Stability of structure

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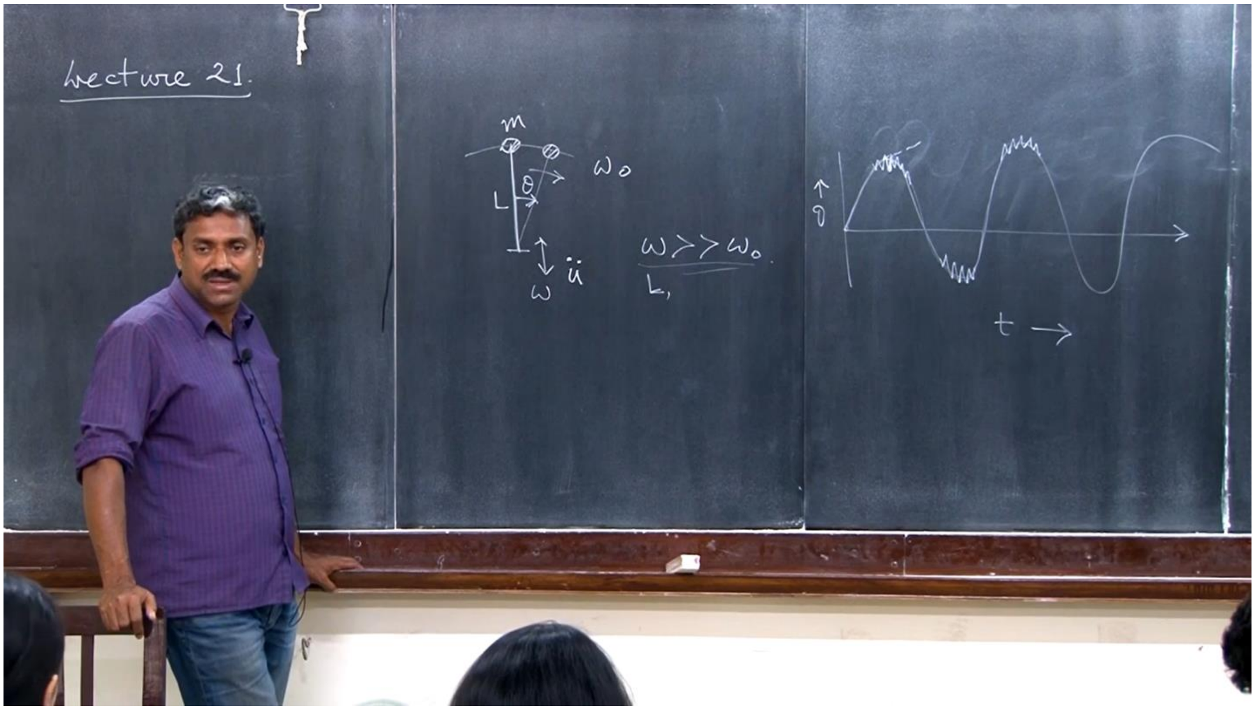
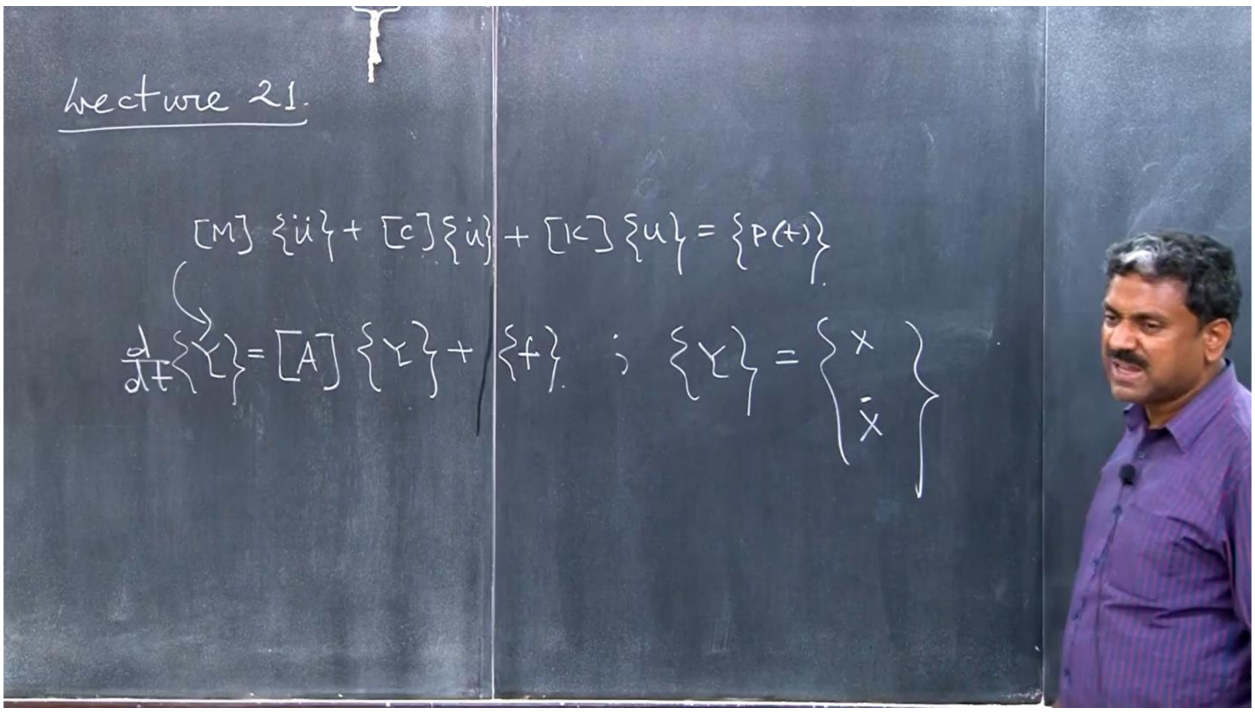
WEEK-11

Lecture 21: Lyapunov Stability and Chaotic System

Let us welcome you to this lecture on the 21st stability of structure. So, let us briefly recapitulate what we have done. So, we started with dynamic stability, and we have almost reached the end of this chapter. So, we have studied a number of cases, you know, starting from follower force and then gyroscopic stability, right? And then parametric resonance, and then aerodynamic flutter, and then we have also considered the noise stabilization of the system. And the last example was noise stabilization of the system. That's what we are illustrating briefly; we are illustrating the stability of this steel, you know, I mean, kept on palm, right? So, what I was just, you know, trying to demonstrate to you is that this is an example of noise stability. You have seen that this stick, when I want to hold it in my palm, right? On my, you know, so then what we have to do, you see, when it is trying to fall right, then I have to take it down right, and when it is trying to, you know. So, the way this works for stabilization is to keep this stick from falling. I have to move this thing in the vertical plane, and then we have established that clearly. So here was the stick that was a simple model that we have considered. So, here was the mass, right? And then this is rigid. This was length L, and then we are giving excitation here, right? So, you $\ddot{y}$ right, and we have considered that the degrees of freedom when we solved it mathematically. This is the θ, right? And then there were these two frequencies of this system; one is, of course, the frequency of oscillation for the stick, right? That's right. This stick, when you are going to put it down, is trying to fall because of its mass. So that will have a frequency ω_0, and then you are going to excite it using some frequency ω. So, for stability, I have shown that there are some conditions; then ω needs to be very, very large compared to ω_0, right? And of course, there were some other parameters involved in this inequality, which were nothing but this length L and the amplitude of this excitation, among others. Amplitude of excess, so essentially what is happening is, you know, whenever this is trying to fall, you know, then you have to take it downward, right? So that this θ reduces, and then when it is trying to come too close, you are just trying, I mean, when it is trying to fall to this side, right? It is trying, getting distant, then you have to take it down, and then when it is coming and crossing, once you try to fall, then you just have to take it upward, right?



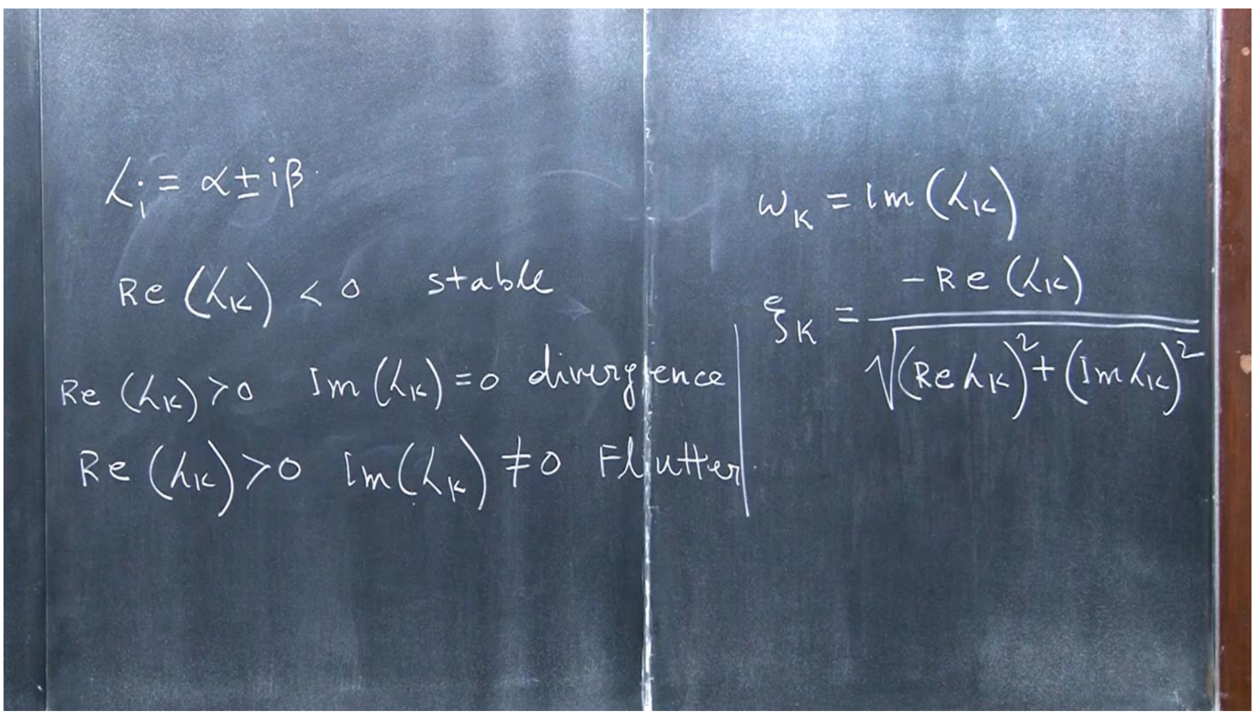
So, essentially what? I was there at two-time scales in the motion that was competing. One is that, of course, we have this periodic motion that is dictated by this frequency ω_0. You know, if you plot θ, and then... You know, with time, this frequency, this time period, this characteristic frequency for this is ω_0, okay. So, this is the slow frequency on top of that; because of this excitation ω, all these small oscillations are riding on these things. Huh. Riding on what they're essentially doing, as soon as this is trying to destabilize it, okay? This fellow, this noise is trying to bring it down, you understand? And that's why this noise stabilization is occurring, right? And this I have established mathematically using a very simple mathematical argument that you must have seen in the previous part. So, this is not just that; see, I was just trying to explain this with a very simple stick example kept in my palm, right? But please note that there are many examples of this kind of noise stabilization, especially. In physical systems, right? In physics, you know, and I mean when we ride a bicycle, that is also nothing but a noise-stabilized system, okay? Good, so now one more thing I'm going to briefly mention to you: we have already briefly touched on this when we discussed aerodynamic instability; we have introduced the concept of this self-exciting force, you know. And then aerodynamic, you know the flutter derivative and others, right? Which basically correlates the exciting force with the response of the structure. Because of all this excitation, we call it parametric excitation. Because excitation is the force that is being applied to the system, it can modify and is basically dependent on the deformation of the structures. So, that basically creates a loop, right? For you know, the feedback loop, right? Through which they interact, basically, the deformation of the elastic structure interacts with the aerodynamic loading, and those are described using a simplified model, and then finding this aerodynamic flutter derivative. And then we have taken the simplified example of course we deal with; this you know reduced order model for the Tacoma bridge. And then we have demonstrated how it collapses; essentially, all the features of the physics we have captured and seen.



We have also demonstrated how the frequencies change. And then how they coalesce, and that coalescence of frequency essentially indicates the onset of flutter, and that also indicates the setting of aerodynamic damping, which is negative, okay, right. that's what and then essentially two modes. So, got coupled right? Right. So, the flexural mode basically pump energy to the to the torsional mode. Right? The torsional mode is the one that is basically triggering the instability. Right? The negative damping is coming in the torsional mode. Right? Because you have seen that, through the mathematical expression of the respective flutter damping, you know that the aerodynamic torsional mode will allow negative resultant damping, right? Anyway, we have also demonstrated the coalescence and separation of the real part and the imaginary part of the frequency, but I will just generalize it. So, we have seen that, in a multi-degree freedom system, the equations can be written as

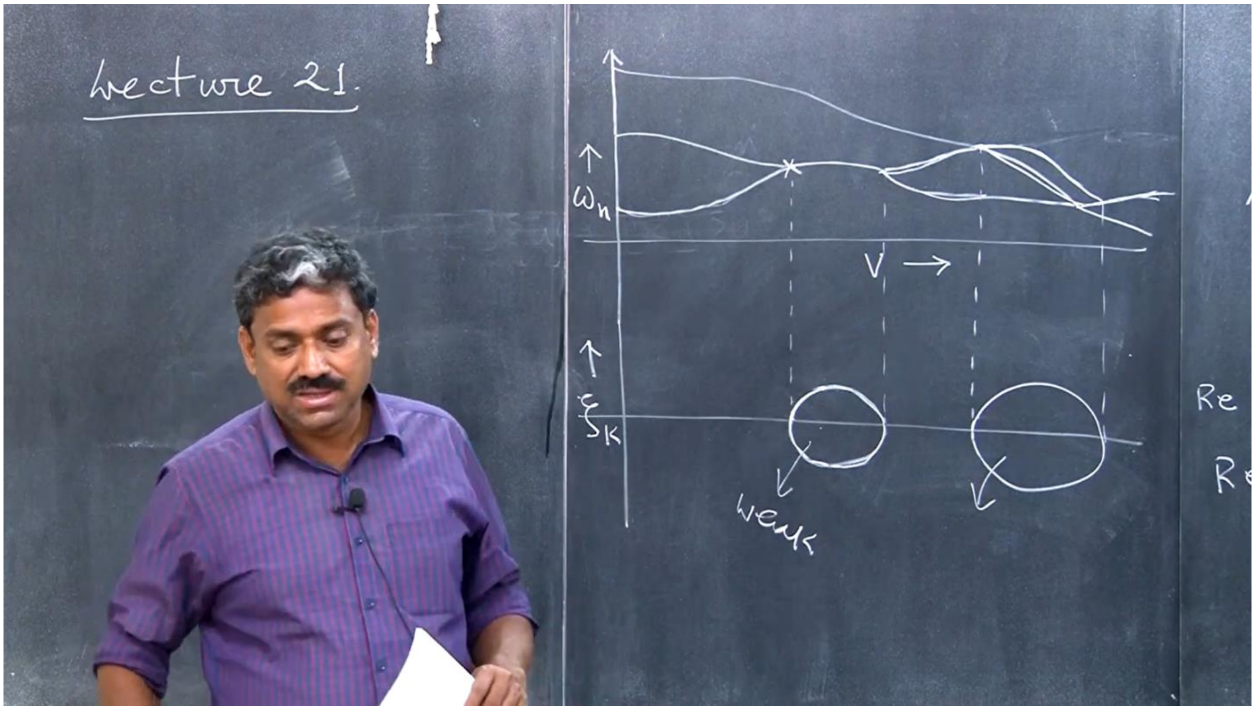
$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{P(t),\}$$

right? Yeah, it is equal to $P(t)$, right? And then we can write it in a state form; I have already written it in the previous example, right? So, a state, and then Y is equal to, you know, this is equal to $\dot{Y}$, right? So, $\frac{d}{dt}\{y\} = [\mathrm{A}]\ \{\mathrm{Y}\} + \{f\}$, and then, of course, there is some excitation, you know, some force, something like this. So, here when we define the state variable, this y is a state variable consisting of both the displacement as well as x and $\dot{x}$, right? So, essentially, if it's a two-degree-of-freedom system, its initial condition basically, and then of course excitation decides exclusively the state of the system, right?

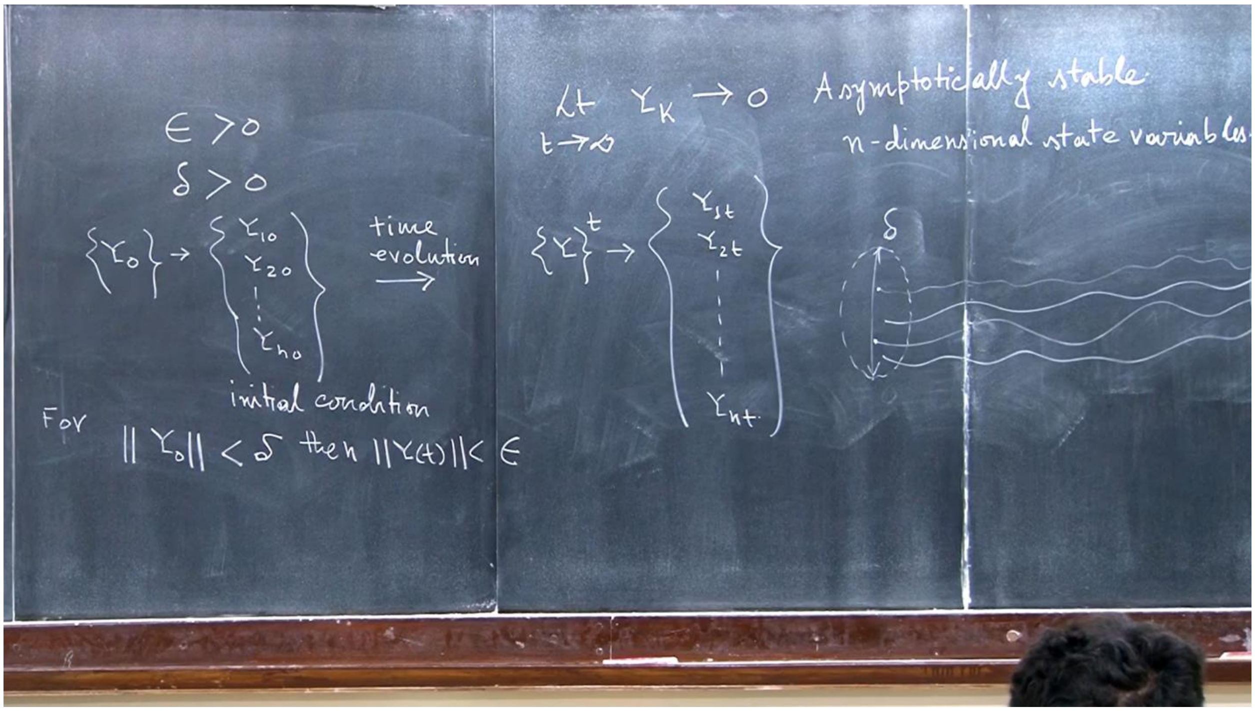


So, and then of course what we have seen is that here it is an asymmetric matrix, you know, in state form, and then if we want to solve it, this will lead to a quadratic eigenvalue problem.

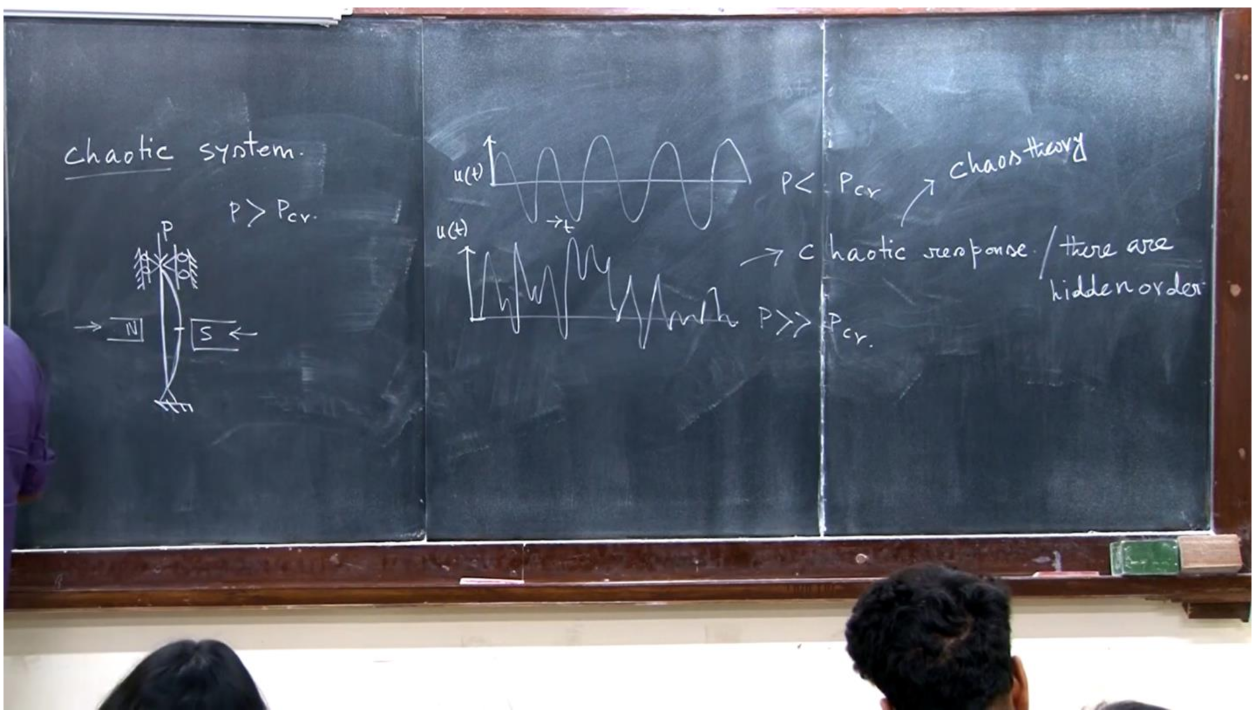
If you can convert it, you know, so in both cases, whatever eigenvalues you get will be the same; this will appear in complex conjugate. So, we are defining these eigenvalues λ_i right. $a \pm i\beta$ right, you know or $\alpha \pm i\beta$ whatever and they will appear in complex conjugate So then what you know. So, either you can write in state form, or you can directly solve the quadratic eigenvalue problem. And then I have demonstrated to you that the real part of (λ_k), if it is less than zero, is of course stable; that means this is positive damping. So, the real part of λ is essentially associated with the damping, right? This I have mentioned to you, right? ω_k is nothing but the imaginary part of (λ_k) ω_k^2, rather, okay? H and ξ_k is the damping; you know the damping ratio is minus the real part of (λ_K). And then you know this one. Huh? Right. So, the real part of (λ_k) is greater than zero and the imaginary part of (λ_k) is zero. This is basically divergence. Divergence means nothing, where there is no frequency. So, system will diverge in one direction. Right? Huh? So, there's no periodic motion, right? And, of course, positive damping, and then the other case for the real part—sorry, negative damping. This is basically Flutter whatever. So that means it is negative damping. This indicates negative damping and then there are frequencies. So it will perform the oscillatory motion, but its amplitude will leave. And then when we draw this, you know, in the phase diagram, you know the change in these two; I have shown you in the previous example, but I will just rewrite it. I'm assuming that this is for aerodynamic instability, so wind velocity V is important, but it is not necessarily something you have to consider. This can be any parameter that controls stability. It can be destabilizing wind speed. It can pertain to gyroscopic motion. It can be the axial force P for the rotating S. It is for parametric resonance; it can be, of course, a dynamic component of P, you know. Okay, something like that. Okay, so here I have shown you how this stick looks, you know. So, I will draw it something like this, you know. So, this you're projecting on the C frequency, you see that flutter is indicated by this, which indicates the onset of flutter because we have plotted the variation of ω with that, you know, V. So, you see that there is a coalescence of this frequency, and then there is a bifurcation once again, and then once again the coalescence of two frequencies, okay. This one and this one are missing, and if you project them on the ξ_k axis, then basically the locus will be a circle. Some, I mean, elliptical or something like that. So, this indicates these are the flutter regimes you see, that okay, and you see that sometimes the smaller one can be very small. The weight for the coalescing zone. Okay. The coalescing zone can be very small. If it is smaller, the strength of that instability is smaller, and these are called weak flutter and strong flutter. So, if this is called smaller, then it is weak flutter. Weak flutter, and this is strong flutter. Weak flutter can be suppressed by increasing the damping in the system. Okay. Where the strong flutter may not be. Okay. So, you will see that later, once this aerodynamic collapse of the Tacoma Bridge.



There has been a huge research effort to really understand what was basically going on. Okay. And people started to understand the aerodynamic instability. And there are various means to control the flutter, one of which is of course by adding additional damping or augmenting the system with damping. So that additional damping will suppress this. So, with this essentially, we cover all the things, but I will just try to briefly touch upon, it's not possible to explain everything and do but many of you must have heard the concept of Lyapunov stability originated in 1893, towards the end of the 18th century. So, the Lyapunov concept of stability analysis, here in civil engineering systems, we do not routinely follow that, because we have an alternative way to analyze the system, right? But please note that this concept of Lyapunov stability is very important, okay, and people have used that if extensibility is used in biology. It is extensively used for the financial market, you know, the economy, and nowadays many of you are also aware of this share trading, right? Share market. Yeah, so you are saving and multiplying by investing in various shares, right? So, what is the stability of the share market, that also through Lyapunov stability? So, there is actually a difference. See what happened in so far as structural analysis and structural systems are concerned. Okay. Structural or mechanical systems, not that you cannot achieve a level of stability, but you have to put, I mean, it's an alternate way to do it. So, in Lyapunov stability, the way they define stability is something like this. So, the way it is written is that there exists a positive number ϵ, for all you know a positive number $\epsilon > 0$, for which there exists another positive number $\delta > 0$ for which, okay, if the system.

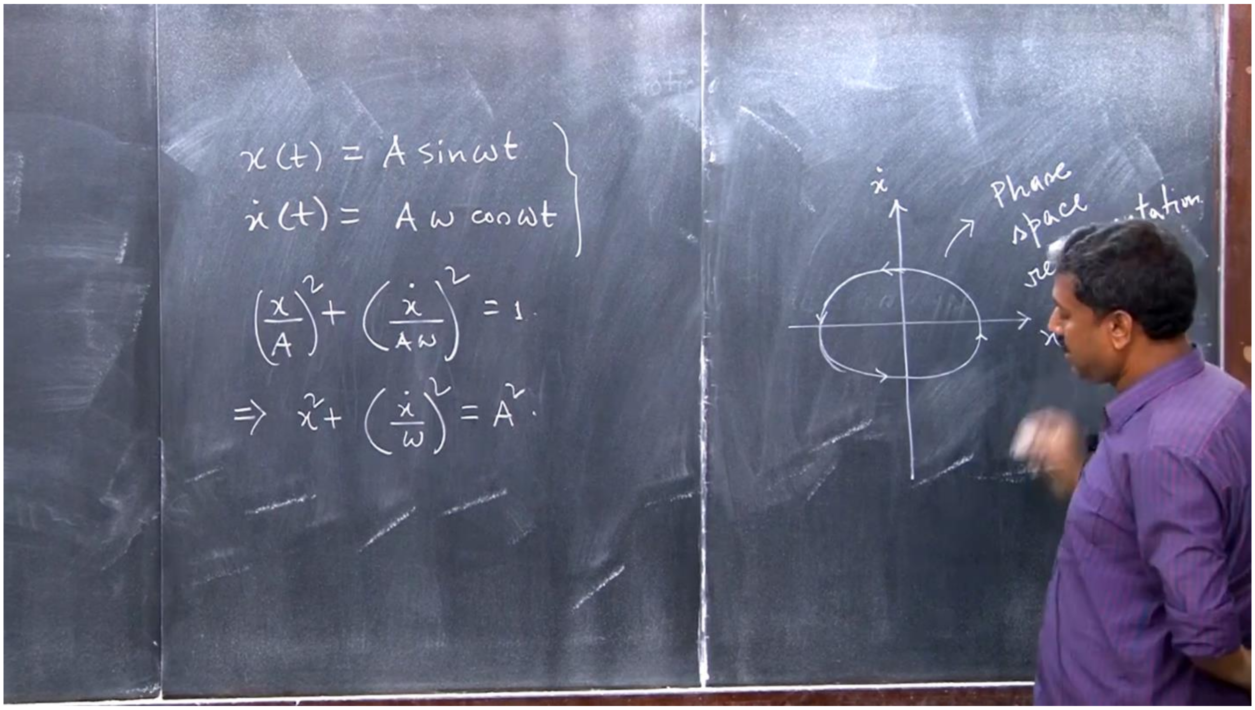


So, Y if the system you know $\{y_0\}$ means of course it's a vector it's basically consist of the state variables right, $\{y_1\}, \{y_2\}$ etc right, for a system. so, any system that, it has a degree of freedom right. And whatever degree of freedom you multiply by two because it's velocity as well as displacement, right? So, it will have, like, you know, $\begin{Bmatrix} Y_{10} \\ Y_{20} \\ \dots\dots\dots. \\ Y_{n0} \end{Bmatrix}$, something like this. So, this system is defined like that. These are the initial conditions, right? Initial conditions, right? For a system's initial conditions, huh? And it is evolving in time. Okay. Time evolution, right? So, when it evolves over time, then of course at any instant, it will have the state variable $\{y\}^t$ which means $\{y\}^t$ will have, you know, things written like $\begin{Bmatrix} Y_{1t} \\ Y_{2t} \\ \dots\dots\dots. \\ Y_{nt} \end{Bmatrix}$okay. So, it's basically an n-dimensional state variable, right? A state variable can be one-dimensional or n-dimensional. Of course, for a single degree of freedom system, how many state variables will there be? For a single degree of freedom system, there are only two state variables: velocity and displacement. So, how are we defining the Lyapunov stability? So, for any system, if there is one positive number ϵ, which is greater than I mean positive, then there exists another positive number δ such that for all $\|Y_0\| < \delta,$ you know if you take the L_2 norm of the initial condition. So, there must exist some δ. So that. If the norm of the state variable is less than δ, then it is less than all ϵ. You understand what I'm trying to say. What does it mean? That means you consider that you have an n-dimensional space; you consider this. So, we have a small, you know, two degrees of freedom system. So, it is a circle, right? So, you have these two state variables, and they're evolving in time. So how are they evolving in time? They are evolving in time maybe something like that. Huh? So, Lyapunov stability is defined in a way. So, that if these two remain within some finite boundary, with some finite. You know this is bounded by some finite boundary; then at any time instant after a time t elapses, all these response variables remain within a circle of radius ϵ. You understand what I'm saying? So, if there exists some ϵ such that all these response variables, when plotted in state, remain within ϵ, then there exists some δ. So, all the initial conditions remain within, you know, this δ. So, this δ helps to understand the concept. So, what it means is that This basically implies that if your initial conditions are bounded, then your response can have any time; they have some bound, and you can find some bound for that you understand. So, it can evolve over time; it can, but it must be bounded within some limit, and that is defined by a positive number. You see that, okay? So, it might have any arbitrary kind of evolution. But as long as it's within δ, it is within ϵ. Now, when it is destabilized, if you go beyond that, then it is not stable in terms of Lyapunov. Understand what I'm trying to say? And when I am defining the L_2 norm, it's basically the square of y_{01} plus the square of y_{02}, right? The L_2 norm is nothing but the square of the norms, right? Fine. So, that is defined as the Lyapunov stability. Now, if in the limit as t tends to zero, you know if y_k tends to zero, then it is called asymptotically stable. This is called asymptotically stable. So, this is the concept of Lyapunov stability, which is widely used in many disciplines, including structural engineering and structural mechanical systems. Now there is, of course, a slightly modified version that it is also applied with. We can define a Lyapunov stability function. Okay. Lyapunov functions will be, once again, a function. Okay. It's a Lyapunov functional, rather. Okay, so a Lyapunov function, most of the time, you will see that it is some form of energy functional. So, if you can define it as positive definite, the way we have formed the Lagrange for a dynamic system, and from there we derive the equation of motion, right? So, the Lagrangian, you know, can sometimes be adopted as a Lyapunov function. You see that the positive definiteness of the Lyapunov function also ensures stability. Okay. Yes. So, Lyapunov functionals are also for some systems, especially the systems that are not discrete continuous systems; there, you can find the kinetic energy, and you can find the potential energy. And you can form a functional, which is a Lyapunov functional, and you can ensure its positive definiteness, and then you can ensure your stability, so I'm not going into much detail about the Lyapunov stability. Or how to find out that, but you see that's the concept essentially; why I wanted to mention that, I mean, without mentioning Lyapunov's stability, this discussion of stability is not complete. Because this notion of Lyapunov stability is extensively used in all other fields. So, we must be aware of its connection with our structural system as well as understand what I'm trying to say. So, I will not go into mathematical details, but let me tell you something. So you see, whatever we have learned until now, I mean for all the linear systems, if you apply some kind of sinusoidal excitation, then its response will also be sinusoidal. Okay. Right. Sinusoidal or a combination of sine and cosine. Okay. But if the system you know has some kind of nonlinearity. Okay. Then sometimes its response can be a perfectly deterministic excitation, but you have an apparently random response. So, this kind of behavior is called a chaotic system. So, I will show you a chaotic example system. So, you take a column. Okay. And this column essentially is You apply this load; compressor P is much greater than $P_{critical}$. So, it will, of course, buckle. Then you put two magnets here. There is this experiment people have done, okay, north and south pole, you know, and then you apply these or assume that these are all electromagnets, okay? And then you apply some alternating current. So that they are using the reversal in the respective and make you assume that this column is made of iron or those kinds of magnetic materials which can be attracted by a magnet, right? Maybe made of iron, okay? So, as soon as it is greater than $P_{critical}$, it will buckle. Therefore, it will be full of geometric nonlinearity. Right? And then you apply some alternating force. You can see that if you apply alternating current, it will have sinusoidal fluctuations in the pulse, right? So, it and then you can put some you know maybe one ledger based you know here, ledger based system you know ledger using you know ledger measurement you can find out the displacement you can plot So you'll see that as soon as p is less than $p_{critical}$, when you are applying this sinusoidal force, you see that exponential; you know what I'm trying to explain to all of you. Did you get what I mean by explanation? This is a column I meant made of magnetic material, okay, which is attracted by a magnet. And by this, you know some kind of arrangement; you can put some bearing, etc., and then you can apply different forces. So you can apply P, you can change P, you can make P; you know, by tightening, this experiment can be very simply done in a structural engineering lab. It's not a big deal; it's very simple. So, P can be greater than $P_{critical}$, P can be less than $P_{critical}$, and if P is larger than $P_{critical}$, it will have larger degrees of geometric nonlinearity, right? It will be, you have all seen that elastica. I discussed the elastica's stability behavior, and then you apply alternating current, you know, by using this magnet, right? To apply so you'll see that when P is much lesser than $P_{critical}$. if you apply excitation force to be Sinusoidal its response will also be Sinusoidal or cosine So if there is a single frequency input excitation, you'll have a single frequency output, but as it is greater than $P_{critical}$ or even more than 1.2 to 1.5 $P_{critical}$, you will see that initially it will be very simple, you know. Something like this assumes that there is no damping essential, but when P is much greater than $P_{critical}$, you will see that as soon as P is very much greater than $P_{critical}$.



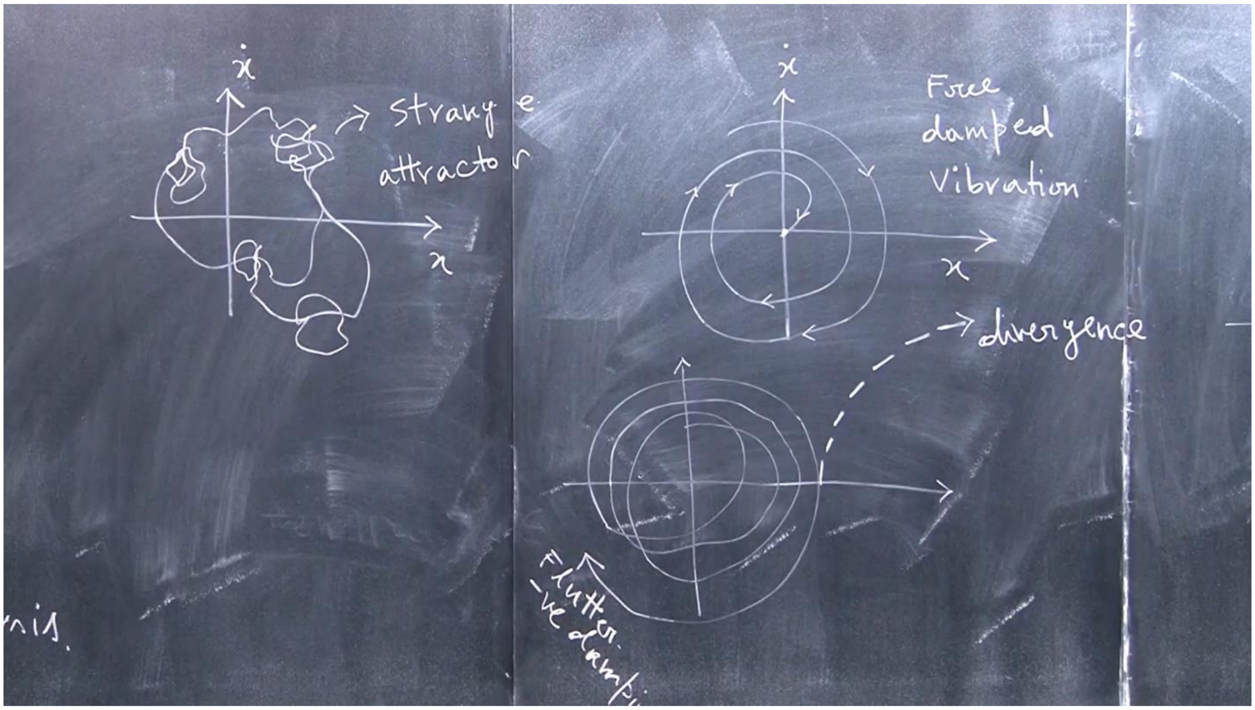
Then, see your input excitation is purely sinusoidal, but what is your output excitation and what is the output excitation? I'm assuming this is the deflection; this is deflection here. That deflection you can measure using some laser-based displacement measurement system; we have one in the lab. So, $U(t)$ so here you are plotting with time, we are plotting $U(t)$ displacement $U(t)$. Okay,

$U(t)$. So, our excitation is purely, you know, single monochromatic, meaning single frequency sinusoidal, but our output excitation is purely random. Do you see that it's really random? What is it called? How can you explain why this is happening? Does anybody have any thoughts? Nonlinearity, of course, that's good that it is attributed to nonlinearity, but then a nonlinear system is so random. Many of you must have studied a little bit of random migration dynamics, right? Hmm, see, random migration means that the input excitation is also random, and the output excess has to be random. But hear, because of system nonlinearity, this becomes apparently so random, right? But actually, this is not apparently random. This response is called a chaotic response, and this gives rise to a completely different theory, which is called chaos theory. Although it looks apparently random, it is actually not random. There is hidden order in this. Okay, so the difference between a random signal and a chaotic signal is that This is the chaotic response, not random. There are hidden orders. How can you explore hidden order? There are various mathematical techniques through which you can find out the hidden order. The hidden order in chaos is basically the subject of chaos. Okay. So, the hidden order in chaos is found out by, you know, of course, one is that, you know, phase representation. There is a technique called the Poincaré map. Then there is something called the Lyapunov exponent. So, any time series or any random signal can be analyzed for its Lyapunov exponent. I will explain what that is, Lyapunov exponent, and there are other things that you can also do, such as the vaporization diagram, and maybe another is power spectrum analysis. So, this itself is a subject to be taught separately. I'm not going there, but what I'm trying to emphasize to you is that although it looks random, it actually isn't; there is a hidden order behind that, and that hidden order can be explored using different kinds of representations such as phase representation, Poincaré map, and Lyapunov exponent, which is why I was particularly interested in pointing out the Lyapunov exponent. That is because, you see, we all studied Lyapunov's theory, right? The Lyapunov concept of stability. So, I tried to establish that connection. the Lyapunov exponent is nothing but what I want to explain to you. Okay. See if you know how to calculate the Lyapunov exponent; it is a little involved, and I mean, in this course, it's not possible to explain that. but any of this system there is some represent which is called phase representation. Okay. So, any random signal can have its phase represented, which I will explain simply, and from there, using some algorithm, we can calculate the Lyapunov exponent. So, Lyapunov exponent shows basically divergence of that signal. Okay, that means how there, see there is a, there are maybe there is one trajectory. How the trajectory is diverging.

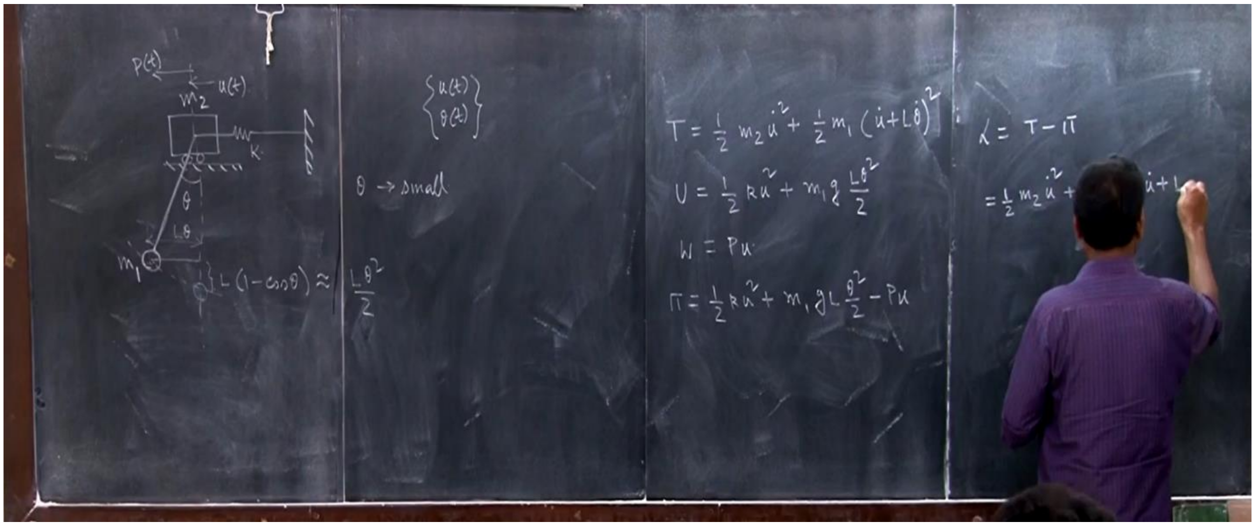


So that is so if you have a negative Lyapunov exponent, that is associated with chaos, that means it implies that trajectories are, well, you know, diverging, okay. So that is, see, chaos doesn't mean that it will lead to instability, but chaos is some form of instability, some form of apparently random response, right? Okay. So, people have studied this chaotic behavior in structural systems; it is not important as far as structural engineering is concerned, but for many mechanical engineering systems, this is important. Now what is phase space, let me explain to you, I'm assuming that, this is a sinusoidal excitation $A\sin\omega t$ okay. then if I take derivative $\dot{x}(t)$ then what you will get $A\omega\cos\omega t$ right. So, if I write$\left(\frac{x}{A}\right)^2$ and then I write $(\dot{x}/a\omega)^2$ is equal to 1, right? So, what it means is $x^2 + (\dot{x}/\omega)^2 = A^2$. So, if I plot it here, you know, I'm putting X here, and I'm putting this as X, and this is $\dot{X}$, right? So, $\dot{X}$, uh. So, here you will see that this kind of thing is getting right so well, isn't it? We don't see, although there is time variation; they are varying in time, but when I'm doing this representation, I'm essentially, I'm putting these two state variables together, and they form a kind of elliptical structure, right? This is called phase space representation. So, you'll see. So now, for a damped system, a three-vibration-damped system, how will it look? See initially started with some dispersion and then gradually, it is spiraling, ultimately merging with the origin. It is a damped free vibration. Free damped vibration. Right?

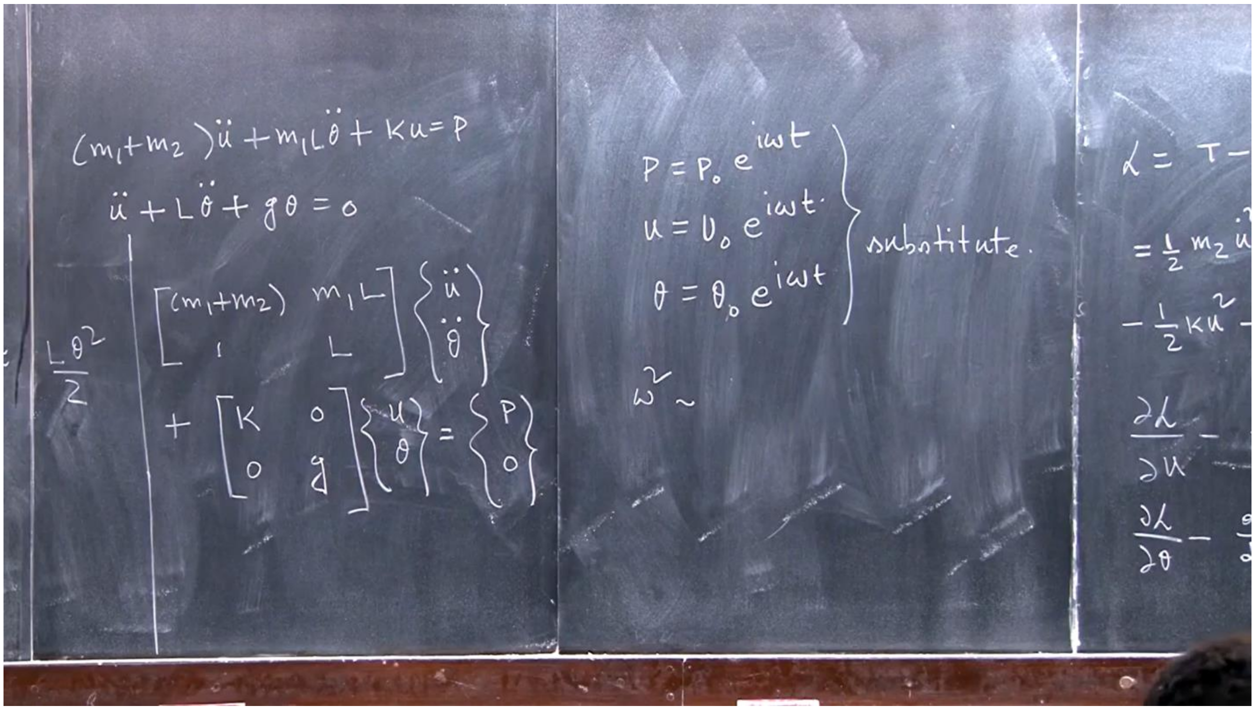
There is no external force. Do you see how the phase stress is referred to? This is coming to a point, right?



So now what happened is that when you want to discuss this system, you see that. So, you understand that all dynamical systems can have an alternative representation other than the time stream. And that representations to phase space, and this is extremely important for nonlinear dynamics, okay. Because that indicates, as far as stability is concerned, you'll see that for any aerodynamic stability, we need to consider what is going to happen. You see, let's assume that you are having this, you know. So ultimately, you see the amplitude is going to be more and more, right? So, it is going to diverge; you see that it will go, so that will be seen. I'm considering a system that is, you know, subjected to dynamic instability. So that you see that, that is increasing, so that means it is, see here, it is damping, positive damping, right? Here it is negative damping, right? So, it is increasing; thus, phase space is expanding. It's an ever-expanding phase space; understand that, right? Now, of course, this indicates its flutter because it's through negative damping. Flutter, negative damping, this is negative damping, huh? Now the same thing from here assumes that it is coming here, but from here it is having these kinds of things; then, of course, this will have divergence. It is no more; it is, see, flutter is associated with what? Periodic motions, right? But divergence is not, so if it is divergent, it is something like that, right? Now what happened with the chaos in a chaotic system? So, if you plot, you will not necessarily see that it will have only two degrees of freedom, x and $\dot{x}$. It is a single degree of phase space. Because only two variables, you'll see that whenever it is getting attracted somewhere, like this, this is called an attractor. You see, in a chaotic system, there will be multiple reasons for the apparently random response. It has multiple equilibria; you know, alternate configurations. Okay. So, depending on the excitation and the amplitude of excitation or nonlinearity, you know the mobilization of nonlinearity will reach different points in the phase space, and then it is a very irregular kind of geometry of the phase space, and that's why these are called strange attractors. So, strange attractor, there are many other ways people try to characterize this strange attractor in the phase space. So, you know, using some fractal geometry and others. Okay, I mean, have you heard of fractal geometry? You know, the dimension of one dimension, two dimensions, three dimensions. But there are, you know, fractal dimensions in which the dimension will be 1.2, 1.5. So, these are called fractal dimensions. So, this will have a fractal dimension, and this has a basin of attraction and others, okay? I'm not going into details, but this is the way it is, okay? Any chaotic system we may have is a chaotic system, so you will see that for a system to be chaotic, the system must be nonlinear. A linear system can never be chaotic. You see that? So, it is the nonlinearity that is basically triggering this apparent randomness because it will have multiple alternate equilibrium configurations. and depending on your excitation and things initial condition it will reach to one of these states. Okay. And then it can jump from one state to another one state to during the course of motion. And that's what you know; there are different kinds of phenomena that are called phenomena, like intermittency; for fluid systems, this kind of phenomenon occurs in turbulence. So, from laminar flow, when you go to turbulent flow, the transition is accompanied by nonlinearity because there is nonlinearity. You know, the convexity of acceleration is nonlinear, right? Because of the nonlinearity, turbulence occurs, and then with increasing Reynolds number, what happens is that you'll see that nice signal, but suddenly there is a huge spike, which is nothing but a shift from one to another in the space of, you know, the space of alternate equilibrium configurations. And then gradually there will be an occurrence of this, you know, this sudden burst, okay? And then ultimately it will lead to this turbulent kind of motion, okay? So, you know it's stable, but there is also some kind of instability, okay? For structural instability, don't go to the sea; what happened is that here I will tell you something about a single degree of freedom system: you have two dimensions. But please consider that you have a multi-degree of freedom if you have, you know, a million or a million degrees of freedom, right? So, you can have a reduced order model, but even if you reduce the model, it has two or three degrees of freedom; you'll have a six-dimensional space. So, those are called hyperspaces, okay? And you know in hyperspace it will not be possible to measure all the responses. So, you have to construct this, you know, phase portrait using several algorithms, okay. Those are called; there is something called the Takens embedding theorem. So those theorems you can use to have a qualitative representation of the phase space. And from there, you can extract the stability information, okay, through Lyapunov analysis and others. So, those are research topics, and a little more advanced, you know, one of my students worked in that area, actually in his PhD. So, some of you may be interested; if you are interested, then you can explore on your own. These are interesting things, and from a civil engineering perspective, I think people can contribute. Because the problem is that not many civil engineers work in this area, I don't find anybody, at least in India, who works in this area. So, the system of chaos and nonlinear dynamics itself is a huge subject okay, anyway. So, till now, so I will just try to, I will not solve but I will just show you another example and please, I will not solve it completely, but I will just summarize, and then with that, I will basically conclude dynamic stability. I think we have done enough dynamic stability correctly. So, this problem you are trying to solve yourself. Huh? But I will essentially write down the expression. Have you solved this example in your dynamics course on the equation of motion? Have you done this one? No. So, this is a spring-mass system; you know the stiffness is K, and this is mass M. You know you are applying some force here; this force is P, you know, $P(t)$. So, you are asked to find out about the stability of the system. Okay, I will not consider the stability, but I will essentially write on the equation of motion, and the rest of the string you can do yourself. Okay. So, of course, there two degrees of freedom system. Let us define angle θ here. Okay. And if this angle is θ, then you can understand that for small θ, we still assume θ to be small. Small θ and then u is also small; you will have this from the equilibrium configuration. It has this deformation $\{U(t)\}$, you know. Then this angle is θ; this is $L\sin\theta$. If you know, of course, this comes from this; this one is $L(1-\cos\theta)$, and then this is nothing, $L\theta^2/2$. If you say that this angle is θ, then we can take this one as $L\sin\theta$, right? So, essentially, there are two degrees of freedom: one is $U(t)$ and the other is $\theta(t)$, right? right $\{U(t)\}$ and $\{\theta(t)\}$. So, I will once again, you see that for this system to, so essentially spring system is basically performing motion okay.



There is some applied force speed; it is resisted by a spring of stiffness K, and then it is attached to a pendulum. This arm is rigid, and this is a concentrated mass. So, you can clearly understand what the kinetic energy T is? Kinetic energy is half. Now assume this is M_1 and this is M_2. Okay. $\frac{1}{2}M_2\dot{U}^2 + \frac{1}{2}M_1(\dot{U} + L\dot{\theta})^2$. So, you see that here it is only U, but here the deformation will be governed by a $u(t) + L\theta$, right? And the strain energy is $\frac{1}{2}ku^2$ strain energy, as you can see. This potential energy will also contribute, right? This one, huh? So, $U = \frac{1}{2}ku^2 + m_1 g\frac{L\theta^2}{2}$, in fact, instead of potential energy. So, $m_1 g\frac{L\theta^2}{2}$contributes to potential energy, and the work done is nothing but p into u, the work done by the external load p. That is being applied, huh? So, then the total potential energy is $\frac{1}{2}ku^2 + m_1 g\frac{L\theta^2}{2} - PU$, so, the Lagrangian, if you define kinetic energy minus potential energy, right? So, $\frac{1}{2}m_2\dot{u}^2 + m_1(\dot{u} + L\dot{\theta})^2 - \frac{1}{2}Ku^2 - m_1 g\frac{L\theta^2}{2} + PU$ equals zero, and you can write down the equation of motion. So, $\frac{\partial L}{\partial u} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) = 0$. There is no dissipation. So, of course, with respect to u, you will write one equation, and then you will get another equation with respect to θ. So, $\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0$. So, these two equations you will get right. Once you form the Lagrangian functional, then you can derive that.



So, I will just write down these five equations, and then So, the first one you will get is essentially

$$(m_1 + m_2)\ddot{u} + m_1 L\ddot{\theta} + Ku = P.$$

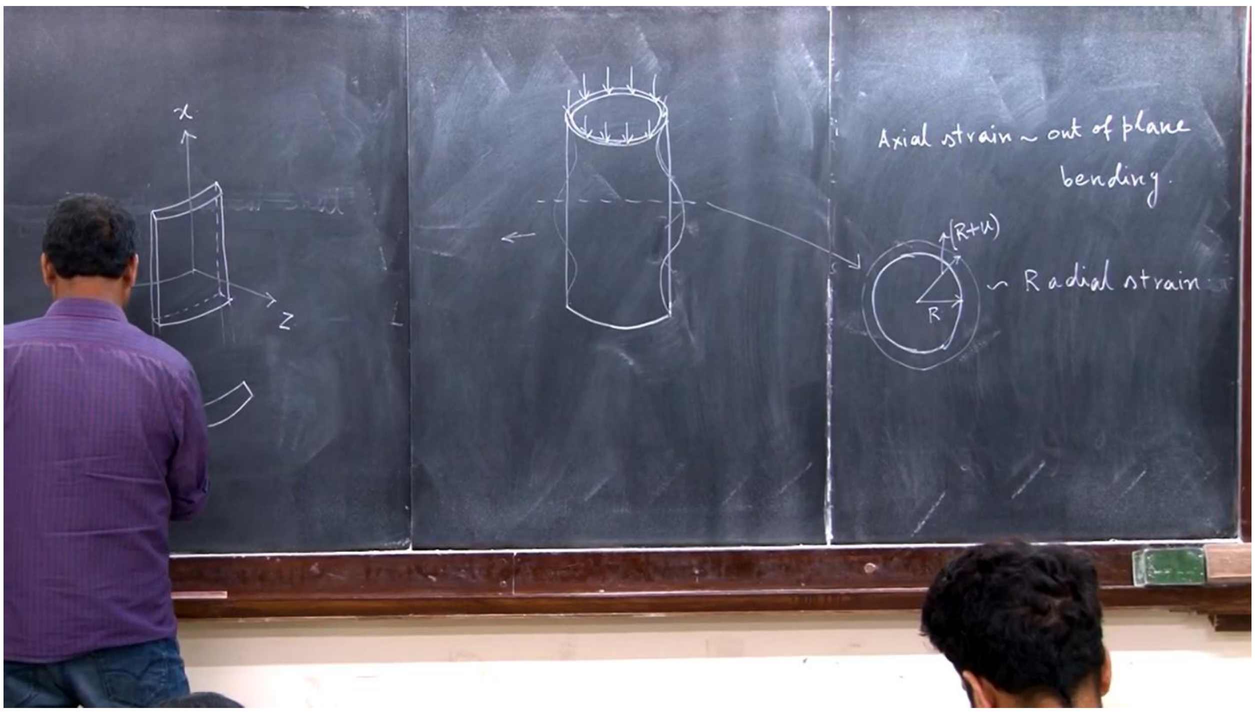
This is one, and another one is

$$\ddot{u} + L\ddot{\theta} + g\theta = 0.$$

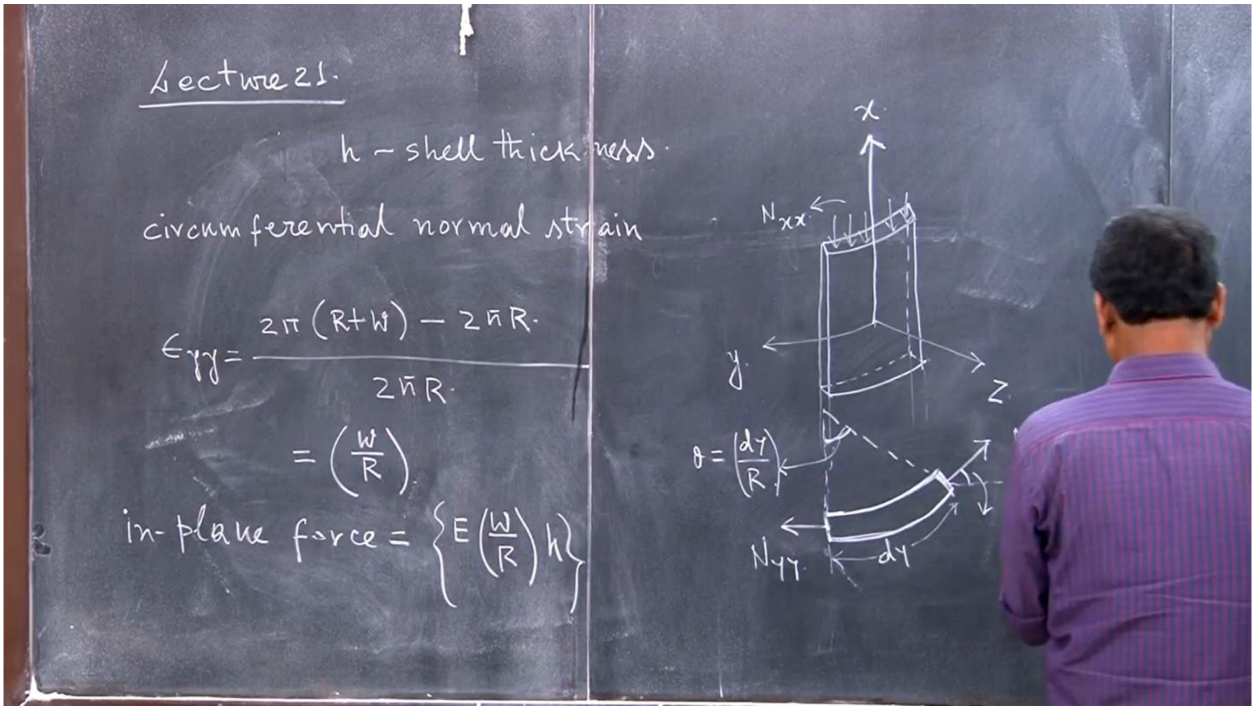
Okay. So, these are two equations. So, if you write this in matrix form, essentially what you will get here is

$$\begin{bmatrix} (m_1 + m_2) & m_1 L \\ 1 & L \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & g \end{bmatrix}\begin{Bmatrix} u \\ \theta \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix},$$

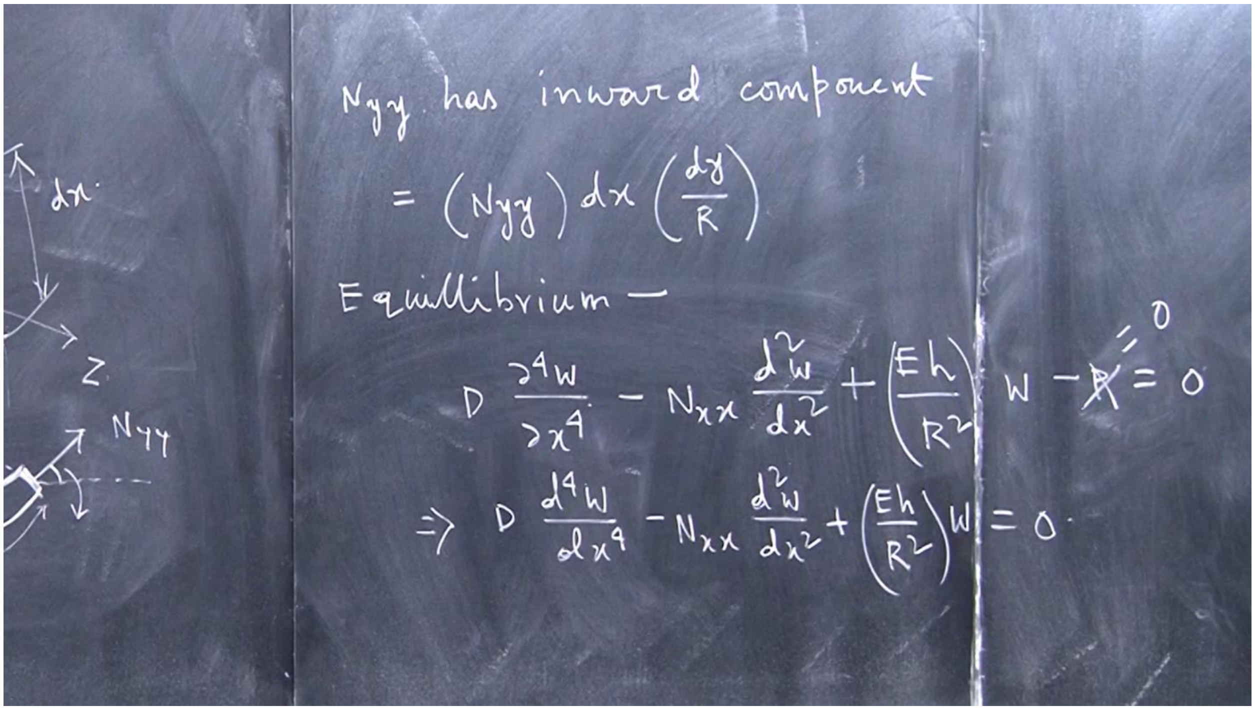
right? See that that's a coupled system of equations, and now you can assume the solution. Of course, you have to find out the ω, right? So, you can assume you see that. So, you are following right, what I'm doing right. So, you can assume we can assume the solution to be in this form. Assumed the P, $P = P_0 e^{i\omega t}$, u as $u_0 e^{i\omega t}$, and θ is $\theta_0 e^{i\omega t}$, and then substitute. If you substitute, then you'll get an equation in ω^2 in terms of other variables, and once you have it in terms of ω^2, you will see whether there is any possibility of dynamic instability. In the form of divergence or flutter, okay? That you can do yourself, right? Okay. So, with this, I would like to conclude the discussion on dynamic instability. Okay, we have solved a number of problems. We have explored the various possibilities and avenues of dynamic instability. We have also discussed, of course very briefly, all the formalisms available for treating the dynamic instability problem and basic concepts on dynamic stability, I think. What is possible, you know, in the class? So, now I know, and I am going to discuss the last chapter. At least, whatever, because we essentially, you know, confined our discussion to, you know, elastic systems; we didn't, we are not going to consider inelastic stability. That doesn't mean that, you know, a system cannot go into an inelastic system or an elastoplastic system, you know, so stability. In fact, the system for elastoplastic stability and others is a huge subject, especially for geotechnical mechanics and to a certain extent in structural mechanics as well. Especially in concrete and other materials in which this strain softening behavior occurs, those things you are a little more, a little more involved, complicated, you know, when separate discussions are required, but at least as far as elastic stability is concerned. I am going to cover the last thing and we'll give you an overview because that has very important implications for design. Okay. So, the last chapter is on the stability of shells. So, we have discussed the stability of plates. Many of you have attended a course in the theory of plates and shells. So, you know the basic mechanics of shells and why shells are important, and you are also exposed to the bending theory of shells, various bending theories, and their simplifications and implications. So, Shell, as you see why cells are important, and then you know it is said that without the discussion of Shell, and if you don't know Shell, then you don't know structural engineering, and also you don't know mechanics. Okay. So shell is the most complicated, and you know it has very integrated mechanics. Okay. You know the discussion on the physics and math involved in shell theory, both in the bending of shells and in the buckling; it's very interesting and very complex. So, there are very interesting phenomena that occur, and that is nothing but the manifestation of stability, you know, the manifestation of nonlinearity. We will cover them gradually, one after another, but what is most important to start with is So shells are two-dimensional structures, you know, and they are characterized by their two dimensions, which are much larger than the third dimension, the thickness, right? It is characterized by curvature, so why is this curvature important? This curvature basically helps in establishing a balance between the bending action and membrane action, right? The way you have seen the bending of the arch, you have seen the right arch bending, so I mean what happened there on Earth that your bending moment is reduced by the axial thrust? So, that helps in reducing the bending moment, right? So, the same thing is basically exploited in two dimensions, right? But the problem with that is that, because of the thinness in one direction, you know, right? Like a plate, it is also susceptible to buckling, and that's why shells must be designed against buckling. So, shell buckling is a very important consideration. And unlike others, we always take the example of shell buckling. Why? Because we have seen, let us go back to our, you know, first discussion that we have shown you four kinds of systems, right? So, a third kind of system, where you have seen that, is a system that shows this stable unstable asymmetric bifurcation behavior. And one limb is stable; another limb, the ascending limb, was stable, while the descending limb was unstable, and it was extremely imperfection-sensitive and notorious because it follows one-half m imperfection sensitivity. Can you recall that example right now? And then we have, you know, like this; these are the kind of, isn't it, this kind of system. So, this system essentially, you know, is although these were for the toy system, a simple system. but Shell demonstrates that kind of behavior. So, another thing is that this and there will be multiple mode you know symmetric mode anti-symmetric mode and they are all coupled through geometric nonlinearity okay and that's because of this modal interaction; I have shown it to you, right? One very simple example we have taken is August's column. A column in which two buckling modes interact, and when I have demonstrated that although individual buckling modes are imperfection insensitive, they follow a two-thirds power law. Buckling critical load increases with imperfections. But as soon as they are coupled because of these modal interactions, they become imperfection-sensitive. That means the critical load reduces with increasing levels of imperfection, making it two-thirds imperfection sensitive. So, all these cascading things, you know, they have a cascading effect on the aggregated Shell behavior. Okay. And that's why there is a huge drop in the critical load, you know. So that was initially not explained by whatever linearized shell theory you developed. It was developed at the time when none of them were able to explain the huge experimental drop that was observed in Shell. So Shell, if you either take a cylindrical shell, a spherical shell, or a conical shell, you do go and test it. And you will see that the kind of critical load that will be shown through the experiment will be much, much below the theoretical critical load obtained through linear analysis. and that was not explained until later, when it was explained by a group of researchers. We have to incorporate, you know, there are three important things in that social, you know, buckling: one is pre-buckling nonlinearity, unlike others from very early before buckling; there is huge nonlinearity in the system. You have to take pre-buckling nonlinearity into account; you have to consider modal interaction because of geometric nonlinearity. So, all these things have a cascading effect on reducing the load. So, the first thing, of course, we will start with is the simple shell theory, and then I will show you the basic example of axisymmetric buckling. But this is purely an academic exercise. Please note that this behavior will never be observed. Okay. And why it will not be observed, I will explain. So, first thing, let us discuss this axi-symmetric buckling of a cylindrical shell. Huh. So, we are going to consider a cylindrical shell geometry. So, here is the way: when I consider axisymmetric buckling, the kind of buckling that you will observe is something like this.



Do you see that? And along with that, oh I mean it has to be axisymmetric, okay? Please note this. So, the first thing is axisymmetric, and then there will, of course, be axial strain along with that because of out-of-plane bending; you know, axial strain due to out-of-plane bending. And there will be another strain that is being compressed. Do you see that? So, if you just cut it, you know, one of the sections you just cut. Okay? And you just see it here. So and so the radius R is increasing to $(R + U)$. Then there will be some radial strength. So, there will be radial strength, right? Or radial strength? So, we are going to consider that here. So, when we are considering axisymmetric buckling of a cylindrical shell, now you know what we are taking—a simple strip here, okay? And please note that in a cylindrical shell, it is characterized by single curvature. The curvature along the circumference is in one direction; in the other direction, there is no curvature, as it has an infinite radius of curvature.



So, I'm just taking a simple strip here. You see that, and my choice of coordinates is a longitudinal x-axis, right? Then circumferentially in y and z is the outward axis, okay? Along the thickness, right? So, you see that there is a single curvature here, right? This curvature, you know, so N_y is basically the plane forces along the circumferential direction, right? This must be an angle if we take a simple strip that $\theta = (dy/r)$. So, you know N_{yy}, and then it is subjected to axial force. Okay, N_x. So here it is subjected to N_x, huh? N_x compression. You see if we consider equilibrium right. So, circumferential normal strain. This is circumferential normal strain, right? And so, if the normal strain is, then the respective in-plane forces are so. E is multiplied by thickness E, and then I'm assuming the thickness is h. Okay. So, $E(w/R)$, h is the shell thickness. Fine. In-plane forces. and this N_{yy} it will have some outward force resultant force right. So, N_{yy} has an inward resultant force. The inward component of that force is N_{yy} times dx. I mean, if you consider this small section, this is dy and this is dx. I'm considering, as you know, the equilibrium of a small section.



So, d is dx, and this was dy, huh? $(N_{yy})dx\,(dy/r)$. Because of this, angle θ is an inward component. So, we can write this effective axial force $P^{*}dxdy$ as nothing but $pdxdy$. You can write down the equilibrium equation. Okay. Equilibrium. We will take help. See, the bending is only with respect to the x-axis, right? Bending, so that's what I will not include. So, you see, $d\nabla^2\nabla^2 W(x,y)$ was something that's what we have derived, right? But here the bending is with respect to xy, so I will only include this term, right? $D\frac{\partial^4 w}{\partial x^4} - N_{xx}\frac{d}{dt}\frac{\partial^2 w}{\partial x^2} + \left(\frac{\mathrm{Eh}}{R^2}\right)\mathrm{W} - \mathrm{P} = 0$

So, of course, you see that this P we are writing, if there are some external forces, but there are no external forces, is essentially zero, right? So, that will be the governing equation. So, this equation can be solved to obtain the next class we will do.