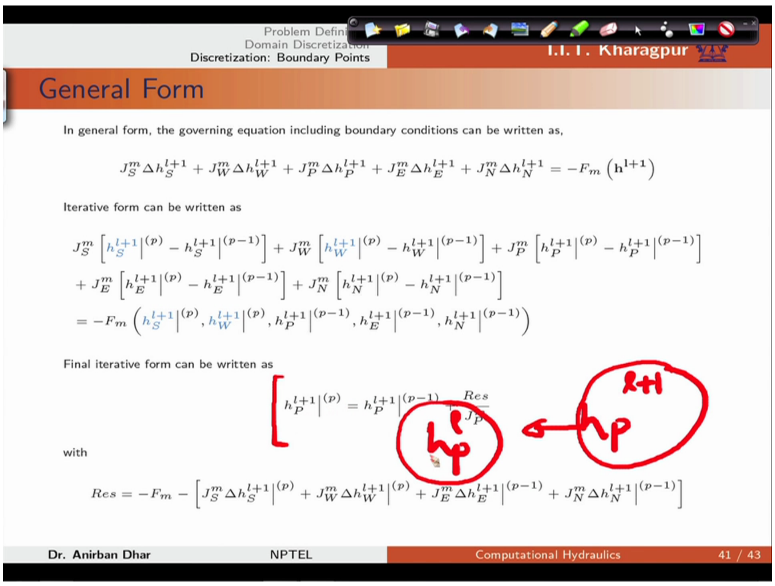

So initially for any problem we have time loop and space loop. Now space loop we can solve using this iterative form and we can update time and for updated time we can transfer this n plus 1 level value to nth level directly.

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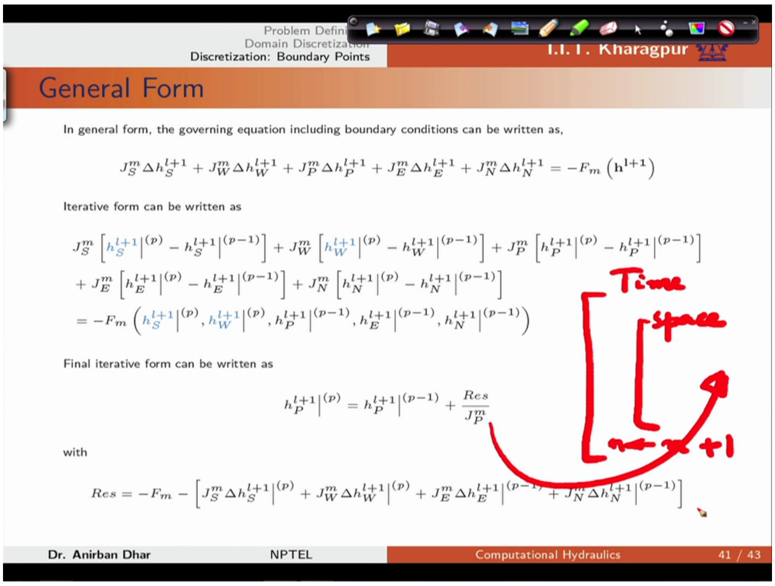

So this is our unsteady 2D finite volume unconfined aquifer implicit iterative format.

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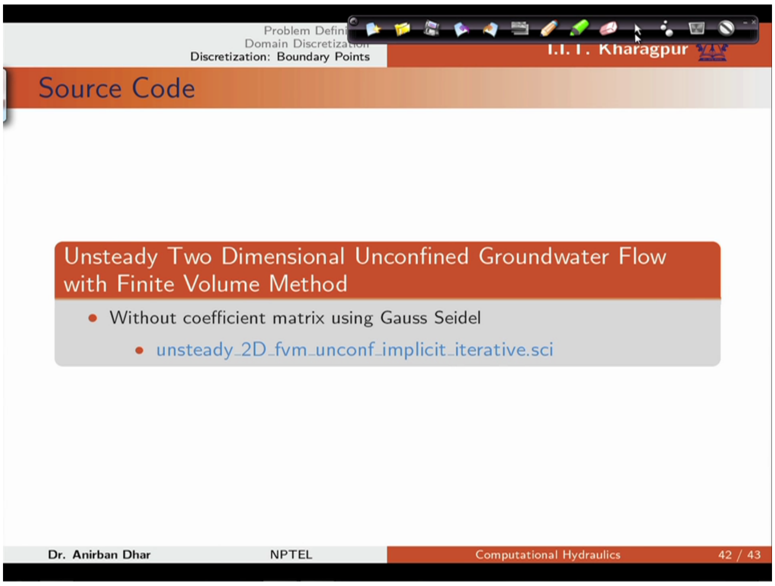

Now what we can do we can solve it using our unsteady equation. Now in this case again we have mnode, mcell, nnode, ncell, these are available. And Lx, Ly, these values are available. Time max I have written as 50. HA, hB, hC, hD, these values are directly available.

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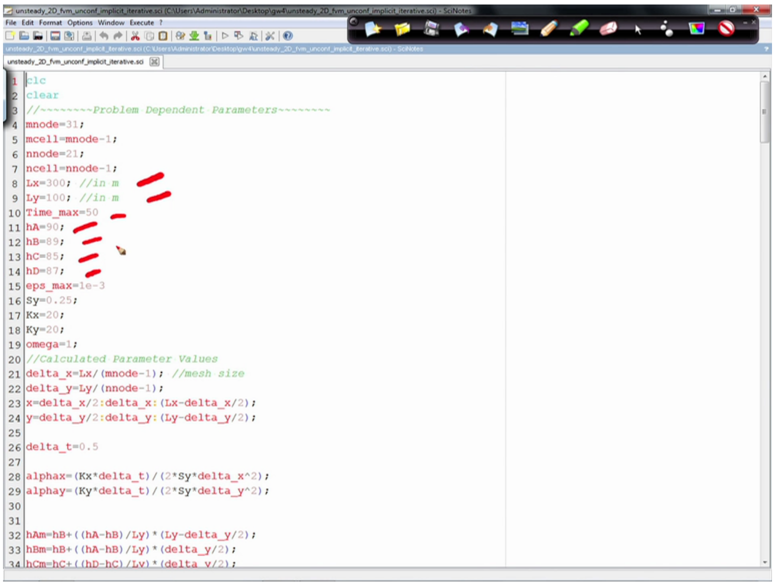

Eps max, this is required for convergence. Sy is point 25. Kx Ky, these values are 20 metres per day.

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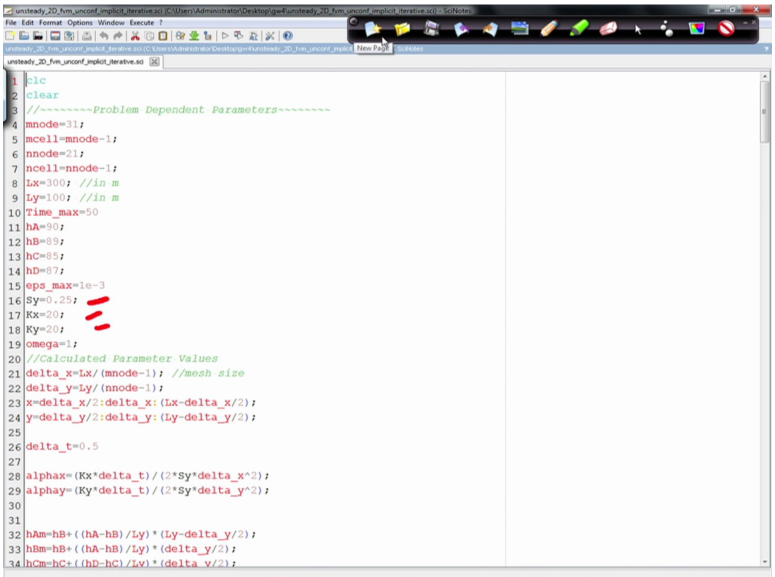

Now for this one we can further calculate Lx Ly values. This is delta x, delta y. So obviously for Lx direction we are discretizing the cells like this. This is equal but we are starting from, this is dx by 2 and we are ending dx by 2 before that. So in this case for x we are starting from delta x by 2 and running up to Lx minus delta x by 2 and we are getting the self centred values here.

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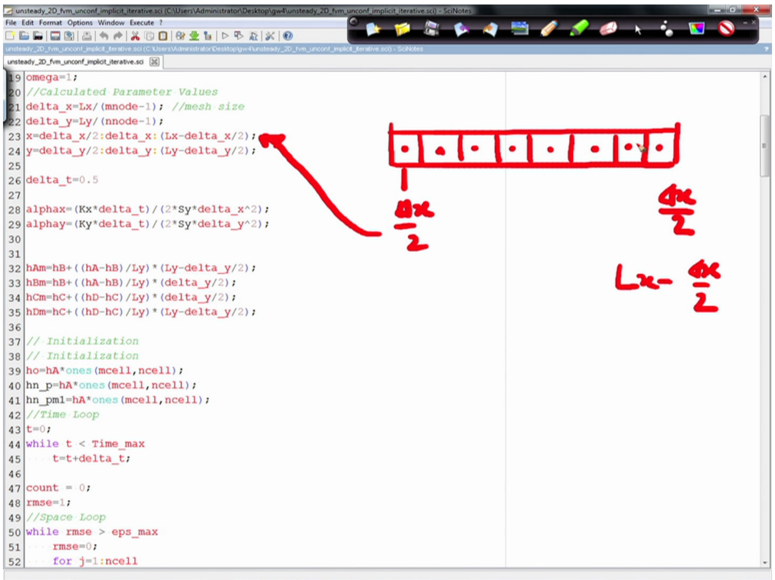

Y similarly we are writing it like this. So from delta y by 2 we are starting and we are going up to Ly minus delta y by 2. So that is why I have written it. Now delta t is point 5 in this case.

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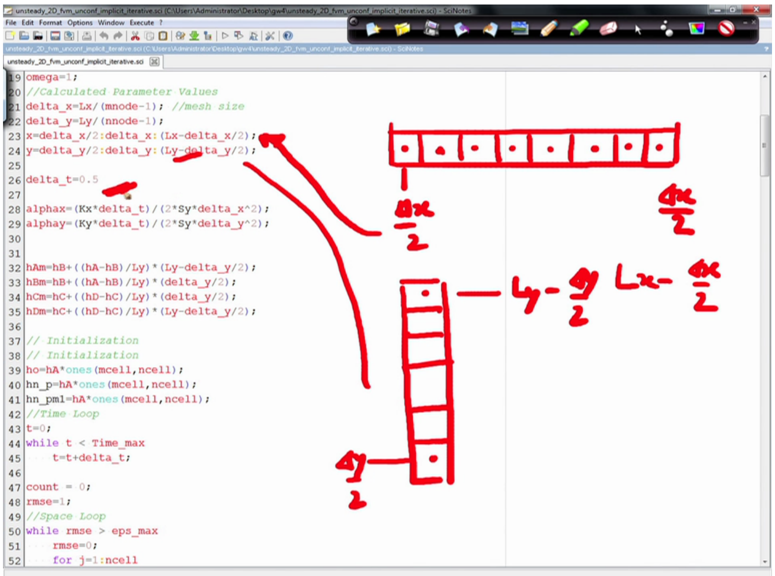

So alpha x, alpha y we have all values available. So now if we consider hA, hA is nothing but this value this corner value. HA is corner level value. This is B, this is A. But we need this value which is not at the centre. Now I can write this as hA modified value and I can write this as hB modified value. So, that I can calculate from here.

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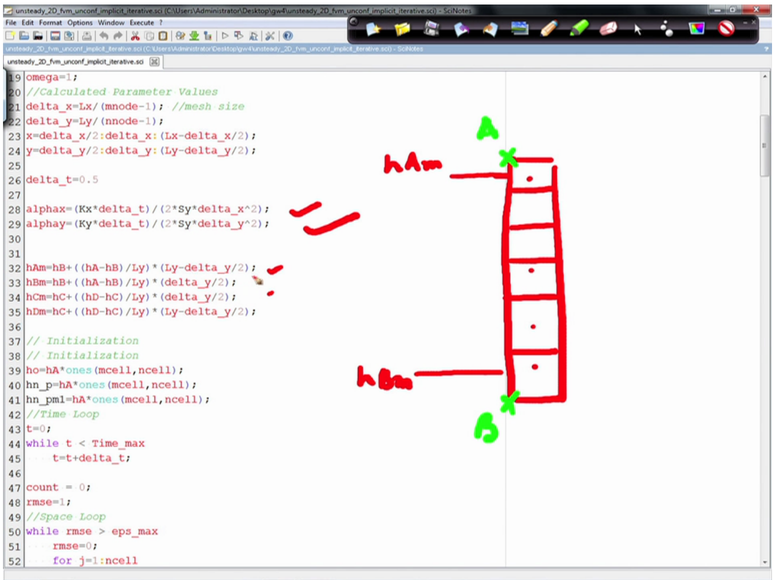

Similarly for C and D, in this case C point will be there. This is C, this is D. So obviously in this case also we will have that hD modified for this level and hC modified for this level.

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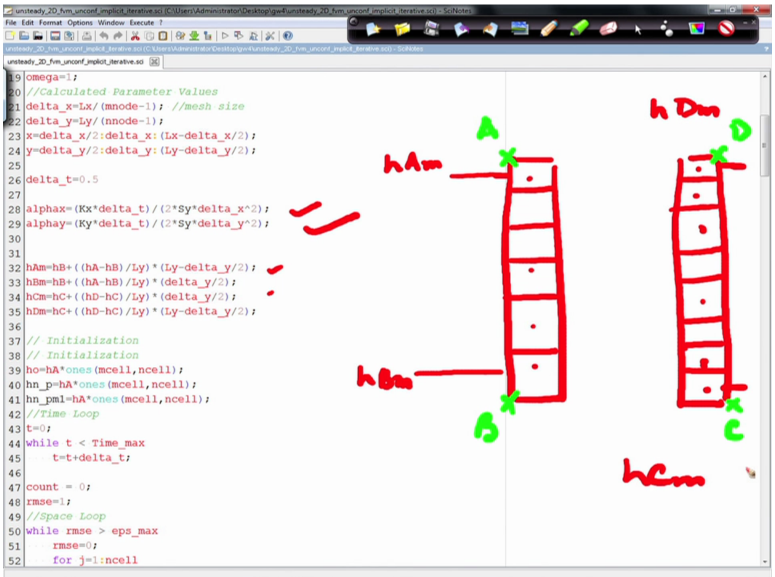

These are the values directly coming from here. Now ho which is old time level, hn P which is for pth iteration. This is hn P minus 1, this is P minus 1 level.

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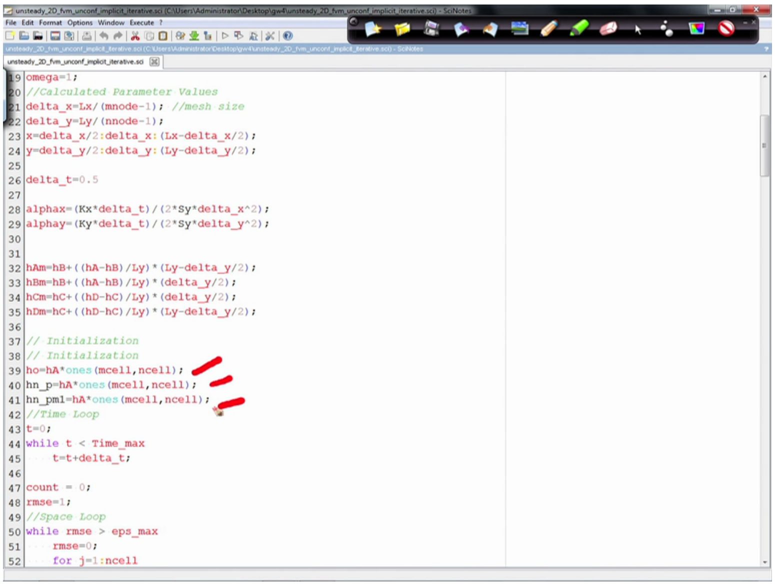

Now in this case again the approach is same. We are writing the JS, JW, JP, JE, JN terms for interior cell. This is for interior cells and this is Fm. Now we need to calculate dS which is the difference between hP and hP minus 1 level for dS, dW, dE, dN.

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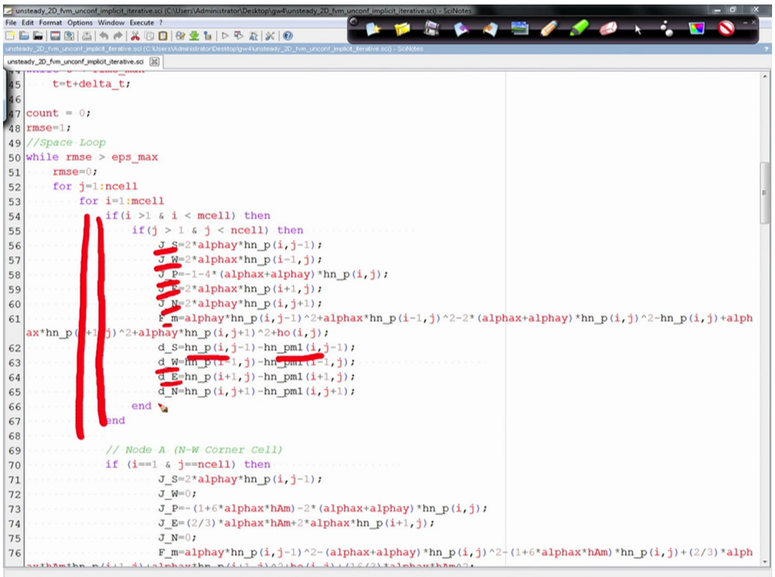

Now in this case we should also calculate the northwest corner, southwest, southeast corners. So depending on that JS, JW, JP, JE, JN values will be there.

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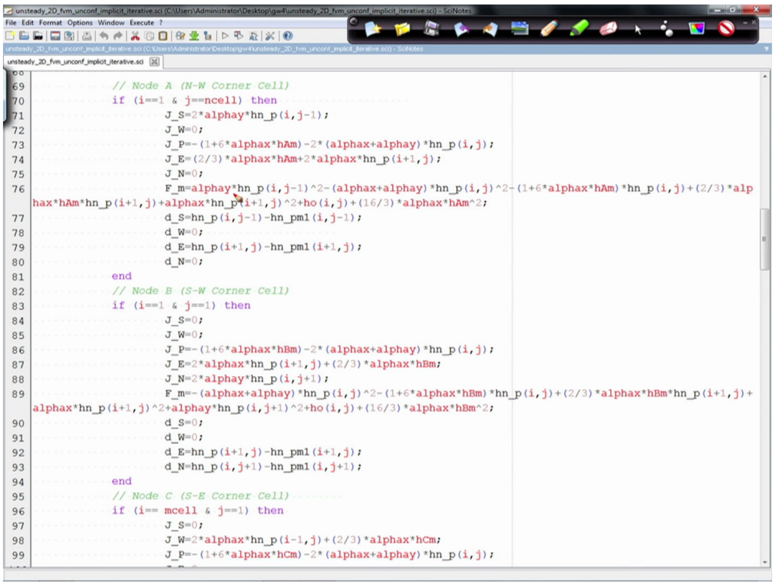

We have to also implement the specified left boundary condition, specified right boundary condition and Neumann boundary condition bottom, Neumann boundary condition on top.

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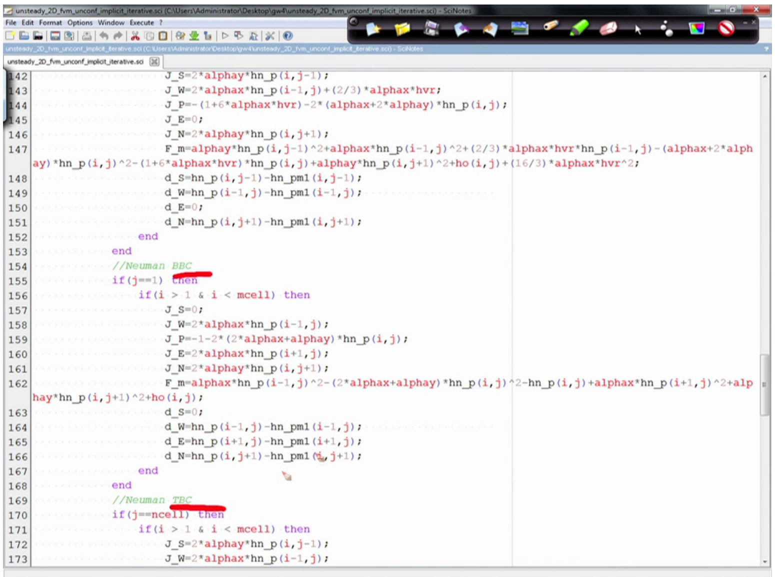

So after implementing these we need to calculate this residual. Residual is nothing but minus m minus JS, dS, JW, dW, JE, dE, JN, dN. Now we have not included JP here. So this is the residual.

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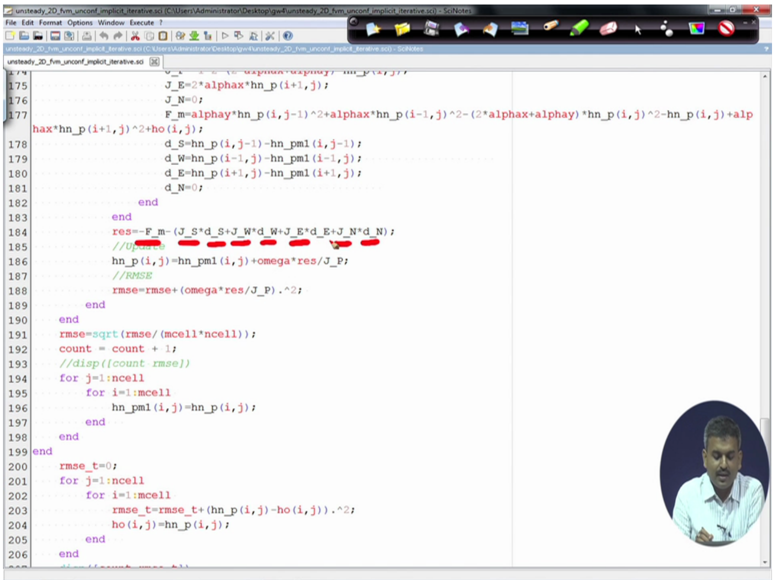

Residual is residual divided by JP and this is hn P minus 1, hn P. And we can calculate this rmse and again we can calculate rmse here which is the final rmse. And this is the count. Now we need to transfer this new values to old values and we can utilise this for space loop.

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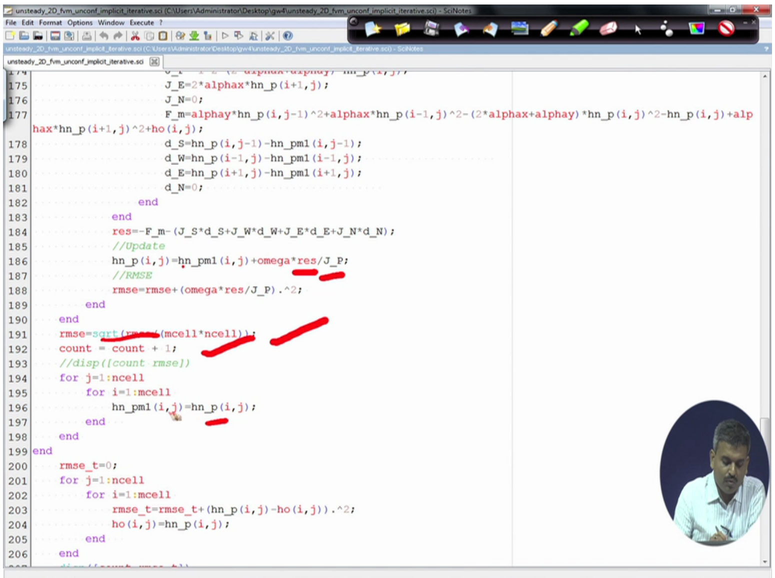

Then comes the time loop. For time loop this new value is old value. This is for steady state condition. This rmse t we need to implement.

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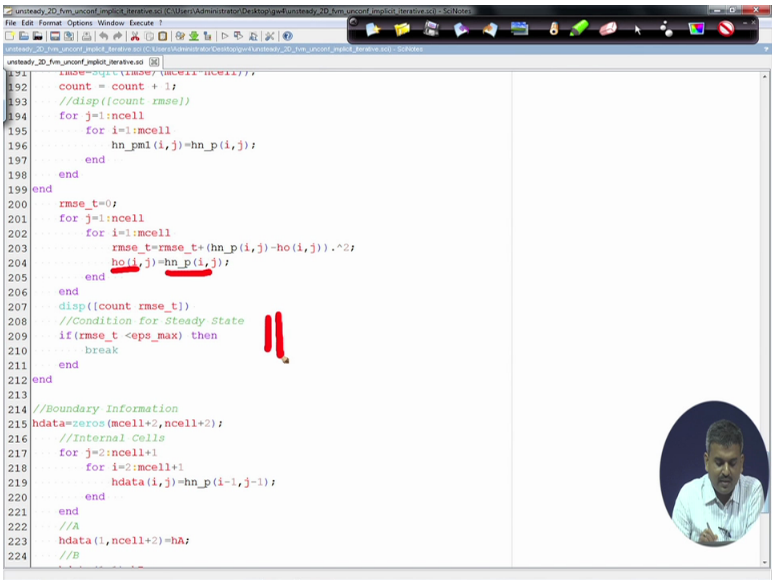

And obviously we know that for our boundary we have got values for only internal nodes. That means we have information about these nodes. This is in simple form. This is the information we have, self centred values.

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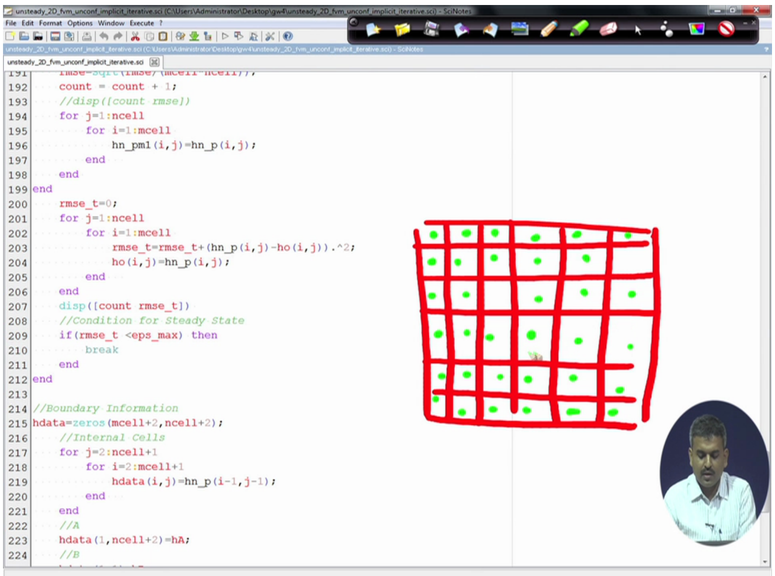

But further we need information regarding these points. So that we can get from the derivative condition. We can equate derivative equals to zero and bN and bS values we can calculate there.

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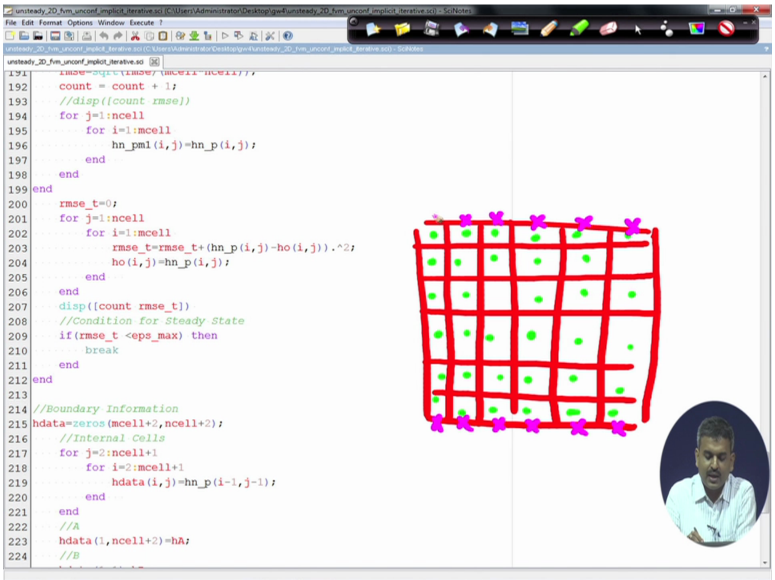

Also on the left hand boundary we can directly get the interpolated value from our specified boundary condition. So after solution which are green dots we need to extend our program to get this pink cross and green cross points so that we can solve the problem for the full domain.

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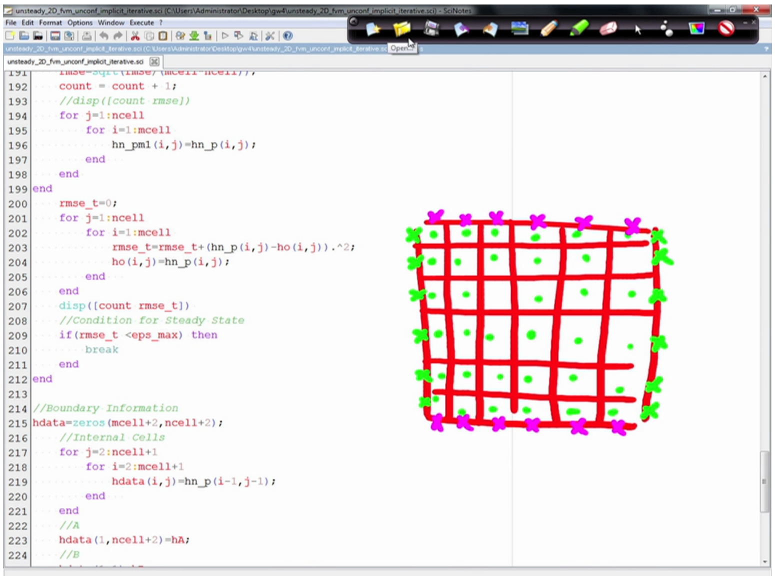

Now if we run this code again it will take some time because in this case we need certain number of iterations for each space loop convergence. In space loop we are solving that Jacobian matrix. If we solve that Jacobian matrix so obviously here you can see that 744 iterations are required to get rmse value of 983.

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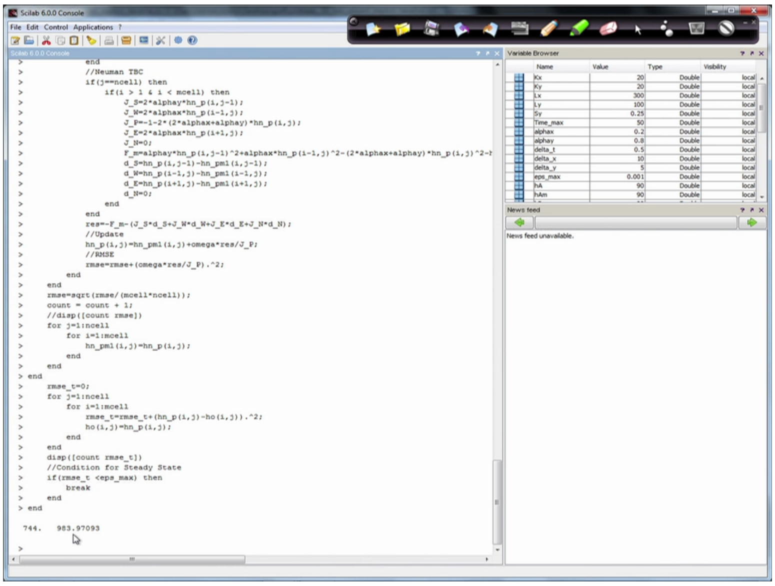

So rmse value of 983 this is absurd value. So obviously further 669 iterations are required to get this 133 rmse value. Now slowly these rmse values will decrease and finally we can get the solution out of this process and we can get the contour value from this program.

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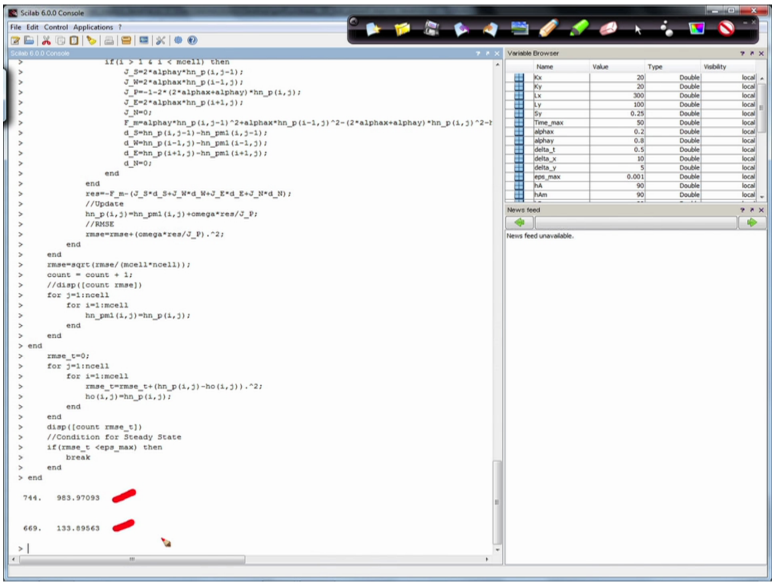

And contour value is similar to our finite (diff) volume or finite difference solution. But the contour is similar but somewhat different compared to our confined aquifer solution because we are solving unconfined aquifer problem here. So you can see that 380 iterations and slowly this rmse value is decreasing. This is 2 point 4, this is 56.

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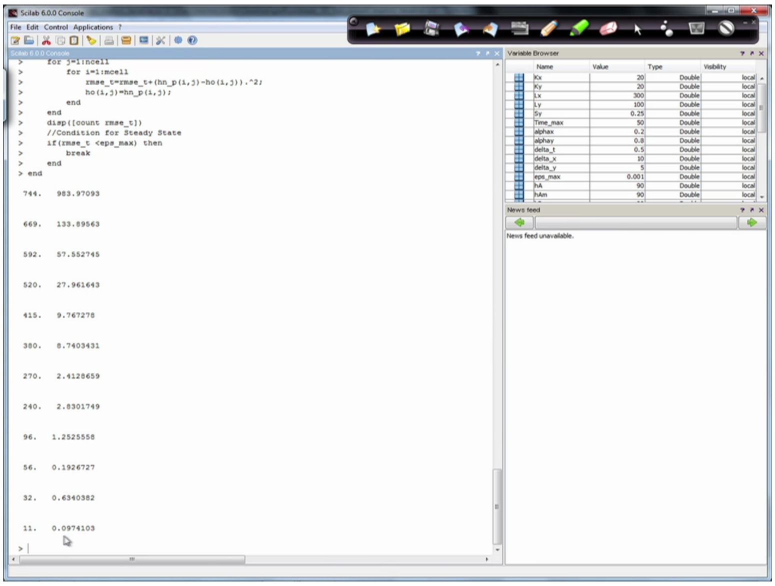

And finally we are getting the solution from the program. So iterative technique we can utilise to solve this unconfined aquifer problem.

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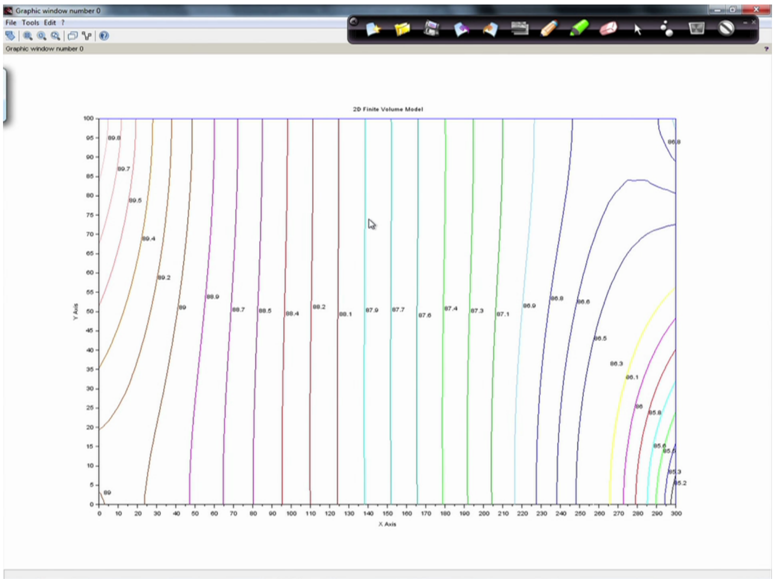

So this is all about the unconfined aquifer problem using finite volume approach. Thank you.