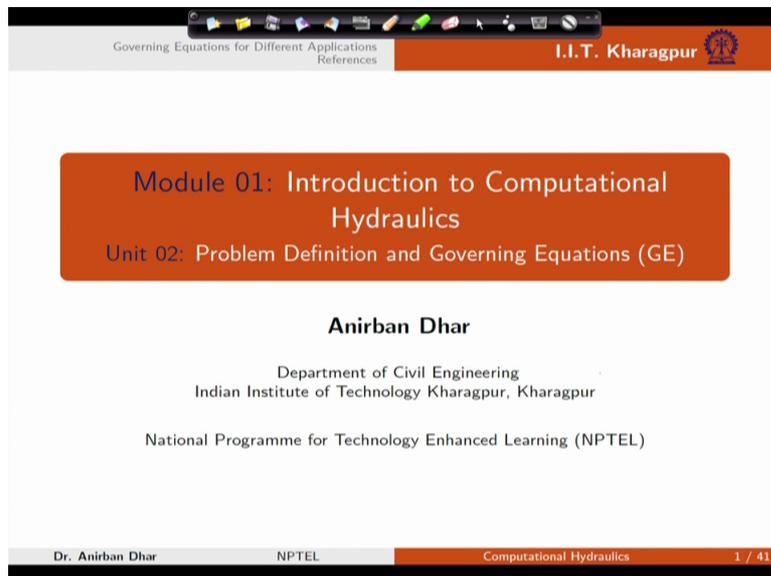


**Computational Hydraulics**  
**Professor Anirban Dhar**  
**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 2**  
**Problem Definition and Governing Equation (GE)**

Welcome to this lecture number 2 of this module 1, introduction to computational hydraulics and this unit is about problem definition and governing equations.

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Governing Equations for Different Applications  
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**Module 01: Introduction to Computational Hydraulics**  
**Unit 02: Problem Definition and Governing Equations (GE)**

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So what are the learning objective? To identify the governing equation for hydraulic system. So in this particular lecture we will try to identify the governing equation related to hydraulic systems.

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## Learning Objectives

- To identify the **Governing Equations** for hydraulic systems

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Introduction, governing equation define the relationship between variables in terms of ordinary differential equation or partial differential equations. And it relates the independent and dependent variables. ODEs or PDEs represent conservation laws that is mass, momentum and energy in general or simplified form. Depending on the nature of the problem we can use a three dimensional form or a simplified one dimensional equation for the problem.

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## Introduction

- Governing equation defines the relationship between the variables in terms of *ordinary differential equations (ODE)* or *partial differential equations (PDE)*.
- ODEs/PDEs represent conservation laws (i.e., mass, momentum and energy) in general or simplified form.



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So what is this ODE? ODE is differential equation with one independent variable. And what is this PDE is? PDE is differential equation with two or more independent variables.

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## Introduction

- Governing equation defines the relationship between the variables in terms of *ordinary differential equations (ODE)* or *partial differential equations (PDE)*.
- ODEs/PDEs represent conservation laws (i.e., mass, momentum and energy) in general or simplified form.

**ODE**  
Differential Equation with **ONE** independent variable.

**PDE**  
Differential Equation with **two** or **more** independent variables.

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Now (wha) what are the basic principles? Basic principles are conservation of mass, conservation of momentum and conservation of energy.

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## Basic Principles

- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy

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If we talk about this conservation equation in general, so for incompressible fluid flow we can write down the differential equation or partial differential equation like this. In this case delu by delx represents the change in x direction and delu by dely change in the y and delu by delz change in the z direction. U, V, and W these are 3 components of the velocity vector in 3 directions.

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**Mass conservation equation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

**Momentum conservation equation**

x-dir:  $\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$

y-dir:  $\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3)$

z-dir:  $\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial ww}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \frac{\mu}{\rho} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$

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And what is this momentum conservation equation? In x direction we can write the momentum conservation equation. This is your temporal term, this is your additive term and this is your pressure term, and this is the body force gravity term. And this is our diffuse center where  $\mu$  is the dynamic viscosity  $\rho$  is density of fluid,  $p$  is pressure and we already know that  $u, v, w$  are the components of velocity.

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**Mass conservation equation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

**Momentum conservation equation**

x-dir:  $\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$

y-dir:  $\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3)$

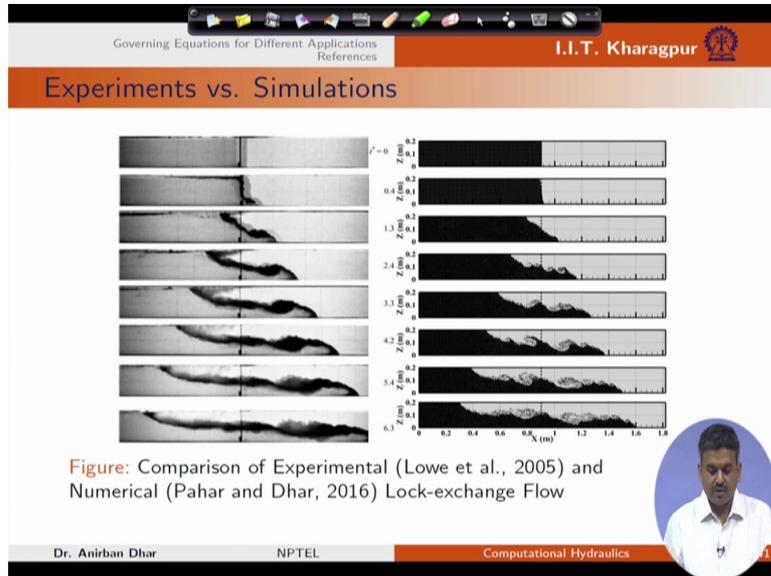
z-dir:  $\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial ww}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \frac{\mu}{\rho} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$

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Now in our previous lecture we have seen that if you perform this experiment of 2 density fluids that is lock exchange flow. So from experiments we can simulate or we can get information about the system and from numerical case we can simulate the corresponding

variation with the help of computers. So to solve this problem we need to solve our conservation of mass and conservation of momentum equations.

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So if we talk about groundwater movement in aquifers then we need to define the conservation of mass that is depth integrated mass conservation equation for groundwater flow.

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Groundwater Movement in Aquifers  
Governing Equations

Depth-integrated mass conservation equation for groundwater flow can be written as,

$$S \frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = f \quad (5)$$

where

- $S$  = storativity (specific yield for unconfined aquifers and storage coefficient for confined aquifers)
- $h = h(x, y, t)$  = groundwater head [L]
- $\mathbf{q} = q_x \hat{i} + q_y \hat{j}$  = flux [ $L^2/T$ ]
- $f$  = source/sink term [ $L/T$ ]
- $\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j}$  = del operator [ $1/L$ ]
- $t$  = time [ $T$ ].

Where  $s$  is our storativity term for unconfined aquifers, it is specific yield. For confined aquifers this is storage coefficient. Interestingly this  $Q$  is Darcy discharge and  $f$  is source or

sink term. So change in head or groundwater head can be represented using this mass conservation equation.

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### Groundwater Movement in Aquifers

Governing Equations

Depth-integrated **mass conservation** equation for groundwater flow can be written as,

$$S \frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = f \quad (5)$$

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- $\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j}$  = del operator [ $1/L$ ]
- $t$  = time [ $T$ ].

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Now if we talk about momentum equation in case of aquifer or aquifer flow thing, then Darcy's law is a momentum conservation equation where  $q$  is the Darcy velocity or Darcy discharge vector and  $T$  is transmissivity,  $K$  is hydraulic conductivity and as we can see that this  $T$  is calculated as minimum value of  $h$  and  $z_u$ ,  $z_u$  is the top aquifer elevation minus the bottom aquifer elevation. That means the saturated thickness portion.

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### Groundwater Movement in Aquifers

Governing Equations

The Darcy's law [**momentum conservation equation**] can be written as,

$$\mathbf{q} = -T \nabla h \quad (6)$$

where

- $T$  = aquifer transmissivity [ $L^2/T$ ]

$$T = K [\min(h, z_u) - z_b] \quad (7)$$

- $K = K(x, y)$  = hydraulic conductivity [ $L/T$ ]
- $z_u$  = top aquifer elevation [L]
- $z_b$  = bottom aquifer elevation [L]

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So if you talk about the one dimensional flow in terms of ordinary differential equation by neglecting the transient term and the variation in y direction we can write it as  $\frac{dq_x}{dx} = f$  and momentum conservation equation can be written as  $q_x = -T \frac{dh}{dx}$ .

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### Steady One-Dimensional Groundwater Flow

Governing Equations: ODE

**Mass Conservation Equation**

$$\frac{dq_x}{dx} = f \quad (8)$$

**Momentum Conservation Equation**

$$q_x = -T \frac{dh(x)}{dx} \quad (9)$$

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So similarly for contaminant transport we need to solve one hydraulic head related equation that is the mass conservation equation. Now  $K_{xx}$ ,  $K_{yy}$  these two are our hydraulic conductivity values in x and y direction, h is the potentiometric head  $q_s$  is the volumetric flux per unit volume,  $S_s$  is the specific storage and this thing is applicable for confined aquifers system.

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### Contaminant Transport

Fluid flow

**Mass Conservation Equation**

$$\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial h}{\partial y} \right) + q_s = S_s \frac{\partial h}{\partial t} \quad (10)$$

where

- $K_{xx}, K_{yy}$  = hydraulic conductivity along x and y axes [L/T]
- $h$  = potentiometric head [L]
- $q_s$  = Volumetric flux per unit volume [1/T]
- $S_s$  = specific storage [1/L]

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We need to solve another extra equation for contaminant transport that is our scalar transport equation where  $c$  is the concentration and  $\eta$  is the effective porosity and  $dx$ ,  $dy$ ,  $d_{xx}$ ,  $d_{yy}$  are the hydraulic dispersion coefficient along  $x$  and  $y$  direction. And  $d_{xy}$ ,  $d_{yx}$  these two are the hydraulic dispersion coefficient along cross directions.  $V_x$  and  $v_y$ , these two are velocity along  $x$  and  $y$  axis. These two values can be calculated from Darcy velocity.

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## Contaminant Transport

### Concentration Equation

#### Scalar Transport Equation

$$\frac{\partial(\eta C)}{\partial t} = \frac{\partial}{\partial x} \left( \eta D_{xx} \frac{\partial C}{\partial x} + \eta D_{xy} \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial y} \left( \eta D_{yx} \frac{\partial C}{\partial x} + \eta D_{yy} \frac{\partial C}{\partial y} \right) - \frac{\partial}{\partial x} (\eta v_x C) - \frac{\partial}{\partial y} (\eta v_y C) + q_s C_s \quad (11)$$

where

- $\eta$  = effective porosity,
- $D_{xx}$ ,  $D_{yy}$  = hydraulic dispersion coefficients along  $x$  and  $y$  axes [ $L^2/T$ ]
- $D_{xy}$ ,  $D_{yx}$  = hydraulic dispersion coefficients along cross-directions [ $L^2/T$ ]
- $v_x$ ,  $v_y$  = velocity along  $x$  and  $y$  axes [ $L/T$ ]
- $C_s$  = concentration of source [ $M/L^3$ ].

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If we talk about channel networks, similarly we need to define two equations. One is for conservation and another for momentum, mass conservation and moment conservation. If we talk about the mass conversation equation, in that point this  $\frac{\partial h}{\partial t}$  that considers the change in the channel water depth with time.  $1/b$  is the free surface width.  $\frac{\partial \eta}{\partial x}$  is a change with respect to  $x$  direction. That means it is one dimensional in space and time both are considered.

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## Channel Networks

Governing Equations

Depth-integrated **mass conservation** equation for surface water flow can be written as,

$$\frac{\partial h}{\partial t} + \frac{1}{B} \frac{\partial Q}{\partial x} = 0 \quad (12)$$

where

- $h = h(x, t)$  = channel water depth [L]
- $B$  = free-surface width [L]
- $Q$  = flux [ $L^3/T$ ]
- $t$  = time [T].

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Momentum conservation equation this equation we can relate the change in  $q$  with respect to time with other variable that is  $h$ . And  $s_f$  and  $s_0$  these two are actually friction slope and bed slope.  $s_f$  can be calculated from Manning's equations and  $a$  is the cross sectional area.

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## Channel Networks

Governing Equations

The **momentum conservation** equation can be written as,

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} + gA(S_f - S_0) = 0 \quad (13)$$

where

- $g$  = acceleration due to gravity [ $L/T^2$ ]
- $A$  = cross-sectional area [ $L^2$ ]
- $S_f$  = friction slope
- $S_0$  = bed slope

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If we consider surface flooding the equation is simple. We need to solve  $\frac{\partial h}{\partial t}$  into  $q$ .  $\frac{\partial h}{\partial t}$  considers the change in depth with respect to time into area. That is actually net inflow of water to the plot. So we should consider all kinds of inflow conditions. It can be inflow from the canal section or rainfall to the surface area. So  $a$  is the cross-sectional area of the plot. So if we can discretize this we can solve the surface flooding problem.

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### Surface Flooding

Variable:  $h(x,y,t)$

Figure: Initial Condition (Biswas, 2016)

$$A \frac{dh}{dt} = Q \quad (14)$$

$A$  = c/s area of the plot [ $L^2$ ]  
 $h$  = depth of water [ $L$ ]  
 $Q$  = net inflow of the water to the plot [ $L^3/T$ ]

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If we consider this open channel flow that is hydraulic jump.  $U$  and  $w$  these two are actually longitudinal and vertical velocity components.

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### Open Channel Flow

#### Hydraulic jump

Variables:  $u(x,z,t), w(x,z,t)$

Figure: Initial condition of hydraulic jump (Pahar and Dhar, 2017)

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So for this one to solve the hydraulic jump we need to consider mass conservation. This is actually vector form of our previous mass conservation equation considering all directions. But in this case only longitudinal and vertical components will be there. And momentum conservation equation it is written in terms of total derivative.  $Du$  by  $dt$ , a total derivative of  $u$  that actually considers aquatic term plus temporal change in the velocity.

And tau by row that is sub particle scale tensor and u is our kinematics viscosity and g is acceleration due to gravity. So depending on equation and direction we need to assign different value.

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### Open Channel Flow

Hydraulic jump

**Mass conservation equation**

$$\nabla \cdot \mathbf{u} = 0 \quad (15)$$

**Momentum conservation equation**

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \nu\nabla^2\mathbf{u} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \quad (16)$$

where

- $\frac{D}{Dt}$  = total derivative
- $P$  = fluid pressure
- $\boldsymbol{\tau}$  = sub-particle-scale tensor
- $\nu$  = viscosity

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If we talk about pressurized conduits we should have equation for h and q. Sodepth integrated mass conservation equation for surface flow can be return in terms of temporal change in the piezometric head and this is change in the discharge where c is the wave speed, q is the discharge, t is time and a again is cross sectional area.

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### Pressurized Conduits

Governing Equations

Depth-integrated **mass conservation** equation for pressurized conduit can be written as,

$$\frac{\partial H}{\partial t} + \frac{c^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad (17)$$

where

- $H = H(x, t)$  = piezometric head [L]
- $c$  = wave speed [ $L^3/T$ ]
- $Q$  = discharge [ $L^3/T$ ]
- $t$  = time [T].

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And if we consider the momentum so momentum is again change of discharge with respect to time and change in h with respect to x direction. So we are basically considering one dimensional flow system. In this case  $J_s$  and  $J_u$  these two are separately defined.

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### Pressurized Conduits

Governing Equations

The momentum conservation equation can be written as,

$$\frac{\partial Q}{\partial t} + gA \frac{\partial H}{\partial x} + J_s + J_u = 0 \quad (18)$$

where

- $g$  = acceleration due to gravity [ $L/T^2$ ]
- $A$  = cross-sectional area [ $L^2$ ]
- $J_s$  = steady friction loss
- $J_u$  = unsteady friction loss

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And  $J_s$  actually that is steady friction loss and unsteady friction loss is the  $J_u$ . We will discuss these equations in individual units during solution process.

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### Pressurized Conduits

Governing Equations

#### Steady Friction Loss

$$J_s = \frac{f_s Q |Q|}{2DA} \quad (19)$$

where

- $f_s$  = Darcy-Weisbach friction factor
- $D$  = pipe diameter [ $L$ ]

#### Unsteady Friction Loss

$$J_u = \frac{k}{2} (Q_t + c \Phi_A |Q_x|) \quad (20)$$

where

- $\Phi_A = sqn(Q)$
- $k$  = Brunone friction coefficient [ $L$ ]

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So if we consider a pressurized conduits and in network system then we need to solve again mass conservation equation that is represented by this particular equation where  $q$  is flow along line  $j$  and  $p$  is pressure along line  $j$ ,  $\rho$  is density,  $\tau_{uj}$  this is

another term which is represented this cross-sectional area for the pipe is  $A_j$  and  $l_j$  is the length of pipe and this capital lambda is a set of fluid lines.

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## Pressurized Conduits

Governing equations

Mass Conservation equation

$$\frac{\partial \hat{q}_j}{\partial t} + \frac{A_j}{\rho} \frac{\partial p_j}{\partial x} + \tau_j(q_j) = 0, \quad x \in [0, l_j], \quad j \in \Lambda \quad (21)$$

where

- $p_j$  = pressure along the line  $j$
- $q_j$  = flow along the line  $j$
- $c_j$  = fluid line wave speed for pipe  $j$
- $A_j$  = cross-sectional area for pipe  $j$
- $l_j$  = length of pipe  $j$
- $\Lambda$  = set of fluid lines

  
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So Momentum conservation equation again it is for change in  $p$  with respect to time and other term is related to  $q$  and  $\tau_j$  is the cross-sectional frictional resistance and  $\lambda_j$  these terms are defined like our previous mass conservation equation.

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## Pressurized Conduits

Governing equations

Momentum Conservation equation

$$\frac{\partial p_j}{\partial t} + \frac{\rho c_j^2}{A_j} \frac{\partial q_j}{\partial x} + \tau_j(q_j) = 0, \quad x \in [0, l_j], \quad j \in \Lambda \quad (22)$$

where

- $\tau_j$  = cross-sectional frictional resistance

  
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Now if we consider surface water groundwater equation and in depth average sense if we want to use those equations. So we can write this mass conservation equation for surface water height that is represented in terms of this equation. This is  $p_s$  again  $p_s$  this is actually

depth average velocity into depth and  $c$  is Chézy's equation and this is our momentum equation it represents the variation of  $\tau_s$ .  $\beta_m$  is the momentum correction factor this is our velocity correction term and  $g$  is acceleration due to gravity in this case.

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### Surface water-groundwater interaction

Surface water

**Mass conservation equation**

$$\frac{\partial \zeta^s}{\partial t} + \frac{\partial p^s}{\partial x} = 0 \quad (23)$$

where  
 $\zeta^s$  = surface water height [L]  
 $p^s$  = surface water unit discharge [ $L^2/T$ ]

**Momentum conservation equation**

$$\frac{\partial p^s}{\partial t} + \frac{\partial}{\partial x} (\beta_m U p^s) = -gH \frac{\partial \zeta^s}{\partial x} - \frac{g(p^s)^2}{H^2 C^2} + \nu \frac{\partial^2 p^s}{\partial x^2} \quad (24)$$

$\beta_m$  = momentum correction factor  
 $\nu$  = kinematic viscosity [ $L^2/T$ ]

$$p^s = \int_0^{\zeta^s} u dz = \bar{U} H$$

$\bar{U}$  = average surface water velocity

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So for groundwater we need to introduce this  $\eta$  or effective porosity term to consider the porous medium. Similarly for momentum conservation we need to use Darcy's law.

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### Surface water-groundwater interaction

Groundwater

**Mass conservation equation**

$$\eta \frac{\partial \zeta^g}{\partial t} + \frac{\partial p^g}{\partial x} = 0 \quad (26)$$

where  
 $\eta$  = effective porosity  
 $\zeta^g$  = ground water height [L]  
 $p^g$  = ground water discharge [ $L^2/T$ ]

**Momentum conservation equation**

$$p^g = -K \zeta^g \frac{\partial \zeta^g}{\partial x}$$

where  
 $K$  = hydraulic conductivity [ $L/T$ ]

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So we can combine these equations with channel flow to solve this surface water groundwater interaction problem. If we consider regional scale modelling then for channel flow we need to solve this relatively general equation where  $q_l$  is the lateral inflow or

outflow,  $m_l$  is the lateral momentum flux and  $a_0$  is the depth storage in channels. So with these two equations if you compare this two this  $q_l$  and  $m_l$  relates it with groundwater equation.

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### Surface water-groundwater interaction

Governing Equations: Channel flow

Depth-integrated mass conservation equation for channel flow can be written as,

**Mass Conservation Equation**

$$\frac{\partial s_c(A + A_0)}{\partial t} + \frac{\partial Q}{\partial x} - q_L = 0 \quad (28)$$

**Momentum Conservation Equation**

$$\frac{\partial s_m Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\beta Q^2}{A} \right) + gA \frac{\partial h_L}{\partial x} + gA(S_f + S_c) + M_L = 0$$

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So we can consider datum and from this datum we can take this  $h_g$  which is groundwater level  $h_r$  is the river level and  $z_r$  the bed level and  $z_b$  is the height of our impervious layer.

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### Surface water-groundwater interaction

Figure: Channel flow/groundwater flow interaction (Gunduz and Aral, 2005)

The diagram shows a cross-section of a river channel and the underlying aquifer. A datum is shown at the bottom. Key variables include:  $h_g$  (groundwater level),  $h_r$  (river level),  $z_r$  (bed level),  $z_b$  (height of impervious layer),  $w_r$  (infiltration), and  $m_r$  (exfiltration). The layers are labeled RIVER, AQUIFER, and Impervious Layer.

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So with this variable we can define our mass conservation equation combining our Darcy's law we can write it and in this case as  $z_b$  is the (impe) height of impervious layer. So we are considering the flow over this impervious layer only. So that is why this  $h_g$  minus  $z_b$  term is

there and I is source sink term that is added and sy is a specific for the unconfined aquifer system.

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**Mass Conservation Equation**

$$\begin{aligned}
 & \frac{\partial}{\partial x} \left[ (h_g - z_b) K_{xx} \frac{\partial h_g}{\partial x} + (h_g - z_b) K_{xy} \frac{\partial h_g}{\partial y} \right] \\
 & + \frac{\partial}{\partial y} \left[ (h_g - z_b) K_{yx} \frac{\partial h_g}{\partial x} + (h_g - z_b) K_{yy} \frac{\partial h_g}{\partial y} \right] + \sum_{k=1}^{n_w} [Q_{w_k} \delta(x - x_k) \delta(y - y_k)] + \\
 & \sum_{m=1}^{n_r} \left[ \int_0^1 q_{L,m} \sqrt{\left(\frac{dg_{xm}}{dt}\right)^2 + \left(\frac{dg_{ym}}{dt}\right)^2} \times \delta(x - g_x(t)) \delta(y - g_{ym}(t)) dt \right] \\
 & + I = S_y \frac{\partial h}{\partial t} \quad (30)
 \end{aligned}$$

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And finally if we talk about this 1D 2D integrated system where we need to consider the channel, water depth, channel discharge, surface flooding, depth or depth of surface water along with velocity components in x and y directions. We can express it in terms of this vector form where u1d, 1d is for channel flow approximation, a and q these two are for one mass and another one is momentum equation, f1d again this q is for mass, q square a and gi1 this is for momentum equation.

H1d, again ql which considers lateral discharge is for mass and this one is for momentum equation.

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### 1D-2D integrated system

One-dimensional Governing Equations

Depth-integrated mass and momentum conservation equations for surface water flow can be written as,

$$\mathbf{U}_{1D,t} + \mathbf{F}_{1D,x} = \mathbf{H}_{1D} \quad (31)$$

$$\mathbf{U}_{1D} = \begin{bmatrix} A \\ Q \end{bmatrix} \quad (32)$$

$$\mathbf{F}_{1D} = \begin{bmatrix} Q \\ \frac{Q^2}{A} + gI_1 \end{bmatrix} \quad (33)$$

$$\mathbf{H}_{1D} = \begin{bmatrix} gI_2 + gA(S_0 - S_f) \end{bmatrix}$$

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In combined form or short form we can write this governing equation and this comma t represents the derivative with respect to t and comma x this is actually derivative with respect to x.

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### 1D-2D integrated system

One-dimensional Governing Equations

Depth-integrated mass and momentum conservation equations for surface water flow can be written as,

$$\mathbf{U}_{1D,t} + \mathbf{F}_{1D,x} = \mathbf{H}_{1D} \quad (31)$$

$$\mathbf{U}_{1D} = \begin{bmatrix} A \\ Q \end{bmatrix} \quad (32)$$

$$\mathbf{F}_{1D} = \begin{bmatrix} Q \\ \frac{Q^2}{A} + gI_1 \end{bmatrix} \quad (33)$$

$$\mathbf{H}_{1D} = \begin{bmatrix} gI_2 + gA(S_0 - S_f) \end{bmatrix}$$

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So in our previous equation i1 and i2 these two are actually are general terms and defined for general cross section like this where h is the depth of flow and i2 and i1 are calculated like this.

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### 1D-2D integrated system

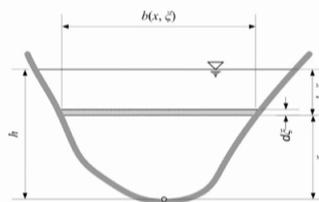


Figure: Channel cross-section

$$I_1 = \int_0^h (h - \xi) b(x, \xi) d\xi$$

$$I_2 = \int_0^h (h - \xi) \frac{\partial b(x, \xi)}{\partial x} d\xi$$

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If we consider the two dimensional mass and momentum conservation equation then again we need to calculate the things.  $U_{2D}$ , the first one is for mass conservation and momentum conservation in u direction  $h v$  is momentum conservation for v direction. Like this h first row represents the mass conservation equation second and third these two are for momentum conservation in or x and y direction. So what is this  $i_R$ ? It is the rainfall and finally we can write it in terms of vector form.

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### 1D-2D integrated system

Two-dimensional Governing Equations

Depth-integrated mass and momentum conservation equations for surface water flow can be written as,

$$\mathbf{U}_{2D,t} + \nabla \mathbf{F}_{2D} = \mathbf{H}_{2D} \quad (37)$$

$$\mathbf{U}_{2D} = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix} \quad (38)$$

$$\mathbf{F}_{2D} = \begin{bmatrix} hu & hv \\ hu^2 + \frac{gh^2}{2} & huv \\ huv & hv^2 + \frac{gh^2}{2} \end{bmatrix} \quad (39)$$

$$\mathbf{H}_{2D} = \begin{bmatrix} i_R \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{bmatrix}$$

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So with this governing equations we can conceptualize our system in terms of 1D 2D or 3D equations. Thank you.