Computational Hydraulics
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Lecture 2
Problem Definition and Governing Equation (GE)

Welcome to this lecture number 2 of this module 1, introduction to computational hydraulics and this unit is about problem definition and governing equations.

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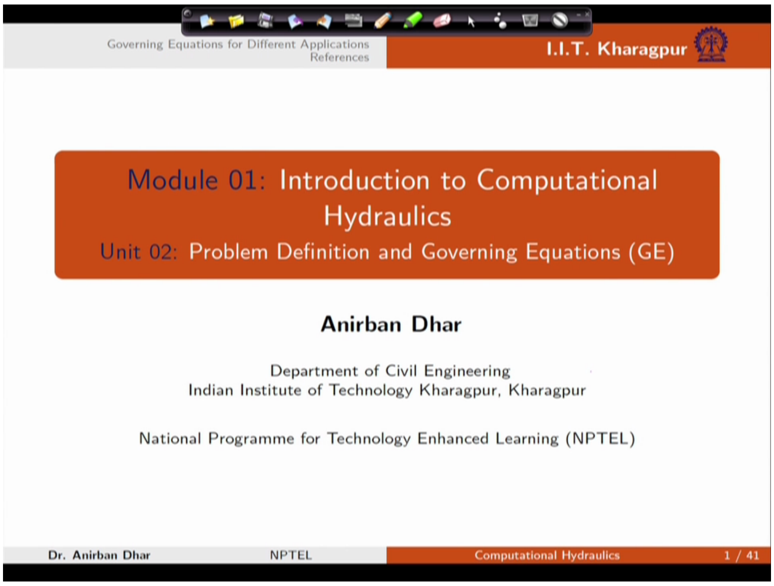

So what are the learning objective? To identify the governing equation for hydraulic system. So in this particular lecture we will try to identify the governing equation related to hydraulic systems.

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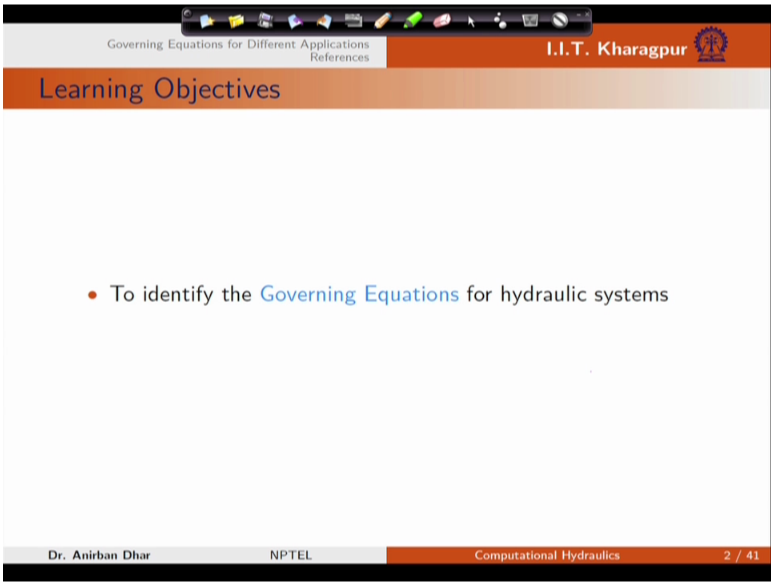

Introduction, governing equation define the relationship between variables in terms of ordinary differential equation or partial differential equations. And it relates the independent and dependent variables. ODEs or PDEs represent conversation laws that is mass, momentum and energy in general or simplified form. Depending on the nature of the problem we can use a three dimensional form or a simplified one dimensional equation for the problem.

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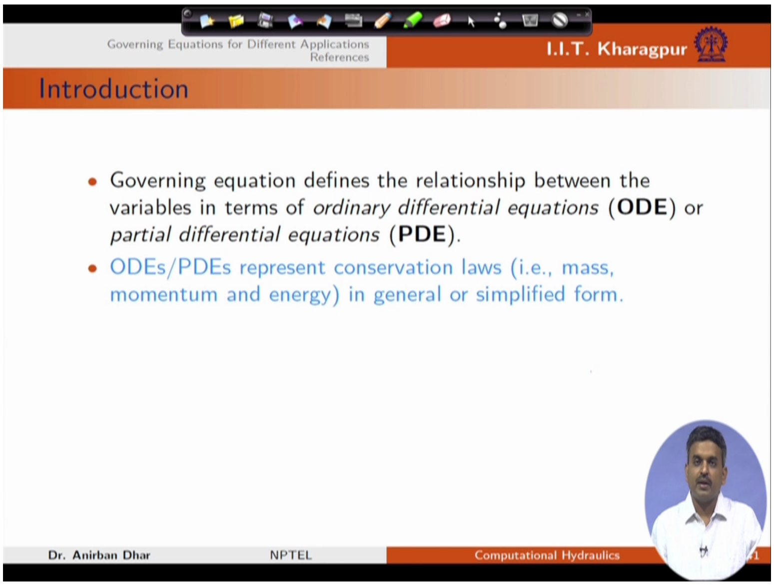

So what is this ODE? ODE is differential equation with one independent variable. And what is this PDE is? PDE is differential equation with two or more independent variables.

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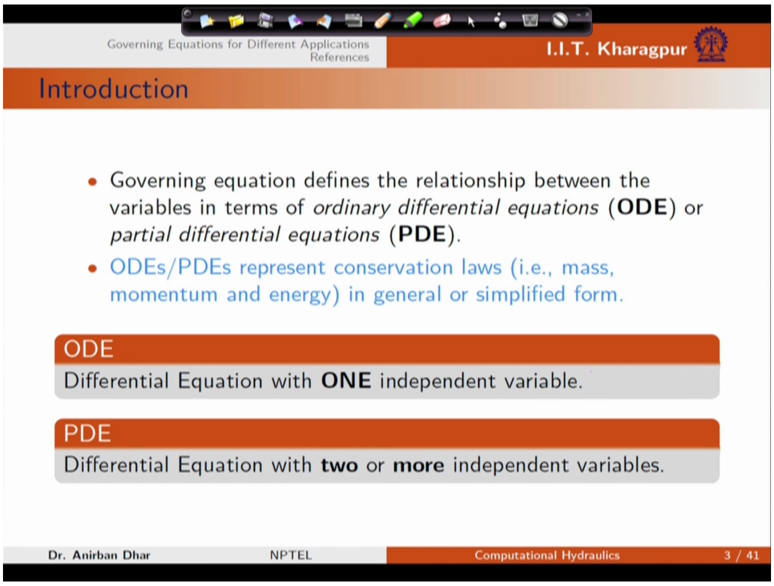

Now (wha) what are the basic principles? Basic principles are conservation of mass, conservation of momentum and conservation of energy.

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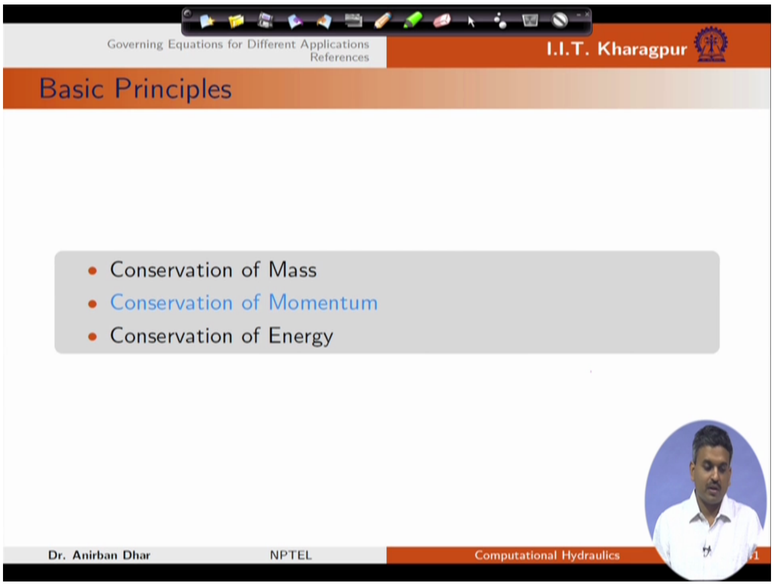

If we talk about this conservation equation in general, so for incompressible fluid flow we can write down the differential equation or partial differential equation like this. In this case delu by delx represents the change in x direction and delu by dely change in the y and delu by delz change in the z direction. U, V, and W these are 3 components of the velocity vector in 3 directions.

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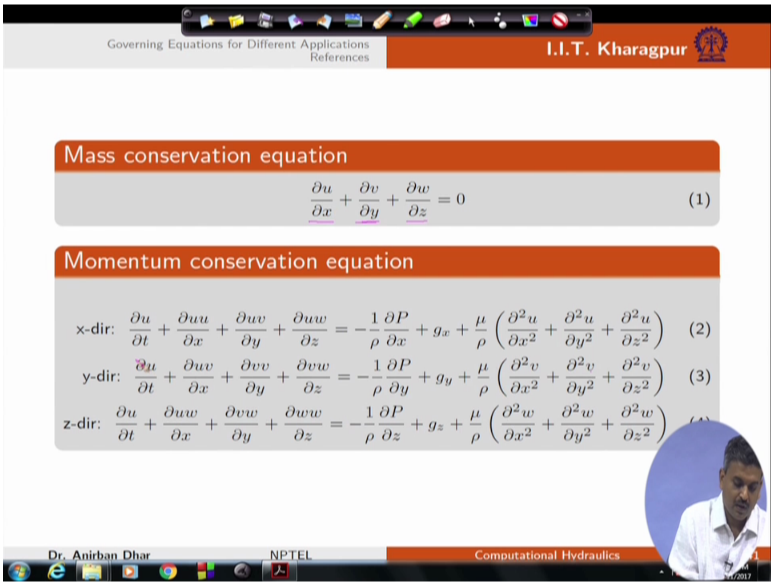

And what is this momentum conversation equation? In x direction we can write the momentum conservation equation. This is your temporal term, this is your additive term and this is your pressure term, and this is the body force gravity term. And this is our defuse center where u is the dynamic viscosity row is density of fluid, p is pressure and we already know that u, v, w are the components of velocity.

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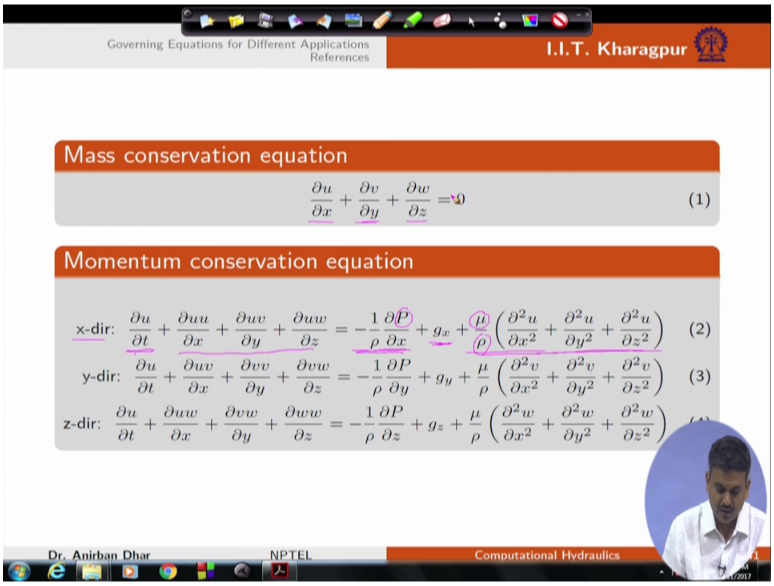

Now in our previous lecture we have seen that if you perform this experiment of 2 density fluids that is lock exchange flow. So from experiments we can simulate or we can get information about the system and from numerical case we can simulate the corresponding variation with the help of computers.So to solve this problem we need to solve our conservation of mass and conservation of momentum equations.

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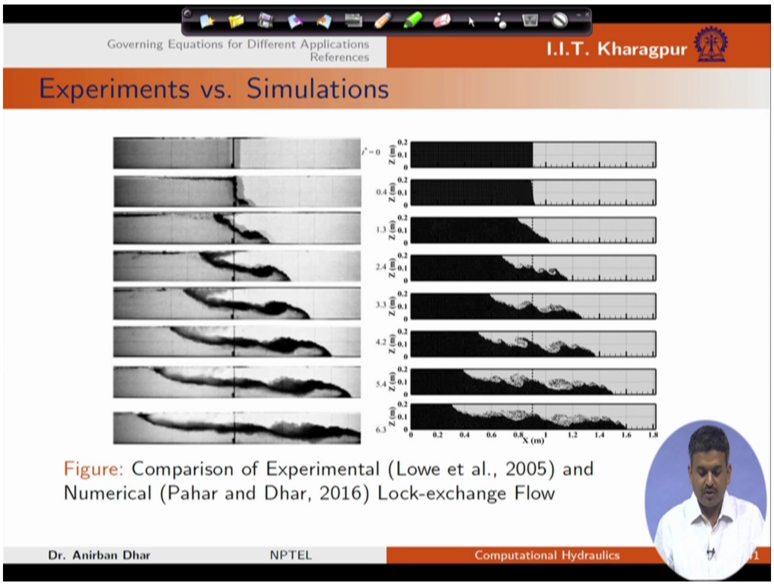

So if we talk about groundwater movement in aquifers then we need to define the conservation of mass that is depth integrated mass conservation equation for groundwater flow.

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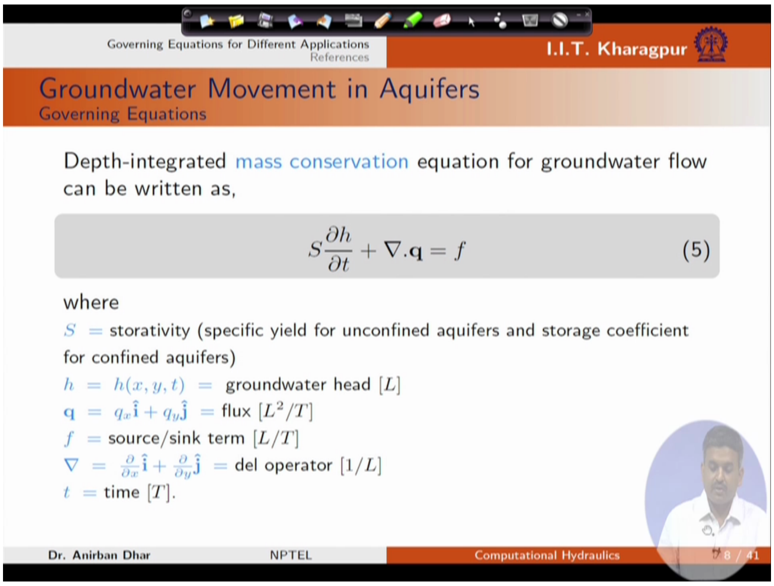

Where s is our storativity term for unconfined aquifers, it is specific yield. For confined aquifers this is storage coefficient. Interestingly this Q is Darcy discharge and f is source or sink term. So change in head or groundwater head can be represented using this mass conversation equation.

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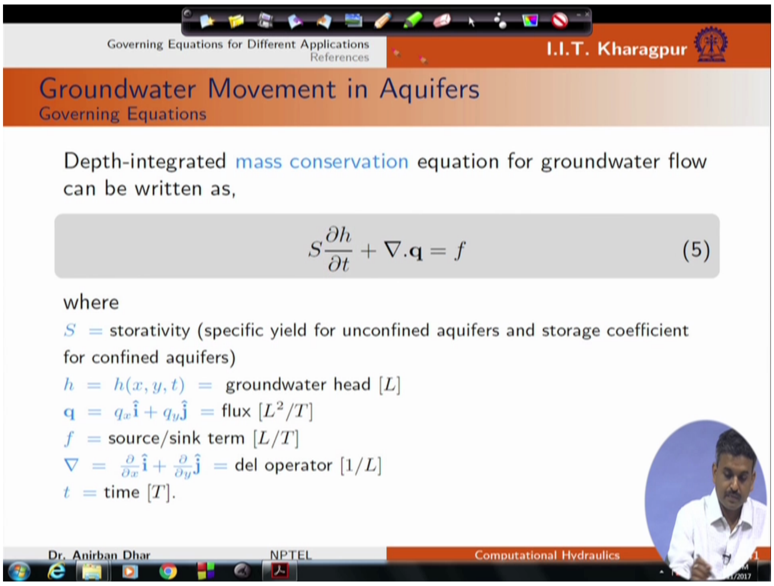

Now if we talk about momentum equation in case of aquifer or aquifer flow thing, then Darcy's law is a momentum conservation equation where q is the Darcy velocity or Darcy discharge vector and T is transmissivity, K is hydraulic conductivity and as we can see that this T is calculated as minimum value of h and zu, zu is the top aquifer elevation minus the bottom aquifer elevation. That means the saturated thickness portion.

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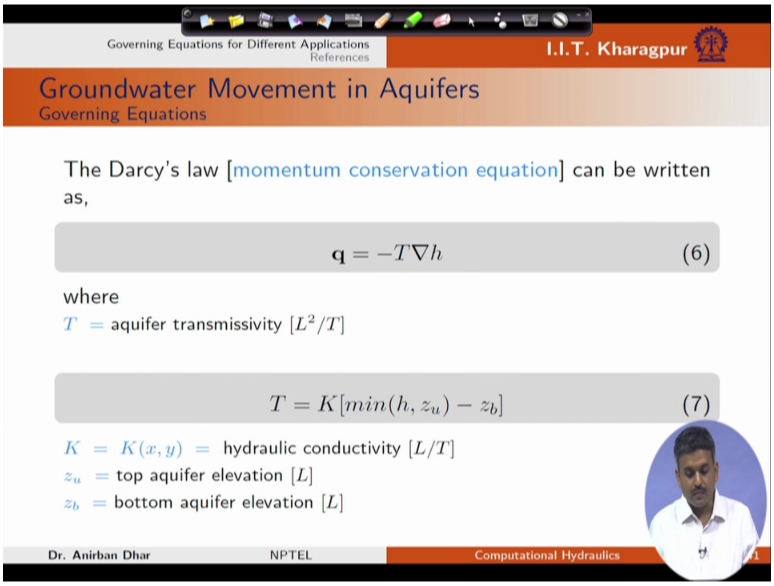

So if you talk about the one dimensional flow in terms of ordinary differential equation by neglecting the transient term and the variation in y direction we can write it as delqx dqx by dx equal to f and momentum conservation equation can be written as qx is equal to minus t del dh by dx.

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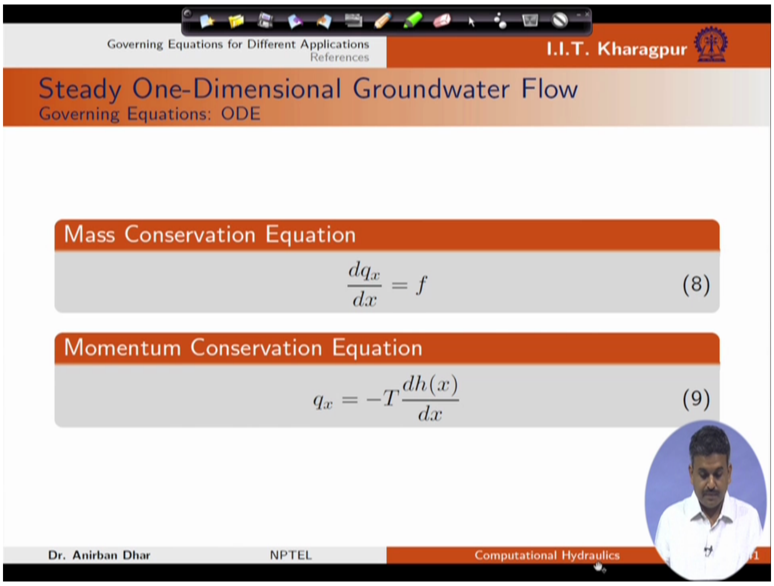

So similarly for contaminant transport we need to solve one hydraulic head related equation that is the mass conversation equation. Now kxx, kyy these two are our hydraulic conductivity values in x and y direction, h is the potentiometric head qs is the volumetric flux per unit volume, ss is the specific storage and this thing is applicable for confined aquifers system.

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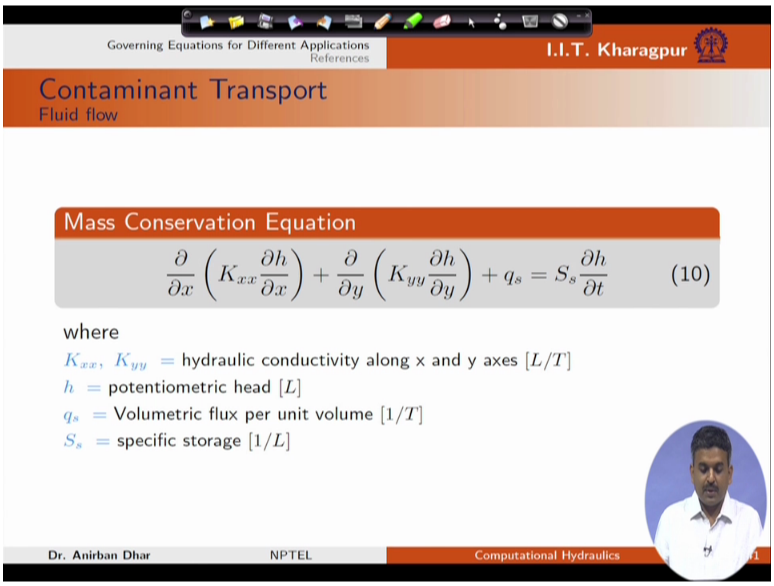

We need to solve another extra equation for contaminant transport that is our scalar transport equation where c is the concentration and eta is the effective porosity and dx, dy, dxx, dyy are the hydraulic dispersion coefficient along x and y direction. And dxy, dyx these two are the hydraulic dispersion coefficient along cross directions. Vx and vy, these two are velocity along x and y axis. These two values can be calculated from Darcy velocity.

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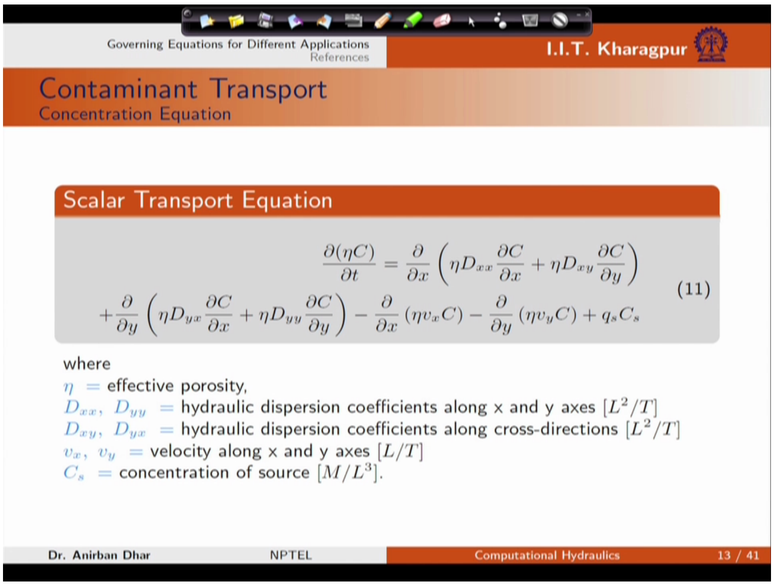

If we talk about channel networks, similarly we need to define two equations. One is for conservation and another for momentum, mass conservation and moment conservation. If we talk about the mass conversation equation, in that point this delh by delt that considers the change in the channel water depth with time. 1 by b is the free surface width. Delq by delx is a change with respect to x direction. That means it is one dimensional in space and time both are considered.

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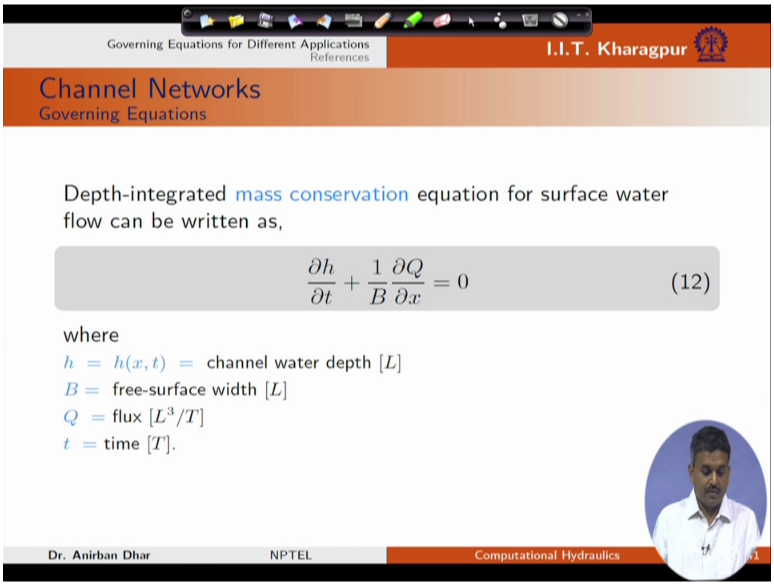

Momentum conversation equation this equation we can relate the change in q with respect to time with other variable that is h. And sf and s0 these two are actually friction slope and bed slope. Sf can be calculated from Manning’s equations and a is the cross sectional area.

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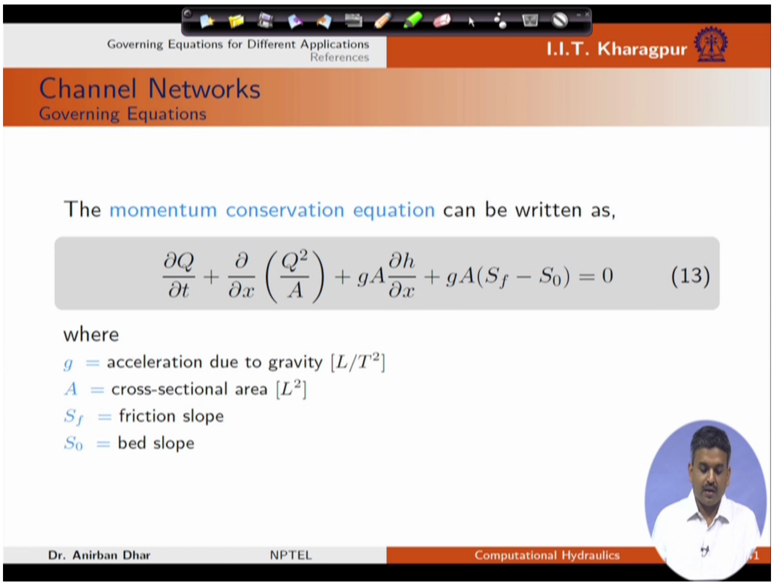

If we consider surface flooding the equation is simple. We need to solve delh by delt into q. Delh by delt considers the change in depth with respect to time into area. That is actually net inflow of water to the plot. So we should consider all kinds of inflow conditions.It can be inflow from the canal section or rainfall to the surface area. So a is the cross-sectional area of the plot. So if we can discretize this we can solve the surface flooding problem.

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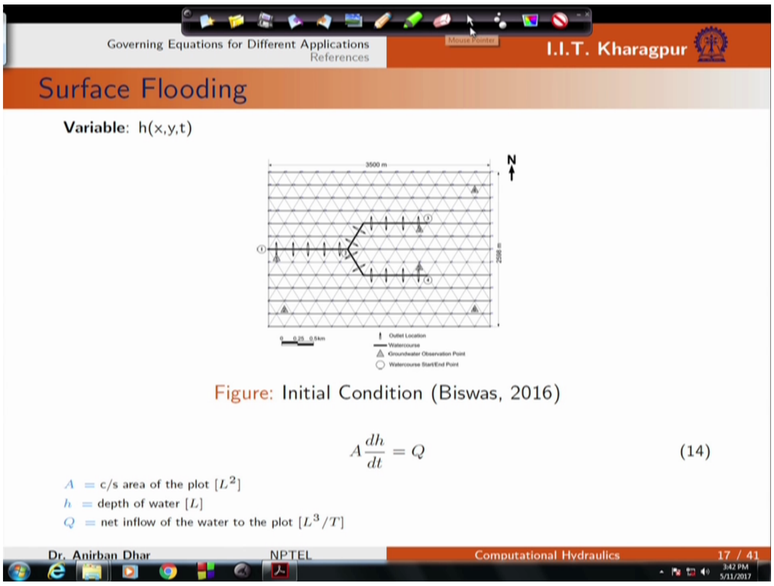

If we consider this open channel flow that is hydraulic jump.U and w these two are actually longitudinal and vertical velocity components.

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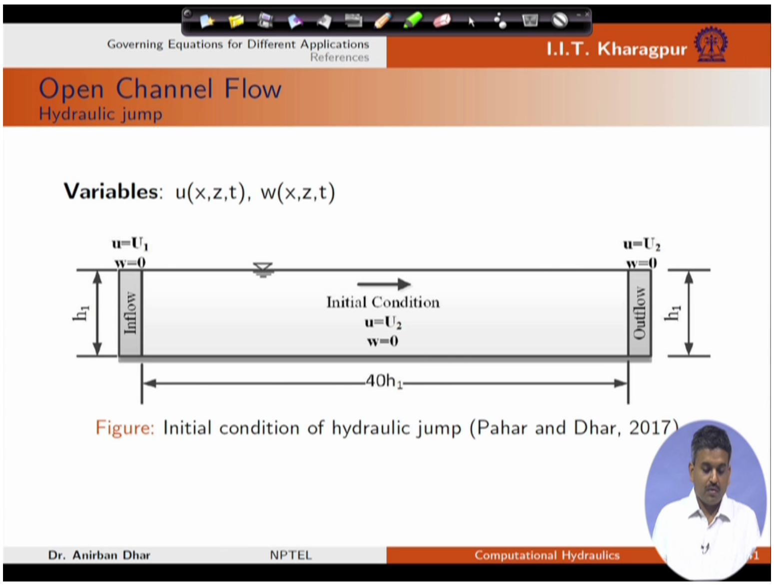

So for this one to solve the hydraulic jump we need to consider mass conservation. This is actually vector form of our previous mass conservation equation considering all directions. But in this case only longitudinal and vertical components will be there. And momentum conversation equation it is written in terms of total derivative. Du by dt, a total derivative of u that actually considers aquatic term plus temporal change in the velocity.

And tau by row that is sub particle scale tensor and u is our kinematics viscosity and g is acceleration due to gravity.So depending on equation and direction we need to assign different value.

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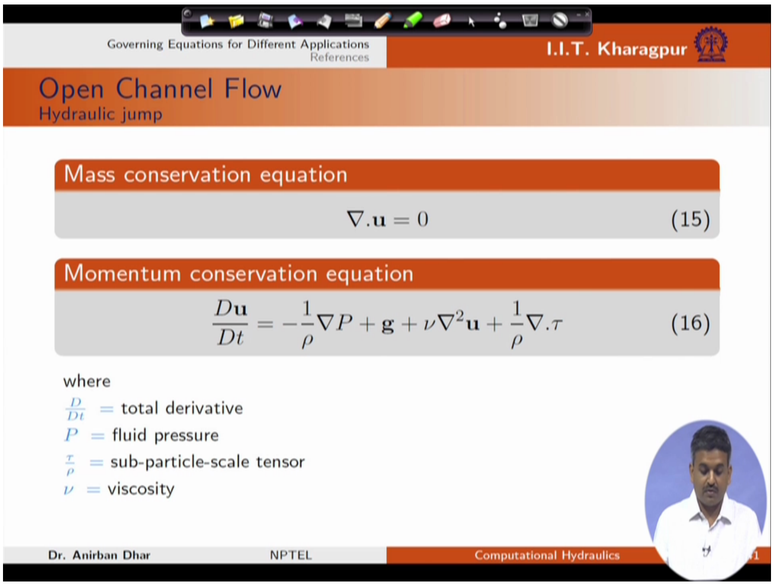

If we talk about pressurized conduits we should have equation for h and q. Sodepth integrated mass conservation equation for surface flow can be return in terms of temporal change in the piezometric head and this is change in the discharge where c is the wave speed, q is the discharge, t is time and a again is cross sectional area.

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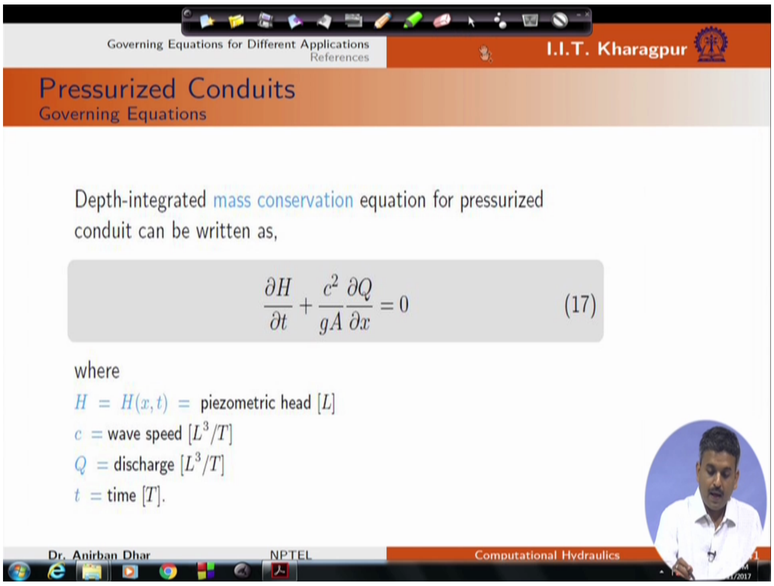

And if we consider the momentum so momentum is again change of discharge with respect to time and change in h with respect to x direction. So we are basically considering one dimensional flow system. In this case js and ju these two are separately defined.

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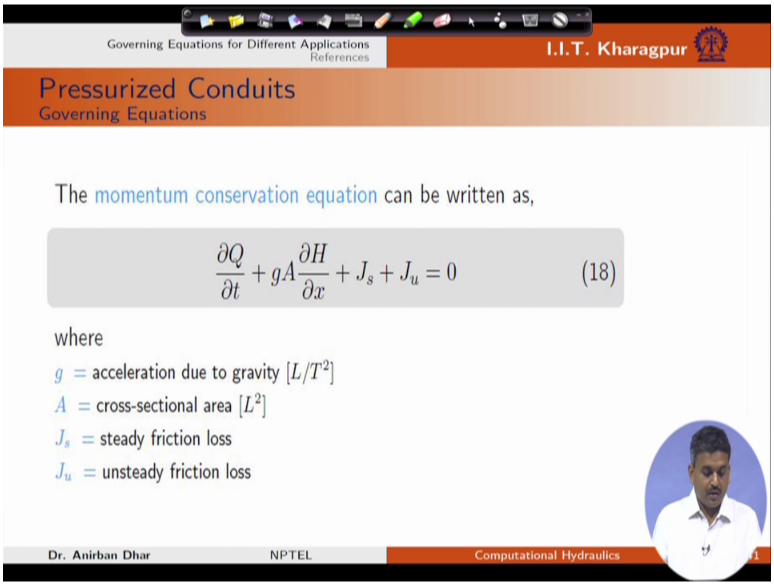

And js actually that is steady friction loss and unsteady friction loss is the ju. We will discuss these equations in individual units during solution process.

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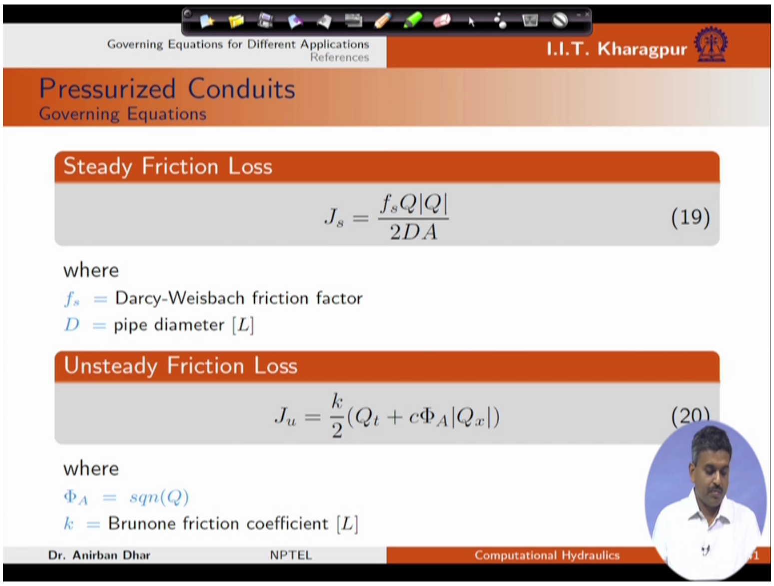

So if we consider a pressurized conduits and in network system then we need to solve again mass conservation equation that is represented by this particular equation where q is flow along line j and p is pressure along line j, area is represented by aj, row is density, tauj this is another term which is represented this cross-sectional area for the pipe is j and lj is the length of pipe and this capital lambda is a set of fluid lines.

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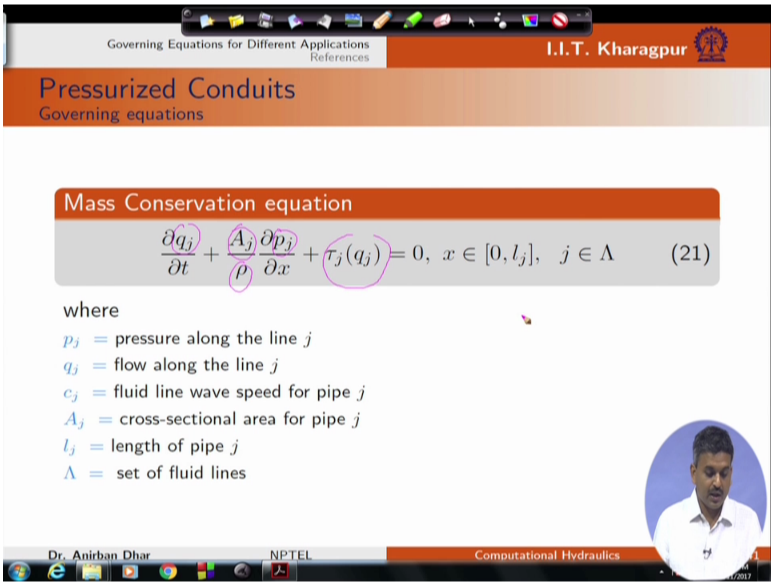

So Momentum conservation equation again it is for change in p with respect to time and other term is related to q and tauj is the cross-sectional frictional resistance and lambda lj these terms are defined like our previous mass conservation equation.

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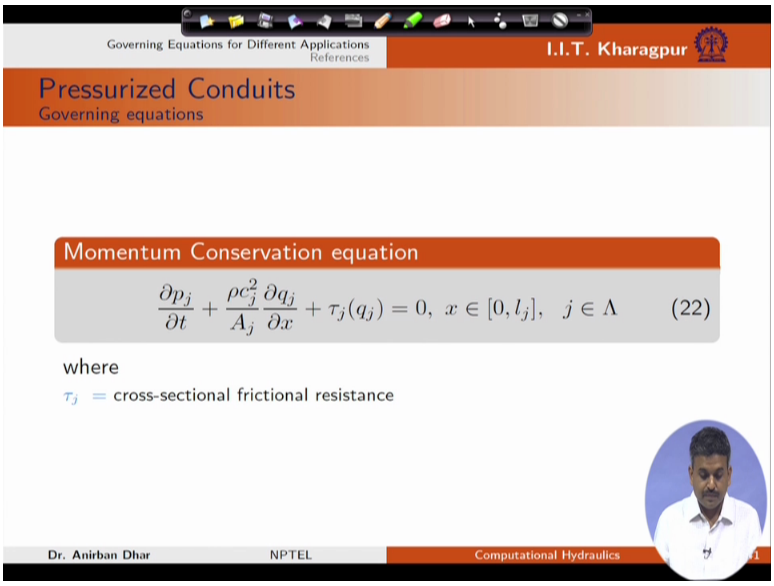

Now if we consider surface water groundwater equation and in depth average sense if we want to use those equations. So we can write this mass conversation equation for surface water height that is represented in terms of this equation. This is ps again ps this is actually depth average velocity into depth and c is chases equation and this is our momentum equation it represents the variation of ts. Beta is the momentum correction factor this is our visual term and g is acceleration due to gravity in this case.

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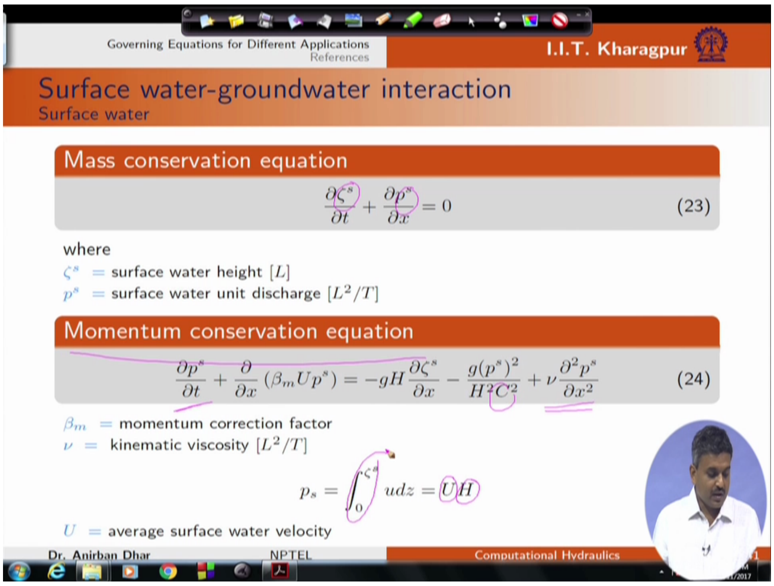

So for groundwater we need to introduce this eta or effective porosity term to consider the porous medium. Similarly for momentum conservation we need to use Darcy's law.

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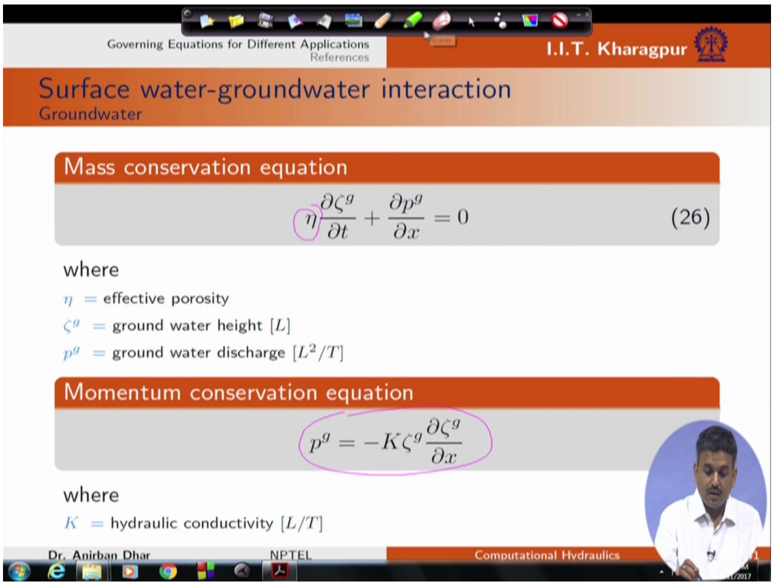

So we can combine these equations with channel flow to solve this surface water groundwater interaction problem. If we considerregional scale modelling then for channel flow we need to solve this relatively general equation where ql is the lateral inflow or outflow, ml is the lateral momentum flux and a0 is the depth storage in channels. So with these two equations if you compare this two this ql and ml relates it with groundwater equation.

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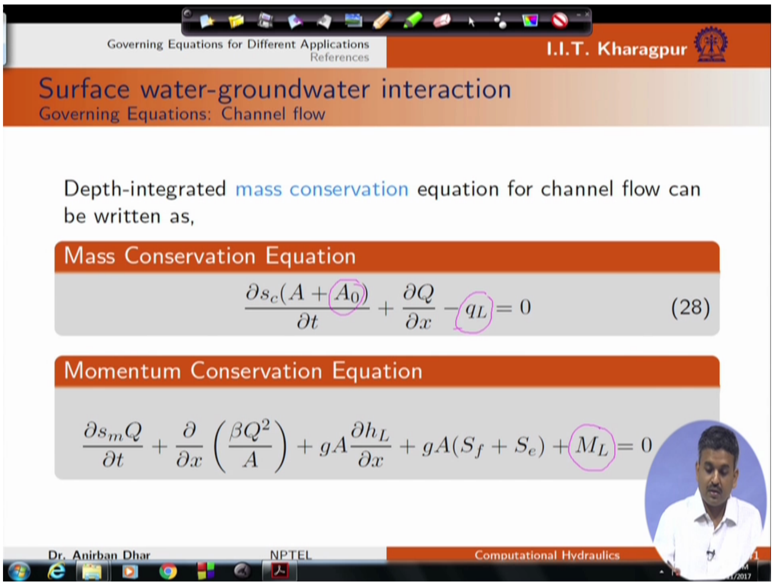

So we can consider datum and from this datum we can take this hg which is groundwater level hr is the river level and zr the bed level and zb is the height of our impervious layer.

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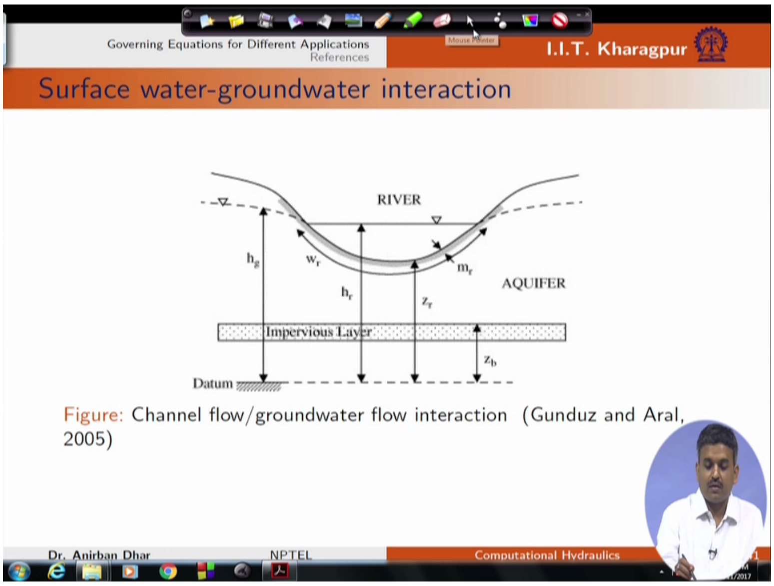

So with this variable we can define our mass conservation equation combining our Darcy's law we can write it and in this case as zb is the (impe) height of impervious layer. So we are considering the flow over this impervious layer only. So that is why this hg minus zb term is there and I is source sink term that is added and sy is a specific for the unconfined aquifer system.

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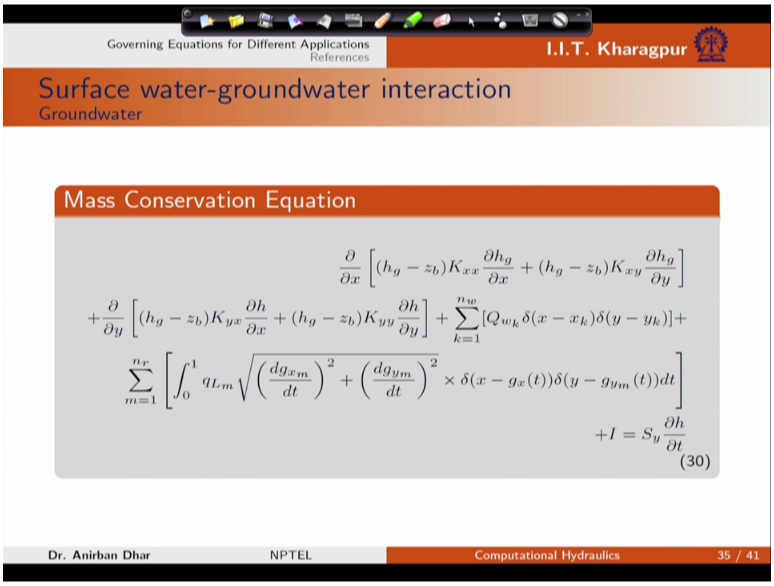

And finally if we talk about this 1D 2D integrated system where we need to consider the channel, water depth, channel discharge, surface flooding, depth or depth of surface water along with velocity components in x and y directions. We can express it in terms of this vector form where u1d, 1d is for channel flow approximation, a and q these two are for one mass and another one is momentum equation, f1d again this q is for mass, q square a and gi1 this is for momentum equation.

H1d, again ql which considers lateral discharge is for mass and this one is for momentum equation.

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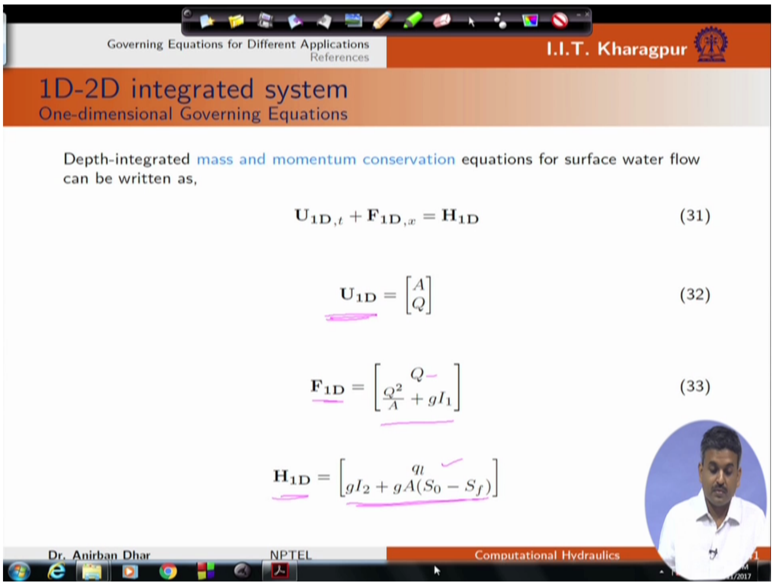

In combined form or short form we can write this governing equation and this comma t represents the derivative with respected to t and comma x this is actually derivative with respect to x.

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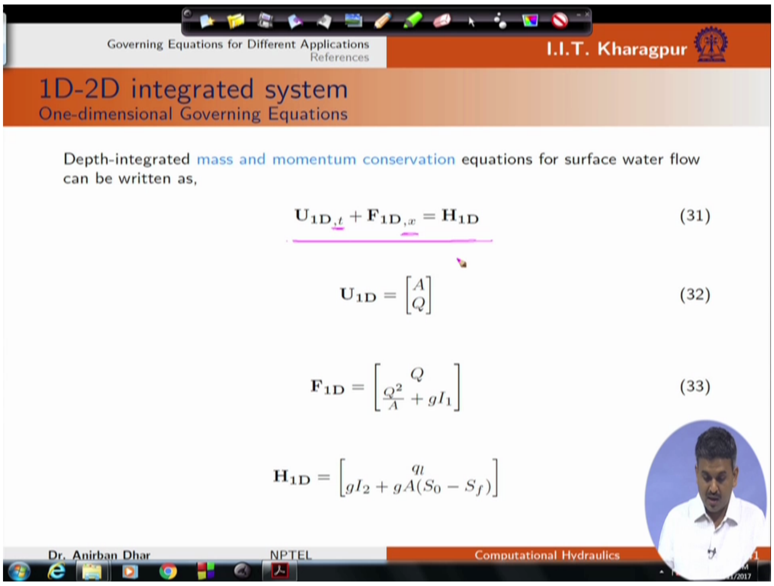

So in our previous equation i1 and i2 these two are actually are general terms and defined for general cross section like this where h is the depth of flow and i2 and i1 are calculated like this.

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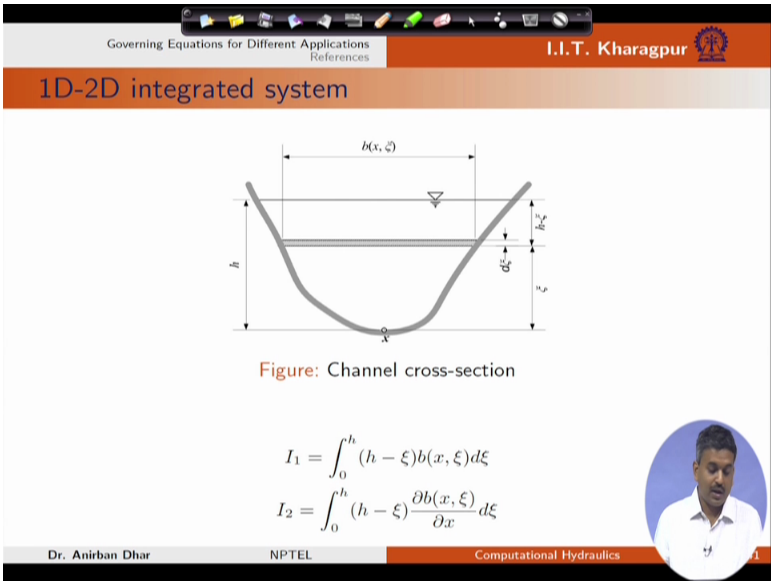

If we consider the two dimensional mass and momentum conservation equation then again we need to calculate the things. U2d,the first one is for mass conservation and momentum conservation in u direction hv is momentum conversation for v direction. Like this h first row represents the mass conservation equation second and third these two are for momentum conservation in or x and y direction. So what is this ir? Ir is the rainfall and finally we can write it in terms of vector form.

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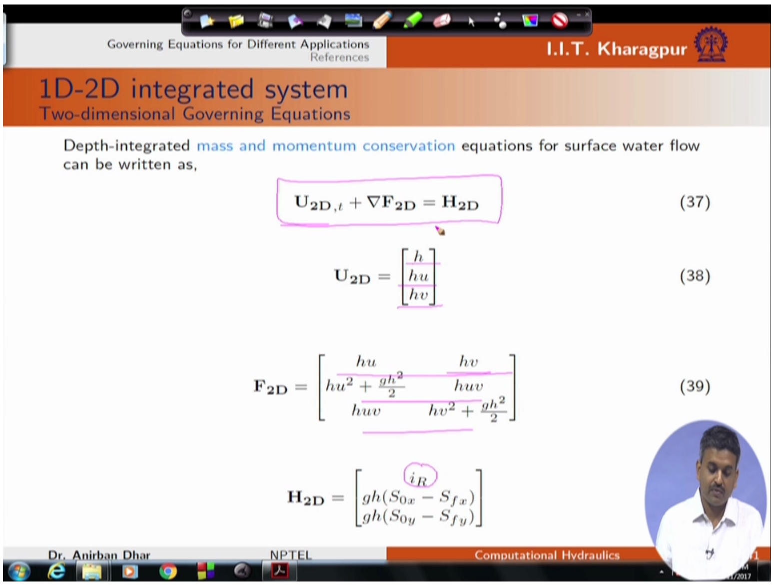

So with this governing equations we can conceptualize our system in terms of 1D 2D or 3D equations. Thank you.