Stability of structure

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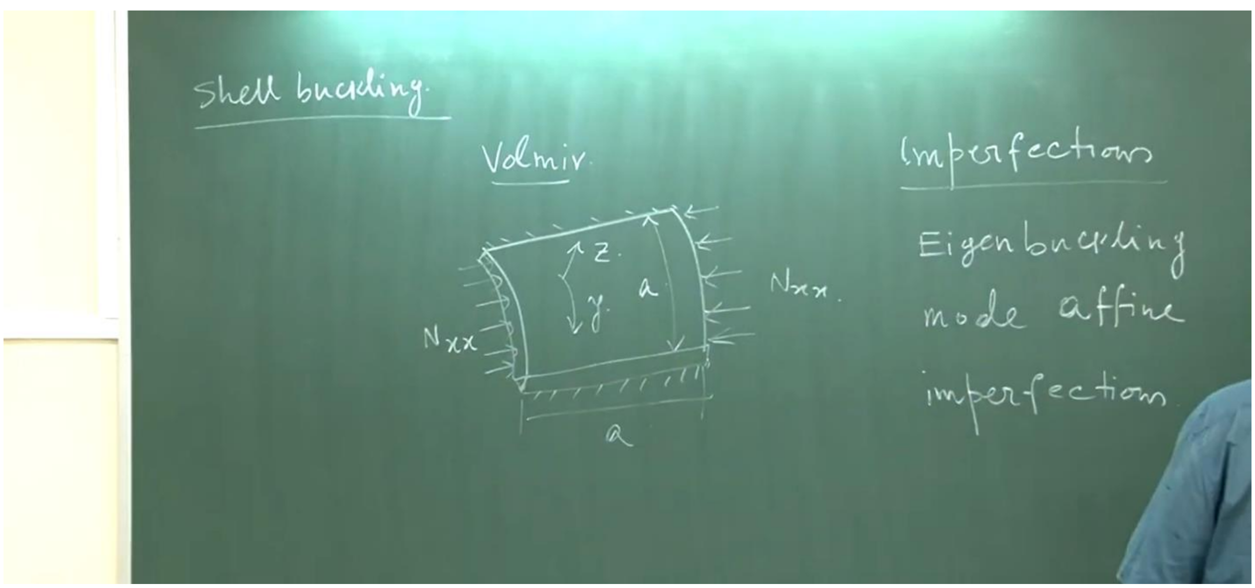
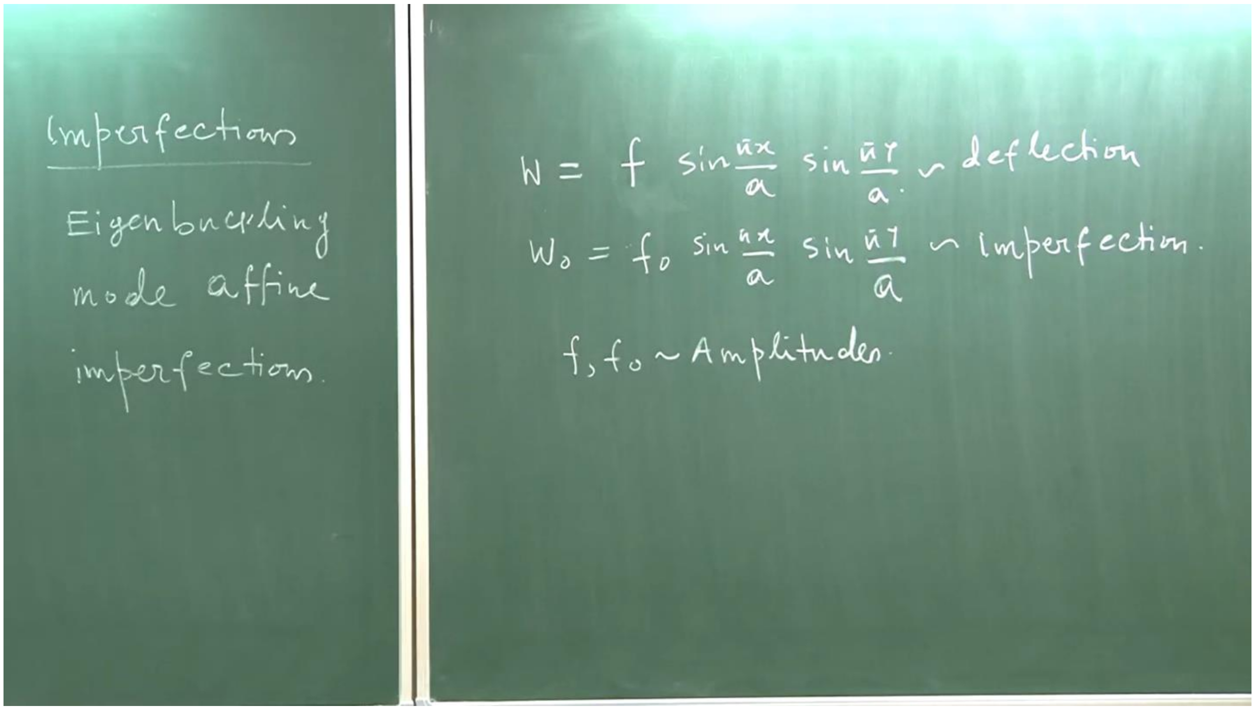
WEEK-12

Lecture 25: Shell Buckling

So, well, this is the last lecture on shell buckling and perhaps the last lecture on the, you know, subject. So, let us briefly recall what we have covered till now. So, we have covered the buckling of shells, axisymmetric buckling of shells, and after that, we have covered the buckling of shells following, you know, Donnell's equation. We have derived the Donnell equation following a simplifying assumption, and then we have solved for the three cases. One case is an axially compressed cylindrical shell, and then axial compression occurs along with external pressure or suction, whichever is applicable, and then the third one is subjected to torsion. We have demonstrated that well Even though there are certain simplifying assumptions involved in all the solutions, there are very few interesting facts that we have uncovered. You know, using this theory, that it is the certain loading condition; for example, an axially loaded cylinder. We have seen that it reveals the closely spaced multiplicity of modes.



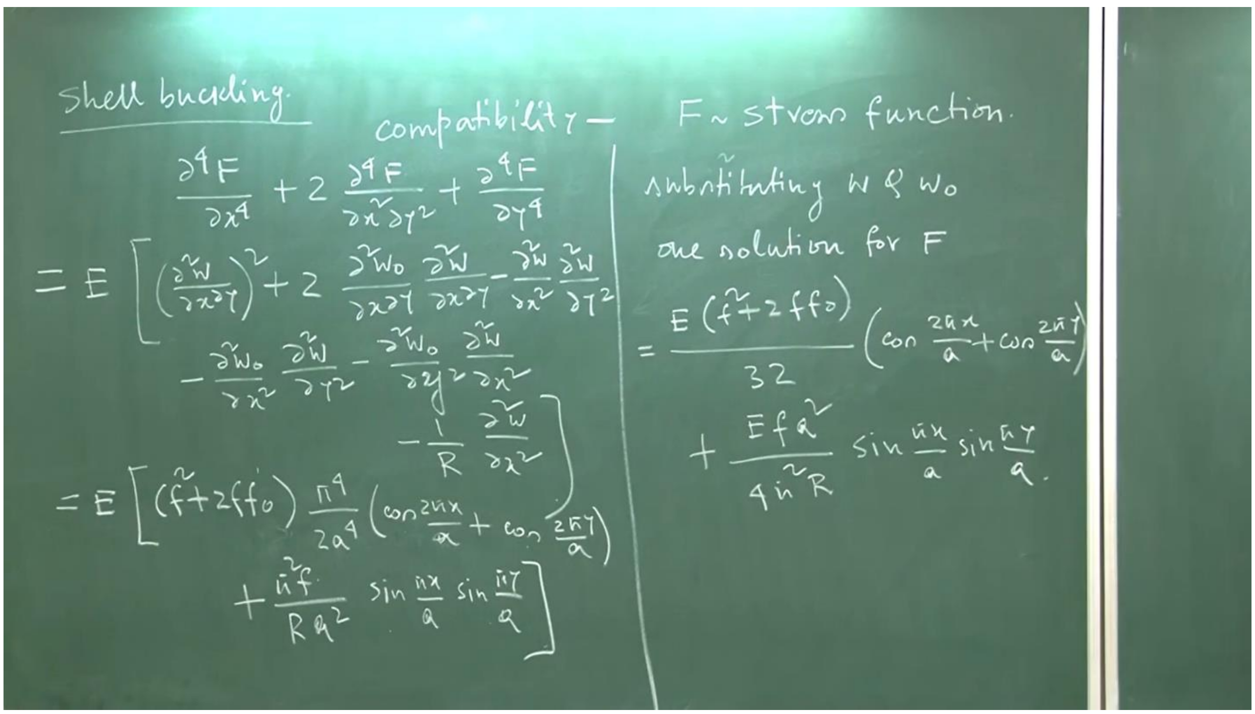
And they allow the modes to interact because they are geometrically coupled, right? And Fso, there is a huge reduction in the critical load that will eventually be captured in post-critical analysis, right? However, we have also seen that this mode of multiplicity is a function of the mode of loading, and the same cylinder is subjected to lateral pressure, you know, external pressure. If suction or the same cylinder is subjected to torsion loading, then it is no longer imperfection sensitive, okay? So, the essential fact is that the closeness of the mode and the post-critical load drop and subsequently experimentally obtained critical load and then analytically obtained critical load shows huge disparity. All these things are related. So now, what we are trying to do the analysis in the post critical regime that means after buckling what really happens. So, for that, we have adapted a similar approach, as you know, in Dernel's equation, but we have kept the equilibrium equation out of the plane equilibrium equation, you know. And then the in-plane equilibrium equation separates, and of course, there will be coupling. So, we have two field variables: one in W that is out-of-plane deflection and the other field variable is essentially the stress function. So, what we know is that axial, you know, in-plane forces can be derived from the stress function, and then the governing equation for the equilibrium, as well as the compatibility equations we derived. We have written it down, and now we are trying to solve it. Of course, because of the complicated non-linear form of the equation, we are going to consider a simple panel, which is a cylindrical panel, okay. And this panel, we are assuming, is subjected to axial force N_{xx} in two directions: this is x, and along this y direction; it is all simply supported. These are all simply supported. All these edges are simply supported except that they are allowed to deform axially; because of this axial loading, you know, axial compression, and then Z is perpendicular to that. So, the solution we are going to present is the volumetric solution, right? This is a simple ad hoc solution, but it still shows essential features. The volmiv solution we also adopted to solve the post critical behavior of plate in that essentially; we recall the steps involved. So, there also we had two equations, one is equilibrium equation and another equation was compatibility equation. So, what did we do? We essentially assumed some deflection profile. And then, from the initial state of stress, we derive some approximation of the stress function, and then we substitute it into the equilibrium equation, right? Because they are coupled equations, we solved them using Galerkin, essentially following the same steps. So, we have also assumed that this is of length a, and in this reaction, this is also of length a. So, why are both a, because of the simplification, okay? Now, we assume, and also the other interesting thing that we must consider is the imperfections, right? So, if you can recall what we discussed in the previous class, we explained the inclusion of the term "imperfection" in the strain displacement relationship, right, and their imperfections. So, the kind of imperfections that we are going to assume are similar to the buckling mode, okay. So, these are eigen buckling mode affine imperfections; this is similar to the previous ones except they differ by scaling factors, okay.



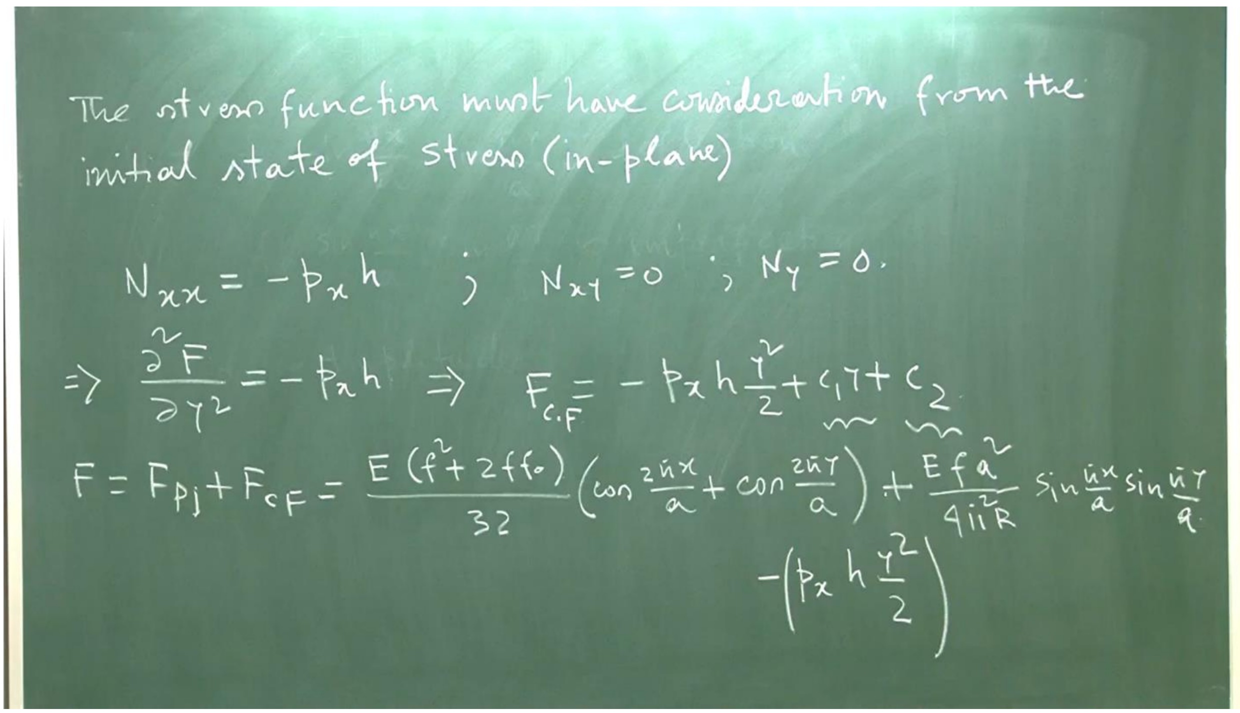
Please note that imperfections can be random as well, but that will quantitatively change it; qualitatively, there will be no change, and we want to demonstrate the qualitative post-critical load drop, okay. So, as you see the assumption, so, let us assume W is equal to W_0, with this kind of boundary condition $\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$ right, $\pi\frac{1}{a}$ right, this is the deflection. Now, imperfection; we can assume imperfection w_0 is, you know, $f_0 \sin\frac{(\pi x)}{a}\sin\frac{(\pi y)}{a}$. So, this is the deflection, okay? Outward or out of deflection, and this is imperfection. Please see why they are self-refined, as this is similar in expression. Special distribution to that of the deflection, right? That is why F and F_0 are nothing but the amplitudes of F and F_0, right? The amplitudes of the deflection and imperfections, okay. So, with these assumptions, you know, let us substitute in the compatibility. First, let us use the compatibility equation, okay? So, I am removing it. Because the equation is a little big, you know, large. So, the equation capital F is nothing but we are considering the compatibility equation, okay. Compatibility, because we have assumed the deflection and imperfection, requires us to know the stress function, as there are two variables that we are considering: compatibility and f, which is the stress function. So, here we will write down the equation:

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2\,\partial y^2} + \frac{\partial^4 F}{\partial y^4}$$
$$= \mathrm{E}\left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 + 2\frac{\partial^2 w_0}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right]$$

So, this is the governing compatibility, okay. So, what will we do? We just take w and w_0 and then substitute them here, okay? So, substituting w and w_0 in this equation, if you substitute in this equation, you can clearly see that the right-hand side will be known, right? And when the right-hand side is known, the right-hand side will be expressed in terms of some kind of series of sine and cosine; sine function sine, these are all double derivatives.



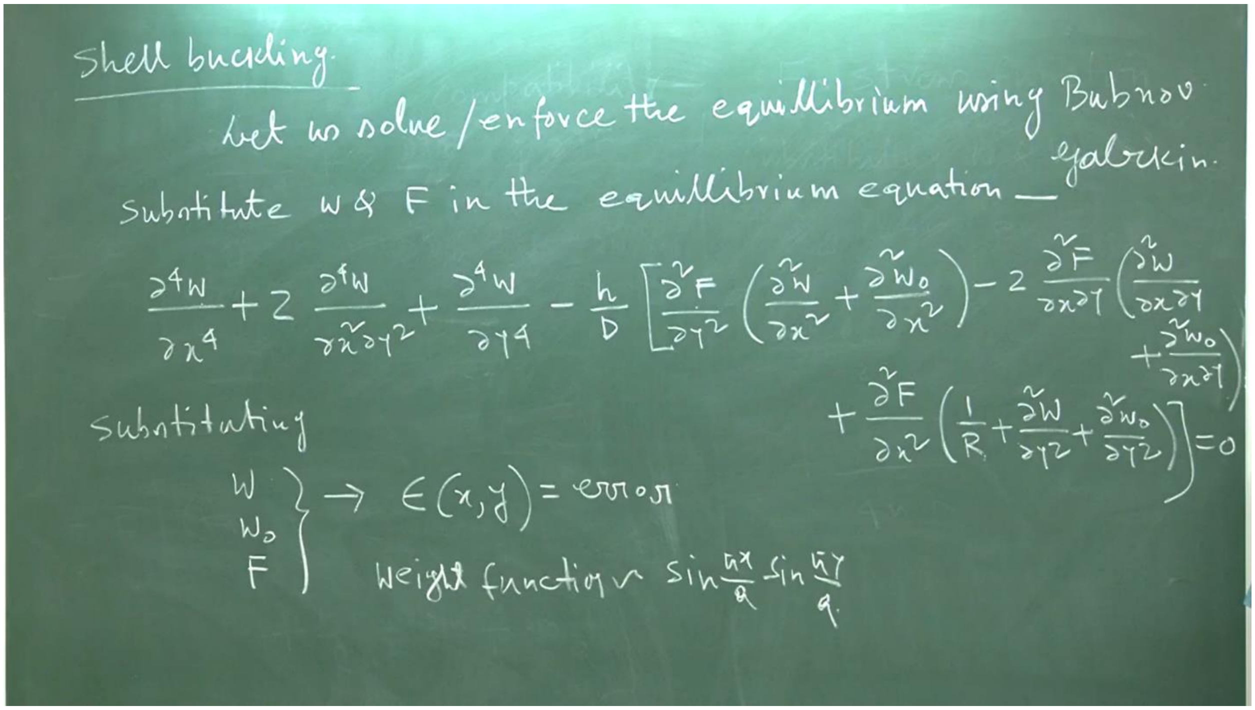
So, it will be all sinusoidal right and when you take you know product of sine you can always express this in terms of addition and difference of sine function right Because I know that we know that $2\sin a \sin b$ is nothing but $\cos(a+b) - \cos(b-a)$, right? So, what we see on the right-hand side is a sum of what? If you know the sum of trigonometric functions, the sum of sine and cosine, sine function, right? So, if the sum of sine functions, then you can see this is a linear differential equation and this is a fourth-order differential equation, right? It must have due consideration of the initial state of stress, right? So, the stress function must take due consideration of the initial state. So, the stress function must have consideration from the initial state of stress. Initial state of stress right or initial state of a plane stress. Stress means these are all in plane stress, right? How do I do that? Now, if you can recall, that initial membrane stress is what? $-T_x h$. I am assuming that it is. And this one is axially compressed in the x direction, right? I am assuming that P_x is the pressure per unit area. So, $P_x h$ is the force per unit length, right? P_x is the pressure, okay? Axial compression pressure. And then N_{xy} is 0 and N_y are 0, right? So, what is N_{xx}? N_{xx} is nothing but $\frac{\partial^2 f}{\partial y^2}$ is nothing but $-P_x h$; h is the thickness of the shell, right? h is the thickness.



So, we integrate it and then you will see F will be $-P_x h \frac{y^2}{2} + c_1 y + c_2$. If you consider this term and this term, of course, this constant stress function does not really give any stress. So, it is not really relevant. This term, if you differentiate it twice, will also be 0. It does not give anything. So, only meaningful contributions come from this, right? So, now this is one solution; maybe F_1 is another solution, right? So, what we have learned for any equations is right; if so, we can treat this particular solution, particular integral, and this complementary function as a complementary function. So, the total solution f is the particular integral solution plus the complementary function, which is the contributed function, because it is a non-homogeneous equation; the solution of a non-homogeneous equation consists of two terms. So, I can write. If the total solution becomes $\frac{E(f^2+2ff_0)}{32}\{\cos(2\pi x/a)+\cos(2\pi y/a)\}+\frac{ef^2}{4\pi^2 R}\sin(\pi x/a)\sin(\pi y/a)$ and then this one is $-(P_x hy^2/2)$ right the whole. So, this is the function of the stress function, right? So, now you see I have the displacement profile dw, but f and f_0 are not known; f and f_0 are unknown, right? But we got a functional expression of f, w, and w_0, right? So, how do we determine f? If you know, because what is f_0? The imperfection's amplitude. If you do measurements, then you should know the imperfection amplitude, right? But what you do not know is that f is the given amplitude of displacement, right? So, how will you do that? Which equation did we make up? We have satisfied stress compatibility, we have satisfied compatibility, and we have satisfied the boundary conditions while assuming this kind of solution, right? Boundary conditions were satisfied, and compatibility was satisfied to obtain the particular solution for the stress function. The only thing that remains is the equilibrium equation. So, we have to solve the equilibrium equation, but do you think that if you directly solve this f, substituting the stress function, the function of xy, for w and w_0, it will satisfy the equation? It will not, right? There will be some residue. So, the best way to do it using residual error technique, right, least square, you know, Galerkin technique, Bobnov-Galerkin, right. To solve the equilibrium. So, let us solve or enforce the equilibrium using the Bubnov-Galerkin technique. Why Bubnov? Bubanov refers to the weight function that you are going to use. will be same as the function you have assumed. So, what is the weight function? The weight function will be the same as whatever is associated with w, $w\sin(\pi x/a)\sin(\pi y/a)$, right? That is the weight function. So, the error terms you know by substituting in the equation, putting w and f in the equilibrium equation. We substitute the equilibrium equation; what you are going to get is, as you see, the equilibrium equation if you can recall what the equilibrium equation was. The equilibrium equation was something like this: you see that

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4} - \frac{h}{D}\left[\frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\right)-2\frac{\partial^2 f}{\partial x\,\partial y}\left(\frac{\partial^2 w}{\partial x\,\partial y}+\frac{\partial^2 w_0}{\partial x\,\partial y}\right) + \frac{\partial^2 f}{\partial x^2}\left(\frac{1}{R}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w_0}{\partial y^2}\right)\right]=0$$

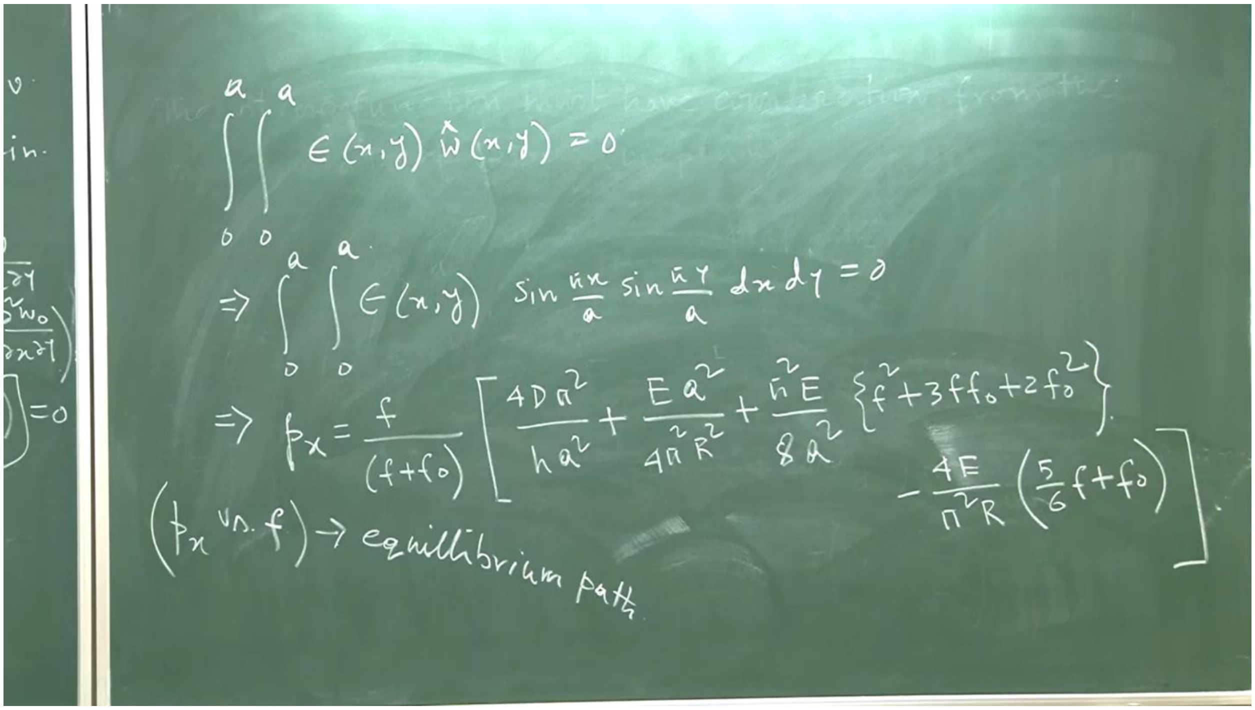
that was the equilibrium equation, right?



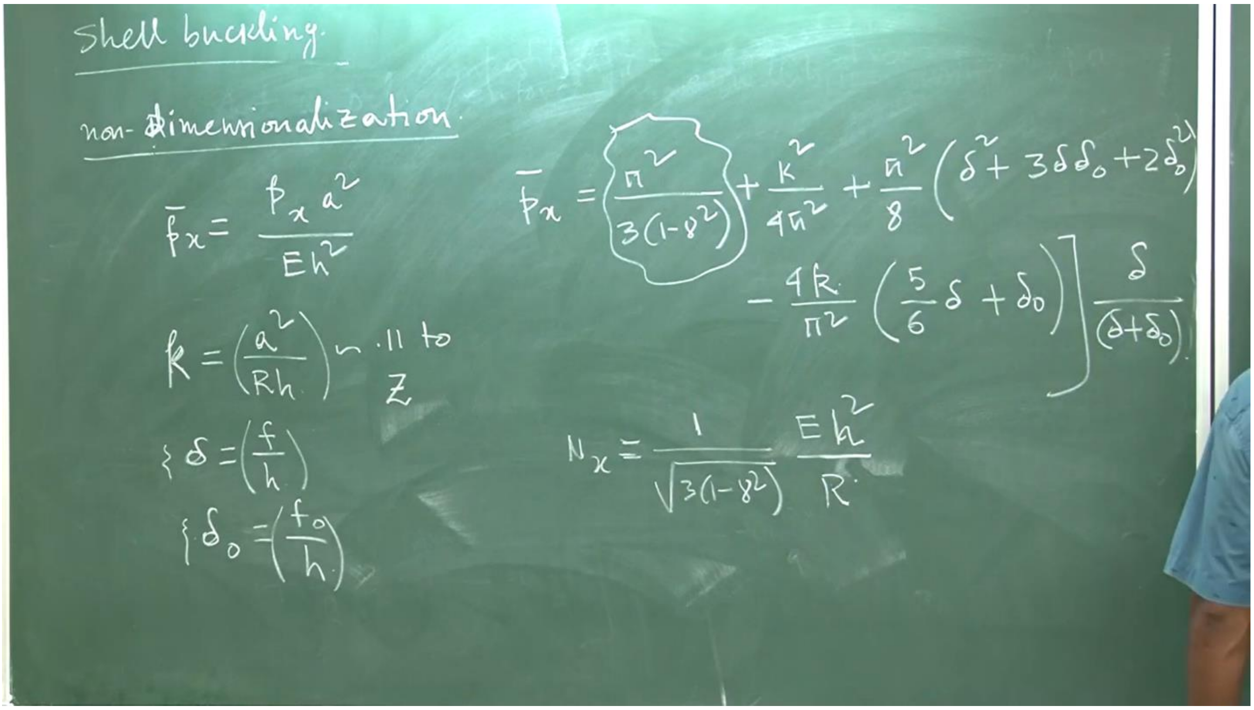
So, you substitute w, you substitute for f, you substitute w_0, right? And then whatever, I am assuming that substituting, you know. You know, you know, you know, 0 and f, okay. Substitute there will be an error term; I am assuming the error ϵ. This is an error, and what will be the weight function? Weight function, you know, the weight function for Bubnov-Galerkin will be what? The same $\sin(\pi x/a)\sin(\pi y/a)$ right that is the error function. So, now if we enforce that error minima, and you know error minima is weighted least squares, right? Weighted least squares, what do we do? We multiply the error by the weight function and make it 0 over the domain. So, integration from 0 to a, integration from 0 to a, $\epsilon(x, y)$ error 1, and then weight function. Function $w(x, y)$ must be equal to 0, right? So here we have the integration from 0 to a; whatever the error is, I do not know, but there will be a big expression of error. It is better to do it using, you know, MAPLE or MATLAB, you know, using symbolic computation. You know that $\sin(\pi x/a)\sin(\pi y/a)$ must be equal to 0. This is one equation by doing so from here. That will give you the equation if you simplify. Then he will get from here; I will get the unknown, which is P_x. P_x will get you. I am just writing the expression:

$$\frac{f}{(f+f_0)}\left[\frac{4D\pi^2}{ha^2}+\frac{Ea^2}{4\pi^2R^2}+\frac{\pi^2E}{8a^2}\{f^2+3ff_0+2f_0^2\}-\frac{4E}{\pi^2R}\left(\frac{5}{6}f+f_0\right)\right]=P_x$$

So, this will be the, you see, this is the, please note that this P_x is beyond the buckling, okay. So, this P_x is beyond buckling. So, it can take both the critical and post-critical regime, right? So, what, are we going to plot? Our ultimate goal is to get the equilibrium path here, right? So, P_x versus what?



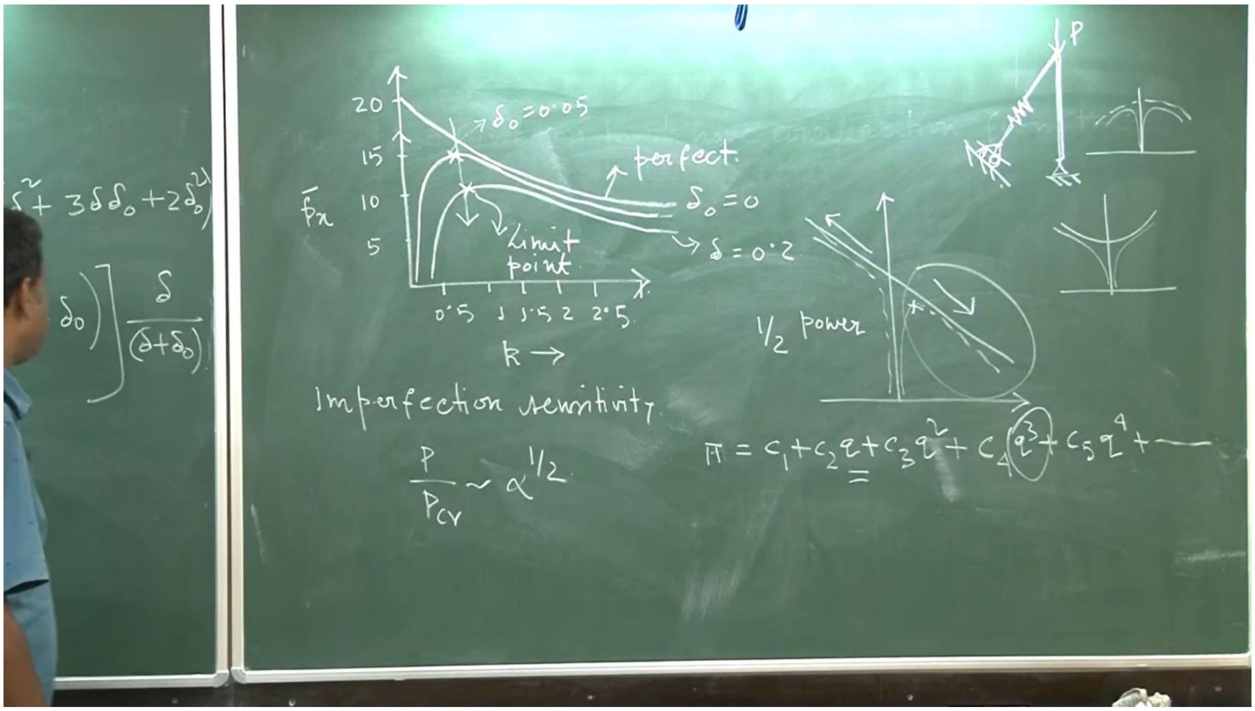
So, we will plot P_x versus F. F is the amplitude of displacement, right? Amplitude of outward deflection, you know, is out-of-plane deflection. So, this is will give you the equilibrium path. So, this is the equilibrium path for the cylindrical shell panels. I will not call it a whole shell because I am considering a panel. For the whole shell, it will be more complicated, okay. But I want to demonstrate how curvature is going to influence the load drop. So, the equilibrium path is the one from the very beginning; you will now go back to the very few initial lectures, right? I was repeatedly emphasizing the equilibrium path because that provides all the information about the system's behavior and stability. So, now I am going to plot it, okay. So, when I am plotting it, please note that there is very important stuff here. That f versus P_x versus, if the equilibrium path is a function of the imperfection amplitude f_0. It will also be a function of the distribution of it, which means here we are assuming that this is self-affine, similar to that of the displacement field. But it will have little imperfections if you do a full-scale finite element simulation, which is available in various research papers or journals published. We also have our papers there that you can see, but here at least one very interesting thing we can see.



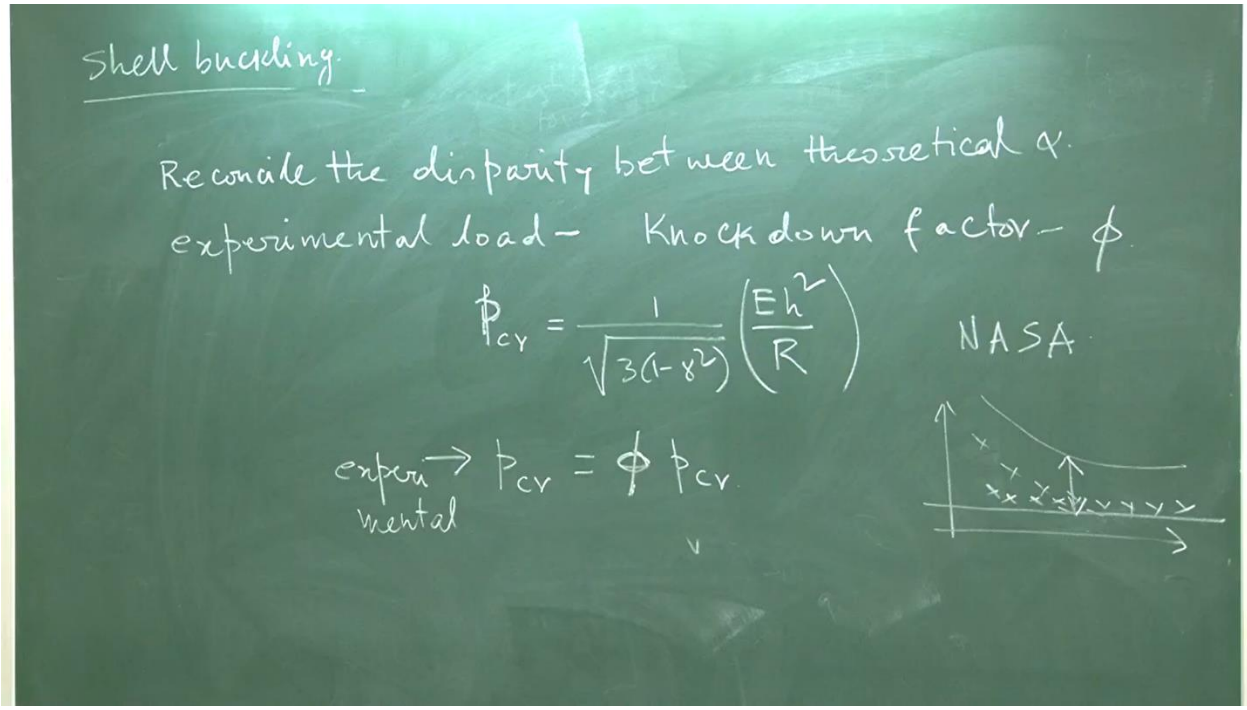
So, I am going to plot this thing for you; there are some kinds of dimensionalization that are possible. So, some dimensionalization is okay, and non-dimensionalization is okay. Non-dimensionalization, so it will not have dimensions, okay. So, if you do non-dimensionalization, then this is the way you will do $\bar{P_x}$, so P_x is being normalized, So it is P_x, whatever that P_x we obtain, $\frac{P_x a^2}{Eh^2}$, okay, $K = \left(\frac{a^2}{Rh}\right)$, $\delta = \left(\frac{f}{h}\right)$ and $\delta_0 = \left(\frac{f_0}{h}\right)$. So, these are all non-dimensional. Please note that f/h, where f is the displacement and h is the thickness of the shell, is non-dimensional with respect to the imperfection amplitude, which is non-dimensionalized by the thickness of the shell. So, these are the non-dimensional deflections, and this is the non-dimensionalized imperfection amplitude, right? And this k is a, you know, parameter that is essentially (a^2/Rh), okay. So, is the length, so it is similar to the Burt-Dorff parameter, right? $L^2/(Rh\sqrt{1-\nu^2})$, right? It is similar to the Z parameter, which we have previously explained, and which distinguishes between a moderately long shell and a short shell. And P_x is the non-dimensionalized axial force and axial stresses. If you do, then finally, equations, you will get $\bar{P_x}$ if you substitute; upon this, it can be non-dimensionalized as:

$$\bar{P_x} = \left[\frac{\pi^2}{3(1-\nu^2)} + \frac{k^2}{4\pi^2} + \frac{\pi^2}{8}(\delta^2 + 3\delta\delta_0 + 2\delta_0^2) - \frac{4k}{\pi^2}\left(\frac{5}{6}\delta + \delta_0\right)\right]\frac{\delta}{(\delta + \delta_0)}$$

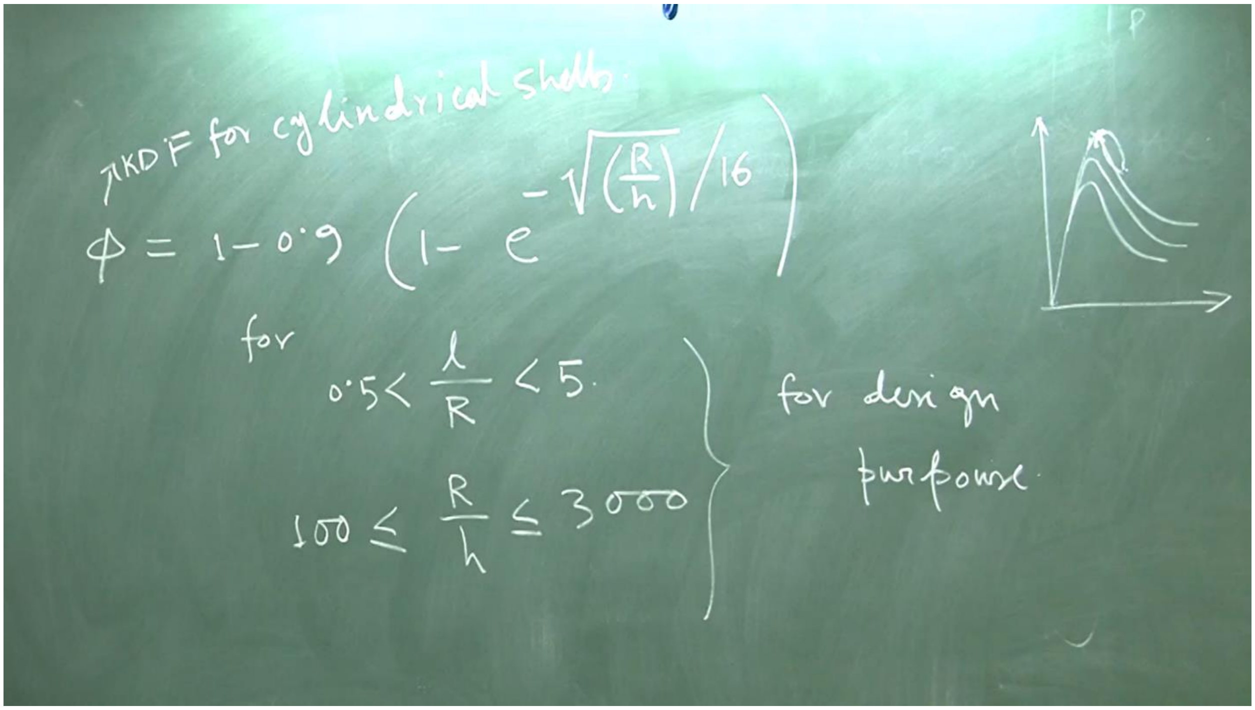
Now, you can clearly see that. when we consider the critical load, critical load if you can recall for the cylindrical shell what it was? For the axial force, if you can recall, N_x was what? N_x was $\frac{1}{\sqrt{3(1-\nu^2)}}\frac{Eh^2}{R}$, something like that, right? Is that not it? But here you see that, if you non-dimensionalize, you see that the $3(1-\nu^2)$ term is coming right. So, this term takes care of the buckling things; you see additional terms take care of what is happening in the post-critical state, okay? So, now I am going to draw it, okay? So, when I am drawing it, please note that. So, you see, not known as the K parameter, because that is similar to the "but" or "parameter," right? With that $\bar{P}$, I am plotting. So, you see that if there are no imperfections, then you see what kind of plot you can see? Of course, as you increase the displacement, as you increase the $\bar{K}$—sorry, $\bar{K}$ means some kind of slenderness measure. So, it will decay, right? The way columns decay with increasing slenderness ratio is similar, but if you fold in little imperfections, this is for a perfect cylinder. Please note that this is for perfect. Perfect cylindrical shell. Perfect. But as soon as you put in little imperfections, then what is happening? It is going up; this is the point where it is buckling because after that, there is a decrease in the load-carrying capacity. Do you see that? So, from the equilibrium path itself, you will get this, correct? That we have seen which one is the critical load. Similarly, what is the maximum load here? That is the critical load. After that, do you see that there is a decay? But what is most interesting to see here is that, you know, for 20 to this one, there is a decay; you see that there is a huge decay in the critical load. And why this decay, why this significant decrease in the critical load? It is because of imperfections, you see that. So, this really shows the imperfection sensitivity, okay. Right. Now and if you see this one, you can recall little bit now here I will go to once again initially when we have considered the cases for a simple system right. Can you recall where you can see this kind of imperfection sensitivity in this kind of equilibrium path? Recall that we are having this one; there is this rigid bar, right? And it is restrained here by the inclined spring, and then it was subjected to some force P, right? And then we can clearly see that you have seen it. On this side, it was, you know, from here to here, and then, you know, so asymmetric. You know the asymmetric equilibrium path and stable-unstable behavior. From this side, it is stable; this table has unstable behavior. So, this was basically similar to that. So, you understand, although this system was a hypothetical system, an idealized toy system. but we now can see a system at that time I used to emphasize that shell shows this kind of behavior, but do you see a similar kind of equilibrium path you can see from here? So, this is stable-unstable bifurcation, asymmetric stable-unstable bifurcation. And when it is unstable, it is imperfection-sensitive because there is a limit point; it is increasing and then it is decreasing. You can clearly see these points are called limit points, right? So, these are limit point instabilities.



So, as soon as we put imperfections, the bifurcation instability gets converted into a limit point instability. Because this sudden transition from a stable to an unstable system is not abrupt, okay. Here it attains maximum load and then goes down, so this is called the limit point. So, whatever we have discussed in any shell and when you started the course or idealized system, you can see qualitatively similar behavior here. Another interesting thing here is that you can see the potential energy function we have written in terms of, $c_1 + c_2q + c_3q^2 + c_4q^3 + \cdots$, you know, q^4, something like this. So, whenever you can see the various, we can see this kind of equilibrium path, right? And then on imperfection, this was stable and symmetric, and then we have also seen, you know. Symmetric but unstable paths are right and right like this, but in all these cases, it will all include symmetric bifurcation, square terms, and quartic terms. q^2 q^4 even power term okay that is why they were symmetric but when there is this asymmetric path is there then it will have this cubic term You know this cubic term becomes important, you see, of course, this. This has to be a quadratic function, right? So, these do not really matter, so the cubic term will be present. So, shell buckling involves this cubic term; that is why it is so asymmetric in bifurcation. And because of this cubic term, it shows imperfection sensitivity and what kind of imperfection sensitivity this shows. Half power imperfection sensitivity, half power right. Half-power imperfection sensitivity. How do you explain that? P/P_{critical}, it varies with imperfection to the power of half, right? So, P is the limit point load, and P_{critical} is the critical load. So, with increasing imperfections, the load-carrying capacity decreases, right? And it is the notorious imperfection sensitivity, okay? And there is no imperfection sensitivity that is captured through this present analysis, which we have simplified a little bit, you know. But it takes into account the coupling, you know, the coupling between the in-plane stresses and then the out-of-plane deflection and things. So, they are geometrically coupled; they are coupled to geometric nonlinearity, okay. And that is what the modes interact; the closely spaced mode or multiple modes interact, and this interaction among different modes is the culprit. See, because of the cubic term, the system itself is unstable and asymmetric, right? But at the same time, when examined closely, individual modes are basically asymmetric and imperfection-sensitive. Now that both this mode interacts, I have shown it to you using a simple column right. Auguste's column, an interacting column where these two modes are imperfect and two modes if they are interacting. If they are coupled, then individual mode may be imperfection insensitive, but because of the coupling, they will become imperfection sensitive. So, that further reduces the load. So, this kind of cascading effect happens, and that is what is key to the imperfection sensitivity of the shell, okay. These are important. Now I will explain some more things, but it is mostly qualitative; what happened is that if you do the experiment. So, in order to reconcile the disparity between theoretical and experimental loads. So, what happened that initially has been a law that poses an enigma to the research community is why there is so much difference between the theoretical load and the experimental load. Because the linear theory cannot predict the interaction among modern things, you know. So, the first analysis that was proposed to explain this fact was a paper by von Karman and T. Cian; I think it was in 1941. And then they have explained this phenomenon, okay. And after that, there was subsequent work by others. There have been, in his thesis, von Karman's thesis, which was written in Dutch; later, it was translated. So, it fell into obscurity and oblivion, you know, but later it was translated for the English-speaking community. Then people started realizing that he also proposed, what? Von Kerman and Cian explained these modal interactions. And then why this disparity of load in these things? But there has been a post-buckling theory by Koiter, you know. So, I have explained to you Koiter's name, the imperfection sensitivity, right? Koiter's theory of imperfection, half power, 2, and 3rd power, and things, okay. So, he has, he is basically, this is in Dutch, and later it was translated into English, you know, so that people can understand, okay. So, Koiter's thing, okay. So, all this contribution basically refers to the post-buckling theory of Koiter, which provided asymptotic solutions to the post-critical regime. What happened is that the asymptotic solution means what happened; that means after buckling, there is a huge reduction of load, right? This kind of thing happens, right? So, post-buckling, asymptotic means he, but his solution was valid only immediately after the post-critical load. So, deep into larger deflection, it is not valid; that is what the asymptotic theory is: some kind of approximate solution to the nonlinear differential equation. So, it uses some kind of perturbation expansion and the asymptotic property of the asymptotic series, basically. They use asymptotic series and perturbation of the field variable to solve the equation, and it is an approximate solution that depends on the smallness of some asymptotic parameters. So, those things have been given by Koiter's post-critical theory, but please note that they are very interesting and it explains this load drop, okay. So, this was to reconcile the disparate exponent load; a knockdown factor was developed, and a knockdown factor was proposed. So, what is the knockdown factor? You see that what we have seen in the theoretical load P_{critical}, you know. So, P_{critical} may be, if you see the repressor P_{critical} pressure axial, is what? It is $\frac{1}{\sqrt{3(1-\nu^2)}}\left(\frac{Eh^2}{R}\right)$, right? This was the formula. So, the experimental load $P_{\text{critical}} = \phi P_{\text{critical}}$. So, this is, you know, theoretical load, and this is experimental. So, experimental. They use some kind of factor to reduce the theoretical load to get the actual load, and that factor is called the knockdown factor. NASA, the National Aeronautics and Space Administration, has investigated this widely, and it is based on the huge experimental program. In fact, around the 1970s or even before that, okay, but recently, there was renewed interest among these people to further, you know, keep the margin lower because the knockdown factor has wide variability, okay.

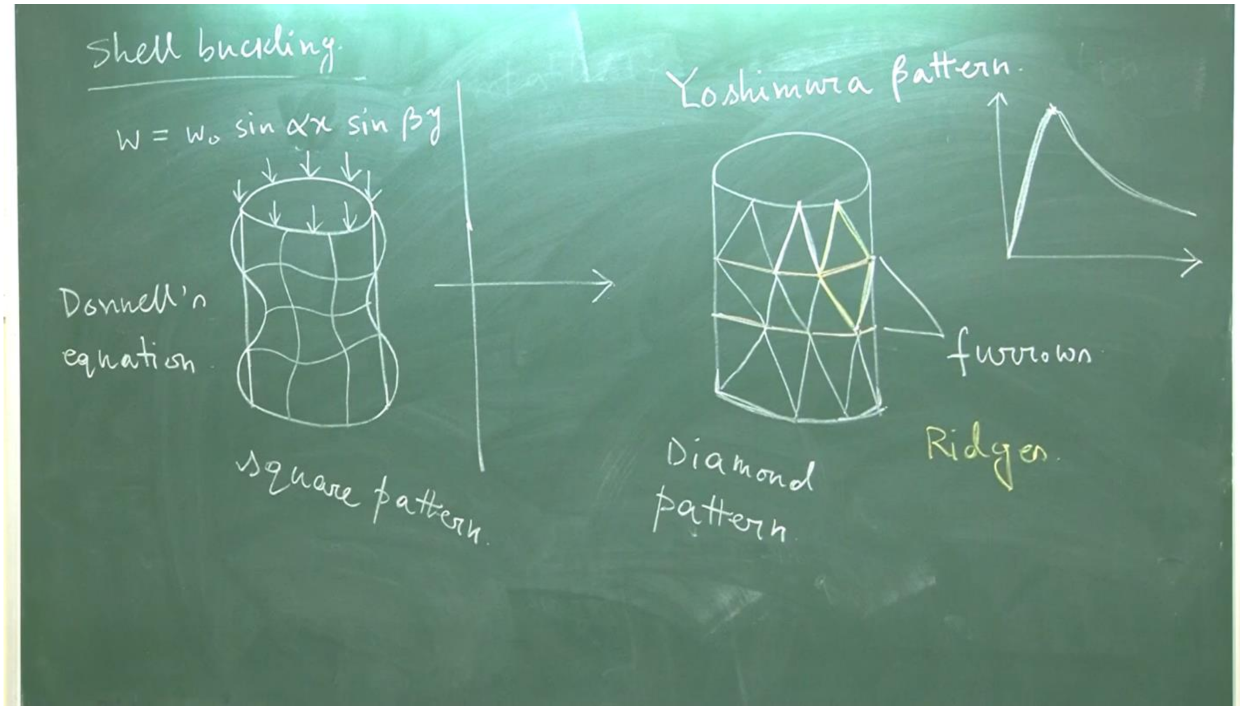


So, there have been some probabilistic analysis also you know based on that that what should be the, so reliability-based design can be done to choose this ϕ Because of the uncertainty involved. So, the basis of this knockdown factor ϕ is, of course, experiments because people have tested a large number of samples, okay. There have been a large number of testing programs and testing campaigns involved. Based on all this, whatever was proposed and adopted by NASA is now in space exploration. They use a very conservative value of this knockdown factor, okay? Very conservative, and essentially, it took as the, if you do lots of experiments, any plot of the theoretical results is here, and then the experimental results are somehow here, okay? So, what they did is they have essentially taken. And the lower bound of that experiment, okay, so that is a huge disparity. That is to say, there was significant scatter among the data. In view of the significant scatter, why is there significant scatter? Because there is so much uncertainty involved in the imperfections, in the geometric imperfections of the shell, see here we have assumed something like You know, the imperfections result in a profile very similar to the deflection profile, but actually, they are random imperfections, okay. There are a number of imperfections that can occur. So, to be safe in design, they have proposed that NASA adopt this knockdown factor, which is very, very safe. Of course, later, you know, there has been a renewed interest around 2017 or so, even from people a little around that. They have tried to reduce and reconcile this margin so that they can have what will allow economic design. So, along with that, you understand what the knockdown factor is. Of course, knockdown factors are proposed as a function of r/t; I will write down the expression for that.

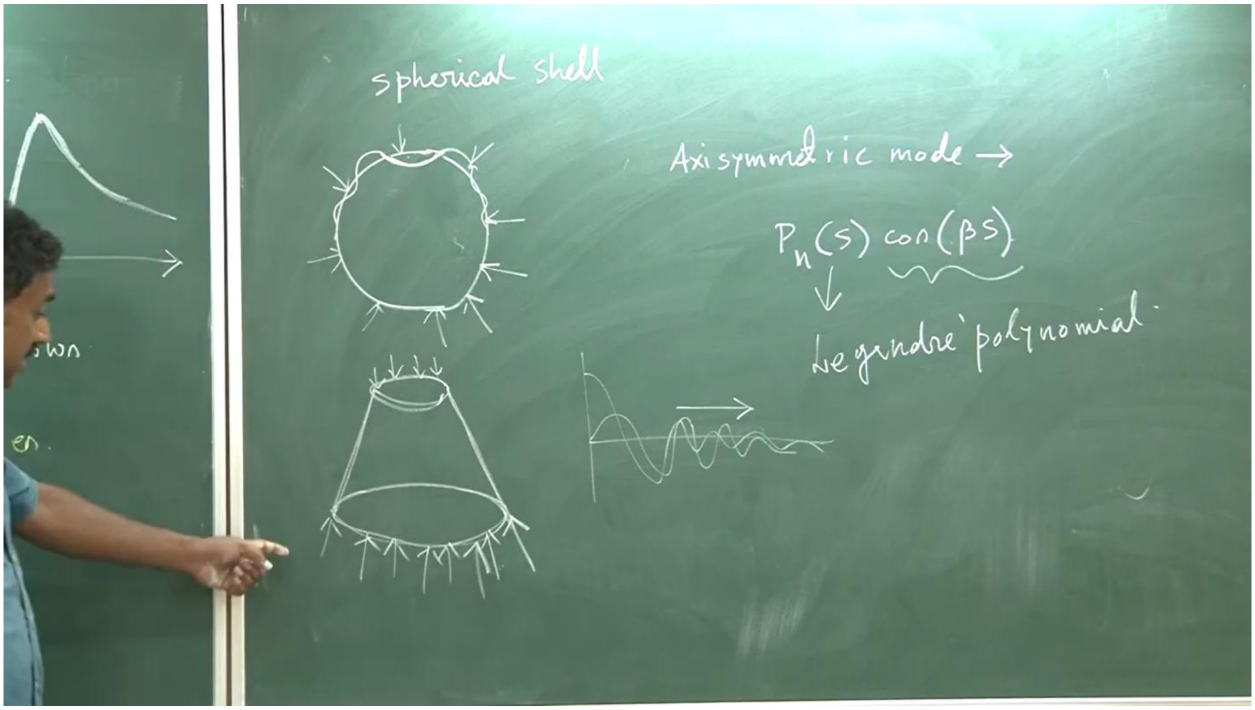


So, this knockdown factor for the shell, you know, So, ϕ is expressed as

$1-0.9\left(1-e^{-\sqrt{\left(\frac{r}{h}\right)}/16}\right)$. This one is the knockdown for the cylindrical shell, knockdown factor KDF; this is KDF for the cylindrical shell, okay. So, this is for $0.5 < l/r > 5$,, and then capital $R_y = 0.5$. So, this is the range of validity because most of the experiments have been conducted in this, all the experiments you know. So, that was basically the empirical formula that has been fitted by NASA, okay. So, this is for the cylindrical shell; this formula has been given, okay? So, other than that, let me explain some other things. So, these are for the design; of course, these are for design purposes, right? Because NASA's space exploration involved not only this, all aircraft and other things, but the space mission also involved this spacecraft, which is essentially made of shell, right? And then you have to be very careful about their design, right? So, what happened if you see? The way to do the experiment is cylindrical shell buckling, right? So, how will it look like? You see, initially in the simulation, whatever we have learned, what we have learned is that this fellow will buckle like this, right? And then here, you know, in something like this, you know. So, we have seen that this, so what kind of pattern is this? This is a square pattern, right?



That is what we have assumed when we simply supported enough fails. So, when we assume $w = w_0 \sin(\pi x/a)$ and then $\sin(\alpha)$, whatever. You know, $\sin(\alpha x)$ and $\sin(\beta y)$, right? That when we assume this is for the koiter's solution, we assume it like that. So, our buckling mode shape is essentially made up of a square pattern, right? That was our assumption for substitution in the Donnell equation, right? Donnell's equation, but when you conduct an experiment, you will hardly see this kind of pattern form. The patterns are completely different, you know, and then I will explain why it is so. So, what happened? Of course, this experiment—whatever I am explaining—I will assume that when doing the experiment. It is not that easy to directly apply load compression, you know. So, this is subject to an axial compressor, right? While you are applying compression, you know that the compressive force comes from the top; there is a mandrel that is placed inside, okay. So, this fellow is giving some kind of loose support to the cylinder. Inside, we have put some kind of mandrel. What happened? No, no, no, some kind of geometry, maybe just a ridge, like a solid cylinder itself. So, that it gives continuous support to this, okay, and then you apply this kind of force, okay. Then you will see that as you approach, So, this pattern is good and this pattern is good until this you see immediately, after buckling but after that what really happens if you go little far then the kind of pattern that is formed But please note that I am explaining to you what I am putting a mandrel inside so that this fellow, which is nothing but a solid cylinder, is clear. That will give a little support to this, you know, this one. So, essentially, the outer cylinder, the cylinder that is being tested, is essentially acting like a sleeve on top of that; you understand that this is the mandrel, okay? And I keep on, you know, compressing it. So, ultimately, you will see. That's what happened, you know; I am not explaining much. So essentially, what happened? Let me explain it to you. So, you see, ultimately this square pattern will convert it into a diamond pattern. And this kind of pattern is called the Yoshimura pattern. This pattern is a diamond-shaped pattern, and it consists of these furrows, you know. These are the furrows which are parallel to the perpendicular of the longitudinal axis. These are furrows, you know? Furrows, you know, these are furrows, and this one, this inclined one, is called a ridge; these are all ridges. Why is this diamond the kind you see? Because you know this one has the furrows and here are the ridges, right? These and these are the ridges, right? Furrows and ridges, alright. What happens essentially is if you cut this thing. So, furrows are going inside, and these ridges are coming outside, okay. So, you take a small, you know, Coca-Cola can or a beer can, and then you compress it. You will not get the whole pattern throughout the length. But you will get its concentration somewhere because that is nothing but the localization of what is called localization, you know, displacement localization. But here, if I put the stand, this beautiful pattern forms; we have to arrange for that mandrel so that it basically supports this. Okay. So that is very important. Okay. You get a similar kind of pattern in the middle, but that is a different thing. That is some kind of displacement localization. But here, what pattern is forming all through? What happened? These furrows and ridges, you know, are forming. What happened initially? Let me tell you. So, if you consider, I am considering one of the circumferences here, you know, then what happened? This, if you consider this one, this circular shape, right? So, furrows are essentially, you know, going to look like this. So, you see that this furrows the length of furrows; some of the lengths of furrows are identical to that of the circumference. So, this represents an inextensible mapping of the surface. So, this kind of thing, when you see that the circumference will become this kind of structure consisting of furrows and then ridges, you know the ridges are inclined; these are all inclined. So, what happened is that, because of its extensible nature, we can clearly understand this can only happen by bending. So, what essentially happened is that this kind of pattern represents that at the end, when it is completely buckled in that post-buckling regime, all the load is essentially taken by the bending; there is no membrane energy. What is membrane energy? I am standing here; you are pulling me, you know, so I am going to, you know, stretch as if out-of-plane displacement is happening, you see that. That is what von Kahneman's non-linearity is, right? $\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$. Because of out-of-plane displacement, there is in-plane strain, right? So, membrane strain is present, but here the membrane strain exists. So, you sense that in the most critical regime, all the membrane strain energy is getting converted into bending energy. Okay, all the strain energy; so that is the reason why you see that first loading increases and then decreases. So, why is this increase happening? Because initially, when you are pushing this, you know, shell, the shell is trying to, you know, deform out of plane, and through that deformation, it is straining membrane strain energy. Out of plane deflection is adding membrane strain energy, not bending, but as soon as this membrane strain energy increases gradually beyond a particular level of deformation. That will not be energetically favorable; the bending energy will be energetically favorable, and it will suddenly drop to this Yoshimura pattern you see. Understand? So, when this…Membrane strain energy occurs in that regime of behavior, where the load increases and then decreases. This is due to the formation of the Yoshimura pattern, you understand. So, at the end, basically, if you go a little deep into the post-critical regime, what happens is that all these furrows have a tendency to go inward. It has a tendency to go inward and to move inward. Why is it so? Because if it goes inward, then it will have all bending energy, no membrane energy, and that is energetically favorable, right? So, that is the reason why there is a tendency for these furrows to grow inward, and if you see when you push a shell, you take a, you can do this experiment, you take either you know coca-cola can, you take a beer can and then you push it, then you will see that you will not get this UFO pattern Because you do not have a mandrel-like arrangement, you can definitely see that in the middle, there is a concentration of displacement, and localization of displacement is happening. and that there is a tendency to appear that it will propagate or move inward. So, that it has, it essentially takes the bending strain energy instead of the membrane strain energy. So, pattern formation is not only common here, or localization of displacement is not only common; please note, all things are elastic here, there is no inelasticity, okay. So, it is localization, but non-linearity is there—geometric non-linearity, right? Because localization and all these things, you know, cannot happen without non-linearity, okay. So, it is not common for cylindrical shells, but similar kinds of things can also happen in spherical shells, okay. So, I will explain to you a little bit. So, what happened in the spherical shell? If you take a spherical shell and then once again subject it to external pressure, the spherical shell. And it is pressurized externally, or you can use a suction pump to take the ring out so that it is compressed. There are suction options; that is another way to do it. So, what happens if you consider here is that there will be, of course, the mode safe if you consider an axis-symmetric mode.



The solution for the axisymmetric mode of buckling is given in the book by Timoshenko and Gere. Timoshenko, and that is "Elastic Stability of Structures," there is a book by Timoshenko, right? There, the solution for the axisymmetric buckling of a spherical shell is given. This looks like you know $P_n(s)\cos(\beta s)$, where p and s, s is basically the meridional coordinate, okay. So, p is nothing but the Legendre polynomial. This is a little more complicated; it is modulated by a cosine function. So, if you see what this buckling mode looks like, it will have this kind of thing at the equator and at the pole; it will deform like this, and outside of this, there will be this thing, but gradually this will come down. Why? Because you see, Legendre polynomials are something like this: the variations, if you can recall, of the Legendre polynomial decay, right? You see that, right? Their magnitude decays with distance, right? So, the Legendre polynomial defines, along the meridional axis, that this will decay. So, maximum displacement occurs here at the pole. So, if you just really go a little beyond in the post-critical regime, what happens? Abruptly, the initial global buckling mode will localize at the top. So, all displacements can be localized. Essentially, we will see that this kind of elastic localization occurs in the spherical shell and that it is kind of abrupt. So, post-critical. Similarly, this analysis has been presented by various people, such as Hutchinson, Odalie, and others. The analysis is a little more complicated for you to follow unless you are conversant with this asymptotic technique, perturbation technique, and related concepts. It is definitely in Timoshenko and Timoshenko's book on stability that they essentially discuss buckling only; they do not go beyond that. But nowadays, with the advancement of the subject, there has been more advanced analysis, especially by Hutchinson and their group, and all this analysis has been presented in significant detail, and it is actually very interesting. The only problem is that you have. With your mathematical background, you have to put in a little more effort for that. But it is a very interesting problem; you see, the localization here of displaced similarly in a cylindrical shell also happens in the middle. So, the localization problem is that the equation has to be non-linear, and then the equation must have a large term compared to the others; only then can you have localization. And then it also allows for an asymptotic and very nice asymptotic or perturbation-based analysis to do that localization. Similarly, you can also have it known in a conical shell. Because conical cells are also very common in space exploration, right? And these are if they are subjected to some kind of this, and then sometimes they are also subjected to this kind of pressure; then this pressure can be whatever is coming from the first vector control. And things, these are sometimes all you know; follower force, and for there, you can have a flutter kind of dynamic instability. But if you consider static instability like buckling, there is also some kind of pattern that can form. People have not studied, but you can do that, okay? So with this, I would like to conclude this, you know, shell buckling. What I want to emphasize is that shell buckling itself is a huge topic, and it is very fascinating. But it is a whole subject in itself. But if you see that, there are many resources, especially the book by Bazant, the book by Bruce and Almorth, the papers written by Caladine, and then Southwell, and then Cian, and then Von Karman, and then the thesis of Koiter. So, all these things give a very detailed explanation, you know. There are phenomena that are well known, but there are still things to explore. So, with this thing, there is actually a group of shell buckling researchers. So, some of you, if you want, can join that group; actually, there are many discussions happening over there. So, I would like to thank you all. So, for the course you know, today is the last lecture for that. If I find that some of my lectures are useful, there may be small mistakes here and there. If you find it, you can always mail me to notify me. But essentially, I have seen that I have consciously tried to avoid mistakes, but you know, human beings are prone to making mistakes, committing mistakes, and sometimes some misconceptions may also occur in my mind as well. But nevertheless, you can send your feedback, clarification, and things. With this, the lectures will be put online, and things will become useful for you, you know that. That is where I think my efforts would reward me; you know, it is not about how much you have learned, but the prime aim of the subject, when I started doing this, was that I wanted to achieve that. Well, it is not about how much you have learned, but the thing is to give you a glimpse of what the subject is, to open the avenues in front of you. I have covered very little. So, I have covered for us the tip of the iceberg, but the whole subject is there, you know. So, that is what I think, you know. It is what I try to ignite, to inspire you, and to ignite the passion in you, you know. so, that you can explore on your own. That is more important than learning, you see. So that's all. Thank you very much.