Design of Reinforced Concrete Structures
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Lecture - 07
Limit State of Collapse Flexure II

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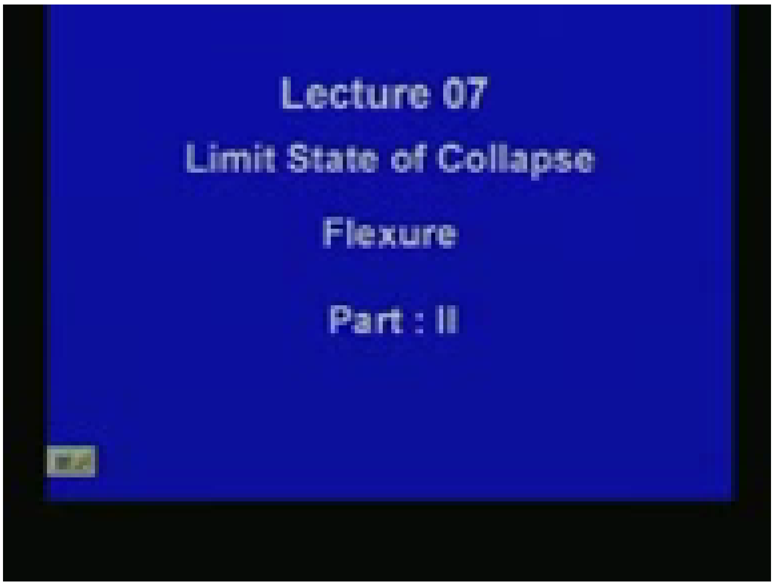

Today, we shall continue limit state of collapse flexure part 2. So, that is our lecture 07 limit state of collapse flexure part 2.

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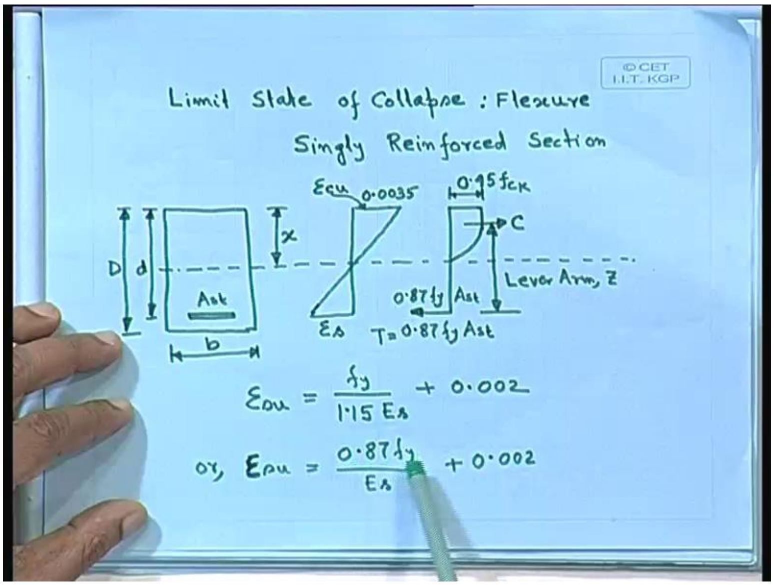

Last class, we have done started we have started limit state of collapse flexure singly reinforced section that where reinforcement in the tension side stress-strain curve the strain diagram and the corresponding stress diagram. The ultimate strain in steel it is dependent on fy we shall get epsilon su equal to 0.87fy taking 1.15 as the partial safety factor for materials. So, 0.87fy by Es plus 002.

We have also done the problem if we know I shall show you that expression of resistance moment for a balanced section. In terms of fy and p we have derived the equations in the last class.

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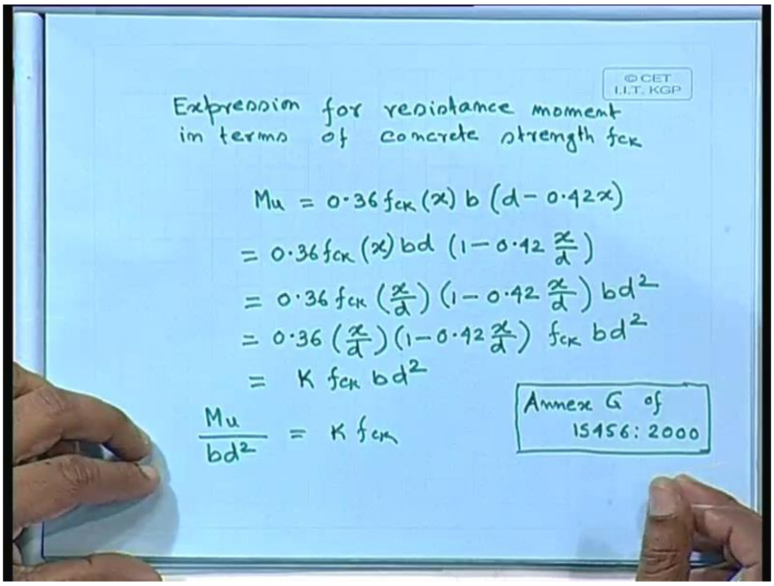

Today, we shall start expression for resistance moment in terms of concrete strength fck Mu equal to 0.36fck times x times. This is the compressive force multiplied by d minus let us put it 0.42 x we can put it 0.41 x we can also find out point four x also in different books. But, we shall take let us take here 0.42 x 0.36fck times x multiplied by bd times 1 minus 0.42 x by d.

We can further write down, 0.36fck x by d times 1 minus 0.42 x by d times bd square. So, here bd another d is coming from here x by d. So, we can get this is 1 expression we can consider or we can we can write down in this fashion also 0.36 x by d multiplied by 1 minus 0.42 x by d times fck bd square.

This is 1 term 1 36 x by d times 1 minus 0.42 x by d fck bd square, for different grade of concrete we shall get this is dependent on this fck. We can also write down say K fck bd square. Further we can write down Mu by bd square equal to k fck we shall get Mu by bd square equal to K fck we can refer annex G of IS 456 2000. So, Mu by bd square k fck where k is nothing, but 0.36 x by d 1 minus 0.42 x by d.

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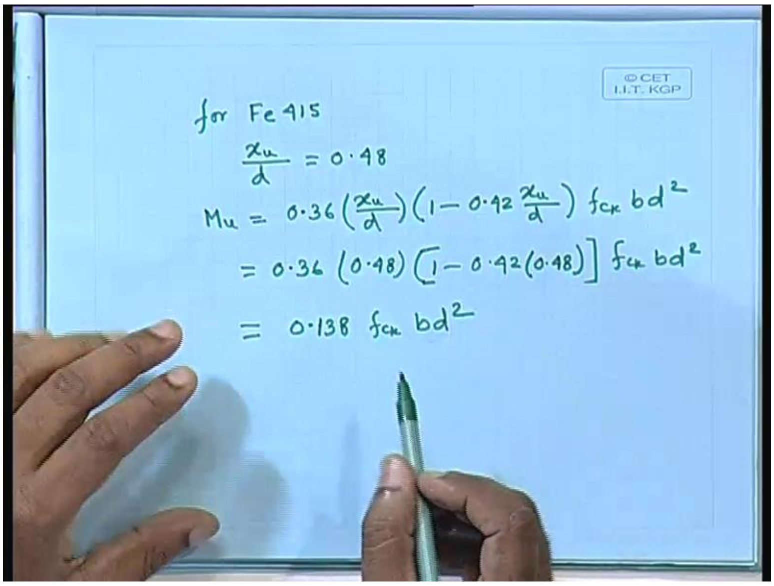

So, let continue for Fe 415 xu by that is the limiting value that we have seen equal to 0.48. Therefore, Mu equal to 0.36 xu by d multiplied by 1 minus zero 0.42 xu by d times fck bd square. Which comes as 0.36 multiplied by 0.48 multiplied by 1 minus 0.42 times let me put it this way 0.48fck bd square, which results 0.138fck bd square. So, for concrete grade for M 20 we can multiply 20 we shall get the resistance for concrete for a particular grade section.

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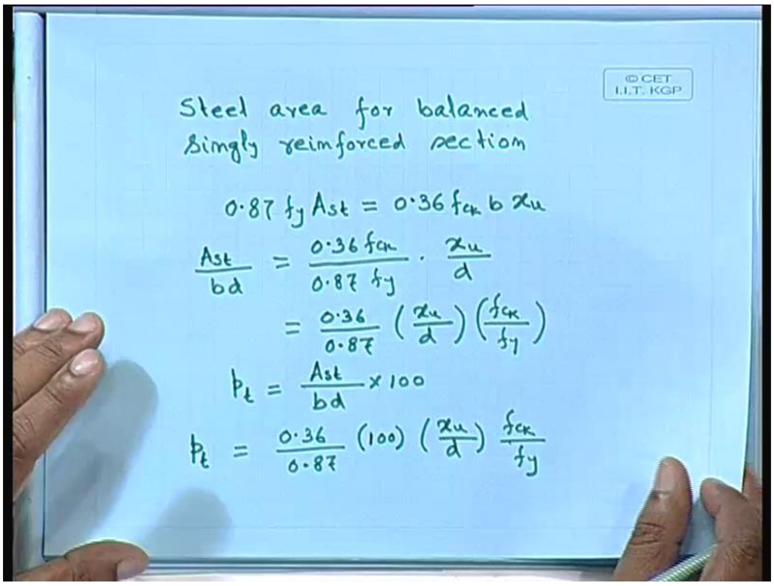

Next, we shall come steel area for balanced singly reinforced section.

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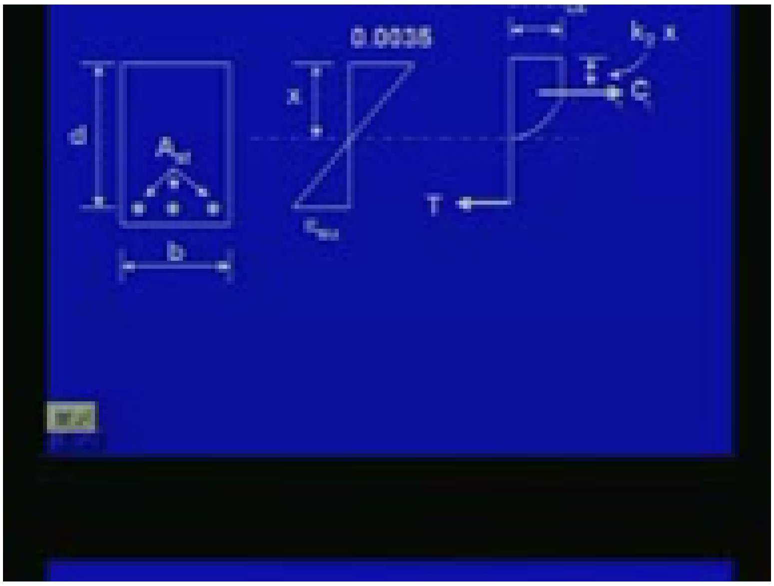

Here, we shall take we always refer we can refer say c and t and this your that lever arm. Lever arm here and this is K 2 x that where from the top compressive fiber that c is acting. For the balanced section we shall get yield strength in the steel that will be 0.87fy. And we have also reached the 0.0035 the strength and the corresponding that stress block.

We shall write down 0.87fy Ast, this is the tensile force 0.87fy times the area of the steel that is your tensile force which will b, equal to compressive force. Since, it has to be in equilibrium 0.36fck b time's xu that is the limiting neutral axis depth. Therefore, Ast by bd we can write down as 0.36fck by 0.87fy multiplied by xu by d. For further we can write down as 0.36 by 0.87 xu by d this is the limiting xu by d because you're considering here, we are taking balanced section multiplied by fck by f y.

Let us say pt equal to that is percentage of steel in tensile zone pt equal to Ast by bd multiplied by 100. We can write down pt will be equal to 0.36 by 0.87 multiplied by … 100 pt equal to 0.36 by 0.87 multiplied by 100 times xu by d fck by fy. We can write down the percentage of steel for balanced section.

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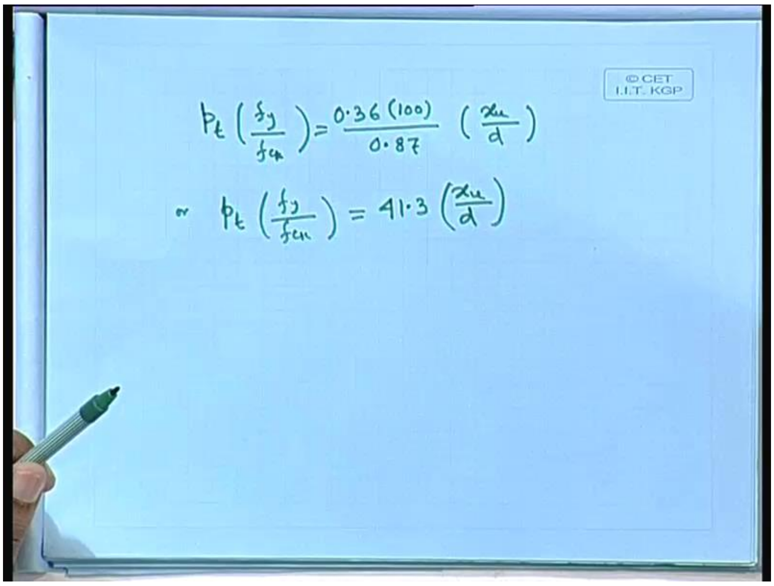

Therefore, we can further write down in this fashion pt fy by fck equal to 0.36 multiplied by hundred divided by 0.87 xu by d or pt fy by fck equal to 41.3 xu by d xu by is the limiting value.

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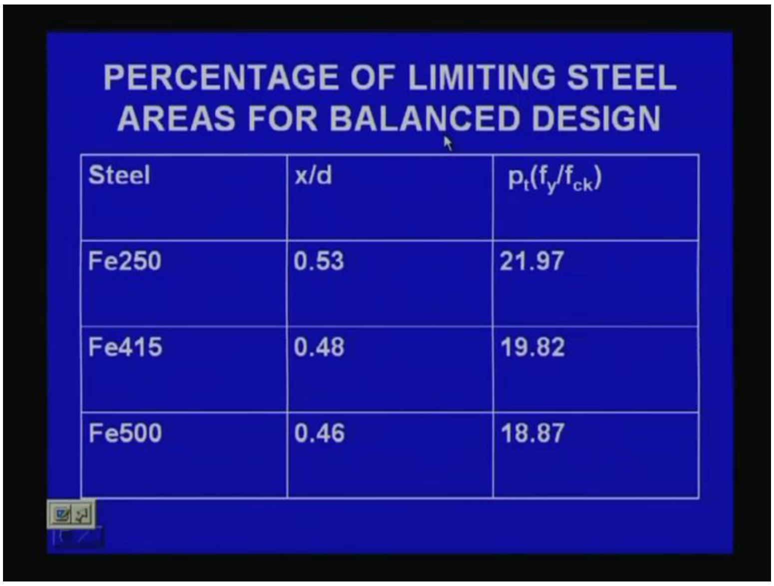

Steel	x/d	$p_t(f_y/f_{ck})$
Fe250	0.53	21.97
Fe415	0.48	19.82
Fe500	0.46	18.87

We can get percentage of limiting steel areas for balanced section for different grades. We shall get this is that we can find out in the IS 456 also x by d. And the corresponding pt fy by fck, we shall get closer to this value depending upon the number of that fractional terms your are considering the decimal the number of decimals your are considering. Depending on that you may defer or otherwise the value will come closer to this 21.97 19.82 and 18.87.

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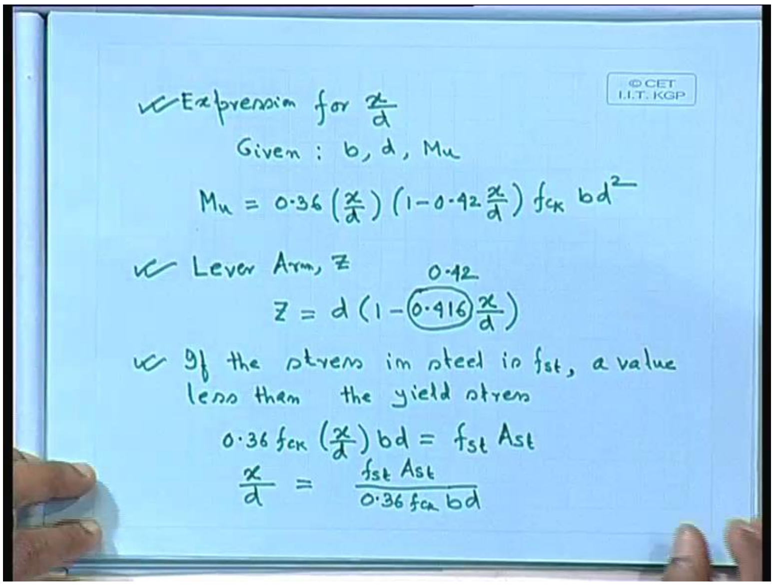

We shall further go expression for x by d given b d Mu. As we know Mu equal to 0.36 we can write down x by d multiplied by 1 minus 0.42 x by d times fcx bd square.

Already we have derived this equation Mu equal to 0.36 x by d 1 minus 0.42 x by d fck bd square. We know b d Mu we know and also fck therefore, we can find out from this expression we can get x by d this is a quadratic equation. In any book you can also get the close form solution, but any way I think that is not required, what you can do you just put the value and give the quadratic equation and find out x by d.

What about lever arm Z? (Refer Slide Time: 09:28) Please this 1 the distance c and t that is your liver arm I should write down here that is your lever arm right. We can write down the equation, possibly let see whether I can write down. So, lever arm Z lever arm Z equal to d 1 minus. We can further write down as 0.42 as you wish 0.416 or 0.42 it is better we should continue with 1 value, but since you have already started with 0.42.

So, let us consider take 0.42 here. If the stress in steel is fst which is a value less than the yields stress. We can get 0.36fck x by d bd equal to fst times Ast, we are taking fst is not necessarily that value not necessarily that yield stress fy or 0.87 fy. If that be the case the steel it should be in equilibrium and from here we can get this expression we can find out x by d equal to fst Ast divided by 0.36fck bd. So, what we are getting then if b and d and Mu known 1 part we can get from this expression Mu we can find out x by d. We can further get Z the lever arm and also we can get x by d if we know fst Ast fck bd then also we can find out x by d. So, these are the 3 different expression that we can get.

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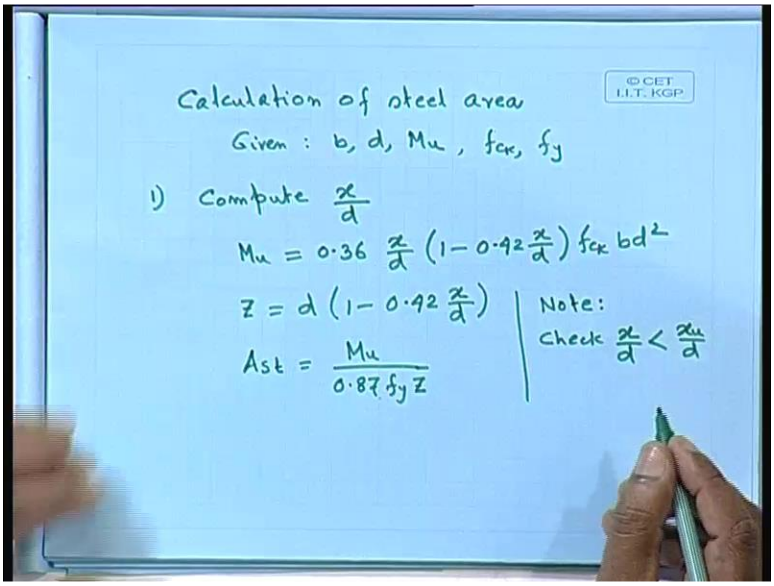

We can continue calculation of steel area given b d MU we can further we can write down here for completeness fck as well as fy. Compute x by d from the expression 0.36 x by d multiplied by 1 minus 0.42 x by d times fck bd square. Z d 1 minus 0.42 x by d Ast equal to Mu by 0.87fy times Z the lever arm. We can get Mu from here we shall get x by d compute the Z the lever arm and then you find out area of steel. Mu is nothing but, 0.87fy the stress multiplied by the area of steel will give you the force multiplied by Z we can get the area of steel.

When you are computing x by d from this expression, Mu 1 36 x by d. From this expression when you are getting computing x by d. Then you check let me just give a note here you check x by d less than xu by d; that limiting xu by d for the balanced section. That should be checked this will be more than the xu by d that you should check from this expression.

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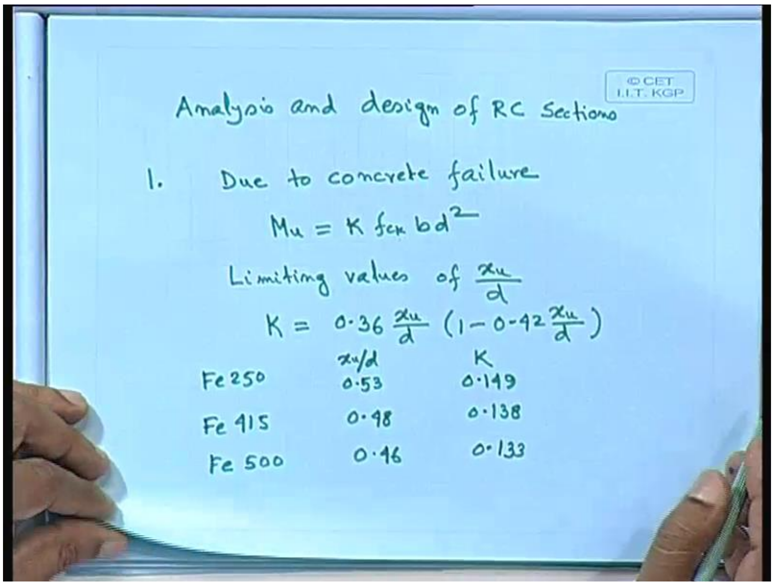

Analysis and design of RC sections, what do you mean by analysis and design? When a section is given the section is given the width depth, steel, the number of bars, bar diameter, then concrete grade, steel grade when it is given. You are finding the moment capacity then we call it analysis analysis of RC section moment capacity of that section.

When the moment is given concrete grade is given, steel grade is given and also we generally assume that width of the beam. But we have to find out the depth of the beam and number of bars and the diameter then we call it design. Analysis is everything given you find out the moment capacity. The other way the moment is given you find out the depth and the steel bars to be provided in that case you consider that design.

Number 1: what shall we do that due to concrete failure; Mu equal to k fck bd square, limiting values; of xu by d K equal 0.36 xu by d 1 minus 0.42 for Fe 250 xu by d 0.53 K 0.149, Fe 415 0.48 0.138 that is K and Fe 500 xu by d 0.46 and K 0.133. We can get Mu for Fe 415 Mu equal to 0.138 fck bd square similarly for Fe 500 we hardly use it 0.133 fck b d square. If it is M 20 grade of concrete immediately you can find out the moment capacity. If it is M 25 or M 30 like that we can get the moment capacity assuming that there is a concrete failure.

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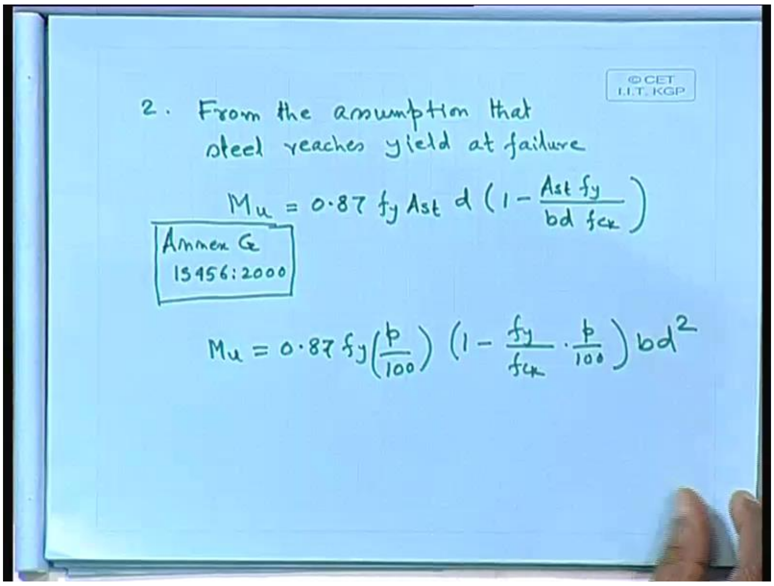

Similarly, we can have the other part; that means steel failure. Let us write down from the assumption that steel reaches yield at failure. Mu equal to 0.87fy Ast times d 1 minus Ast fy by bd fck, you will get this expression in annex G IS 456 2000. We have already derived this expression Mu equal to 0.87fy times p by 100. That percentage of steel is known 1 minus fy by fck times p by 100 bd square.

If we know the percentage of steel from there we can find out that Mu that also we can find out.

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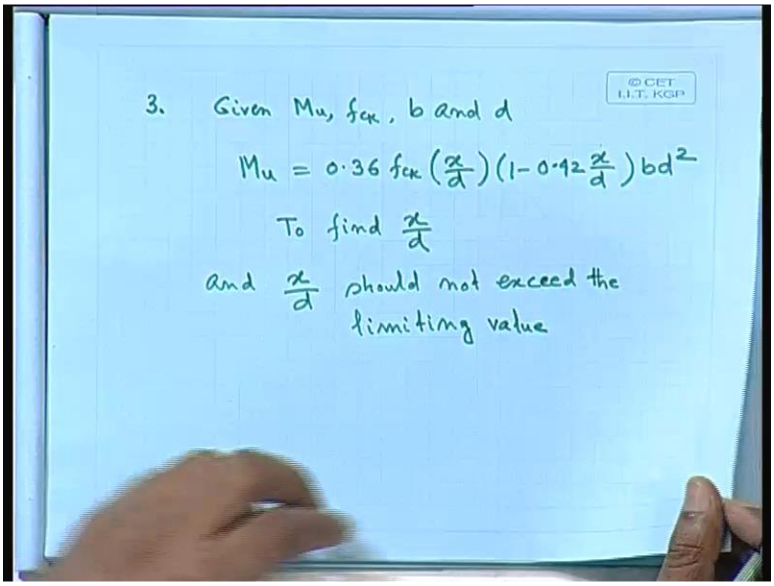

Number 3: given Mu fck b and d, we have already done it Mu equal to 0.36fck x by d 1 minus 0.42 x by d multiplied by bd square. Use this expression to find x by d and x by d should not exceed the limiting value.

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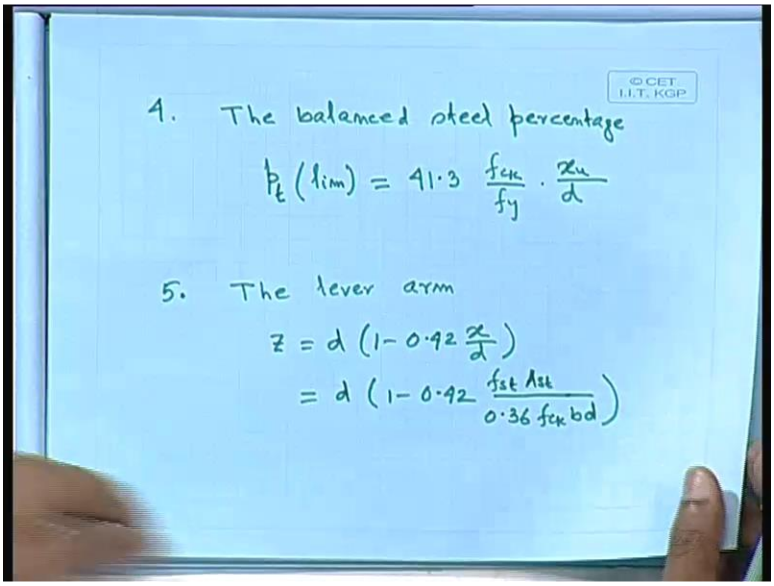

Number 4: we are giving different cases concrete failure steel failure this is another condition. Number 4: the balanced steel percentage we write down that pt let us say limiting value. This is the limiting value equal to 41.3 fck by fy multiplied by xu by d this is the limiting percentage of steel for the balanced section.

We can get that we should not exceed if it is under reinforced the percentage of steel you have to provide less than this pt limit less then this value. If it is over reinforced then it will be more than that the lever arm Z d 1 minus 0.42 x by d equal to d 1 minus 0.4 fst Ast divided by 0.36fck bd. So, we can get lever arm and then we can continue we can find out the area of steel if it is required. We can compute the area of steel if we know Z we can find out the area of steel also.

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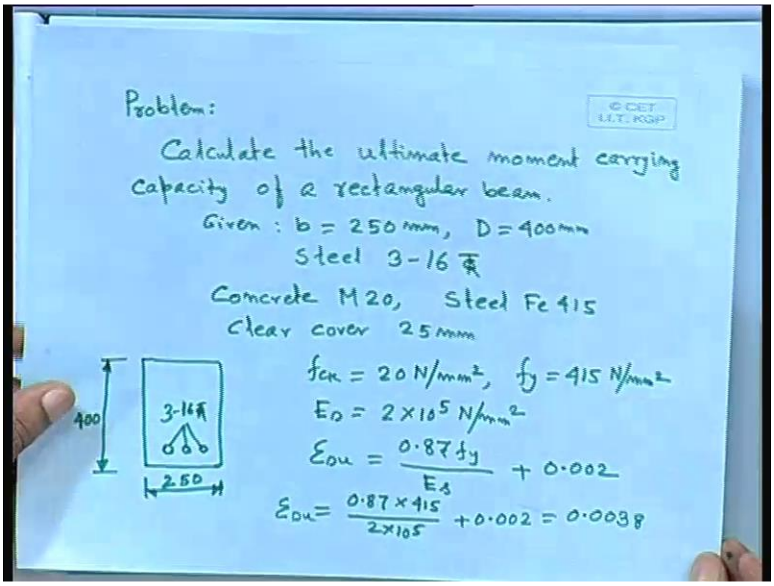

Let us solve 1 problem then. Calculate the ultimate moment ultimate moment carrying capacity of a rectangular beam. Given b equal to 250 millimeter d D 400 millimeter, steel 3 16 tor; we shall use this symbol to show that it is high yield strength deformed bar. High for mild steel the tor this 1 for high yield strength deformed bar, concrete M 20, steel Fe 415, clear cover 25 millimeter.

The section looks like 3 16 tor overall depth 400 overall depth 400 width 250. Fck here 20 Newton per square millimeter, fy 415 newton per square millimeter, Es: modulus of elasticity of steel 2 into 10 to the power of 5 Newton per square millimeter. Unless otherwise specified unless otherwise specified we shall Es equal to 2 into 10 to the power of 5 Newton per square millimeter.

Epsilon su 0.87fy divided by Es plus 0.002 as per our code which comes as 0.87 into 415 divided by 2 into 10 to the power of 5 plus 0.002 equals 0.0038; epsilon su 0.0035. You can remember that 1 also for Fe 200 15 0.0038 t shall I continue. Compute 3 16 tor we call it tor. So, we are writing t o r we call it tor steel we can say. The other 1 we say phi whenever, we say phi sixteen phi; that means, it is mild steel if we specify this symbol then we shall assume if the grade of steel is not given we shall assume that is Fe 415 at least.

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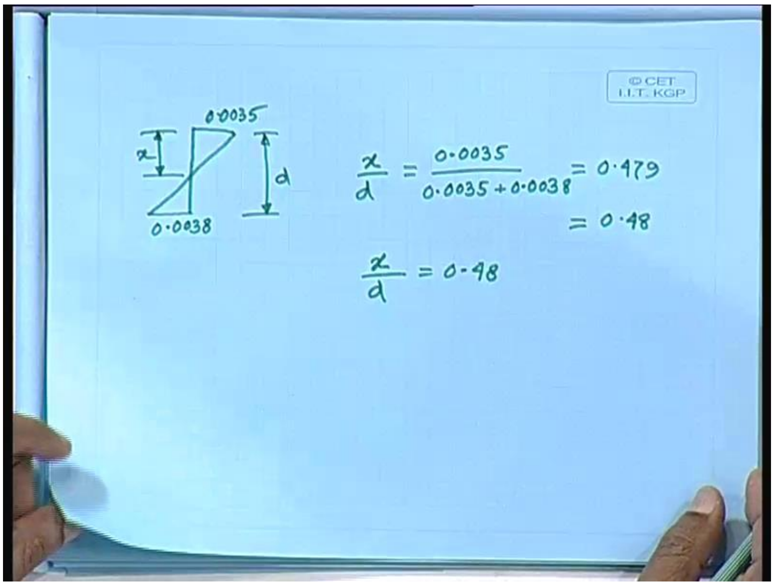

What about the neutral axis? 0.0035 concrete strength 0.0038 steel x and d. Therefore, x by d equal to 0.0035 divided by 0.0035 plus 0.0038 which comes as 0.479 and we know it is nothing but, 0.48 x by d equal to 0.48. We can compute the balanced percentage of steel.

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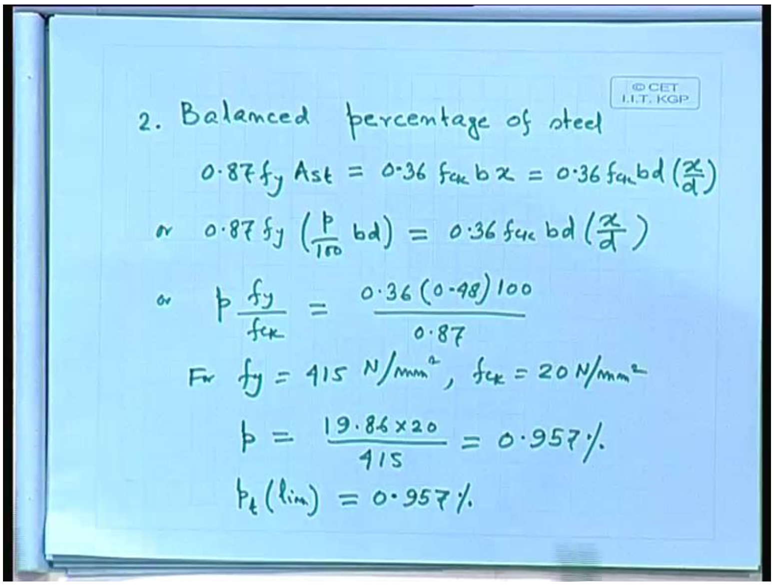

Balanced percentage of steel as it this is number 2. Let us write down the expression at least in 1 case where from we are getting 0.87fy equal to 0.36fck b times x equal to 0.36fck bd multiplied by x by d. We can further we can write down 0.87fy p by 100 bd equal to we are taking here percentage of steel 0.36fck bd times x by d. Or p fy by fck equal to 0.36 0.48 times 100 divided by 0.87 0.36 bd bd cancelled x by d is nothing, 0.48 times 100 divided by 0.87.

For fy equal to 415 newton per square millimeter that is the grade specified in this problem fcx equal to 20 Newton per square millimeter which we have to take in this problem. P comes as computing all those things equal to 0.957 percent. That balanced percentage of steel 0.957 percent we can write down since, we have specified areas pt lim; limiting value we can say pt lim equal to 0.957 percent. That balanced percentage of steel limiting value.

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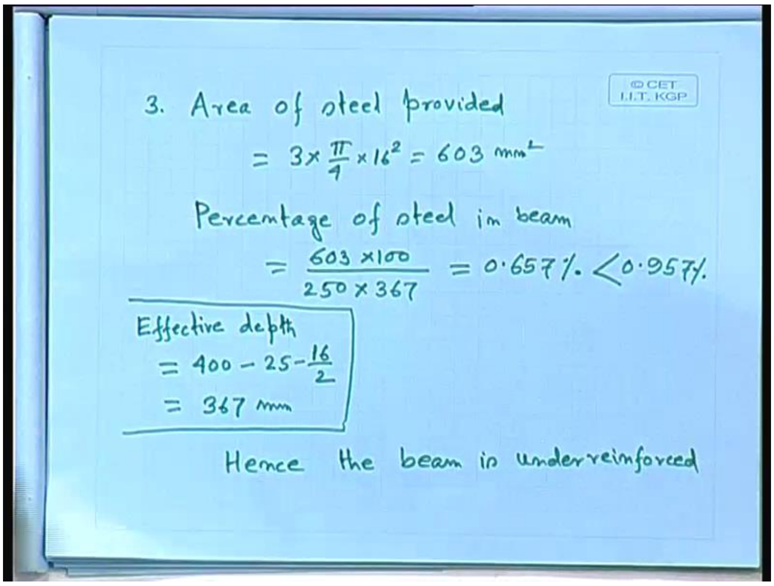

Area of steel provided 3 16 tor 603 square millimeter. Percentage of steel equal to 603 multiplied by 100 divided by 250 into what is the effective depth; so far we have not come across that. Let us find out the effective depth effective depth equal to 400 minus clear cover 25 minus 16 by 2 comes as 367.

So, 250 is the width of the beam multiplied by 367 equal to 0.657 percent less than the limiting value 0.957 percent. Hence, the beam is under reinforced hence the beam is under reinforced.

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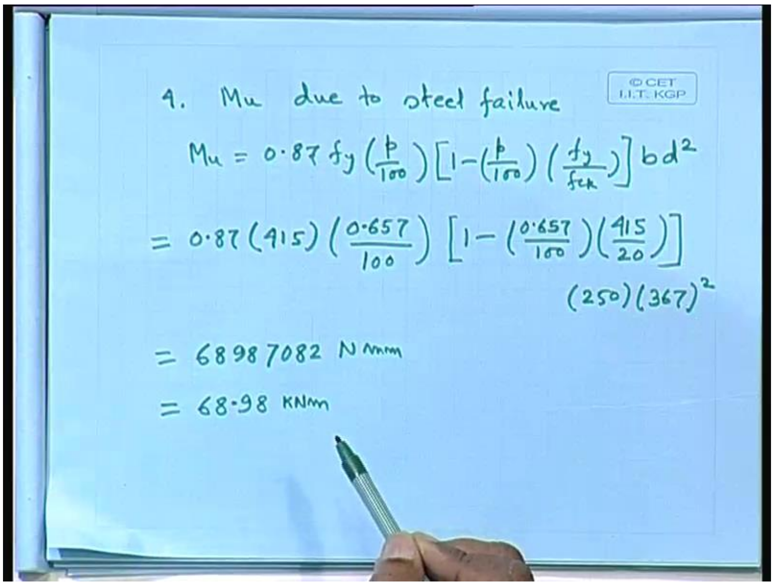

We can compute, Mu due to steel failure Mu equal to 0.87fy multiplied by p by 100 the expression we have already derived 1 minus p by 100 times fy by fcx times bd square. At least once for all let us write down the full expression 0.657 is the percentage of steel we have computed which is provided 3 16 bar 1 minus 0.657 by 100 415 by 20 times b is how much 250 d is 367 equals 68987082 Newton millimeter or 68.98 kilo Newton meter. Moment carrying capacity due to steel failure we shall get 68.98 kilo Newton meter.

We can check whatever due to concrete failure we can take whatever due to concrete failure.

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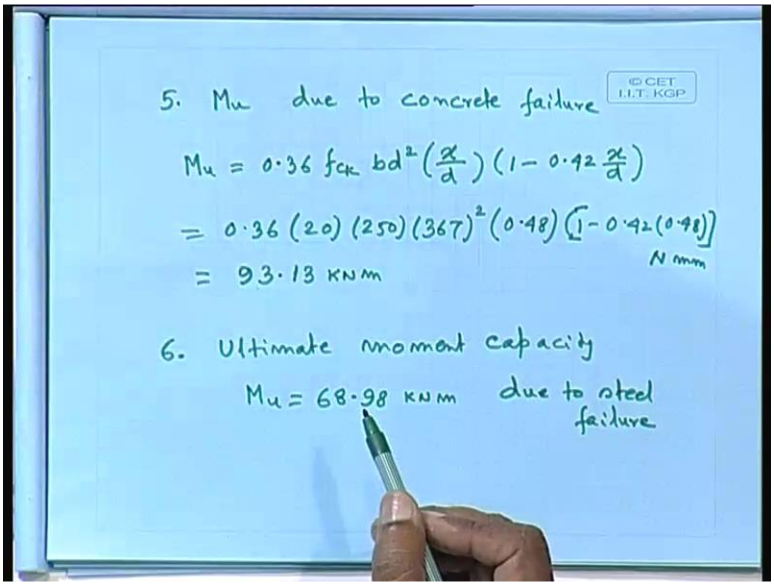

So, 5 Mu due to concrete failure, Mu equal to 0.36fck bd square x by d 1 minus 0.42 x by d equals 0.36 250 367 0.48 1 minus which comes as 93.13 kilo Newton meter. This 1 in Newton millimeter and we are getting after the segregation that 93.13 kilo Newton meter. Ultimate moment capacity Mu 68.98 kilo Newton meters due to steel failure. So, the problem we have done in the very beginning that is you're that with took this problem b, any way we took this problem anyway we can write down there is nothing wrong in it.

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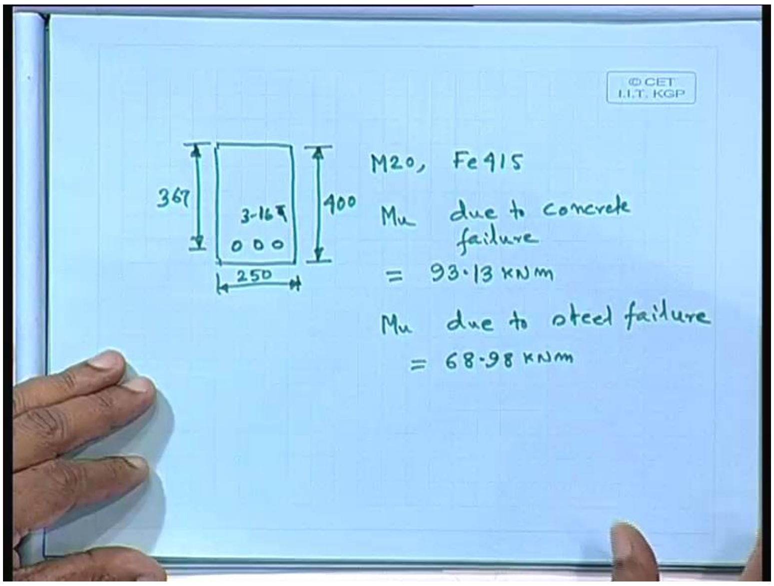

We have taken the problem to summarize the whole thing that 3 16 bars and effective depth 367 when computed, Mu 20 Fe 415. We got Mu due to concrete failure equals 93.13 kilo Newton meter and Mu due to steel failure 68.98 kilo Newton meter. We have taken the section the overall depth 400; we have taken the section and we have got these 2 values.

Our objective should be that, we have to provide the section that 93.13 kilo Newton meter and the other case we are getting 68.98. Almost you can say 25 difference of 25 kilo newton meter; that means, we can add 1 more bar we can find out we can check we can add 1 more bar and we can find out that, whether it is coming closer to 93.13 or the other way also it may exceed also. In that case we have to choose that bar also that bar that 16 that we can use to have whole numbers. So, that design is such that where the design comes that we have to optimize this section.

The steel and concrete failure the moment capacity due to that steel failure and due to concrete it should come closer. Otherwise, it is not an optimum design this 1 we have done this particular problem we have done this problem we have taken it as a analysis everything has given we are finding the moment carrying capacity.

The other alternative that what you have to do that we have to do the design. Therefore, we have to do a lot of calculation if you do it with the calculator then you have to do lot of calculation again you change the depth not the 1 process, not the single process that you will go and solve it and you will get the required depth not like that. Maybe you have to change the depth or maybe you have to change the bar diameter. So, the process is a little bit you can say iterative and it comes in that fashion.

What we shall do in the next class we shall solve the problem; few more problem we shall solve and there we shall do the design.

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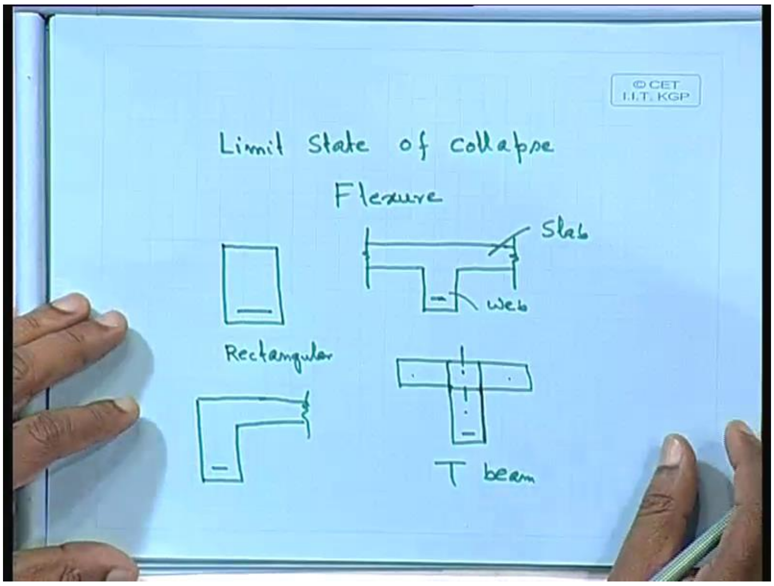

We shall summarize our beam problem let us summarize it. We call it limit state of we are considering only flexure only bending we are considering. So far, we have taken singly reinforced section and rectangular singly reinforced section and rectangular.

It is possible this 1 slab, this is the part of the slab this is the rib or web of the beam. If you take this part this is your slab, this is web we take it we can also take it as a T-beam where we shall take the effect of the slab. Because, when we cast the slab and beam we cast altogether in the same time we cast it we can consider this 1 as 1 unit. And when we take this 1 unit our code says that how far you can go. That means, this is the beam whether shall we take that or shall we take this portion only no, shall we take this portion only the rectangle whatever you are considering. And we are not taking the effect of this plane spot or the slab part.

We can go certain distance from both sides from the central line of the beam we can go and we can take the flange width. When you are considering that when you taking that part then we consider as a T-beam. Similarly, it is possible as say at the corner at the side at the end the beam could be like this; that means, the slab ends and the beam either you can take it as L-beam or inverted L-beam whatever you call; it can take this effect. This is all related to your the singly reinforced section where, all the reinforcement in the tensile side. It is also possible that we have to consider the other part that is called doubly reinforced section.

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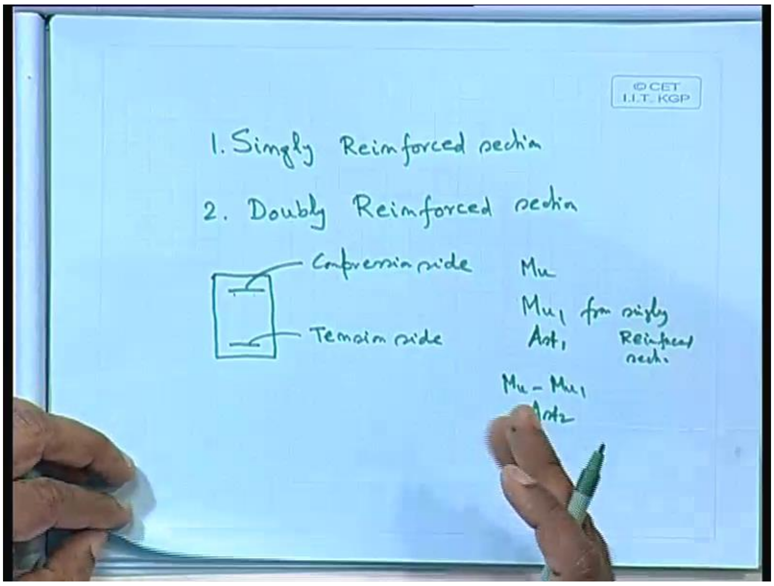

So, 1 part is the singly reinforced, so far whatever we are doing we have done so far the rectangular 1 only. And the other 1 doubly reinforced section. In this case, we provide reinforcement this is your tension side, compression side. What is the basic philosophy? The basic philosophy is that, tension side you are providing the reinforcement. Concrete can take compression at the top for this usual case we are talking concrete can take compression at the top. And from the equilibrium consideration we can provide that your say equivalent your tension reinforcement. So, that it will be in equilibrium and we can calculate the moment carrying capacity.

Sometimes it may happen due to some problem that we cannot increase the depth; the moment is such that we cannot increase the depth. The moment has come due to computation due to your load external load we have got whatever moment we have got it. But we cannot increase the moment of resistance. That means, we cannot increase the depth if it always preferable that increase the depth because, if you increase the depth that is coming square this square whereas, if we increase the width then it is simply multiplied by b only.

So, it is always preferable you increase the depth. So, that is why you always increase the depth. If we increase the depth we can get that all section make it singly reinforced section, but sometimes it happens that we cannot increase the depth. Therefore, what you have to do the moment carrying capacity due to singly reinforced section we can get certain say Mu 1 the moment is Mu that which you have to provide. Mu 1 we have got from singly reinforced section the remaining balance Mu minus Mu 1, we shall provide providing compression steel.

So, if you provide the steel in the compression side similarly, you have to provide for equilibrium we have to provide in the tension side also we have to provide the reinforcement. And then we shall get tension side that something this 1 for this say Ast 1 the remaining part say Ast 2. So, Ast 1 plus Ast 2 you have to provide in the tension side so that we shall do it in 1 class that we shall do. So, this is the whole thing we shall do. So, far the limit state of collapse taking flexure only. There are other part also that we have to be taken care of CR also we have to be taken care of, when we shall design the beam.

Thank you.