Computational Hydraulics
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Lecture 13
Finite Volume Method: Overview

Welcome to this lecture number 13 of the coursecomputational hydraulics. We are in module 2, numerical methods. And in this particular lecture I will be covering unit 9, finite volume method -overview.

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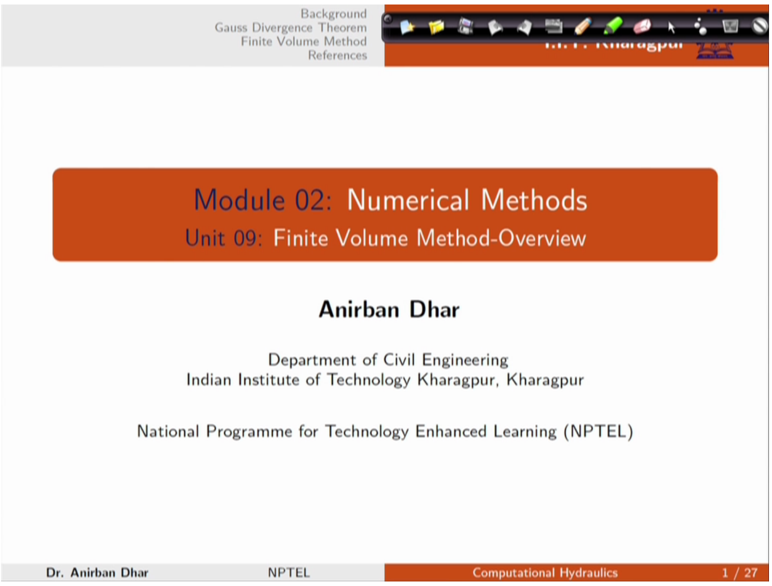

Learning objective for this particular unit. At the end of this unit students will be able to discretize the derivatives of single valued one-dimensional functions using finite volume method. And they will be able to derive the algebraic form using discretized ordinary differential equations and boundary conditions.

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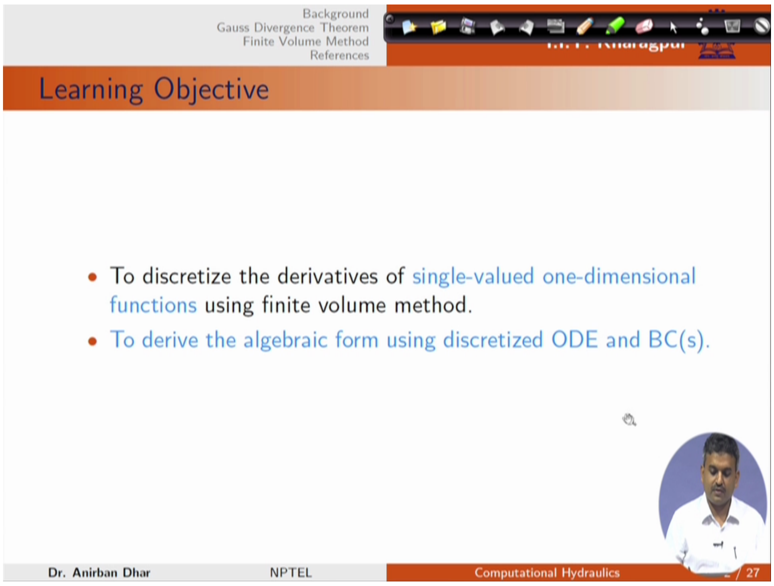

Let us consider our general equation in terms of phi where phi is a general variable in terms of x, y, z and t. The first term is temporal term, second term is advection relative term and first term in the right hand side this is related to diffusion. F is relatedto other forces and S phi is related to source sink term. And this lambda phiand upsilon phi, these are problem dependent parameters. And gamma phi is the tensor or coefficient tensor for the right hand side.

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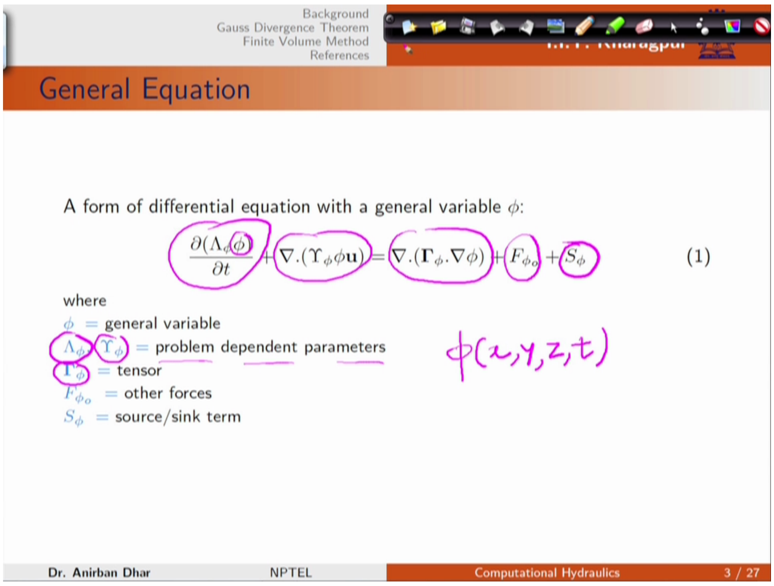

Now this is a general equation. We can write this general equation as R, where this bold x represents x, y, z directions. In the method of weighted residual, the residual R can be written asthis one. This is our left hand side of the equation. With minus sign we can include the right hand side. Now with this if we write this weighted integral of this residual and we equate this to zero for all wl corresponding to a particular L, we can write this where this omega represents the domain.

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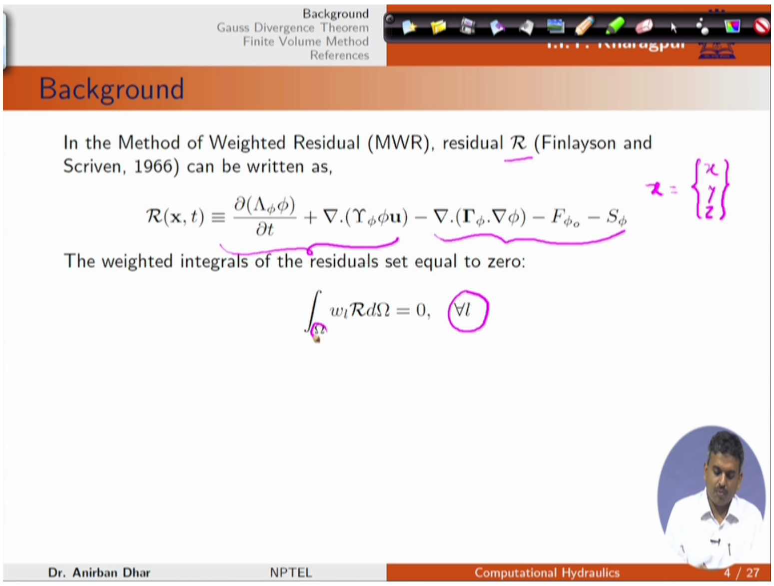

Now with this if we approximate this wl or weight function as Dirac delta, that means that x is equal to L, this is 1 Dirac delta equals to 1. Otherwise this is zero. So with this weight function in collocation method, if we apply this Dirac delta instead of wl, we can write this one as R xl t 0. That means this integral can be approximated as R residual xlt. That means if we evaluate the residual at point L, then and we equate it to zero then we can get the approximation in terms of method of weighted residual.

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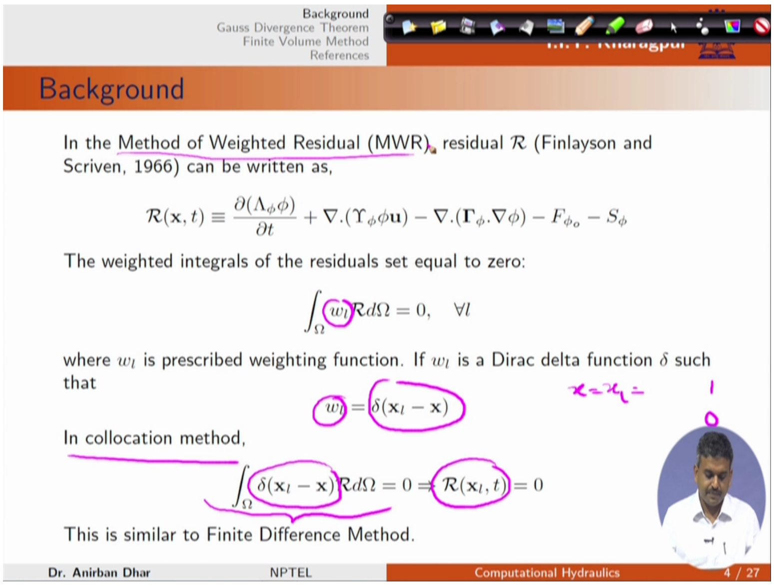

Interestinglyin our finite difference method we have used similar concept. For a particular location L we have used Taylor series expansion to approximate the derivatives with the help of neighboring points. So this is similar to our finite difference method.

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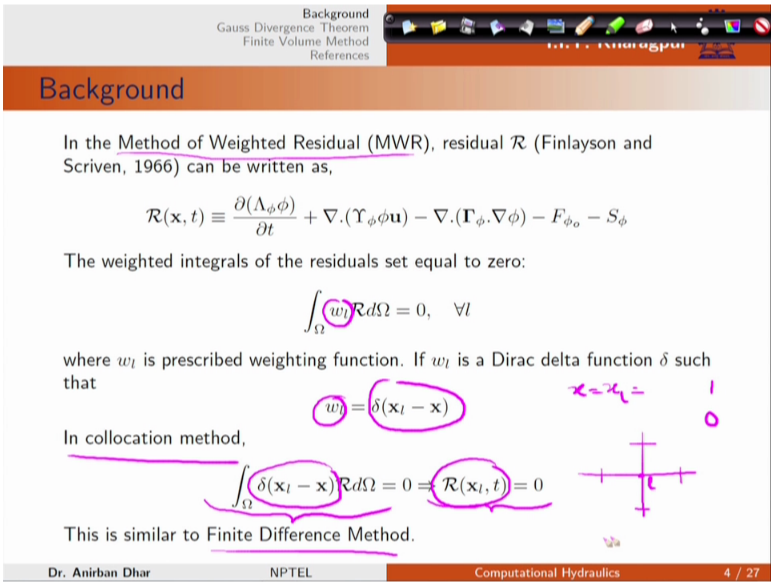

Now with this if we apply this wl such that for a particular subdomain omega L, omega is the domain and omega L is the subdomain. For that subdomain we have weight function 1. Outside that subdomain our weight function is zero.

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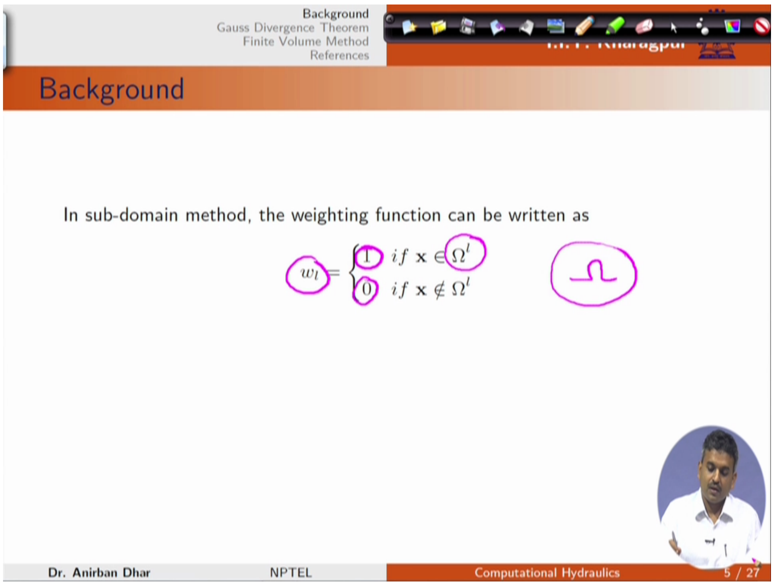

So with this information if you proceed and write our method of weighted residual, so we can write it as this, where wl for small domain or subdomain we have this equal to zero. And this is similar to finite volume method. Infinite volume method our approach is to reduce the elemental error for a particular problem with given differential equation. R is in terms of given differential equation and this limit is for subdomain. So with this we can start our finite volume method.

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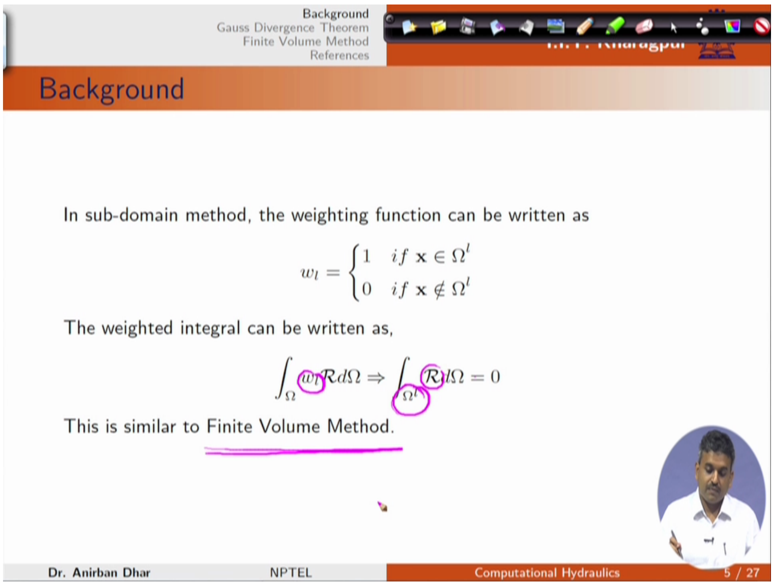

Now we need to define this Gauss divergence theorem. As per Gauss divergence theorem, suppose this omega is the volume bounded by a closed surface S. And A vector field defined inthis omega and on S.If S is piecewise smooth with outward normal. This is unit normal and A vector is continuously differentiable then with triple integral which is actually volume integral over whole subdomain. This is our domain.

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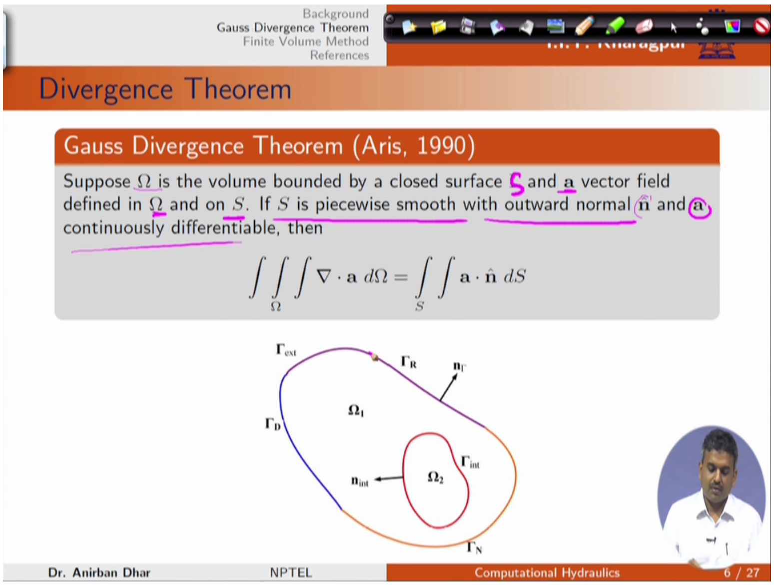

For individual subdomain, if we apply this we can write this one in terms of surface integral where this del dot A will be transferred to A dot n dS. So in this case we are transforming the volume integral to surface integral. And n is outward normal, A is any arbitrary vector and this is our del operator.

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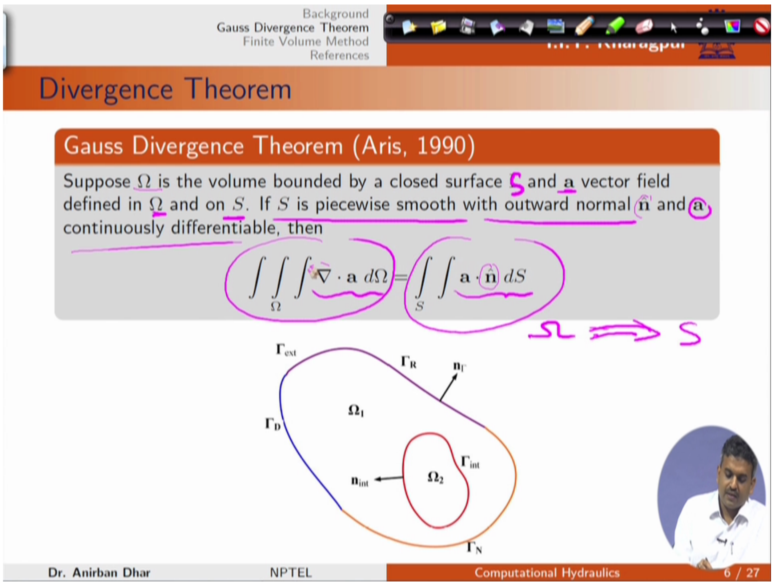

Now let us utilize this concept for finite volume formulation. In divergence theorem we have talked about the surface. If you consider a particular small elemental volume where this is our delta x size. This size is delta y and vertical side is delta z. Then we can write individual components for this particular domain and surface.

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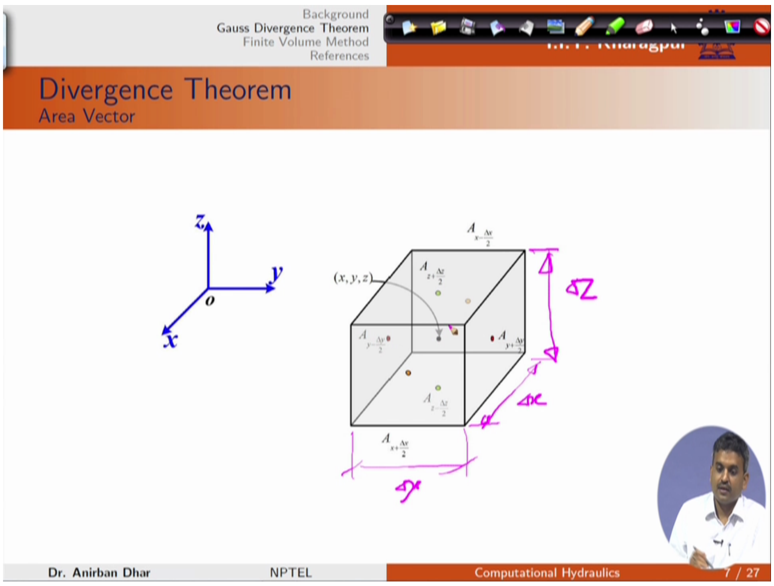

In this case basic assumption is that area is having direction and direction of area is always outward positive. So outward is z direction with green arrow. These are positive. Red arrow in y direction and orange arrow in x direction, these values are positive. If we utilize this,so area in z direction, this is A z plus del z by 2 into k hat. This one is positive and positive z direction.

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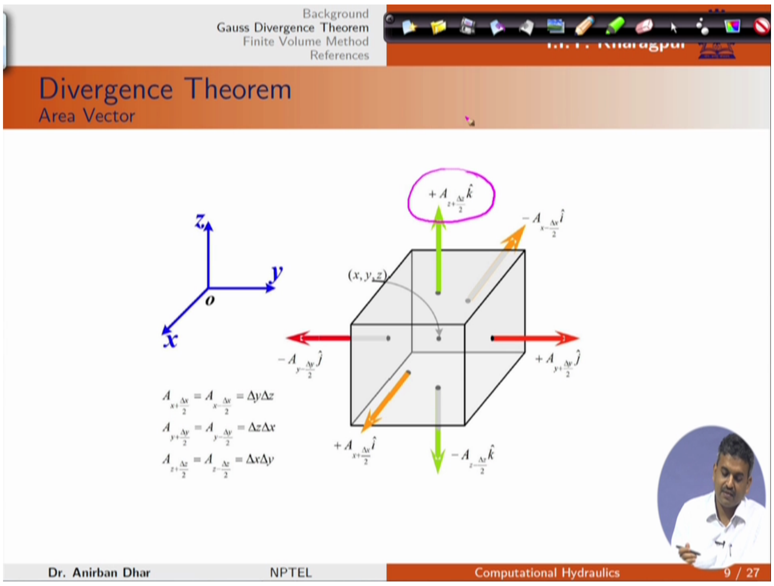

This is negative because it is pointing towards negative z direction. Again this is pointing towards positive y direction. This is pointing towards negative y direction. This is pointing towards positive x direction. This one is pointing towards negative x direction. Now individual values of x plus delta x by 2, this is equal to z A x minus delta x by 2. This is actually del y del z.

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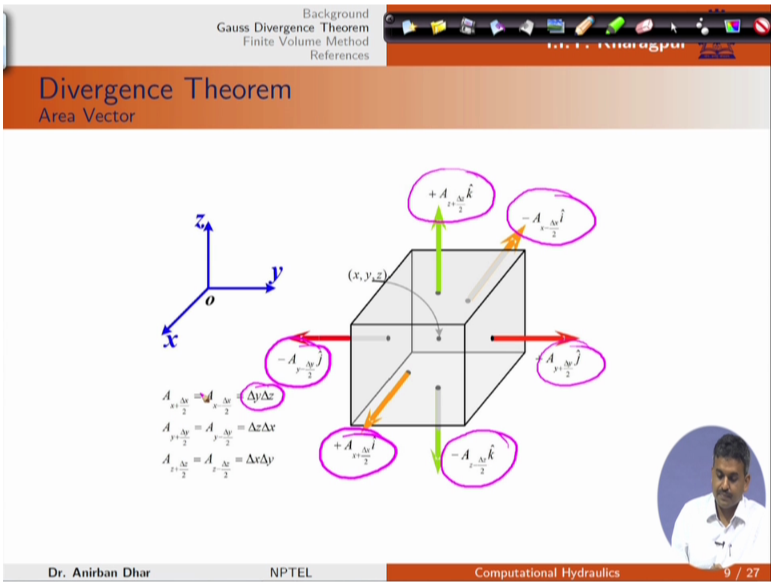

Now whatever problem we have solved in our finite difference approach, we can also solve those problems in finite volumemethod. So let usconsider first our ordinary differential equations and boundary value problem with two boundary conditions. One side this is specified boundary, this side is an impermeable boundary condition. We have one leaky confining layer, one unconfined aquifer. Bottom we have confined aquifer and at the base we have impervious bottom. This is groundwater table position.

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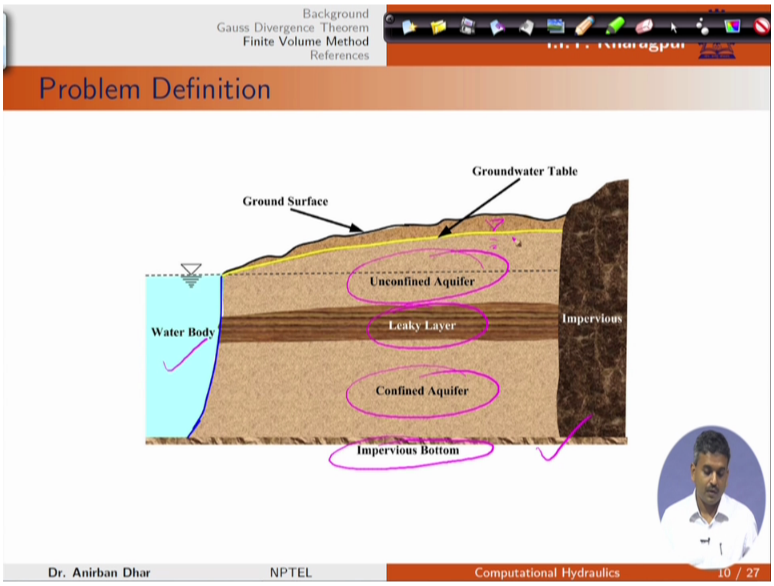

So with this we can mathematically conceptualize the problem in terms of this differential equation. In our finite difference approach we have utilized this differential equation. But the basic differential form for a particular confined aquifer system, this will be T del h by del x, d by dx. Now in this case other things are known. That is left hand side we have Dirichlet,right hand side we have zero Neumann. And H wt is represented in terms of C0, C1, C2, in nonlinear form.

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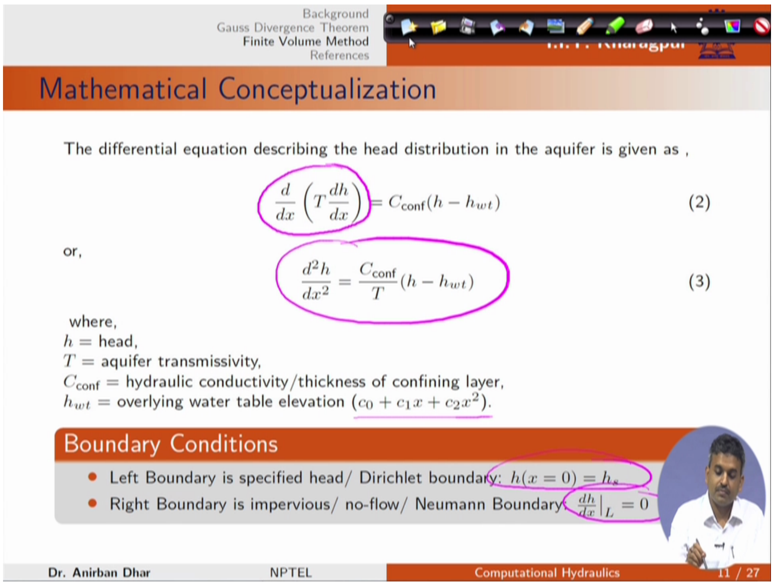

In case of finite difference we have discretized our domain in terms of some node points. And we have represented this nodes as x0, x1, xi minus 1, xi, xi plus 1, xn minus 1, xn. Thus in this case we have n number of segments available. N number of segments starting from 1 to n.

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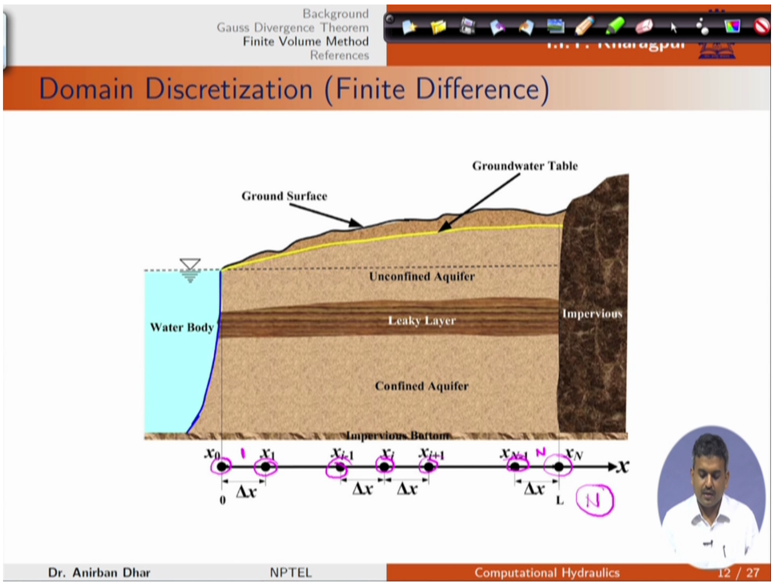

But in finite difference approach we divide this length 0 to L into number of elements. So first case, this is our xi. Xi is defined at the center of the element. So this is actually del x. Again xn minus 1, this is with del x length. And individually for xi, xi minus 1, xi plus 1 we have used the same discretization with elements. Now these individual lines, these are actually element boundaries. For each element we will have element boundary. For n point we have this del x by 2 length extra. So for this del x by 2, we need to define separate elements.

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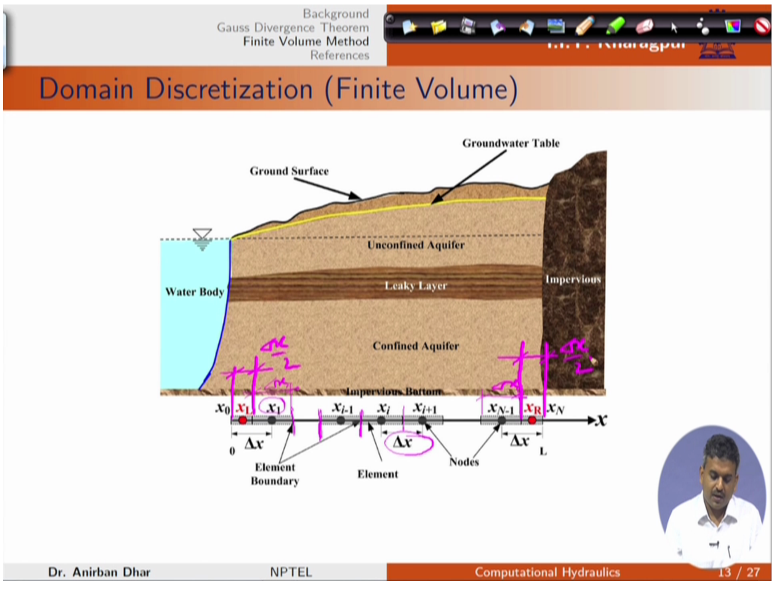

So for 1 to n minus 1, we have n minus 1 number of elements and for xl and xr, we have two extra elements. So all total we will have n plus 1 number of elements in this case.

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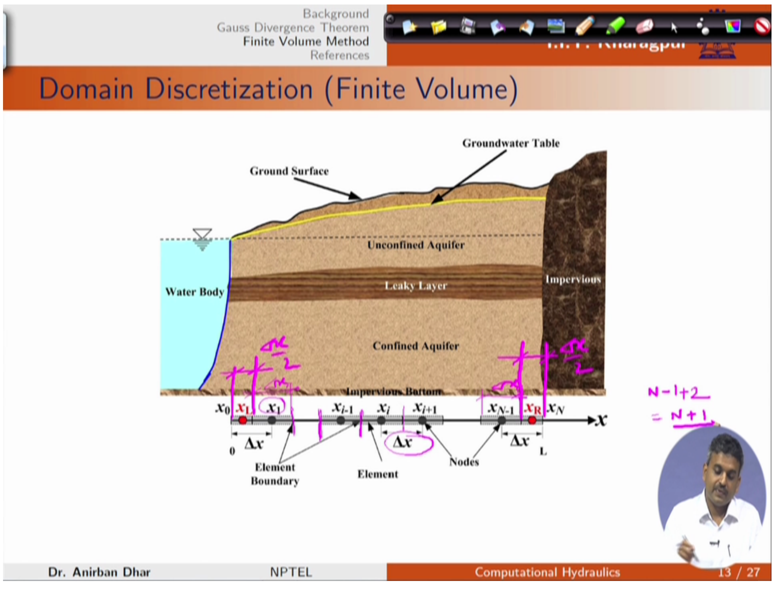

However we can utilize a different concept where we can divide this 0 to L domain into different elements of different sizes. Starting this cell center for the first element as x1. Now for discretization let us use this concept, first one. Because this is equivalent to our finite difference domain discretization.

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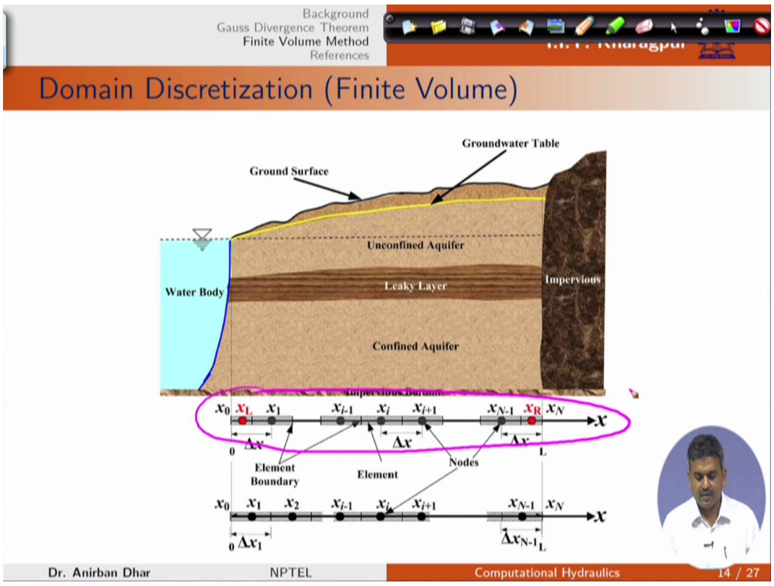

Let us discuss our governing equation. Center node or cell center for the middle element is P and cell center forleft element,that is W and east side we have E. So for this one if we integrate our main governing equation with this limits, omega P. This omega P represent this central element. And we can equate this to 0.

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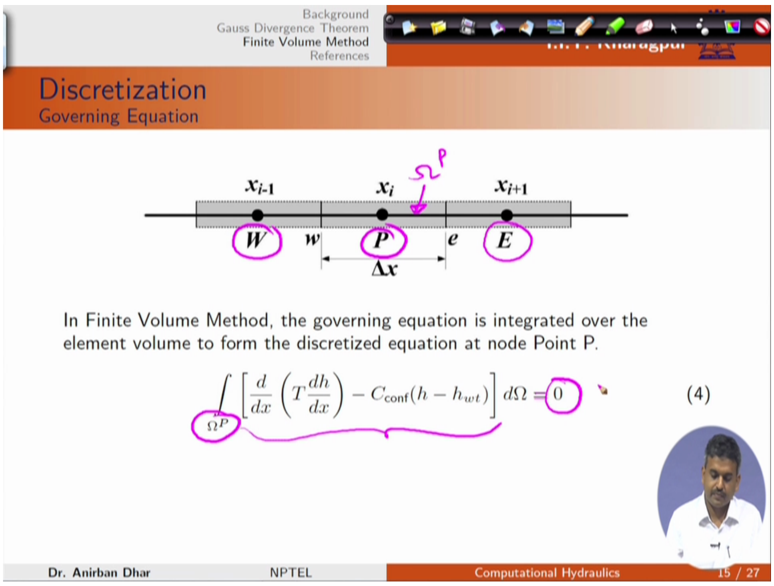

So if utilize this, then we can write this into two parts. First part is with derivatives and right part is equivalent to source sink term.

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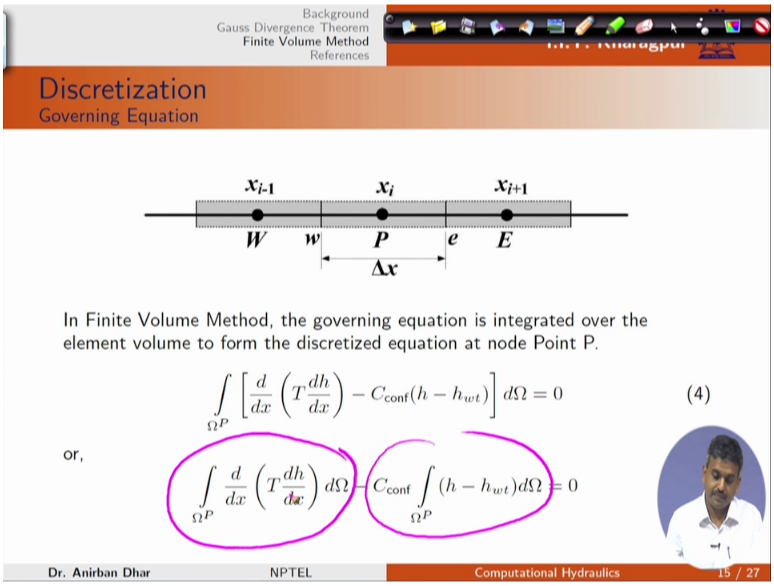

Now with this if we discretize the left hand derivative part using this Gauss divergence theorem, this is equivalent to area integral. And area integral with T dh by dx i. This is our vector. This vector dot n into dS. N is the outward normal for this element from our divergence to or volume integral to surface integral. In this case for n dS and i dot hat, we can write it with Ath component in x direction. Area component in x direction. So this integral can be written as summation where T dh by dx, this is evaluated at element boundary.

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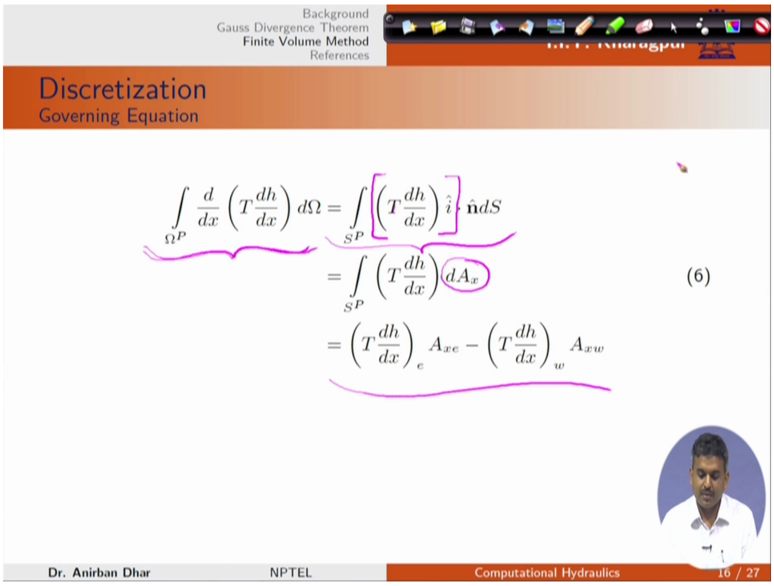

This is at P,this is element boundary on east side. This is at west side. So A xe and A xw, these are area values corresponding to this A xw and A xe.

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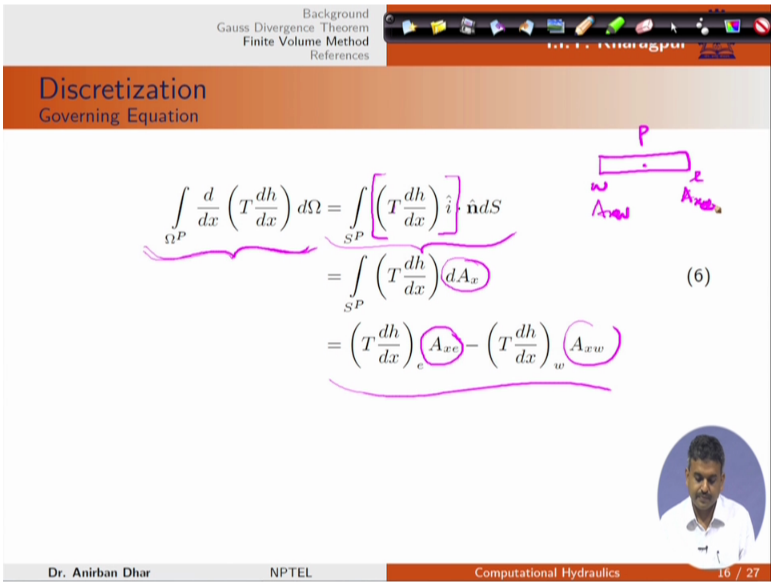

So with this in uniform grid system, this is equal to Te. T evaluated at e and this is hE minus hP. This is forward derivative. This is again evaluated at W. At W we can write it as hP minus hW divided by del x.

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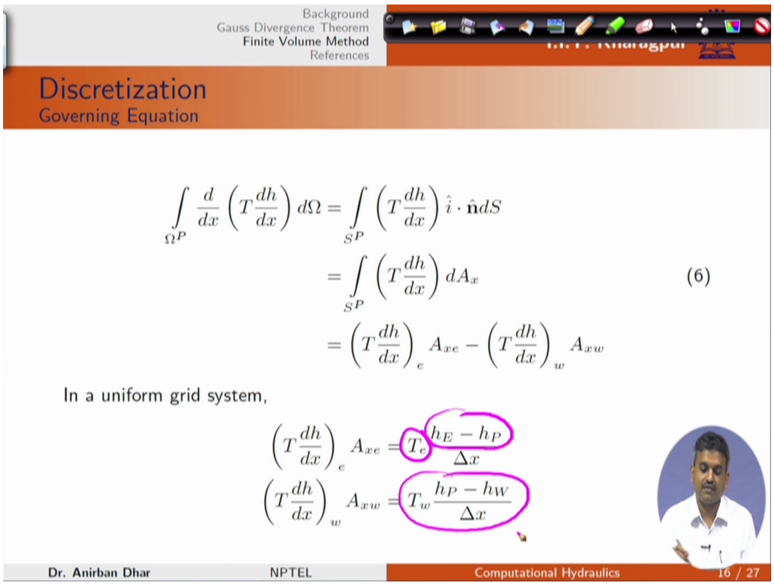

With this derivatives and right hand side source sink term, that is actually omega P hd omega, this is equivalent to hP into del x 1 into 1 which is the volume of this element. So we can write this as hP into del x and the right hand side second term h wt, we can use this C0,C1, C2x squaredx. In this case xw to xe, this is the limit. So with thiswe can integrate it.

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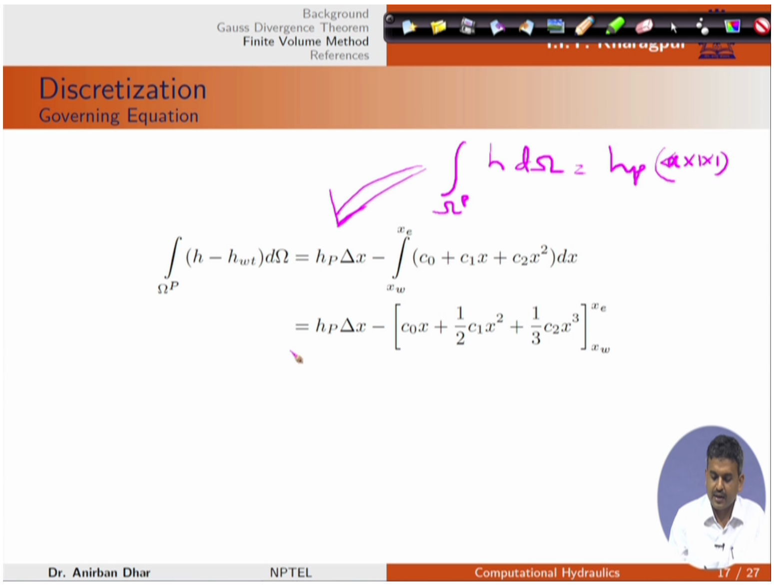

And in compact form this can be written as Te hE minus hP by del x. Tw hP minus hW by del x. And this Cconf is multiplied with this one.

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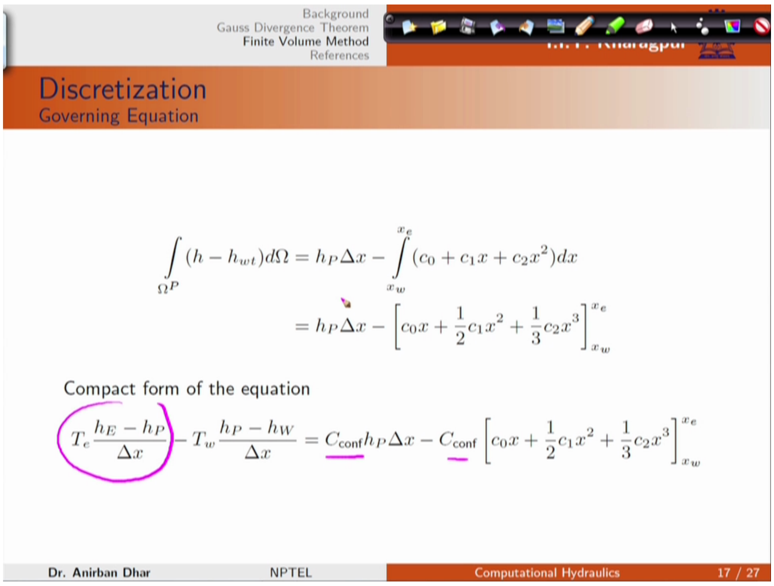

So with this if we proceedwith the approximation that Te equals to Tw T and xe minus xw which is del x, we can write this that hE minus 2 hP plus hW by del x square, this is Cconf by T hP because in this caseputting the integration limits we can approximate the right hand term.

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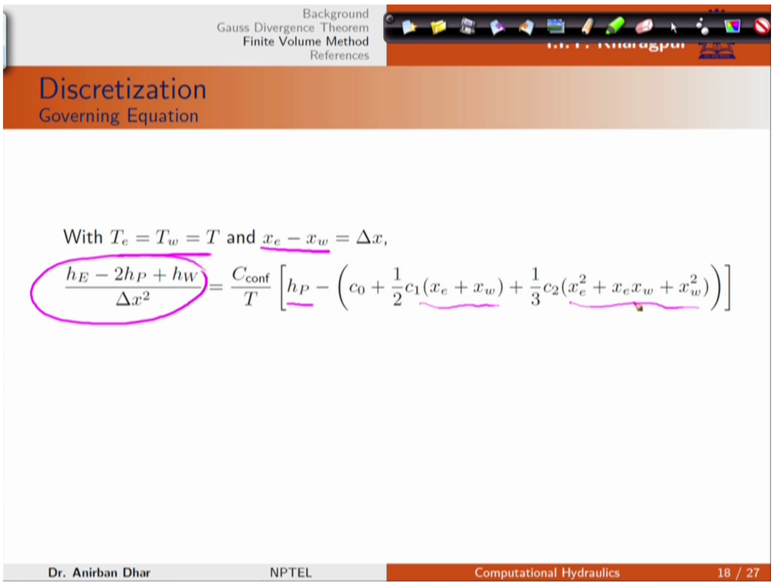

In this case if we compare our equation with the finite difference one, so this is for ith one, this is P, E is equivalent to i plus 1 and W is equivalent to i minus 1. So left hand side is similar.

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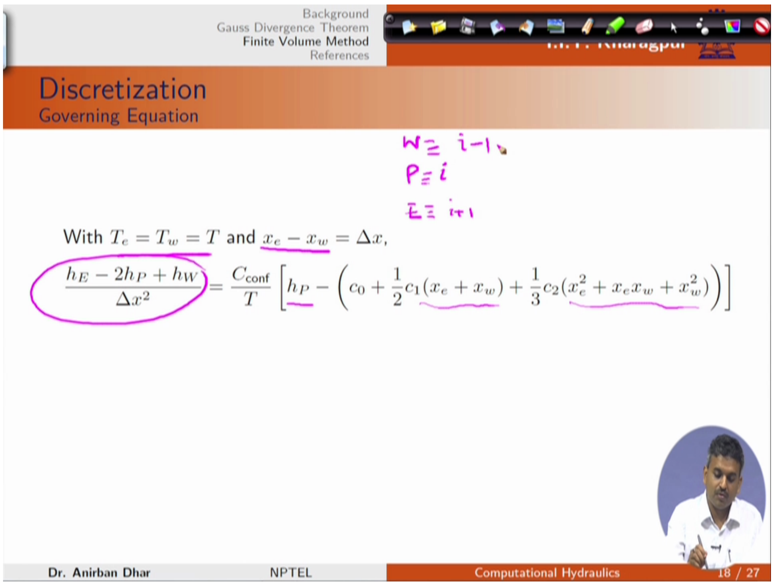

So let us look into the right hand side. Right hand side 1 extra time is coming. This is extra if we approximate this. Because xe is nothing but this xp plus del x by 2 and xw is nothing but xp minus del x by 2. With this if we approximate then we will get 1 by 12 del x square.

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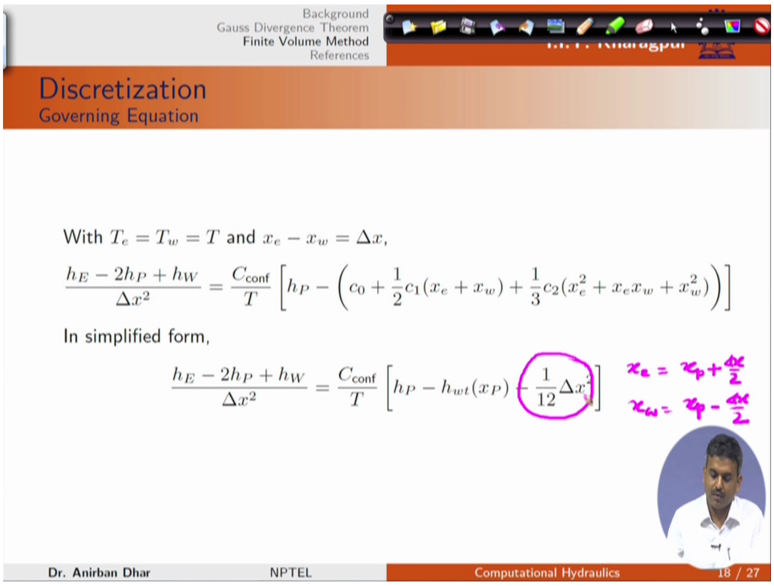

Now for left boundary the situation is same but slightly different. In this case this length is del x by 2. So xL is situated at del x by 4 distance. So distance between for this e face we have distance between e and P. This is actually 3 del x by 4. And left hand side we are taking derivative using P and h0. H0 is a specified value at this point.

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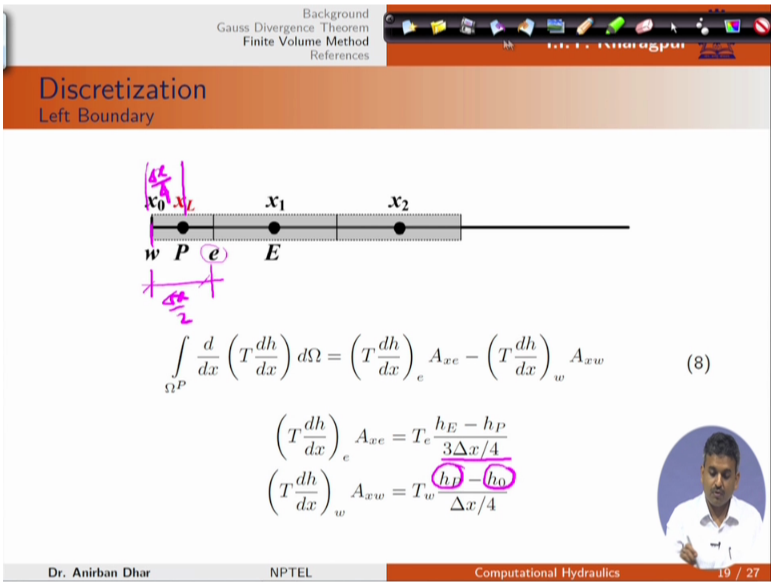

So with this if we approximate the thing for left hand boundary, we can get similar discretization. But h0 is known. HP and hE are unknown. So we need to transfer this h0on the right hand side.

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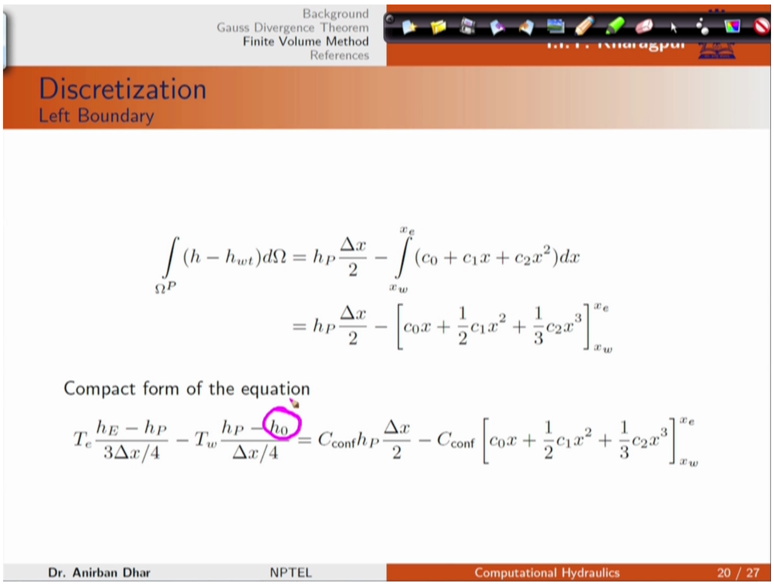

So final equation we will get like this. In terms of hE and hP on the left hand side and these values are known on the right hand side.

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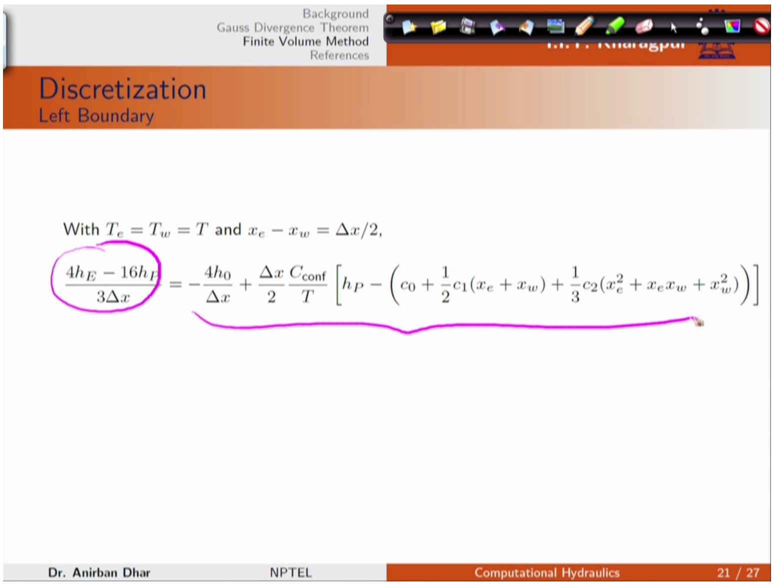

So in simplified form this is the thing. But in this case 1 by 48 del x square is the extra term.

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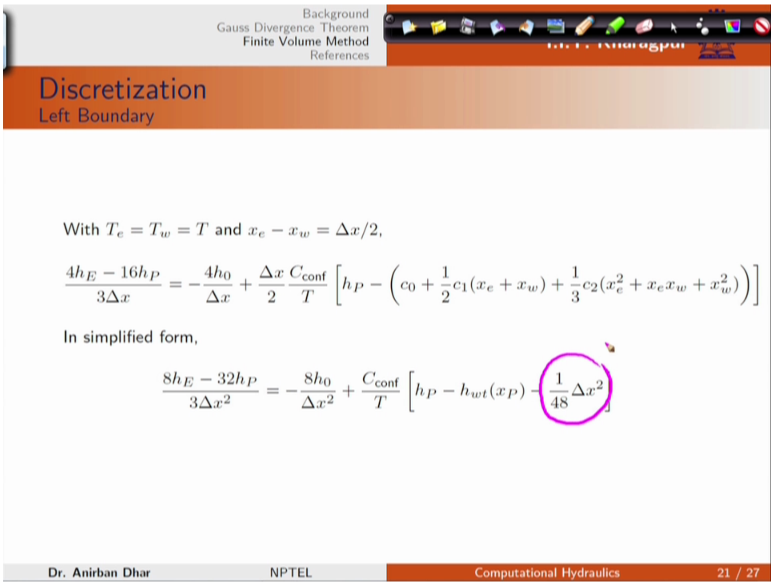

Now if weutilize the same concept for the right boundary, so we will have only xe term e face. And e face, we're getting this one will be zero for east face. And W term will be there. So we can write this in terms of hP and hW.

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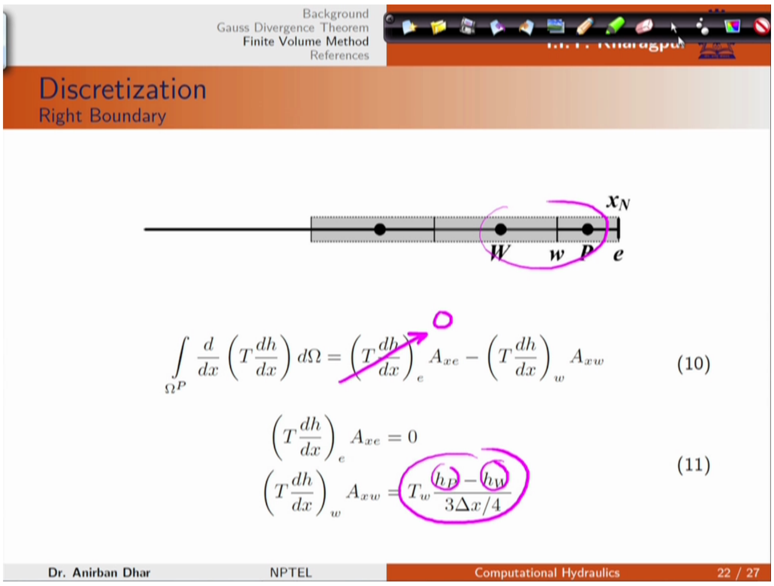

So in this case we can again approximate this and for east right boundary, we can again get the simplified equation in terms of hW and hP. So with the interior equation left boundary and right boundary we can solve the actual problem. This is similar to finite difference technique.

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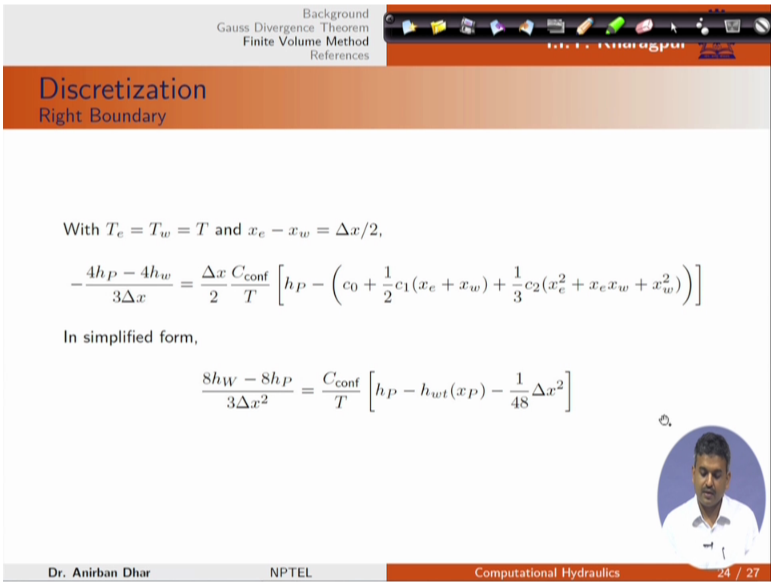

But we need to determine the values at the end points, x0 and xN. We can use our Neumann boundary condition and we can involve this xr point, xn and xn minus 1. And we can relate this xn, xr, xn hN minus 1 like this. So in this case we can get the hN value from this equation. So we have got all the values.X At point x0 this is defined. At point xn we can get from this equation. For interior points starting from xL to xr, we can use the boundary condition and interior governing equations.

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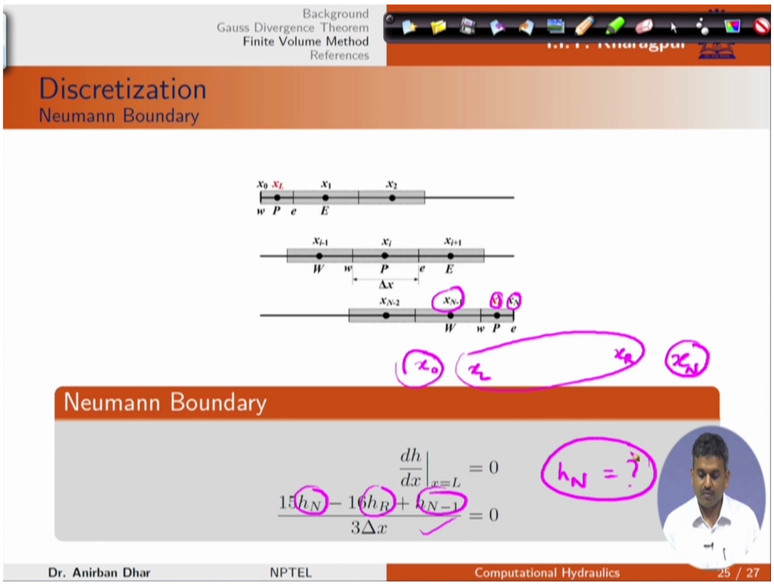

So now with this we can solve the problem. Thank you.