

Computational Hydraulics
Professor Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 13
Finite Volume Method: Overview

Welcome to this lecture number 13 of the course computational hydraulics. We are in module 2, numerical methods. And in this particular lecture I will be covering unit 9, finite volume method -overview.

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The slide features a navigation menu at the top left with the following items: Background, Gauss Divergence Theorem, Finite Volume Method, and References. The main content area has a central orange box containing the text "Module 02: Numerical Methods" and "Unit 09: Finite Volume Method-Overview". Below this, the presenter's name "Anirban Dhar" is listed, followed by his affiliation: "Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur". At the bottom, it mentions "National Programme for Technology Enhanced Learning (NPTEL)". The footer contains "Dr. Anirban Dhar", "NPTEL", "Computational Hydraulics", and "1 / 27".

Learning objective for this particular unit. At the end of this unit students will be able to discretize the derivatives of single valued one-dimensional functions using finite volume method. And they will be able to derive the algebraic form using discretized ordinary differential equations and boundary conditions.

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Background
Gauss Divergence Theorem
Finite Volume Method
References

Learning Objective

- To discretize the derivatives of **single-valued one-dimensional functions** using finite volume method.
- To derive the **algebraic form using discretized ODE and BC(s)**.

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Let us consider our general equation in terms of phi where phi is a general variable in terms of x, y, z and t. The first term is temporal term, second term is advection relative term and first term in the right hand side this is related to diffusion. F is related to other forces and S phi is related to source sink term. And this lambda phi and upsilon phi, these are problem dependent parameters. And gamma phi is the tensor or coefficient tensor for the right hand side.

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General Equation

A form of differential equation with a general variable ϕ :

$$\frac{\partial(\Lambda_\phi \phi)}{\partial t} + \nabla \cdot (\Upsilon_\phi \phi \mathbf{u}) = \nabla \cdot (\Gamma_\phi \cdot \nabla \phi) + F_{\phi_o} + S_\phi \quad (1)$$

where

- ϕ = general variable
- $\Lambda_\phi, \Upsilon_\phi$ = problem dependent parameters
- Γ_ϕ = tensor
- F_{ϕ_o} = other forces
- S_ϕ = source/sink term

$\phi(x, y, z, t)$

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Now this is a general equation. We can write this general equation as R, where this bold x represents x, y, z directions. In the method of weighted residual, the residual R can be written as this one. This is our left hand side of the equation. With minus sign we can include the right

hand side. Now with this if we write this weighted integral of this residual and we equate this to zero for all w_l corresponding to a particular L , we can write this where this ω represents the domain.

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In the Method of Weighted Residual (MWR), residual \mathcal{R} (Finlayson and Scriven, 1966) can be written as,

$$\mathcal{R}(\mathbf{x}, t) \equiv \frac{\partial(\Lambda_\phi \phi)}{\partial t} + \nabla \cdot (\Upsilon_\phi \phi \mathbf{u}) - \nabla \cdot (\Gamma_\phi \cdot \nabla \phi) - F_{\phi_o} - S_\phi$$

The weighted integrals of the residuals set equal to zero:

$$\int_{\Omega} w_l \mathcal{R} d\Omega = 0, \quad \forall l$$

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Now with this if we approximate this w_l or weight function as Dirac delta, that means that x is equal to L , this is 1 Dirac delta equals to 1. Otherwise this is zero. So with this weight function in collocation method, if we apply this Dirac delta instead of w_l , we can write this one as $\mathcal{R} \times l \neq 0$. That means this integral can be approximated as \mathcal{R} residual $\times l$. That means if we evaluate the residual at point L , then and we equate it to zero then we can get the approximation in terms of method of weighted residual.

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In the Method of Weighted Residual (MWR), residual \mathcal{R} (Finlayson and Scriven, 1966) can be written as,

$$\mathcal{R}(\mathbf{x}, t) \equiv \frac{\partial(\Lambda_\phi \phi)}{\partial t} + \nabla \cdot (\Upsilon_\phi \phi \mathbf{u}) - \nabla \cdot (\Gamma_\phi \cdot \nabla \phi) - F_{\phi_o} - S_\phi$$

The weighted integrals of the residuals set equal to zero:

$$\int_{\Omega} w_l \mathcal{R} d\Omega = 0, \quad \forall l$$

where w_l is prescribed weighting function. If w_l is a Dirac delta function δ such that

$$w_l = \delta(\mathbf{x}_l - \mathbf{x})$$

In collocation method,

$$\int_{\Omega} \delta(\mathbf{x}_l - \mathbf{x}) \mathcal{R} d\Omega = 0 \Rightarrow \mathcal{R}(\mathbf{x}_l, t) = 0$$

This is similar to Finite Difference Method.

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Interestingly in our finite difference method we have used similar concept. For a particular location L we have used Taylor series expansion to approximate the derivatives with the help of neighboring points. So this is similar to our finite difference method.

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Background

In the Method of Weighted Residual (MWR), residual \mathcal{R} (Finlayson and Scriven, 1966) can be written as,

$$\mathcal{R}(\mathbf{x}, t) \equiv \frac{\partial(\Lambda_\phi \phi)}{\partial t} + \nabla \cdot (\Upsilon_\phi \phi \mathbf{u}) - \nabla \cdot (\Gamma_\phi \cdot \nabla \phi) - F_\phi - S_\phi$$

The weighted integrals of the residuals set equal to zero:

$$\int_{\Omega} w_l \mathcal{R} d\Omega = 0, \quad \forall l$$

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This is similar to Finite Difference Method.

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Now with this if we apply this w_l such that for a particular subdomain ω , ω is the domain and ω_L is the subdomain. For that subdomain we have weight function 1. Outside that subdomain our weight function is zero.

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Background

In sub-domain method, the weighting function can be written as

$$w_l = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega^l \\ 0 & \text{if } \mathbf{x} \notin \Omega^l \end{cases}$$

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So with this information if you proceed and write our method of weighted residual, so we can write it as this, where w_l for small domain or subdomain we have this equal to zero. And this

is similar to finite volume method. Infinite volume method our approach is to reduce the elemental error for a particular problem with given differential equation. R is in terms of given differential equation and this limit is for subdomain. So with this we can start our finite volume method.

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Background

In sub-domain method, the weighting function can be written as

$$w_l = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega^l \\ 0 & \text{if } \mathbf{x} \notin \Omega^l \end{cases}$$

The weighted integral can be written as,

$$\int_{\Omega} w_l \mathcal{R} d\Omega \Rightarrow \int_{\Omega^l} \mathcal{R} d\Omega = 0$$

This is similar to Finite Volume Method.



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Now we need to define this Gauss divergence theorem. As per Gauss divergence theorem, suppose this omega is the volume bounded by a closed surface S. And A vector field defined in this omega and on S. If S is piecewise smooth with outward normal. This is unit normal and A vector is continuously differentiable then with triple integral which is actually volume integral over whole subdomain. This is our domain.

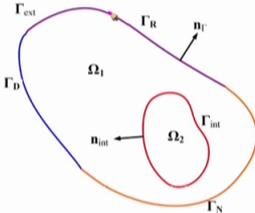
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Divergence Theorem

Gauss Divergence Theorem (Aris, 1990)

Suppose Ω is the volume bounded by a closed surface S and \mathbf{a} vector field defined in Ω and on S . If S is piecewise smooth with outward normal $\hat{\mathbf{n}}$ and continuously differentiable, then

$$\iiint_{\Omega} \nabla \cdot \mathbf{a} \, d\Omega = \iint_S \mathbf{a} \cdot \hat{\mathbf{n}} \, dS$$



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For individual subdomain, if we apply this we can write this one in terms of surface integral where this del dot A will be transferred to A dot n dS. So in this case we are transforming the volume integral to surface integral. And n is outward normal, A is any arbitrary vector and this is our del operator.

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Divergence Theorem

Gauss Divergence Theorem (Aris, 1990)

Suppose Ω is the volume bounded by a closed surface S and \mathbf{a} vector field defined in Ω and on S . If S is piecewise smooth with outward normal $\hat{\mathbf{n}}$ and \mathbf{a} continuously differentiable, then

$$\iiint_{\Omega} \nabla \cdot \mathbf{a} \, d\Omega = \iint_S \mathbf{a} \cdot \hat{\mathbf{n}} \, dS$$

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Now let us utilize this concept for finite volume formulation. In divergence theorem we have talked about the surface. If you consider a particular small elemental volume where this is our delta x size. This size is delta y and vertical side is delta z. Then we can write individual components for this particular domain and surface.

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Divergence Theorem

Area Vector

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In this case basic assumption is that area is having direction and direction of area is always outward positive. So outward is z direction with green arrow. These are positive. Red arrow in y direction and orange arrow in x direction, these values are positive. If we utilize this, so area in z direction, this is A_z plus Δz by 2 into \hat{k} . This one is positive and positive z direction.

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Divergence Theorem

Area Vector

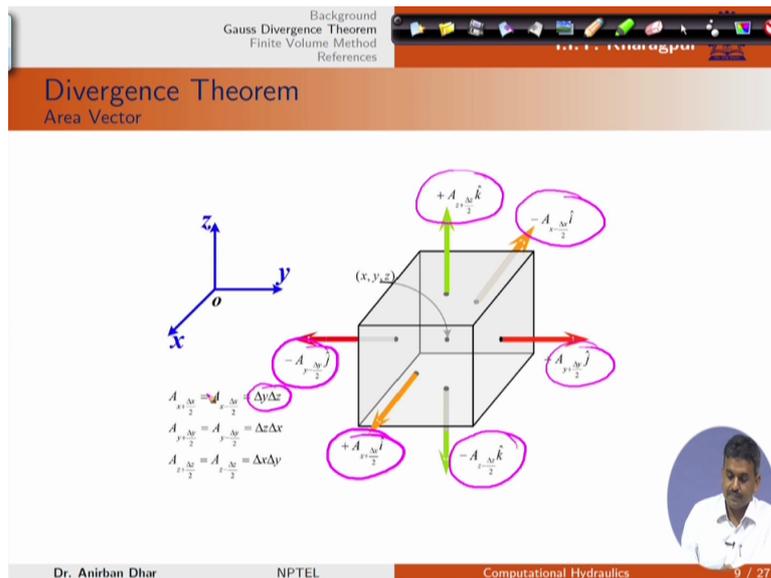
(x, y, z)

$A_{x+\frac{\Delta x}{2}} = A_{x-\frac{\Delta x}{2}} = \Delta y \Delta z$
 $A_{y+\frac{\Delta y}{2}} = A_{y-\frac{\Delta y}{2}} = \Delta z \Delta x$
 $A_{z+\frac{\Delta z}{2}} = A_{z-\frac{\Delta z}{2}} = \Delta x \Delta y$

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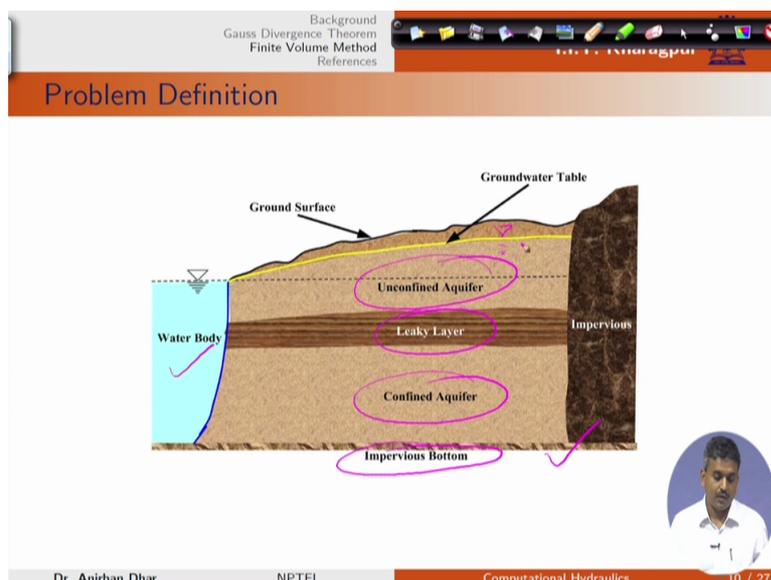
This is negative because it is pointing towards negative z direction. Again this is pointing towards positive y direction. This is pointing towards negative y direction. This is pointing towards positive x direction. This one is pointing towards negative x direction. Now individual values of x plus Δx by 2, this is equal to $z A_x$ minus Δx by 2. This is actually $\Delta y \Delta z$.

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Now whatever problem we have solved in our finite difference approach, we can also solve those problems in finite volumemethod. So let usconsider first our ordinary differential equations and boundary value problem with two boundary conditions. One side this is specified boundary, this side is an impermeable boundary condition. We have one leaky confining layer, one unconfined aquifer. Bottom we have confined aquifer and at the base we have impervious bottom. This is groundwater table position.

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So with this we can mathematically conceptualize the problem in terms of this differential equation. In our finite difference approach we have utilized this differential equation. But the basic differential form for a particular confined aquifer system, this will be $T \text{ del } h \text{ by del } x, d$

by dx. Now in this case other things are known. That is left hand side we have Dirichlet, right hand side we have zero Neumann. And H wt is represented in terms of C0, C1, C2, in nonlinear form.

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Mathematical Conceptualization

The differential equation describing the head distribution in the aquifer is given as ,

$$\frac{d}{dx} \left(T \frac{dh}{dx} \right) = C_{\text{conf}}(h - h_{wt}) \quad (2)$$

or,

$$\frac{d^2 h}{dx^2} = \frac{C_{\text{conf}}}{T} (h - h_{wt}) \quad (3)$$

where,
 h = head,
 T = aquifer transmissivity,
 C_{conf} = hydraulic conductivity/thickness of confining layer,
 h_{wt} = overlying water table elevation ($c_0 + c_1 x + c_2 x^2$).

Boundary Conditions

- Left Boundary is specified head/ Dirichlet boundary: $h(x=0) = h_a$
- Right Boundary is impervious/ no-flow/ Neumann Boundary: $\left. \frac{dh}{dx} \right|_L = 0$

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In case of finite difference we have discretized our domain in terms of some node points. And we have represented this nodes as $x_0, x_1, x_i - 1, x_i, x_i + 1, x_n - 1, x_n$. Thus in this case we have n number of segments available. N number of segments starting from 1 to n.

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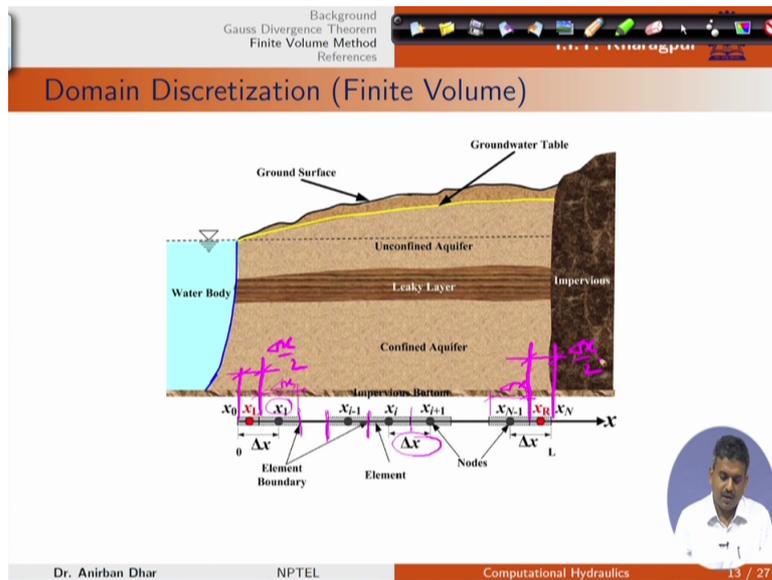
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Domain Discretization (Finite Difference)

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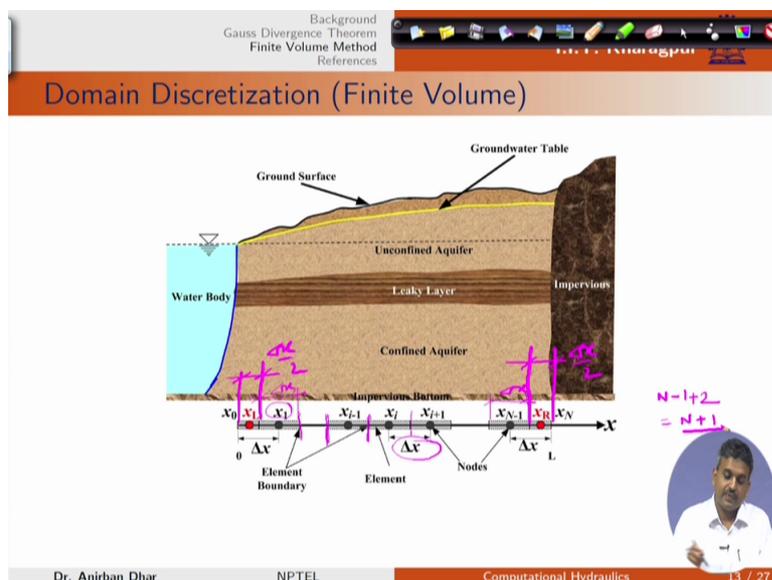
But in finite difference approach we divide this length 0 to L into number of elements. So first case, this is our x_i . x_i is defined at the center of the element. So this is actually Δx . Again x_{i-1} , this is with Δx length. And individually for x_i , x_{i-1} , x_{i+1} we have used the same discretization with elements. Now these individual lines, these are actually element boundaries. For each element we will have element boundary. For n point we have this Δx by 2 length extra. So for this Δx by 2, we need to define separate elements.

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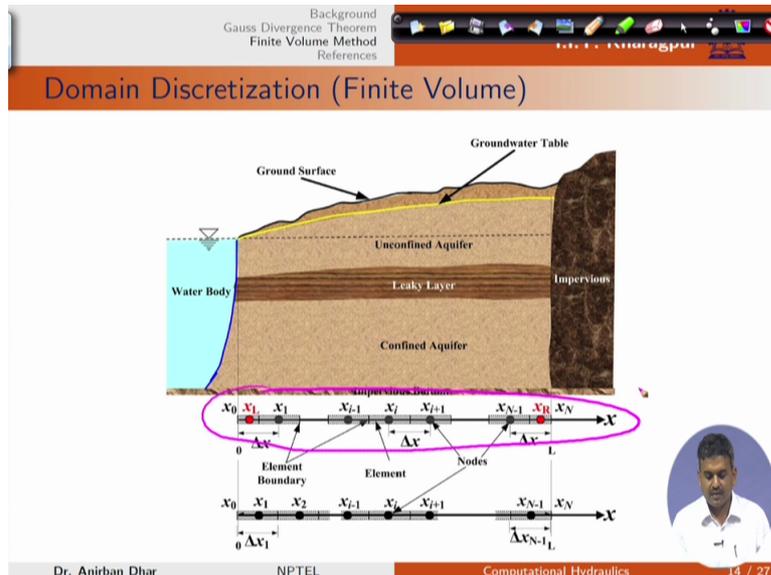
So for 1 to n minus 1, we have n minus 1 number of elements and for x_l and x_r , we have two extra elements. So all total we will have n plus 1 number of elements in this case.

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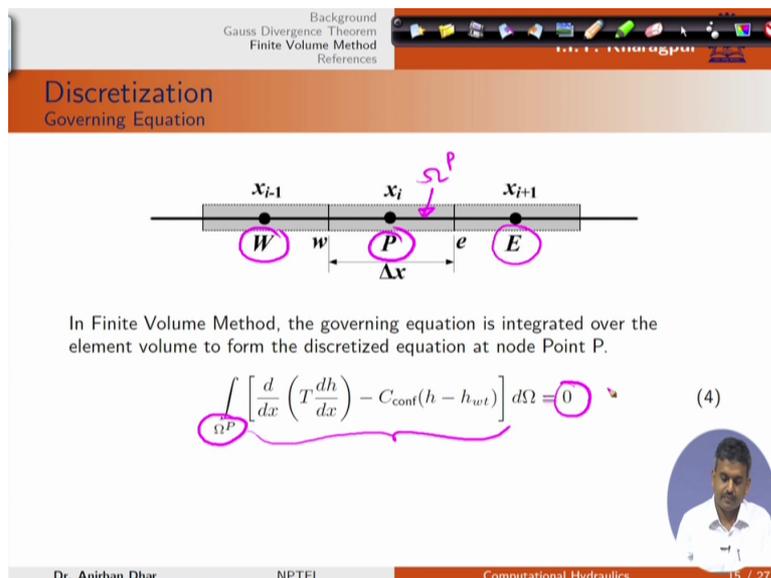
However we can utilize a different concept where we can divide this 0 to L domain into different elements of different sizes. Starting this cell center for the first element as x_1 . Now for discretization let us use this concept, first one. Because this is equivalent to our finite difference domain discretization.

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Let us discuss our governing equation. Center node or cell center for the middle element is P and cell center for left element, that is W and east side we have E . So for this one if we integrate our main governing equation with this limits, Ω_P . This Ω_P represent this central element. And we can equate this to 0.

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So if utilize this, then we can write this into two parts. First part is with derivatives and right part is equivalent to source sink term.

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Discretization Governing Equation

In Finite Volume Method, the governing equation is integrated over the element volume to form the discretized equation at node Point P.

$$\int_{\Omega^P} \left[\frac{d}{dx} \left(T \frac{dh}{dx} \right) - C_{\text{conf}}(h - h_{wt}) \right] d\Omega = 0 \quad (4)$$

or,

$$\int_{\Omega^P} \frac{d}{dx} \left(T \frac{dh}{dx} \right) d\Omega - C_{\text{conf}} \int_{\Omega^P} (h - h_{wt}) d\Omega = 0$$

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Now with this if we discretize the left hand derivative part using this Gauss divergence theorem, this is equivalent to area integral. And area integral with T dh by dx i. This is our vector. This vector dot n into dS. N is the outward normal for this element from our divergence to or volume integral to surface integral. In this case for n dS and i dot hat, we can write it with Ath component in x direction. Area component in x direction. So this integral can be written as summation where T dh by dx, this is evaluated at element boundary.

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Discretization Governing Equation

$$\begin{aligned} \int_{\Omega^P} \frac{d}{dx} \left(T \frac{dh}{dx} \right) d\Omega &= \int_{S^P} \left[\left(T \frac{dh}{dx} \right) \hat{i} \right] \cdot \hat{n} dS \\ &= \int_{S^P} \left(T \frac{dh}{dx} \right) dA_x \\ &= \left(T \frac{dh}{dx} \right)_e A_{xe} - \left(T \frac{dh}{dx} \right)_w A_{xw} \end{aligned} \quad (6)$$

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This is at P, this is element boundary on east side. This is at west side. So A_{xe} and A_{xw} , these are area values corresponding to this A_{xw} and A_{xe} .

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So with this in uniform grid system, this is equal to T_e . T evaluated at e and this is h_E minus h_P . This is forward derivative. This is again evaluated at W . At W we can write it as h_P minus h_W divided by Δx .

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With this derivatives and right hand side source sink term, that is actually $\Omega^P \frac{dh}{dx}$, this is equivalent to h_P into Δx into 1 into 1 which is the volume of this element. So we can

write this as h_P into Δx and the right hand side second term h_w , we can use this $C_0, C_1, C_2 x^2$. In this case x_w to x_e , this is the limit. So with this we can integrate it.

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Discretization
Governing Equation

$\int_{\Omega^P} h d\Omega = h_P (\Delta x)$

$$\int_{\Omega^P} (h - h_w) d\Omega = h_P \Delta x - \int_{x_w}^{x_e} (c_0 + c_1 x + c_2 x^2) dx$$

$$= h_P \Delta x - \left[c_0 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_2 x^3 \right]_{x_w}^{x_e}$$

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And in compact form this can be written as $T_e h_E$ minus h_P by Δx . $T_w h_P$ minus h_W by Δx . And this C_{conf} is multiplied with this one.

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Governing Equation

$$\int_{\Omega^P} (h - h_w) d\Omega = h_P \Delta x - \int_{x_w}^{x_e} (c_0 + c_1 x + c_2 x^2) dx$$

$$= h_P \Delta x - \left[c_0 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_2 x^3 \right]_{x_w}^{x_e}$$

Compact form of the equation

$$T_e \frac{h_E - h_P}{\Delta x} - T_w \frac{h_P - h_W}{\Delta x} = C_{conf} h_P \Delta x - C_{conf} \left[c_0 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_2 x^3 \right]_{x_w}^{x_e}$$

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So with this if we proceed with the approximation that T_e equals to T_w and x_e minus x_w which is Δx , we can write this that h_E minus $2 h_P$ plus h_W by Δx^2 , this is C_{conf} by $T h_P$ because in this case putting the integration limits we can approximate the right hand term.

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With $T_e = T_w = T$ and $x_e - x_w = \Delta x$,

$$\frac{h_E - 2h_P + h_W}{\Delta x^2} = \frac{C_{\text{cont}}}{T} \left[h_P - \left(c_0 + \frac{1}{2} c_1 (x_e + x_w) + \frac{1}{3} c_2 (x_e^2 + x_e x_w + x_w^2) \right) \right]$$

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In this case if we compare our equation with the finite difference one, so this is for ith one, this is P, E is equivalent to i plus 1 and W is equivalent to i minus 1. So left hand side is similar.

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$W \equiv i-1$
 $P \equiv i$
 $E \equiv i+1$

With $T_e = T_w = T$ and $x_e - x_w = \Delta x$,

$$\frac{h_E - 2h_P + h_W}{\Delta x^2} = \frac{C_{\text{cont}}}{T} \left[h_P - \left(c_0 + \frac{1}{2} c_1 (x_e + x_w) + \frac{1}{3} c_2 (x_e^2 + x_e x_w + x_w^2) \right) \right]$$


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So let us look into the right hand side. Right hand side 1 extra time is coming. This is extra if we approximate this. Because x_e is nothing but this x_P plus Δx by 2 and x_w is nothing but x_P minus Δx by 2. With this if we approximate then we will get 1 by $12 \Delta x^2$.

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Discretization Governing Equation

With $T_e = T_w = T$ and $x_e - x_w = \Delta x$,

$$\frac{h_E - 2h_P + h_W}{\Delta x^2} = \frac{C_{\text{conf}}}{T} \left[h_P - \left(c_0 + \frac{1}{2}c_1(x_e + x_w) + \frac{1}{3}c_2(x_e^2 + x_e x_w + x_w^2) \right) \right]$$

In simplified form,

$$\frac{h_E - 2h_P + h_W}{\Delta x^2} = \frac{C_{\text{conf}}}{T} \left[h_P - h_{\text{wt}}(x_P) \left(\frac{1}{12} \Delta x \right) \right]$$

$x_e = x_P + \frac{\Delta x}{2}$
 $x_w = x_P - \frac{\Delta x}{2}$



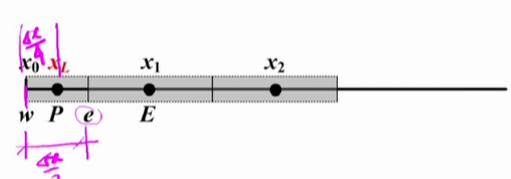
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Now for left boundary the situation is same but slightly different. In this case this length is Δx by 2. So x_L is situated at Δx by 4 distance. So distance between for this e face we have distance between e and P. This is actually $3 \Delta x$ by 4. And left hand side we are taking derivative using P and h_0 . h_0 is a specified value at this point.

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Discretization Left Boundary



$$\int_{\Omega^P} \frac{d}{dx} \left(T \frac{dh}{dx} \right) d\Omega = \left(T \frac{dh}{dx} \right)_e A_{xe} - \left(T \frac{dh}{dx} \right)_w A_{xw} \quad (8)$$

$$\left(T \frac{dh}{dx} \right)_e A_{xe} = T_e \frac{h_E - h_P}{3\Delta x/4}$$

$$\left(T \frac{dh}{dx} \right)_w A_{xw} = T_w \frac{h_P - h_0}{\Delta x/4}$$


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So with this if we approximate the thing for left hand boundary, we can get similar discretization. But h_0 is known. h_P and h_E are unknown. So we need to transfer this h_0 on the right hand side.

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Left Boundary

$$\int_{\Omega^P} (h - h_w) d\Omega = h_P \frac{\Delta x}{2} - \int_{x_w}^{x_e} (c_0 + c_1 x + c_2 x^2) dx$$

$$= h_P \frac{\Delta x}{2} - \left[c_0 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_2 x^3 \right]_{x_w}^{x_e}$$

Compact form of the equation

$$T_e \frac{h_E - h_P}{3\Delta x/4} - T_w \frac{h_P - h_0}{\Delta x/4} = C_{\text{conf}} h_P \frac{\Delta x}{2} - C_{\text{conf}} \left[c_0 x + \frac{1}{2} c_1 x^2 + \frac{1}{3} c_2 x^3 \right]_{x_w}^{x_e}$$

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So final equation we will get like this. In terms of h_E and h_P on the left hand side and these values are known on the right hand side.

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Discretization
Left Boundary

With $T_e = T_w = T$ and $x_e - x_w = \Delta x/2$,

$$\frac{4h_E - 16h_P}{3\Delta x} = -\frac{4h_0}{\Delta x} + \frac{\Delta x}{2} \frac{C_{\text{conf}}}{T} \left[h_P - \left(c_0 + \frac{1}{2} c_1 (x_e + x_w) + \frac{1}{3} c_2 (x_e^2 + x_e x_w + x_w^2) \right) \right]$$

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So in simplified form this is the thing. But in this case $1/48 \Delta x^2$ is the extra term.

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Discretization

Left Boundary

With $T_e = T_w = T$ and $x_e - x_w = \Delta x/2$,

$$\frac{4h_E - 16h_P}{3\Delta x} = -\frac{4h_0}{\Delta x} + \frac{\Delta x}{2} \frac{C_{\text{conf}}}{T} \left[h_P - \left(c_0 + \frac{1}{2}c_1(x_e + x_w) + \frac{1}{3}c_2(x_e^2 + x_e x_w + x_w^2) \right) \right]$$

In simplified form,

$$\frac{8h_E - 32h_P}{3\Delta x^2} = -\frac{8h_0}{\Delta x^2} + \frac{C_{\text{conf}}}{T} \left[h_P - h_{wt}(x_P) - \frac{1}{48}\Delta x^2 \right]$$

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Now if we utilize the same concept for the right boundary, so we will have only x_e term on the east face. And on the west face, we're getting this one will be zero for the west face. And W term will be there. So we can write this in terms of h_P and h_W .

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Right Boundary

$$\int_{\Omega^P} \frac{d}{dx} \left(T \frac{dh}{dx} \right) d\Omega = \left(T \frac{dh}{dx} \right)_e A_{xe} - \left(T \frac{dh}{dx} \right)_w A_{xw} \quad (10)$$

$$\left(T \frac{dh}{dx} \right)_e A_{xe} = 0 \quad (11)$$

$$\left(T \frac{dh}{dx} \right)_w A_{xw} = T_w \frac{(h_P - h_W)}{3\Delta x/4}$$

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So in this case we can again approximate this and for the east right boundary, we can again get the simplified equation in terms of h_W and h_P . So with the interior equation, left boundary, and right boundary, we can solve the actual problem. This is similar to the finite difference technique.

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Discretization Right Boundary

With $T_e = T_w = T$ and $x_e - x_w = \Delta x/2$,

$$-\frac{4h_P - 4h_w}{3\Delta x} = \frac{\Delta x}{2} \frac{C_{\text{conf}}}{T} \left[h_P - \left(c_0 + \frac{1}{2}c_1(x_e + x_w) + \frac{1}{3}c_2(x_e^2 + x_e x_w + x_w^2) \right) \right]$$

In simplified form,

$$\frac{8h_w - 8h_P}{3\Delta x^2} = \frac{C_{\text{conf}}}{T} \left[h_P - h_{wt}(x_P) - \frac{1}{48}\Delta x^2 \right]$$

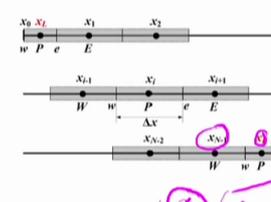

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But we need to determine the values at the end points, x_0 and x_N . We can use our Neumann boundary condition and we can involve this x_r point, x_n and x_n minus 1. And we can relate this x_n , x_r , x_n h_N minus 1 like this. So in this case we can get the h_N value from this equation. So we have got all the values. At point x_0 this is defined. At point x_n we can get from this equation. For interior points starting from x_L to x_r , we can use the boundary condition and interior governing equations.

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Discretization Neumann Boundary



Neumann Boundary

$$\frac{dh}{dx} \Big|_{x=L} = 0$$

$$\frac{15h_N - 16h_P + h_{N-1}}{3\Delta x} = 0$$

$h_N = ?$



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So now with this we can solve the problem. Thank you.