Stability of Structure

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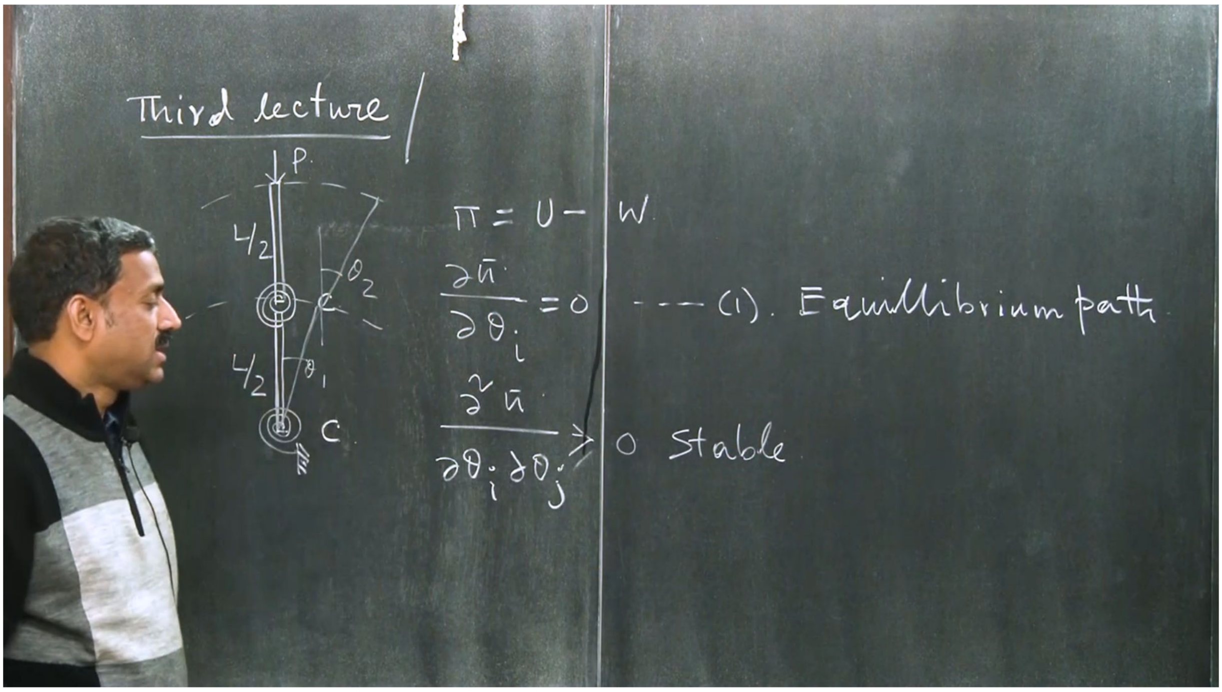
WEEK-02

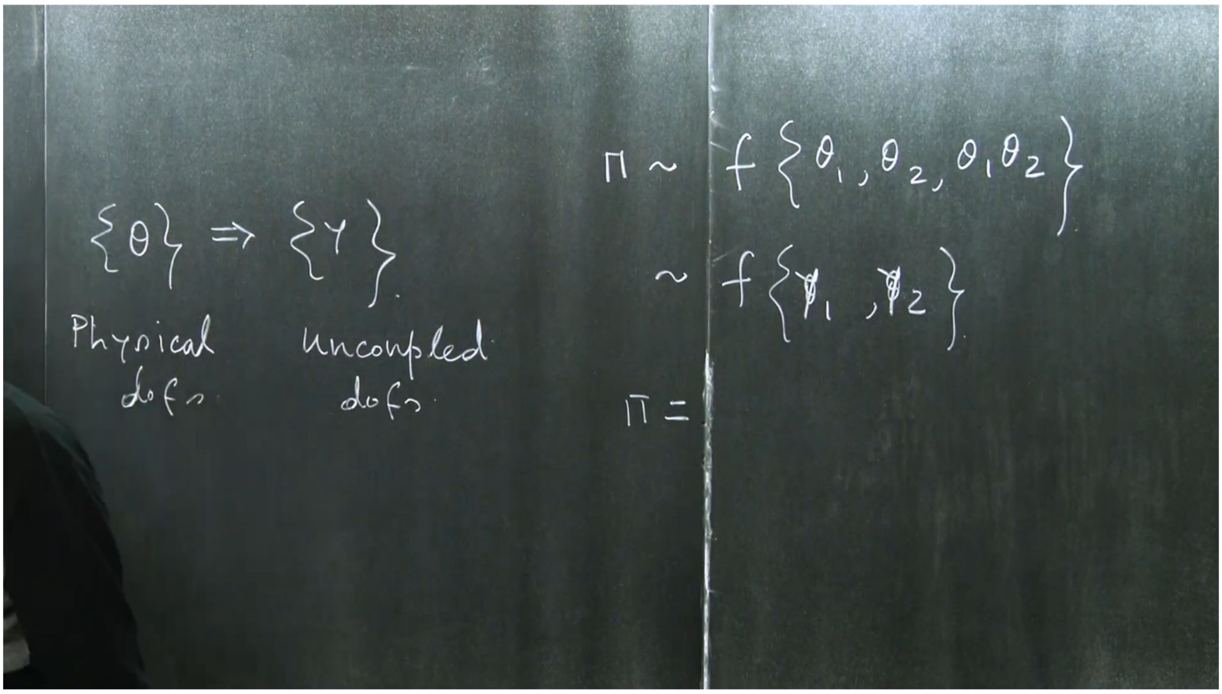
Lecture 03: Stable Symmetric Bifurcation Behavior

So, this is the third lecture, stability of structure. Let us briefly recapitulate what we have done in the previous class. So, we introduced the concept of stability and the first lesson that we have learned is that in order to analyze the stability, the system must be perturb, we must be looking for an alternative configuration, right. So, while evaluating equilibrium we are trying to minimize the potential energy here, we have to explore the higher order derivative of the potential energy function ok.

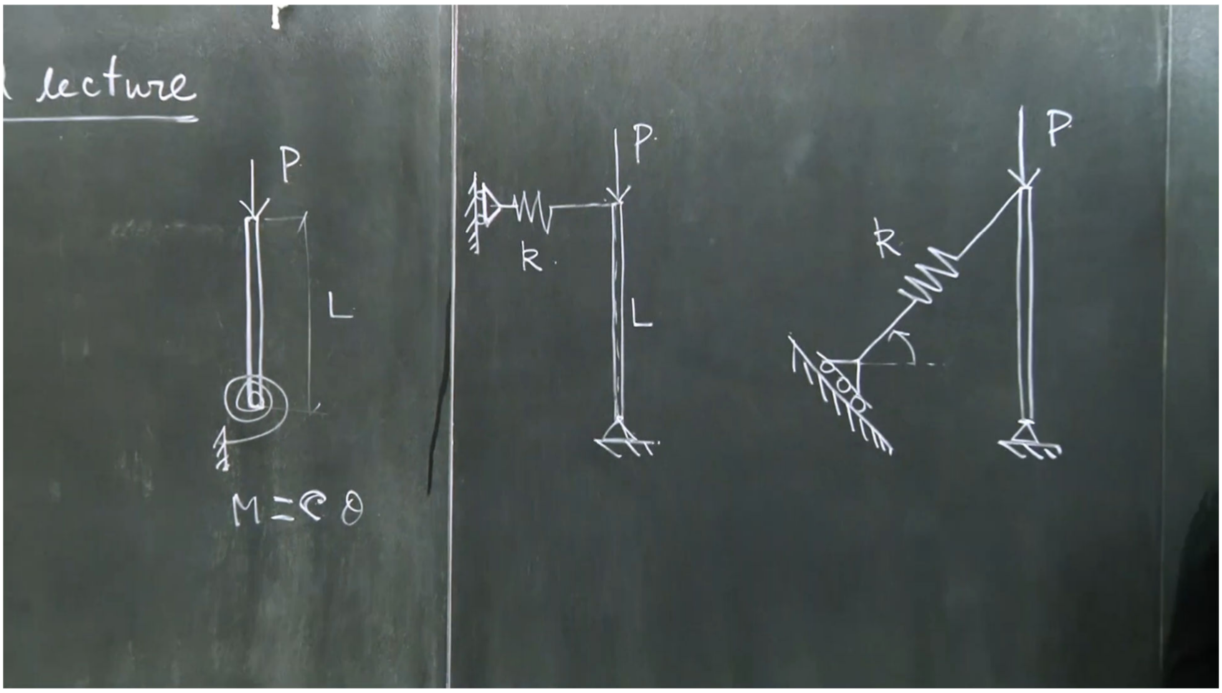


So, we have considered a two degree of freedom system and we have demonstrated that how we read into the equilibrium path for the system, and then what is the nature of equilibrium whether the equilibrium is stable, unstable or neutral and the approach we followed that was the energy approach ok. Energy approach we have learned in other courses as well right.

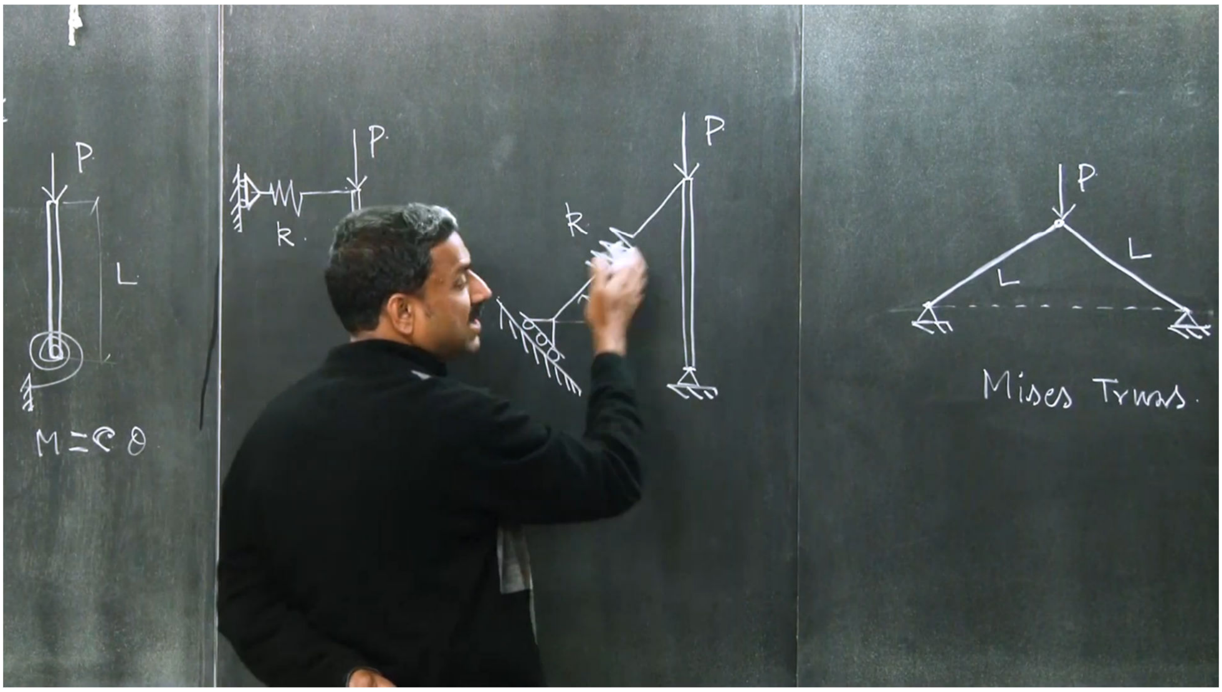
One such example is of course the finite element in order to derive the equilibrium equation the systems of linear simultaneous equations or systems of equation are derived using minimization of the potential energy functional. So, I will start with the potential energy functional because I think it is easier and most intuitive to master the concept of stability ok. So, that was the reason. So, we can recall that the system that we have considered was a consisting of 2 rigid bar systems right. they were connected to each other by rotational spring at the end also they were connected to a rotational spring and it was subjected to axial load compression load P and this were all dimension L by 2, L by 2, and they were I mean for simplification simplicity we assume the identical rotational stiffness for both the rotational spring right. So, we have derived the potential energy functional π which is strain energy minus work done and in order to obtain the equilibrium path. Equilibrium path we have made this we have differentiated with respect to the degrees of freedom θ_i. So, we have perturbed it. So, θ_i is nothing but the system, and these were the degrees of freedom basically right.



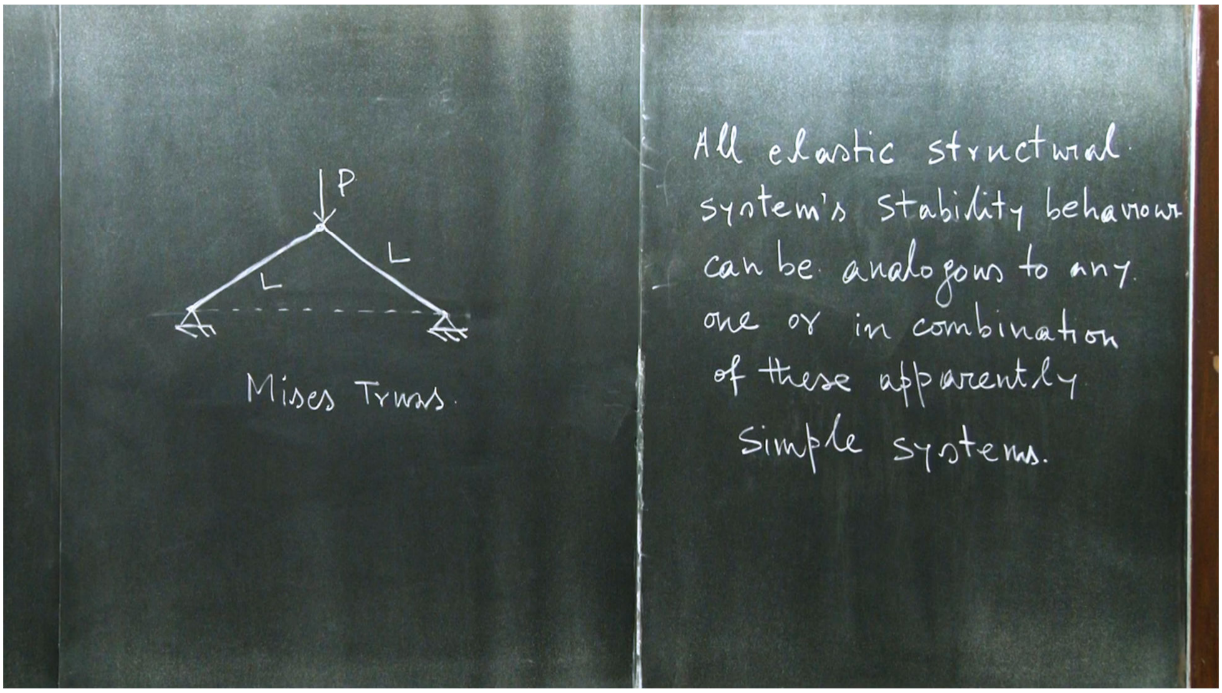
So, this is θ_1, this was θ_2 right. So, we have in the perturbed configuration, we have written down the potential energy functional and in order to find out the equilibrium path, we have found out, we obtain the potential, the gradient of the potential energy functional and then for the stability. So, this was giving us the equilibrium path. So, you have obtained the critical load and then we have considered the higher order derivative, there basically we have tried to make it greater than 0 for stable configuration for searching for stable configuration, if it is 0 then it is neutral equilibrium and if it is less than 0 it is unstable configuration. So, this was leading us to the hessian matrix of the put in set energy function right. and we have investigated it is the determinant of this and its principal minor must be greater than 0 for stable equilibrium ok. So, we have also learned that potential energy functional gives us all the information of the equilibrium path and then critical load, and whether the configuration is stable or unstable and we all we have also seen the influence of imperfections on that ok. Now other than that when we are finding out the critical load, we obtain that the two sets of legitimate bases for it each, because these are two degrees of freedom system, we obtain two critical load and respective critical load is associated with a something called analogous to mode shape, we call them buckling mode shape and those buckling mode shapes have as legitimate basis. So, from the physical coordinate θ we can obtain, we can go to a uncoupled coordinate. So, this is the physical coordinate, physical degree of freedom and we can go to an uncoupled degree of freedom right. And thereby we have also seen that the potential energy functional π that is what consisting of the terms like θ_1, θ_2 and the cross term like θ_1, θ_2. Whereas when we are transforming this using linear transformation because these are nothing but eigenvectors right. So, they form a legitimate basis. So, potential energy functional become a function of θ_1 and θ_2 only. And we have seen that y_1, y_2 only and the potential energy function for this simple case, we have seen that it can be expressed in terms of quadratic term, quartic term or even higher order term like 6th order term, 8th order term because all are odd event terms right. And if we do not require to find out in that case in the Heisian matrix or determinant or principal minor rather by looking into the coefficient sign of the coefficients. of the potential energy function, we can comment on the nature of the equilibrium, whether it is stable unstable or neutral. So, what we did not see in this example is that we did not consider the post buckling behavior right we have only considered the critical behavior not the post critical behavior, and we have also not investigated the nature of imperfections what is the influence of imperfections on its stability behavior that we are going to consider using simple system ok. But this was just an illustration just to introduce to you the concept of Hessian of the potential energy function, and how various matrix of it such as determinant and principal binaries are helpful to ascertain the nature of equilibrium ok. Now onwards we are going to consider simple system and their behavior to show important insight about the stability of system ok. you may wonder that why we are considering only this simple system ok. So, rigid bar connected with spring that means rigid system with concentrated elasticity. Why cannot we directly jump into structure like maybe column, maybe plate, maybe shell or things like that. We can definitely do so but please you have to be patient ok because, using simple system we are going to demonstrate the different class of behavior okay, and when you do this age of finite element you see all the thing skill sets analytical skill sets what people used to have perturbation. theory and then asymptotic expands and different kind of approximation to solve the equation and now all everything is has gone into finite element. Take a commercial software and you start analyzing stuff ok, knowingly or unknowingly ok. In the process you do mistake ok. So, since we can do that using finite element but understanding and interpretation of the result is more complicated ok. So, that is what I will try to demonstrate the physics of stability that we need to learn essentially ok. So, for that we are going to consider several simple systems which will essentially demonstrate the physical aspect of stability ok. So, we are going to classify the behavior of all elastic structural system into four categories, okay.



So, I am going to show you to four simple system and all these systems are single degree of freedom system that means they are motion. or they are deformation can be essentially described using a single variable ok. So, what are the three different system? This I will call them as toy system because they look like toy ok. So, I am going to consider first a rigid bar, and this rigid bar is connected with a rotational spring right and which is subjected to a load concentrated load P and maybe this one is a playing tail. And the rotational stiffness has a c ok. So, that means this is related with c into θ, this is going to experiencing a rotation θ then v is equal to c into θ then the moment develop is the rotational stiffness of the spring. So, this is one system. Next system I am going to consider is like this. is a rigid bar connected with a spring here and then it is attached to a spring ok and this spring is allowed to to displace, it is subjected to concentrated load p this is length l this is maybe stiffness k so this is like a roller kind of support so let me see this is this system you can perturb it right ok and here this system also you can perturb it because if you perturb in this way or that way this has to be flexible they has to move right. So, this is a rigid bar which is hinged at one end and other end is connected with a spring. translational spring right, this is the second system.

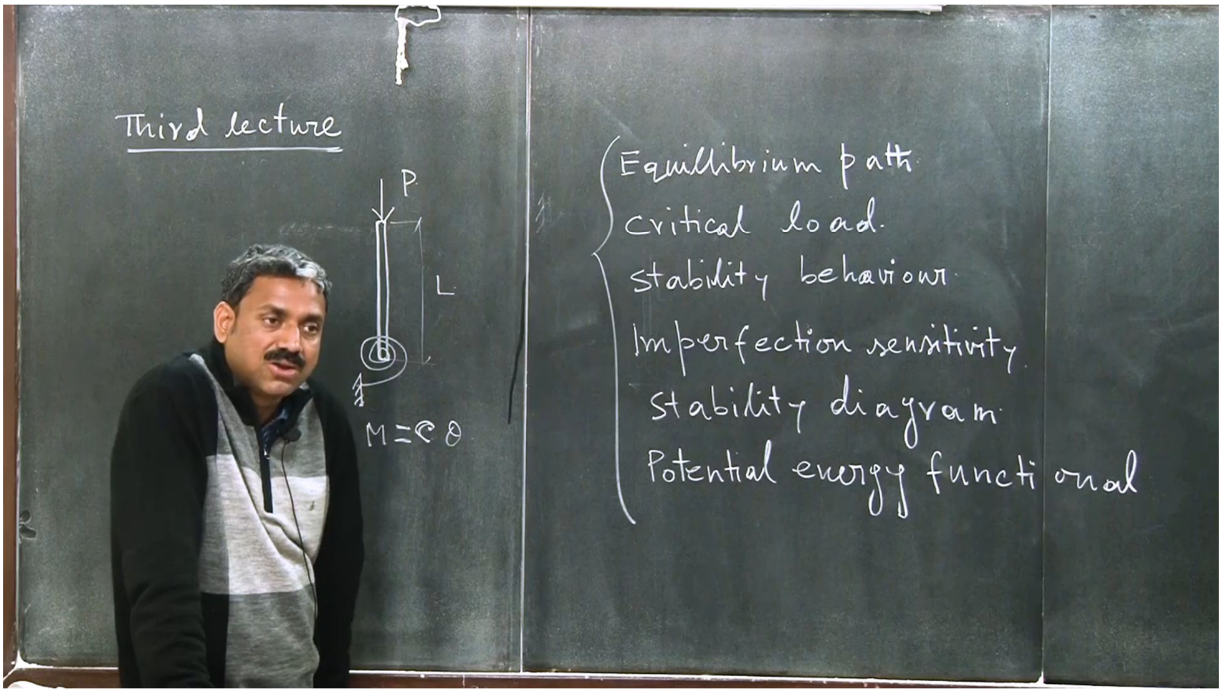


The third system we are going to consider is once again a rigid spring, a rigid bar which is connected to a spring and it is subject to a concentrated load P once again, but it is in it is attached with a translational spring with a restrained by a spring and this is but that is inclined spring ok and that is that is free to move on a inclined spring. so, that means this spring is hinged at one end another end it is rested by inclined spring. So, this inclined spring has important implication that I will show you ok. And the fourth system I am going to consider is 2 bar trusses, so maybe something like this and this is called misses truss ok. These are all hinge ok, this is a truss, these are all hinge joint, this is a hinge joint, this is a subjected to concentrated load P, this is a link tail ok and this is maybe it is a truss member ok. And here it is this is called Mises's truss ok. And these are all a rigid bar connected to a rotational spring subjected to a load P. This is rigid bar connected to a which is hinged at the bottom and connected to a translational spring with horizontal orientation. The rigid bar which is hinged at one end but it is connected with inclined spring right which is free to move either surface because when it is going to deflect it should be free to move. The other one the last one is a basically truss member ok and subjected to load B ok. So, all these four systems this simple system 1, system 2, system 3 all these four systems can display the stability behavior.

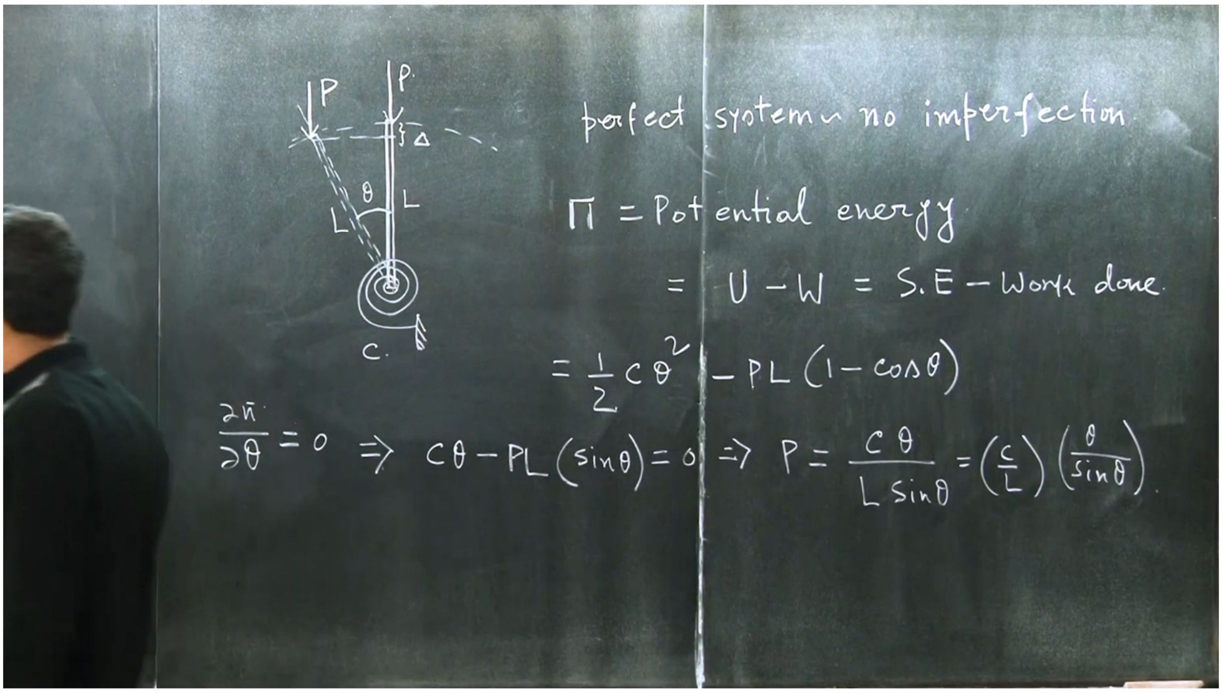


All the elastic structural system into certain extent elastoplastic structural system or inelastic system will follow the will demonstrate or display the similar stability behavior ok. So, please note that all elastic structural systems stability behavior can be analogous, to any one or in combination of this system of this apparently simple system. Please note that.

So, four classes of behavior we are going to see. So, you understand why because you should not get bored that okay what the hell we are doing is this too simple system, the rigid bar only and only one degree and associated with the idealized spring. But the importance is that this will reveal a greater picture that means all the stability behavior essentially is demonstrated by this system in combination or alone will show you the behavior of the real structural system. Real structural system means it is either buckling of column, buckling of beam column or buckling of plate or not buckling only post buckling as well, and then buckling or post buckling or post critical behavior of shell and nowadays there are other systems as well nano micro systems, where the stability behavior you are harnessed for the fabrication for efficient devices and things like that ok for adaptive behavior ok. So, with this thing we are going to start analyzing stability of each one of these rights. So, first let us start with the first system ok and here we will see what. So, what we are going to study? We are going to study the four things,

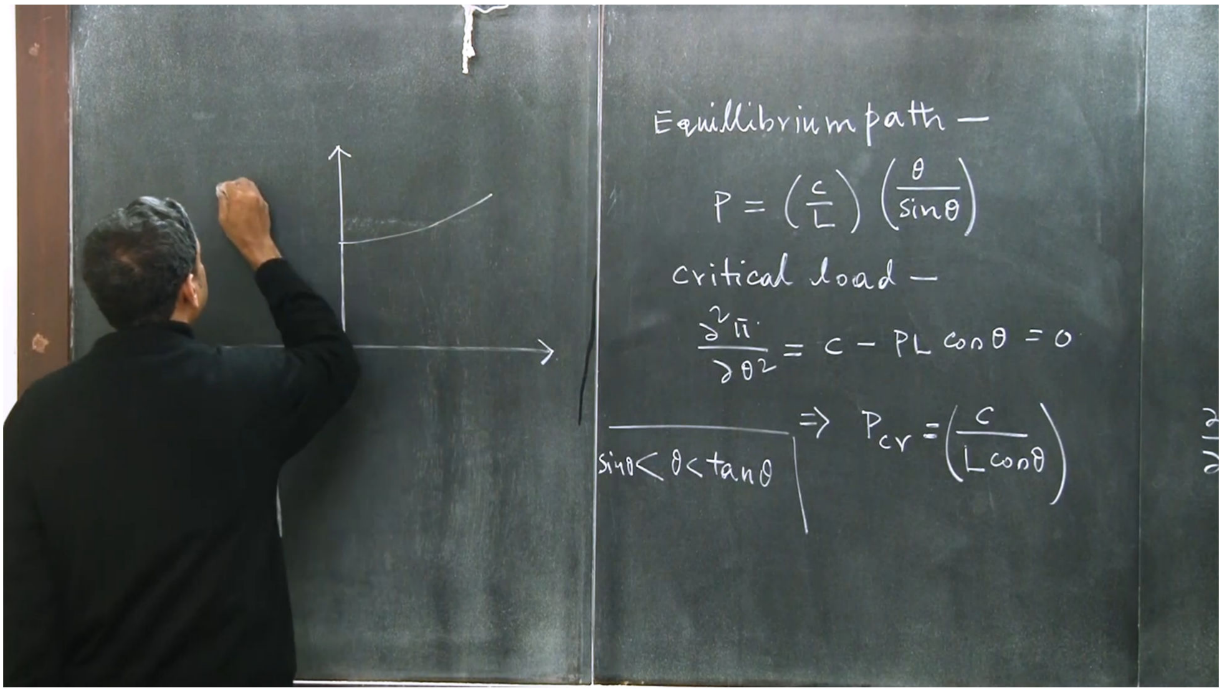


we are going to study equilibrium path, we are going to study critical load, we are going to study stability behavior we are going to study imperfection sensitivity, we are going to study stability diagram. I am going to study potential energy functional. things which we must study and then it is their behavior. So, here first we are going to obtain equilibrium path, then we are going to find the critical load, then stability behavior, imperfection sensitivity, stability diagram and potential energy. The nature of the potential energy functional and how we can comment on the behavior of the system without going into detail analysis, clear ok. So, for this system to find out and analyze its stability, what are the first things to do? The first thing to do is I am redrawing the figure here ok. So, here this is a rigid bar right and this part is connected to a rotational spring and subjected to a load P and so rotational C and length L. So, what is going to happen, we are perturbing it. So, it is going to go here, going to go there and then I am basically going to this angle is θ. I mean in any direction it can be irrespective of any perturbation can be anti-clockwise, clockwise and then please note that because it is rigid, but load P is going to remain always vertical right. It is not going to follow the deform thing. So, perturbation can be any one of these.



I mean in any direction it can be irrespective of any perturbation can be anti-clockwise, clockwise and then please note that because it is rigid, but load P is going to remain always vertical right. It is not going to follow the deform thing. So, perturbation can be any one of these. So, what is going to happen? This fellow, if it is θ, then there will be moment developer, resisting moment and then this fellow is going to do some work. So, what is that work? This is the δ, load, you will do work by deforming this, right. So, if you see potential first, we will assume that it is a perfect system first we are going to study the perfect system that means no imperfection system, and also no geometric imperfection. If we want to put imperfection what we would have done, we would put little initial rotation that we are not going to do later we will study. Potential, we are following the energy approach. So, let us find out the potential energy, what is the potential energy? strain energy minus work done, ok.

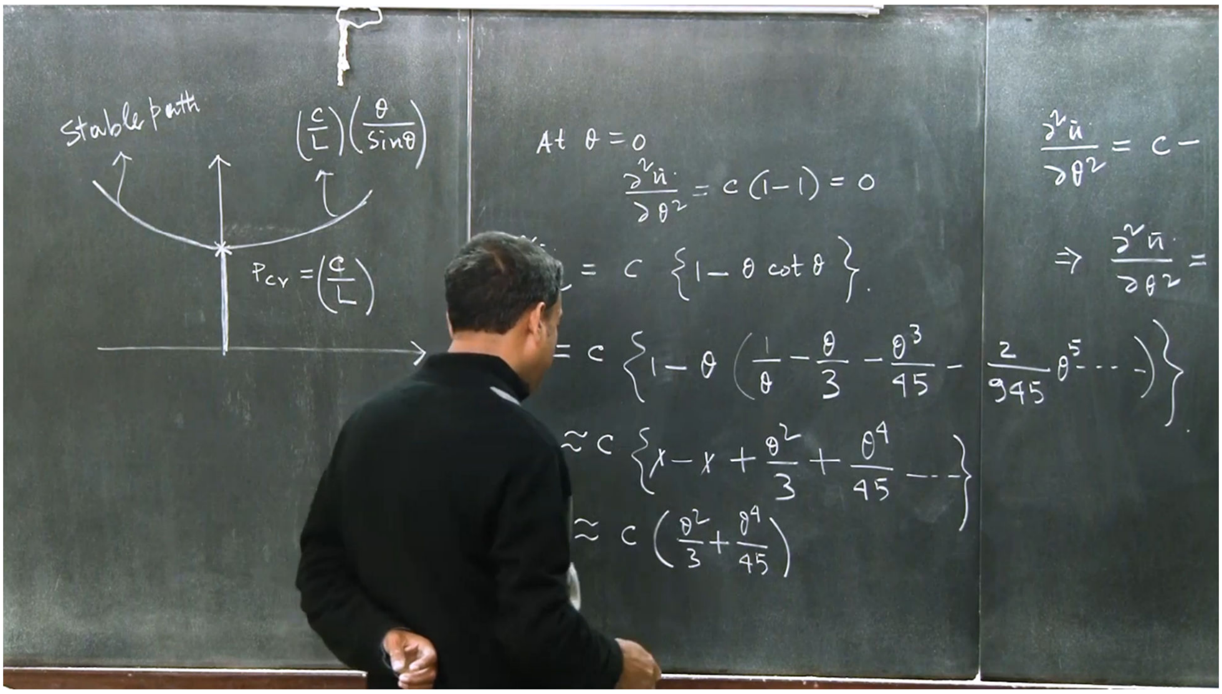
Strain energy will be what? Of course, you see that half moment into rotation, so half m into θ, m is nothing but c into θ, so half c into θ^2 right, and what is the work done? Work done is this p, p into δ and this δ is nothing but what? If this is length l, this l cosine of θ and this l into 1 minus cosine of θ right, fine. Now one thing I can do okay, so when I am doing that, I will first find out equilibrium path right. So, from there what I am going to do, $\frac{\partial \pi}{\partial \theta}$ must be equal to 0 for the equilibrium right. So, $\frac{\partial \pi}{\partial \theta}$ is how much? C into θ minus P into L sinθ equal to 0 So, P must be equal to $\frac{C\theta}{L\sin\theta}$ or $\frac{C}{L}\frac{\theta}{\sin\theta}$. So, the equilibrium path is nothing but …Equilibrium path, if equilibrium has to be satisfied of course θ is equal to 0 is a fundamental path because that is without perturbation right. It will always remain in equilibrium θ is equal to 0 right. But we are going into alternate configuration right. So that is non-zero θ. So that will be what? The equilibrium path will be $\frac{C}{L}$, $\frac{\theta}{y\sin\theta}$ right.



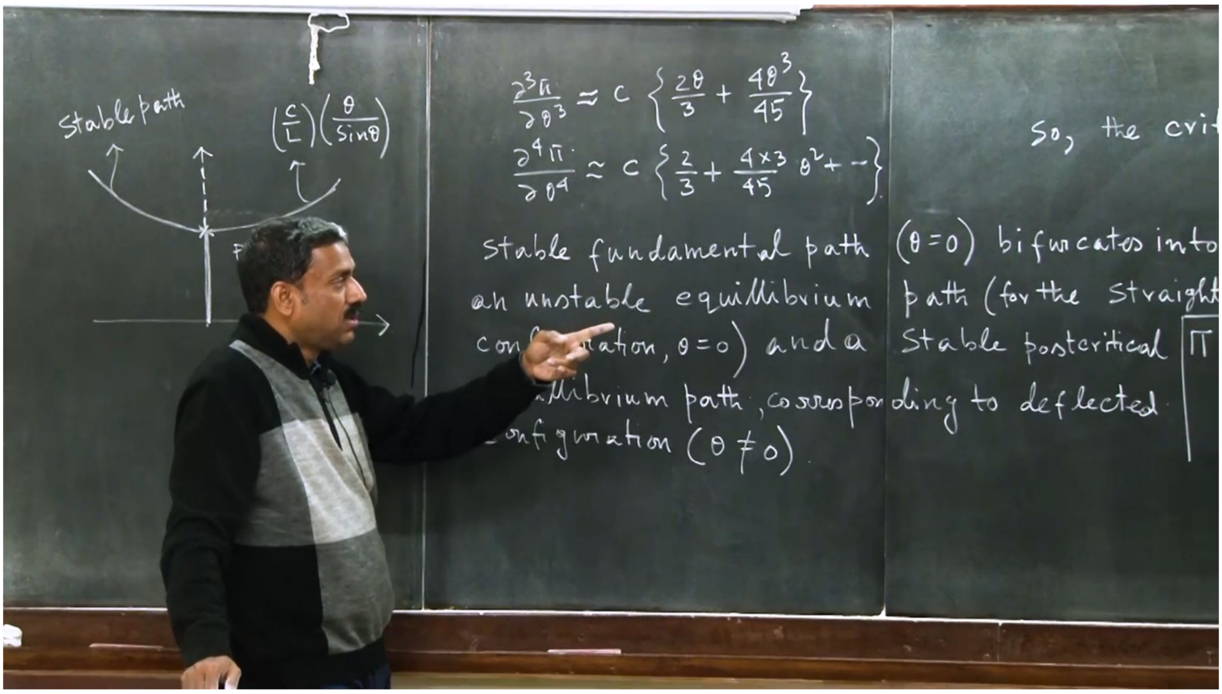
Now, I am going to draw it, how it will look like? an equilibrium, this equilibrium for now I will find out the critical. So, what is the critical, stable unstable, second order derivative we have to find out right, critical means transition between stable to unstable right. So, if you want to find out the critical load. critical load right. So, what we are supposed to do? Let us differentiate double $\frac{\partial^2\pi}{\partial\theta^2}$ is how much? $\frac{\partial^2\pi}{\partial\theta^2}$ is how much? c minus p l cosine of θ must be equal to 0. So, p critical is nothing but c divided by l cosine of θ. So, $\frac{c}{l}\frac{1}{\cos\theta}$ right this is the critical load yeah. So, what we can see that θ tending to 0, so when θ tending to 0, cos θ is what? If it is $\frac{c}{L}$, so until and here θ tending to 0, $\frac{\theta}{\sin\theta}$ is what? Limit θ tends to 0, $\frac{\sin\theta}{\theta}$ is 1, So, until $\frac{c}{L}$, it will fine. this is stable path but after that it has to bifurcate right because then for θ is equal to 0 it is not a stable path right, not an equilibrium path even. So, it will have to have some deflection and that equilibrium path is given by what $\frac{c}{n}\frac{\theta}{\sin\theta}$. So, how it will look like $\frac{\theta}{\sin\theta}$? Of course this is a curve right. Now, all of this you know the identity of θ is greater than $\sin\theta$ less than $\tan\theta$, when you are in first year mathematics differential calculus you must have done this θ, when you express the radian this inequality θ greater than $\sin\theta$ less than $\tan\theta$ for all θ, so, that means $\frac{\theta}{\sin\theta}$, for any θ it is always increase ratio, right. See, when θ tending to 0, θ will be $\sin\theta$. But as θ is increasing, θ will be much higher than $\sin\theta$. So, it will be an increasing type of you know, so it will look like something like this. You see that? It will look something like this. Are you understanding? And then it is symmetric. It is symmetric, right? So, what we see that my path was that this was the fundamental part. After that, it is bifurcating, right? Clear? So, this is my critical load and then P is basically critical is nothing but $\frac{C}{L}$ here this point and then the equilibrium path is here what we are plotting $\frac{C}{L}\frac{\theta}{\sin\theta}$. So, these paths are nothing but equilibrium path and we are do not know till now that why whether it is stable or unstable we will see there clear ok. Now next thing that if this is the equilibrium path, we have to see the nature of equilibrium path whether it is stable or unstable ok. So, how to do that ok. So, to do that what we will do let us preserve it as the potential energy functional we will do that. So, the post critical path, we want to see. So, what we will do for the stable and unstable, $\frac{\partial^2\pi}{\partial\theta^2}$ is nothing but C minus E L cosine of θ right. This is nothing but stability path right because that will decide if it is 0 then it is boundary, greater than 0 stable, less than 0 unstable. Please note that on the top of it, the equilibrium must be satisfied, right. We want to study the stability of the equilibrium path. First the system has to be stable and has to be in equilibrium, right. So, equilibrium condition must be imposed on this, isn't it? So, for that, this is C minus and this P is what? $\frac{C}{L}\frac{\theta}{\sin\theta}$, then L cosine of θ and this is nothing but C minus C $\frac{\theta}{\tan\theta}$, equals to C $\left(1-\frac{\theta}{\tan\theta}\right)$ or

I can write, I will just simplify it $\frac{\partial^2\pi}{\partial\theta^2}$ = C $(1-\theta\cot\theta)$, Now I will tell you something look what is the inequality we know θ is greater than $\sin\theta$ less than $\tan\theta$. So, $\frac{\theta}{\tan\theta}$ is what? $\frac{\theta}{\tan\theta}$ less than 1 that means this is always greater than 0. Always for θ is not equal to 0. that means my post critical path what? that means this path is stable, So, my post critical path is stable. So, this is a stable path for all θ this is a stable path all of you understood right, one more information I would like to give it to you that whether you know what will happen at θ is equal to 0, there $\frac{\theta}{\tan\theta}$ is 1. So, $\frac{\partial^2\pi}{\partial\theta^2}$ is basically what c (1 - 1) is basically 0. That means a θ is equal to 0, you do not know the configuration, θ is equal to 0 means when pcr is equal to cl this is the critical node. You do not know the nature of equilibrium where p critical is reached just. So, for that what will you do? You have to look into the higher order. "It is important to understand how to determine maxima and minima using differential calculus" So, what we should do ok. Let us do one thing ok. $\frac{\partial^2\pi}{\partial\theta^2}$ is nothing but c $(1-\theta\cot\theta)$, and equals to c $(1-\theta)$ and what is the expansion of $\cot\theta$, the $\cot\theta$ expansion is nothing but let I am writing it $\frac{1}{\theta}$, you can do Maclaurin Series expansion minus $\frac{\theta}{3}$ - $\frac{\theta^3}{45}$ - . then higher terms if you want to include - $\frac{2}{945}\theta^5$ and so on…

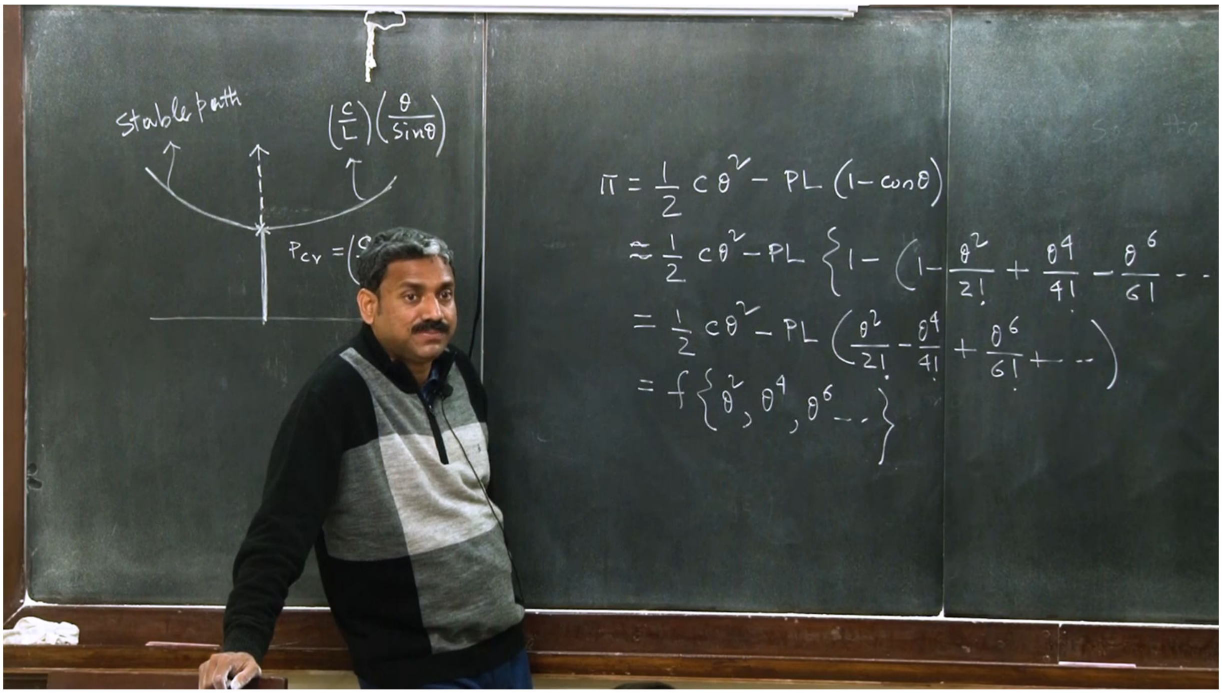
So, this I am going to write like this C. {1 - $\left(1+\frac{\theta^2}{3}+\frac{\theta^4}{45}+\cdots\right)$ }ok. I am going to neglect all these things ah. So, this this will go. So, I am going to write c and then C.($\frac{\theta^2}{3}+\frac{\theta^4}{45}$) and things like that whatever ah. So, what I will do then let us first then. So, from here I am going to come there. So, $\frac{\partial^3\pi}{\partial\theta^3}$ will be C $\left(\frac{2\theta}{3}+\frac{4\theta^3}{45}\right)$ approximate please note that is approximate right. Once again this you will not be able to decisively say that, because all this contains θ, so when you put θ ending to 0, this will all be 0 to take further derivative $\frac{\partial^4\pi}{\partial\theta^4}\frac{2}{3}$ +. Now you see that there are some constant terms plus square all this term, so, that means if other terms also vanishes but this $\frac{2}{3}$ term will stay so, that means $\frac{\partial^4 \pi}{\partial \theta^4}$ that is greater than equal that means, the critical state is stable. So, that means this is stable, and this critical load is $\frac{C}{L}$ right.



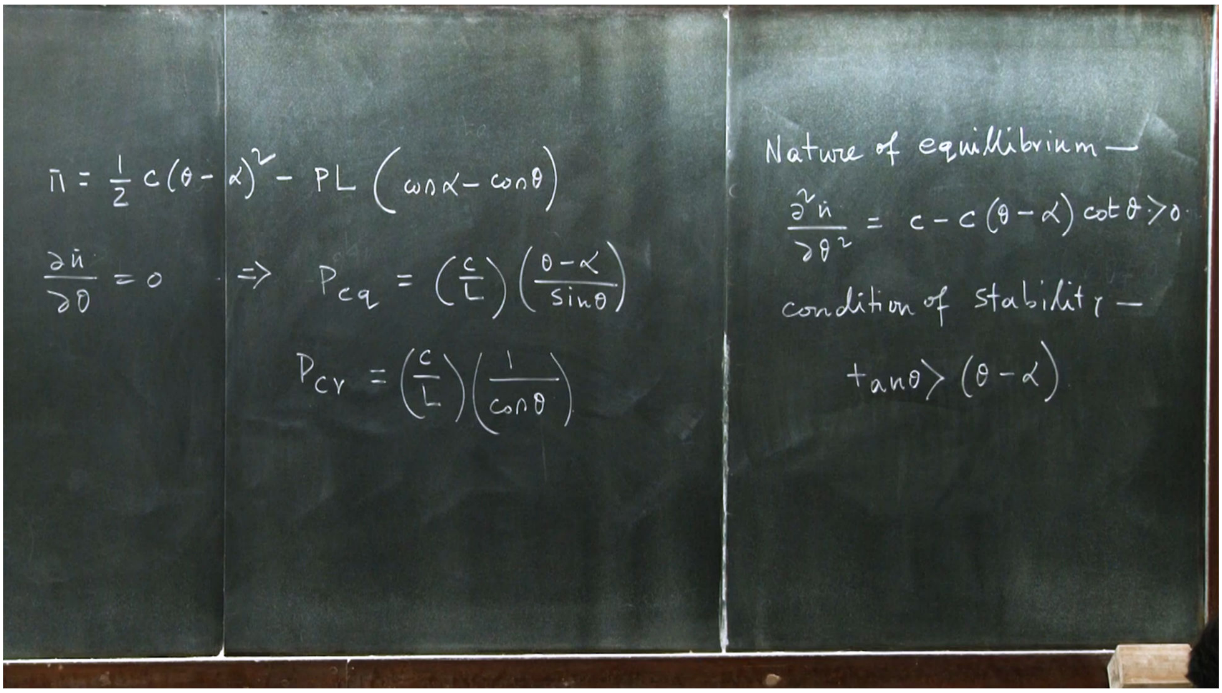
So, what we can see that the stable fundamental path θ is equal to 0 goes and bifurcate and it has results into a stable critical state, and then subsequently it bifurcated into a symmetric stable path, which is θ non zero symmetric stable deflected. And then an unstable fundamental path because here, this is broken line is unstable. because when the load is greater than $\frac{C}{L}$ this, and then this cannot be stable anymore, right? It has to deflect, right? So, I am just writing something. So, this statement you have to try to understand. So, I am writing down something. So, when you will not be able to find out, using lower derivative, then you should go for higher derivatives, okay? Please note that, huh? So, I am just writing. that stable fundamental path is nothing but θ is equal to 0, bifurcates into an unstable equilibrium path for the given straight configuration of the path, straight means configuration that means θ is equal to 0, unstable equilibrium path and a stable post-critical equilibrium path, which corresponding to deflected configuration, or perturb configuration That means, θ is equal to non-zero, okay, is fine. I am reading it out, please note this. Stable fundamental path θ is equal to 0, bifurcate into an unstable equilibrium path for the straight configuration θ 0 and a stable post critical equilibrium path corresponding to the deflection configuration. This was stable, this is bifurcating into a stable path, this is unstable.



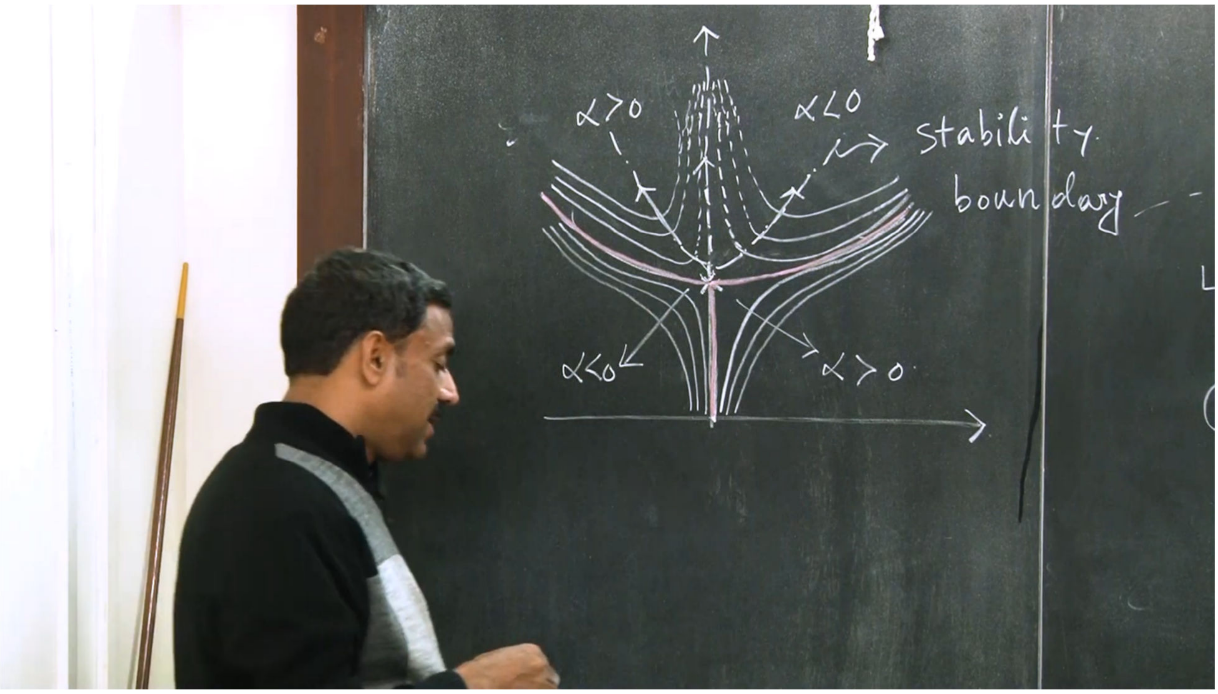
Clear to all of you? Now, so what we have done? We have obtained the equilibrium path, we have obtained the critical load, we have obtained the nature of the equilibrium, nature of the stability of the equilibrium path. We have demonstrated that this system demonstrates a stable path, right. stable post critical path because load is increasing as θ is increasing load is increasing. The kind of system which shows this kind of behavior is basically buckle column, and this kind of behavior although very little post critical strength. See like buckling does not mean that the load carrying capacity is depleted eliminated completely for the system. Post critical stage also that can structure can carry load, but for column so very little post critical load for 30-degree deflection it can only have 4 percent, 5 percent increase in load, but for plate it is significant. So, this kind of behavior is this portrayed by column and plate ok. all of you are learning plate theory right. Another thing what is important here, I would just try to write this potential energy function which I just write. So, if you expand it, how does it look? So, what do you see that potential energy function becomes a function of what θ^2 θ^4 θ^6 do you see that. So, it is a function of all what quadratic term quartic term and then others right all are even terms right there is no order of terms. So, this is a very important in very interesting. and if you substitute over there then you will also see that equilibrium path P, it remains same.



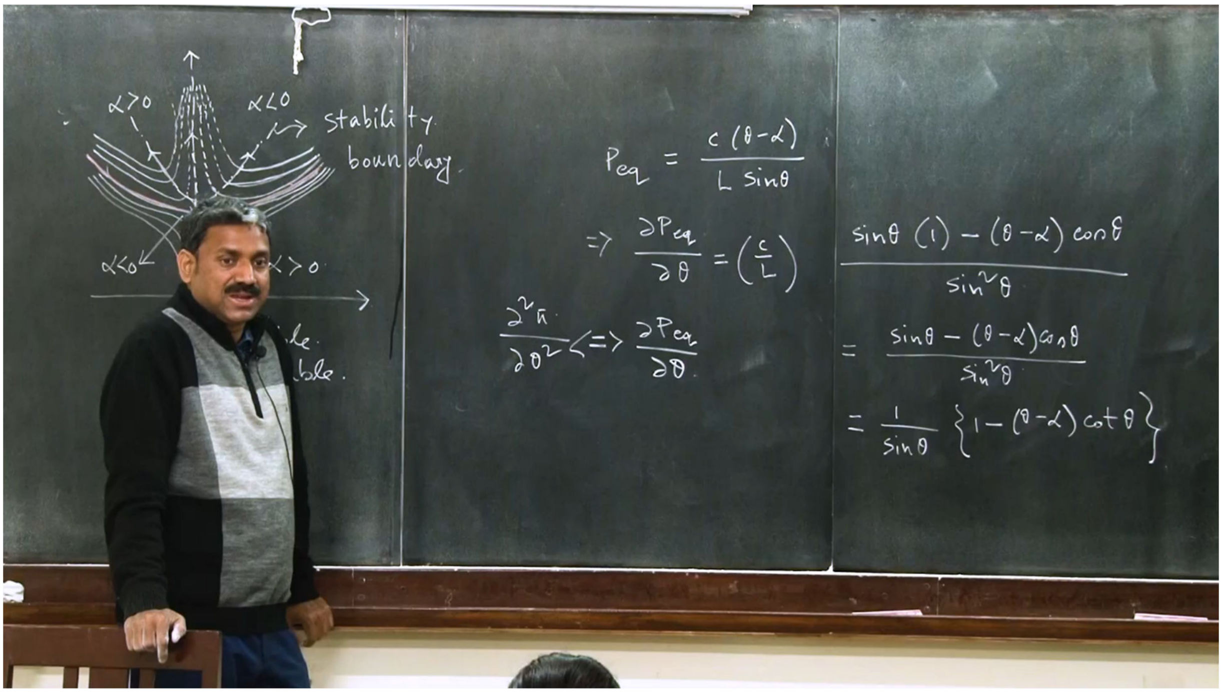
So, this is a characteristic of the system which shows the symmetric bifurcation. Whether it is stable or unstable that is a different matter, but symmetric bifurcation it is a system that essentially characterized by an even term in the potential energy functional. So, by looking into expression of the potential energy functional, you can comment on the stability behavior of the system. And that is what now it is if you see, why on that, what is the concept is such instance that, you know the energy landscape theory, I think when you are qualitatively constructed that using probing load technique, you can essentially, you can essentially fit a functional like that ok. So, a system by looking into this functional expression you will be able to comment on the stability behavior. So, you understand the nature of the potential now I will just put little imperfection in that ok. So, if I want to put little imperfection p and then maybe here, I am assuming that this is having some imperfection α ok and then further I am just putting it ok θ and this is p. So, what will be the potential energy functional π ½ c $(\theta - \alpha)^2$ - pl (cosα - cosθ) right, once again you do all these calculations equilibrium path $\frac{\partial \pi}{\partial \theta}$ is not doing it but I am writing down the expression that will be equilibrium path will be $\frac{C}{L}\frac{\theta - \alpha}{\sin\theta}$, α is the imperfection ok. And then this equilibrium path and you will also see the critical load you can find out. So, Critical load will not depend on the imperfections, so it will have the similar expression as you earlier, but now we want to find out the stable or unstable, nature of stability right, nature of equilibrium, higher order information, nature of equilibrium path, write $\frac{\partial^2 \pi}{\partial \theta^2}$, then this expression you will get c - c $(\theta - \alpha)$ cotθ , and then there you will see for stability.



So, the condition for stability this must be greater than 0 for condition stability. that this if you simplify tanθ must be greater than $\theta - \alpha$ right. So, now it is not only θ that is important, it is also α that is coming into picture. Look equilibrium path depend on α, but critical route does not depend on α. However, the stability condition and stability boundary will depend on α right. Magnitude of imperfections right. So how I will draw that? See if I want to draw that I will draw the top of it I will superimpose that here and the way it will look like this look like a flowery picture you see that, how I am going to do that you see please. what I am trying to show you, please note that okay. See, what I told you that, when there is imperfection, the imperfections basically destroy the bifurcation path. when if you see the main diagram was the stable fundamental path is bifurcating into this stable too, right. So, bifurcation was occurring. when there is imperfection see this is α greater than zero, imperfection so path there is no but please note that this path is drawn in a way so that one of its ends are asymptotic to the equilibrium path you see that. So, the equilibrium path of imperfect system asymptotically approaches to the equilibrium path of the stable system do you see that. So, please I am writing it down here, please note.



So, you have to take different value of α and you have to plot. So, what the α value will take 0.01, 0.02, 0.03 something like. So, you see that, the equilibrium path of imperfection system asymptotically approaches, to the perfect system ok, So, that is a see it destroy the structure so incorporation look. Please note the diagram here, you see this is the perfect structure and these are the equilibrium path for the imperfect structure. These are all stable, so all the solid line are stable, all the broken line are unstable. and this particular where there is this particular is the stability boundary because this basically defines the stable and unstable boundary. So, what you see that I have broken line here you see that and here this limb of the equilibrium path is asymptotically approaches towards θ is fundamental path do you see that, and fundamental path itself was unstable see that this was unstable these are all even imperfection system are also unstable and they are approaching towards this unstable fundamental path θ is equal to 0 you see that, where as beyond critical of this was a stable path after bifurcation it is a stable path. Imperfection system equilibrium path is approaching them asymptotically. You know what is an asymptote? How to find out asymptote right? And these are different for different imperfections α greater than 0 α less than 0. Now in the top these are basically reverse this is α greater than 0 α less than 0. Now you may wonder that how I am obtaining this? You put some numerical value and if you plot some plotting software and then you will get the same thing ok. Take typical value of that ok. But one important thing you should must understand that stability and unstability stable and unstable are separated by the boundary and this is the boundary over which $\frac{\partial^2\pi}{\partial\theta^2}$ is 0 right. because that is why from stable to unstable you see that from here it is solid line but here these are all broken line you understand the unstable to stable transition boundary that is what I am defining at the stability boundary clear. One more important stuff that I am going to explain it to you is that you see how you are obtaining the stability boundary by the making the Hessian or by second derivative to be There is another way we can obtain it. You know how? You see that. Stability also implies, let me explain it here. So, this is the condition of stability, we are getting something like this, right? Here, stability boundary, right? Could you please give me the equilibrium path? Equilibrium path, what was the expression for the equilibrium path?



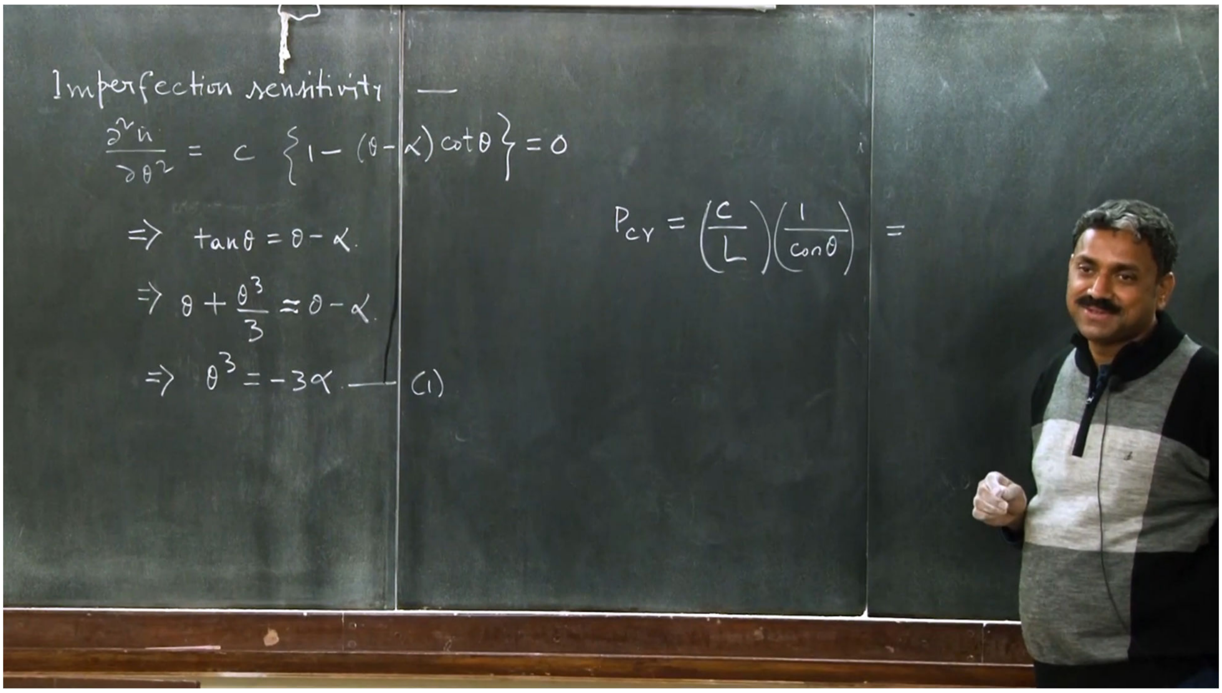
Equilibrium path was C (θ - α)/ L sinθ, right? C (θ - α)/ L sinθ right, you see I will just do one thing, so, $\frac{\partial p_{eq}}{\partial\theta} = \frac{c}{l}$

you see that $\frac{\sin\theta(1)-(\theta - \alpha)cos\theta}{sin^2\theta}$

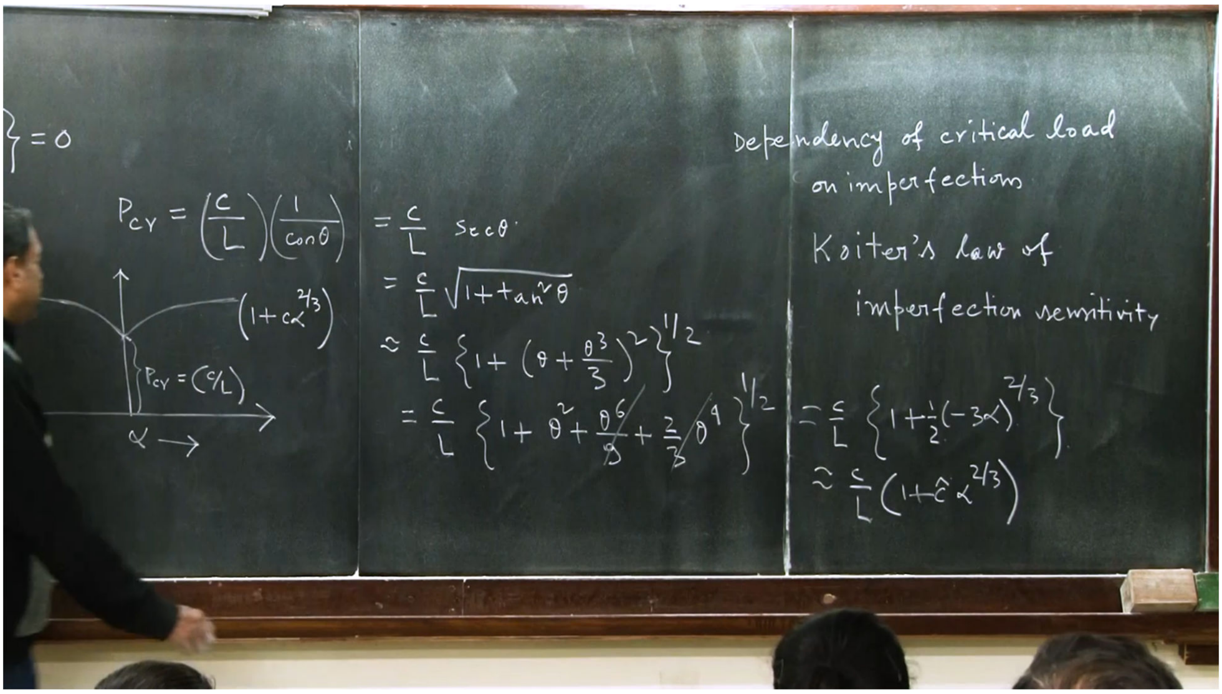
$\frac{1}{\sin\theta}$ $\{1 - (\theta$ - $\alpha)$ cot $\theta\}$, you see this is analogous to this $\frac{1}{\sin\theta}$ $\{1 - (\theta$ - $\alpha)$ cot $\theta\}$, $\frac{\partial^2\pi}{\partial\theta^2} = \{C - C(\theta - \alpha)\cot\theta\} > 0$, these two expressions are analogous only the difference is that there is sinθ. So, now the equivalence between this and that expression you can understand right.

So, what I can say that $\frac{\partial^2\pi}{\partial\theta^2}$ it is a analogous to $\frac{\partial p_q}{\partial\theta}$ right, then see for ascertaining the stability boundary or stable and unstable transition, you have to explore on the second derivative of the potential energy function right and when you are doing that when you are obtaining $\frac{\partial^2\pi}{\partial\theta^2}$ then on that P you have to impose the equilibrium path as well because see the first system has to satisfy equilibrium right. So, finding out $\frac{\partial^2\pi}{\partial\theta^2}$, see for this system it is simple because that is what $\frac{\partial^2\pi}{\partial\theta^2}$ you can easily find out. But for complicated system you will see when destroy system there will be problem to establish. Instead of that whatever the equilibrium path if you take the derivative of that, that will be analogous to this. So, that will also reveal the information. Now could you please tell me why this and this are same? I mean these are kind of equivalent. See how infinite element what you did? In order to find out the equilibrium equation right, when you obtain the matrix equation assume that, I mean you have considered linear finite element right. In linear finite element you have minimized the potential energy and you get sets of linear simultaneous sequence right. Now consider, so that was, that was the way you obtained right. Now consider it is a non-linear system. In non-linear system what do you do? How do you analyze non-linear system? You write the equilibrium in incremental form, right. So, instead of making the potential energy minimization, you just incremental potential, okay, $\delta\pi$ is equal to 0, right. So, in incremental, you write down the equilibrium in incremental form ok. Then incremental form that leads to what? That leads to nothing but you just give and I mean you can write down the same incremental in two different of course one is energy but in nonlinear finite element is easier and convenient to write down in what? In using the virtual work ok because also virtual work equation is more generalized. You know in many cases, this energy minimization might not work especially some non-linear systems, where it will not work and sometimes you must have learned that instead of potential energy function you sometimes work with the complementary functional and this and there are many functional alternating alternatives functional that people have proposed. But when you write down the virtual equation right using virtual work, then you basically whatever the equilibrium you just apply some virtual displacement and you write down virtual work equation right. That give you incremental equation right, incremental equilibrium equation. So, see in linear finite element what you have obtained? You obtained the force is equal to stiffness into deflection right. and incremental if you want to write then you are going to write δf ok. So, I am going to write δf is equal to k_t into d, I mean I am going to write here instead of δ. So, here it is I am writing δ, here basically you see these are the what forces and this is displacement right k is equal to k into in linear in incremental form these are the internal forces and these are the pivotal deformation and this is tangent stiffness matrix tangent right. So, this is what $\frac{\partial p_q}{\partial \theta}$ this is nothing but k_t. equilibrium. So, first order equilibrium you have obtained right equilibrium part, but now you are differentiating further that means what will happen if there is little change in the in θ what is going to happen in the internal in the P ok. So, that is $\frac{\partial^3 \pi}{\partial \theta^3}$. So, both of these whether it is Hessian of the potential energy functional or it is the gradient of the force with respect to θ, both are equivalent simply because they correspond to a increment, I mean, incremental, equations, okay, incremental equilibrium equations and they basically ascertain the determinant of K_T, the tangent stiffness, okay. I will explain it here. You see here, why it is stable? Because you draw a tangent here, tangent is always positive. $\delta\theta$ is what nothing but tangent stiffness right.

This positive, positive means what? You see that because here it is scalar right. If it is a multi-dimensional space, multi-dimensional system you have to take of course determinant and principal minus. Positive means it is stable, these are all stable. See in negative, you said here also for negative θ in the slope will be negative. so, ultimately $\frac{\partial p}{\partial \theta}$ is equal to positive, ok. So, that is why it is positive. Here it is positive, here it is negative, that is why it is unstable. You understand that? So, slope of the equilibrium path also reveals the nature of equilibrium. Slope of the equilibrium path reveals the nature of equilibrium, whether stable or unstable. Please note it down. The slope of the equilibrium path reveals the nature of stability, clear. And then the equivalency must keep in your mind that the equivalency of the hessian and then the gradient of the equilibrium path with respect to the displacement this will come sandy ok. So, you see this flowery diagram, it reminds you maybe the. I do not know what it reminds me something else in biology, when you used to cut this flower right, then if you will remove the petals right and then you will see that I think there will be egg and then these things will come and then fertilization etc. So, it reminds me that figure actually in biology right isn't it anyway. One more thing is still remaining the imperfection sensitivity. So, you please see the simple $\frac{\partial^2 \pi}{\partial \theta^2}$, we are going to study in the last aspect that is imperfection and I think it is most important okay, imperfection sensitivity. So, $\frac{\partial^2 \pi}{\partial \theta^2}$ I have derived it, what was that that was nothing but C 1 - θ - α, what θ right that was 0 that is the stability boundary. So, I want to study stability boundary. So, from here part you will get $\tan\theta$ is equal to θ - α and this is on the stability boundary ok. So, I will just expand this $\tan\theta$ as $\theta + \frac{\theta^3}{3}$, and then you will see, it is equal to θ - α okay and that means θ^3 will all approximate X. So, θ^3 is nothing but - 3 α okay this is one increase right. Now you tell me that what was the critical load? Critical load was $(\frac{C}{L})(\frac{1}{\cos\theta})$ right and I am trying to study the imperfection sensitivity.



what is imperfection sensitivity? Imperfection sensitivity is nothing but dependency of critical load on imperfection. Dependency of critical load on imperfection. Imperfection sensitivity implies the dependency of critical load on imperfection So, why it is important you will see from here we will generalize our notion of imperfect sensitivity. Koiter back in 1940s, koiter ok, he developed imperfection, his theory of imperfection sensitivity. And you see those time there were no finite element right, finite element appears only around 1950s, I mean that was I think the development of computer and then the development of matrix method whatever is the precursor of this finite element were similar right ok. So, I mean if development of computers digital computer was played a synergistic role right in both. At that time there were no this no finite element and things and all these great minds basically worked and koiter was one of them. Of course, initially when he wrote down his thesis I mean, it was in the German language. So, later it was translated by someone who translated it I think and then he was a I cannot recall his name but he said that he has rediscovered koiter okay koiters. Koiter's law of imperfection sensitivity because why I am mentioning because quite sometimes people will think that what I mean a simple system I am doing all this unnecessarily mathematical juggler i am doing, but that is not actually correct there are much more interesting thing to do, ah recently one of my student PhD students has worked on this direction and then he got a couple of good publication on that. actually, we have exploited the koiter's notion in safety assessment and probabilistic and imparting what is the influence of proper random uncertainty on cell buckling and things like that. So, koiter basically develop the law of imperfection, I will state that what it, so these are called power law of imperfection sensitivity ok. So, koiter as when he discovered his thesis went into oblivion. So, this was not noted by later people when it was translated it got a wider attention actually ok. So, from here see, his treatment was different I mean but if we cannot generalize to that extent in classroom setting. But using simple system we will see we will lead to those kinds of things ok. And you will see you will be able to appreciate that what we are doing ok. So, I have seen that ok. let us we have to get rid of this θ, we have to relate piece here with α right. So, we want to study whatever is going on in the stability boundary later. So, in the stability boundary it is 0. So, that basically gives us what? That basically gives us relation with θ gives us - 3 α rights. on the stability boundary I am assuming that θ is small that because θ is small that is what I mean we can truncate the $\tan\theta$ and I will introduce it to x right ok. So, then I am just writing this $(\frac{C}{L})$ $(\frac{1}{\cos\theta})$ is nothing but $\frac{C}{L}\sec\theta$ and $\frac{C}{L}\sec\theta$ is what $\sec\theta$ is $\frac{C}{L}\sqrt{1+\tan^2\theta}$ right. So, I am going to write it , $\frac{C}{L}\{1+\left(\theta+\frac{\theta^3}{3}\right)^2\}^{1/2}$ and then $\frac{C}{L}\{1+\theta^2+\frac{\theta^6}{9}+\frac{2}{3}\theta^4\}^{1/2}$ and here what we are going to neglect the half these terms just take one what is $\frac{C}{L}\left\{1+(-3\alpha)^{\frac{2}{3}}\right\}$. So, you can also write some constant $\frac{C}{L}\ (1+\hat{e}\alpha^{\frac{2}{3}})$ So, what we can see that P critical. So, what does it mean? The critical load is going to increase with imperfection and what two-third power load. So, this I will now draw the imperfection sensitivity diagram. So, here I am going to write α is the imperfection. So, P critical was $\frac{C}{L}$ for but then this will be is going to increase. You see that so P critical basically it increases. it is being 1 + some order of some constant α to the power $\frac{2}{3}$. So, this is koiter's two-third power law of imperfections. So, a system which one we do? Well, I mean you just take it like this series expansion. so, we just take half okay and two binomial okays, so that is what.



because 1 + θ^2 to the power 1 + $\frac{1}{2}$ θ^2 ok. That is what this half and these all these things I am punishing. So, what we see that a system which demonstrate stable bifurcation and then it potential energy functional contains this all-quadratic term. so, it increasing critical load with so imperfections rather increase the critical load right. and that is follow so this is called koiter's two-third power law, koiter's two-third power law. So, this kind of system follows two-third power law ok. So, why it is happening? Why do you think that imperfection is increasing the load? The reason is that if you see consider the system. it was restoring force is coming by rotational spring C into θ where the destabilizing force is what C into L, L into sinθ right. So, the destabilizing force and sinθ is always less than θ you see especially for higher θ. So, θ is equal to 0 sinθ tending to θ you see so higher θ, what is happening your restoring force is increasing in much a larger rate than that of the disturbing force. You see your restoring force of the order of c θ and your destabilizing forces of the order of sinθ something ok. beca use for θ increases in the larger rate than the sinθ for increasing θ the restoring force is being more dominant than that of driving force and that is what is leading to this, not only the stable bifurcation but also the imperfection sensitivity ok. and imperfection, this is not imperfection sensitive that means when the critical load is increasing this is called imperfection insensitive, and two-third is not very significant power load. I will come their worst imperfection sensitive. So, this kind of structure because the critical load increases with imperfection. these are referred as not imperfection sensitive. When the critical load decreases with imperfection that is called sensitive. But then there is mild sensitivity and worst sensitivity. So, two-third power means it is mild sensitivity and here it is insensitive because it is increasing. Clear? Okay, thank you for today's lecture.