Probability Methods in Civil Engineering

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Module No. # 04

Lecture No. # 15

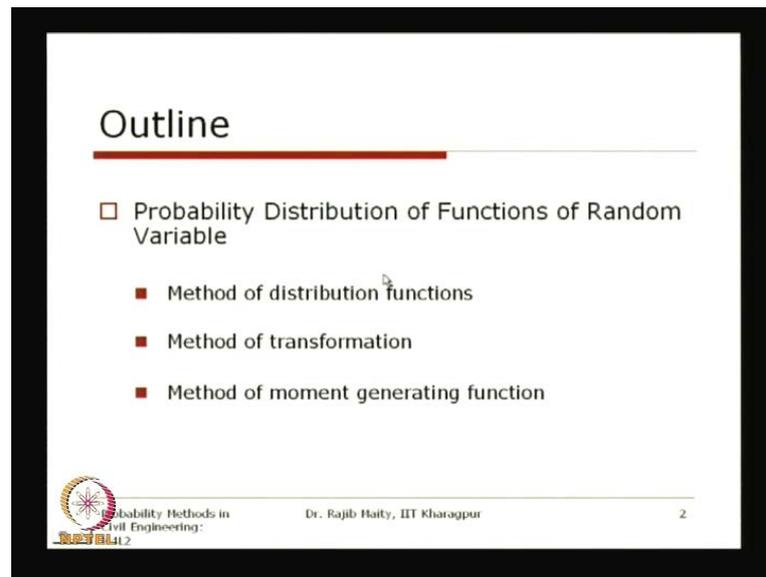**Functions of Random Variables-Different Methods**

Welcome to this lecture on this 2nd lecture on this module 4. In this module, we are discussing on Function of Random Variable, and in today's lecture what we will do, we will just learn about this different method.

So, if you recall that ourlast lecture in that, last lecture, we discussed about the fundamental theorem and in that fundamental theorem we have seen that, if there are differentroots of a functionof random variable; then how we can make a general equation to find out the probability density of that function of therandom variable, from the information which is available with thisoriginal random variable, its density is known and the functional formwith this new random variable is known.

So, using these two equation we have seen that, what are the,what is the fundamental relationship between the probability density function of this two random variables, and keeping that theorem in mind today what we will do is that,we will find outdifferent methods. And these different methods are basically based on the different assumption, but so far as the fundamental concept is concerned is same what we discussed in this last class.

So, what in today's class, what we are discussing is that we will discuss about different methods to get the probability density of the functions ofsingle random variable.

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So, to get that probability distribution both the PDF and CDF there are different methods are available, and as a I told that all these methods arebased on the same fundamental concept of the fundamental theorem, that we discuss in the last class. And we will use the same concept to discuss about our different methods which, this methods arebased on certain assumption which are realistic, so far as application is concerned.

So, the first thing is that method of distribution function, then the method of transformation and then method of moment generating function, we will discuss; the first we will discuss that method of distribution function, so if Ibefore I go to that that theoretical thing, if I justdraw here the concept of that,what is the fundamental thing is that just relating the fundamental theorem of ourlast class, that what we have to find is that if the, there are two random variables one is this x and other one is your y, and there could be somesome relationship of thisequation, and what we are supposed to do is that, we have to find out, so we know what is what is the density function is for this variable x is known.

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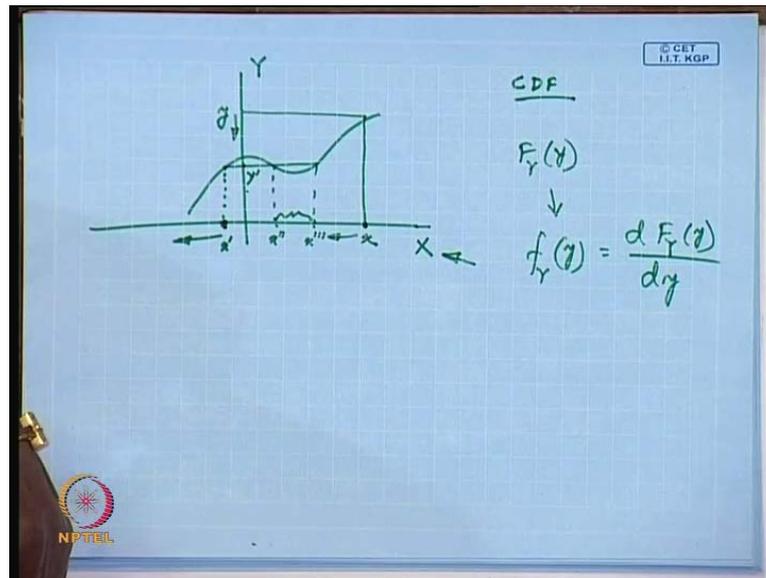
So, what we have to find out is that, for a particular value of y we have to find out<mark>what is</mark> <mark>that</mark>what is the domain for that particularvalue x, if I am first I am that considering that probability the method of distribution function, herethe method of distribution function means, what we are talking about that CDF that Cumulative Distribution Function. So, to now if this is a specific value of this random variable y is thissmall the lower case letter y, than what I have to find out is that, what is the corresponding domain of the random variable x. So, if I want to know this CDF that means, this that y is less than equals tothespecific value y, so that is thissideso these domain is the equivalent domain for this x is that specific value of that random variable x and its domain.

So, the CDF that is probabilityup to that particular value y is equals to probability of the random variable x up to this specific, this specific value of this x. Now, if there are in the last class we have also seen that, if incase<mark>so</mark>here these domains are same and continuous domain, from this minus infinity to this y and for x this minus infinitive to the x, now for some other value it may, so happened that there could be some<mark>some</mark> different zones.

For example, if I just take the anothervalue of this y, let us say that we give name as this yprime, then there could be that this domaincan <mark>at</mark> different domains for the random variable x. The first one this could be this x prime, so which is less than equals to x prime which is corresponding to this point, plus another domain which is your, this one if I just give this names as x double prime and this is x triple prime, then these zone as well

as from minus infinity to x prime summation of this probability for the random variable x should be equal to the probability of therandom variable y less than equals to y prime.

So, this is theconcept where what we aretalking about this method of distribution function, so what we have to do is that, we have to find out what is the infers relationship and that should be express in terms of this y to get thatthatthe probability, this cumulative density function for that new random variable y; and once we get, this one we will we can get that probability density function by their differentiation.

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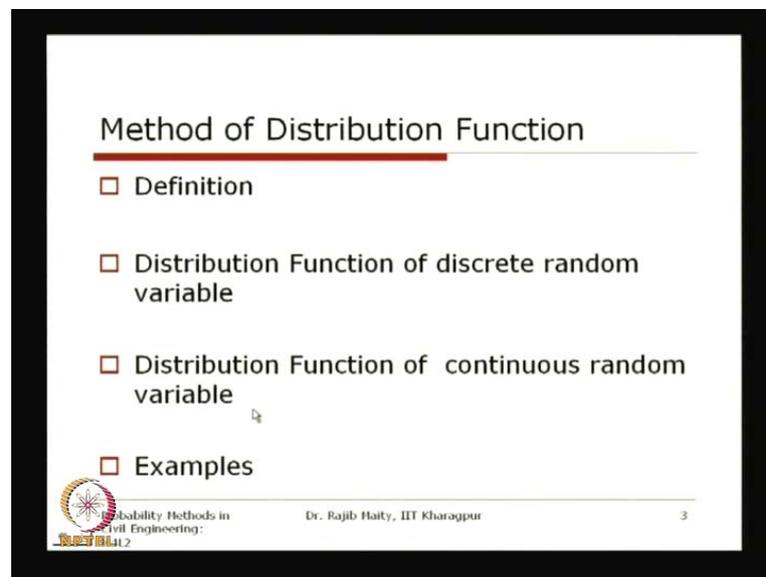
So, this is the keeping, this one is the basic concept we will now, see that method of distribution function. So, first what the so far as the fundamental definition is concerned that is what we are meaning is that, whathow this two this two variables are related that we have discuss, then the distribution function of thatdiscrete random variable as well as this distribution function of the continuous random variable, there will be a little bitlittle difference between these two, when we are talking about that discrete as well as this continuous, we will discuss this one and we will see it through some example.

Basically what we will be doing, we will be using the same example that we have used in the last class, and we will show that in the last class from the fundamental theorem we have solved that equation. Now, with the help of this method of distribution also we will solve the same problem to show that, howwhat the result comes just to differentiate what

is thisdifference is there, if any what is the difference, so we will use same problem that we discuss.

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So, what we just now is discuss is that that the distribution function that is the cumulative distribution function F y, y of a random variable is the probability of the eventthat this y less than equals to a specific value y consisting of all outcomes xi such that, y xi is equals to g x xi, which is less than equals to yso, that means here what we are trying to say this is the fundamental thing of this cumulative density, at cumulative distribution function that we discuss earlier is that, so if I want to know this F y, then I have to find out the design, I have to find out the probability that this is less than equals to y.

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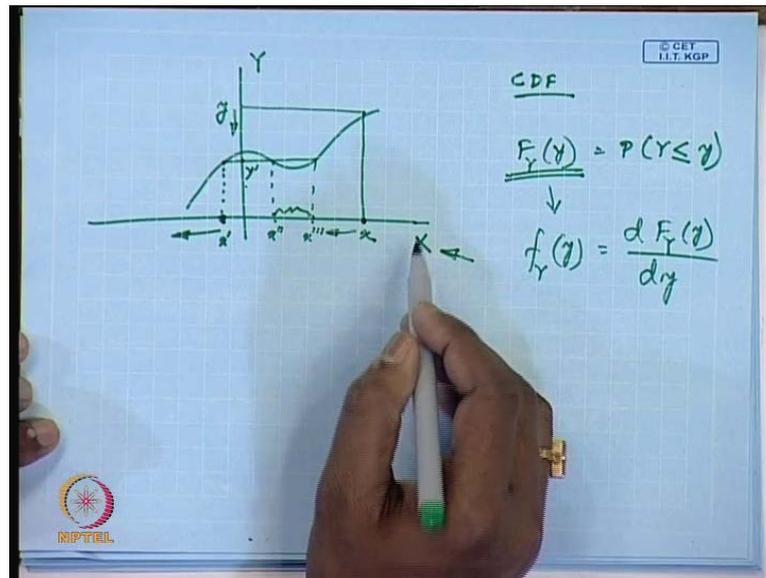
Now, referring to the figure that just now I have drawn isthat, what we are we are going to get that for this F y y is that, this F y y is nothing but, the probability of the random variable less than equals to a particular value y. So, this is from the fundamental, this is from the basicdefinition of this cumulative distribution function that we discussin the long backearlier classes. So, some specific value y is there I have to find out this probability, what is known to usthusproperties of this x is known, so that is why I have to related to this random variable which is known to us, which is the x here.

So, thus if I now, so if I now want to know this F y which is equals to the probability of y less thanthe specific value y, which will be equal to now I am relating it to the functional form, the functional dependence between x and y that is y is equals to g x, so I am replacing that one, so this g x should be less than equals to y. So, I have to find out some domain in the x which corresponds to thata for the random variable y which is less than this specific value of y. So, it follows thatprobability of y less than equals to y is equals to x less than equals to g inverse of y directly from this equation, so this g inverse y is anotheranotheryeah function which I am just taking the inverse function of this g x, the functional form is there. So, what is here is that, I have to express that functionalthatfunctionaldependence in terms of in terms of y.

So, if I know this one now probability of x less than equal to a specific value, we we can easily get it from thatthe properties of this random variable x which is we knowthat this

cumulative distribution is F x for that specific value g inverse y. Now, this distribution function is <mark>you know</mark> that, this is a cumulative distribution function and once I get this cumulative distribution function, then we can differentiate it to get that probability density function.

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Method of Distribution Function...Contd

☐ Therefore the distribution function technique is that to find the value of x such that $g(x) \leq y$ form a set of on the x axis denoted by R_Y.

☐ The probability that $g(x) \leq y$ was obtained i.e. $P[g(x) \leq y]$ by integrating the density function $f_X(x)$.

☐ Once $F_Y(y)$ is obtained, the density was found out by differentiation
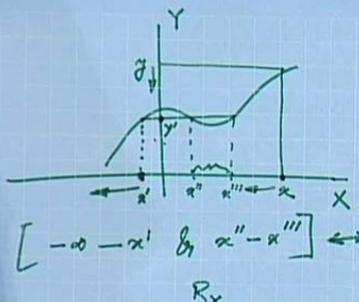
$$f_y(y) = \frac{d}{dy} F_y(y)$$

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So, in brief thedistribution function technique that is cumulative distribution function technique is that to find the value of x such that, theg x is less than equals to y from <mark>from</mark> a set on the x axis and which is denoted by R y.

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So, here if we refer again that for any specific any specific value for this y, for a specific value of this random variable y, I have to first identify what is the set of x that corresponds to that specific value ofy. So, this one for this here, this y prime that set is nothing but, one is that from minus infinity to x prime and xdouble prime to x prime. So, this is the full set of the x, that I have which is corresponds to that minus infinity to y prime, so this is this is that set which we are talking about that R y that is the corresponding set for thisspecificvalue of the y, this is what is explainexplain here.

So, first of all this for this in this in this technique the distributionfunction technique, we have to find the value of x such that, this g x is less than y form a set on the x axis and this is denoted by R y. Now, theprobability that g x less than equals to the specific value y isobtained that is the probability of g x less than equals to yby integrating thedensity function of F x, so integrating this density function of F x. So this is, this F x is yourprobability density function of x, now if I integrate it,we will get the probability that cumulativedistribution function that CDF of x and from that CDF of x we will get that, what is thatdistribution ofdistribution of x for aspecific value thatthis probability will be obtained, which wediscussin this slide that is; if I know that specific value of this y and if I know the probabilitythat cumulative distribution function of x, then from this form we can get that probability that is probability of this specific less than specific value of y is equals to from that cumulative distribution function of x.

Now, once we get this F yY is obtained then we know that its density that is PDF, probability density function can be identified, can be found out by the differentiation with respect to y, so we know this one from thisbasic principle of this CDF and PDF.

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Now, so we will just see that two differentthing for this discrete random variable, and for this continuous randomvariable; there is a slight difference in the sense may be when we are talking about thiseducational form the difference may not be, so may not be may easily understood, but in the next method when we aregoing to talk that is method oftransformation, that time it will moremore clear, so but here what we are trying to say just we will just differentiate from in terms of integration or in terms of sum summation what the case maybe some may be that integration is for the continuous random variable and summation for the discrete random variable, because we know thatfor the discrete random variable the probability is are concentratedat a particular value, so we have to sum them up.

So, here so that is why it says the for the discrete random variable x that F y, when we are getting that another function is equals to the summation of the probabilities of this x i for all such x i which is less than that g inverse y; that g inverse y means that inversefunction of that functional relationship between x and y. So, we have to first identify that which are that specificvalues of the x which is less than this g inverse y and for those specific values whatever the probabilities are their for the x should be summed up sorry should be summed up, and that should be equal to the probability of that specific value of this y.

Now, for the continuous, onecontinuous randomvariable we know that,<mark>we if</mark> we have this probability density function F x of x then to find out this F y of y, it should be integrated <mark>for the</mark>for the set, for which this x is less thanequals to g inverse y, so <mark>you know</mark> that, so this set this from just now discussin this pictorial form this is your basically that R y, that setfor which this condition is satisfied. So, if we integrate over thisregion than we will get, so other words are from this left most support that is from the minus infinity to that inverse we will get thisformand this will be equal to the probability of y up to the specific value small y (Refer Slide Time: 17:02).

So, here means even though this is shown <mark>in a</mark> in a verycompact form, it may not be depending on this, how many roots are available here for example, here that we have shown in this figure that for this one the integration must be the summation of this, so minus infinity to the x prime.

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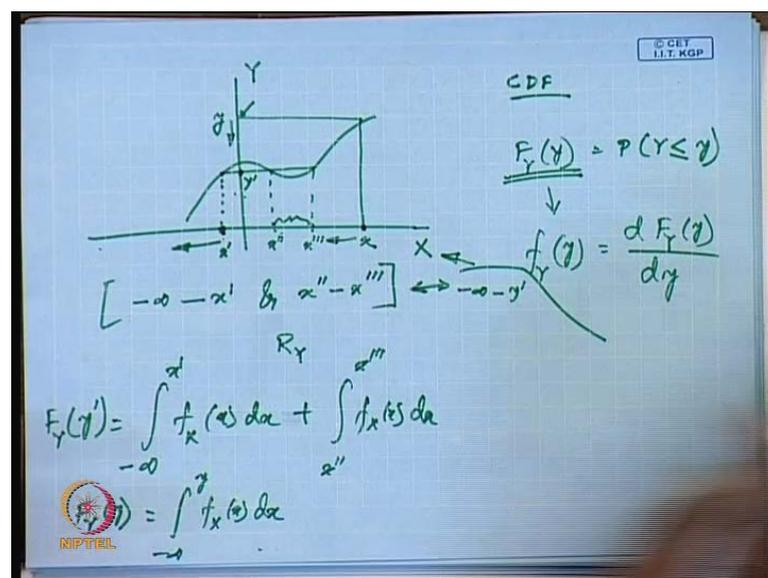
So, the integration from this minus infinitive to x prime that F x ofthis x d x plus I have to take another integration that is from the x, double prime to x triple prime that is thiszone F x d x. So, if we get this form <mark>then we</mark> then this will corresponds to that F y of that value F yprime here, this F y prime we will get, but if I just want to take thatthis pointthat is F y of y, this issimple you can get that from the minus infinitive to y F x d x, so this will be equal to these probabilities.

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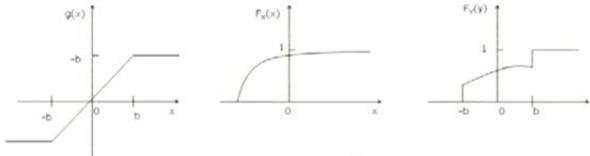
So, nowwe will just showthis isnot very, not specific to any particular civil engineering problem, but this is very important tounderstand the concept that we here discussed, so far and this is taken from the Pepulas book Pepulas and Pillai book. So, here what is shown here, is that a relationshipbetween this x and ythis g x is equals to yyou know that, this is the functional form, so this is if, this is the type of relationship then we will see whathow it happens, because whenever we are talking about, we are just giving some values tothis g xcorresponding toa specific value of y then here in this form we can see, we have taken this example just to demonstrate that if these things become becomes flat, this functional relationship becomes flat at some some point then what will happen this, at flat at some point means mathematically I can say that the rate of change of y that is g x, the rate ofchange of y with respect to rate of with respective to x it is constant.

So, it becomes when it becomes flatthen how thisthis, seethis distribution lookslike that that we will see, so here what we have to do the basically is that we have to have to first of all find out the three different region of this function. So, in such suchcases, so where there you can see that there are three distant, different, dependence forever this three different zones, there is one is that with respective to x, if I see that one the first zone is that less than equals to minus b, second zone is from this minus b to plus b and the third zone isfrom the from the that greater than equal to b.

So, <mark>for the</mark> for the first region that is when the x is less than equals to minus b then the value of this g x or the y is minus b, and forthisregion that is when thisx is greater than equal to b the value of y is bthis will not be minus b this is b. So, <mark>value is</mark> value of y is b and for the zone from the minus b to plus b it is varyinglinearly.

Now, in this case how if we know the distribution of this x what should be the distribution of g y? So, for this region it is whatever we have discussed, so far is a iseasy to get, because I just get any specific value of thisg x that is of this y find out what is the corresponding value of this, value for this x and find out their density function. So, that is easy, but here we are more interested to know that how it is for this zone where this relationship becomes, relationship is constant; so this two zone we will see, so this figure what it isshowing is that how this F x of x looks likeanda function is taken, so that function the pictorial view of that function is looks like the, this obviously it starts from 0 we know that it is CDF cumulative distribution function, and it is shown that it is assume that to 1 at infinity.

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Now, <mark>so</mark>what we have to first as I told that, we have to find out that three differentregions; one is that if this y equals to g x that with respect to thisfigure that we have shown just now, it follows that this F y y is discontinuous for two cases, one is that for thisat this minus b is less than minus b and at another point which is equals to b; so at this point this function is discontinuous, because to <mark>you know</mark> that mathematically two

straight lines is meeting that point. So, according to the method of distribution the first if I this is a general form as I just explaining for the continuous random variable, that F y y is equals to the probability of y less than equals to a specific value ofy, so and this y is replaced by its functional form g x g x y now, keeping this one as this as this basic equation now there are three situationscan happen.

The first is that y is greater than b, if y is greater than b then for the g x less than equals toless than equals to y that is specificvalue of the random variable y, for every x F y y is equals to 1, because that is the maximum value that it can takes, so atfor this greater than equal to b this F y is 1.

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Similarly, if I just take that y less thanequals to b, then for this g x less than equals to y for no x, so that is why this cumulative distributionfunction, the value of the cumulative distribution function for this zone that is y is less than b is equals to 0. Now, for this b that is from the minus b to plus b, if I come to this region thenby the nature of this functional dependence you can see this isjust one specific value you will get for the x for a specific value of y. So, any value you take for this g x that is for this y from this minus b to this plus b, then it will corresponds to a specific value of x, so less than equals to that value of y will be equal to the less than equals to x of that correspondingvalueon thisx axis means that for the random variable x.

So, for this minus b to plus b that distribution of this y will be same as this value for this F x, and for this when it is becoming greater than b it should be equal to 1 and when it is becoming less than minus b it should be equals to 0. So, what now, if you see it here pictorially then <mark>from this</mark> from the minus infinity to minus b the value of this F y is 0, then atthis value that atminus b this F y is becomingis equal to the value of that F x at minus b, and it is going on following the same curve up to the point b and at point b this is equals to value 1. Now,<mark>if you</mark> if you recall our description of the CDF means, few more theearlier classes we discuss, so this straight line basically is an indicative line, if I just <mark>draw it</mark>draw it here, thenyou recall that the staircase function that wehat we drawn while describing <mark>that</mark>that CDF for this discrete random variable then it looks like this.

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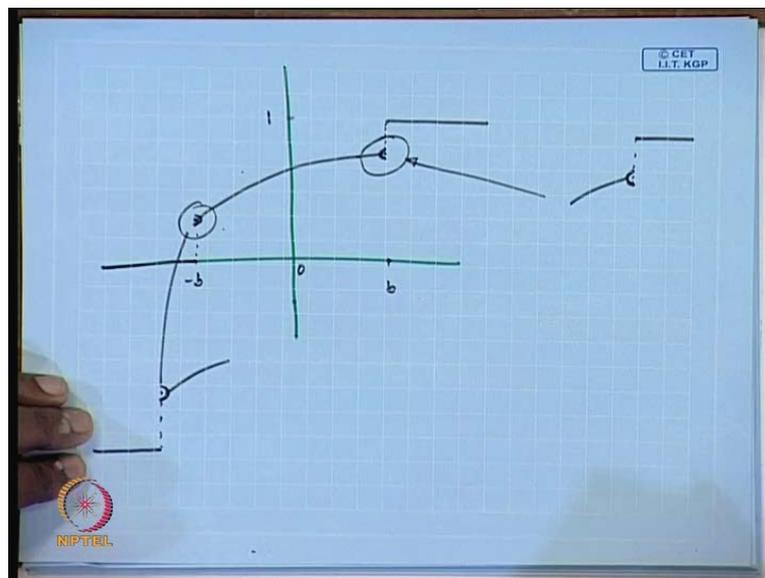
So, what happens now I am using another thing, so if this point is your minus b, this point is your plus b then the CDF comes up to here, then this is just an indicativeline, because at b this is equal to thatparticular value of this y, so it follows from here and go up goes up to this plus b and then this becomes equals to 1 this is 0.

Now, theway the function is defined here, I will just come back to this figure once again refer to this slide now, thatway this function is defined here that is the g x is constant for x less than equals to be and x greater than equal to b, so the constant <mark>form is</mark> from this equals to be, so when we are taking this form equal to that the similar form of this F x for this region from this minus b to plus b that means, is b that constant value is included

forthe value up to minus b (Refer Slide Time: 27:34). So, what I canshow here is that, so at minus b this value is 0 and at plus b the value is 1, so this line which is the similar form of this F x cannot take this value so I am just, if I just zoom this part is it looks like thisso it can go as close as plus b, but it cannot take the value plus b just recall our description of the CDF <mark>for thus</mark> for the throwing a dice in few previous class we discuss, it cango and we cannot touch that b, because at b the value is 1 similarly, for thisminus b there will be another, it cannot touch this minus b, so it comes like this, because at minus b the value is 0.

So, this one I think a making this thingclear, so what we are trying to drive is a, what we are trying to convey hereis that when this function the rate of change of that y, rate of change of y become constant with respect to x at that point, we can observe the discontinuity in that cumulative distribution function of y which is shown <mark>in this</mark> in this specific example.

To illustratethe theory that we explain now, we will take asimilar problem, similar means that we took in thisin the last class also, and we will see that how we can solve that particular thingin this context.

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Here, we will take <mark>that</mark>that problem there are two variables, one is that this z which is basicallythat your wave height when there is a windis blowing on the free water surface, <mark>that</mark> that wave height depends on various factors and also it isrelated to the square of the

windvelocity, and so this two are relatedlike this and obviously, there will besomeconstant will behere that we can see, so so that that this constantis some a, now here the question is that this V if you know that distribution of this V whether we can get the distribution of this z; now this wind velocity generally what we see that this is having a distribution ofdistribution of this wind wave velocity is an exponentialdistribution say that this is looks like you know the exponentialdistribution like this, and here the support of this one is greater than equal to 0

In the last class the problem that we took, this one is having a support over this minus infinity to the plus infinitythey are, we are talking about some stain energy and force which was a standard normal distribution. So, that that is having theentire support of this real axis minus infinity to plus infinity, but here it is the wind velocity, so this can take only positivevalues with respect to as, when we are talking about that wave height. So, it towards thetowards the obstaclewhere we are measuring the wave height, and this is the distribution and it is 0 elsewhere, so ifwe know now, so here also we can get that similar to this earlierthe last class problemthat is this g is equals to plus minus square root of z by a we will get, and obviously this minus, thus minus root thenegative which is notis not a valid case. And that modulus of this d v by d z will be equals to 1 by 2 square root a z, now this f z of this one the distribution of this new random variable z is equals to that, this distribution of this known variable at those roots that is z by a, if we want to writethe complete expression, if b minus square root z by a it is 1 by 2 square root of a z which is the value of this 1.

Now, this is the basically this is not valid, so we have to take only thisvalue and what we can write that 1 by 2 square root a z f v square root z by a and this we can express as 1by 2 v naught square root a z exponential minus 1 by v naughtsquare root z by a, where again that z should be greater than equal to 0 which is the support of this distribution. So, here, we have seen that even though they are having other roots we should not consider that root, which is the outside of thesupport of this random variable b, so the distribution of this z is 1 by 2 v naught square root a z exponential minus 1 by v naught square root zby a at the support is that z is greater than equal to 0.

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Discuss about the anothermethod, that is call method of transformation; and this method of transformation is popular in the sensethat in most of the engineering application problem and most of the practical applicationproblems as well, we have seen that whenever some function of a random variable is taken inmany casesthe correspondence is always one-to-one. So, what is mean by this one-to-one correspondencemeans that, for a specific value of theoriginal random variable, here we are using the notationas the x, so there will be only one value of the of the value of this depended random variable that is here we are using y. So, for a specific value of x there is one and only one value of y and vice versa, so this kind of transformation is called thatthat one-to-one transformation.

So, if this is the one-to-one transformation then using that fundamental theorem that what we discussin this last class, that can be reduced to a very, so very effective equation and that equation can be directly used to find out, what is the probability density function directly by using the that equation. So, that is what our secondmethod we are going to talk that method of transformation, and obviously, we have to remember that method of transformation means, what we are referring here now is the one-to-one transformation.

So, here, that this case one is the one-to-onetransformation and we will just show one example of this discrete and another example on this continuous, and thatcould be another case that the transformation is not one-to not one not one-to-one, so we are talking about this when the transformation is one-to-one.

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So, suppose that we know the densityfunction of x also, suppose that the function that y is equals to g x <mark>sorry</mark> there will be one g that is the functionalform that y equals to g x is differentiable, and monotonic for the values within its range for which the densityf x is not equal to 0, so this is basically what is what we are referringis that to the one-to-one.

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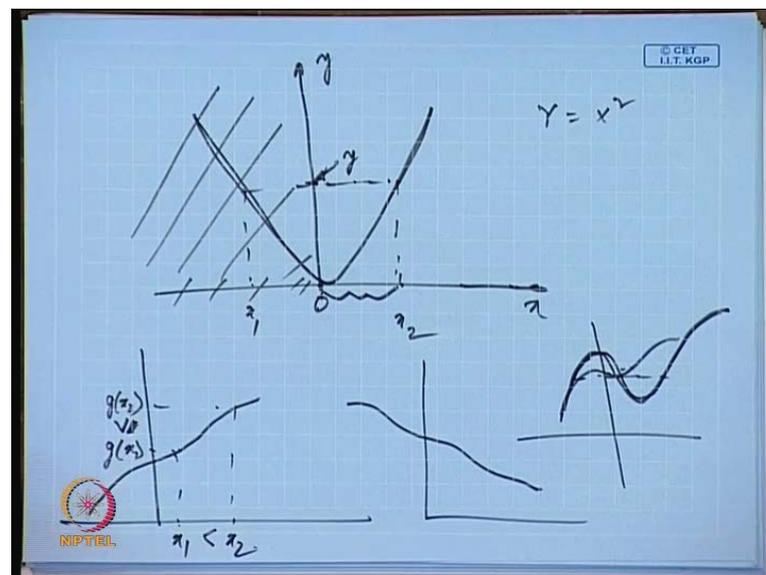
So, we will justdiscuss about this monotonic,what is this monotonic is talking about is that, if the function either is increasing, either if it is increase, so it should increase over the range of this f x or if it is decreasing it is <mark>it is</mark> decreasing. Now, this monotonic means

it can never it can never, if it is the increasing function in this it can never decrease, if the x 1 so fine, so if this is the value x 1 and if this is the value x 2, and if this x 1 is less than x 2 then this value that is, this is your that g x 1 and if this is your g x 2 then g x 1 always should be less than less than equals to g x 2, if I just put this sign that less than equals to g x 2, that means I am including this sign the equal to that means; it can even be flat that is called the monotonically increasing, but if I exclude this equal to sign sign that is known as the strictly monotonically increasing. So, strictly monotonically increasing means it can even not be equal, it will always increase for the for some higher values of the of the x otherwise if the if the equal to it is include that is known as that monotonic.

So, this function is monotonic that means, that this transformation is always one-to-one now we recall that, we for the general discussion when we are talking about this kind of equation, so there is possible the possibility of having more than one roots is only is can happen, only in case of when this function can increase and then again decrease and then again can increase, so in this kind of situation only there it is possible to have more than one root, so what we discuss in this fundamental theorem. So, here what we are talking about it can never increase that it is it is monotonic, that means there is always one-to-one relationship of that from this x to that g x, this is what is what is explained here.

So, once again I read that suppose we know the density function of x also suppose that the function y equals to g x is differentiable and monotonic for the values within its range for which the density f x is not equal to 0. So, monotonic we have explained y the requirement of this monotonic, and what is monotone monotonic and there is another requirement is that differentiable. So, we will just see if it is not differentiable at a particular point, what will happen and it will be clear, when we are referring to the fundamental theorem. When the equation that is g equals to sorry the y equals to g x can be solved for x as a function of y, this transformation that is the inverse mapping is then used to find the density function of this y.

Similarly, we can do the same when there is more than one variable x and then that case there is more than one mapping. So, this one basically we will be discuss when we are, when will be discussing that multiple random variable, here we are discussing in this lecture the single random variable. So, we will deal only from one random variable to another random variable, that is from x to y.

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So, in order to apply the transformation there should be the one-to-one correspondence between the X and Ythat is what we discuss, here is one example, that y equals to 1 plus 9 X. So, if I take this example, then you can see that for a specific value of Y only one value of Y is there and vice versa, so this is an example of the one-to-onetransformation. Now, y is equals to sine X is not a one-to-one transformation, since more than one value of X willgive the same value of same values of Y.

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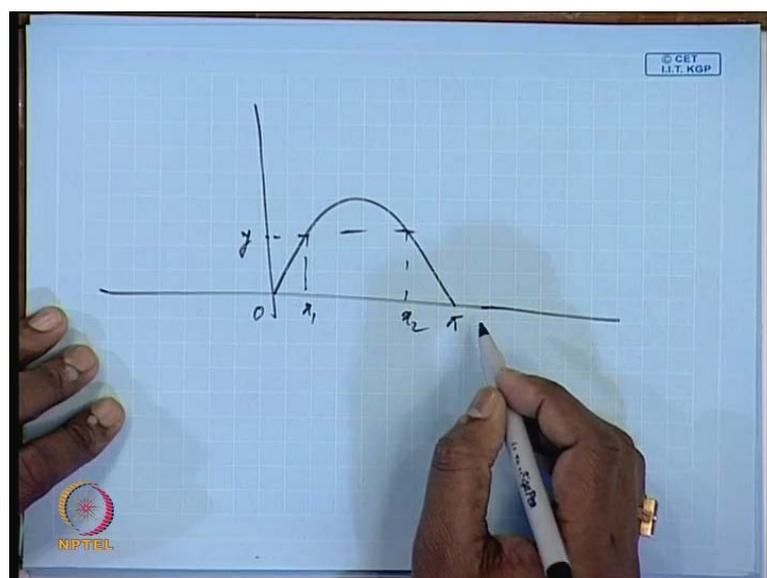
So, you can see that, so if this is that if so if the function is valid for this 0 to pi then you knows that for a specific for this same value of this y there are two values, this is x 1 and this is x 2. So, two values of this x are possible, so that is why this not a one-to-one transformation.

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Now, for the discrete variable let the random variable x with the P m f P x of x be transformed such that, that yis equals to g x, andthe inverse function of y can be written asthis x equals to g inverse y, if I just give that new function of theh y. So, h y equals to this g inverse y corresponding to y equals to g x.

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Then this P m F for this y that is P y y is given by this P y y is equals to that P x h y, where this h y denotes the inverse function such as that, x equals to h y ify is equals to g x that is the this h y is nothing but, that g inverse of y.

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So, with this yeah one example we cantalk about for this discrete random variable just we will see that one-to-one transformation; obviously we are talking again here the one-to-one transformation, that is if the transformationhave ys only one specific value.

So, this one this probability of overtopping a guide bund after x years from now can be modeled as a geometric distribution. Now, we discuss in previous lectures that what is geometric distribution and this in this geometric distribution the random variable that x, that we are talking about this is a either the probability of success or maybe the success and failure you know that wegenerally attribute is arbitrarily.

So, here the overtopping of the guide bund is one event, so that event isthat event is has taken place or not that is if I say the success, means we should not means sociallywe should not say that the overtopping a guide bund is a is a success, but it is just mathematically whether the event has taken place or not that is, that the guide bund is overtopped or not. So, that one that can happen in after x years, that means so from this point onwards what is the probability that the event will occur after one year, what is the probability that event will occur after two years similarly,what is what is theprobability thatit will occur after anyanyyears. So, this random variable is modeled as this geometric distribution that we discuss earlier.

So, here is this geometric distribution form that is P x of x is equals to your pi 1 minus pi power x minus 1 for this x is the discrete value here 1, 2, 3. So, whatever the year that you will take and this pi you know is the probability ofsuccess, in the sense that that pi between 0 and one and the probability of the overtopping in a particular year, now the situation is this that tomake the guide bund more effective, it is strengthened in such a way that if I transformthat, if I want know that want do some new analysis, then the transformation y equals to 2 x can besuitablesuitable transformation. Now, if this is assess then if I want toget some probability for this, get want to know the nature of this y then, obviously I have to refer to I have to refer to the probability distribution for this new random variable y, so I have to know what the distribution of this y is.

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So, following the sameprinciple that is first of all we have to find out what is the inverse function, because you know this I have to explain in terms of this original random variable. So, this is your x is equals to y by 2, because y is equals to 2 x just we discussed, thus the PDF forthe y that is probability of y is equals to thatfunction express in terms of that inverse inverse function of thaty that is g inverse y which is y by 2. So, this one we have just put, and we have to we have to show that what is the possible value that y can take. So, this is valid for thaty equals to this is a mistake that should not be x now this is y, the y is equals to 2, 4, 6 thisonwards, because you know that y is equals to twice, of the x now if we just compare that earlier distributional form this present distribution form that, if earlier it was P x x equals to that is, so this is pi this pi multiplied by 1 minus pi power x minus 1.

And this new 1 is the P y y equals to pi 1 minus pi power y by 2 minus 1. So, this probability of failures under the old scheme x after 1 year, 2 years, 3 years, and 4 years for example, is equivalent to the probability of failure after 2 years, 4 years, 6 years and 8 years respectively and in this way it can go.

So, even though this is a very straight forward application that what we have shown here,for this is for this discrete random variable, but if weshow but the concept is here is also same. So, what when we are talking about the one-to-one transformation we are

finding out the respective domain for the new random variable and finding out their probabilities.

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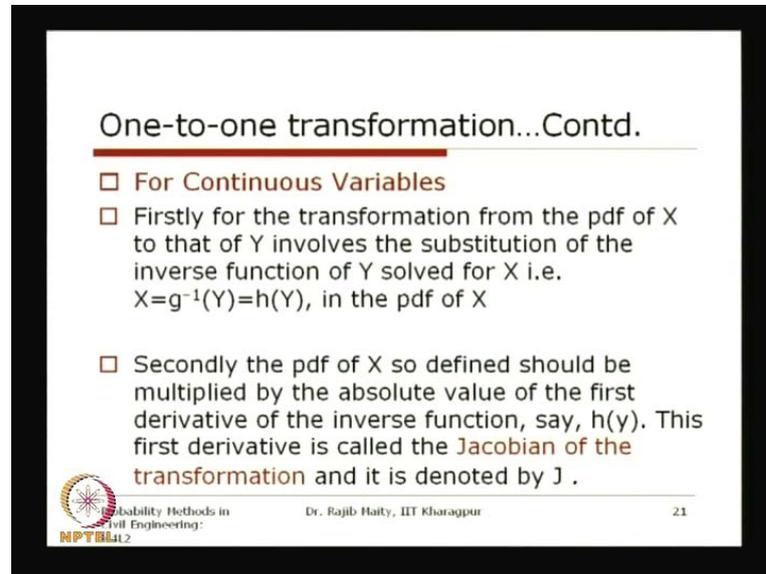
Now, we will discuss about for thefor thecontinuous random variable, now this continuous random variable if I just want to explain firstmechanically then the firstly the transformationfrom the PDF of x to that of y involves the substitution of the inverse function of y solved for x, that is x equals to g inverse y. So, this we have to substitute in terms of the x g inverse y what exactly we did for this, just now that for the discreterandom variable example.

So, secondly the PDF of x, so defined should be multiplied by the absolute value of the first derivative of the inverse function, say thath y. So, this first derivative is called the Jacobian of the transformation and it is denoted by j. So, the absolute value of the Jacobianshould be multiplied with that new with that PDF of this y express in terms of this,we have shown as the y is that x equals to g inverse y that is to be that is to be multiplied. So, mechanically whatwhat this two steps is that first of all you express the same the PDF, in terms of this the new random variable and multiply it by a Jacobian,Jacobian is nothing but, the first derivative of that, of the first derivative of the inverse function of h y that is g inverse y. Now, the same thing if I want to represent with reference to the fundamental theorem that we discussed in the last class, that we will seethen it will be more clear to us.

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One-to-one transformation...Contd.

☐ Let us recall from fundamental theorem. If $f_X(x)$ is known and Y=g(X), to determine $f_Y(y)$ a specific value of y, $y=g(x)$ is solved. If there are n real roots, $x_1, x_2, ..., x_n$,

Then, $f_Y(y) = \dfrac{f_X(x_1)}{|g'(x_1)|} + \dfrac{f_X(x_2)}{|g'(x_2)|} + ... + \dfrac{f_X(x_n)}{|g'(x_n)|} + ...$

where $g'(x)$ is the derivative of $g(x)$

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Just recall, that the fundamental theorem that we discuss in the last class, that f x is known and y equals to g x to determine the f y, a specific value of y equals to g x is solved solved, now if there are n roots that is for this equation if there n roots x 1, x 2, x n then we have seen and we have proven that this equation holds, and we have also refer to this populism and pillar where this proof isavailable as well. So, now this g prime x is also we have defined that this is the derivative, as a first derivative of this is g x.

Now, here what we are doing of, what we are saying is that this is the one-to-one transformation, that means; this root will always be one root, so when we are taking a specific value of y then there will be one and only one value of this x, because we have started with the assumption that this is the one-to-onetransformation,so there will be only one.

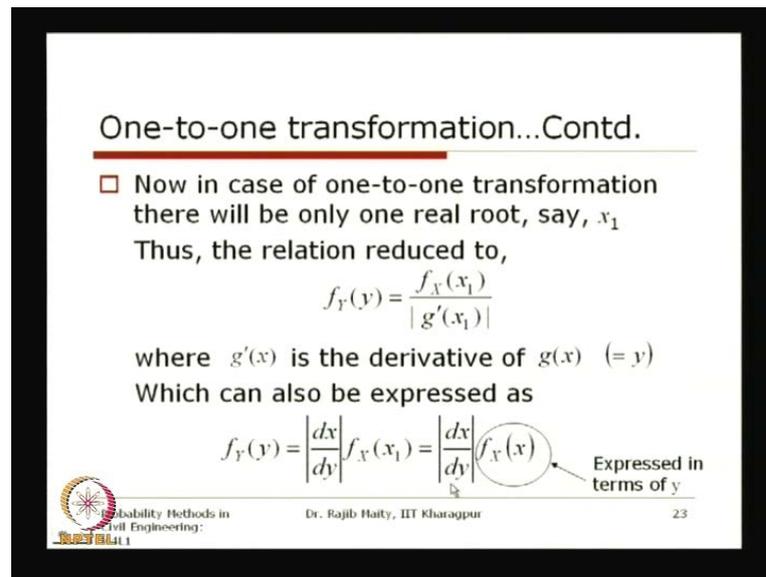(Refer Slide Time: 50:30)

One-to-one transformation...Contd.

□ Now in case of one-to-one transformation there will be only one real root, say, x_1
Thus, the relation reduced to,

$$f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|}$$

where $g'(x)$ is the derivative of $g(x)$ $(= y)$
Which can also be expressed as

$$f_Y(y) = \left|\frac{dx}{dy}\right| f_X(x_1) = \left|\frac{dx}{dy}\right| f_X(x)$$

Expressed in terms of y

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So, straight forward what we can say is that, we can just reduce this other components this is the first this one is the valid thing what is explained here, so this f y y is equals to this f x x 1 divided by g prime x 1, nowthis g x we know that this is a derivate g x is equals to your y. So, now if I just this can also be expressed in terms of this f y y is equals to d x d y, y is your g x that is just I am multiplying that is why I am just taking it inverse, so multiplied by f x x 1 as there is only one root just to make it that, so there is no x 1,x 2, x 3, x 4, so just we are replacing it byinters of this x.

Now, what we have to do is that, because this is the f y, so you have to explain this full function should be with respect to the y, so this should be express in terms this y; now what is this, this is the PDF of this x which is express in terms of this y multiplied by this factor, which is that first derivative of the inverse <mark>inverse</mark>function, because this we have taken that inverse the d x by d y this is nothing but, the Jacobian this is what we have discuss in this first slide of thiscontinuous random variable.

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So, this f y y equals to this absolute value of this Jacobian multiplied by this one which is expressed in terms of this y, so this multiplication it can be proven that this multiplication with this Jacobian is nothing to ensure that this integrates to unity, which is the requirement of this PDF you know.

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So, we can take some example that is first and foremost that example, that we can take is that this for, this from the normal distribution to this standard normal distribution; we know that this is known to us earlier we explain that is for this standard normal

distribution, if you just take this one this relationship that is x minus mu by sigma wheremu is the mean and sigma is the standard deviation. So, this distribution of y will become a standard normal distribution with mean 0 and standard deviation 1.

Now, so this is one, one-to-one transformation and if this is the one-to-one transformation, how it will happen that whether using the same equation that you have used it can be shown very easily, that how it is it is transforming to this standard normal distribution. Similarly, for this log normal distribution we know that y is equals to log x ==if that== if that==that== y is equals to yourthat normal distributionthen x is the log normal distribution. So, we know thatdistributional form is normal then from that equation y equals to log x, this is also one-to-one transformation. So, whether using this one, whether you get that distribution of this log normaldistribution that we have already discuss, now we will see that whether from the normal distribution, whether that that log normal distribution can be achieved or not.

Similarly, for the others distributions also, we will see a through different examples of this civil engineering problem in thenext class,thank you.