

Computational Hydraulics
Professor Anirban Dhar
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Indian Institute of Technology Kharagpur
Lecture 34
Unsteady Two-Dimensional Flow Using Finite Volume Method

Welcome to this lecture number 34 of the course computational hydraulics. We are in module 3, groundwater hydraulics. And in this particular lecture class I will be covering unsteady two dimensional flow using finite volume method and specifically the formulation is for confined aquifer flow and this is unit number 4.

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The image shows a presentation slide with a white background and a dark blue header. The header contains the text "Problem Definition", "Domain Discretization", and "Discretization: Boundary Points" on the left, and "I.I.T. Kharagpur" with a logo on the right. The main content area features a large blue rounded rectangle with the text "Module 03: Groundwater Hydraulics" and "Unit 04: Unsteady Two-Dimensional Flow using Finite Volume Method". Below this, the name "Anirban Dhar" is displayed, followed by "Department of Civil Engineering" and "Indian Institute of Technology Kharagpur, Kharagpur". At the bottom, it says "National Programme for Technology Enhanced Learning (NPTEL)". The footer of the slide includes "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "1 / 28".

Learning objective for this particular lecture. At the end of this lecture students will be able to solve unsteady state two dimensional groundwater flow equation using finite volume method and specifically confined aquifer flow.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Learning Objective

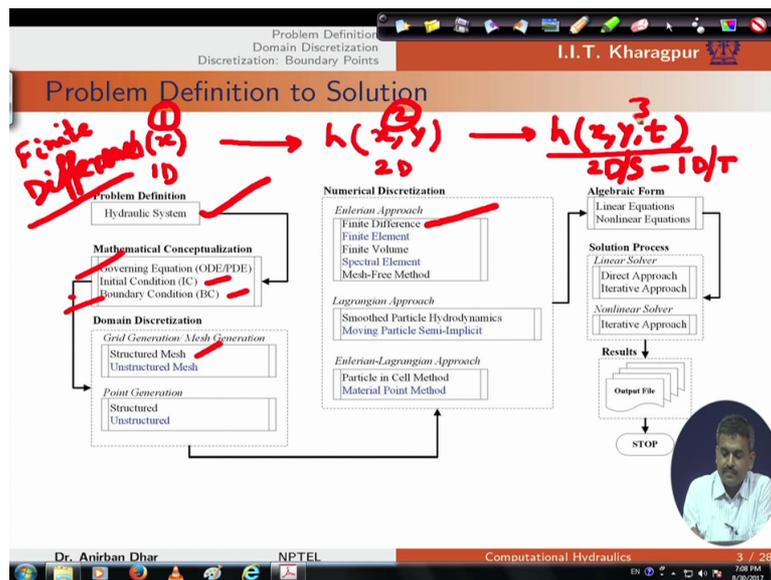
- To solve unsteady state two dimensional groundwater flow equation using Finite Volume Method.

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We have seen that from finite difference formulation we have h_x which is one dimensional, then we have solved the equation for xy , then we have introduced the time component in formulation. So starting from 1D then 2D in space, then 2D in space and 1D in time we have formulated using finite difference formulation. So again that is one kind of Eulerian approach. So starting from our hydraulic system we have mathematically conceptualized the problem in terms of governing equation.

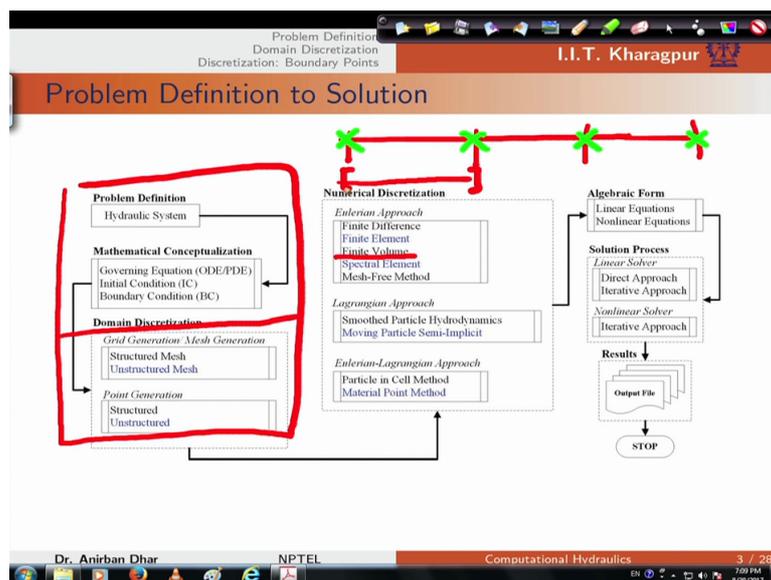
Then for 1D case we have utilised boundary condition, for 2D case we have utilised both the boundary condition on four sides for a two dimensional system and this $h(x,y,t)$ problem which is having one temporal dimension, that is two dimensional space and one dimensional time, we have utilised initial condition, boundary condition both. And in this finite difference approach we have utilised this structured grid for all the problems. This is problem number 1, 2 and 3.

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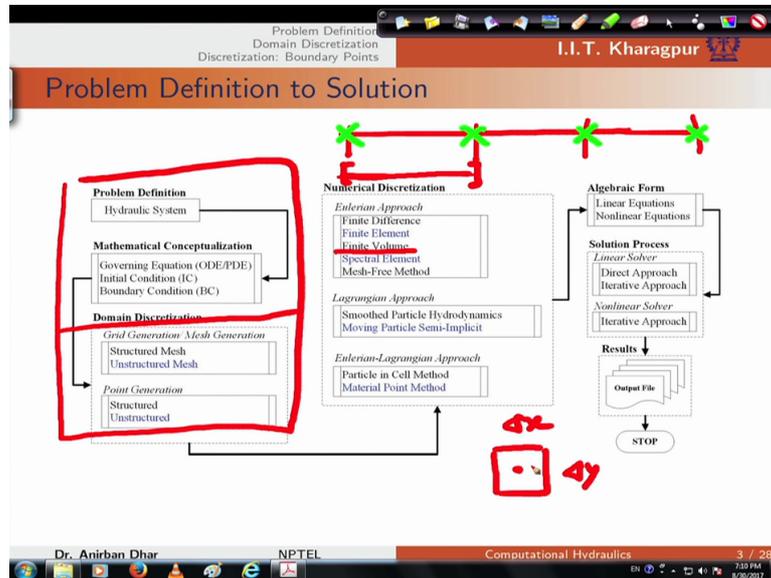
So now we will try to utilise the concept of finite volume to discretize the same equation but we will try to see what are the differences exists in the solution process. So discretization obviously up to this we can utilise the same governing equation. Discretization wise if we consider finite difference method then we need to consider certain number of grid points or node points. But in case of our finite volume approach we need to consider this cell thing. So, one dimensional cell will look similar to this.

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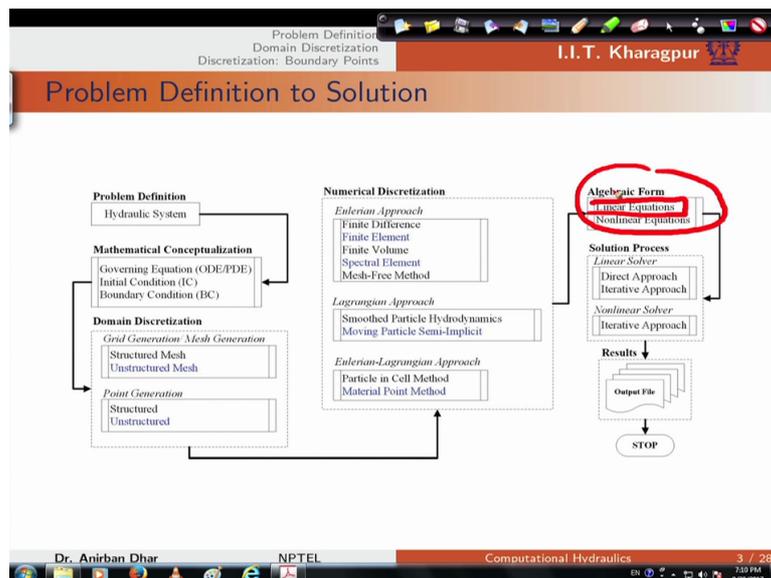
But if we have two dimensional case then cell will be similar to this where this will be del y dimension, on this side this will be del x dimension for the cell. And we will try to utilise the self centred value for our problem.

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For case of algebraic form again we will try to utilise linear equation because our problem is linear in nature.

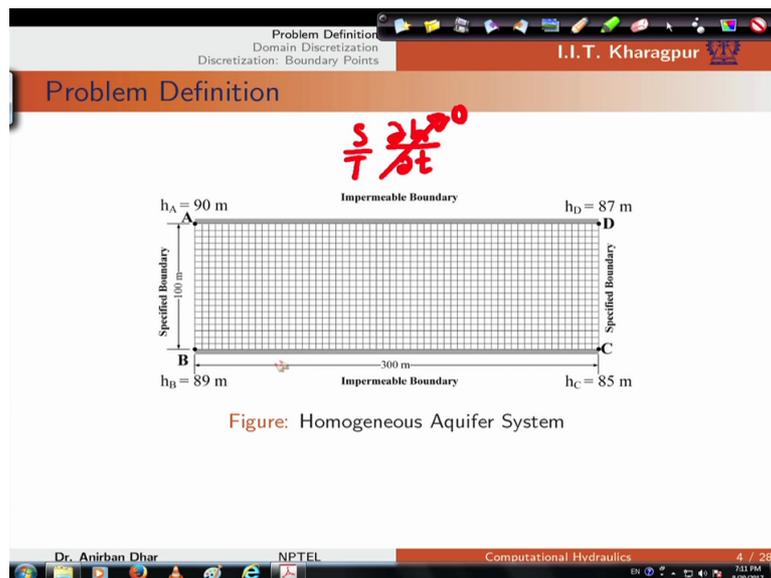
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Problem definition, this is the same problem we have defined for our steady state case and we have utilised the transient case to solve the steady state problem. Obviously in transient case

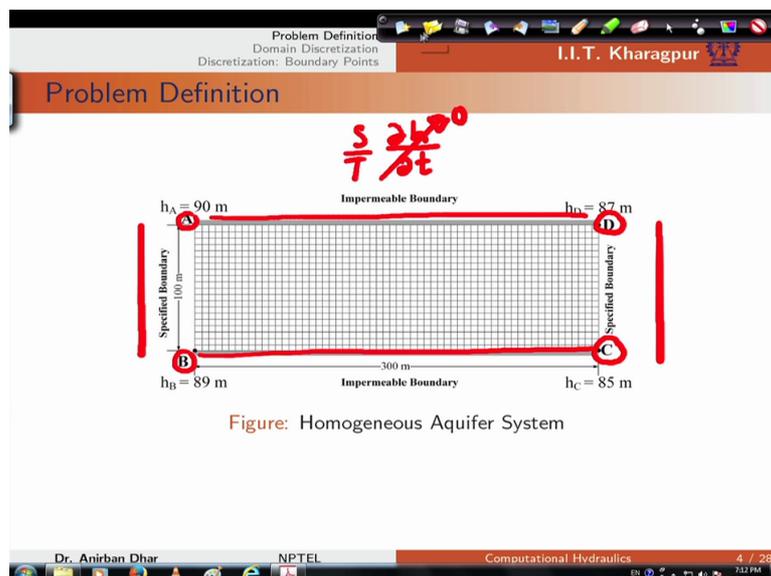
we have utilised the time stepping approach and for the consecutive time step if there is not much change in the h value then we can get equivalent steady state solution from the transient approach. So this approach is called as Quasi steady state approach.

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So in our case we have left hand boundary as specified boundary, right hand boundary as specified boundary and this point values are specified at the point A, B, C, D for these four points. And the bottom and the top boundaries, these boundaries are impermeable boundaries. Now we will try to see how we can discretize the same problem using finite volume approach.

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So this is again the same governing equation. Two dimensional in space IBVP that is initial boundary value problem and this equation is valid for homogeneous isotropic confined aquifer condition. So in this case S is storativity and our T is transmissivity.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

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Problem Definition

Governing equation

A two-dimensional (in space) IBVP can be written as,

$$\Omega : \frac{S}{T} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$$

$S = 5 \times 10^{-5}$
 $T = 200 \text{ m}^2/\text{day}$

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So let us see how we can discretize it. This equation is subject to initial condition which is an initial distribution in the aquifer. If we have transient problem then we need to exactly specify the actual initial condition in the field. But if we are solving steady state problem then we can start with any arbitrary initial guess for the steady state condition. However if the guess value is close to the actual solution then the number of iteration required will be relatively small.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

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Problem Definition

subject to

Initial Condition

$$h(x, y, 0) = h_0(x, y)$$

and

Boundary Condition

$$\Gamma_D^1 : h(0, y, t) = h_1(y)$$
$$\Gamma_D^2 : h(L_x, y, t) = h_2(y)$$
$$\Gamma_N^3 : \left. \frac{\partial h}{\partial y} \right|_{(x,0,t)} = 0$$
$$\Gamma_N^4 : \left. \frac{\partial h}{\partial y} \right|_{(x,L_y,t)} = 0$$

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Then comes this boundary condition. Left hand boundary obviously h_1 which is varying on the left side, this is Dirichlet boundary on the right side, this is Neumann which is zero Neumann. That means no flow boundary conditions on top and bottom.

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The slide is titled "Problem Definition" and is part of a presentation from I.I.T. Kharagpur. It shows the following conditions:

subject to

Initial Condition

$$h(x, y, 0) = h_0(x, y)$$

and

Boundary Condition

$$\begin{aligned} \Gamma_D^1 : h(0, y, t) &= h_1(y) \\ \Gamma_D^2 : h(L_x, y, t) &= h_2(y) \\ \Gamma_N^3 : \frac{\partial h}{\partial y} \Big|_{(x, 0, t)} &= 0 \\ \Gamma_N^4 : \frac{\partial h}{\partial y} \Big|_{(x, L_y, t)} &= 0 \end{aligned}$$

The slide also includes a small video inset of a speaker and a footer with "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "6 / 28".

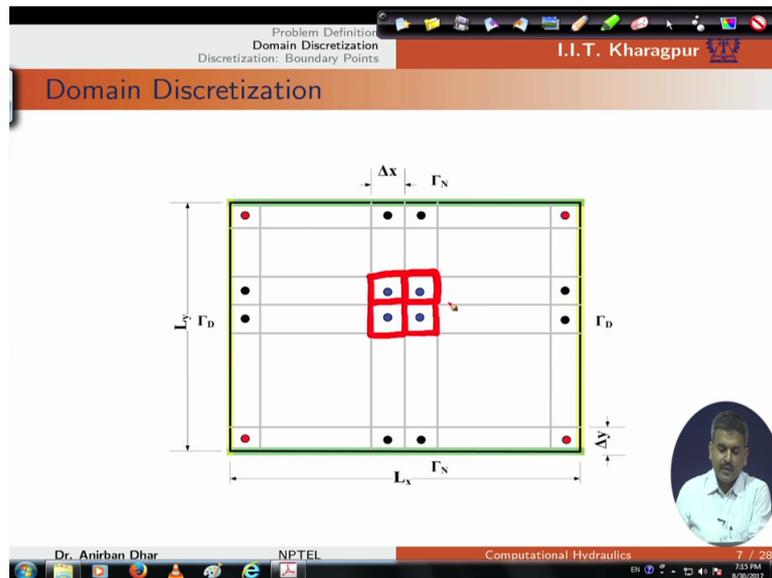
This is a domain discretization. In case of our finite difference approach we have considered this point as central point and we have utilised the surrounding nodes for discretization of space and time derivative. And this is for spatial derivative if we consider another dimension which is for time then we can consider the time discretization.

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The slide is titled "Domain Discretization" and shows a 2D grid of nodes. The grid is rectangular with dimensions L_x and L_y . The horizontal spacing between nodes is Δx and the vertical spacing is Δy . The boundaries are labeled Γ_D (Dirichlet) and Γ_N (Neumann). The nodes are represented by red dots at the corners and black dots at the midpoints of the edges. Blue dots are placed at the cell centers, and red crosses are placed at the midpoints of the edges, indicating a central point and its surrounding nodes for discretization.

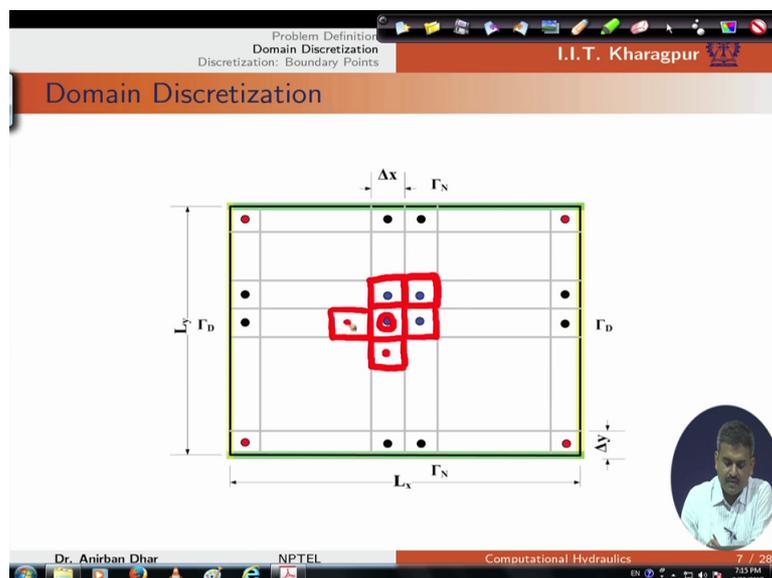
So but as we can see that blue dots are placed at the cell centres which is rectangular in nature. So these are individual cells. So based on self centred values we will try to solve this finite volume problem.

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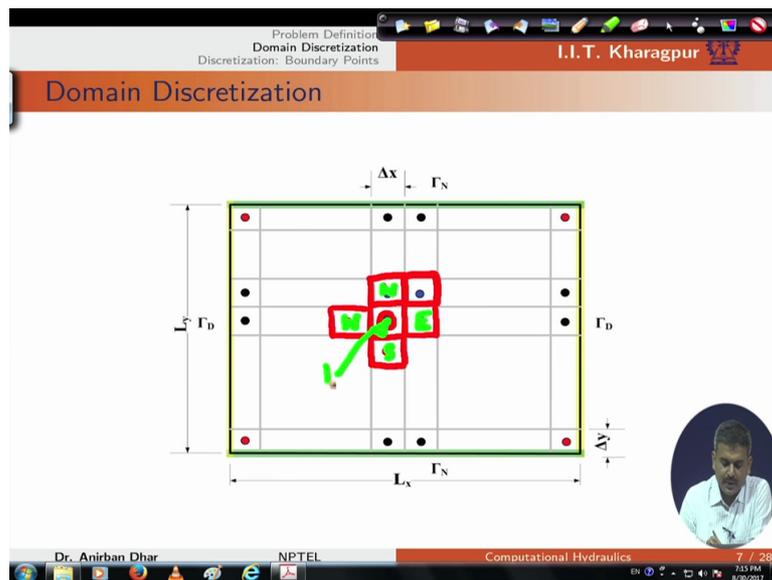
Now there can be different type of cells. maybe this particular cell which is having all neighbouring cells present. This is the self centred for neighbouring cells.

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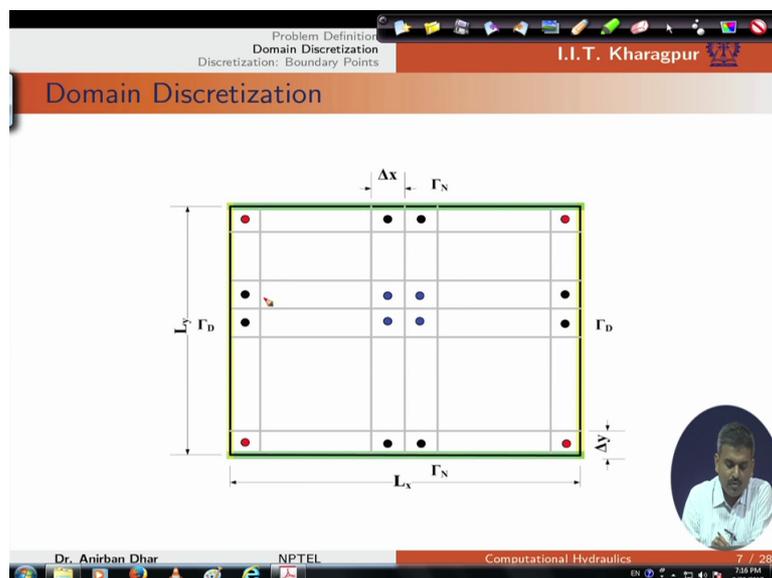
So on the northern side this is the cell and this is on the southern side and this is on the western side and this is on the eastern side of the central cell which is the cell P.

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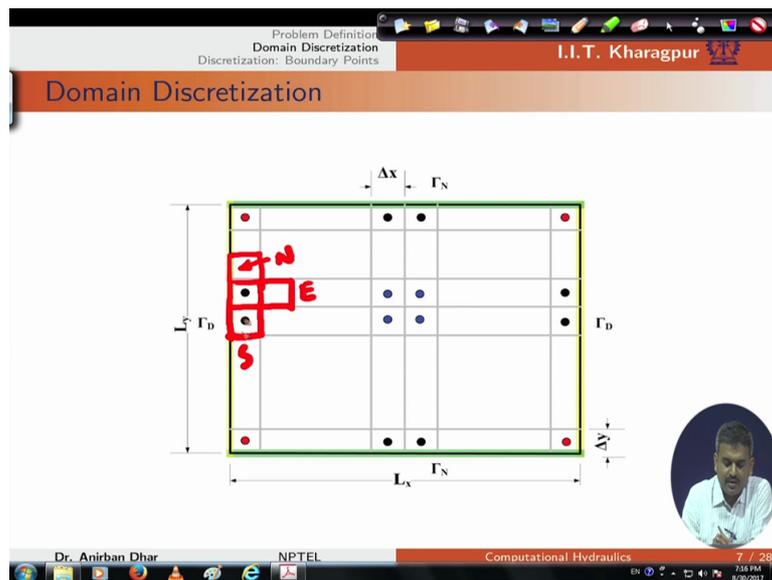
Now if we consider our left boundary and which is very close to our left boundary. So this is a cell with black dots, the cells which are adjacent to the boundary.

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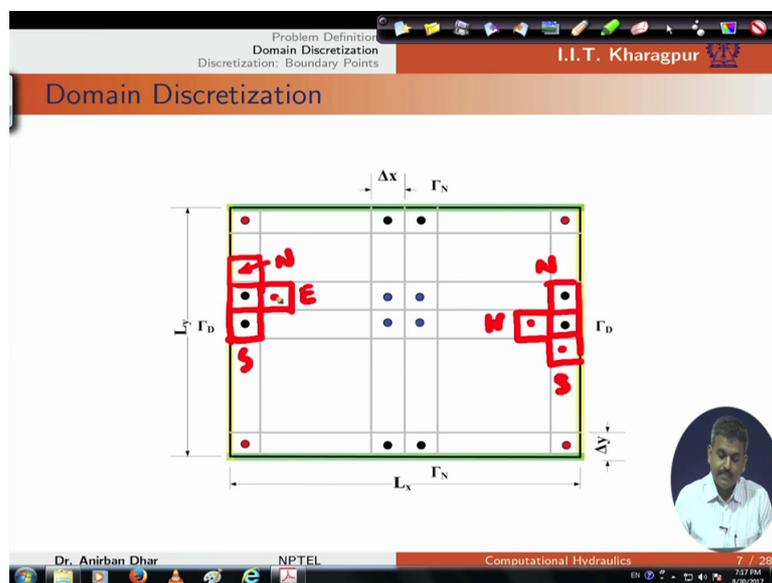
Now on the left boundary interestingly in this case there is no west cell present. obviously this is northern cell, this is eastern one, this is southern one. But there is no western cell present here.

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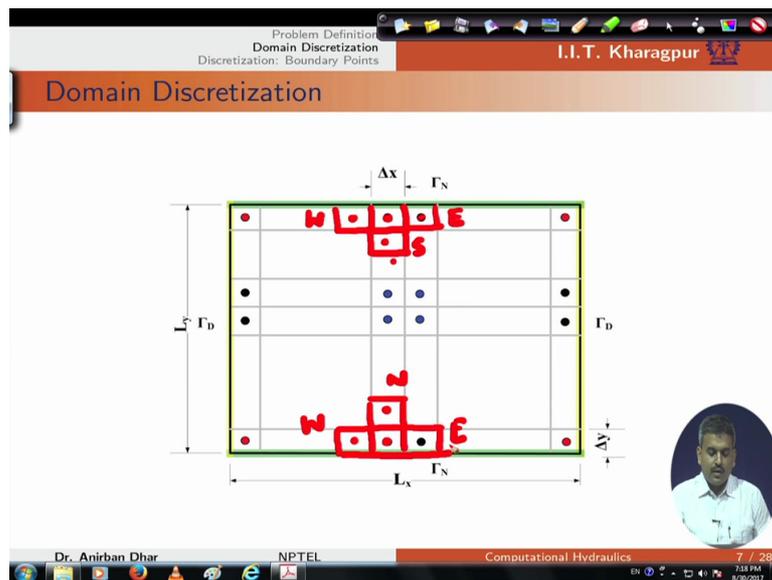
And similarly on the right hand side we do not have any eastern cell. So on this side this is northern, this is western, this is southern cell. So these are two different conditions on left boundary and right boundary.

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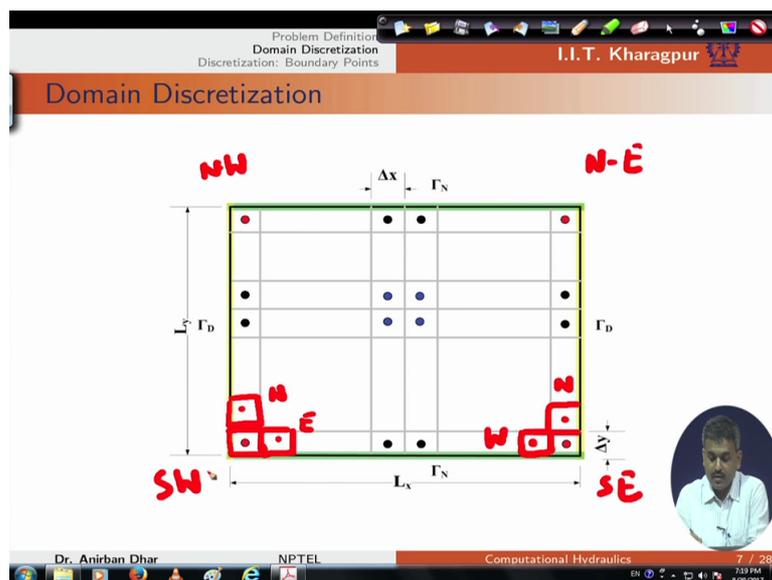
Now if we consider the situation for the top boundary again we have no northern boundary present for this case. If this is the central cell then we have western, eastern and southern cells. On the bottom if this is the central cell then we have western, eastern and northern cell present. But no southern cell present here.

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Another situation that is very much important for this case if we consider our corner points. For corner points, corner points are represented with red dots. Now if this is the central cell then we have eastern cell present and northern cell. There is no western or southern cell present in this case. So on this boundary also we have western, northern. But no eastern and no southern cell present there. This is for northwest boundary, this is northeast, this is southeast and south west.

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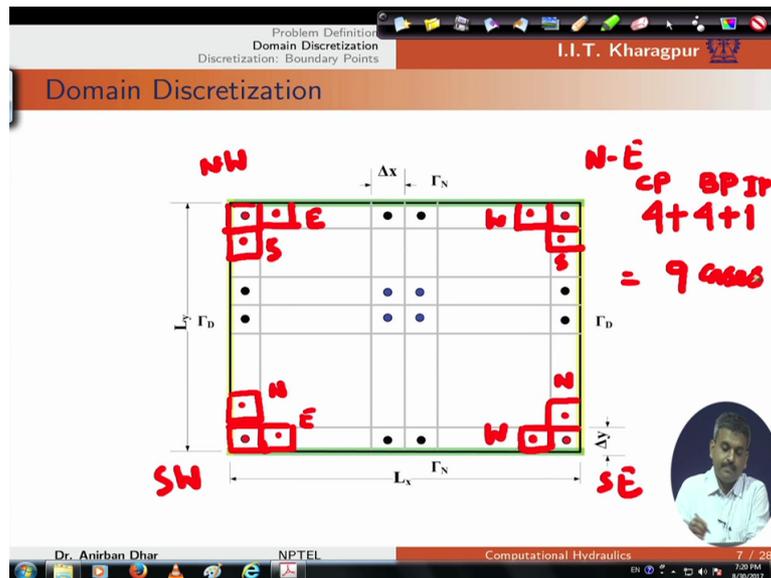


So in this case we have only southern and western boundary. No eastern and no northern boundary. Similarly for this case we have southern and eastern boundary cells presents but no

northern or western cell present in this case. So all total considering four corner points plus four boundary points, this is for CP corner points, BPR boundary points plus 1 for IP or interior point.

So total 9 cases or 9 situations maybe there for solving this finite volume problem.

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Now from our lecture 15 if we see the discretization of this problem then simply we can discretize this temporal term like this. So obviously no Δt will be there. Δt will be coming from the right hand side. This is simple eastern and western cases. Ideally speaking we have this kind of formulation on the right hand side and the left hand side we have dS by $dT \Delta h$ by Δt .

(Refer Slide Time: 16:28)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, $\frac{\delta h}{\delta t} = \nabla^2 h$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now we can represent it with ∇h by ∇t and $\nabla \cdot$ this one. So now for this one we can apply the integration over spatial domain and temporal one t plus ∇t . And we can apply the same integration in this case t to t plus ∇t and from this one and obviously this d terms will be there. $D \omega$ and dt . In this case also $d \omega dt$ terms will be there on the right hand side.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

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Implicit Scheme

From Lecture 15, $\frac{\delta h}{\delta t} = \nabla^2 h \Rightarrow \int \int \frac{\delta h}{\delta t} = \int \int (\nabla \cdot (\nabla h)) d\omega dt$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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So from this one this left hand side is very much clear. On the right hand side what we are getting? We are getting first order derivatives. We have four faces. One is northern, one is eastern, one is southern, another one is western.

(Refer Slide Time: 18:19)

Problem Definition
Domain Discretization
Discretization: Boundary Points

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Implicit Scheme

From Lecture 15, $\frac{\sum \partial h}{T \partial t} = \nabla^2 h \Rightarrow \iint \frac{\sum \partial h}{T \partial t} = \iint (\nabla \cdot (\nabla h)) d\Omega$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now we will consider that area is always outward positive. So if area is always outward positive then area on this face is nothing but del y into i, on this face this is minus del y into i, on this face this is del x into j, on this face it is del x into j.

(Refer Slide Time: 19:17)

Problem Definition
Domain Discretization
Discretization: Boundary Points

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Implicit Scheme

From Lecture 15, $\frac{\sum \partial h}{T \partial t} = \nabla^2 h \Rightarrow \iint \frac{\sum \partial h}{T \partial t} = \iint (\nabla \cdot (\nabla h)) d\Omega$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now if we consider that this divergence or Gauss divergence theorem so this will be multiplied with area vector or surface vector for this case. This is face. F can be eastern or northern, southern, western.

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Problem Definition
Domain Discretization
Discretization: Boundary Points

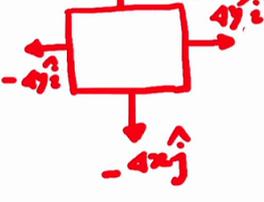
I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, $\frac{\partial h}{\partial t} = \nabla^2 h \Rightarrow \int_V \frac{\partial h}{\partial t} dV = \int_V (\nabla \cdot (\nabla h)) dV$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

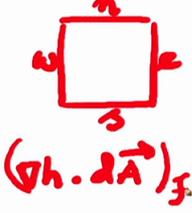
For interior points:



$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$


$(\nabla h \cdot d\vec{A})_f$

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Now if we consider del h, del h we can represent it as del h by del x into i plus del h by del y into j.

(Refer Slide Time: 20:09)

Problem Definition
Domain Discretization
Discretization: Boundary Points

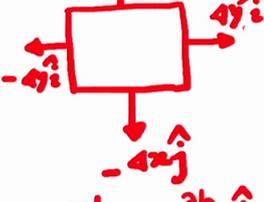
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Implicit Scheme

From Lecture 15, $\frac{\partial h}{\partial t} = \nabla^2 h \Rightarrow \int_V \frac{\partial h}{\partial t} dV = \int_V (\nabla \cdot (\nabla h)) dV$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

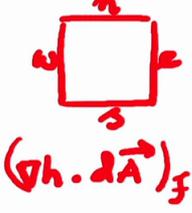
For interior points:



$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$


$(\nabla h \cdot d\vec{A})_f$

$\nabla h = \frac{\partial h}{\partial x} \hat{i} + \frac{\partial h}{\partial y} \hat{j}$

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So if we take this del h dot dA for eastern face so this is nothing but del y multiplied here into del h by del x.

(Refer Slide Time: 20:27)

Problem Definition
Domain Discretization
Discretization: Boundary Points

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Implicit Scheme

From Lecture 15, $\frac{\sum \partial h}{T} = \nabla^2 h \Rightarrow \int \int \frac{\sum \partial h}{T} = \int \int (\nabla \cdot (\nabla h)) dA$

$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$

For interior points:

$\nabla h = \frac{\partial h}{\partial x} \hat{i} + \frac{\partial h}{\partial y} \hat{j}$

$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$

$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$

$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$

$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$

$(\nabla h \cdot \vec{dA})_f$

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So only this term will be there. On western face we are multiplying with minus del y and we are multiplying it with delta h dot dA vector and F. So obviously in this case we will get a negative component. So that is why we have del h by del x e L plus 1 and del h by del x L plus 1 w.

(Refer Slide Time: 22:28)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, $\frac{S}{T} \frac{\partial h}{\partial t} = \nabla \cdot \mathbf{h} \Rightarrow \int_V \frac{S}{T} \frac{\partial h}{\partial t} = \int_V \nabla \cdot (\mathbf{h}) \, dV$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points:

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

$\nabla \mathbf{h} = \frac{\partial h}{\partial x} \hat{i} + \frac{\partial h}{\partial y} \hat{j}$

$(\nabla \mathbf{h} \cdot d\mathbf{A})_f$

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So in this case we have discretized the right hand side thing and we can get the discretized form of that in terms of derivatives. Now we need to see how we can discretize it.

(Refer Slide Time: 22:52)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, $\frac{S}{T} \frac{\partial h}{\partial t} = \nabla \cdot \mathbf{h} \Rightarrow \int_V \frac{S}{T} \frac{\partial h}{\partial t} = \int_V \nabla \cdot (\mathbf{h}) \, dV$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points:

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

$\nabla \mathbf{h} = \frac{\partial h}{\partial x} \hat{i} + \frac{\partial h}{\partial y} \hat{j}$

$(\nabla \mathbf{h} \cdot d\mathbf{A})_f$

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Time discretization is straight forward. It is starting from L to L plus 1. L corresponds to t. L plus 1 means we are considering t plus delta t time level.

(Refer Slide Time: 23:16)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, $\frac{S}{T} \frac{\partial h}{\partial t} = \nabla^2 h \Rightarrow \int_V \frac{S}{T} \frac{\partial h}{\partial t} = \int_V (\nabla \cdot (Vh)) dx dt$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

$\nabla h = \frac{\partial h}{\partial x} \hat{i} + \frac{\partial h}{\partial y} \hat{j}$

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Now with this discretization which is our first level discretization using finite volume method we can use our knowledge from finite difference to discretize individual derivatives on the eastern, western, northern and southern sides.

(Refer Slide Time: 23:47)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15, **FVM**

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now this is our northern side, this is southern, this is eastern, this is western. So let us consider the neighbouring cells. So we have all neighbouring cells for governing equation. Because governing equation is applicable for interior cells. So in this case this is the central one, this is western, eastern, northern, southern. So this is cell P, this is E, this is west, this is north, this is south.

(Refer Slide Time: 25:12)

Problem Definition
Domain Discretization
Discretization: Boundary Points

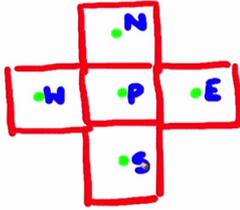
I.I.T. Kharagpur

Implicit Scheme

From Lecture 15,

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

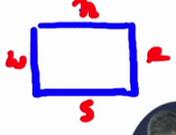
For interior points,



$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$


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Now if we want any discretization of the eastern face, this is northern, this is southern, this is western. If you want any discretization on the eastern face we should consider P and E. So del h by del x from our knowledge of finite difference we can say that hE minus hP divided by del x. Del x is the distance between the central nodes E and P.

(Refer Slide Time: 25:57)

Problem Definition
Domain Discretization
Discretization: Boundary Points

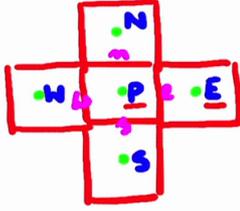
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Implicit Scheme

From Lecture 15,

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

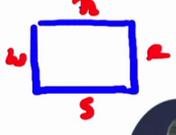
For interior points,



$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$


Dr. Anirban Dhar NPTEL Computational Hydraulics 8 / 28

Del h by del x at western face. Western face we should consider point P and point W. So hP minus W divided by del x. Del h by del y at northern face. Northern face we should consider this P and N. So hN minus hP, this one.

(Refer Slide Time: 26:32)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15,

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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On the southern face $\frac{\partial h}{\partial y}$ this is h_P minus h_S . So we should consider this h_P and h_S for southern boundary.

(Refer Slide Time: 26:45)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

From Lecture 15,

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

For interior points,

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now we have got information regarding the face values of these derivatives. I should say these are face values because at face these derivatives are defined. So individually we have calculated the discretized form of the derivatives. Now you can utilise these derivatives to from the final governing equation.

(Refer Slide Time: 27:17)

So if we substitute those derivatives then we will get this form of discretization. This is similar to our finite difference formulation. In finite difference formulation we have seen that for ij point we have i plus 1j, this is i minus 1j, this is ij minus 1, this is ij plus 1. Now in case of our finite volume we are considering this one as eastern, this one as western, this one as northern, this one as southern cell.

(Refer Slide Time: 28:17)

So our cell centre. Now this discretization is equivalent to over finite difference formulation. Now if we further simplify it using these two parameters. Let us say that alpha x can be defined in terms of T delta t divided by S delta x square and alpha y is T delta t divided by S

delta y square. Now with these two if we transfer all unknown quantities on the left hand side we will get hP, hE, hN, hW, hS because all are having L plus 1 as index for time component.

(Refer Slide Time: 29:24)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

FDM (M) i,j (N) $i,j+1$ (E) $i+1,j$ (S) $i,j-1$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{h_E^{l+1} - 2h_P^{l+1} + h_W^{l+1}}{\Delta x^2} + \frac{h_N^{l+1} - 2h_P^{l+1} + h_S^{l+1}}{\Delta y^2}$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} + \alpha_x h_W^{l+1} - [1 + 2(\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l$$

with $\alpha_x = \frac{T \Delta t}{S \Delta x^2}$ and $\alpha_y = \frac{T \Delta t}{S \Delta y^2}$.

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So we have hS, hW these components. On the right hand side we have some known quantities. In this particular problem we do not have any pumping or injection component. That is why I have not considered the W here which is usual source sink term.

(Refer Slide Time: 29:53)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

FDM (M) i,j (N) $i,j+1$ (E) $i+1,j$ (S) $i,j-1$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{h_E^{l+1} - 2h_P^{l+1} + h_W^{l+1}}{\Delta x^2} + \frac{h_N^{l+1} - 2h_P^{l+1} + h_S^{l+1}}{\Delta y^2}$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} + \alpha_x h_W^{l+1} - [1 + 2(\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l$$

with $\alpha_x = \frac{T \Delta t}{S \Delta x^2}$ and $\alpha_y = \frac{T \Delta t}{S \Delta y^2}$.

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Now this is the discretized form of the governing equation.

(Refer Slide Time: 30:02)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} = \frac{h_E^{l+1} - 2h_P^{l+1} + h_W^{l+1}}{\Delta x^2} + \frac{h_N^{l+1} - 2h_P^{l+1} + h_S^{l+1}}{\Delta y^2}$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} + \alpha_x h_W^{l+1} - [1 + 2(\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l$$

with $\alpha_x = \frac{T\Delta t}{S\Delta x^2}$ and $\alpha_y = \frac{T\Delta t}{S\Delta y^2}$.

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Now we need to see how we can discretize our boundary points? So if you consider this as left, left boundary we have northern, eastern, and southern cells present but no western cell.

(Refer Slide Time: 30:27)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Then we need to calculate this value at western face. So western face if we consider the derivative $\partial h / \partial x$, so we should use three point calculation. That means the value which is h_{BW} at face only. Then P, E. So these three points are utilised for calculation of derivative which is at the western face.

(Refer Slide Time: 31:11)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

Otherwise the eastern, northern and southern faces we have same discretization as we have utilised for our interior points.

(Refer Slide Time: 31:29)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

$$\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

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Now if I consider this western boundary then I have this information that at western this is at P, this is at E. So this is the point where I need to calculate the derivative. So i, i plus half because distance between these two is Δx by 2. So i plus half and this distance is again Δx . So Δx plus Δx by 2, this is 3 by 2.

(Refer Slide Time: 32:15)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

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Now if I consider the situation where h_i , I am writing it as h_i only. h_i plus half, this is $h_{i+1/2}$. We need to consider this $\frac{\partial h}{\partial x}$ because another side I am not writing the index for the j . So $\frac{\partial h}{\partial x}$, this is evaluated at i and we need to multiply this Δx by 2 thing.

(Refer Slide Time: 33:06)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$h_i = h_i$
 $h_{i+1/2} = h_i + \frac{\partial h}{\partial x} \Delta x$

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

Plus there will be $\frac{\partial^2 h}{\partial x^2} \Delta x$ by 2, this is again calculated at i th point and this Δx by 2 square plus some terms will be there.

(Refer Slide Time: 33:25)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$h_i = h_i$
 $h_{i+1/2} = h_i + \frac{\partial h}{\partial x} \Big|_i \frac{\Delta x}{2} + \frac{1}{2!} \frac{\partial^2 h}{\partial x^2} \Big|_i \left(\frac{\Delta x}{2}\right)^2 + \dots$

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

Then i plus 3 by 2 , this is a h_i plus $\frac{\partial h}{\partial x}$ by $\frac{\Delta x}{2}$. This is $3 \Delta x$ by 2 plus this is $\frac{\partial^2 h}{\partial x^2}$ evaluated at i point this is $3 \frac{\Delta x}{2}$ squared and there will be 1 by factorial 2 in this case.

(Refer Slide Time: 34:12)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$h_i = h_i$
 $h_{i+1/2} = h_i + \frac{\partial h}{\partial x} \Big|_i \frac{\Delta x}{2} + \frac{1}{2!} \frac{\partial^2 h}{\partial x^2} \Big|_i \left(\frac{\Delta x}{2}\right)^2 + \dots$
 $h_{i+3/2} = h_i + \frac{\partial h}{\partial x} \Big|_i \frac{3\Delta x}{2} + \frac{\partial^2 h}{\partial x^2} \Big|_i \left(\frac{3\Delta x}{2}\right)^2 + \dots$

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

So now what I will do, I will multiply α with the first one, β with the second one and γ with the third one. So if I multiply α , β , γ with the first, second and third equation so the first term that is coefficient of h_i that will be $\alpha + \beta + \gamma$. This should be zero because we do not need this h_i on the right hand side. But we need this $\frac{\partial h}{\partial x}$ by $\frac{\Delta x}{2}$ because $\frac{\partial h}{\partial x}$ by $\frac{\Delta x}{2}$ at i , this component is required.

(Refer Slide Time: 35:04)

So we need beta into del x by 2 plus gamma into 3 del x by 2 and this should be 1 because coefficient of del h by del x this should be 1.

(Refer Slide Time: 35:26)

And third one which is important because we have three unknowns so we need three equations. Now will consider the third term here.

(Refer Slide Time: 35:43)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$\alpha + \beta + \gamma = 0$
 $\beta \times \frac{\Delta x}{2} + \gamma \times \frac{3\Delta x}{2} = 1$

$\alpha h_i = h_i$
 $\beta h_{i+\frac{1}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{\Delta x}{2}$
 $\gamma h_{i+\frac{3}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{3\Delta x}{2}$

$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$ $\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$
 $\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$ $\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

So ideally speaking in this case we will get third equation which is beta into del x square by 4 plus this is gamma into 9 del x square by 4. 1 by factorial 2 that will be there but we can multiply it. This should be zero.

(Refer Slide Time: 36:14)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$\alpha + \beta + \gamma = 0$
 $\beta \times \frac{\Delta x}{2} + \gamma \times \frac{3\Delta x}{2} = 1$
 $\beta \times \frac{\Delta x^2}{4} + \gamma \times \frac{9\Delta x^2}{4} = 0$

$\alpha h_i = h_i$
 $\beta h_{i+\frac{1}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{\Delta x}{2}$
 $\gamma h_{i+\frac{3}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{3\Delta x}{2}$

$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$ $\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$
 $\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$ $\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

Now we have reduced the problem to these three equations. Now in this case we need to solve these three equations. So if we solve these three equations we will get these components because this is h at W this is nothing but we have h BW. Then at P we have h P and at E we have hE. So alpha multiplied with BW, beta multiplied with hP and gamma multiplied with hE and we need to add this.

(Refer Slide Time: 37:15)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions
Left and Right Boundary

$\alpha h_i = h_i$
 $\beta h_{i+\frac{1}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{\Delta x}{2}$
 $\gamma h_{i+\frac{3}{2}} = h_i + \frac{\partial h}{\partial x} \cdot \frac{3\Delta x}{2}$

$\alpha + \beta + \gamma = 0$
 $\beta \times \frac{\Delta x}{2} + \gamma \frac{3\Delta x}{2} = 1$
 $\beta \times \frac{\Delta x^2}{4} + \gamma \frac{9\Delta x^2}{4} = 0$

$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$
 $\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$
 $\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$
 $\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

So we will get the discretization for Δh by Δx at west face for this problem. So for left boundary we have defined everything because we cannot directly utilise the face value on the western boundary directly here. So we will indirectly utilise this derivative or this h value which is available at this level.

(Refer Slide Time: 37:54)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions
Left and Right Boundary

$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x}$
 $\left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$
 $\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y}$
 $\left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 10 / 28

Now if we see the discretization we need to change only west face. This one needs spatial treatment. So I have changed this value here.

(Refer Slide Time: 38:25)

Now h_{BW}^{l+1} plus 1, this is specified value. Obviously this should be transferred to the right hand side. So in this case we are getting again h_S . In this case we are also getting the components which are directly coming from α_x and α_y . Because α_x in our case this is defined as Δt divided by $S \Delta x^2$ and α_y this is defined as Δt $S \Delta y^2$.

(Refer Slide Time: 39:28)

Now if we simplify this we do not have west face. So that is why no w component here. W component is zero. Now we need to see what is this h_P ? One h_P is coming here, so this 1 is for this h_P .

(Refer Slide Time: 39:53)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{4 \Delta x} - \frac{8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3 \Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + 2(2\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 11 / 28 7:45 PM 8/20/2017

Another one which will come here that is $2\alpha_y$ that will come from this second del y square term. Then we will have this α_x into $4y$. That will be coming from here because this is 3 and that will be 4 if we add. This is 3 and this is 4.

(Refer Slide Time: 40:45)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{4 \Delta x} - \frac{8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3 \Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + 2(2\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 11 / 28 7:45 PM 8/20/2017

So this is okay. And 4 by $3\alpha_x h_E$ because we have extra emphasis on h_E component so this is there. And α_y into h_N again it is coming from here.

(Refer Slide Time: 41:04)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + 2(2\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

$\alpha_x = \frac{T \Delta t}{S \Delta x^2}$
 $\alpha_y = \frac{T \Delta t}{S \Delta y^2}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 11 / 28 7:46 PM 8/30/2017

And we need to transfer this term to the right hand side because it is a known component. So 8 by 3 alpha x h BW, I have transferred it on the right hand side.

(Refer Slide Time: 41:23)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + 2(2\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

$\alpha_x = \frac{T \Delta t}{S \Delta x^2}$
 $\alpha_y = \frac{T \Delta t}{S \Delta y^2}$

Dr. Anirban Dhar NPTEL Computational Hydraulics 11 / 28 7:46 PM 8/30/2017

Now on the left hand side we have all unknown terms except hW L plus 1. We do not have any component hW L plus 1 for left boundary.

(Refer Slide Time: 41:45)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + 2(2\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 11 / 28

Now if we have right boundary similar to this we can again define the problem. We can define it as i, this one as i minus half and this one at i minus 3 by 2. This is corresponding to E, this P, this is W. and the corresponding values this is BE, this is hP, this is hW. So with this information we can discretize the first order derivative of h with respect to x at the face E using these three points, eastern face value, self centred point P and self centered point W.

(Refer Slide Time: 42:58)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Left and Right Boundary

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 12 / 28

And other derivatives are similar to our interior points. Now in this case I have changed only the eastern face value because we are considering right boundary. In right boundary we need to change this eastern face thing.

(Refer Slide Time: 43:26)

Now if we change this eastern face derivative with spatial terms that we have utilised for derivation. Now in this case we are getting h_S, h_W, spatial emphasis on W, h_P and alpha_y. But there is no eastern term E L plus 1 in this case. Obviously we need to transfer this first term which is specified term and we need to transfer it to the right hand side. And this is our right hand side which is again specified boundary condition.

(Refer Slide Time: 44:17)

Now if we see the similar thing for top boundary, so top boundary there is no northern cell. But in this case the situation is different because this boundary is impermeable boundary. Obviously del h by del y is zero.

(Refer Slide Time: 44:45)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Top and Bottom Boundary

$$n$$

$W \bullet w$	$\bullet P$	$e \bullet E$
\bullet	$\bullet S$	\bullet

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_E^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 14 / 28

In case of our left hand and right hand boundary we have not considered this term as zero but in this case we have a zero Neumann condition. And other derivatives are same but again h_{BN} value we can write in terms of h_P and h_S . So we need to utilise this zero value in our formulation. But this expression will be required for calculation of boundary value after calculation of all internal self centred values from our solution process because we need complete information at boundary also.

So to get this green cross value or boundary value which is h_{BN} , we cannot get this h_{BN} or this value is not specified.

(Refer Slide Time: 45:56)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

Top and Bottom Boundary

$$n$$

$W \bullet w$	$\bullet P$	$e \bullet E$
\bullet	$\bullet S$	\bullet

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_E^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 14 / 28

So after getting h_P and h_S we can calculate h_{BN} based on this secondary condition which is zero. So from this condition we can get the value but that is after the solution process.

(Refer Slide Time: 46:15)

The slide shows a grid of nodes for a 2D domain. The nodes are labeled as follows:

n		
$W \bullet w$	$\bullet P$	$e \bullet E$
\bullet	s \bullet S	\bullet

Below the grid, the following equations are shown:

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{Sh_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Now approach is same. In this case we have used northern boundary as zero. We have directly specified as zero. Now corresponding equation we can get there is no h_{NL}^{l+1} term. In this case h_W , h_S , h_P is there but we do not need to transfer any spatial term on the right hand side.

(Refer Slide Time: 46:47)

The slide is titled "Implicit Scheme" and shows the following equations:

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x}\right)_e^{l+1} - \left(\frac{\partial h}{\partial x}\right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y}\right)_n^{l+1} - \left(\frac{\partial h}{\partial y}\right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right] \Delta y + \left[0 - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} + \alpha_x h_W^{l+1} - [1 + (2\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_x h_E^{l+1} = -h_P^l$$

Now this is (bo) bottom boundary condition. Bottom boundary condition again we can write this zero condition and for other faces we have same condition like (con) interior conditions.

(Refer Slide Time: 47:09)

The slide shows a 2x2 grid of nodes. The top node is labeled 'N' and 'n'. The bottom node is labeled 'P'. The left node is labeled 'W' and 'w'. The right node is labeled 'E' and 'E'. The bottom boundary is labeled 's'. Below the grid, the following equations are shown:

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y}$$

The last equation is circled in green. A small circular inset image of a man is visible on the right side of the slide.

Now in this case we have put this as zero because this is zero. We do not have S term. Western, P, eastern, northern and the right hand side again specified term is there.

(Refer Slide Time: 47:31)

The slide shows the following equation:

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x}\right)_e^{l+1} - \left(\frac{\partial h}{\partial x}\right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y}\right)_n^{l+1} - \left(\frac{\partial h}{\partial y}\right)_s^{l+1} \right] \Delta x$$

Below this, another equation is shown:

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - 0 \right] \Delta x$$

The '0' in the second equation is circled in green. Below the equations, it says "In simplified form, this can be written as" followed by:

$$\alpha_x h_W^{l+1} - [1 + (2\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l$$

The terms $\alpha_x h_W^{l+1}$, h_P^{l+1} , $\alpha_x h_E^{l+1}$, and $\alpha_y h_N^{l+1}$ are underlined in green.

Now spatial treatment for northwest corner. So northwest corner, now we need to combine the impermeable boundary condition and specified boundary condition because northern side you have impermeable boundary condition, on the western side we have specified boundary

condition. So western side we can directly utilise this derivative that we have derived for our left boundary or western boundary. And on the right hand side we can directly use this zero value.

(Refer Slide Time: 48:15)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

N-W Corner

	n		
w	P	e	E
	s		
	S		

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 18 / 28

So if you directly use these values, this is for western boundary, this is for northern boundary, so we can get the corresponding equation. But in this case there will be no $h_N L$ plus 1 term and h_S or $h_W L$ plus 1 term, $h_N L$ plus 1 term. So there will be no such term in this case.

(Refer Slide Time: 48:53)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x}\right)_e^{l+1} - \left(\frac{\partial h}{\partial x}\right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y}\right)_n^{l+1} - \left(\frac{\partial h}{\partial y}\right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x} \right] \Delta y + \left[0 - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} - [1 + (4\alpha_x + \alpha_y)] h_P^{l+1} + \frac{4}{3} \alpha_x h_E^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BW}^{l+1}$$

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Now if we discretize our northeast corner, northeast corner or top side is impermeable boundary, eastern side we have specified boundary. So this is for specified boundary, this is for again northern boundary which is zero boundary.

(Refer Slide Time: 49:16)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

N-E Corner

•	W • w	P •	n
•	•	s	e
•	•	S	

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} = 0 \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 20 / 28

Now we can use the same concept here. This is for eastern boundary, this is for northern boundary and we can get the corresponding equation here.

(Refer Slide Time: 49:33)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x}\right)_e^{l+1} - \left(\frac{\partial h}{\partial x}\right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y}\right)_n^{l+1} - \left(\frac{\partial h}{\partial y}\right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} - \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right] \Delta y + \left[\frac{8h_{BN}^{l+1} - 9h_P^{l+1} + h_S^{l+1}}{3\Delta y} - \frac{h_P^{l+1} - h_S^{l+1}}{\Delta y} \right] \Delta x$$

In simplified form, this can be written as

$$\alpha_y h_S^{l+1} + \frac{4}{3} \alpha_x h_W^{l+1} - [1 + (4\alpha_x + \alpha_y)] h_P^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BE}^{l+1}$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 21 / 28

And in this case we can get the equation. So another one is southeast corner, southeast corner this is impermeable, this is specified.

(Refer Slide Time: 49:51)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

S-E Corner

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0$$

Dr. Anirban Dhar NPTEL Computational Hydraulics 22 / 28
7:53 PM 8/26/2017

So obviously we can get the boundaries which is, this is specified, this is zero boundary.

(Refer Slide Time: 50:03)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

S-E Corner

$$\left(\frac{\partial h}{\partial x}\right)_e^{l+1} = \frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} \quad \left(\frac{\partial h}{\partial x}\right)_w^{l+1} = \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial y}\right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y}\right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0$$

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7:53 PM 8/26/2017

So corresponding to that we can get the governing equation. This is specified boundary, eastern face and southern boundary we have zero boundary condition. So this is again our condition.

(Refer Slide Time: 50:19)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{8h_{BE}^{l+1} - 9h_P^{l+1} + h_W^{l+1}}{3\Delta x} - \frac{h_P^{l+1} - h_W^{l+1}}{\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - 0 \right] \Delta x$$

In simplified form, this can be written as

$$\frac{4}{3} \alpha_x h_W^{l+1} - [1 + (4\alpha_x + \alpha_y)] h_P^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3} \alpha_x h_{BE}^{l+1}$$

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Now on southwest corner, southwest corner this is zero. Southern boundary impermeable, this is specified. So specified and zero boundary conditions. We can directly use this concept.

(Refer Slide Time: 50:37)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Boundary Conditions

S-W Corner

$$\left(\frac{\partial h}{\partial x} \right)_e^{l+1} = \frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} \quad \left(\frac{\partial h}{\partial x} \right)_w^{l+1} = \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x}$$

$$\left(\frac{\partial h}{\partial y} \right)_n^{l+1} = \frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} \quad \left(\frac{\partial h}{\partial y} \right)_s^{l+1} = \frac{-8h_{BS}^{l+1} + 9h_P^{l+1} - h_N^{l+1}}{3\Delta y} = 0$$

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So this is for western and this is for southern. So we will get corresponding equation here.

(Refer Slide Time: 50:50)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Implicit Scheme

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\left(\frac{\partial h}{\partial x} \right)_e^{l+1} - \left(\frac{\partial h}{\partial x} \right)_w^{l+1} \right] \Delta y + \left[\left(\frac{\partial h}{\partial y} \right)_n^{l+1} - \left(\frac{\partial h}{\partial y} \right)_s^{l+1} \right] \Delta x$$

$$\frac{S}{T} \frac{h_P^{l+1} - h_P^l}{\Delta t} \Delta x \Delta y = \left[\frac{h_E^{l+1} - h_P^{l+1}}{\Delta x} - \frac{-8h_{BW}^{l+1} + 9h_P^{l+1} - h_E^{l+1}}{3\Delta x} \right] \Delta y + \left[\frac{h_N^{l+1} - h_P^{l+1}}{\Delta y} - 0 \right] \Delta x$$

In simplified form, this can be written as

$$-[1 + (4\alpha_x + \alpha_y)]h_P^{l+1} + \frac{4}{3}\alpha_x h_E^{l+1} + \alpha_y h_N^{l+1} = -h_P^l - \frac{8}{3}\alpha_x h_{BW}^{l+1}$$

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Now in general format we can write this equation including the boundary condition. Obviously a particular term, let us say W is not present. Then we will write aW is equal to zero and coefficient is zero in this case. So we have this thing. On the left hand side we have this penta diagonal structure. On the right hand side we have known quantity.

(Refer Slide Time: 51:25)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$a_S h_S^{l+1} + a_W h_W^{l+1} + a_P h_P^{l+1} + a_E h_E^{l+1} + a_N h_N^{l+1} = r_P$$

$a_W = 0$

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So we can solve this using our Gauss Seidel iterative technique. In Gauss Seidel iterative technique we can calculate this hP L plus 1 at P which is equals to hP L plus 1, this is at P minus 1 plus omega into residual which is right hand side minus left hand side divided by ap.

Now based on this calculation we can iterate and we can get the solution for a particular time level.

(Refer Slide Time: 52:23)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

General Form

In general form, the governing equation including boundary conditions can be written as,

$$a_S h_S^{l+1} + a_W h_W^{l+1} + a_P h_P^{l+1} + a_E h_E^{l+1} + a_N h_N^{l+1} = r_P$$
$$h_p^{l+1} = h_p^{(l)} + \omega \frac{r_p}{a_p}$$

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So we will have time level and within space level, within at particular time we need to iterate using Gauss Seidel iterative technique and we can formulate it using unsteady or we can solve it using this Gauss Seidel approach. Unsteady 2D finite volume confined implicit iterative dot SCI. So this is for confined aquifer using finite volume, 2D in space, unsteady, implicit and iterative technique.

(Refer Slide Time: 53:03)

Problem Definition
Domain Discretization
Discretization: Boundary Points

I.I.T. Kharagpur

Source Code

Unsteady Two Dimensional Groundwater Flow with Finite Volume Method

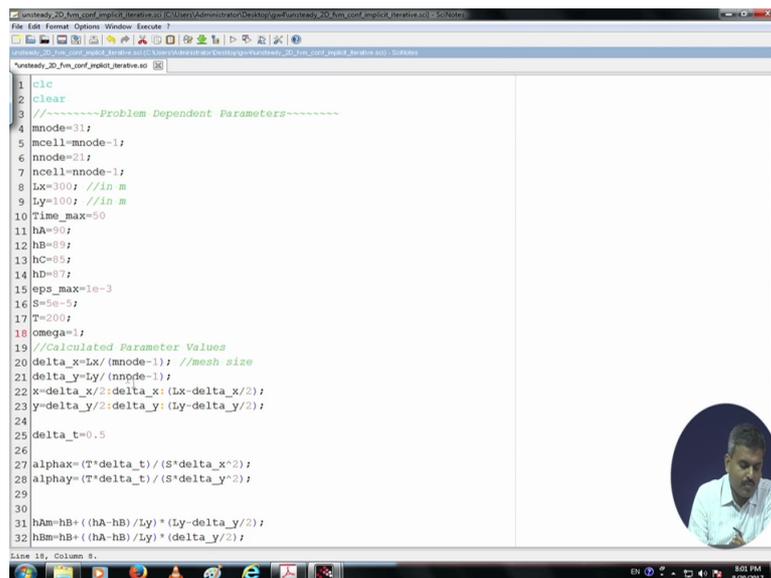
- Without coefficient matrix using Gauss Seidel
- [unsteady_2D_fvm_conf_implicit_iterative.sci](#)

Dr. Anirban Dhar NPTEL Computational Hydraulics 27 / 28 7:58 PM 6/26/2017

Now let us see how can we solve it using our scilab because we are coding everything in scilab. So let us open it. Okay, open file. So we can start our thing with this one. So in this case we have this code. So first line starts with clc clear. So clc is clear console, then clear in the variables. M node is nodes in x direction. So we can get number of cells. So obviously number of cells will be one number less than the number of nodes, so m node minus 1.

N node 21. Ncell, cell on y direction that is n node minus 1. Lx is 300, Ly 300, time maximum is 5. We can write it as 50 also because we are solving steady state problem. So that is not important. HA this 90, hB 89, hC 85, hD 87, epsilon max 1 into 10 to the power minus 3, this is required for Gauss Seidel. S is 5 into 10 to the power minus 5, T is 200, omega this is for Gauss Seidel, we can use as 1. So delta x, obviously this should be mesh size or cell size, Lx divided by m node minus 1. And delta y, this is Ly divided by n node minus 1.

(Refer Slide Time: 56:18)



```

1 clc
2 clear
3 //-----Problem Dependent Parameters-----
4 mnode=31;
5 ncell=mnode-1;
6 nnode=21;
7 ncell=nnode-1;
8 Lx=300; //in m
9 Ly=100; //in m
10 Time_max=50
11 hA=90;
12 hB=89;
13 hC=85;
14 hD=87;
15 eps_max=1e-3
16 S=5e-5;
17 T=200;
18 omega=1;
19 //Calculated Parameter Values
20 delta_x=Lx/(mnode-1); //mesh size
21 delta_y=Ly/(nnode-1);
22 x=delta_x/2:delta_x:(Lx-delta_x/2);
23 y=delta_y/2:delta_y:(Ly-delta_y/2);
24
25 delta_t=0.5
26
27 alpha=(T*delta_t)/(S*delta_x^2);
28 alphas=(T*delta_t)/(S*delta_y^2);
29
30
31 hAm=hB+((hA-hB)/Ly)*(Ly-delta_y/2);
32 hBm=hB+((hA-hB)/Ly)*(delta_y/2);

```

Now we can run this x which is our total covered our covered coordinate. This is running from del x by 2 because self centered on the x direction that is starting from del x by 2. And on the other side it is starting from del y by 2. So this is del y by 2 and with del y or delta y as distance between two points we can calculate everything. So that will be running from delta x by 2 to Lx minus delta x by 2 and Ly minus delta y by 2. And delta t is point 5.

(Refer Slide Time: 57:41)

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Unsteady_2D_fm_conf_implicit_iterative.scilab
File Edit Format Options Window Execute ?
Unsteady_2D_fm_conf_implicit_iterative.scilab
Unsteady_2D_fm_conf_implicit_iterative.scilab
Unsteady_2D_fm_conf_implicit_iterative.scilab
11 hA=91;
12 hB=89;
13 hC=85;
14 hD=87;
15 eps_max=1e-3
16 S=5e-5;
17 T=200;
18 omega=1;
19 //Calculated Parameter Values
20 delta_x=Lx/(mnode-1); //mesh size
21 delta_y=Ly/(mnode-1);
22 x=delta_x/2:delta_x:(Lx-delta_x/2);
23 y=delta_y/2:delta_y:(Ly-delta_y/2);
24
25 delta_t=0.5
26 I
27 alpha_x=(T*delta_t)/(S*delta_x^2);
28 alpha_y=(T*delta_t)/(S*delta_y^2);
29
30
31 hAm=hB*((hA-hB)/Ly)*(Ly-delta_y/2);
32 hBm=hB*((hA-hB)/Ly)*(delta_y/2);
33 hCm=hC*((hD-hC)/Ly)*(delta_y/2);
34 hDm=hC*((hD-hC)/Ly)*(Ly-delta_y/2);
35
36 // Initialization
37 // Initialization
38 ho=hA*ones(mcell,ncell);
39 hn=hA*ones(mcell,ncell);
40
41 //Time Loop
42 t=0;
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unsteady_2D_fm_conf_implicit_iterative.scf
unsteady_2D_fm_conf_implicit_iterative.scf
39 hn=HA*ones(mcell,ncell);
40
41 //Time Loop
42 t=0;
43 while t < Time_max
44     t=t+delta_t;
45
46 count = 0;
47 rmse=1;
48 //space Loop
49 while rmse > eps_max
50     rmse=0;
51     for j=1:ncell
52         for i=1:mcell
53             if (i > 1 & i < mcell) then
54                 if (j > 1 & j < ncell) then
55                     a_S=alpha;
56                     a_W=alpha;
57                     a_P=-1-(2*(alpha+alpay));
58                     a_E=alpha;
59                     a_N=alpha;
60                     r_P=ho(i,j);
61                     res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
62                 end
63             end
64
65             // Node A
66             if (i==1 & j==ncell) then
67                 a_S=alpha;
68                 a_W=0;
69                 a_P=-1-(4.0*alpha+alpha);
70                 a_E=(4.0/3.0)*alpha;
71
72                 r_P=ho(i,j)-(8.0/3.0)*alpha*hAm;
73                 res=r_P-(a_S*hn(i,j-1)+a_P*hn(i,j)+a_E*hn(i+1,j));
74             end
75
76             // Node B
77             if (i==1 & j==1) then
78                 a_S=0;
79                 a_W=0;
80                 a_P=-1-(4.0*alpha+alpha);
81                 a_E=(4.0/3.0)*alpha;
82                 a_N=alpha;
83                 r_P=ho(i,j)-(8.0/3.0)*alpha*hBm;
84                 res=r_P-(a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
85             end
86
87             // Node C
88             if (i==mcell & j==1) then
```

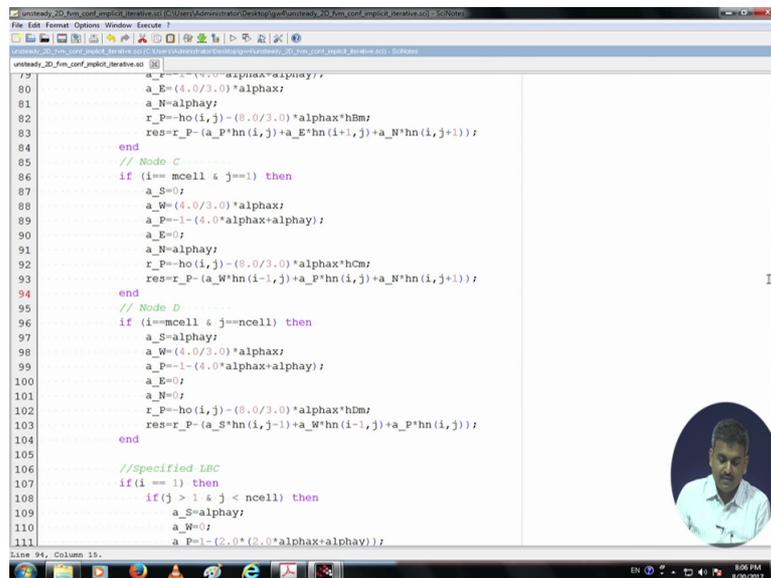
So we can calculate the residual. Now based on this and nodes A or cell A, this is not node directly, this is cell A. We need to consider which is on the northwest corner. Again there is no aW term and aN term.

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```

aN and aS is zero, aE is zero for C at cell which is corner cell which is near to the C point.

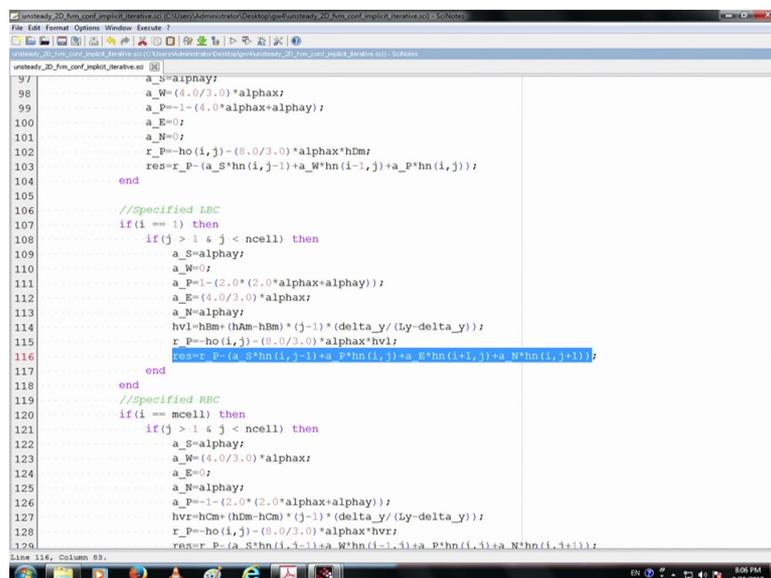
(Refer Slide Time: 01:00:55)



```
79      a_r=-1+(4.0*alpha)*alphay;
80      a_E=(4.0/3.0)*alphax;
81      a_N=alpha;
82      r_P=ho(i,j)-(8.0/3.0)*alpha*hBm;
83      res=r_P-(a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
84      end
85      // Node C
86      if (i==mcell & j==1) then
87      a_S=0;
88      a_W=(4.0/3.0)*alphax;
89      a_P=-1-(4.0*alpha+alphay);
90      a_E=0;
91      a_N=alpha;
92      r_P=ho(i,j)-(8.0/3.0)*alpha*hCm;
93      res=r_P-(a_W*hn(i-1,j)+a_P*hn(i,j)+a_N*hn(i,j+1));
94      end
95      // Node D
96      if (i==mcell & j==ncell) then
97      a_S=alphay;
98      a_W=(4.0/3.0)*alphax;
99      a_P=-1-(4.0*alpha+alphay);
100     a_E=0;
101     a_N=0;
102     r_P=ho(i,j)-(8.0/3.0)*alpha*hDm;
103     res=r_P-(a_W*hn(i,j-1)+a_P*hn(i,j)+a_N*hn(i,j+1));
104     end
105
106     //Specified LBC
107     if(i == 1) then
108     if(j > 1 & j < ncell) then
109     a_S=alphay;
110     a_W=0;
111     a_P=-1-(2.0*(2.0*alpha+alphay));
```

And corner cell which is near to D point, this is aE, aN. Now on the left hand boundary we have the specified hvl which is left hand value on the left hand boundary which is intermediate value between A and B and rP. And this is the residual value.

(Refer Slide Time: 01:01:31)



```
97      a_S=alphay;
98      a_W=(4.0/3.0)*alphax;
99      a_P=-1-(4.0*alpha+alphay);
100     a_E=0;
101     a_N=0;
102     r_P=ho(i,j)-(8.0/3.0)*alpha*hDm;
103     res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j));
104     end
105
106     //Specified LBC
107     if(i == 1) then
108     if(j > 1 & j < ncell) then
109     a_S=alphay;
110     a_W=0;
111     a_P=-1-(2.0*(2.0*alpha+alphay));
112     a_E=(4.0/3.0)*alphax;
113     a_N=alpha;
114     hvl=hBm*(hBm-hCm)*(j-1)*(delta_y/(Ly-delta_y));
115     r_P=ho(i,j)-(8.0/3.0)*alpha*hvl;
116     res=r_P-(a_S*hn(i,j-1)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
117     end
118
119     //Specified RBC
120     if(i == mcell) then
121     if(j > 1 & j < ncell) then
122     a_S=alphay;
123     a_W=(4.0/3.0)*alphax;
124     a_E=0;
125     a_P=alpha;
126     a_R=-1-(2.0*(2.0*alpha+alphay));
127     hvr=hCm*(hDm-hCm)*(j-1)*(delta_y/(Ly-delta_y));
128     r_P=ho(i,j)-(8.0/3.0)*alpha*hvr;
129     res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_R*hn(i,j+1));
```

And the right hand side we have right hand boundary condition. Again we do not have E in this case.

(Refer Slide Time: 01:01:40)

```
110 a_W=0;
111 a_P=1-(2.0*(2.0*alpha+alpha));
112 a_E=(4.0/3.0)*alpha;
113 a_N=alpha;
114 hvl=hCm+(hM-hCm)*(j-1)*(delta_y/(Ly-delta_y));
115 r_P=ho(i,j)-(8.0/3.0)*alpha*hvl;
116 res=r_P-(a_S*hn(i,j-1)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
117 end
118 end
119 //Specified BBC
120 if(i == mcell) then
121     if(j > 1 & j < ncell) then
122         a_S=alpha;
123         a_W=(4.0/3.0)*alpha;
124         I=S0;
125         a_N=alpha;
126         a_P=1-(2.0*(2.0*alpha+alpha));
127         hvr=hCm+(hM-hCm)*(j-1)*(delta_y/(Ly-delta_y));
128         r_P=ho(i,j)-(8.0/3.0)*alpha*hvr;
129         res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_N*hn(i,j+1));
130     end
131 end
132 //Neuman BBC
133 if(j==1) then
134     if(i > 1 & i < mcell) then
135         a_S=0;
136         a_W=alpha;
137         a_P=1-(2.0*alpha+alpha);
138         a_E=alpha;
139         a_N=alpha;
140         r_P=ho(i,j);
141         res=r_P-(a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
142     end
143 end
```

Then Neumann bottom boundary we do not have S.

(Refer Slide Time: 01:01:45)

```
123 a_W=(4.0/3.0)*alpha;
124 a_E=0;
125 a_N=alpha;
126 a_P=1-(2.0*(2.0*alpha+alpha));
127 hvr=hCm+(hM-hCm)*(j-1)*(delta_y/(Ly-delta_y));
128 r_P=ho(i,j)-(8.0/3.0)*alpha*hvr;
129 res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_N*hn(i,j+1));
130 end
131 end
132 //Neuman BBC
133 if(j==1) then
134     if(i > 1 & i < mcell) then
135         a_S=0;
136         a_W=alpha;
137         a_P=1-(2.0*alpha+alpha);
138         a_E=alpha;
139         a_N=alpha;
140         r_P=ho(i,j);
141         res=r_P-(a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
142     end
143 end
144 //Neuman TBC
145 if(j==ncell) then
146     if(i > 1 & i < mcell) then
147         a_S=alpha;
148         a_W=alpha;
149         a_P=1-(2.0*alpha+alpha);
150         a_E=alpha;
151         a_N=0;
152         r_P=ho(i,j);
153         res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j));
154     end
155 end
```

Then we have this boundary which is top boundary condition. This is aS but there is no northern boundary on top boundary.

(Refer Slide Time: 01:02:04)

```
unsteady_2D_fm_conf_implicit_iterative.scd
File Edit Format Options Window Execute ?
unsteady_2D_fm_conf_implicit_iterative.scd
unsteady_2D_fm_conf_implicit_iterative.scd
135 a_S=0;
136 a_W=alpha;
137 a_P=-1-(2.0*alpha+alpha);
138 a_E=alpha;
139 a_N=alpha;
140 r_P=ho(i,j);
141 res=r_P-(a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j)+a_N*hn(i,j+1));
142 end
143 end
144 //Neuman BC
145 if(j==ncell) then
146 if(i > 1 & i < mcell) then
147 a_S=alpha;
148 a_N=alpha;
149 a_P=-1-(2.0*alpha+alpha);
150 a_E=alpha;
151 a_N=0;
152 r_P=ho(i,j);
153 res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j));
154 end
155 end
156 //Update
157 hn(i,j)=hn(i,j)+omega*res/a_P;
158 //RMSE
159 rmse=rmse+(omega*res/a_P).^2;
160 end
161 end
162 rmse=sqrt(rmse/(mcell*ncell));
163 count = count + 1;
164 disp((count rmse))
165 end
166 rmse=0;
```

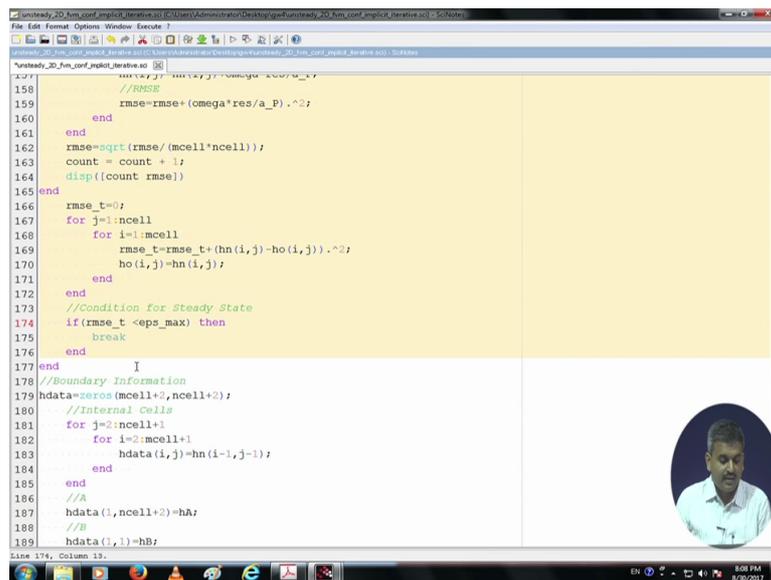
And finally we can update this hn equals to hn. We are storing the same value in the same matrix because we can get the updated value. Residual by aP and rmse equals to omega, we can calculate rmse. And outside we can calculate rmse divided by mcell into ncell.

(Refer Slide Time: 01:02:27)

```
unsteady_2D_fm_conf_implicit_iterative.scd
File Edit Format Options Window Execute ?
unsteady_2D_fm_conf_implicit_iterative.scd
unsteady_2D_fm_conf_implicit_iterative.scd
145 if(j==ncell) then
146 if(i > 1 & i < mcell) then
147 a_S=alpha;
148 a_N=alpha;
149 a_P=-1-(2.0*alpha+alpha);
150 a_E=alpha;
151 a_N=0;
152 r_P=ho(i,j);
153 res=r_P-(a_S*hn(i,j-1)+a_W*hn(i-1,j)+a_P*hn(i,j)+a_E*hn(i+1,j));
154 end
155 end
156 //Update
157 hn(i,j)=hn(i,j)+omega*res/a_P;
158 //RMSE
159 rmse=rmse+(omega*res/a_P).^2;
160 end
161 end
162 rmse=sqrt(rmse/(mcell*ncell)); I
163 count = count + 1;
164 disp((count rmse))
165 end
166 rmse=0;
167 for j=1:ncell
168 for i=1:mcell
169 rmse=rmse+(hn(i,j)-ho(i,j)).^2;
170 ho(i,j)=hn(i,j);
171 end
172 end
173 //Condition for Steady State
174 if(rmse < eps_max) then
175 break
176 end
```

So these are the number of cells required. Now this is outside rmse which is required for or time calculation because within a time if things are not advancing then we can skip and we can calculate the thing. That means the steady state value.

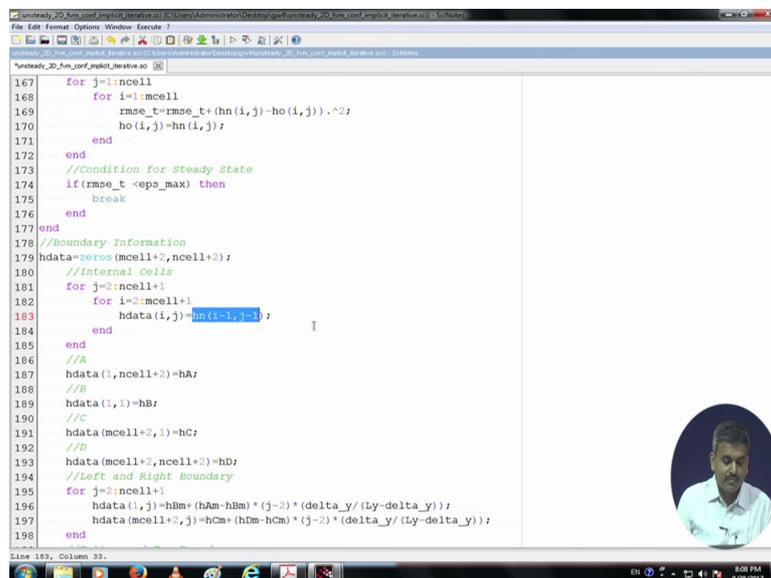
(Refer Slide Time: 01:03:05)



```
158     rmse_t=0;
159     rmse=rmse+(omega*res/a_P).^2;
160 end
161 end
162 rmse=sqrt(rmse/(mcell*ncell));
163 count = count + 1;
164 disp((count rmse))
165 end
166 rmse_t=0;
167 for j=1:ncell
168     for i=1:mcell
169         rmse_t=rmse_t+(hn(i,j)-ho(i,j)).^2;
170         ho(i,j)=hn(i,j);
171     end
172 end
173 //Condition for Steady State
174 if (rmse_t < eps_max) then
175     break
176 end
177 end
178 //Boundary Information
179 hdata=zeros(mcell+2,ncell+2);
180 //Internal Cells
181 for j=2:ncell+1
182     for i=2:mcell+1
183         hdata(i,j)=hn(i-1,j-1);
184     end
185 end
186 //A
187 hdata(1,ncell+2)=hA;
188 //B
189 hdata(1,1)=hB;
```

And for boundary condition we need to consider two more cells because boundaries are not included. So mcell plus 2 and ncell plus 2. That means on both side we need to consider points so that we can include the boundary. So for internal cells we can directly transfer the internal values.

(Refer Slide Time: 01:03:30)



```
167     for j=1:ncell
168         for i=1:mcell
169             rmse_t=rmse_t+(hn(i,j)-ho(i,j)).^2;
170             ho(i,j)=hn(i,j);
171         end
172     end
173 //Condition for Steady State
174 if (rmse_t < eps_max) then
175     break
176 end
177 end
178 //Boundary Information
179 hdata=zeros(mcell+2,ncell+2);
180 //Internal Cells
181 for j=2:ncell+1
182     for i=2:mcell+1
183         hdata(i,j)=hn(i-1,j-1);
184     end
185 end
186 //A
187 hdata(1,ncell+2)=hA;
188 //B
189 hdata(1,1)=hB;
190 //C
191 hdata(mcell+2,1)=hC;
192 //D
193 hdata(mcell+2,ncell+2)=hD;
194 //Left and Right Boundary
195 for j=2:ncell+1
196     hdata(1,j)=hBm+(hAm-hBm)*(j-2)*(delta_y/(Ly-delta_y));
197     hdata(mcell+2,j)=hCm+(hDm-hCm)*(j-2)*(delta_y/(Ly-delta_y));
198 end
```

For external or for h data at corner we can transfer the values and left boundary and right boundary we can transfer by interpolating the specified values. And the bottom boundary we can calculate from our derivatives available.

