Stability of Structure

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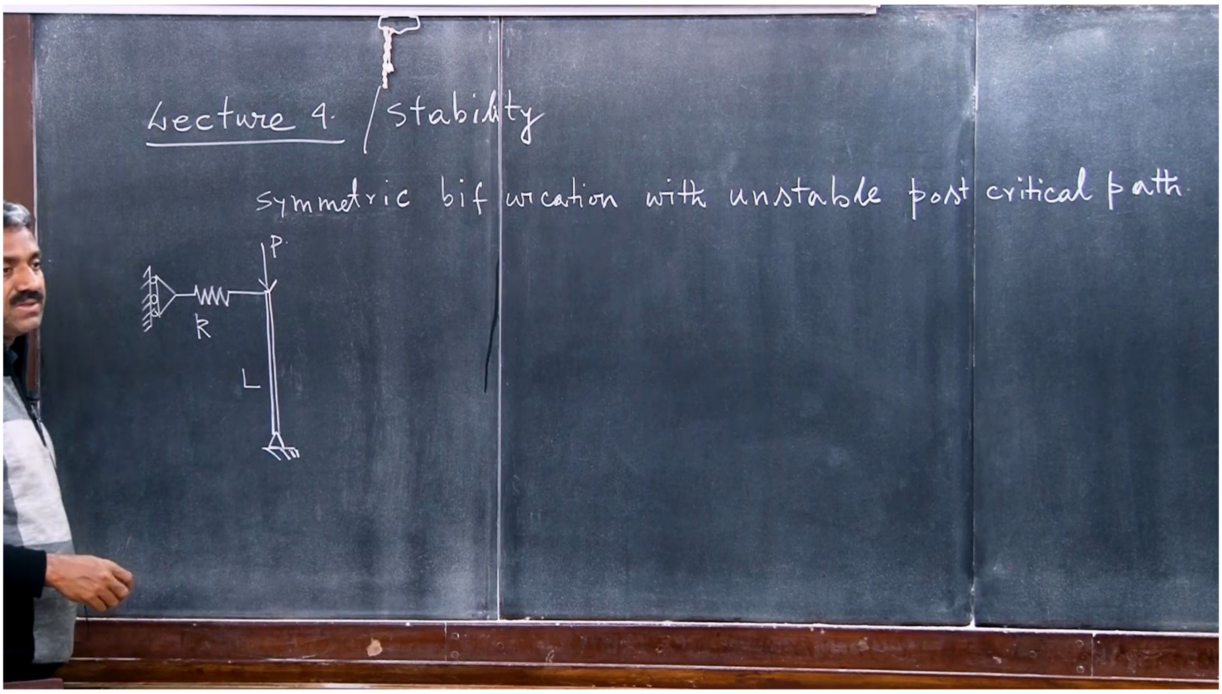
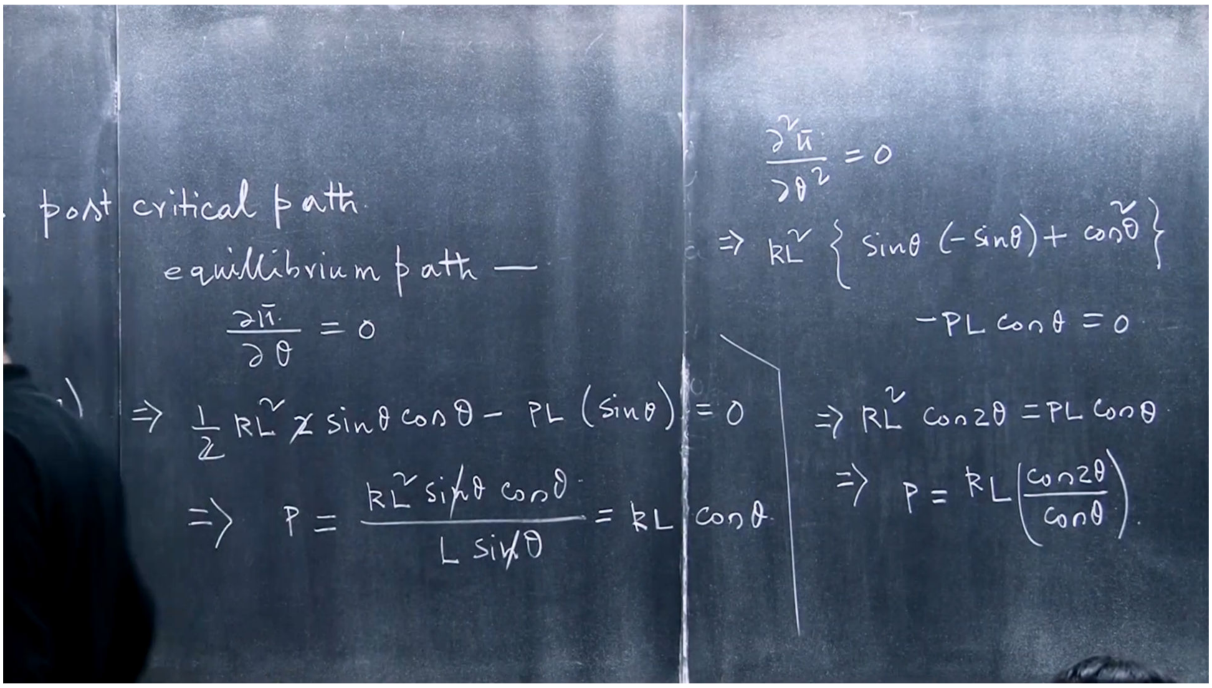
WEEK-02

Lecture 04: Bifurcation Stability: Symmetric Unstable and Asymmetric Mixed Paths

Okay welcome to fourth lecture on stability of structure. So, let us briefly recapitulate what we have done. So, we introduced the concept of stability. We have also learned that how we can ascertain about nature of stability through the energy functional and we have solved a simple problem, thereafter we have considered 4 different kind of system which are simple system consisting of rigid bar with concentrated spring right, concentrated elasticity. restoring force are offered by this concentrated spring either rotational spring or translational spring right. And then we have seen that the stability behavior of all the structural system, elastic structural system can be approximated by considering this apparently simple system. The first system we have considered. a rigid bar restrained by rotational spring which is subjected to a low concentrated load P and we have seen that the system demonstrates symmetric bifurcation behavior. and the post critical path is stable right. So, a stable fundamental path bifurcates into an unstable into a stable post critical path and an unstable fundamental path right. And then we have shown that this kind of system which is showing this stable bifurcation also demonstrate we have also investigated its imperfection sensitivity and we have seen that it follows two-third power law of imperfection sensitivity. but it is not imperfection sensitive meaning the critical load increases with increasing imperfection okay which is a hallmark of this kind of stable bifurcation right. So, now we are going to consider we have also seen in a previous example that example that. The bifurcation is eliminated on incorporation of imperfections. So, bifurcation behavior is only noted when it is a perfect system, right.

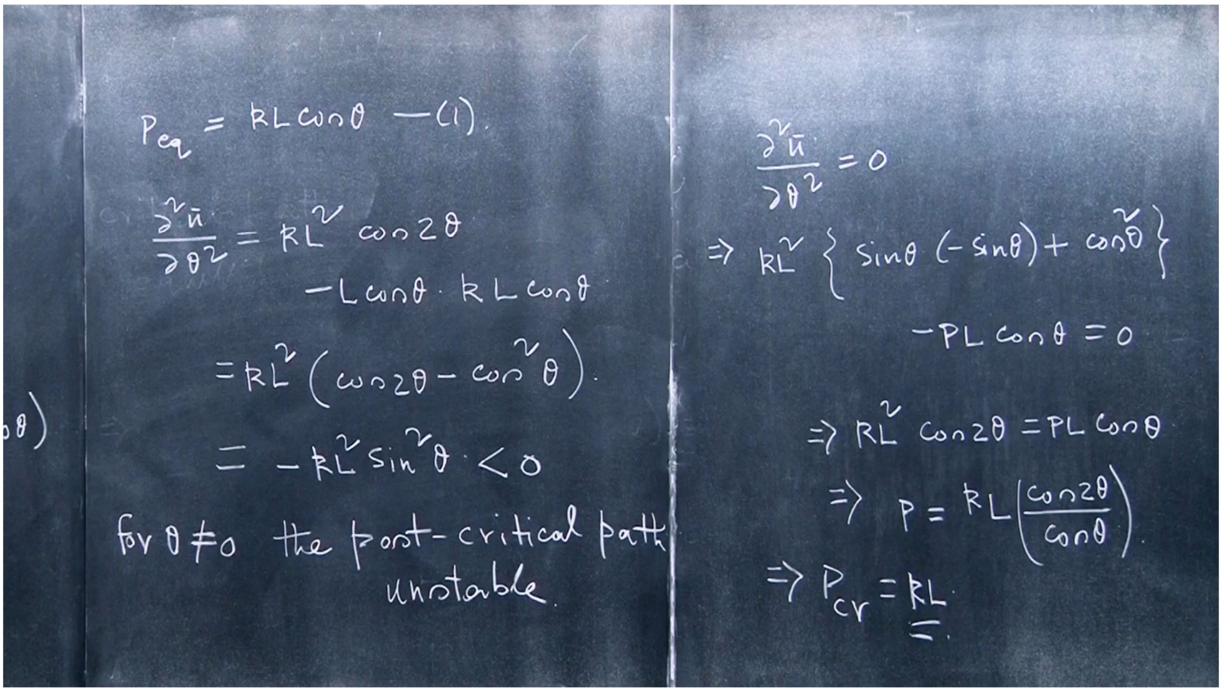


So, stable fundamental path is bifurcating, okay. fundamental path means where the equilibrium is for $\theta = 0$ for non-zero value of θ okay. this is the path which basically showing the bifurcation right but on incorporation of imperfections the bifurcation ceases to exist rather the stability behavior the stable you know the equilibrium path asymptotically approaches the the equilibrium path of the perfect system that we have also demonstrated in the stability diagram. Now we are going to consider the second category of system which shows symmetric bifurcation but an unstable post critical path ok. So, the hallmark of second kind of system is symmetric bifurcation with unstable post critical path. So, this system you may recall we have all discussed about this toy system. So, this system is once again this is a rigid bar and this rigid bar is pinned here and then it is restrained by so here it is restrained by you know. spring of stiffness k and this is P and the length of this bar is L okay. This bar is rigid. You see this support has to be you know ruler. Because it must allow you know the translation along in its vertical plane right. So, now the first lesson that we have learned in any stability analysis is we have to perturb the system. We have to look for alternate configuration. So, the system degree of freedom. this is a single degree of freedom system. Because all the displacement here can be described essentially by rotation right because it is rigid. So, let us perturb it. So, this fellow will you know its locus will be alternate locus will be here right and then it will come down and then you know this thing goes like this right. So, I am assuming this, let us first study the perfect system and then we will impart imperfection right. So, this angle is θ right. So, then load P is going there and if it is L you know, so I am going to write down the potential energy function. So, potential energy is equal to strain energy minus work done. So, strain energy is what? What is the strain energy? Now strain energy is contributed by this spring which is deformed right and what is the displacement between these two points. if this angle this is L, then $L\sin\theta$ this one right $L\sin\theta$ so you can write $\frac{1}{2}kL^2\theta$ right. So, the restoring force is k into $L\sin\theta$ right, is not it? And then so that is $\frac{1}{2}kL^2\sin^2\theta$ right. So, minus and then what is the work done? The work done is the vertical displacement, and it is nothing L and this is $L\cos\theta$. So, P into $L(1-\cos\theta)$ right, this is fine right. So, let us first find out the equilibrium path. So, a system needs to be in equilibrium. For equilibrium we must have the potential energy function minimum right. Minimization of potential energy function $\frac{\partial \Pi}{\partial \theta}$. so, what we obtain is $\frac{1}{2}kL^2 \cdot 2\sin\theta\cos\theta - PL\sin\theta = 0$ right. So, $P = \frac{kL^2\sin\theta\cos\theta}{L\sin\theta}$ and then this will go so $P = kL\cos\theta$ right. This is equilibrium path right. and then we are also going to find out critical value.

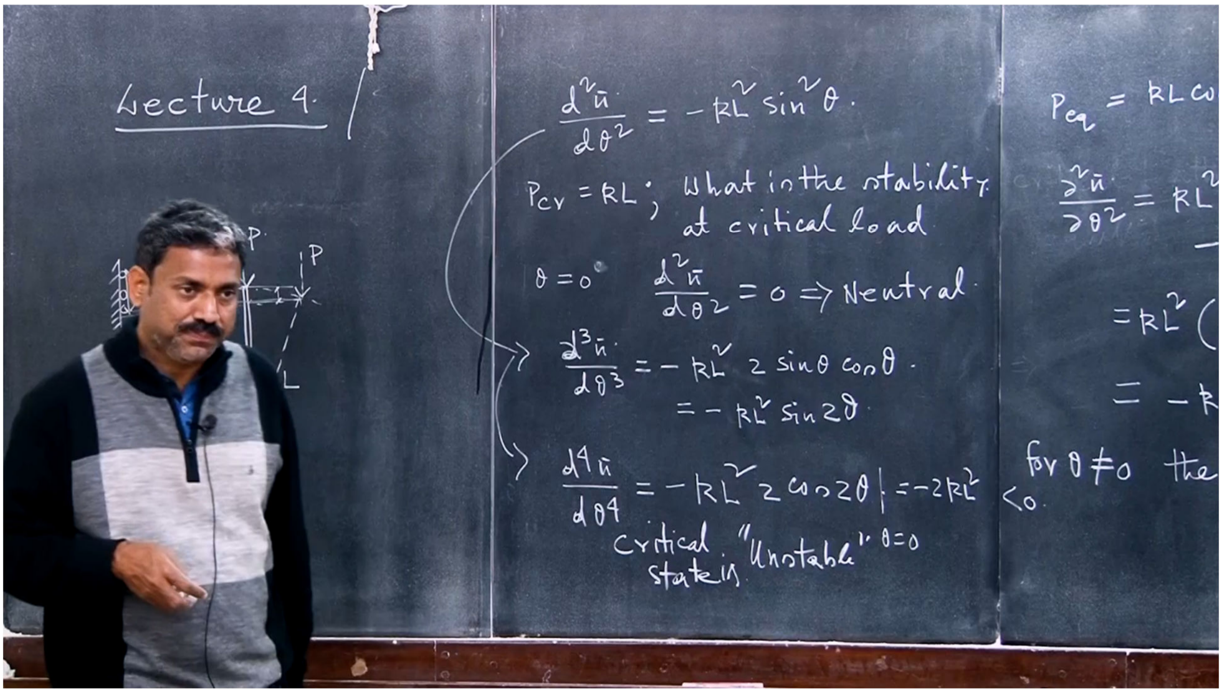


So, for that $\frac{\partial^2 \Pi}{\partial \theta^2} = 0$, $\frac{\partial^2 \Pi}{\partial \theta^2}$ to further differentiate what we are going to get is $kL^2(\cos^2\theta - \sin^2\theta) - PL\cos\theta = 0$ right. Then it is $kL^2\cos2\theta = PL\cos\theta$. or $P = \frac{kL\cos2\theta}{\cos\theta}$ right fine.

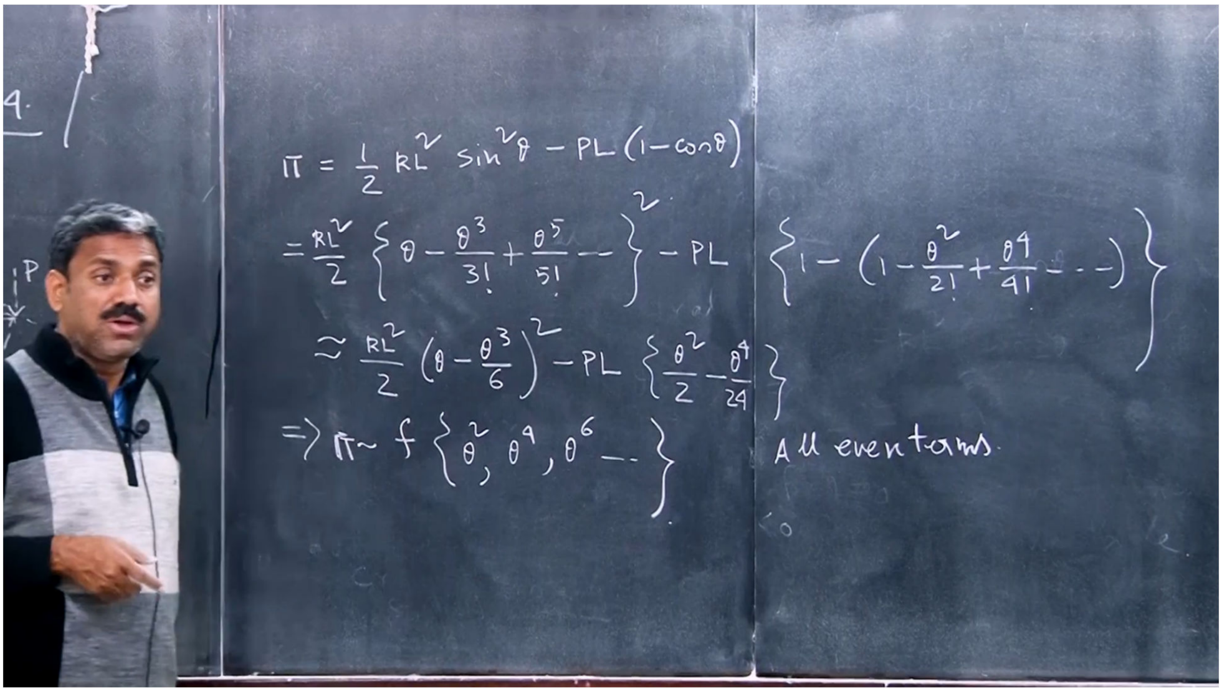
Now ok. let us keep the equilibrium path. So, here we will put this you know $P_{\text{eq}} = kL\cos\theta$ ok and what we will do we will substitute this equilibrium path on this condition right. So, $\frac{\partial^2 \Pi}{\partial \theta^2}$. So, you see that here when $\theta \to 0$ right. So, what it will $P = kL$ right. So, $\theta = 0$ right. So, you know that means you know P_{cr} it will be kL right fine and $\theta = 0$ that is what the critical load is ok. and $\frac{\partial^2 \Pi}{\partial \theta^2}$ is will be nothing but $kL^2\cos 2\theta - PL\cos\theta$ and P is nothing but $kL\cos\theta$ ok. So, here if you $kL^2(\cos^2\theta - \sin^2\theta) - kL^2\cos^2\theta = -kL^2\sin^2\theta$ right. So, what we see $\frac{\partial^2 \Pi}{\partial \theta^2}$ is always less than 0 right. So, that means it is always unstable that means you know for $\theta \neq 0$. For $\theta \neq 0$ that means you know for the any perturb configuration. that means the perturb configuration it is always unstable so the post critical path is unstable.



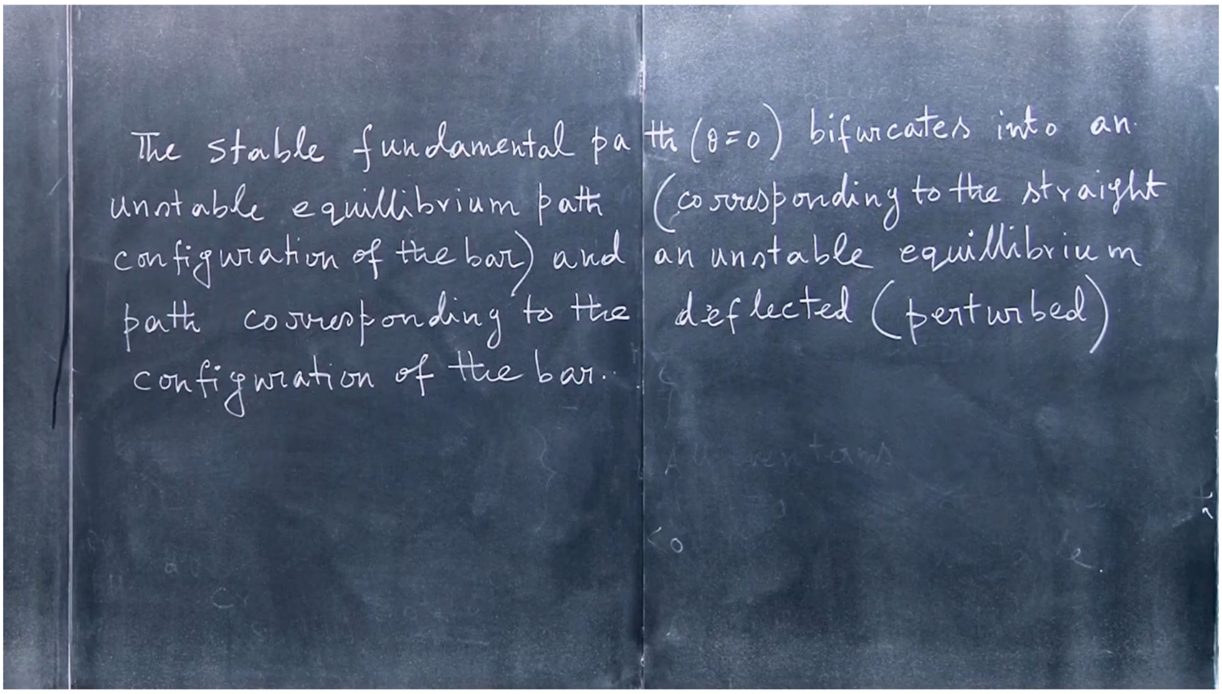
So, you understand bifurcation will occur at critical load kL right and then it will lead to a stable post critical unstable post critical path right clear. Now what will happen at $\frac{\partial^2 \Pi}{\partial \theta^2}$, Please note that although I am writing partial it is basically it can be total derivative because only one variable right. So, this is $-kL^2\sin^2\theta$. So, what about in the critical load? So, $P_{\text{cr}} = kL$ and you want to you know check what is the state of equilibrium at. So, at the when it is just reaching the critical load right. So, what is the stability at critical load right. So, if without you know investigating on the higher order derivative. if we take $\theta = 0$ then what is happening $\theta = 0$ then $\frac{\partial^2 \Pi}{\partial \theta^2}$ is what? It is 0 that means it is indecisive. So, a prior you may comment. So, that which implies that, it is in neutral equilibrium, neutral but that is not always correct. So, for that you have to investigate higher derivative. You will see that is kind of confusing you know that will lead to a wrong conclusion ok. So, what will happen? You just differentiate it twice. So, you just $\frac{\partial^3 \Pi}{\partial \theta^3} = -kL^2 \sin\theta cos\theta$ right. That means $-kL^2 \sin 2\theta$ right here also if you take $\theta = 0$. it is 0 or anything. it will not give any clue right once again I will differentiate it further. what is going to happen $\frac{\partial^4 \Pi}{\partial \theta^4} = -kL^2 2\cos 2\theta$. What is this for $\theta = 0$? $\theta = 0$. it is $-4kL^2 < 0$ what does it mean? that means at critical load it is unstable. So, critical that means at critical load this is unstable, you see it is still unstable. whenever "A critical state is inherently unstable.



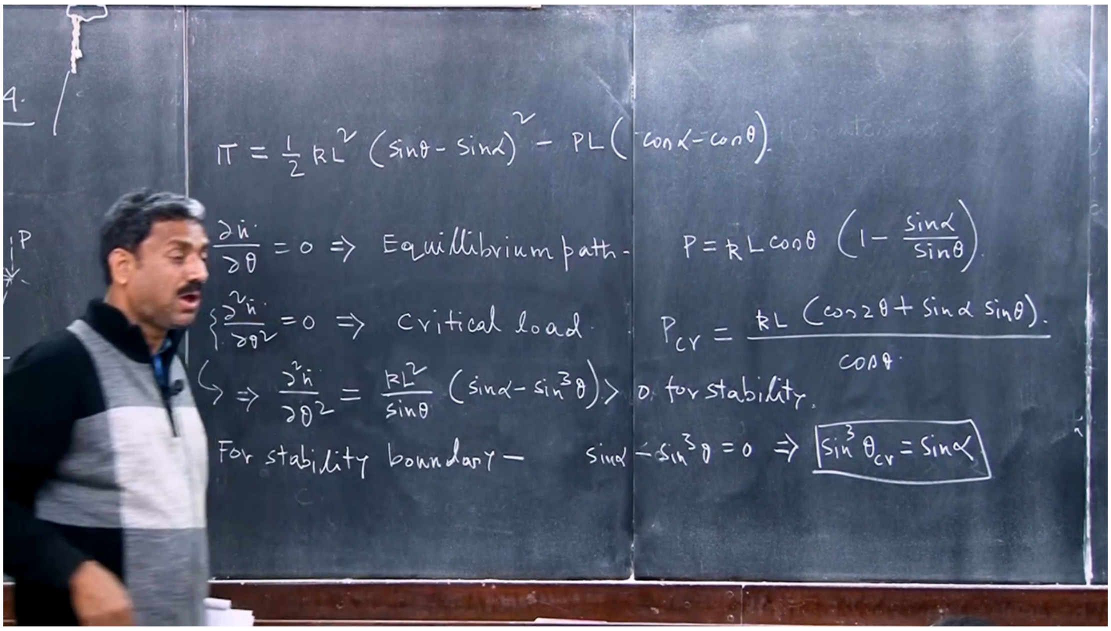
"So, if we try to assess the state and conclude the stability from the secondary medium, we find that it leads to a strong notion." So, you have to check for higher derivative and we can see indeed like the post critical path. you know is unstable at the critical load that equilibrium is also unstable clear. So, you understand a stable fundamental path bifurcate into an unstable critical path right. Post critical path unstable critical path as well as unstable post critical path right which is symmetry ok. So, now what will happen we just draw how does it look like. So, we have studied the behavior of the system. Also, another thing you would like to you know, you have noted down things right ok. So, what was the expression of Π, Π is what $\frac{1}{2}kL^2\sin^2\theta - PL(1-\cos\theta)$ that was the expression right. So, I just want to $kL^2/2\sin\theta$. if your $\theta - \theta^3/3! + \theta^5/5!$, whatever right highest we can always neglect $PL(1-(1-\theta^2/2!+\theta^4/4!)$ right. So, what we see, we neglect the higher order term but we just keep it. and this is a square. So, $\frac{1}{2}kL^2\left(\theta - \frac{\theta^3}{6}\right)^2 - PL(\theta^2/2 - \theta^4/24)$. this will go and this is something like that right, now you see that, so Π is a function of what? It becomes a function of quartic term, so θ, θ^2, θ^4, θ^6 right, all are square you know square term.



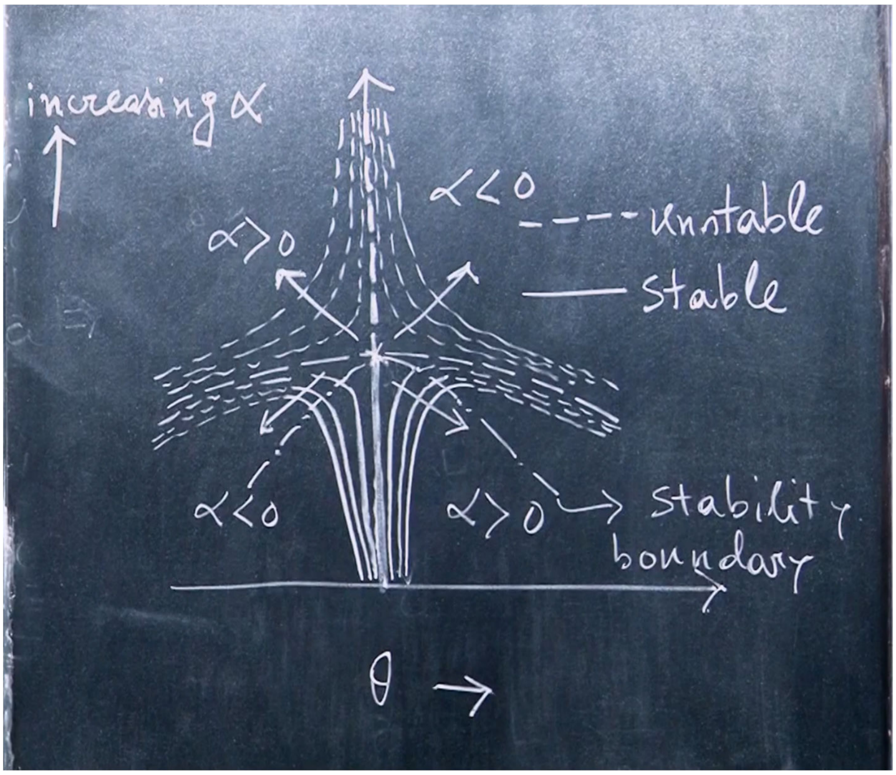
So, like earlier these are all what? it is always positive definite potential energy functional right. this is Π. it is always consisting of, there is no odd term, all are event term. because all are event terms right that is why it leads to what symmetric bifurcation, please note that. The difference with the previous system where the rigid bar was restrained by rotational spring is that both lead to stable symmetric bifurcation but one is stable but this one is unstable. But in both the cases the potential energy functional consist of term. which are all appearing in square stick like this ok, clear. So, please note that because there is no word term that is why symmetry ok. So, if you draw the equilibrium path and if you draw the stability diagram. the way it will look like is θ and then I am just right here and then you see all these are unstable and from there you know these are you see that it is the critical load. this value is nothing but kL okay, and after that you see earlier it was concave surface here and now it is convex. this is so all this dotted line is unstable right and this is stable.



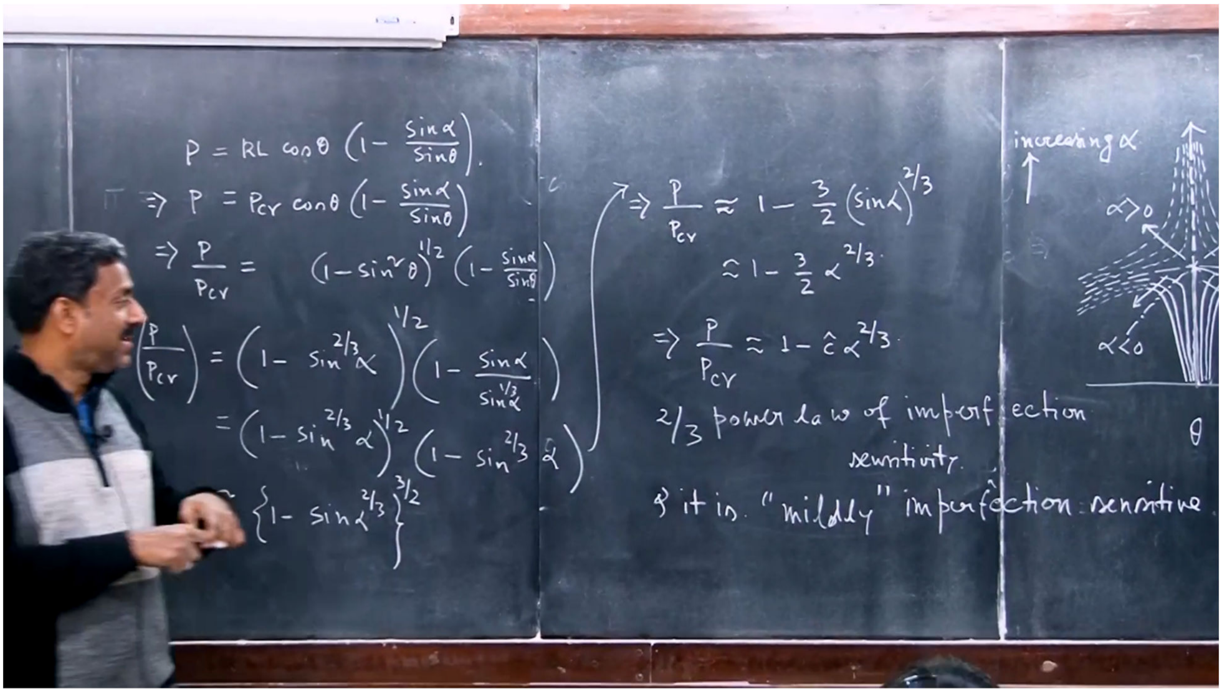
So, a stable fundamental path that $\theta = 0$, and bifurcate into an unstable fundamental path and unstable post critical path okay right fine. So, if you want me, I can just write it. so, the way previously I have written. I assume you have noted that right, be careful about the potential energy functional okay. So, I will just write here I am just removing these things all of you have noted down right. So, the stable fundamental path that is $\theta = 0$ bifurcate into and unstable equilibrium path. which is corresponding to the straight configuration bar and an unstable equilibrium path. Corresponding to the deflected (you may use perturbed) okay. These are not deflected for the perturbed. Configuration of the bar. So, now we have to study the imperfection sensitivity right. So, what we have obtained? So, you are understanding that what bifurcation is. So, from here this is going but at $\theta = 0$ and this load is reaching. it is unable to stay you know this in its equilibrium. So, in order to have an alternate equilibrium we have to deflect it has to have θ is the non-zero value of θ right. And that the equilibrium path is unstable right fine and so this is bifurcation because this you know path is followed and then here it is bifurcating into a different path alternative path right. So, that is why bifurcation is happening right and it is symmetric with respect to this and this is also associated with the symmetry breaking. why symmetry breaking because what is happening you see until this when the load is less than that you take the bar and perturb it. So, you take any perturbation like θ positive negative. It will remain its $\theta = 0$ right. So, it does not distinguish between $\theta = 0$ or θ positive or negative right. So, it was symmetric basically with respect to its stability right. and it will bring back to this. But as soon as this is reached then it intends to diverge it is unstable. So, when you are perturbing. it will what? it will diverge to any particle. if you are giving a positive perturbation it will tend to divert towards that. So, that means it is breaking and it is losing its symmetric behavior with respect to its deflection you see that. So, that is the notion of symmetry breaking although this is a more generalized term that is used in physicists' physics actually. this symmetry breaking for the bifurcation system and others for elastic system stability. This is also relevant ok. So, now we are going to so all of you have taken a note of this right. Now we are going to consider the imperfection to see its imperfection behavior. So, the potential energy for the imperfection, when we are treating it to be imperfect right. So, what we are assuming that initially there will be some reflection right. this angle is α; α is the imperfection here and then α is the magnitude of imperfection right. So, then potential energy once again it is very simple earlier, whatever we have used L the deflection δ will be the spring deflection. and it become $L\sin\theta$. So, $L\sin\theta$ whatever earlier but because of imperfection it was $L\sin\alpha$ so it is $L(\sin\theta - \sin\alpha)$ right. and the vertical deflection of under this P right that will be what earlier it was L you know $\cos\alpha$ minus now it is $L\cos\theta$. So, $L(\cos\alpha - \cos\theta)$ right α is the imperfection right. So, I am writing the expression. So here $\frac{1}{2}kL^2(\sin\theta - \sin\alpha)^2 - PL(\cos\alpha - \cos\theta)$. So, I am not going to do the differences of all these things. now I will just write the expression just to know how to do that right. fine this was the only difference right one incorporation of the imperfection α. So, α is nothing but the initial perturbation ok. So, then $\frac{\partial \Pi}{\partial \theta}$ I want to find out the equilibrium path. so, equilibrium path how it is given? Equilibrium path will be $kL\cos\theta\left(1 - \frac{\sin\alpha}{\sin\theta}\right)$. it is simple if you just substitute you will get it ok this is the equilibrium path. and from for the critical load $\frac{\partial^2 \Pi}{\partial \theta^2}$. So, that is the critical load right critical load value of P. So, then P_{cr} will be $kL\frac{(\cos 2\theta + \sin\alpha\sin\theta)}{\cos\theta}$ see if you substitute $\alpha = 0$ then it will mimic the perfect system right and now I am not doing the simple manipulation calculations that you can do yourself right now what I will do for the critical whatever $\frac{\partial^2\Pi}{\partial\theta^2}$ we have got the equation right. So $\frac{\partial^2\Pi}{\partial\theta^2}$ will essentially be we will combine these two see what we are going to do $\frac{\partial^2\Pi}{\partial\theta^2}$ we will definitely we will get in terms of P right but then we must impose the equilibrium on that



So, then the way we did earlier we will try to get the expression in terms of θ and P right. sorry it will be k right this k ok because it is the not rotational spring but transverse spring the stiffness of the spring ok k is the stiffness. So, here this if you substitute the way you are doing. you are making it 0 then you are double differentiating it and then you are substituting whatever the equilibrium. So, combining you will see that simple expression I am going to get $\frac{kL^2}{\sin\theta}(\sin\theta - \sin^3\theta)$, So, now you tell me what will be the stability condition? it is greater than 0 right this for stability condition right. and we want to find out the stability boundary now so stability boundary what? stability boundary means when $\frac{\partial^2\Pi}{\partial\theta^2}$ double derivative is going from positive to negative. then we said 0 that means when it will so for Stability boundary $(\sin\alpha - \sin^3\theta)$ must be 0. And then from here we get some critical value. So, $\sin^3\theta_{\mathrm{cr}} = \sin\alpha$ ok.

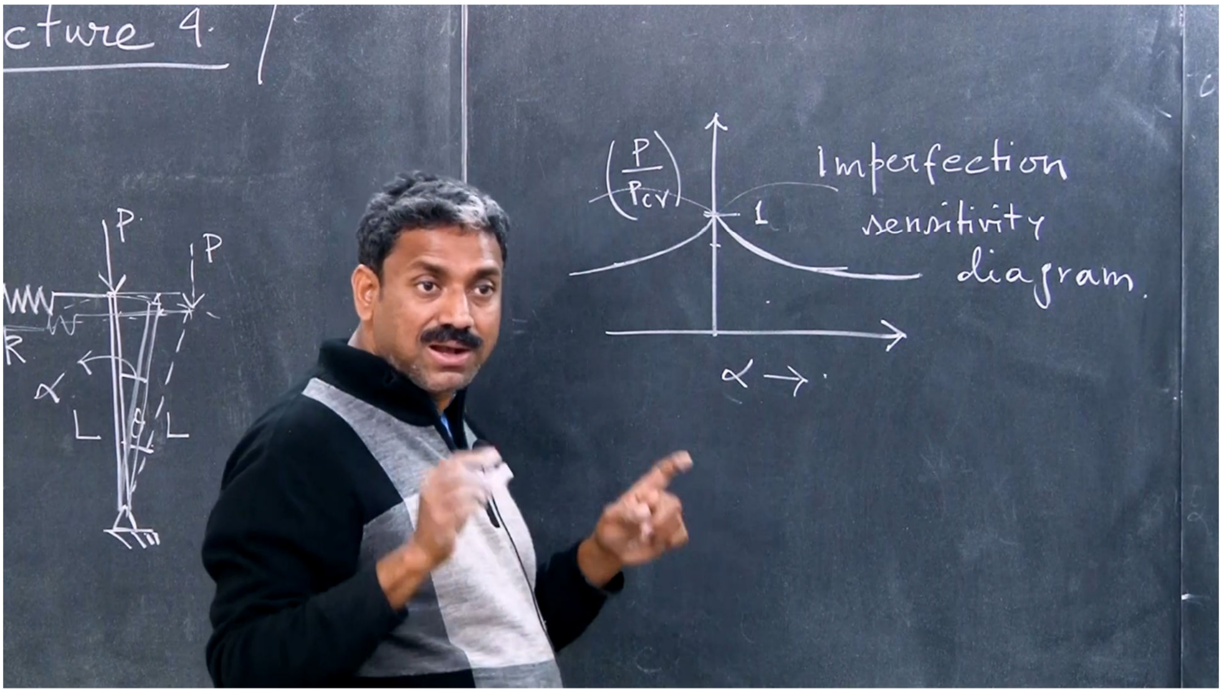


So, that basically gives us the you know stability boundary ok. Now of course how will you do the path, of course we will take different value of θ and we will take little imperfections and we will plot there. "If you plot it the way you learned earlier, it will represent an imperfect system and will be asymptotic to this. " So, let me see how it will look like. Thank you. So, this is $\alpha > 0$ and $\alpha < 0$. So, you see that so here this is the stability boundary ok. So, by now, you all know why this is shifting — it's because this needs to be positive, right?" So, you can understand that for different value of α the stability boundary will change right you see that. So, these are for increasing α. when I am you know putting this so that means you know increasing α in increasing α ah imperfects. You see this little flowery structure you know another thing so please draw this figure first. Now so you understand this diagram, from here you can clearly see that. So, hessian positive also implies that $\frac{\partial P}{\partial \theta}$ is also positive. $\frac{\partial P_{\mathrm{eq}}}{\partial \theta}$ is also positive right because hessian it is equivalent to be the to the tangent stiffness right. So, you see that here you know if you take this slope is positive until this here also negative slope is negative. So, essentially this is positive ok $\frac{\partial P}{\partial \theta}$. "So, that's why it's positive at first, and then it becomes negative — $\frac{\partial P}{\partial \theta}$. This indicates instability, and it depends heavily on the imperfection magnitude, α. Okay?" Now, we will see the imperfection sensitivity. how will you do that? the way you did previously.



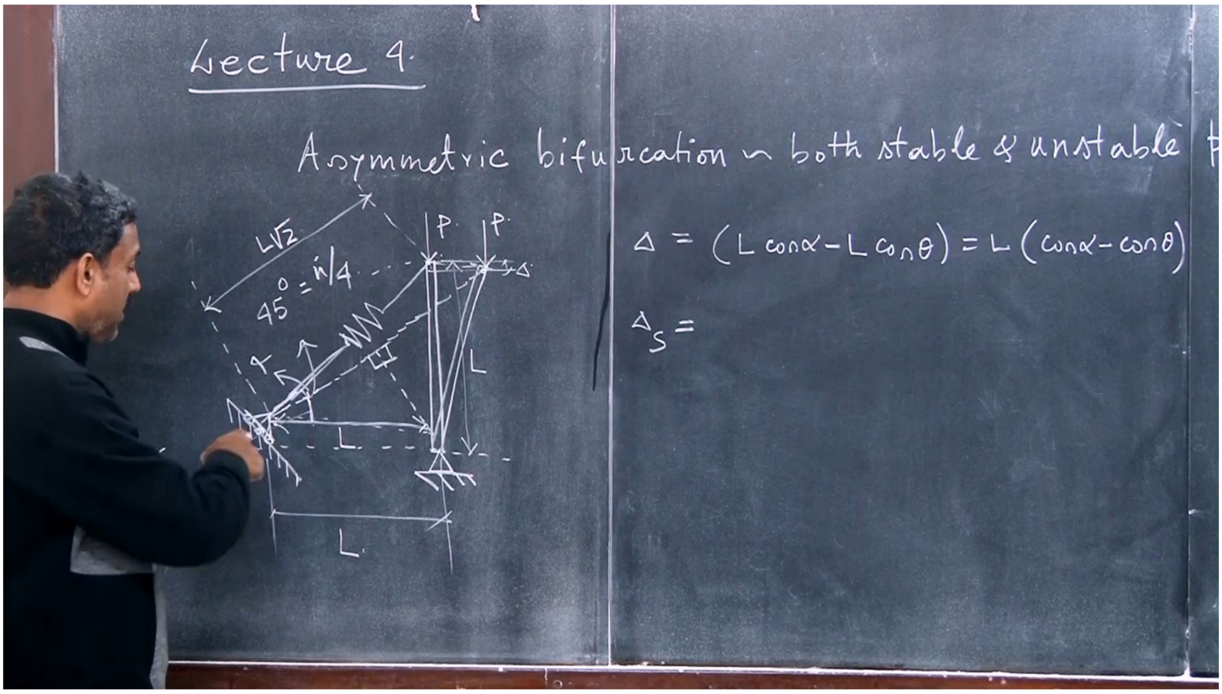
So, earlier we directly attack this P_{cr} right and then we made that from the stability boundary. How we have reached at the imperfection sensitivity.

We consider the stability boundary and, by eliminating another variable. we established a relation between α and θ. For some value of θ, the stability boundary essential related θ to α. That means α plays a critical load, in terms of the stability boundary ok. So, we identified that value and substituted it into P_{cr} That's how we found it in the previous example, right?" "Here, we can do the same thing. In the previous example, where we had a stable post-critical path, we saw that it followed a power law — the critical load increased with increasing imperfection." There is another format to the imperfection sensitivity that directly start with the equilibrium path itself. So, instead of any substitute here, I am going to substitute this here, okay. And you will see I will bring in a different, you understand that pkL is nothing but p_{cr}. Critical means that $\theta = 0$, right, okay. So, P is equal to $kL\cos\theta\left(1 - \frac{\sin\alpha}{\sin\theta}\right)$ right and what we have seen $\sin^3\theta = \sin\alpha$. We want to relate P in terms of α right and what I know kL is critical. So, P if we see $P = P_{\text{cr}}\cos\theta\left(1 - \frac{\sin\alpha}{\sin\theta}\right)$. Fine now, $\frac{P}{P_{\text{cr}}}$ is the instability boundary. I can see $(\sin^3\theta = \sin\alpha)$ right. So, I will substitute here. so, $cos\theta$ is nothing but $(1-\sin^2\theta)^{1/2}$ and $(1-\frac{\sin\alpha}{\sin\theta})$ this ok. $P/P_{cr}=(1-\sin^2\theta)^{1/2}\left(1-\frac{\sin\alpha}{\sin\theta}\right)$ then what I will do, we will do you know $(1-\sin^{\frac{2}{3}}\alpha)^{\frac{1}{2}}(1-\frac{\sin\alpha}{\sin^{\frac{1}{3}}\alpha})$,Then $(1-\sin^{\frac{2}{3}}\alpha)^{\frac{1}{2}}(1-\sin^{\frac{2}{3}}\alpha)$, then it will become $(1-sin\alpha^{\frac{2}{3}})^{\frac{3}{2}}$ So, then I will just further simplify it from here. so, P/P_{cr} will here if you take binomial $1-\frac{3}{2}(\sin^{\frac{2}{3}}\alpha)$ and $then\ it\ is$ $(1-\frac{3}{2})\alpha^{2/3}$ so what we see that P/P_{cr}= $1-\hat{c}\alpha^{2/3}$. So, what do you see in imperfection if we increase the load carrying capacity then it will decrease, "So, this shows that it follows a two-thirds power law of imperfection sensitivity. It's an imperfect system and is mildly imperfection-sensitive. I'll explain to you why it's considered mild, okay?" Why it is mildly that I will explain to you. The previous example P_{cr} was $1+$. that means with imperfection critical load was increasing but here it will be decreasing, load carrying capacity will decrease. So, that is what earlier one it was not imperfection sensitive, but this is imperfection sensitive. okay and it is decreasing. So, we will also draw the imperfection sensitivity diagram. So. imperfection sensitivity diagram will look like α here and here P/P_{cr}. and this is P_{cr}, I mean this value is 1 maybe you can put it 1, and this is decrease and it is called the imperfection sensitivity diagram right. The previous one how it was looking like? "From this point, it was going upward — remember? Earlier, the path was going up like this for the stable post-bifurcation case.



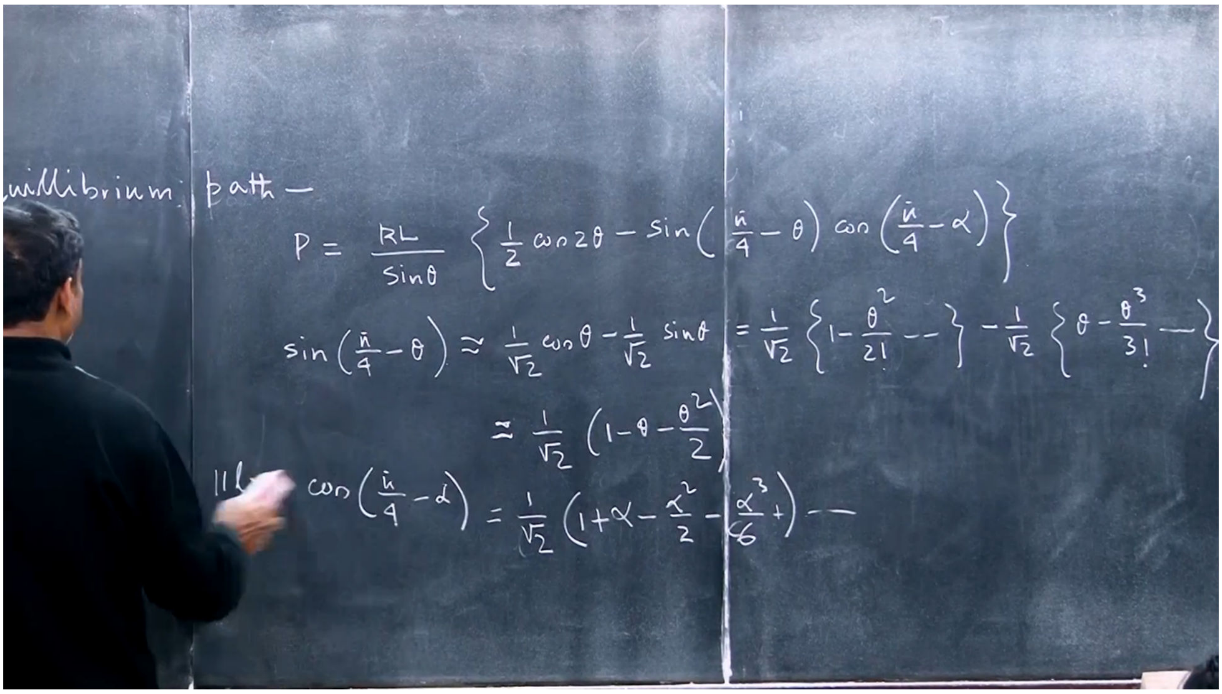
But here, it’s going downward, which indicates an unstable bifurcation. Okay?" And what is this I told you we have mentioned that of course the way we are doing is little ad hoc by using very simple system toy system and with simple or expression simple energy potential energy functional but this there was a rigorous derivation by von Karman. Koiter imperfection sensitivity ok. So, we have learned the second system and its behavior. What kind of system will show this kind of behavior? Can you associate this physically? load it will be mildly imperfection sensitive ok. So, some of you know in the previous example, where it was imperfection insensitive and the stable post critical bifurcation.

"This kind of behavior is observed in large deformation buckling of columns, buckle columns, and even buckling plates." And here where it is the unstable post critical path. this is demonstrated by some of the buckling mode ok. Buckling mode means you know this is single reference system right. So, you will see that when you will study the shell buckling. buckling will have multiple mode and some of these modes are very closely spaced. So, it will have symmetric mode, it will have anti symmetric mode. Some of the buckling mode they are especially the one which are you know symmetric. that mode will be mild imperfection sensitive ok, where the load will drop with increasing imperfection ok. various mode will show different but some of the symmetric mode may show this kind of cell buckling ok. "So, that is the physical example, and you can mimic the equilibrium path here using finite element analysis." What you have also understood that when you are incorporating imperfection the bifurcation is disturbed right. It asymptotically approaches the bifurcation behavior, following the bifurcating path. However, when imperfections are incorporated, the path no longer exhibits any bifurcation. It follows a different path, but it is asymptotic to the bifurcation path. This is the fundamental behavior of the imperfect system, clear?" So now we are going to consider a third category of system which will so both the system we have discussed. are symmetric bifurcation, they give symmetric bifurcation. now we will consider asymmetric bifurcation. So, please recall that for both are symmetric bifurcation. but in one case it is stable post critical path whether in other case it is unstable post critical path. "In one case, the equilibrium at the critical load is stable, while in the other case, the critical equilibrium is unstable.

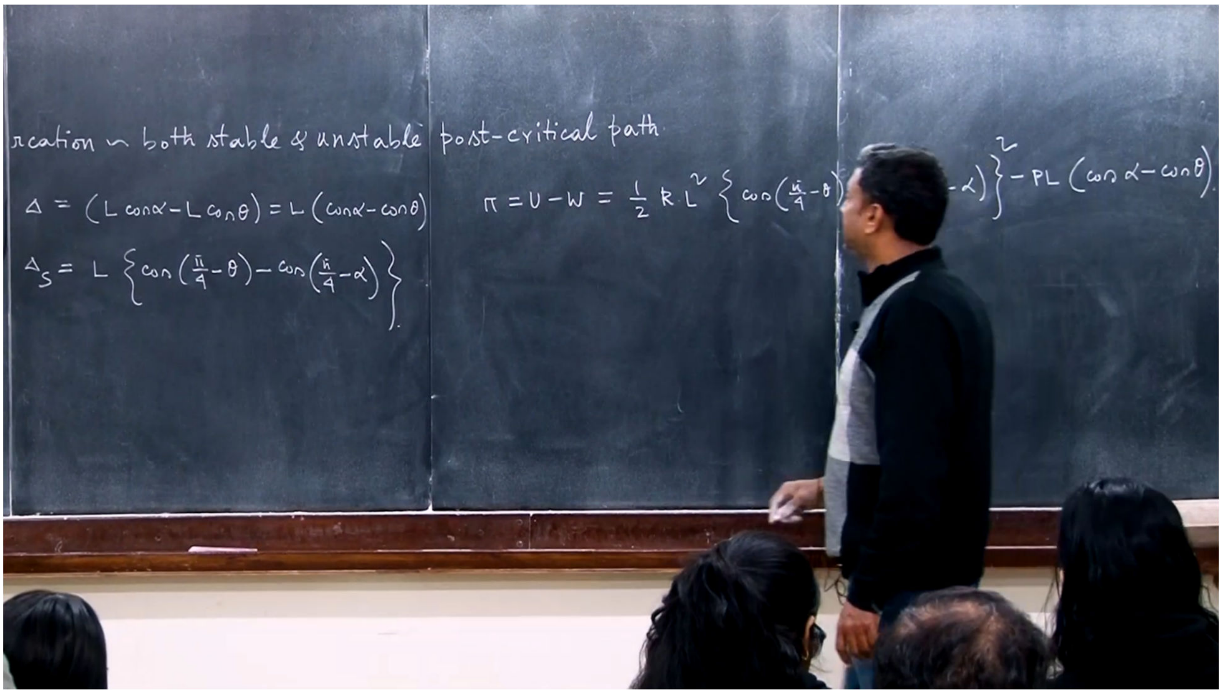


This instability cannot be captured using just the Hessian; you need to consider higher-order derivatives." Now, with the incorporation of imperfections, we have seen that one system does not exhibit imperfection sensitivity, while the other does. Both follow a power law of imperfection sensitivity and are mildly imperfection-sensitive

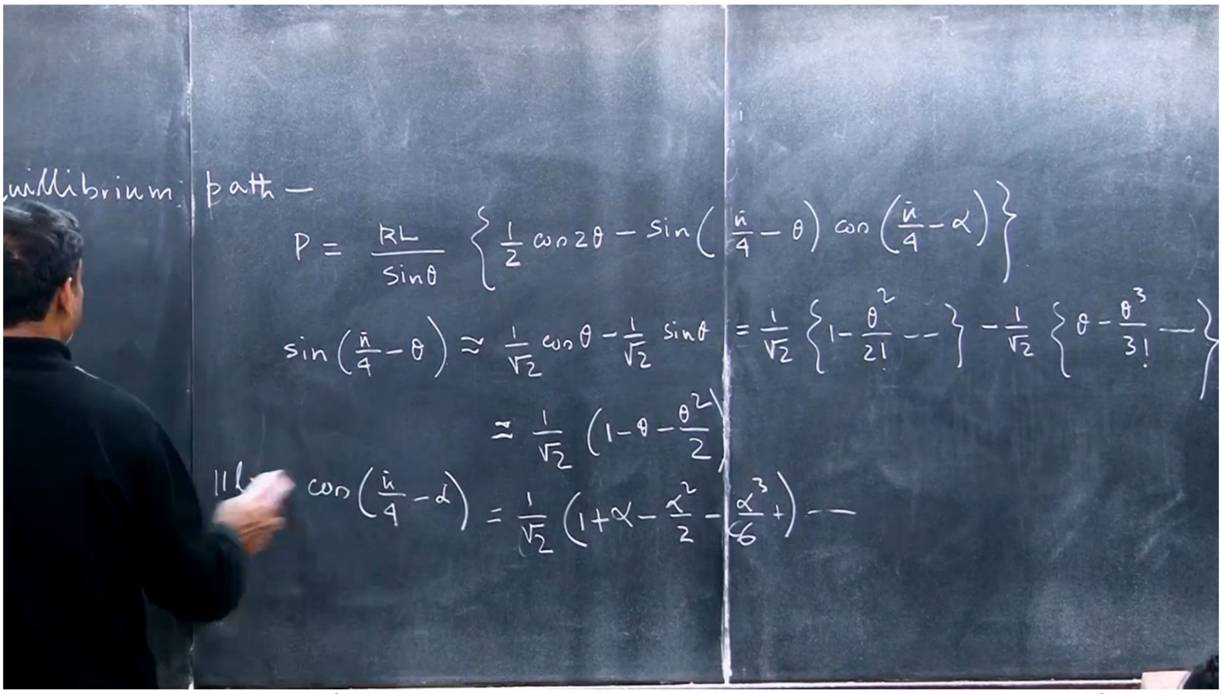
" So, you will see that whenever the post critical path is unstable. it will be imperfection sensitive. So, it is a general conclusion okay. "Okay, now we will discuss asymmetric bifurcation. Because it is asymmetric, both the stable and unstable post-critical paths occur. " So, it shows the same system because it is asymmetric. it shows both the stable as well as unstable ok. So, you know that there is a multiple behavior in the same system ok. So, for that we are going to consider a system. and now I am not going to consider the perfect system ok. I am going to consider the imperfect system directly and from there we will specialize for the perfect system ok. "Here, you see a rigid bar subjected to a load P. It is restrained by an inclined translational spring. You can understand that... have to have a roller support from the inclined plane okay. And then this angle we will put this angle to be you know 45 degrees okay for simplification okay. this is 45 degree that means you know $\pi/4$ ah this is length L.



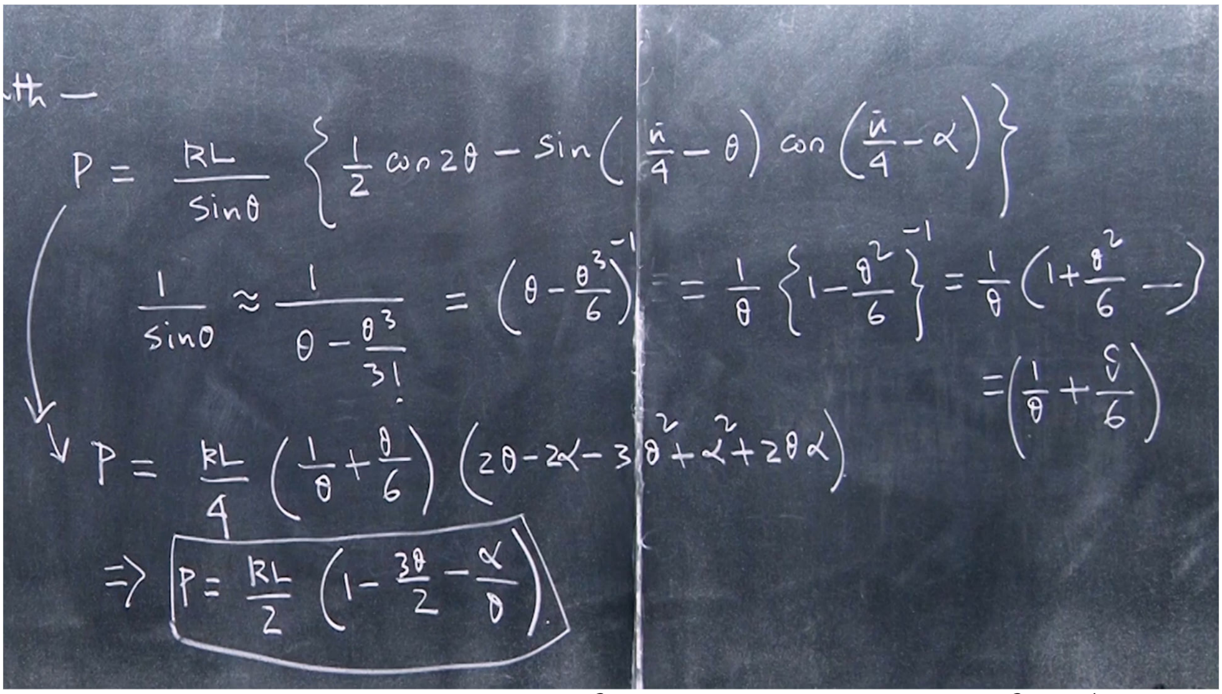
So, if this is L then this length is also L if it is 45 degree and then this length is $L\sqrt{2}$ right from here, this length is nothing but $L\sqrt{2}$. This is the hypotenuse for the you know right angle triangle right. So, we are assuming initially. "The first step in studying stability is to perturb the system." So, I am perturbing it ok. So, when I am perturbing it, it is reaching to this configuration as in that reaching to this configuration and then this is here ok. So, we are assuming that initially maybe there is some imperfection. this perfection is α and then after that there is θ you know initially imperfection is α and then we have the perturbation is θ ok. So, you can understand. So, the way we are going to assess the potential energy function right. So, potential strain energy minus work done. You can understand this easily. Earlier, if you consider the deflection under load P, it will be $\Delta = (L\cos\alpha - L\cos\theta)$ right, then $L(\cos\alpha - \cos\theta)$ but what about the spring, the displacement in the spring will be what? So, the way we are going to do is we are considering this length is L. and from here we are going to draw perpendicular this angle okay. then $\Delta_s = L\left\{\cos\left(\frac{\pi}{4}-\theta\right) - \cos\left(\frac{\pi}{4}-\alpha\right)\right\}$.



and then when it is θ then same $2L\cos(\pi/4 - \theta)$ ok. So, ultimately the expression the elongation is spring will be $L[\cos(\pi/4 - \theta) - \cos(\pi/4 - \alpha)]$ ok. Understood right all of you understood right why it is so? because we are doing dropping a perpendicular from this point to here. And then this if this angle is α, then αcos of this angle this angle is nothing but $\pi/4 - \alpha$. So, two times of this okay that is what it is. So, potential energy function Π is energy minus work done. So, $\Pi = \mathrm{U} - \mathrm{W} = \frac{1}{2}kL^2[\cos(\pi/4 - \theta) - \cos(\pi/4 - \alpha)]^2 - PL(\cos\alpha - \cos\theta)$.

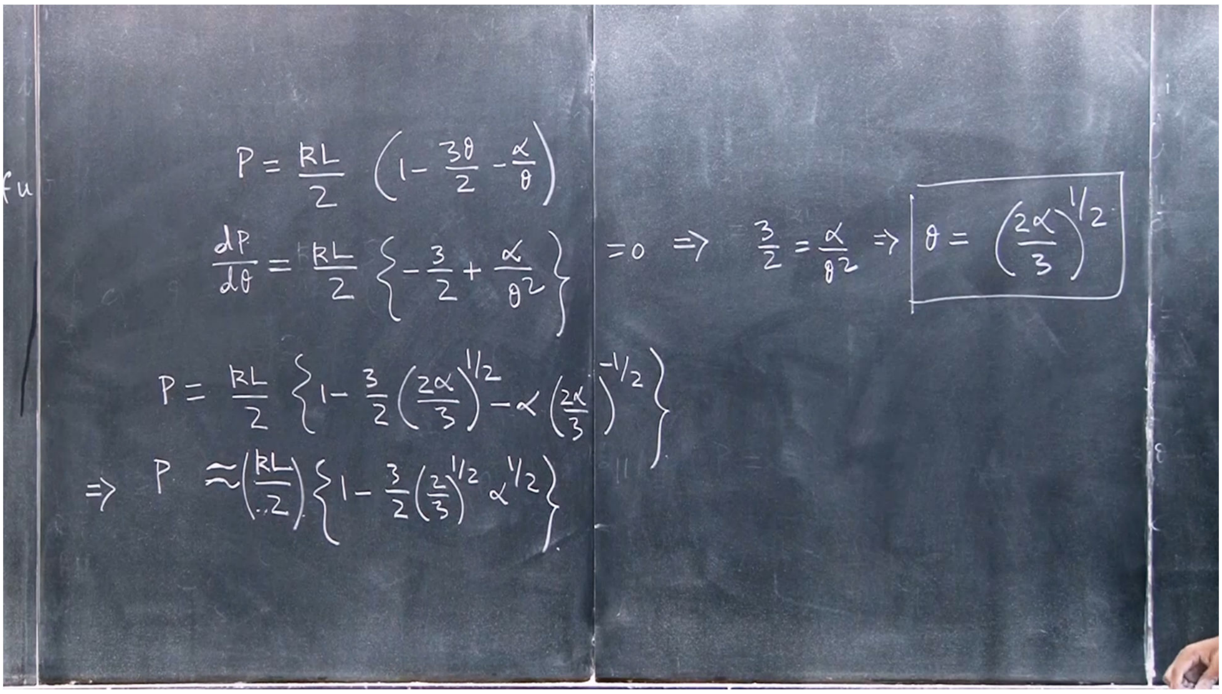


So, let us do differentiate $\frac{\partial \Pi}{\partial \theta}$ is equal to 0. I will just write it ok. Simplified expression $\frac{\partial \Pi}{\partial \theta}$ will be equilibrium path. and it will be $P = \frac{kL}{\sin\theta}[\frac{1}{2}\cos 2\theta - \sin(\pi/4 - \theta)\cos(\pi/4 - \alpha)]$. this is the equilibrium path. So, I will write another version of P ok. So, you have noted these things right ok. So, this is little involved to simplify but we will do that ok. and we will also make use of the relationship that you know $\frac{\partial^2 \Pi}{\partial \theta^2}$ is equivalent to $\frac{\partial P}{\partial \theta}$ equilibrium. ok that is what we will make sure. So, the equilibrium path P is $\frac{kL}{\sin\theta}[\frac{1}{2}\cos 2\theta - \sin(\pi/4 - \theta)\cos(\pi/4 - \alpha)]$. we will directly simplify this ok by series expansion. So, for that, I'm using $\sin\pi/4$ as part of the simplification — please take note of that." So, $\sin(a - b) = (\sin a \cos b - \cos a \sin b)$ $\sin 45°$ $\text{Sin}\pi/4$ is nothing $1/\sqrt{2}$ right. So, if you just write $\sin\left(\frac{\pi}{4} - \theta\right) = (\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta)$. and then it is $1/\sqrt{2}\{1 - \frac{\theta^2}{2!}\}$-$1/\sqrt{2}\{\theta - \frac{\theta^3}{3!}\}$. why I am doing this thing it will be clear to you because this is an asymmetric system. why it is asymmetric? Because you see when you are pulling it apart right see this is the restoring force. So, the destabilizing force is nothing but $PL\sin\theta$. The restoring force is equal to this value, and the restoring moment about this point is the restoring force multiplied by the lever arm. However, when you are perturbing it in right in clockwise then with increasing θ this distance is what decreasing whereas when you are taking it to anticlockwise θ then this distance is increasing that means the stabilizing force is varying with positive or negative θ you understand. "This system is chosen because it provides a stabilizing force, ensuring the desired equilibrium." Angle 45 degree is for just for simplifying the calculation. But this inclination is because of that we want to make a system in which the system will distinguishes between the positive θ and negative θ and the way it is distinguishing is that it is basically in one direction it is the restoring force is increases. increasing another direction, it is restoring force is decreasing. whereas the destabilizing moment is not basically changing. whether it is this and this it does not really matter. In one case it will show post critical stable behavior. whether is other case it will post critical unstable behavior. When it will be stable when the restoring force restoring moment will be dominating over the destabilizing moment then that time it will be stable. The unstable case will be the opposite, which will be reflective. However, we are not directly considering equilibrium or similar notions; rather, we aim for a more objective assessment, which is why we are applying our energy concept. But ultimately what is happening that you must see that and there is another important difference that we are going to anticipate. See in all the previous system where it is showing symmetric bifurcation ok. Stable or unstable, our potential energy functional consists of quadratic, quartic, sixth-order, eighth-order, and other even-power terms. Due to the asymmetry, the functional must be able to distinguish this asymmetry, and this will be incorporated accordingly. It will include odd-power terms, not just even-power terms—that is the key point here. Because of these odd terms, the system will exhibit interesting behavior, including the well-known imperfection sensitivity and half-power sensitivity. All this thing we are going to explore and that for that instead of trigonometric function we are going to do polynomial function. So, I am not always doing by I am continuing with the you know trigonometric you know manipulations with trigonometric function. but one thing I can suggest to you know examine wherever you know try to simplify trigonometric in terms of polynomial. by series expansion that largely simplify the problem not only that that give you know intuitive kind of notions about the stability behavior of the system, clear. So, these if you I am just you know. And you do not require to retain all the term you just retain this term this term you know first two terms ok. So, you will see that it is $1/\sqrt{2}(1-\theta-\theta^2/2)$ oks. Similarly, you know $\cos(\pi/4-\alpha)$ will look like $1/\sqrt{2}(1+\alpha-\alpha^2/2+\alpha^3/6)$ although we are not going to consider all the term only square term, we will keep ok. But please note this thing ok. we will simplify $\cos 2\theta$, it is this term this term is like simplify like that this term simplifies like that although we are not going to consider that we will retain until cube term ok. So, $\cos 2\theta$ you know thing $1-2\cos^2\theta = 1-2(1-\theta^2/2!+\cdots)$ all terms. then we have to take square root ultimately you will see $1-2\theta^2$ right. So, if you substitute everything and $\sin\theta$ means θ $\sin\theta$. the expansion is what? $\theta-\theta^3/3!$ right factorials 3 plus $\theta^3/3!$. So, we are going to consider $\theta-\theta^3/6$ ok and we will just take it further. So, all of you please note these things ok. otherwise maybe later it will find it difficult clear. please note it down you have already noted this two now note this,



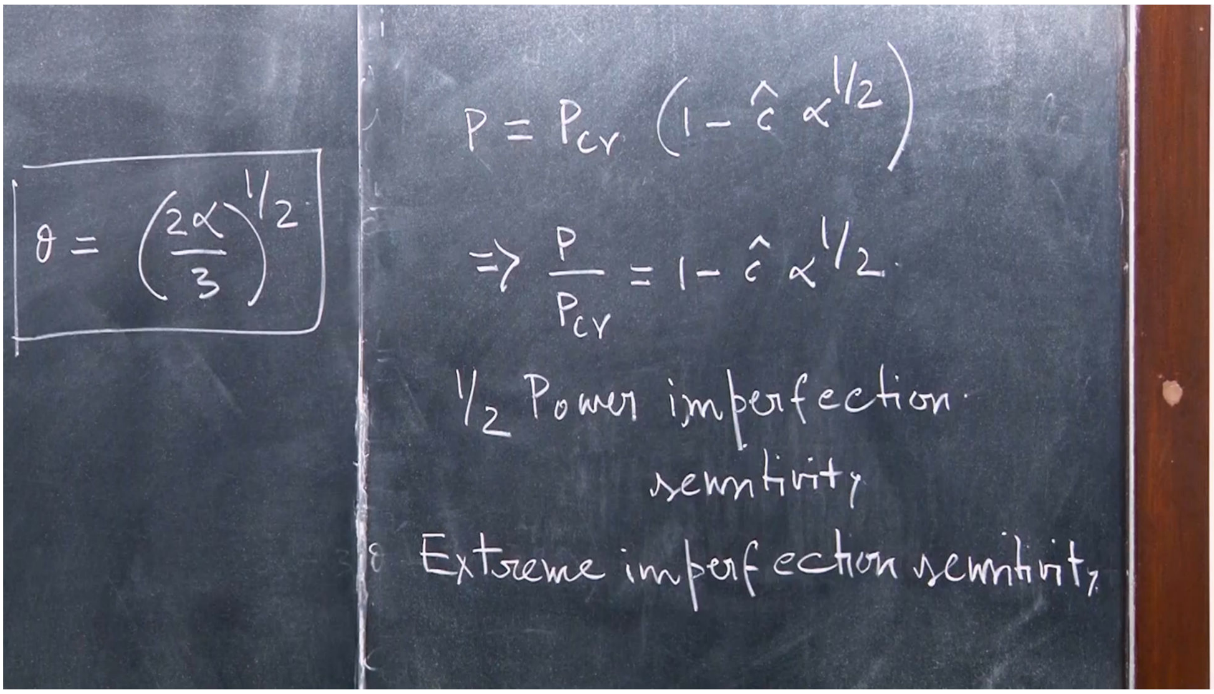
So, $1/\sin\theta$ we are approximating as $1/(\theta-\theta^3/3!)$ and then this being $(\theta-\theta^3/6)^{-1}$ and then that means $1/\theta(1-\theta^2/6)^{-1}$ and $1/\theta(1+\theta^2/6)$ and then $(\frac{1}{\theta}+\frac{\theta}{6})$ So, P_{cr}, so what we are going to do? P we are going to write as kL that is the equilibrium path.

this P is the equilibrium path please note that. This will be kL you know on substituting all this polynomial expansion, we are going to get $P = kL/4(1/\theta+\theta/6)\ (2\theta-2\alpha-3\theta^2+\alpha^2+2\theta\alpha)$. here and $P=\frac{kL}{2}(1-\frac{3\theta}{2}-\frac{\alpha}{\theta})$.

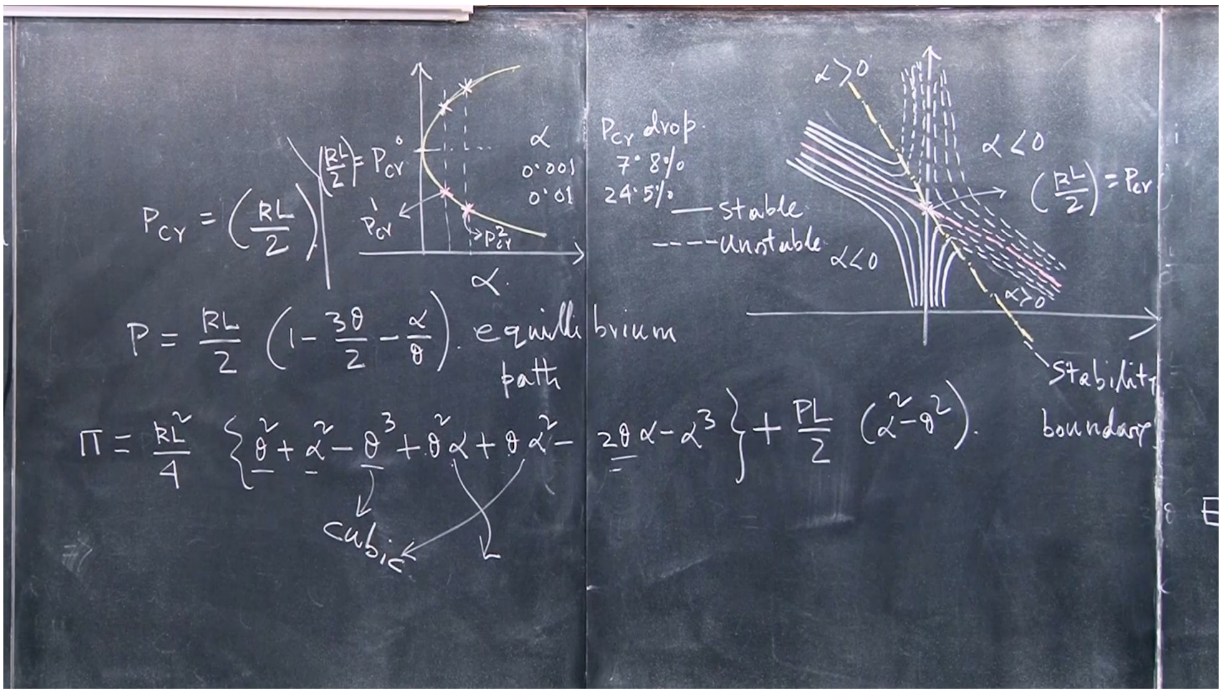


This is the equilibrium path. ultimately you know doing all these things see we are simplifying this. you see trigonometric is much easier to deal with this. And of course, higher order terms we neglected but there it is what we will be able to do lots of stuff ok. and what you can clearly see that you want to equilibrium path and if you take double derivative, you know $\frac{\partial P}{\partial \theta}$ if you do So $\frac{\partial P}{\partial \theta}$ how what will it come? Some of the higher order terms are neglecting. Rewriting $P = \frac{kL}{2}(1 - \frac{3\theta}{2} - \frac{\alpha}{\theta})$. So $\frac{dP}{d\theta}$ is what? $\frac{dP}{d\theta}$ neglect this minus θ is equal to 0. it will be $-\alpha/\theta^2$ and then for very small θ we can neglect this thing right. So, it will be minus you know $3/2$. you see ok. So, $\frac{dP}{d\theta}$ is what? $\frac{dP}{d\theta} = \frac{kL}{2}(-\frac{3}{2} + \frac{\alpha}{\theta^2})$. if I make it 0 from here, you will get $\frac{3}{2} = \frac{\alpha}{\theta^2}$ or $\theta = (2\alpha/3)^{1/2}$ right. Then I will just write $P = \frac{kL}{2}\{1 - \frac{3}{2}\left(\frac{2\alpha}{3}\right)^{\frac{1}{2}} - \alpha\left(\frac{2\alpha}{3}\right)^{-\frac{1}{2}}\}$, okay you see that. this we can approximate it you can approximate it $\frac{kL}{2}$. So of course, this term will be dominating over this term because α is small right. So, you will take square root okay you see that. So, I can write

$P = \frac{kL}{2}\{1 - \frac{3}{2}\left(\frac{2}{3}\right)^{\frac{1}{2}}(\alpha)^{-\frac{1}{2}}\}$, okay. and so ultimately if you do this, $\frac{kL}{2}$ ok that $\frac{kL}{2}$ is nothing but, that is the critical load for this case ok. In this perfect system if you study that $\frac{kL}{2}$ is nothing but the critical load ok. that I have not demonstrated but you can do ok $\frac{kL}{2}$. this $\frac{kL}{2}$ ok this one So you know this piece so from here I am just removing this please note that. this condition we have obtained from the transition that means the stability transition right. So, where we can see $P = P_{cr}$. so, P_{cr} is nothing but $\frac{kL}{2}$. $\frac{kL}{2}$ this is the basically critical load that means where the bifurcation is occurring in the perfect system ok. That I have not demonstrated but if you for perfect system. if you do put the $\alpha = 0$ that is what you will get $\frac{kL}{2}$ ok. you put $\theta = 0$ ok $\theta = 0$ $\alpha = 0$ then you can see $P = \frac{kL}{2}$ you see that ok.



So, critical load will be $\frac{kL}{2}$. So, $P = P_{cr}(1 - \hat{c}\alpha^{1/2})$. Or I can write $P/P_{cr} = 1 - \hat{c}\alpha^{1/2}$. What do you see? Half power imperfection sensitivity. that is called extreme imperfection sensitivity. So, I will put here $P_{cr} = \frac{kL}{2}$. So, now I am removing all these things all of you have noted down must have noted down this and what I am going to write here is I will draw now. So, P_{cr} was nothing but $\frac{kL}{2}$ imperfection sensitivity. and I have said it half power. so, I will draw it and I have also shown this equilibrium path. where x you know, that was nothing but $\frac{kL}{2}(1 - \frac{3\theta}{2} - \frac{\alpha}{\theta})$ that was the equilibrium path. And I will also write the potential energy expression okay. I will write down the potential energy expression. the potential energy expression Π will be $\frac{kL^2}{4}$. In fact, we have neglected many I mean some higher order term here. because it is in terms of P but if you take the energy then it will be square.



So, I will write it you know reasonably with reasonable approximation okay. $\Pi = \frac{kL^2}{4}\{\theta^2 + \alpha^2 + \theta^3 + \theta^2\alpha + \theta\alpha^2 - 2\alpha\theta - \alpha^3\} + \frac{PL}{2}(\alpha^2 - \theta^2)$. So, in all these two previous examples where it was symmetric bifurcation. the potential energy functional content term which was all in even power but here these are odd power and symmetry asymmetric terms are populated all over clear. I am going to draw this now put how then stability diagram. So, please note that, please note carefully. So, this was the perfect system, this was the perfect system, okay. And this is the stability boundary. ok. So, you see that P is then the value of this corresponds to this value. this value corresponds to $\frac{kL}{2}$ oks. which is critical $\frac{kL}{2}$ that is basically P_{cr} ok. So, do you understand the bifurcation? So, once again this table fundamental path is going please draw it and then I am going to explain to you. will just finish with that. So, let me explain to you this is the imperfection stability diagram. you see that asymmetry is clear here you see that when this is $\alpha > 0$ this is you know this is θ. So, $\theta > 0$ $\theta < 0$ this is negative θ ok. when θ is negative the critical load is increasing you see that here the value is $\frac{kL}{2}$ but they are you know it will increase. because θ is negative and it will be positive. but here you see that, this is odd term right and similarly I am neglecting but cubic term will also be there ok similarly the denominator there is also another term. So, in the right-hand side you know left side this is increasing right hand this is decreasing and here at this one the bifurcation is occurring perfect system which is scale by 2. So, stable fundamental path is going and bifurcating into an asymmetric path. and then fundamental path is being unstable right. So, the perfect system for the imperfection system. what is happening for different imperfection $\alpha > 0, \alpha < 0$ there are 4 quadrants right. Here you see this is once again asymptote see this path the stable path is asymptote to the fundamental path that the unstable path is asymptote to the post critical path. see that imperfect system and these are all increasing imperfection please note that you see that so imperfection imperfect system. They create equilibrium path you know it is asymptote to the fundamental path and another limb it is asymptote to the post critical path stable. here it is stable, here it is unstable. and you know I explained what is happening why that size is stable because the restoring force will have him higher you know moment arm okay. that is what it is resulting in much larger more restoring moment comparing the destabilizing force which is not changing whether the attar case it is it is not it is being dominated by the destabilizing forces okay. And there you see another important thing that here you see this positive slope, here negative, negative slope. So, essentially, you know, tangent steepness $\frac{\partial P}{\partial \theta}$ is being positive, right. So, and then the stable to unstable transition is given by this line you see that here it is positive slope but here it is negative slope this is the line and this line please note that this line is basically bisector of this angle. this stability boundary is nothing but a bisector of this angle bisector of this angle clear. Okay now having said that let us come to the imperfection sensitivity diagram what is happening it is half power sensitivity. So, when it you taking square root, it cannot be negative value of α it is said to be positive value. So, I am just putting only positive value of α. α is positive and when I am taking the root there will be 2 root 1 root is extraneous only the you know less root 1 minus root So this, you see that P_{cr}, upper one I am not taking, the lower one, this, this, it is diminishing very fast, very fast. See, if you take α is a much less number than 1, right. So, what will be less, two-third or half? Which one will be less? Well, which one will be more? half will be more right rather than two-third. So, that means $1 - \alpha^{1/2}$ right. So, that will be more. So, that is what you see the 0.001 imperfection that 7.8 percent decrease in critical load. So, the 0.01 in 24 percent decrease in critical load. So, this is extreme level of imperfection sensitivity and that is what you can clearly see that right. Clearly see this one, this one equals to the first one, this one equals to the second one. So, this is the imperfection sensitivity. So, half power sensitivity, extreme level imperfection sensitivity. Now what I mentioned to you earlier that which structure will portray this kind of behavior? Analogous elastic system, elastic cell buckling will portray this behavior. There will be symmetric mode, there will be asymmetric mode. I will show you at the end of this course, okay. Now asymmetric mode will show this kind of behavior, extreme level of imperfection sensitivity. Not only that, there are additional things that happen. Okay two modes can interact and apparently that interaction we are going to demonstrate to you. So individual mode might not be imperfect sensitive. but due to imperfection interaction or modal interaction there can be imperfect sensitive in imparted. Okay so this is the third system. So, I am not writing the behavior but what happened? A stable fundamental path bifurcates into a unstable and stable asymmetric path right. and the system become imperfection sensitive, extreme level of imperfection sensitive. On incorporation of the imperfection the bifurcation structure breaks. and then symmetry breaking occurs right and then for the imperfection system the equilibrium path asymptotically approaches the perfect system. Thank you for today.