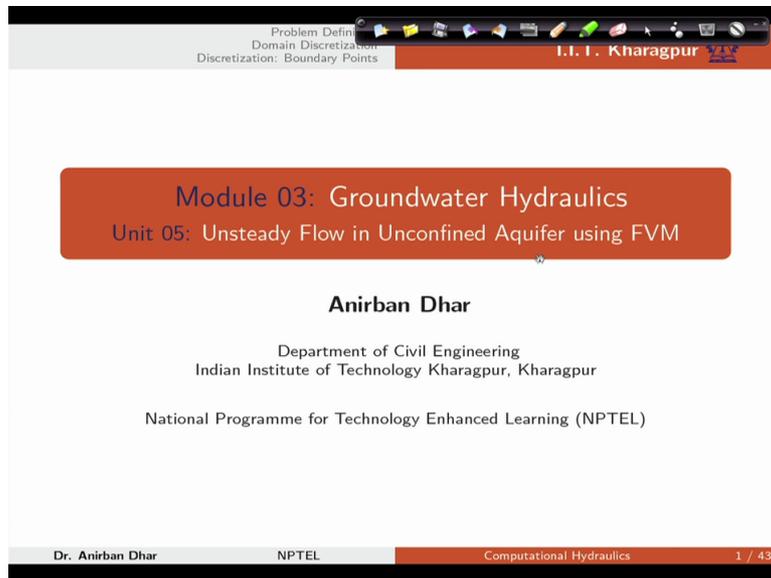


Computational Hydraulics
Professor Anirban Dhar
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Lecture 35
Unsteady Flow In Unconfined Aquifer Using FVM

Welcome to this lecture number 35 of the course computational hydraulics. We are in module 3 groundwater hydraulics. This is unit number 5, unsteady flow in unconfined aquifer using finite volume method.

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The image shows a presentation slide with a white background and a red header bar. The header bar contains the text "I. I. T. Kharagpur" and a logo. Below the header, there is a red box with white text that reads "Module 03: Groundwater Hydraulics" and "Unit 05: Unsteady Flow in Unconfined Aquifer using FVM". Below this box, the name "Anirban Dhar" is displayed, followed by "Department of Civil Engineering" and "Indian Institute of Technology Kharagpur, Kharagpur". At the bottom of the slide, it says "National Programme for Technology Enhanced Learning (NPTEL)". The slide is framed by a black border, and a taskbar is visible at the top with various application icons.

In our last lecture class we have discretized this unsteady flow equation for confined aquifers using finite volume method. So what is the learning objective for this particular lecture? At the end of this particular unit students will be able to solve unsteady two dimensional groundwater flow in unconfined aquifer using finite volume method.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Learning Objective

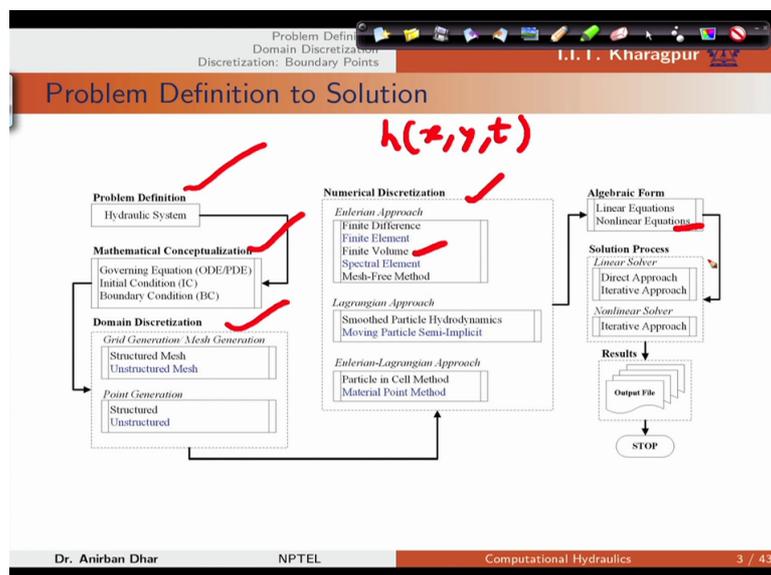
- To solve unsteady two dimensional groundwater flow in unconfined aquifer using Finite Volume Method.

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Problem definition to solution, we will be utilising $h(x,y,t)$ because in unconfined aquifer flow, the flow we will consider only in horizontal direction. So in this case again we will be utilising this problem definition, mathematical conceptualization, domain discretization, numerical discretization, obviously in this case we will be utilising finite volume method. But the main difference with the confined aquifer flow is the nonlinear equations.

In case of this unconfined aquifer flow we will be utilising nonlinear equations and now solution approach will be different compared to our linear equation that we have utilised for confined aquifer flow.

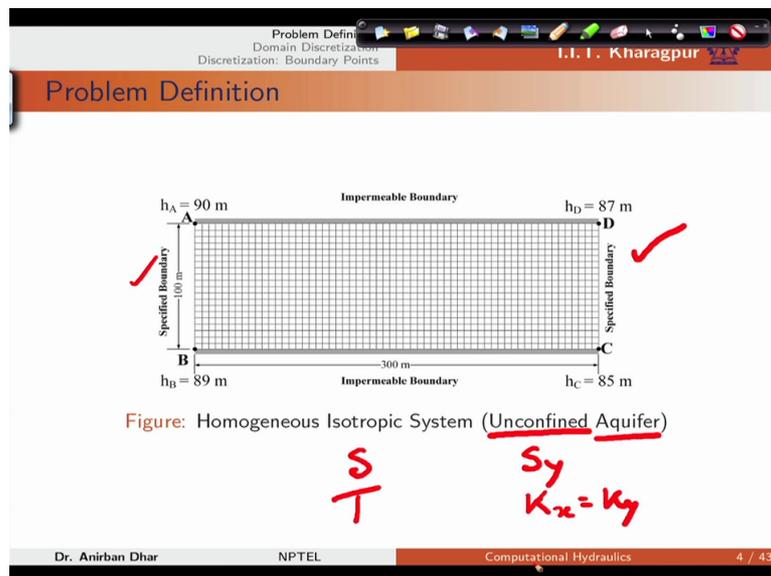
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So problem definition, I am not going to the details of the problem definition. Now in this case again our system is homogeneous isotropic system. Boundary conditions are same like our previous confined aquifer flow. But only difference is that we are considering it as unconfined aquifer flow. In confined aquifer flow we need to define storativity but in unconfined aquifer we need to define specifically, obviously transmissivity is required.

We will define the hydraulic conductivity in terms of K_x , K_y . In isotropic system K_x will be able to K_y in this case.

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Let us define our two dimensional problem. In our confined aquifer problem we have utilised this equation $\frac{\partial h}{\partial t}$ and $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$. Now in this case we have extra term this h . And because of this h , the equation is nonlinear in nature. Now let us consider that S_y specifically is point 25 and K is 20 metres per day. Obviously in this case K_x equals to K_y equals to 20 metres per day.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. I. Kharagpur

Problem Definition

$$\frac{S}{T} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$$

Governing equation
A two-dimensional (in space) IBVP can be written as,

$$\Omega: S_y \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left(K_x h \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y h \frac{\partial h}{\partial y} \right) + W$$

$S_y = 0.25$
 $K = 20 \text{ m/day}$
 $K_x = K_y = 20 \text{ m/day}$

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Now we need to see how we can solve this using our finite volume approach. Initial condition, again the conditions are same like over finite difference approach or finite volume approach that we have utilised for our confined aquifer system.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. I. Kharagpur

Problem Definition

subject to

Initial Condition

$$h(x, y, 0) = h_0(x, y)$$

and

Boundary Condition

$$\Gamma_D^1: h(0, y, t) = h_1(y)$$

$$\Gamma_D^2: h(L_x, y, t) = h_2(y)$$

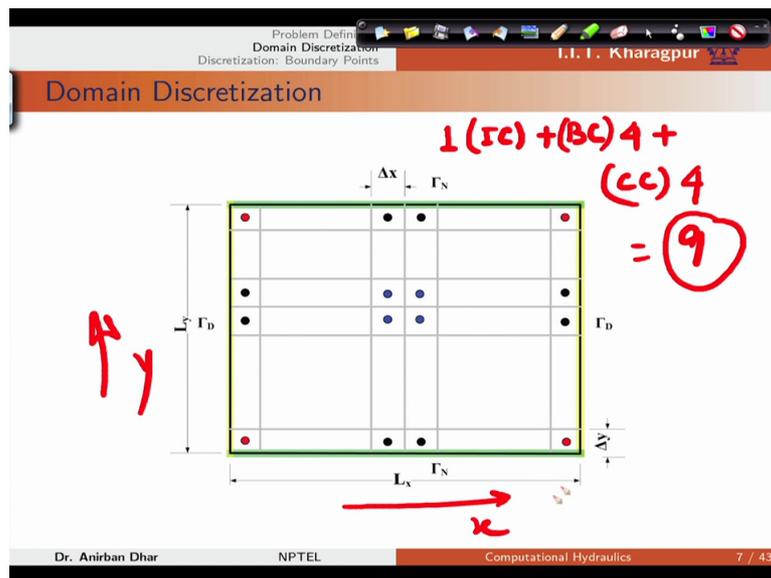
$$\Gamma_N^3: \frac{\partial h}{\partial y} \Big|_{(x, 0, t)} = 0$$

$$\Gamma_N^4: \frac{\partial h}{\partial y} \Big|_{(x, L_y, t)} = 0$$

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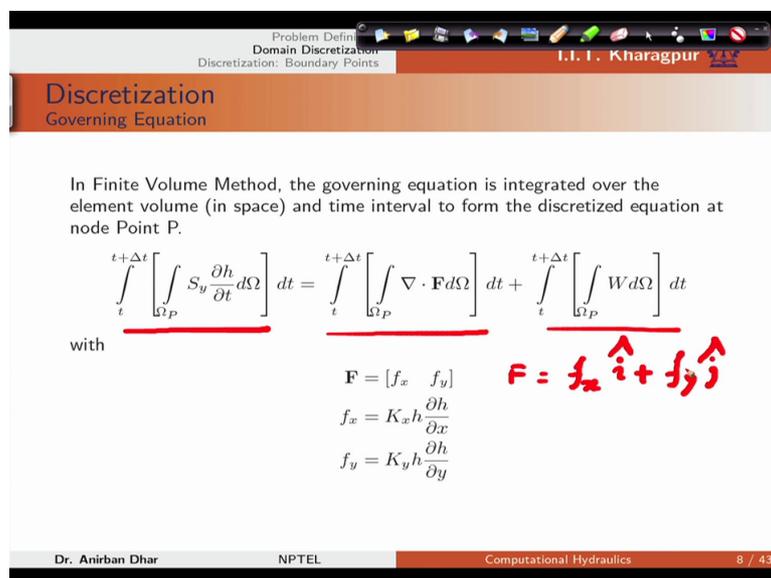
Domain discretization, again we will have interior nodes. So again 1 for interior node or I should say interior cell plus boundary cell we will have 4. And corner cell we will have 4 condition. So all total we need to consider 9 conditions for this problem. Obviously this is L_x and L_y . This is y direction, this is x direction.

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Now in finite volume method the governing equation is integrated over the element in space and time interval to form the discretized governing equation. Now in this case we have space and time derivatives. This F vector is $f_x \hat{i} + f_y \hat{j}$. Obviously F vector if you consider, this is nothing but $f_x \hat{i} + f_y \hat{j}$ where \hat{i} and \hat{j} , these are two unit vectors corresponding x and y directions.

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So this f_x is this quantity. So initially this was the case where $K_x h \frac{\partial h}{\partial x}$ plus this is $f_y = K_y h \frac{\partial h}{\partial y}$. Initially let us consider that our K_x K_y are different and those are varying with x . So we will formulate the general problem and during solution we will consider K_x and K_y , those are having equal values. So this is nothing but f_x plus f_y . So we can write this as divergence of F.

Now if we write it in terms of divergence of F again we can utilise Gauss divergence theorem for the first term on the right hand side and we can use the integration concept for other terms to get the final discretization.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Discretization

Governing Equation

In Finite Volume Method, the governing equation is integrated over the element volume (in space) and time interval to form the discretized equation at node Point P.

$$\int_t^{t+\Delta t} \int_{\Omega_P} S_y \frac{\partial h}{\partial t} d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega_P} \nabla \cdot \mathbf{F} d\Omega dt + \int_t^{t+\Delta t} \int_{\Omega_P} W d\Omega dt$$

with

$$\mathbf{F} = [f_x \ f_y] \quad \mathbf{F} = f_x \hat{i} + f_y \hat{j}$$

$$f_x = K_x h \frac{\partial h}{\partial x}$$

$$f_y = K_y h \frac{\partial h}{\partial y}$$

$$\frac{\partial}{\partial x} (K_x h \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (K_y h \frac{\partial h}{\partial y}) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} = \nabla \cdot \mathbf{F}$$

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Now the first term on the left hand side. This is $S_y \frac{\partial h}{\partial t}$. If we consider that no variation of h within this Ω_P volume, we are considering that Ω_P is the volume in this case. So within this Ω_P there is the variation of h. Now we are considering constant value for this one.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Discretization

Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[\int_{\Omega_P} S_y \frac{\partial h}{\partial t} d\Omega \right] dt$$

$$= S_y \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left(\int_{\Omega_P} h d\Omega \right) dt$$

Ω_P

$-P$

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Now with this consideration if we further write this so this will be nothing but h_P into $\text{del } \Omega_P$ which is a volume for this cell. Now we need to integrate. So obviously if we integrate it over time period or time interval will get something.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Discretization

Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[\int_{\Omega_P} S_y \frac{\partial h}{\partial t} d\Omega \right] dt$$

$$= S_y \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left(\int_{\Omega_P} h d\Omega \right) dt$$

$$= S_y \int_t^{t+\Delta t} \frac{\partial}{\partial t} (h_P \Delta \Omega_P) dt$$

Ω_P

$-P$

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So what is that? So we will get this $h_P L$ plus 1 to L because L , this corresponds to t and L plus one, this corresponds to t plus Δt . Now if we use this finally we will get that our $\text{del } \Omega_P$. This is nothing but $\text{del } x \text{ del } y$ and unit value on the other direction. We are considering 2D volume only in this case. So that is why this $\text{del } x$ into $\text{del } y$.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

Discretization

Governing Equation: Temporal Term

$$\int_t^{t+\Delta t} \left[\int_{\Omega_P} S_y \frac{\partial h}{\partial t} d\Omega \right] dt$$

$$= S_y \int_t^{t+\Delta t} \frac{\partial}{\partial t} \left(\int_{\Omega_P} h d\Omega \right) dt \quad t \rightarrow t$$

$$= S_y \int_t^{t+\Delta t} \frac{\partial}{\partial t} (h_P \Delta \Omega_P) dt \quad [t+1 \rightarrow t+\Delta t]$$

$$= S_y (h_P^{t+1} - h_P^t) \Delta \Omega_P \quad \Delta \Omega_P = \Delta x \Delta y$$

$$= S_y (h_P^{t+1} - h_P^t) \Delta x \Delta y$$

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Now let us consider the first component on the right hand side. So this is the divergence form. Again divergence form of this one. As we have discussed that area is always outward positive. If area is outward positive then this is nothing but del y i. This is minus del y i. This is del x j, obviously negative sign will be there. On the upper side this will be del x j and this is Pth cell.

Now for the cell if we consider that F dot dA from over Gauss divergence theorem, so F dot dA, this component for all faces that means we need to consider this east, north, west and south faces here.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

Discretization

Governing Equation: Spatial Term

$(F \cdot d\vec{A})_f$

$$\int_t^{t+\Delta t} \int_{\Omega_P} \nabla \cdot \mathbf{F} d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega_P} \nabla \cdot (f_x \hat{i} + f_y \hat{j}) d\Omega dt$$

$$= [(f_x)_e^{t+1} A_{xe} - (f_x)_w^{t+1} A_{xw} + (f_y)_n^{t+1} A_{yn} - (f_y)_s^{t+1} A_{ys}] \Delta t$$

$$= \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{t+1} A_{xe} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{t+1} A_{xw} + \left(K_y h \frac{\partial h}{\partial y} \right)_n^{t+1} A_{yn} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{t+1} A_{ys} \right] \Delta t$$

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So for all faces if we consider, this quantity dot area, for eastern face obviously this will be positive. In this case A_{xe} , this is nothing but Δy . This is A_{xw} , this is again Δy because negative sign is already there so we should not consider this thing. So A_{yn} north, again Δx . A_{ys} south again this is Δx in this case. So we have discretized the second term in terms of these derivatives. East, west, north, south and Δt will be there.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization
Governing Equation: Spatial Term

$A_{xe} = \Delta y$
 $A_{xw} = \Delta y$
 $A_{yn} = \Delta x$
 $A_{ys} = \Delta x$

$(F \cdot d\vec{a})_f$

$\int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot \mathbf{F} d\Omega dt = \int_t^{t+\Delta t} \int_{\Omega^P} \nabla \cdot (f_x \hat{i} + f_y \hat{j}) d\Omega dt$

$= [(f_x)_e^{l+1} A_{xe} - (f_x)_w^{l+1} A_{xw} + (f_y)_n^{l+1} A_{yn} - (f_y)_s^{l+1} A_{ys}] \Delta t$

$= \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} A_{xe} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} A_{xw} + \left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} A_{yn} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} A_{ys} \right] \Delta t$

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Now if we consider the uniform grid system as I have discussed. So this will be the case and source term, if we consider the source term obviously this is valid for cell value. W is valid for cell value. So obviously Δx , Δy , Δt , this should be multiplied here.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization
Governing Equation

In a uniform grid system,

$$A_{xe} = A_{xw} = \Delta y$$

$$A_{yn} = A_{ys} = \Delta x \tag{1}$$

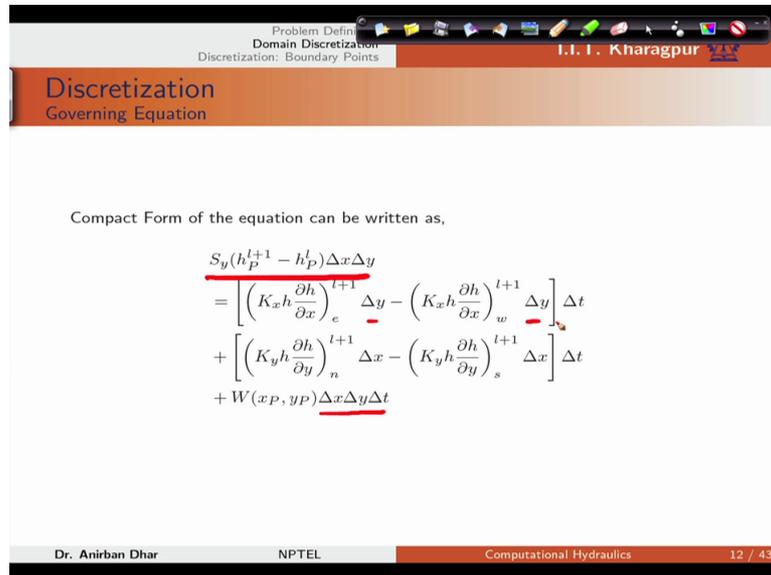
Source Term:

$$\int_t^{t+\Delta t} \int_{\Omega^P} W(x, y) d\Omega dt = \underline{W(x_P, y_P) \Delta x \Delta y \Delta t} \tag{2}$$

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Now in compact form if we combine all terms we can get this. On the left hand side this is the thing and on the right hand side del y will be multiplied with eastern and western face. Del x will be multiplied here and del x, del y, del t on this side.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization

Governing Equation

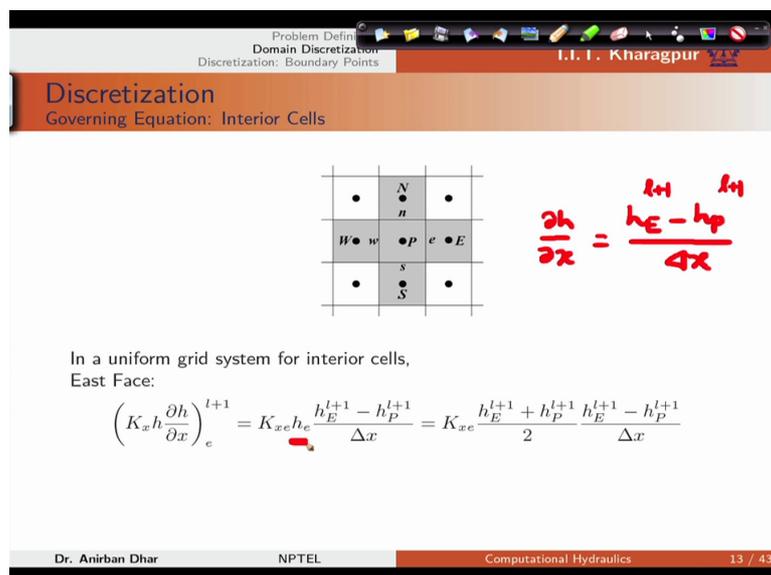
Compact Form of the equation can be written as,

$$S_y(h_p^{l+1} - h_p^l) \Delta x \Delta y = \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} \Delta y - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \Delta y \right] \Delta t + \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} \Delta x - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \Delta x \right] \Delta t + W(x_P, y_P) \Delta x \Delta y \Delta t$$

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So if we see the case for interior cells, now interior cell east face, this is important. East face we are already know that del h by del x at east face we can write it in terms of hE L plus 1 minus hE L plus 1. This comes from the concept of implicit scheme because we are using L plus 1 index for time. And spatial derivative we are (usili) utilising the concept of finite difference.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

•	N	•
•	n	•
W • w	• p	e • E
•	s	•
•	S	•

$$\frac{\partial h}{\partial x} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

In a uniform grid system for interior cells,
East Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} = K_{x,e} h_e \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} = K_{x,e} \frac{h_E^{l+1} + h_P^{l+1}}{2} \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

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Now this is h_e . What is this h_e ? h_e is the h value at the face. Now how to calculate this h value at the face? So what is the readymade approximation? Readymade approximation is that if we can consider this h_e plus h_p by 2. That means the average value between this P and E cell that can be considered as our face value for the east face. So this is our face value and this is a derivative value multiplies by K_{xe} . That means hydraulic conductivity at the face.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Discretization

Governing Equation: Interior Cells

$$h_e = \frac{h_E + h_p}{2}$$

•	N	•
•	n	•
W • w	• p	e • E
•	s	•
•	S	•

$$\frac{\partial h}{\partial x} = \frac{h_E - h_p}{\Delta x}$$

In a uniform grid system for interior cells,
East Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} = K_{x_e} h_e \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} = K_{x_e} \left(\frac{h_E^{l+1} + h_P^{l+1}}{2} \right) \left(\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \right)$$

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Now we are considering that we have homogeneous isotropic system. But in this case we are considering generalized formulation. So we will simply use K_x instead of this K_{x_e} .

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Discretization

Governing Equation: Interior Cells

$$h_e = \frac{h_E + h_p}{2}$$

•	N	•
•	n	•
W • w	• p	e • E
•	s	•
•	S	•

$$\frac{\partial h}{\partial x} = \frac{h_E - h_p}{\Delta x}$$

In a uniform grid system for interior cells,
East Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} = K_{x_e} h_e \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} = K_x \left(\frac{h_E^{l+1} + h_P^{l+1}}{2} \right) \left(\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \right)$$

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Now let us see what is the situation for west face? West face is again similar. We will consider h_w as $h_P \Delta x + 1$ plus $h_W \Delta x + 1$ divided by 2. And this derivative we already know that $\frac{\partial h}{\partial x}$ at western face $h_P - h_W$ divided by Δx .

(Refer Slide Time: 17:08)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

•	N	•
W	•	E
•	S	•

$h_w = \frac{h_p + h_W}{2}$

In a uniform grid system for interior cells,
East Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} = K_{xe} h_e \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} = K_{xe} \frac{h_E^{l+1} + h_P^{l+1}}{2} \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$
West Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} = K_{xw} h_w \left(\frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right) = K_{xw} \frac{h_P^{l+1} + h_W^{l+1}}{2} \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

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So this is the average value, this is the derivative. Now we can write the east face and west face like this.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

•	N	•
W	•	E
•	S	•

$h_w = \frac{h_p + h_W}{2}$

In a uniform grid system for interior cells,
East Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} = K_{xe} h_e \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} = K_{xe} \frac{h_E^{l+1} + h_P^{l+1}}{2} \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$
West Face:

$$\left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} = K_{xw} h_w \left(\frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right) = K_{xw} \left(\frac{h_P^{l+1} + h_W^{l+1}}{2} \right) \left(\frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right)$$

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Now if you have northern face, northern face again we can discretize or approximate this h_N in terms of h_N h_P and the derivative as $h_N - h_P$ by Δy .

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Problem Definition
Domain Discretization
Discretization: Boundary Points

Discretization

Governing Equation: Interior Cells

North Face:

$$\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} = K_{yn} h_n \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} = K_{yn} \left(\frac{h_N^{l+1} + h_P^{l+1}}{2} \right) \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

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Now if we have southern face, for southern face we have hS, hS is nothing but hP and hS average and hP minus hS divided by del y.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

Discretization

Governing Equation: Interior Cells

North Face:

$$\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} = K_{yn} h_n \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} = K_{yn} \frac{h_N^{l+1} + h_P^{l+1}}{2} \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

South Face:

$$\left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} = K_{ys} h_s \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} = K_{ys} \left(\frac{h_P^{l+1} + h_S^{l+1}}{2} \right) \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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So with this we can further simplify our compact form of the equation. On the right hand side interestingly we are getting these square terms, hE minus hP square, hP minus hW square, hN, hP, hP, hS, all square terms. So this is one non linear equation.

(Refer Slide Time: 18:41)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

North Face:

$$\left(K_y h \frac{\partial h}{\partial y}\right)_n^{l+1} = K_{yn} h_n \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} = K_{yn} \frac{h_N^{l+1} + h_P^{l+1}}{2} \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

South Face:

$$\left(K_y h \frac{\partial h}{\partial y}\right)_s^{l+1} = K_{ys} h_s \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} = K_{ys} \frac{h_P^{l+1} + h_S^{l+1}}{2} \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Compact Form of the equation can be written as,

$$S_y (h_P^{l+1} - h_P^l) \Delta x \Delta y = \left[\frac{K_{xe} (h_E^{l+1})^2 - (h_P^{l+1})^2}{2 \Delta x} \Delta y - \frac{K_{xw} (h_P^{l+1})^2 - (h_W^{l+1})^2}{2 \Delta x} \Delta y \right] \Delta t + \left[\frac{K_{yn} (h_N^{l+1})^2 - (h_P^{l+1})^2}{2 \Delta y} \Delta x - \frac{K_{ys} (h_P^{l+1})^2 - (h_S^{l+1})^2}{2 \Delta y} \Delta x \right] \Delta t + W(x_P, y_P) \Delta x \Delta y \Delta t$$

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Now in compact form what we can do, we can transfer this S_y on the right hand side and del t we can include here. So del x square obviously in this case we will get this parameter kind of thing. Multiplied by 1 by del x square. Also K_x del t by 2 S_y , this should be multiplied by 1 by del x square.

(Refer Slide Time: 19:28)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

Compact Form of the equation can be written as,

$$h_P^{l+1} - h_P^l = \frac{K_x \Delta t}{2S_y} \frac{h_E^{l+1})^2 - (h_P^{l+1})^2}{\Delta x^2} - \frac{K_x \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{\Delta x^2} + \frac{K_y \Delta t}{2S_y} \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{\Delta y^2} - \frac{K_y \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{\Delta y^2} + \frac{W(x_P, y_P)}{S_y} \Delta t$$

In simplified form, this can be written as,

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{W(x_P, y_P) \Delta t}{S_y}$$

with

$$\alpha_x = \frac{K_x \Delta t}{2S_y \Delta x^2} \quad \alpha_y = \frac{K_y \Delta t}{2S_y \Delta y^2}$$

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Now we can simplify it. So if we further simplify this thing with the approximation that K_x del t by 2 S_y del x square, K_y del t 2 S_y into del y square we can write it in approximate form.

(Refer Slide Time: 19:57)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

Compact Form of the equation can be written as,

$$h_P^{l+1} - h_P^l = \frac{K_x \Delta t}{2S_y} \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{\Delta x^2} - \frac{K_x \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{\Delta x^2} + \frac{K_y \Delta t}{2S_y} \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{\Delta y^2} - \frac{K_y \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{\Delta y^2} + \frac{W(x_P, y_P)}{S_y} \Delta t$$

In simplified form, this can be written as,

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{W(x_P, y_P) \Delta t}{S_y}$$

with

$$\alpha_x = \frac{K_x \Delta t}{2S_y \Delta x^2} \quad \alpha_y = \frac{K_y \Delta t}{2S_y \Delta y^2}$$

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So now approximate form is this alpha y into hS square plus alpha x into hW square minus 2 alpha x alpha y. These are coming from here because coefficients two for alpha x, two for alpha y.

(Refer Slide Time: 20:22)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

Compact Form of the equation can be written as,

$$h_P^{l+1} - h_P^l = \frac{K_x \Delta t}{2S_y} \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{\Delta x^2} - \frac{K_x \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{\Delta x^2} + \frac{K_y \Delta t}{2S_y} \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{\Delta y^2} - \frac{K_y \Delta t}{2S_y} \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{\Delta y^2} + \frac{W(x_P, y_P)}{S_y} \Delta t$$

In simplified form, this can be written as,

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{W(x_P, y_P) \Delta t}{S_y}$$

with

$$\alpha_x = \frac{K_x \Delta t}{2S_y \Delta x^2} \quad \alpha_y = \frac{K_y \Delta t}{2S_y \Delta y^2}$$

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And one which is coming from this side. This is minus hP L plus 1. This is not square term. Remember that this is not a square term here. But this hP is a square term.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

Compact Form of the equation can be written as,

$$h_P^{t+1} - h_P^t = \frac{K_x \Delta t}{2S_x} \frac{h_E^{t+1} - (h_P^{t+1})^2}{\Delta x^2} - \frac{K_x \Delta t}{2S_y} \frac{(h_P^{t+1})^2 - (h_W^{t+1})^2}{\Delta x^2} + \frac{K_y \Delta t}{2S_y} \frac{(h_N^{t+1})^2 - (h_P^{t+1})^2}{\Delta y^2} - \frac{K_y \Delta t}{2S_y} \frac{(h_P^{t+1})^2 - (h_S^{t+1})^2}{\Delta y^2} + \frac{W(x_P, y_P) \Delta t}{S_y}$$

In simplified form, this can be written as,

$$\alpha_y (h_S^{t+1})^2 + \alpha_x (h_W^{t+1})^2 - [2(\alpha_x + \alpha_y) (h_P^{t+1})^2 - h_P^{t+1}] - \alpha_x (h_E^{t+1})^2 + \alpha_y (h_N^{t+1})^2 = h_P^t - \frac{W(x_P, y_P) \Delta t}{S_y}$$

with

$$\alpha_x = \frac{K_x \Delta t}{2S_y \Delta x^2} \quad \alpha_y = \frac{K_y \Delta t}{2S_y \Delta y^2}$$

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Now another term will be there alpha x hE square, alpha y hN square. So in this side we have all known quantities. Now this is a discretized form for interior points.

(Refer Slide Time: 20:58)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Discretization

Governing Equation: Interior Cells

Compact Form of the equation can be written as,

$$h_P^{t+1} - h_P^t = \frac{K_x \Delta t}{2S_x} \frac{h_E^{t+1} - (h_P^{t+1})^2}{\Delta x^2} - \frac{K_x \Delta t}{2S_y} \frac{(h_P^{t+1})^2 - (h_W^{t+1})^2}{\Delta x^2} + \frac{K_y \Delta t}{2S_y} \frac{(h_N^{t+1})^2 - (h_P^{t+1})^2}{\Delta y^2} - \frac{K_y \Delta t}{2S_y} \frac{(h_P^{t+1})^2 - (h_S^{t+1})^2}{\Delta y^2} + \frac{W(x_P, y_P) \Delta t}{S_y}$$

In simplified form, this can be written as,

$$\alpha_y (h_S^{t+1})^2 + \alpha_x (h_W^{t+1})^2 - [2(\alpha_x + \alpha_y) (h_P^{t+1})^2 - h_P^{t+1}] - \alpha_x (h_E^{t+1})^2 + \alpha_y (h_N^{t+1})^2 = h_P^t - \frac{W(x_P, y_P) \Delta t}{S_y}$$

with

$$\alpha_x = \frac{K_x \Delta t}{2S_y \Delta x^2} \quad \alpha_y = \frac{K_y \Delta t}{2S_y \Delta y^2}$$

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Now if we have non linear equation, obviously we need to solve it using some nonlinear technique and one of such technique is Newton Raphson. In Newton Raphson we need to define the Fm or our function which is function of h L plus 1, all unknown quantities. So I have written all left hand side terms here and transferred this right hand side to the left hand side and equated this one with zeros. So this is my functional form or function form.

(Refer Slide Time: 21:58)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

LHS RHS

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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Now I need to calculate the derivative term or elements of Jacobian matrix. So to calculate the element of Jacobian matrix so I need to take derivative of Fm with respect to hS which is 2 alpha y hS. It is directly coming from here.

(Refer Slide Time: 22:26)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$


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Now next one is hW which is coming from here. This is 2 alpha y hW L plus 1.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P - \frac{W(x_P, y_P) \Delta t}{S_y} \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y) h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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Then comes this h_P which is again minus 1. Minus 1 means because in this case there is no coefficient available. You can simply take derivative here. We will get minus 1. Minus 4 because 2 is already, the square term, this is 2. So 4 alpha x alpha y into h_P . Then h_E on the eastern side we have 2 alpha x into h_E , 2 alpha y into h_N . This is there.

(Refer Slide Time: 23:17)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P - \frac{W(x_P, y_P) \Delta t}{S_y} \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y) h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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And this is our right hand side term. Obviously this is a constant term for function calculation because we are considering this unknown at L plus 1 level.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^{l+1} - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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So with this if you proceed then we can construct one row of the Jacobian matrix. Now for all cells that means if you have M cells, Mc and N cells in y direction. This is in x, this is in y direction. For M Nc into Mc Nc this will be the size of Jacobian matrix. And we will have penta diagonal structure of the Jacobian matrix because you will have P term, W term, E term, N term and S term. So obviously no other term will be associated with the central point P.

(Refer Slide Time: 24:40)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^{l+1} - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

x y
Mc Nc
(Mc Nc x Mc Nc)

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So obviously we will get this P. This is for our other ones on the right hand side which is eastern one, this is western and this is our northern, this is southern. So like this we will get the terms for interior points terms for interior points.

(Refer Slide Time: 25:24)

Problem Definition
Domain Discretization
Discretization: Boundary Points

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

Handwritten notes: x y , M_e N_e , $(M_e N_e \times M_e N_e)$

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These points you will get from these equations. Obviously this will be JS, so S means this one. This is W, W is this one, P is this one, E is this one and N is this one. So for all interior nodes we need to calculate this.

(Refer Slide Time: 25:57)

Problem Definition
Domain Discretization
Discretization: Boundary Points

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{W(x_P, y_P)\Delta t}{S_y}\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y)h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

Handwritten notes: x y , M_e N_e , $(M_e N_e \times M_e N_e)$

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But this is a node number including the boundary nodes. For boundary nodes we need spatial treatment.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2(\alpha_x + \alpha_y)] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P^l - \frac{W(x_P, y_P) \Delta t}{S_y} \right] = 0$$

Elements of Jacobian matrix can be calculated as

P E N

W S

$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$
 $J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$
 $J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 4(\alpha_x + \alpha_y) h_P^{l+1}$
 $J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$
 $J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$

x y

M_c N_c

(M_cN_c × M_cN_c)

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So let us see how we can utilise the concept for boundary nodes. Now we have left boundary, left boundary this discretization is exactly same as we have utilised for confined aquifer system because the derivative term will be same. But we need to multiply hW or value on the right hand side. This is our original equation which is in discretized form.

(Refer Slide Time: 26:47)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})$$

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On the right hand side if we consider eastern face we can directly use the form that we have utilised for interior nodes. For northern face again we can use the same thing. For the southern face also we can use that node.

(Refer Slide Time: 27:12)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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But for the western face we have something different. This is applicable for derivative then we need to multiply h BW which is the height at the face. Height at the face is equal to the specified value for this case. So if we have specified value then we can directly use that because that is a known value.

(Refer Slide Time: 27:53)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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So if you simplify this again we will get terms with hS, hP, hP, hE, hE and hN. In this case we do not have any hW term because for hW term we do not have W cell in case of left boundary. And the right hand side we have this 16 by 3 alpha x BW. This is the first term.

(Refer Slide Time: 28:35)

Problem Definition
Domain Discretization
Discretization: Boundary Points I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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We need to transfer this on the right hand side. And again our approximation is based on this parameter alpha y equals to del t. This is nothing but alpha x equals to del t into Kx and this is Sy 2 into delta x square. This is alpha y equals 2 Ky del t divided by 2 Sy del y square.

(Refer Slide Time: 29:18)

Problem Definition
Domain Discretization
Discretization: Boundary Points I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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So further if we discretize this or if you write it in terms of functional format or a function format then we need to transfer the terms on the right hand side to the left hand side so that we can write it as equal to zero.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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On the left hand side interestingly we have 2 alpha y which is coming from these two.

(Refer Slide Time: 29:57)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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But only one alpha x which is coming from here.

(Refer Slide Time: 30:00)

Problem Definition
Domain Discretization
Discretization: Boundary Points

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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And the coefficient this 6 alpha x, this is coming from here. 3 into 3, this is 3 and from that 3 because we have considered this alpha x equals to this one. We need to consider this. Now this is 2 into 3 that is 6 here, 6 alpha x that is coming here.

(Refer Slide Time: 30:56)

Problem Definition
Domain Discretization
Discretization: Boundary Points

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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Now on the right hand side we have this alpha x. Whatever is there on the left hand side and this is from the right hand side directly coming.

(Refer Slide Time: 31:21)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P^{l+1} - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BW}^{l+1}] - 2[\alpha_x + 2\alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3} \alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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Now we have this derivative hS which is corresponding to this one. Then we have hP or JP. JP is minus 2 alpha x and alpha x plus 2 alpha y. This is 1 plus alpha x h BW. And JE again this is coming as 2 3rd alpha x h BW. And hE again there is another term with square. And there will be northern term or northern boundary.

(Refer Slide Time: 32:11)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P^{l+1} - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BW}^{l+1}] - 2[\alpha_x + 2\alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3} \alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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So in this case obviously we have JW and this is equal to zero. So we do not have this term in our Jacobian matrix.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BW}^{l+1}] - 2[\alpha_x + 2\alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3} \alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

$J_N^m = 0$

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Now if we consider our right boundary, again the discretizations are exactly same as per our confined aquifer system.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Boundary Conditions

Right Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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But in the next page where we have utilised the equations, the equations are different. So for eastern face this is the change. This is the only change required because other terms these are exactly coming from the interior node discretizations.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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So in this case this BE term which is again specified thing is coming here on the right hand boundary. This 3 term this is coming here as 6 alpha x.

(Refer Slide Time: 33:35)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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So we have this thing for our case where we are getting all terms except this hE because there should be no hE here. Only hP, hW, hP, hW, hS, hP, hN and all known values are there on the right hand side.

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Problem Definition
Domain Discretization
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I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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So alpha y, alpha x, this is coefficient of hS, hW and extra emphasis on hW here. And 2 alpha y, this is coming from this one, this one. But only one alpha x, this is coming from here.

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Problem Definition
Domain Discretization
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I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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Now this is a functional form. This is left hand side, this is our right hand side.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 + \frac{2}{3}\alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y](h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{16}{3}\alpha_x(h_{BE}^{l+1})^2\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1} + \frac{2}{3}\alpha_x h_{BE}^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BE}^{l+1}] - 2[\alpha_x + 2\alpha_y] h_P^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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Now if you take individual elements of the Jacobian matrix we will get like this. This is 2 alpha y hS. JW, this is 2 alpha x, again 2 3rd h BE, this will be there. HP which is exactly same, 2 alpha x plus 2 alpha y hP. And finally we have hN but no hE or this case this is zero. We should remember this. This should be zero in this case because we are considering right boundary.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 + \frac{2}{3}\alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + 2\alpha_y](h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} + \alpha_y(h_N^{l+1})^2 - \left[-h_P^l - \frac{16}{3}\alpha_x(h_{BE}^{l+1})^2\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1} + \frac{2}{3}\alpha_x h_{BE}^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BE}^{l+1}] - 2[\alpha_x + 2\alpha_y] h_P^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

$J_t^m = 0$

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Now if we consider our top boundary. Top boundary again this point is important because we need to apply the zero condition on the derivative.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Top Boundary

n		
$W \bullet w$	$\bullet P$	$e \bullet E$
\bullet	s \bullet S	\bullet

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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And if we apply this condition so obviously in this case on the northern boundary this is zero. And as we are considering other derivatives similar to interior points, these are exactly same like interior points and we will get like this.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[0 - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2\alpha_x + \alpha_y] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l$$

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So obviously 2 alpha x because two P terms are there and 1 hP L plus 1 which is coming from here. And hP L which is there on the right hand side which is a known quantity.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[0 - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2\alpha_x + \alpha_y] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l$$

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Now if I write it in function form, this is our left hand side, this is right hand side.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 - [2\alpha_x + \alpha_y] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 - [-h_P^l] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 2[2\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

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Now derivative for hS, hW, hS, hW. Then hP, for hP we have minus 1 then 2 alpha x plus alpha y into hP. This is there. And hE, obviously there should be no JNm term. This is zero in the Jacobian matrix.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y(h_S^{l+1})^2 + \alpha_x(h_W^{l+1})^2 - [2\alpha_x + \alpha_y](h_P^{l+1})^2 - h_P^{l+1} + \alpha_x(h_E^{l+1})^2 - [-h_P^{l+1}] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 2[2\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

J_N^m = 0

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If we have bottom boundary, again for bottom boundary we need to apply this zero condition. Other terms are same like our interior points.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Boundary Conditions

Bottom Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0$$

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Now if we apply the same thing here so this is zero. For other faces we have these forms. Now we have 2 alpha x coming from here and alpha y only one term here and again h_P L plus 1 which is coming here and alpha x E, N, W. But no h_S L plus 1 term in this case.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

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Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - 0 \right]$$

In simplified form, this can be written as

$$\alpha_x (h_W^{l+1})^2 - [2\alpha_x + \alpha_y] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l$$

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So we have left hand side, right hand side and W obviously this is 2 alpha x into hW L plus 1. Now JP is minus 1, this is 2 into 2 alpha x alpha y hP L plus 1. JE is 2 alpha x directly coming from here. JN, this is 2 alpha y hN L plus 1 from this one.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_x (h_W^{l+1})^2 - [2\alpha_x + \alpha_y] (h_P^{l+1})^2 - h_P^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - [-h_P^l] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -1 - 2[2\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

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Now we need to consider this northeast or northwest corner. Northwest corner, in northern side we have impermeable boundary. Obviously we need to put the zero condition and the western boundary we need to put this Dirichlet or specified boundary condition.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Boundary Conditions

N-W Corner

	n		
w	$\bullet P$	$e \bullet E$	\bullet
s	$\bullet S$	\bullet	\bullet
	\bullet	\bullet	\bullet

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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So if we apply that in this case interestingly we do not have any W term. Only E, P and P, S. That means no northern term or h_N^{l+1} or h_W^{l+1} here.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[0 - K_y \frac{(h_S^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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So only one here alpha x is coming here. From here we are getting one alpha y.

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Problem Definition
Domain Discretization
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Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[0 - K_y \frac{(h_S^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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Again from here we are getting this $(\alpha_p) 6 \alpha_x \times h_{BW}$ and we need to transfer on the right hand side.

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Problem Definition
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I.I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[0 - K_y \frac{(h_S^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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In this case this $\alpha_y h_S^2$, this term is there.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[0 - K_y \frac{(h_S^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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Now similarly we can write it in function format. So we have hS. This is JS. JS is 2 alpha y hS L plus 1. P is this one, 2 alpha x plus alpha y. This is 2 by 3 alpha x h BW L plus 1, 2 alpha x hE L plus 1.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 - \left[-h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3} \alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1}$$


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So we do not have JWm, this is zero and JNm, this is zero. So in Jacobian matrix we will not find these two terms.

(Refer Slide Time: 41:43)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BW}^{l+1}] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 - \left[-h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1} \quad J_N^m = 0$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BW}^{l+1}] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3} \alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1} \quad J_N^m = 0$$

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Now if you consider our northeast term, if we have northeast, on the northern side we have zero and eastern side we need to specify the specified boundary condition.

(Refer Slide Time: 41:58)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

N-E Corner

		n	
•	$W \bullet w$	$P \bullet$	e
•	•	s S	
•	•	•	

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now this is exactly same as our specified boundary condition. And northern side we need to put zero condition. And this is the condition that we have utilised from the specified boundary condition.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[0 - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] h_P^{l+1} = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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Now we will get hS, hW, hW, hP, hP. That means no N L plus 1 or E L plus 1 term here.

(Refer Slide Time: 42:41)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[0 - K_y \frac{(h_P^{l+1})^2 - (h_S^{l+1})^2}{2\Delta y^2} \right]$$

In simplified form, this can be written as

$$\alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] h_P^{l+1} = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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So if we consider our function format again 2 alpha y hS. This is coming as western. So from western side 2 alpha x hW L plus 1 and 2 3rd alpha x hBE L plus 1, this is there. This is JP and this is 1 plus 6 alpha x BE. This is directly coming from here. And this is 2 alpha x plus alpha y. This is the term that is coming from here.

(Refer Slide Time: 43:27)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} - \left[-h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1} + \frac{2}{3} \alpha_x h_{BE}^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BE}^{l+1}] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

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Now we do not have any JEm. That means this is zero. Then JNm, this is again zero. These two terms are zero terms.

(Refer Slide Time: 43:43)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_y (h_S^{l+1})^2 + \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} - \left[-h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_S^m = \frac{\partial F_m}{\partial h_S^{l+1}} = 2\alpha_y h_S^{l+1}$$

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1} + \frac{2}{3} \alpha_x h_{BE}^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -[1 + 6\alpha_x h_{BE}^{l+1}] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

$J_E^m = 0$
 $J_N^m = 0$

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Now the southeast corner. On the southern side this will be impermeable. So obviously this will be zero. On the eastern side we have specified boundary condition.

(Refer Slide Time: 44:03)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

S-E Corner

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0$$

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So for specified boundary condition this is zero on the southern side. On the eastern side we have a specified condition which is directly coming from here. So we will have western term or hW term, hP, hP, hN. So no hS and no hE term here.

(Refer Slide Time: 44:41)

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x h_{BE}^{l+1} \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x^2} - K_x \frac{(h_P^{l+1})^2 - (h_W^{l+1})^2}{2\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - 0 \right]$$

In simplified form, this can be written as

$$\alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - [1 + 6\alpha_x h_{BE}^{l+1}] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2$$

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Again we can get equivalent function form. So from this JW, this is directly coming as 2 alpha x hW L plus 1, 2 3rd alpha x BE. JP, this is coming as 2 into alpha x hP L plus 1 and this term is there 1 plus 6 alpha x BE. And on the northern side we have 2 alpha y, this hN L plus 1. And this is our right hand side which is the known quantity. Now we do not have any JSm that is zero and we do not have any JEm zero. So these two terms are zero terms.

(Refer Slide Time: 45:56)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(\mathbf{h}^{l+1}) = \alpha_x (h_W^{l+1})^2 + \frac{2}{3} \alpha_x h_{BE}^{l+1} h_W^{l+1} - [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] h_P^{l+1} + \alpha_y (h_N^{l+1})^2 - \left[-h_P^{l+1} - \frac{16}{3} \alpha_x (h_{BE}^{l+1})^2 \right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_W^m = \frac{\partial F_m}{\partial h_W^{l+1}} = 2\alpha_x h_W^{l+1} + \frac{2}{3} \alpha_x h_{BE}^{l+1}$$

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = - \left[1 + 6\alpha_x h_{BE}^{l+1} \right] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

RHS
 $J_S = 0$
 $J_E = 0$

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Now if you consider southwest corner, the approach is similar. We need to put zero condition here.

(Refer Slide Time: 46:07)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I. I. I. Kharagpur

Boundary Conditions

S-W Corner

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0^{\oplus}$$

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For southern boundary this is zero and for western case we have transferred these values and derivatives.

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Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - 0 \right]$$

In simplified form, this can be written as

$$- [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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So for western boundary we do not have any hS L plus 1 or hW L plus 1 term. So directly we can get this discretized form.

(Refer Slide Time: 46:38)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Implicit Scheme

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{1}{\Delta x} \left[\left(K_x h \frac{\partial h}{\partial x} \right)_e^{l+1} - \left(K_x h \frac{\partial h}{\partial x} \right)_w^{l+1} \right] + \frac{1}{\Delta y} \left[\left(K_y h \frac{\partial h}{\partial y} \right)_n^{l+1} - \left(K_y h \frac{\partial h}{\partial y} \right)_s^{l+1} \right]$$

$$S_y \frac{h_P^{l+1} - h_P^l}{\Delta t} = \left[K_x \frac{(h_E^{l+1})^2 - (h_P^{l+1})^2}{2\Delta x^2} - K_x h_{BW}^{l+1} \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x^2} \right] + \left[K_y \frac{(h_N^{l+1})^2 - (h_P^{l+1})^2}{2\Delta y^2} - 0 \right]$$

In simplified form, this can be written as

$$- [\alpha_x + \alpha_y] (h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1} \right] h_P^{l+1} + \frac{2}{3} \alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 = -h_P^l - \frac{16}{3} \alpha_x (h_{BW}^{l+1})^2$$

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And this is the function in the form of function discretized form can be written. This is our form. Again JP, JP is coming from here, this is 2 alpha x, this is there. This is 2 3rd BW hW, 2 alpha x hWE, 2 alpha x hE coming from here and 2 alpha y hN L plus 1, this is directly coming. So we do not have HSm equals to zero and HWm, this is equal to zero. So this is the condition for our Jacobian matrix.

(Refer Slide Time: 47:44)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

Function and Jacobian

In the form of function discretized form can be written as,

$$F_m(h^{l+1}) = -[\alpha_x + \alpha_y](h_P^{l+1})^2 - \left[1 + 6\alpha_x h_{BW}^{l+1}\right] h_P^{l+1} + \frac{2}{3}\alpha_x h_{BW}^{l+1} h_E^{l+1} + \alpha_x (h_E^{l+1})^2 + \alpha_y (h_N^{l+1})^2 - \left[-h_P^l - \frac{16}{3}\alpha_x (h_{BW}^{l+1})^2\right] = 0$$

Elements of Jacobian matrix can be calculated as

$$J_P^m = \frac{\partial F_m}{\partial h_P^{l+1}} = -\left[1 + 6\alpha_x h_{BW}^{l+1}\right] - 2[\alpha_x + \alpha_y] h_P^{l+1}$$

$$J_E^m = \frac{\partial F_m}{\partial h_E^{l+1}} = \frac{2}{3}\alpha_x h_{BW}^{l+1} + 2\alpha_x h_E^{l+1}$$

$$J_N^m = \frac{\partial F_m}{\partial h_N^{l+1}} = 2\alpha_y h_N^{l+1}$$

Handwritten note in red circles: $J_S^m = 0$, $J_W^m = 0$

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So we have defined all four corners or corner cells and all boundary cells here. Now we need to see how we can solve this problem. In general form the governing equation including the boundary conditions we can write like this. So this is nothing but JS into del hS L plus 1, this is JW, this is JP, JE, JN. So this is same. On the right hand side we have this minus Fm.

(Refer Slide Time: 48:34)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I.I.T. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m(h^{l+1})$$

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Now in this case we can solve this through iterative form. This hS can be expanded like this, hW can be expanded like this, hP, hE, hN. And on the right hand side Fm, this can be expanded in terms of hS, hW, hP, hE hN. So I have changed the colour for hS hW in this case.

(Refer Slide Time: 49:14)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$J_S^m \left[\frac{h_S^{l+1}(p) - h_S^{l+1}(p-1)}{\Delta x} \right] + J_W^m \left[\frac{h_W^{l+1}(p) - h_W^{l+1}(p-1)}{\Delta x} \right] + J_P^m \left[\frac{h_P^{l+1}(p) - h_P^{l+1}(p-1)}{\Delta x} \right] + J_E^m \left[\frac{h_E^{l+1}(p) - h_E^{l+1}(p-1)}{\Delta x} \right] + J_N^m \left[\frac{h_N^{l+1}(p) - h_N^{l+1}(p-1)}{\Delta x} \right] = -F_m \left(h_S^{l+1}(p), h_W^{l+1}(p), h_P^{l+1}(p-1), h_E^{l+1}(p-1), h_N^{l+1}(p-1) \right)$$

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Why? Because if we consider our problem, then we have central cell which is our cell P. This is northern, this is southern cell, this is western, this is eastern. Now during calculation process we will start discretizing our domain into number of cells and we will start our calculation from this point.

(Refer Slide Time: 50:09)

Problem Definition
Domain Discretization
Discretization: Boundary Points

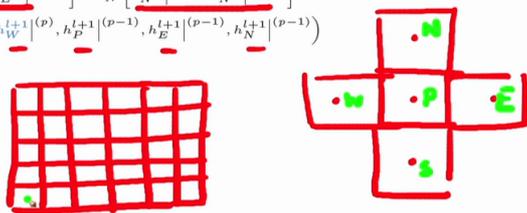
I.I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$J_S^m \left[\frac{h_S^{l+1}(p) - h_S^{l+1}(p-1)}{\Delta x} \right] + J_W^m \left[\frac{h_W^{l+1}(p) - h_W^{l+1}(p-1)}{\Delta x} \right] + J_P^m \left[\frac{h_P^{l+1}(p) - h_P^{l+1}(p-1)}{\Delta x} \right] + J_E^m \left[\frac{h_E^{l+1}(p) - h_E^{l+1}(p-1)}{\Delta x} \right] + J_N^m \left[\frac{h_N^{l+1}(p) - h_N^{l+1}(p-1)}{\Delta x} \right] = -F_m \left(h_S^{l+1}(p), h_W^{l+1}(p), h_P^{l+1}(p-1), h_E^{l+1}(p-1), h_N^{l+1}(p-1) \right)$$


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So from this point we will move towards this. Again we will start from this, again move like this. So in this case what is happening, first we are solving S then moving to this direction and we are coming back here and going to this W cell.

(Refer Slide Time: 50:41)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$J_S^m \left[\frac{h_S^{l+1}(p) - h_S^{l+1}(p-1)}{\Delta x} \right] + J_W^m \left[\frac{h_W^{l+1}(p) - h_W^{l+1}(p-1)}{\Delta x} \right] + J_P^m \left[\frac{h_P^{l+1}(p) - h_P^{l+1}(p-1)}{\Delta x} \right] + J_E^m \left[\frac{h_E^{l+1}(p) - h_E^{l+1}(p-1)}{\Delta x} \right] + J_N^m \left[\frac{h_N^{l+1}(p) - h_N^{l+1}(p-1)}{\Delta x} \right] = -F_m \left(h_S^{l+1}(p), h_W^{l+1}(p), h_P^{l+1}(p-1), h_E^{l+1}(p-1), h_N^{l+1}(p-1) \right)$$

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So for calculation of P, the value of S cell and W cell, these two are already updated values available from the calculation process. So I have mark these values as h_S^p , h_W^{l+1} at pth iteration. This is again pth iteration, this is again at pth iteration. We have updated values available.

(Refer Slide Time: 51:16)

Problem Definition
Domain Discretization
Discretization: Boundary Points
I. I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$J_S^m \left[\frac{h_S^{l+1}(p) - h_S^{l+1}(p-1)}{\Delta x} \right] + J_W^m \left[\frac{h_W^{l+1}(p) - h_W^{l+1}(p-1)}{\Delta x} \right] + J_P^m \left[\frac{h_P^{l+1}(p) - h_P^{l+1}(p-1)}{\Delta x} \right] + J_E^m \left[\frac{h_E^{l+1}(p) - h_E^{l+1}(p-1)}{\Delta x} \right] + J_N^m \left[\frac{h_N^{l+1}(p) - h_N^{l+1}(p-1)}{\Delta x} \right] = -F_m \left(h_S^{l+1}(p), h_W^{l+1}(p), h_P^{l+1}(p-1), h_E^{l+1}(p-1), h_N^{l+1}(p-1) \right)$$

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Now in this case further I can just simplify this and write it in final iterative form like this. So I can consider this value and transfer all other terms on the right hand side. So I will write this residual. So in this case residual excludes the central term. So we have JS, JW, JE, JN. Obviously in this case it is a pth iteration value itself available.

So I have written in terms of pth. And this is P minus 1 level values are available. That is why I have written in terms of P minus 1. So I can calculate this residual and divide it by JpM that is directly coming from here.

(Refer Slide Time: 52:35)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$\begin{aligned} & J_S^m [h_S^{l+1}|^{(p)} - h_S^{l+1}|^{(p-1)}] + J_W^m [h_W^{l+1}|^{(p)} - h_W^{l+1}|^{(p-1)}] + J_P^m [h_P^{l+1}|^{(p)} - h_P^{l+1}|^{(p-1)}] \\ & + J_E^m [h_E^{l+1}|^{(p)} - h_E^{l+1}|^{(p-1)}] + J_N^m [h_N^{l+1}|^{(p)} - h_N^{l+1}|^{(p-1)}] \\ & = -F_m (h_S^{l+1}|^{(p)}, h_W^{l+1}|^{(p)}, h_P^{l+1}|^{(p-1)}, h_E^{l+1}|^{(p-1)}, h_N^{l+1}|^{(p-1)}) \end{aligned}$$

Final iterative form can be written as

$$h_P^{l+1}|^{(p)} = h_P^{l+1}|^{(p-1)} + \frac{Res}{J_P^m}$$

with

$$Res = -F_m - [J_S^m \Delta h_S^{l+1}|^{(p)} + J_W^m \Delta h_W^{l+1}|^{(p)} + J_E^m \Delta h_E^{l+1}|^{(p-1)} + J_N^m \Delta h_N^{l+1}|^{(p-1)}]$$

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So this is the iterative form. Now what I can do? I can start the iteration and get the converged value of hP L plus 1. After getting convergence through the iterative format I can transfer this value to hP L. Then this will become my new old time level value which is calculated from the future time level and I can iterate for the time.

(Refer Slide Time: 53:25)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I. I. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$\begin{aligned} & J_S^m [h_S^{l+1}|^{(p)} - h_S^{l+1}|^{(p-1)}] + J_W^m [h_W^{l+1}|^{(p)} - h_W^{l+1}|^{(p-1)}] + J_P^m [h_P^{l+1}|^{(p)} - h_P^{l+1}|^{(p-1)}] \\ & + J_E^m [h_E^{l+1}|^{(p)} - h_E^{l+1}|^{(p-1)}] + J_N^m [h_N^{l+1}|^{(p)} - h_N^{l+1}|^{(p-1)}] \\ & = -F_m (h_S^{l+1}|^{(p)}, h_W^{l+1}|^{(p)}, h_P^{l+1}|^{(p-1)}, h_E^{l+1}|^{(p-1)}, h_N^{l+1}|^{(p-1)}) \end{aligned}$$

Final iterative form can be written as

$$h_P^{l+1}|^{(p)} = h_P^{l+1}|^{(p-1)} + \frac{Res}{J_P^m}$$

with

$$Res = -F_m - [J_S^m \Delta h_S^{l+1}|^{(p)} + J_W^m \Delta h_W^{l+1}|^{(p)} + J_E^m \Delta h_E^{l+1}|^{(p-1)} + J_N^m \Delta h_N^{l+1}|^{(p-1)}]$$

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So initially for any problem we have time loop and space loop. Now space loop we can solve using this iterative form and we can update time and for updated time we can transfer this n plus 1 level value to nth level directly.

(Refer Slide Time: 54:00)

General Form

In general form, the governing equation including boundary conditions can be written as,

$$J_S^m \Delta h_S^{l+1} + J_W^m \Delta h_W^{l+1} + J_P^m \Delta h_P^{l+1} + J_E^m \Delta h_E^{l+1} + J_N^m \Delta h_N^{l+1} = -F_m (h^{l+1})$$

Iterative form can be written as

$$J_S^m [h_S^{l+1}|^{(p)} - h_S^{l+1}|^{(p-1)}] + J_W^m [h_W^{l+1}|^{(p)} - h_W^{l+1}|^{(p-1)}] + J_P^m [h_P^{l+1}|^{(p)} - h_P^{l+1}|^{(p-1)}] + J_E^m [h_E^{l+1}|^{(p)} - h_E^{l+1}|^{(p-1)}] + J_N^m [h_N^{l+1}|^{(p)} - h_N^{l+1}|^{(p-1)}] = -F_m (h_S^{l+1}|^{(p)}, h_W^{l+1}|^{(p)}, h_P^{l+1}|^{(p-1)}, h_E^{l+1}|^{(p-1)}, h_N^{l+1}|^{(p-1)})$$

Final iterative form can be written as

$$h_P^{l+1}|^{(p)} = h_P^{l+1}|^{(p-1)} + \frac{Res}{J_P^m}$$

with

$$Res = -F_m - [J_S^m \Delta h_S^{l+1}|^{(p)} + J_W^m \Delta h_W^{l+1}|^{(p)} + J_E^m \Delta h_E^{l+1}|^{(p-1)} + J_N^m \Delta h_N^{l+1}|^{(p-1)}]$$

Handwritten red annotations: A vertical line labeled 'Time' and a horizontal line labeled 'Space' with an arrow pointing right labeled 'n+1'.

So this is our unsteady 2D finite volume unconfined aquifer implicit iterative format.

(Refer Slide Time: 54:21)

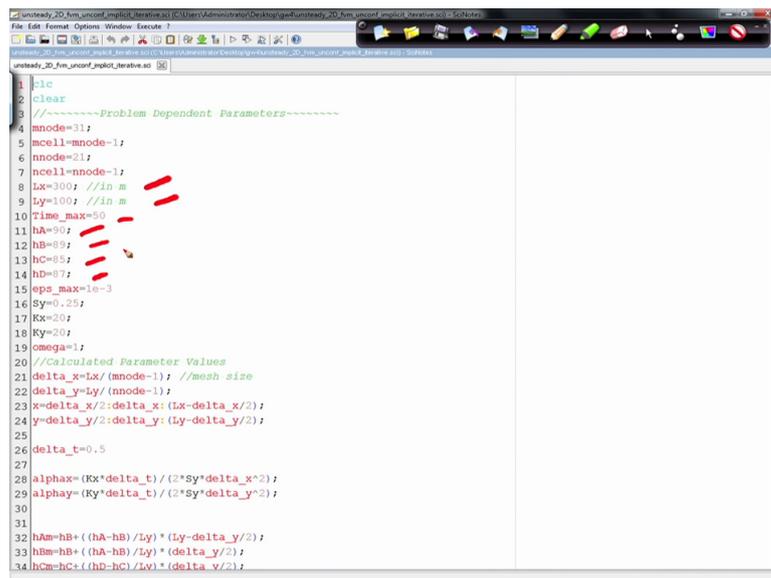
Source Code

Unsteady Two Dimensional Unconfined Groundwater Flow with Finite Volume Method

- Without coefficient matrix using Gauss Seidel
 - [unsteady_2D_fvm_unconf_implicit_iterative.sci](#)

Now what we can do we can solve it using our unsteady equation. Now in this case again we have mnode, mcell, nnode, ncell, these are available. And Lx, Ly, these values are available. Time max I have written as 50. HA, hB, hC, hD, these values are directly available.

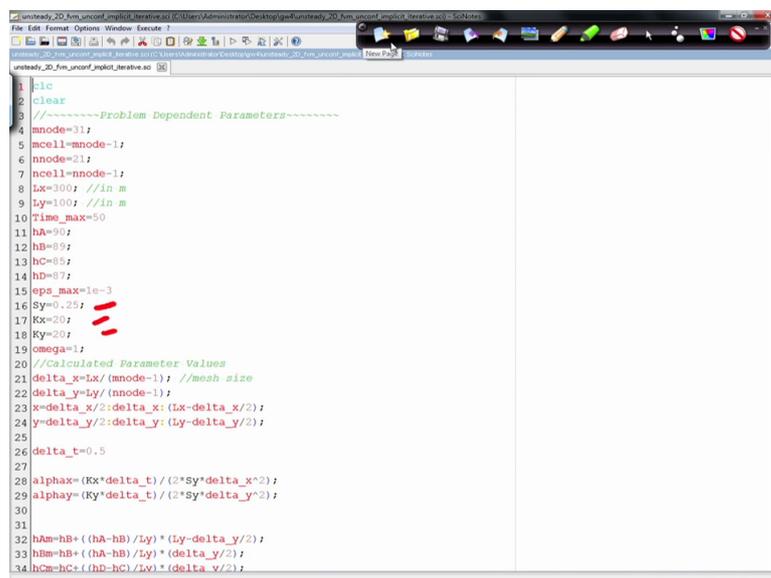
(Refer Slide Time: 54:54)



```
1 tic
2 clear
3 //-----Problem Dependent Parameters-----
4 mnode=31;
5 ncell=mnode-1;
6 nnode=21;
7 hcell=nnode-1;
8 Lx=300; //in m
9 Ly=100; //in m
10 Time_max=50
11 hA=90;
12 hB=89;
13 hC=85;
14 hD=87;
15 eps_max=1e-3
16 Sy=0.25;
17 Kx=20;
18 Ky=20;
19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=Ly/(nnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sy*delta_y^2);
30
31
32 hAm=hB+((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB+((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC+((hD-hC)/Ly)*(delta_y/2);
```

Eps max, this is required for convergence. Sy is point 25. Kx Ky, these values are 20 metres per day.

(Refer Slide Time: 55:09)



```
1 tic
2 clear
3 //-----Problem Dependent Parameters-----
4 mnode=31;
5 ncell=mnode-1;
6 nnode=21;
7 hcell=nnode-1;
8 Lx=300; //in m
9 Ly=100; //in m
10 Time_max=50
11 hA=90;
12 hB=89;
13 hC=85;
14 hD=87;
15 eps_max=1e-3
16 Sy=0.25;
17 Kx=20;
18 Ky=20;
19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=Ly/(nnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sy*delta_y^2);
30
31
32 hAm=hB+((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB+((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC+((hD-hC)/Ly)*(delta_y/2);
```

Now for this one we can further calculate Lx Ly values. This is delta x, delta y. So obviously for Lx direction we are discretizing the cells like this. This is equal but we are starting from, this is dx by 2 and we are ending dx by 2 before that. So in this case for x we are starting from delta x by 2 and running up to Lx minus delta x by 2 and we are getting the self centred values here.

(Refer Slide Time: 56:07)

```

19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=Ly/(mnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sx*delta_y^2);
30
31
32 hAm=hB*((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB*((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC*((hD-hC)/Ly)*(delta_y/2);
35 hDm=hC*((hD-hC)/Ly)*(Ly-delta_y/2);
36
37 // Initialization
38 // Initialization
39 ho=ha*ones(mcell,ncell);
40 hn_p=ha*ones(mcell,ncell);
41 hn_pml=ha*ones(mcell,ncell);
42 //Time Loop
43 t=0;
44 while t < Time_max
45     t=t+delta_t;
46
47     count = 0;
48     rmse=1;
49 //Space Loop
50 while rmse > eps_max
51     rmse=0;
52     for j=1:ncell

```

Y similarly we are writing it like this. So from delta y by 2 we are starting and we are going up to Ly minus delta y by 2. So that is why I have written it. Now delta t is point 5 in this case.

(Refer Slide Time: 56:34)

```

19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=Ly/(mnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sx*delta_y^2);
30
31
32 hAm=hB*((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB*((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC*((hD-hC)/Ly)*(delta_y/2);
35 hDm=hC*((hD-hC)/Ly)*(Ly-delta_y/2);
36
37 // Initialization
38 // Initialization
39 ho=ha*ones(mcell,ncell);
40 hn_p=ha*ones(mcell,ncell);
41 hn_pml=ha*ones(mcell,ncell);
42 //Time Loop
43 t=0;
44 while t < Time_max
45     t=t+delta_t;
46
47     count = 0;
48     rmse=1;
49 //Space Loop
50 while rmse > eps_max
51     rmse=0;
52     for j=1:ncell

```

So alpha x, alpha y we have all values available. So now if we consider hA, hA is nothing but this value this corner value. HA is corner level value. This is B, this is A. But we need this value which is not at the centre. Now I can write this as hA modified value and I can write this as hB modified value. So, that I can calculate from here.

(Refer Slide Time: 57:23)

```
19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=ly/(mnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sy*delta_y^2);
30
31
32 hAm=hB*((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB*((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC*((hD-hC)/Ly)*(delta_y/2);
35 hDm=hC*((hD-hC)/Ly)*(Ly-delta_y/2);
36
37 // Initialization
38 // Initialization
39 ho=ha*ones(mcell,ncell);
40 hn_p=ha*ones(mcell,ncell);
41 hn_pml=ha*ones(mcell,ncell);
42 //Time Loop
43 t=0;
44 while t < Time_max
45     t=t+delta_t;
46
47 count = 0;
48 rmse=1;
49 //Space Loop
50 while rmse > eps_max
51     rmse=0;
52     for j=1:ncell
```

Similarly for C and D, in this case C point will be there. This is C, this is D. So obviously in this case also we will have that hD modified for this level and hC modified for this level.

(Refer Slide Time: 57:59)

```
19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=ly/(mnode-1);
23 x=delta_x/2:delta_x:(Lx-delta_x/2);
24 y=delta_y/2:delta_y:(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sy*delta_y^2);
30
31
32 hAm=hB*((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB*((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC*((hD-hC)/Ly)*(delta_y/2);
35 hDm=hC*((hD-hC)/Ly)*(Ly-delta_y/2);
36
37 // Initialization
38 // Initialization
39 ho=ha*ones(mcell,ncell);
40 hn_p=ha*ones(mcell,ncell);
41 hn_pml=ha*ones(mcell,ncell);
42 //Time Loop
43 t=0;
44 while t < Time_max
45     t=t+delta_t;
46
47 count = 0;
48 rmse=1;
49 //Space Loop
50 while rmse > eps_max
51     rmse=0;
52     for j=1:ncell
```

These are the values directly coming from here. Now ho which is old time level, hn P which is for pth iteration. This is hn P minus 1, this is P minus 1 level.

(Refer Slide Time: 58:17)

```
19 omega=1;
20 //Calculated Parameter Values
21 delta_x=Lx/(mnode-1); //mesh size
22 delta_y=Ly/(mnode-1);
23 x=delta_x/2;delta_x=(Lx-delta_x/2);
24 y=delta_y/2;delta_y=(Ly-delta_y/2);
25
26 delta_t=0.5
27
28 alphax=(Kx*delta_t)/(2*Sy*delta_x^2);
29 alphay=(Ky*delta_t)/(2*Sx*delta_y^2);
30
31
32 hAm=hB+((hA-hB)/Ly)*(Ly-delta_y/2);
33 hBm=hB+((hA-hB)/Ly)*(delta_y/2);
34 hCm=hC+((hD-hC)/Ly)*(delta_y/2);
35 hDm=hC+((hD-hC)/Ly)*(Ly-delta_y/2);
36
37 // Initialization
38 // Initialization
39 ho=ha*ones(mcell,ncell);
40 hn_p=ha*ones(mcell,ncell);
41 hn_pml=ha*ones(mcell,ncell);
42 //Time Loop
43 t=0;
44 while t < Time_max
45     t=t+delta_t;
46
47 count = 0;
48 rmse=1;
49 //Space Loop
50 while rmse > eps_max
51     rmse=0;
52     for j=1:ncell
```

Now in this case again the approach is same. We are writing the JS, JW, JP, JE, JN terms for interior cell. This is for interior cells and this is Fm. Now we need to calculate dS which is the difference between hP and hP minus 1 level for dS, dW, dE, dN.

(Refer Slide Time: 58:54)

```
53         for j=1:ncell
54             for i=1:mcell
55                 if (i > 1 & i < mcell) then
56                     if (j > 1 & j < ncell) then
57                         JS=2*alphax*hn_p(i,j-1);
58                         JW=-1+4*(alphax+alphay)*hn_p(i,j);
59                         JP=2*alphax*hn_p(i+1,j);
60                         JE=2*alphay*hn_p(i,j+1);
61                         JN=alpha*hn_p(i,j-1)^2+alphax*hn_p(i-1,j)^2-2*(alphax+alphay)*hn_p(i,j)-2*hn_p(i,j)+alph
62                         ax*hn_p(i+1,j)^2+alphay*hn_p(i,j+1)^2+ho(i,j);
63                         d_S=hn_p(i,j-1)-hn_pml(i,j-1);
64                         d_W=hn_p(i-1,j)-hn_pml(i-1,j);
65                         d_E=hn_p(i+1,j)-hn_pml(i+1,j);
66                         d_N=hn_p(i,j+1)-hn_pml(i,j+1);
67                     end
68                 end
69             // Node A (N-W Corner Cell)
70             if (i=1 & j=ncell) then
71                 JS=2*alphax*hn_p(i,j-1);
72                 JW=0;
73                 JP=-1+6*alphax*hAm-2*(alphax+alphay)*hn_p(i,j);
74                 JE=(2/3)*alphax*hAm+2*alphax*hn_p(i+1,j);
75                 JN=0;
76                 Fm=alpha*hn_p(i,j-1)^2-(alphax+alphay)*hn_p(i,j)^2-(1+6*alphax*hAm)*hn_p(i,j)+(2/3)*alph
```

Now in this case we should also calculate the northwest, southwest, southeast corners. So depending on that JS, JW, JP, JE, JN values will be there.

(Refer Slide Time: 59:18)

```

80 // Node A (N-W Corner Cell)
81 if (i==1 & j==ncell) then
82   J_S=2*alpha*hn_p(i,j-1);
83   J_W=0;
84   J_P=-(1+6*alpha*hm)-2*(alpha+alpha)*hn_p(i,j);
85   J_E=(2/3)*alpha*hm+2*alpha*hn_p(i+1,j);
86   J_N=0;
87   F_m=alpha*hn_p(i,j-1)^2-(alpha+alpha)*hn_p(i,j)^2-(1+6*alpha*hm)*hn_p(i,j)+(2/3)*alp
88   hax*hm*hn_p(i+1,j)+alpha*hn_p(i+1,j)^2+ho(i,j)+(1/3)*alpha*hm^2;
89   d_S=hn_p(i,j-1)-hn_p(i,j);
90   d_W=0;
91   d_E=hn_p(i+1,j)-hn_p(i,j);
92   d_N=0;
93 end
94 // Node B (S-W Corner Cell)
95 if (i==1 & j==1) then
96   J_S=0;
97   J_W=0;
98   J_P=-(1+6*alpha*hm)-2*(alpha+alpha)*hn_p(i,j);
99   J_E=2*alpha*hn_p(i+1,j)+(2/3)*alpha*hm;
100  J_N=2*alpha*hn_p(i,j+1);
101  F_m=(alpha+alpha)*hn_p(i,j)^2-(1+6*alpha*hm)*hn_p(i,j)+(2/3)*alpha*hm*hn_p(i+1,j)+
102  alpha*hn_p(i+1,j)^2+alpha*hn_p(i,j+1)^2+ho(i,j)+(1/3)*alpha*hm^2;
103  d_S=0;
104  d_W=0;
105  d_E=hn_p(i+1,j)-hn_p(i,j);
106  d_N=hn_p(i,j+1)-hn_p(i,j);
107 end
108 // Node C (S-E Corner Cell)
109 if (i==mcell & j==1) then
110   J_S=0;
111   J_W=2*alpha*hn_p(i-1,j)+(2/3)*alpha*hm;
112   J_P=-(1+6*alpha*hm)-2*(alpha+alpha)*hn_p(i,j);

```

We have to also implement the specified left boundary condition, specified right boundary condition and Neumann boundary condition bottom, Neumann boundary condition on top.

(Refer Slide Time: 59:42)

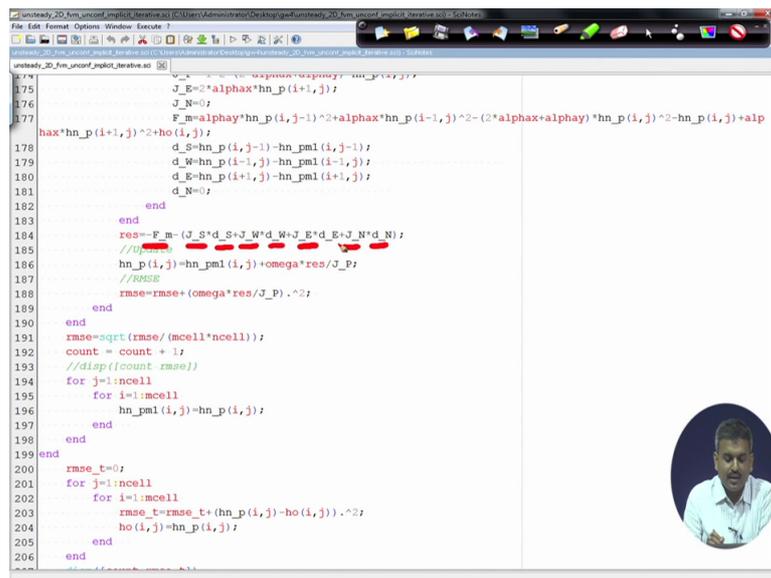
```

142   J_S=2*alpha*hn_p(i,j-1);
143   J_N=2*alpha*hn_p(i-1,j)+(2/3)*alpha*hm;
144   J_P=-(1+6*alpha*hm)-2*(alpha+2*alpha)*hn_p(i,j);
145   J_E=0;
146   J_N=2*alpha*hn_p(i,j+1);
147   F_m=alpha*hn_p(i,j-1)^2+alpha*hn_p(i-1,j)^2+(2/3)*alpha*hm*hn_p(i-1,j)-(alpha+2*alp
148   hay*hn_p(i,j)^2-(1+6*alpha*hm)*hn_p(i,j)+alpha*hn_p(i,j+1)^2+ho(i,j)+(1/3)*alpha*hm^2;
149   d_S=hn_p(i,j-1)-hn_p(i,j);
150   d_W=hn_p(i-1,j)-hn_p(i,j);
151   d_E=0;
152   d_N=hn_p(i,j+1)-hn_p(i,j);
153 end
154 //Neuman BBC
155 if (j==1) then
156   if (i > 1 & i < mcell) then
157     J_S=0;
158     J_N=2*alpha*hn_p(i-1,j);
159     J_P=-1-2*(2*alpha+alpha)*hn_p(i,j);
160     J_E=2*alpha*hn_p(i+1,j);
161     J_N=2*alpha*hn_p(i,j+1);
162     F_m=alpha*hn_p(i-1,j)^2-(2*alpha+alpha)*hn_p(i,j)^2-hn_p(i,j)+alpha*hn_p(i+1,j)^2+alp
163     hay*hn_p(i,j+1)^2+ho(i,j);
164     d_S=0;
165     d_E=hn_p(i+1,j)-hn_p(i,j);
166     d_N=hn_p(i,j+1)-hn_p(i,j);
167   end
168 end
169 //Neuman TBC
170 if (j==ncell) then
171   if (i > 1 & i < mcell) then
172     J_S=2*alpha*hn_p(i,j-1);
173     J_W=2*alpha*hn_p(i-1,j);

```

So after implementing these we need to calculate this residual. Residual is nothing but minus m minus JS, dS, JW, dW, JE, dE, JN, dN. Now we have not included JP here. So this is the residual.

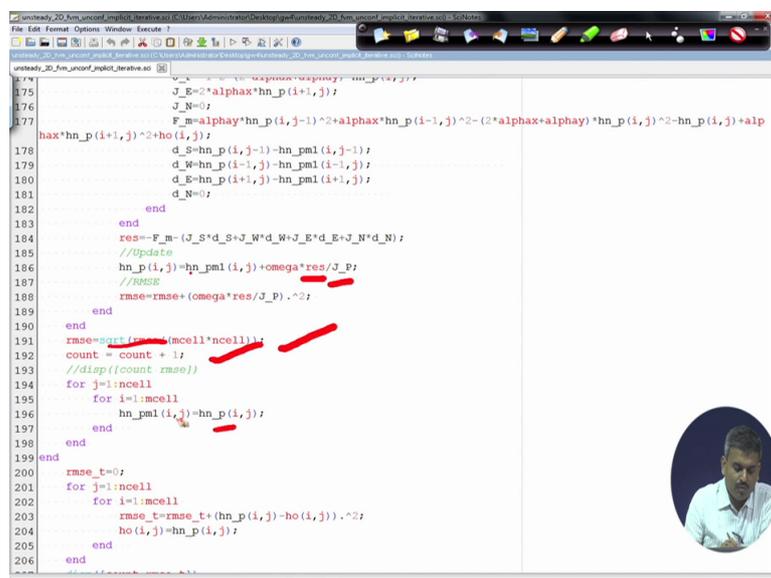
(Refer Slide Time: 01:00:03)



```
175 J_E=2*alpha*hn_p(i+1,j);
176 J_N=0;
177 F_m=alpha*hn_p(i,j-1)^2+alpha*hn_p(i-1,j)^2-(2*alpha+alpha)*hn_p(i,j)^2-hn_p(i,j)+alp
hax*hn_p(i+1,j)^2+ho(i,j);
178 d_S=hn_p(i,j-1)-hn_pml(i,j-1);
179 d_W=hn_p(i-1,j)-hn_pml(i-1,j);
180 d_E=hn_p(i+1,j)-hn_pml(i+1,j);
181 d_N=0;
182 end
183 end
184 res=-F_m-(J_S*d_S+J_W*d_W+J_E*d_E+J_N*d_N);
185 //Update
186 hn_p(i,j)=hn_pml(i,j)+omega*res/J_P;
187 //RMSE
188 rmse=rmse+(omega*res/J_P).^2;
189 end
190 end
191 rmse=sqrt(rmse/(mcell*ncell));
192 count = count + 1;
193 //disp(count rmse)
194 for j=1:ncell
195     for i=1:mcell
196         hn_pml(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:ncell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
```

Residual is residual divided by JP and this is hn P minus 1, hn P. And we can calculate this rmse and again we can calculate rmse here which is the final rmse. And this is the count. Now we need to transfer this new values to old values and we can utilise this for space loop.

(Refer Slide Time: 01:00:30)



```
175 J_E=2*alpha*hn_p(i+1,j);
176 J_N=0;
177 F_m=alpha*hn_p(i,j-1)^2+alpha*hn_p(i-1,j)^2-(2*alpha+alpha)*hn_p(i,j)^2-hn_p(i,j)+alp
hax*hn_p(i+1,j)^2+ho(i,j);
178 d_S=hn_p(i,j-1)-hn_pml(i,j-1);
179 d_W=hn_p(i-1,j)-hn_pml(i-1,j);
180 d_E=hn_p(i+1,j)-hn_pml(i+1,j);
181 d_N=0;
182 end
183 end
184 res=-F_m-(J_S*d_S+J_W*d_W+J_E*d_E+J_N*d_N);
185 //Update
186 hn_p(i,j)=hn_pml(i,j)+omega*res/J_P;
187 //RMSE
188 rmse=rmse+(omega*res/J_P).^2;
189 end
190 end
191 rmse=sqrt(rmse/(mcell*ncell));
192 count = count + 1;
193 //disp(count rmse)
194 for j=1:ncell
195     for i=1:mcell
196         hn_pml(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:ncell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
```

Then comes the time loop. For time loop this new value is old value. This is for steady state condition. This rmse t we need to implement.

(Refer Slide Time: 01:00:46)

```
unsteady_2D_fm_uncoupled_iterative.m
191 rmse=0; %Time step (msecell-hcwx1-hcwx1);
192 count = count + 1;
193 //disp((count rmse))
194 for j=1:mcell
195     for i=1:mcell
196         hn_pml(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:mcell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
207 disp((count rmse_t))
208 //Condition for Steady State
209 if (rmse_t < eps_max) then
210     break
211 end
212 end
213
214 //Boundary Information
215 hdata=zeros(mcell+2,ncell+2);
216 //Internal Cells
217 for j=2:ncell+1
218     for i=2:mcell+1
219         hdata(i,j)=hn_p(i-1,j-1);
220     end
221 end
222 //A
223 hdata(1,ncell+2)=hA;
224 //B
```

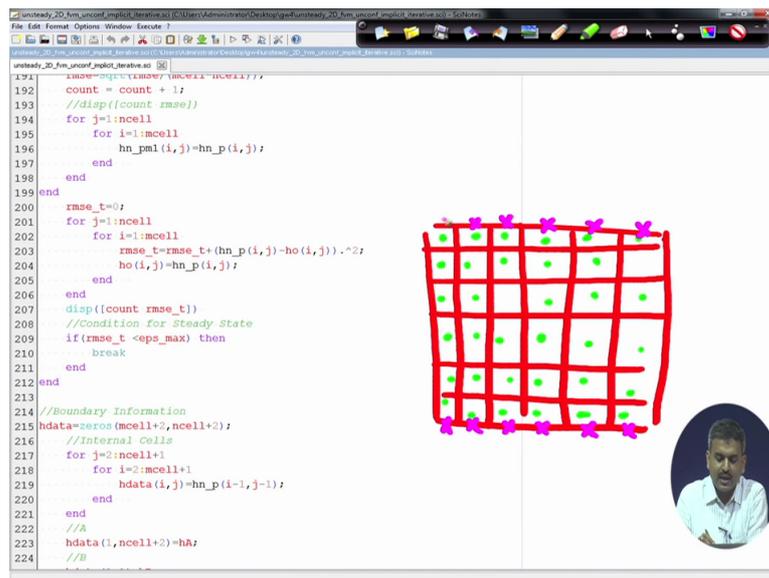
And obviously we know that for our boundary we have got values for only internal nodes. That means we have information about these nodes. This is in simple form. This is the information we have, self centred values.

(Refer Slide Time: 01:01:30)

```
unsteady_2D_fm_uncoupled_iterative.m
191 rmse=0; %Time step (msecell-hcwx1-hcwx1);
192 count = count + 1;
193 //disp((count rmse))
194 for j=1:mcell
195     for i=1:mcell
196         hn_pml(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:mcell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
207 disp((count rmse_t))
208 //Condition for Steady State
209 if (rmse_t < eps_max) then
210     break
211 end
212 end
213
214 //Boundary Information
215 hdata=zeros(mcell+2,ncell+2);
216 //Internal Cells
217 for j=2:ncell+1
218     for i=2:mcell+1
219         hdata(i,j)=hn_p(i-1,j-1);
220     end
221 end
222 //A
223 hdata(1,ncell+2)=hA;
224 //B
```

But further we need information regarding these points. So that we can get from the derivative condition. We can equate derivative equals to zero and bN and bS values we can calculate there.

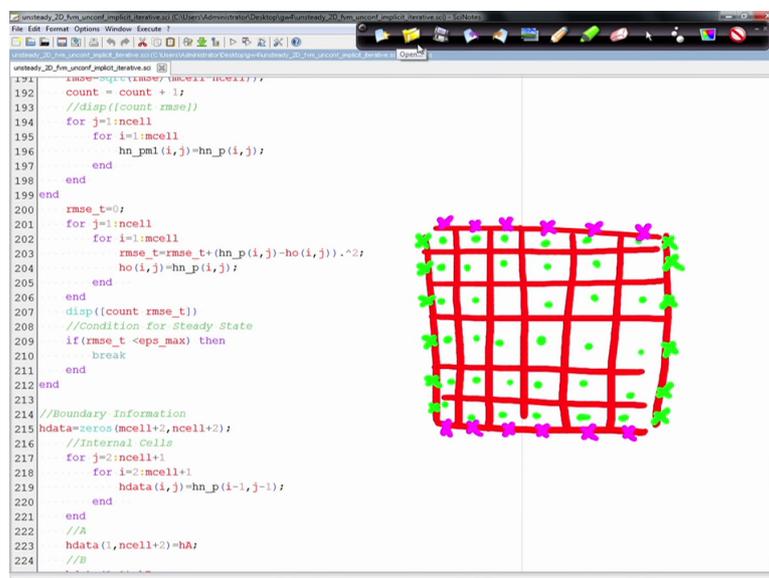
(Refer Slide Time: 01:01:49)



```
191 hdata=zeros(mcell+2,ncell+2);
192 count = count + 1;
193 //disp((count rmse))
194 for j=1:ncell
195     for i=1:mcell
196         hn_p(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:ncell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
207 disp((count rmse_t))
208 //Condition for Steady State
209 if(rmse_t < eps_max) then
210     break
211 end
212 end
213
214 //Boundary Information
215 hdata=zeros(mcell+2,ncell+2);
216 //Internal Cells
217 for j=2:ncell+1
218     for i=2:mcell+1
219         hdata(i,j)=hn_p(i-1,j-1);
220     end
221 end
222 //A
223 hdata(1,ncell+2)=hA;
224 //B
```

Also on the left hand boundary we can directly get the interpolated value from our specified boundary condition. So after solution which are green dots we need to extend our program to get this pink cross and green cross points so that we can solve the problem for the full domain.

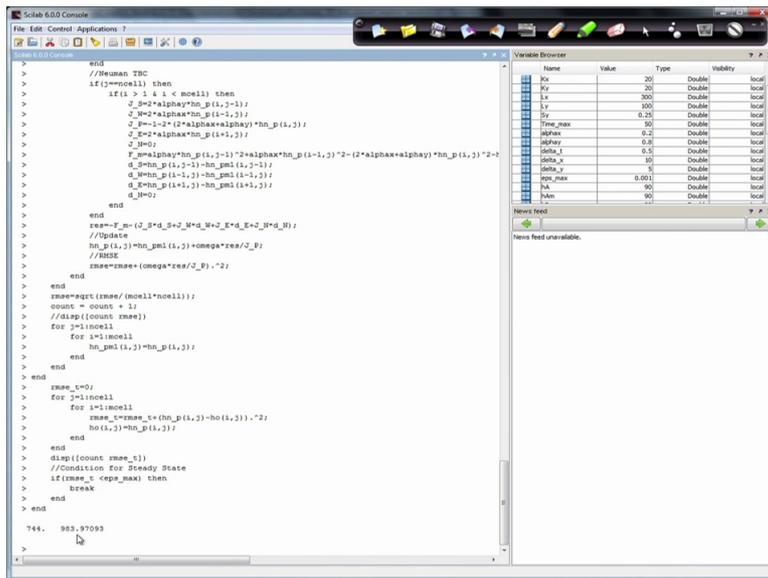
(Refer Slide Time: 01:02:22)



```
191 hdata=zeros(mcell+2,ncell+2);
192 count = count + 1;
193 //disp((count rmse))
194 for j=1:ncell
195     for i=1:mcell
196         hn_p(i,j)=hn_p(i,j);
197     end
198 end
199 end
200 rmse_t=0;
201 for j=1:ncell
202     for i=1:mcell
203         rmse_t=rmse_t+(hn_p(i,j)-ho(i,j)).^2;
204         ho(i,j)=hn_p(i,j);
205     end
206 end
207 disp((count rmse_t))
208 //Condition for Steady State
209 if(rmse_t < eps_max) then
210     break
211 end
212 end
213
214 //Boundary Information
215 hdata=zeros(mcell+2,ncell+2);
216 //Internal Cells
217 for j=2:ncell+1
218     for i=2:mcell+1
219         hdata(i,j)=hn_p(i-1,j-1);
220     end
221 end
222 //A
223 hdata(1,ncell+2)=hA;
224 //B
```

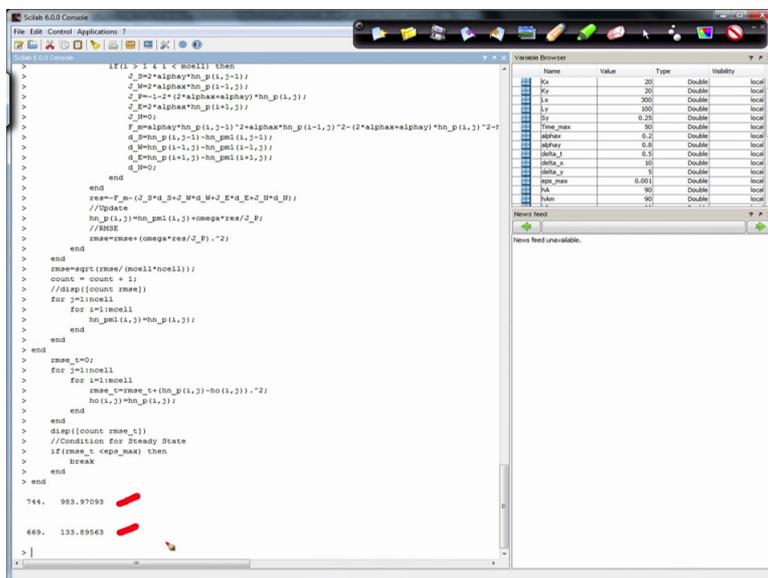
Now if we run this code again it will take some time because in this case we need certain number of iterations for each space loop convergence. In space loop we are solving that Jacobian matrix. If we solve that Jacobian matrix so obviously here you can see that 744 iterations are required to get rmse value of 983.

(Refer Slide Time: 01:03:17)



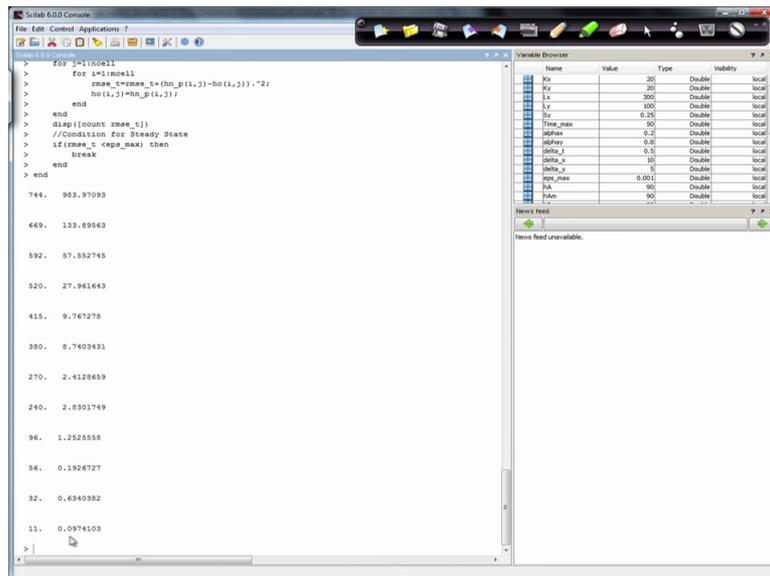
So rmse value of 983 this is absurd value. So obviously further 669 iterations are required to get this 133 rmse value. Now slowly these rmse values will decrease and finally we can get the solution out of this process and we can get the contour value from this program.

(Refer Slide Time: 01:04:01)



And contour value is similar to our finite (diff) volume or finite difference solution. But the contour is similar but somewhat different compared to our confined aquifer solution because we are solving unconfined aquifer problem here. So you can see that 380 iterations and slowly this rmse value is decreasing. This is 2 point 4, this is 56.

(Refer Slide Time: 01:04:48)



And finally we are getting the solution from the program. So iterative technique we can utilise to solve this unconfined aquifer problem.

(Refer Slide Time: 01:05:00)

