Stability of structure

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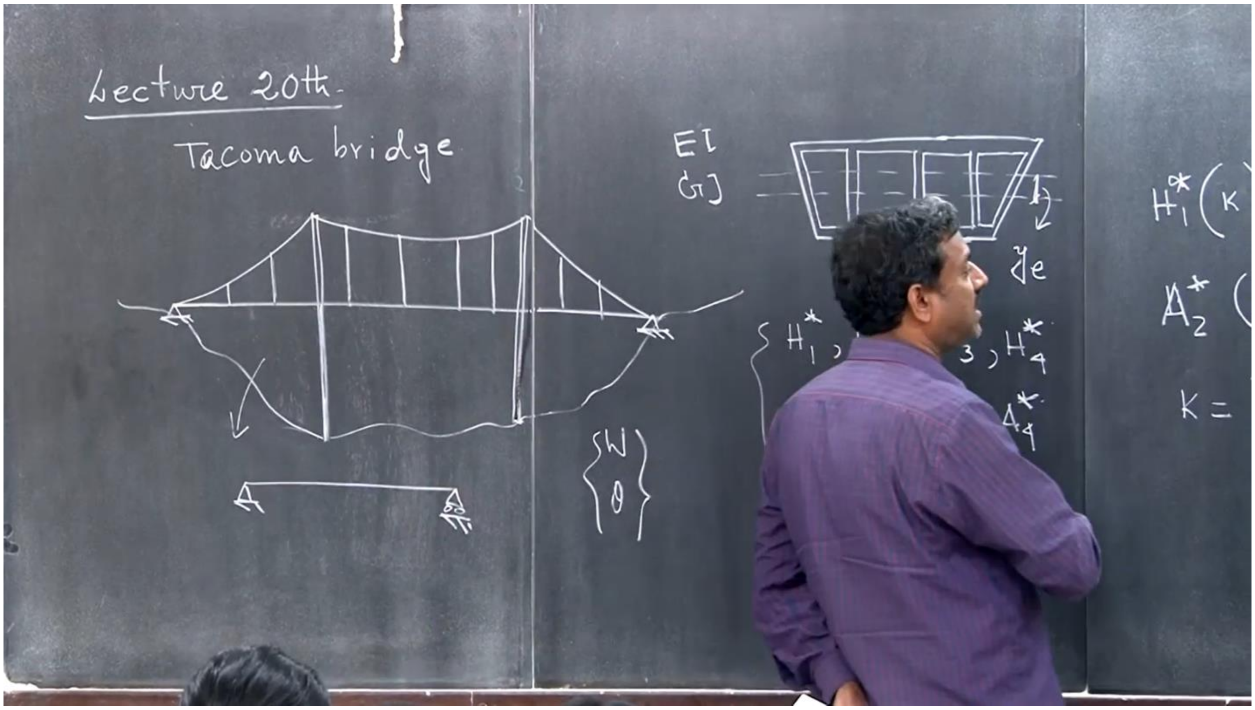
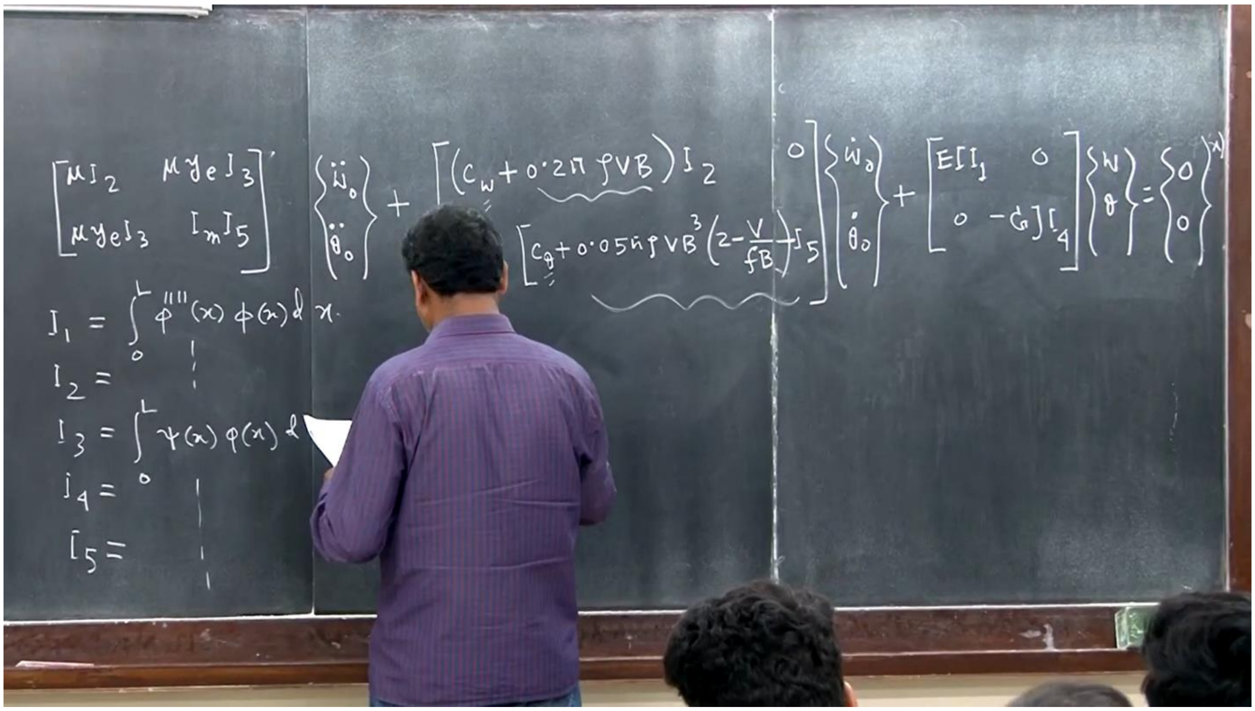
WEEK-10

Lecture 20: Flutter Instability Analysis and Noise Stabilization of System

Welcome to the 20th lecture on the Stability of Structures. Please note that in the previous lecture, I mistakenly labeled it as Lecture 18; it should have been Lecture 19. Let's briefly recall what we were discussing — we were talking about the dynamic part of instability; that is, dynamic instability. So, the one we were, you know, focusing on was the aerodynamic instability in bridges, and then we took the example of the collapse of the Tacoma Narrows Bridge in Washington State in 1940 that happened, right? Because at that time, it was the knowledge of aerodynamic instability that had not actually matured, and people were not incorporated into this design. So, we started discussing this topic. So, this was a table suspension bridge, and then we considered a reduced-order model for this bridge, which we also used when we tested it in the wind tunnel. Always, even though we have a good predictive model, we always want to do physical modeling, experimental modeling, and test it internally to check its critical velocity for fluid. So, we use a sectional model. Okay. We consider a beam model with scaled equivalent properties. Okay. Those are referred to as aeroelastic models. Okay. With distributed elasticity. So, in this simple model, the bridge structure is considered a simply supported beam. Then, the respective properties of the section were also determined: EI and GJ. Here, EI represents the flexural rigidity, while GJ represents the torsional rigidity. J is the torsion constant. Here, warping is considered negligible. The distance between the inertial axis and the elastic axis is denoted as y. We have also introduced several flutter derivatives. Out of all these flutter derivatives, only two remain significant. These flutter derivatives are essentially used to express the aerodynamic loading on the structure as a function of the structural response, which is the displacement of the structure. We have to find out all these flutter derivatives through experimental investigations or computational fluid dynamics (CFD) simulations involving fluid-structure interaction (FSI). So, here the model has two degrees of freedom, involving both the vertical deflection and the torsion of the deck. We denote w and θ; out of all this flutter derivative, I have written down the equation of motion. Then you have seen that out of all the flutter derivatives, except for these two, the others remain. These are expressed as a function of reduced frequency, which is the width of the deck and the frequency of oscillation, and v is the wind velocity.



We have considered the average wind speed; we didn't consider the fluctuations in the first part of it. Of course, wind fluctuations will have some influence, but if we consider the mean wind speed, it is sufficient to determine the critical value at which instability is triggered. This approach is adequate for design purposes. That will be sufficient for the design because we want to design it in a way that the critical velocity for flute triggering flow must be above the design wind speed. Now, we consider that we have this governing partial differential equation. Okay, two simultaneous partial differential equations. So, we solved using the Galerkin approach, and then we used this kind of approximation, similar to the separation of variables for the time-dependent part and then the spatial part. So, in Spatial One, we use some trigonometric functions and a combination of trigonometric and algebraic functions that satisfy, you know, simply supported boundary conditions. Okay. And with all these things, they are basically denoted as $\Psi(x)$ and $\varphi(x)$. Okay. And then, with all these simplifications, what we obtain is the following set of equations. I have also emphasized that, because we have considered a two-degree-of-freedom model, vertical deflection and torsion are just to make the system a little lower in dimension. We could have considered the lateral deflection as well, but we didn't, and if you consider the lateral dimension, it will unnecessarily increase the dimensionality of the problem. I mean, it will increase the number of flutter derivatives from 8 to 18, okay? So, then I will write the matrix equation, if you recall.

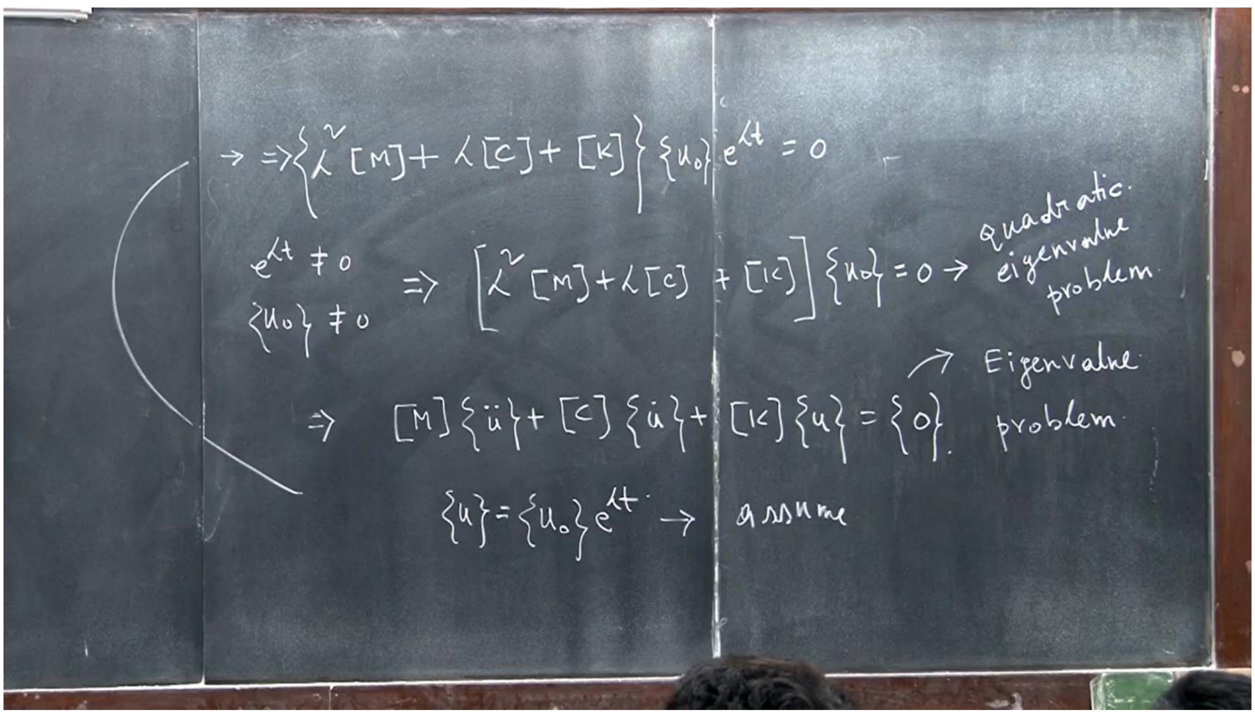


It will be $\begin{bmatrix} \mu I_2 & \mu y_e I_3 \\ \mu y_e I_3 & I_m I_5 \end{bmatrix}$, I have defined, and this is. Please note that I_M is the mass moment of inertia, right? And these are integral: $\begin{Bmatrix} \ddot{w}_0 \\ \ddot{\theta}_0 \end{Bmatrix}$ and then plus here we capitalize

$$\begin{bmatrix} (C_W + 0.2\pi\rho VB)I_2 & 0 \\ 0 & \left[C_\theta + 0.05\pi\rho VB^3(2 - \frac{V}{f_b})i_5\right] \end{bmatrix}$$

, and then this is zero. And here it is zero, and then here it is, and here it is $\begin{Bmatrix} \dot{w}_0 \\ \dot{\theta}_0 \end{Bmatrix}$ plus

$\begin{bmatrix} EII_1 00 & 0 \\ 0 & -GJI_4 \end{bmatrix}$, and $\begin{Bmatrix} W \\ \theta \end{Bmatrix}$ is equal to $\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. And all these I_1, I_2, I_3, and I_5 are nothing but some integrals. I have written about them in the previous class, and I will just express one of them. You know these are nothing but what you obtain by integrating the stress. You know this approximation function that we used for the displacement field. $\varphi(x)$ is the spatial derivative, and the dot is the time derivative. And similarly, all the others, you know, for $i_3 = \int_0^L \varphi\,(x)\Psi(x)dx$, okay, and so on. Okay, all these things I have noted previously. So, what we can clearly see here is that the aerodynamic loading has essentially been incorporated into the damping term on the right-hand side of a homogeneous system of equations. So, these were like damping in material terms, right? This, you know, damping, which is viscous damping, comes okay, but additionally, because of this aerodynamics, you know, effectively. Because the force being exerted on the deck by the fluid-structure interaction is expressed in terms of the response of the structure, right? So, it is the damping that is coming; effectively, they are contributing to the damping, okay? So that becomes a function of the wind speed and the density of the air, and then b is the width of the depth. What we can see is that, "The aerodynamic damping associated with the flexural mode is always positive, whereas this other mode has the potential to be negative." Because it involves the $\left(2 - \frac{V}{f_b}\right)$ term, right?



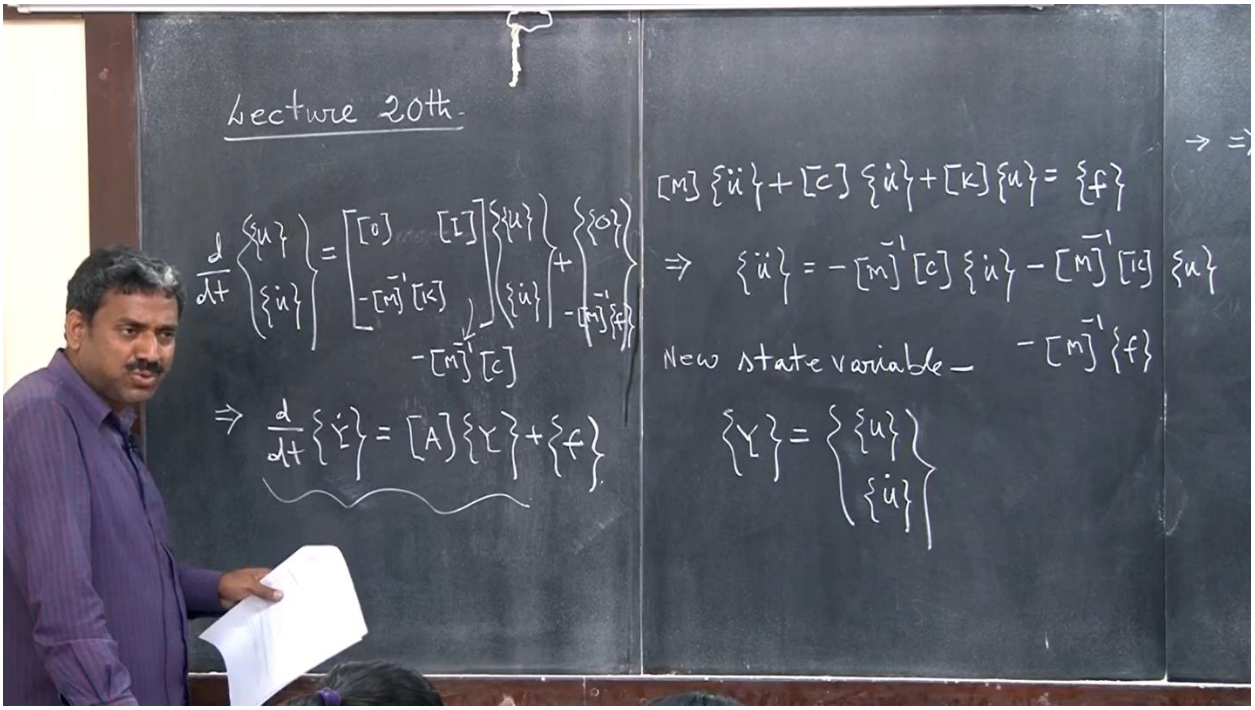
So, for some value of φ, it will be negative, and that will reduce the damping and contribute to negative damping, which will trigger instability. Do you understand? So, ultimately, when we reach that point, we can express it in a simplified form; this represents the mass matrix. And we can write the equation as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{0\}$$

This is a homogeneous system of equations, representing the standard form of the equation of motion. This will lead to an algebraic eigenvalue problem. Since we have used the Galerkin approach to convert the differential operator into an algebraic expression, the resulting system can now be analyzed in terms of its eigenvalues. Now, you might wonder how we can convert this into an eigenvalue problem. To do that, we can assume a solution of the form $u = \{u_0\}e^{\lambda t}$. By assuming this type of solution, when we substitute, what we will get is $\ddot{u}$, which means $\{\lambda^2[M] + \lambda[C] + [K]\}$. Then here we will get $\{u_0\}e^{\lambda t} = 0$. For a non-trivial solution, $e^{\lambda t}$ can never be zero, and u_0 must also be non-zero. Therefore, the condition for a non-trivial solution becomes:

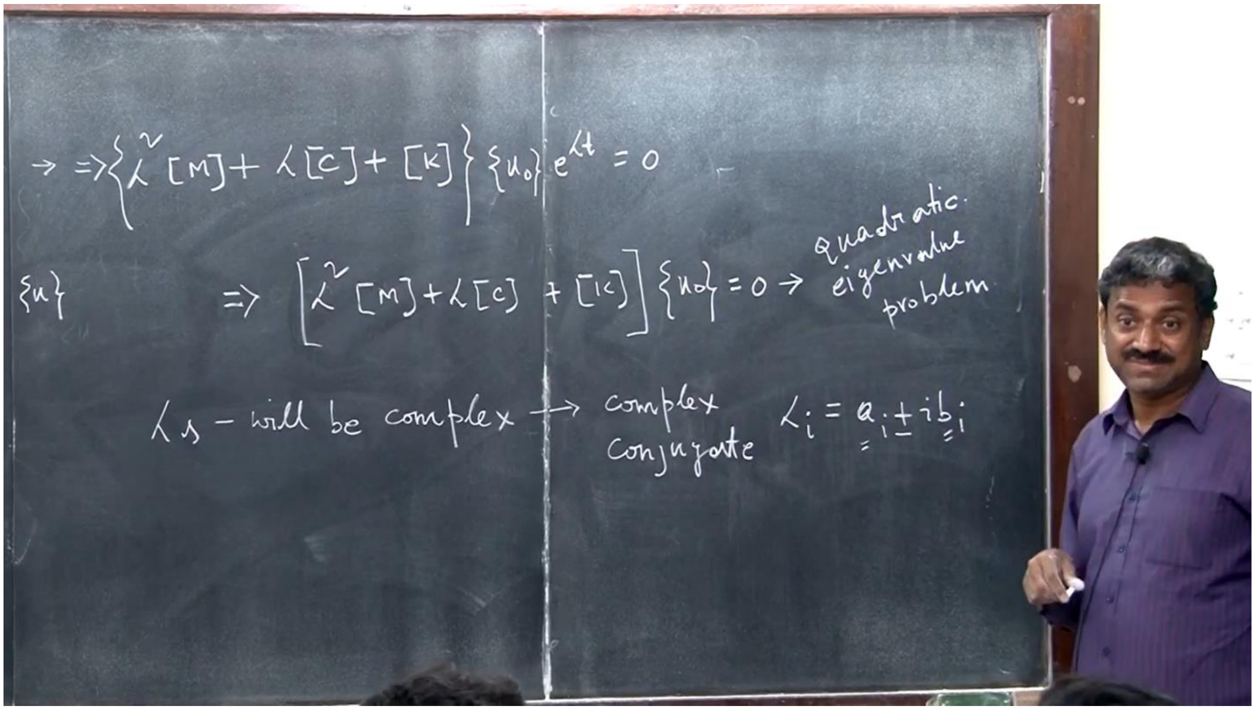
$$[\lambda^2[M] + \lambda[C] + [K]]\{u_0\} = 0.$$

So, this basically leads to an eigenvalue problem. We have this, so one and zero must be equal to zero. So, this is nothing but a quadratic eigenvalue problem. Because this lambda is the value, of course, the determinant needs to vanish, right? So, this leads to a quadratic eigenvalue problem. Why is it quadratic? Because conventionally, the eigenvalue problem we come across is basically a quadratic eigenvalue problem. Which is a little different, you know, from the standard form eigenvalue problem. The standard form is something like this, right? If you can recall that in the stability class. We have learned that $[k + k_g]\{u\} = 0$, right? That was the eigenvalue problem that we came across for static buckling, right? And in dynamics, what you've learned is that $[K]\{\phi\} = \omega^2[M]\{\phi\}$, which represents the generalized eigenvalue problem. Which you come across in dynamics. So, in both cases, you can see that these are first-order equations, right? But here, lambda is appearing only in the ω^2. Okay. But here it is lambda squared. That's why it is quadratic. Nevertheless, you can convert this quadratic eigenvalue problem into first-order form by using the state-space representation. How can we do that? All of you are familiar with the state-space form, right? So, you can either solve the quadratic eigenvalue problem directly or convert it to something like this. The way you convert it, all of you learned the an. Okay, I'll come back to that part a little later. So, any n-th order differential equation can be converted into n first-order differential equations, right? Similarly, any algebraic expression of nth order that can express n numbers of first-order equations is what dynamics is basically about. In dynamics, it always gives you the quadratic eigenvalue problem. But if you write the equation of motion in state form, you can convert it into a first-order value problem. All of you are aware of state-space form.

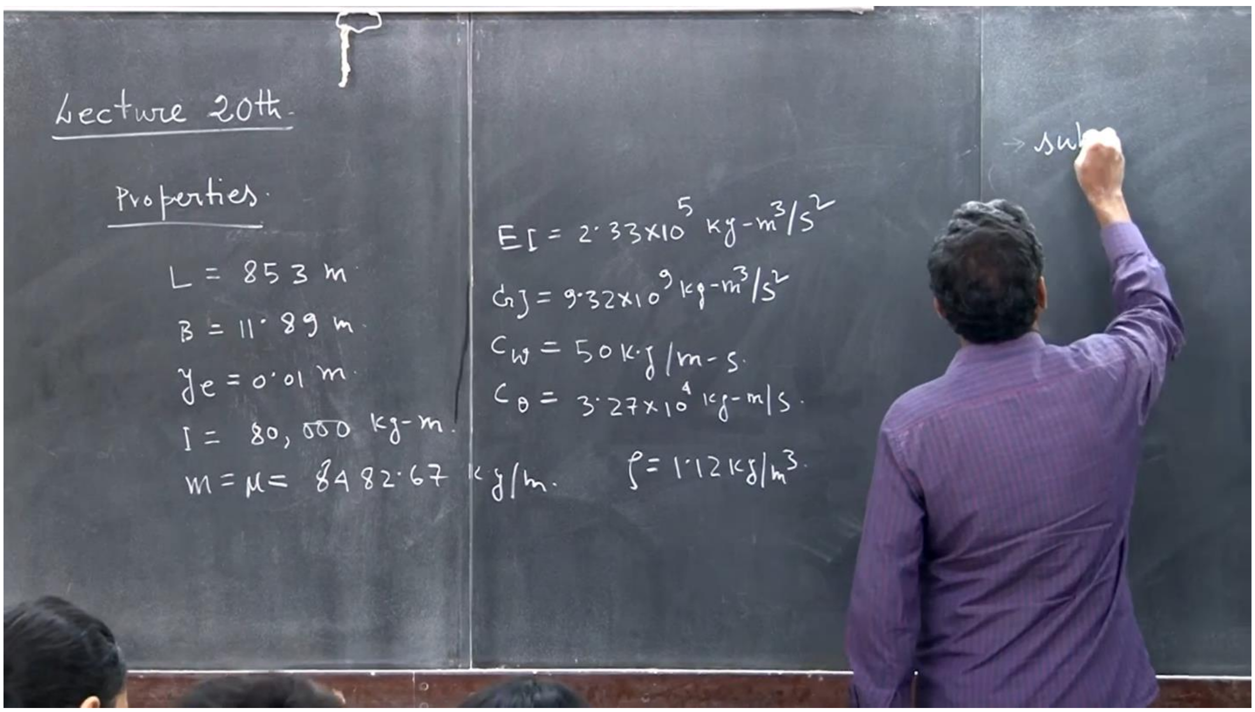


No, no — I taught this in dynamics. You've all forgotten about complex mode analysis, and that's the problem. See, $[M]\{\ddot{U}\}+[C]\dot{u}+[K]\{u_0\}=\{f\}$. Okay. Now you write down $\{\ddot{U}\}=-[M]^{-1}[C]\{\dot{u}\}-[M]^{-1}K\{u\}-[M]^{-1}\{f\}$. Next, we can define a new state variable. New state variable: state. What are the state variables for this? State variables must include the variables that can completely define the entire motion. So, I am defining a state variable that basically contains both $\{u\}$ as well as $\{\dot{u}\}$. Do you see that even $\{\dot{u}\}$? So now the dimensional problem has increased, right, but at the expense of the reduced order. So, can I write like this? $\frac{d}{dt}\begin{Bmatrix}\{u\}\\\{\dot{u}\}\end{Bmatrix}=\begin{bmatrix}[0] & [I]\\-[M]^{-1}[K] & -[M]^{-1}[C]\end{bmatrix}\begin{Bmatrix}\{u\}\\\{\dot{u}\}\end{Bmatrix}+\begin{bmatrix}\{0\}\\-[M]^{-1}\{F\}\end{bmatrix}$. Do you see that? Here is the way I have written it. so, now it is $\frac{d}{dt}\{\dot{y}\}=[A]\{\dot{y}\}+\{f\}$. For the time being, at least for this problem, we assume that the external excitation has already been incorporated into the aerodynamic damping term. So, only these two terms will remain, right? This problem can also be expressed in the state-space form. In other words, you can convert a second-order problem into a first-order problem. Not quadratic, huh? But in that case, the dimensionality of the system will increase from n to 2n. Understood?

So, either you solve form this either you solve this form or you solve this form your Eigen values will be same. The only difference will be a slight change in the eigenvectors. But they are still equivalent. Do you understand what I'm trying to say? As for the eigenvalues, you can see that this matrix is asymmetric.

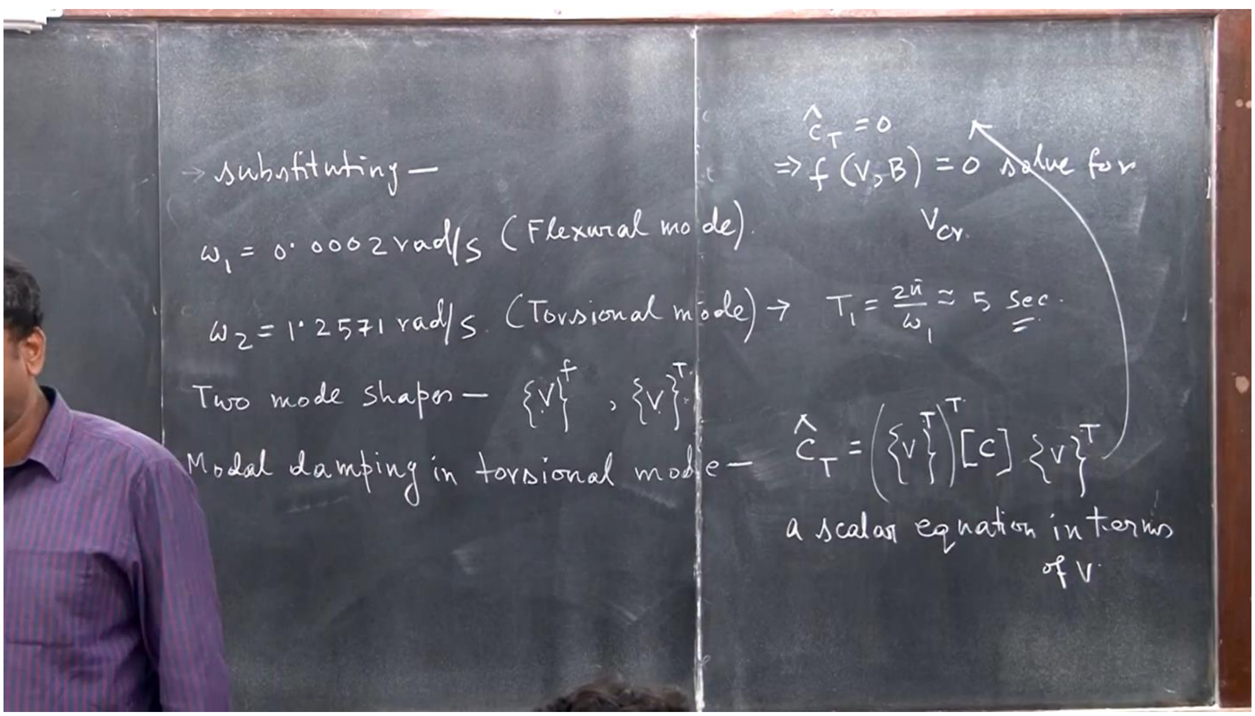


Eigenvalues λ will be complex, so when there is a complex eigenvalue, it will appear in complex conjugate pairs. Complex conjugate means any λ_i will be expressed as $(a_i + ib_i)$, something like that: the real part and the imaginary part. The real part can be associated with the damping because it basically acts as an envelope function. The imaginary part will be associated with the ω frequency, right? Because $e^{\lambda t}$ means $e^{(a+ib)t} = e^{at}e^{ibt}$, and e^{ibt} is nothing but $\{\cos(bt) + i\sin(bt\})$, which will contribute to the absolute. Thus, b_i is associated with the frequency. Whereas the real part of a_i is associated with damping. You understand what the roles of the real part and the imaginary part are. The imaginary part of the eigenvalues will be associated with the oscillatory (vibratory). You can take the square root of that, and taking the square root is very simple for any complex number; you know anything to the power of n is nothing but what? $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$. You know that, right? That's Euler's formula. I don't mean De Moivre's theorem; you understand that the real part of this ω is associated with the damping, while the imaginary part corresponds to the oscillation frequency. From here, the eigenvalues will appear as complex conjugate pairs—plus and minus forms, clear? We'll see what really happens shortly. So, you understand the equivalence between the quadratic eigenvalue problem and the problem that can always be converted into that form. So, in MATLAB and other mathematical software like Mathematica or Maple, there are standard built-in routines to solve this kind of problem. Nowadays, you don't need to write the entire code yourself — you can simply call the appropriate routine, and it will directly give you the solution. The only thing is that, because it appears in a complex where it's an asymmetric matrix, you will have a right set of eigenvectors and a left set of eigenvectors. Okay, we are not talking about that. But sometimes there will be left eigenvectors.



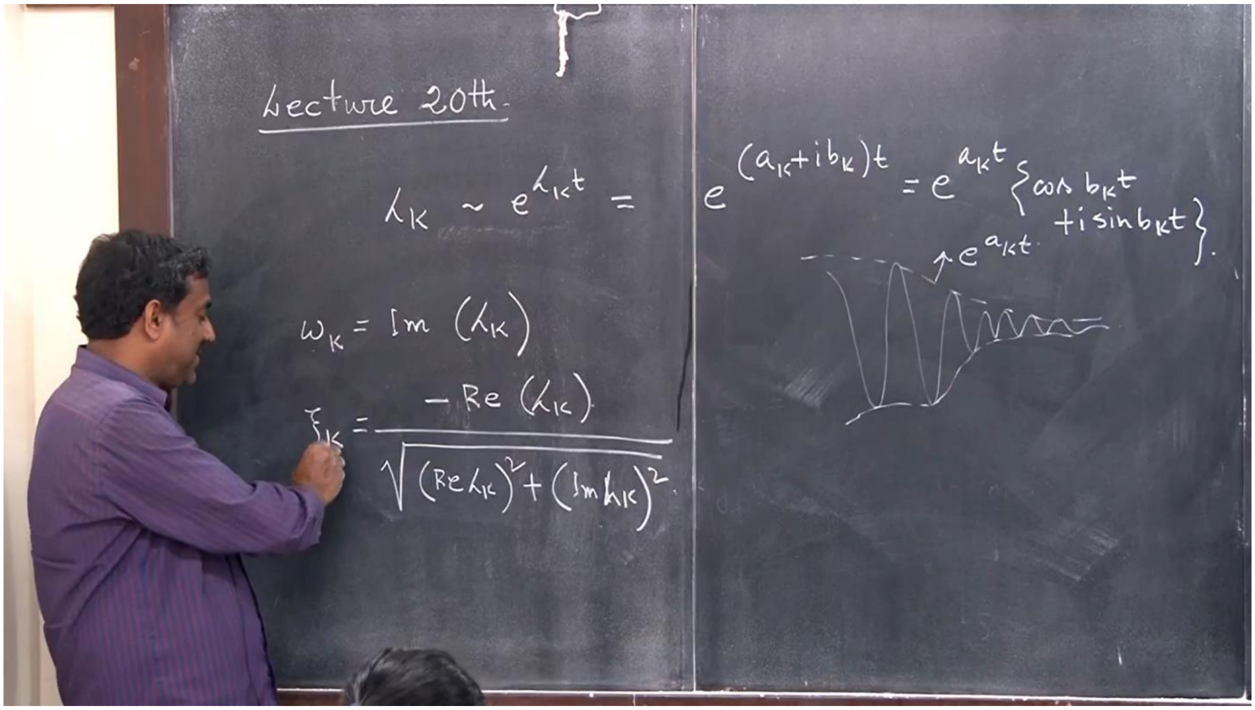
Okay, and the right eigenvector. Okay, it's a little different. So, once you get it, what will you do? In this case, if we solve this, we will evaluate it. Now we will do a numerical evaluation. So, properties of this system will be the bridge deck. For the reduced-order two-degree-of-freedom model for the Tacoma Bridge, the reduced properties are given. So, let me write it down: the span is 853 m — you see, that's almost a kilometer. B, the width of the deck, was 11.89 m — about 12 m. Depth, you know, so it can easily accommodate quite a few lanes, you see. And then y, the distance between the inertia and elastic axes, is 0.01 m, and the moment of area is 80,000 km·m. Please note that, instead of force, these are expressed, okay? I mean, this is the mass moment of inertia. Here, m or μ — we also denote it as μ — is basically the mass per unit length, which is

8482.67 kg/m. Huh? And then EI, I will write here now. EI, the flexural rigidity, is 2.33×10^5 kg·m^3/s^2. You have to multiply it by g to get it in newtons. Right? But what you can do is not a problem. GJ flexural torsional rigidity is 9.32×10^9 kg per meter cubed second squared; do you see that the torsional rigidity is significantly higher right here? Because it has a lot of torsional rigidity, which is higher because it consists of multiple cell sections, and it is a closed section, that's why it is similar. C_w is nothing but the viscous part of the damping in the flexural mode, and C_w is 50 kg/m·s. C_θ, the viscous part of the damping in the torsional mode, is 3.27×10^4 kg·-m/s. The density of air, ρ, is 1.12 kg/m^3. So, all these properties you take, and then you substitute them in the matrices. You substitute these values into the matrix, and then you will obtain the mass matrix, the damping matrix, and the stiffness matrix, right? After that, you will have all the numerical values.

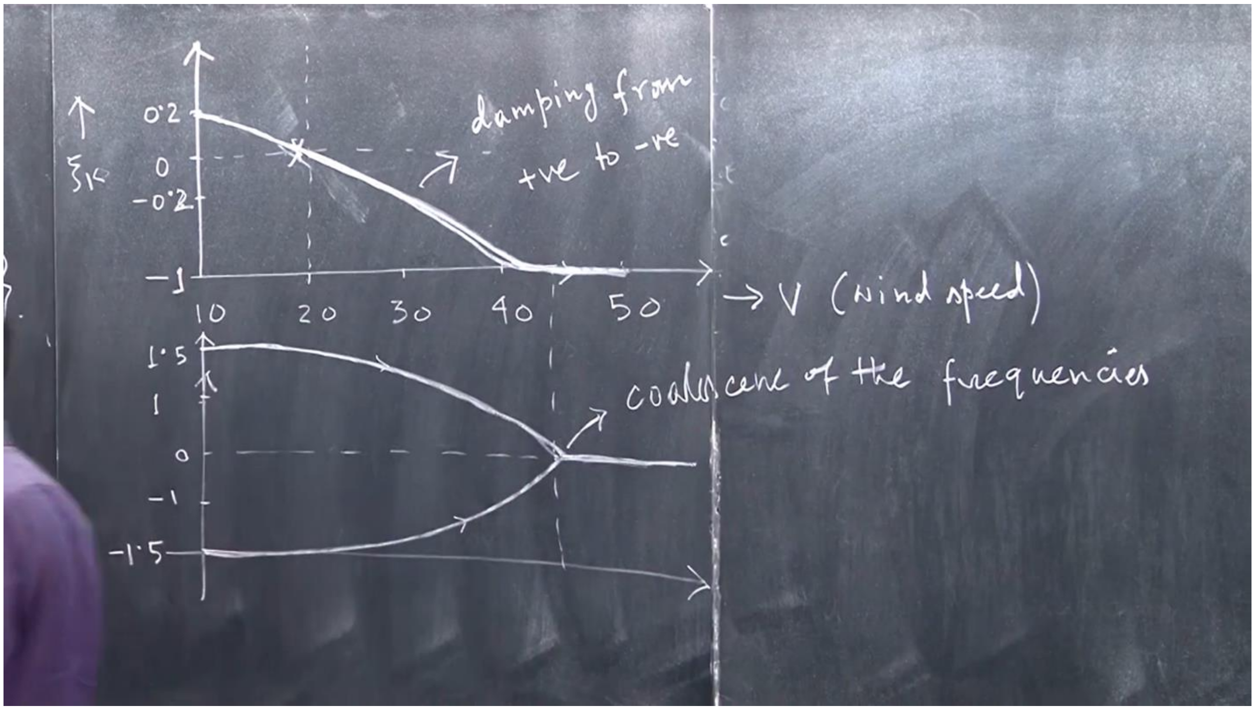


So, with all these things, by substituting all these values, you will get the matrices, and then we solve them. Of course, here we have solved it using MATLAB for the two-degree-of-freedom system.

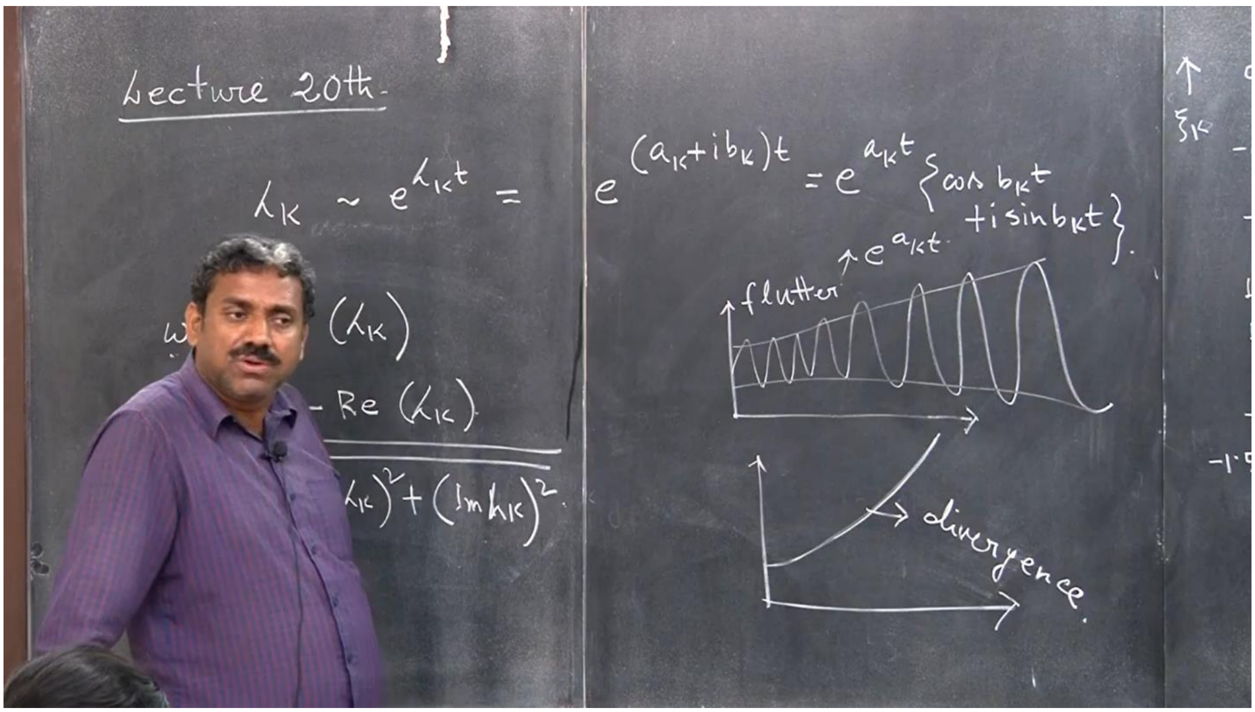
So, the 2 × 2 matrix is very simple. How many natural frequencies will there be? Two, right? Uh, two, right. But since it's a quadratic eigenvalue problem, you'll get four natural frequencies. So, yes; however, the imaginary parts will be identical. Okay, I understand; otherwise, if there are four lambda values, will there be four? Anyway, you can associate them with the two modes, right? Here, you can write that ω_1 is, in fact, something you can solve separately to get the natural frequency, neglecting the damping, and you can solve a generalized eigenvalue problem. Okay, to get the frequencies separately, you can do that as well. Here, by solving separately, we are considering only the mass and stiffness matrices, without regard to damping. So, ω_1 for the flexural mode will be 0.0002 rad/s, and ω_2 will be 1.2571 rad/s. Could you please tell me which one is the flexural mode and which one is the torsional mode? The higher one will be the torsional mode, and the lower one will be the flexural mode. Come on, these are the torsional and flexural modes, right? You people are experts in dynamics—you've done dynamics, advanced dynamics, and even earthquake engineering. Anyway, for the flexural mode, Anyway, if you want, this time period will be what? It can be calculated as $T = \frac{2\pi}{\omega_1}$, which comes out to around 5 seconds. I have calculated it to be around 5 seconds. That's very flexible, huh? Even the stiffer mode is still relatively flexible — one mode is more flexible, and the other is also flexible. Now what we will do is You have seen that negative damping can only occur in the torsional mode, right? So, what we will do is say that the fractural mode can never have negative damping. We can find the modal damping for the torsional mode. How do we find it? By solving the system, we can also obtain the respective mode shapes. Therefore, there will be two mode shapes. So, we are not solving the quadratic eigenvalue problem here, at least in this case. I'll come to that later. First, we are solving the eigenvalue problem with the respective modes—okay, considering only the mass and stiffness matrices. As for the mode shapes, I didn't calculate them here. So, if you want to write them as V, the first mode will be $\{V\}^1$ and the second mode will be $\{V\}^2$. Okay. So, this one for the torsional one is $\{v\}^2$ and $\{V\}^1$; this is for the flexural one. Now, how much is the modal damping in the torsional mode? How do you calculate modal damping in the torsional mode? Huh, maybe I'm denoting it as C_T (capital T) for the torsional mode. So, you have the damping matrix C, right? If you want to calculate modal damping, this is what $(C_T)^T = (V)^T[C]\{V\}$ means; this is how it's calculated. If we consider modal damping, these terms are uncoupled because the off-diagonal terms are zero, so you don't need to worry about classical versus non-classical damping. But yes, it's technically non-classical damping. Here, this is for torsional; since we are calculating torsional damping, this factor should be applied specifically to the torsional mode. Is that fine? Oh, no, no, sorry. There will be another transpose; then if you do it, you will get a scalar expression. Because this is a matrix vector, you should know that if you do this, you will get an equation of C. Okay? A scalar equation in terms of V, right? What kind of equation will that be? You'll get a quadratic equation for V right from here, right? Because the expression for V is coming, you can solve it. So now, when you get this equation for $C_{\text{cap}T}$, it is zero. you set it up to solve it. Here, of course, it is a function of wind velocity, and then there will be B and other things right — initially starting from zero. Here you can see that modal damping can go from positive to negative. So, when it goes from positive to negative while crossing zero, can we say that is our critical velocity that will trigger? We can do that right. So, you can solve it and it is zero. Solve for V_{cr}. When the model damping is zero, find out the respective wind speed for which the model damping for the torsional mode is crossing zero. Because after that, from positive damping, it is going to negative damping. Huh? And then it will trigger unbounded response. Right? So, by solving this you know $C_T = 0$ and you will get V_{CR} That critical velocity I'm mentioning to you will be 18.77 m/s, or it is around 72 km/h. So, when the wind speed is greater than 72 km/hour, it will trigger a flutter unbounded response because of the negative damping. So, I'll just show you what really happens if you solve it a little. So, all of you have followed this until now, whatever I'm saying, okay? One more thing I would like to say: if you plot, you will see that there are two ways to do this kind of stuff. One is, you know, so when you solve for the λ, right, if you see all these things, whatever we did from our basic understanding of dynamics, okay? We solve the eigenvalue problem, find the frequency and modal damping; these are things we really don't solve in the quadratic eigenvalue problem. But in any other system, the system is simple. It's a toy problem. A simple system you can solve by hand. You going to if you have a system which has multiple degree of freedom. Okay, thousands and thousands of degrees of freedom Then what will you do at the time? You should have automatize this process, right? The process should be automatic, and there are search algorithms to find the respective λ for which the flutter instability is triggered. Okay, so how do I do that? I am just showing it. Okay. So you see that λ_k is associated with $e^{\lambda_k t}$, and it is nothing but $e^{(\alpha_k + ib_k)t} = e^{\alpha_k t}(\cos(b_k t) + i\sin(b_k t))$. So, you see, this is the envelope function. So, what really is happening, you see, is that it will have some kind of effect; that means sine and cosine are basically dictating the fluctuations, right?



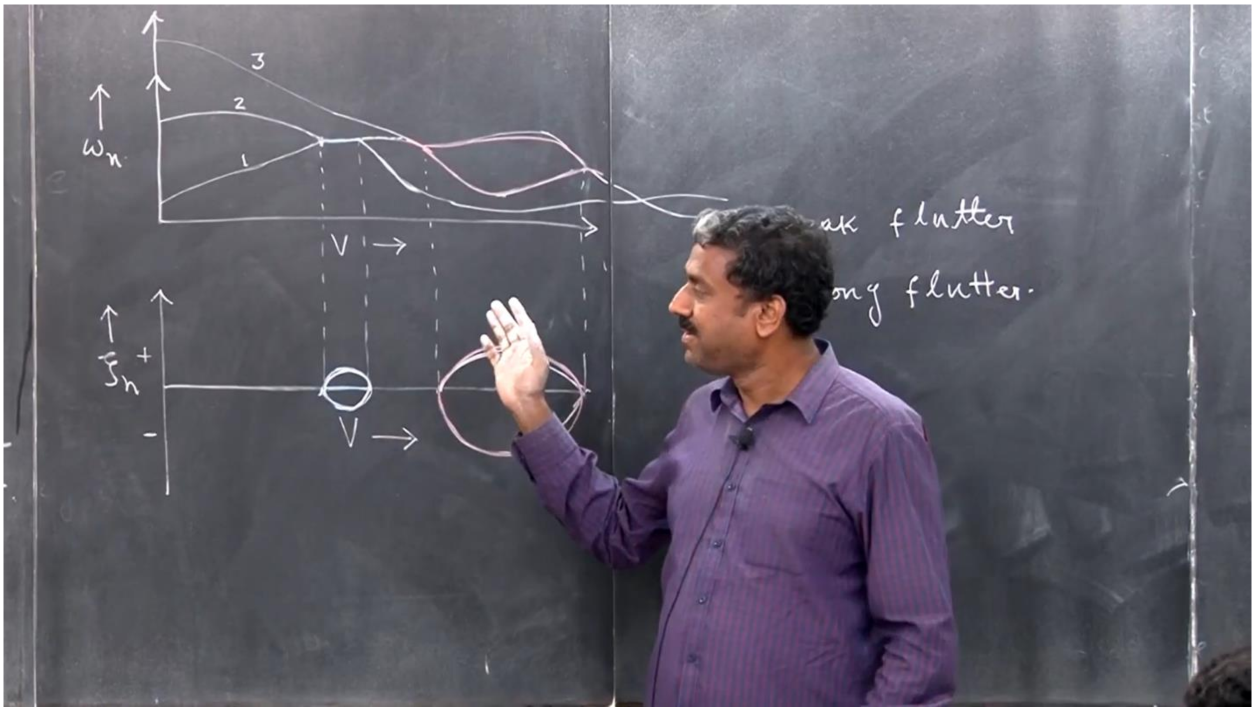
The periodicity of the solution, whereas these are basically the decay functions, right? Do you understand the concept of the decay function? So, this is $e^{\alpha_k t}$, right? So, what I understand is that one part is associated with damping, while the other is associated with frequency. So, how do we find that out? Okay. So, the way we write it, ω_k is basically the imaginary part of (λ_k), and $\zeta(k)$ is the damping, which is $-\frac{\mathrm{Re}(\lambda_k)}{\sqrt{(\mathrm{Re}(\lambda_k))^2+(\mathrm{Im}(\lambda_k))^2}}$. You see that this is the frequency, and this is the damping. You understand why the imaginary part of λ_K is associated with frequency, right? I've explained why damping is like this because it is the involved function. So, now, recall the logarithmic decrement. How do we find it? Similarly, in the real part, it has to be normalized because we are dealing with the damping ratio. Here, ζ is the damping ratio, which is the ratio of actual damping to critical damping. And if you do this, you will obtain these values. Once you have ω_k and ζ_k for every λ, you can estimate ω_k and ζ_k. For at least this case, where you have a quadratic eigenvalue problem, you will obtain four eigenvalues. For the four eigenvalues, you will see that each mode has its corresponding ζ_k and ω_k. Now, if we plot them here, I will plot them, and then we can try to generalize it. So, you'll see that this is ζ_k. So, for different values, this is nothing but a wind speed V. For increasing wind speed, you see that from positive damping, and this is the damping ratio; the positive damping is coming and crossing zero damping. What is the value of zero damping? 18 point something, that's what it is for the Tacoma Bridge, right? and after that, it is coming to be this and then it is coming and then beyond 40, Once again it wil go around one. This is minus one. That means 100% damping. You see how damping is changing, from positive to negative, and negative damping can be as high as 100%. Do you see that? And what will happen to the respective omega values? See, here we are plotting the imaginary part of λ_k.



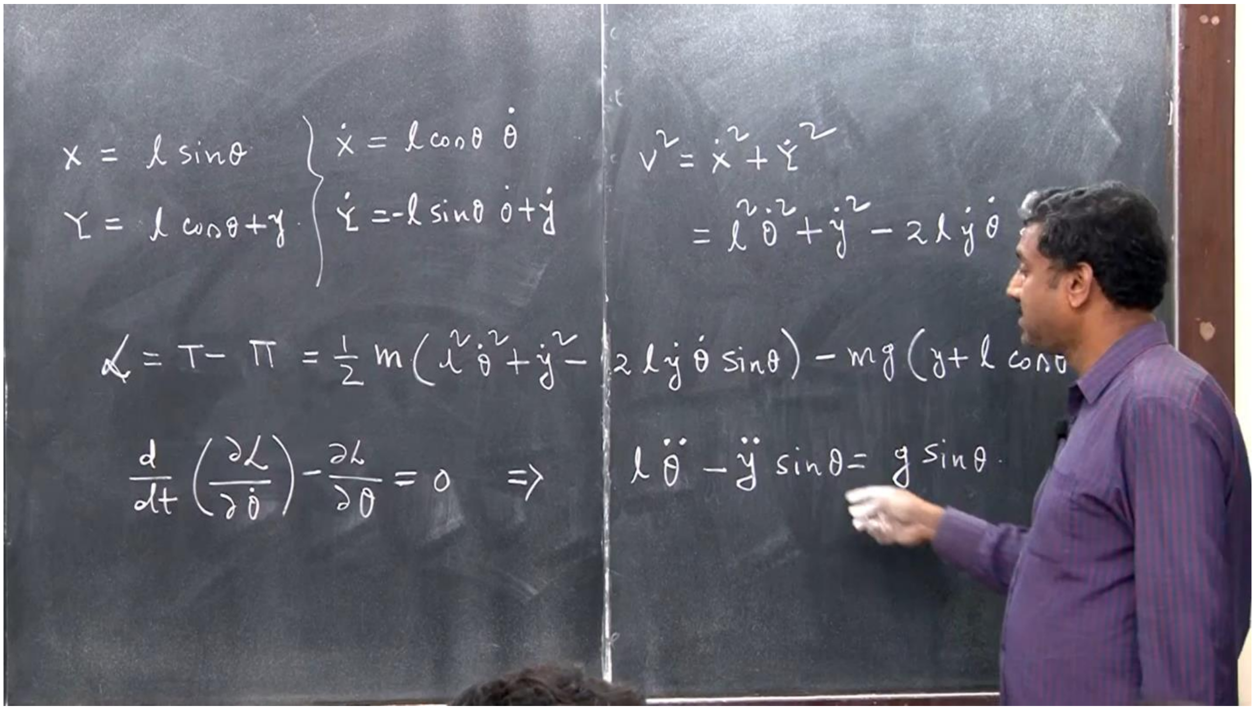
So, one of the positive values is coming and another of the negative values is coming. That's basically coalescence, the coalescence of the natural frequencies. So, when aerodynamic flutter is triggered, the damping is, of course, negative; but at the same time, it is accompanied by the coalescence of the two frequencies. Do you see that this frequency keeps decreasing while the other one keeps increasing, and eventually they meet—that's the coalescence point for the present case? After that point, both frequencies become zero. Zero means the structure can no longer vibrate. It can diverge because if omega is zero, that means there is no periodic oscillation; it will diverge in one direction because with negative damping it will diverge to one side, right? But beyond that, there will be negative oscillation; you see that even if you come, this regime's fluctuations will still be there. But the fluctuation will increase with time, right? So, what will happen? Let me tell you what will happen under flutter, you know. This will increase, you see that, but when it is zero frequency, what will happen then? It will increase like this. So, it will increase like this. Please note that there are no periodic oscillations. This is aerodynamic flutter, whereas this is called divergence. Now you have also learned about divergence, where the frequency becomes zero, right? And when that happens, the geometric stiffness matrix completely overshadows the stiffness matrix that comes from the material contribution — in other words, the material stiffness is entirely counteracted by the geometric stiffness. So, buckling, which is a static instability, is also manifested, you know, dynamically in the form of divergence. Divergence is basically equivalent to buckling because, essentially, you know, its stiffness is all eroded by the geometric stiffness.



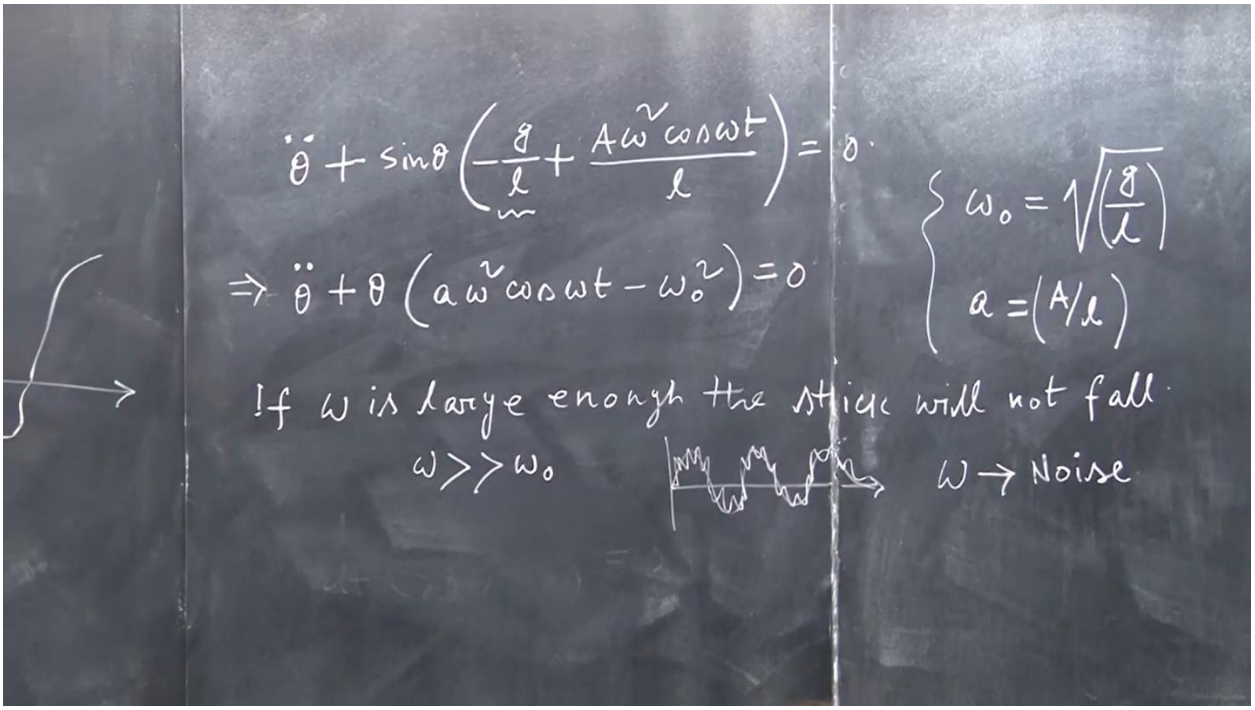
Okay, because geometric stiffness is being subtracted, and that's essentially what is happening. You see that these two modes are quelling, and this is at zero frequency. Okay. Now this is happening for this case. Now, for the more generalized case, what really happens is that I will just show it. So, you understand the difference between flutter and divergence, right? Flutter is associated with vibration with increasing amplitude, and divergence is associated with increasing amplitude, but there are no periodic solutions, right? What really happens here is that we draw the stability diagram by plotting ω_n and ζ_n. For any system that results in a quadratic eigenvalue problem, you can determine λ_k, and from that, you can estimate ω_k and ζ_k. And then you can plot as a function of what the controlling parameter is; here is the velocity, here is the velocity v, and right wind speed. So, here, you'll see what happens — you may have multiple frequencies. One of these frequency branches starts from here: one begins here, another starts there, and so on. Maybe two of these frequencies coalesce and then continue together. Then maybe you know that after these frequencies come, they coalesce, and this will happen once again. Okay, after the coalescence of these modes, they move together from that point. You can see how this happens—the coalescence is indicated here, and between these two branches: one is for positive damping and the other for negative damping. So, here they will go, and then ultimately, they will have these kinds of circular things, and then once again from here to here, there is nothing. Okay. Once again, from here to here, there is nothing. Okay. And once again, maybe these two will once again, then you see that another flutter is happening over there. So, here also, there will be this kind of, you see how the stability diagram with increasing flutter is triggered by the coalescence of these two modes where the omega is basically coalescence. Then, after that, at some point, this is bifurcating, and if you project it onto the damping axis, what is happening here is that if you plot them, this will give you a circular kind of shape. Okay. "And then, once again, bifurcating. Maybe if I denoted the first mode, second mode, and third mode, you could see that it is the coalescence of the first and second modes." Which is triggering flutter here, you see that it is the coalescence of the first and second modes — this part is clear. After that, the coalescence doesn't last long, and then what happens is that mode three and mode two start coalescing. Here, you can see that mode two and mode three are coalescing, and then they separate again — this bifurcation is indicated by the circular marks on the damping axis. Later on, there may be further coalescences and bifurcations as well. So, this is a stability diagram, where we can see the variation of the frequency and the effective damping as a function of the changing parameter. Which parameter controls the stability behavior? In this case, it is the wind speed. So now I understand why the frequencies are coalescing, why they are bifurcating, and why they are projecting. You can see that these circular things are coming. Please note that they always appear in complex conjugate pairs. So, when one of these values becomes positive, but both are coalescing, even the coalescing part has positive damping while the other has negative damping. That's why this kind of locus appears. The size of the locus—whether smaller or larger—depends on the span of these coalescences and bifurcations. And if it is a small one, then it is called weak flutter. If it is a large one, it is called strong flutter. So please try to understand, you know, weak flutter and strong flutter. What happens? You see, the one that is weak flutter can be suppressed by increasing some kind of damping. But the strong one is difficult to suppress; if you increase the damping by some artificial means, weak flutter can be eliminated. But strong flutter cannot occur, so this discussion is essentially concentrated around the aerodynamic stability of cable-stayed or cable-suspension bridges. Please note that other types of bridges are not susceptible to wind. Okay, because they're too stiff. Cable suspension or cable stays is flexible enough to allow flutter.



But what about girder bridges and other things? There, flutter generally doesn't manifest as a global failure mode. There might be localized flutter of some components, which is a different thing. Okay. So that's why the steel bridges, you know, which are steel girder bridges, etc., are not checked against the product. You know wind tunnel tests are generally conducted, and it is mandatory to conduct wind tunnel tests for cable-stayed and cable-suspension bridges that have a larger span, and because of the suspension cables and the load-resisting system, they are inherently flexible. That's what is done. Okay. But it's not just bridges — this is also very important for aircraft wings and other structures. Flutter used to be a common mode of failure in aircraft, which led to the development of an entire branch of study called aeroelasticity. Well, this isn't exactly flutter, but flutter can also happen. And it's not only flutter — other types of aerodynamic instabilities can occur as well, for example, in chimneys. That's why chimneys also need to be tested for aerodynamic instability. So once again you do the aeroelastic modeling for the chimney, and they put it in the wind tunnel. Then you slowly increase the wind speed, and suddenly you see a huge jump in the vibration, okay? And then that basically, that critical velocity is triggered as the velocity that triggers. So, you have to avoid that, and in design, you want to push it through by modifying the geometry and things like that, or sometimes there are artificial means; you just attach some kind of structure around the chimney and things like that, which will break the vortex structure. So, what happened in that for a chimney when the flows, see, a chimney, there is a basic difference between bridges and chimneys: all bridges are basically what? Streamlined objects, but a chimney is a bluff body. So incoming airflow got struck; there is a technician and a joint, and at the end downstream, there is a vortex. Because of the vortex, it is not the along-wind component but the across-wind component that is more important; there is a huge vigorous vibration across the wind and transverse vibrations, which are across the wind. So, in the code, you will see that there is a huge discussion on across wind and along wind, and there is a question of how much force is going to come. So, when you design, you have to do a little check on how much air force is coming from the wind tunnel test and how much is coming from your design value. Also, you have to identify what the critical load for the flutter is. Not only flutter—there are other kinds of things that happen as well, which are called buffeting, overhauling, and other kinds of instability. But please note that these are the basics; I mean the formalism for the analysis of dynamic stability. "Okay, Now I'll consider another case, which is a bit uncommon and not as important for practical purposes. This is called noise stabilization of a system. What is that? I will give you a simple example. You see this stick, right? If I place this stick, it tends to fall, right? Uh-huh. But it's not actually falling. Why? Because we're changing the base, that's why. This is exactly how an inverted pendulum system works. I'll give you an example of an inverted pendulum. So, let's consider this setup: there is a massless base, and on top of it, there's a rigid rod with a mass m attached to the top. Now, this base is subjected to some kind of vertical acceleration, right? I am considering this for simplification; we believe it to be rigid, okay? Mass is concentrated there. So, I mean, this is not exactly fixed, so it can rotate. So, you know I am considering it; you know it's degrees of freedom as θ essentially when I'm trying to stabilize it. What I mean is this fellow is going to fall. Why? Because if you perturb it, then the component of weight is basically destabilizing it. But as soon as I'm trying to, you know, I'm essentially using my hand to apply a vertical motion to this, okay? So, when it tries to fall, it is a lateral disturbance, right? It is trying to vibrate in this plane, and that vibration I'm going to stabilize with a vertical thing. See, when it tries to fall, I am trying to lower my hand, and when it is trying to come, you know, I am just taking it, so through my hand, I am applying nothing but noise. You see why that is called noise. Because whatever I'm applying is at a much higher frequency than the frequency at which it is falling. That's what I'm terming, you know, the excitation I'm giving with my hand as noise. You see that this kind of stabilization is called noise stabilization. This is an example; not only might this be a simple example, but it can also be utilized. You know, sometimes you see that the structures are subjected to earthquake excitations, especially vertical excitation. Okay. So, both types are horizontal as well as vertical. So, consider a system that behaves like an inverted pendulum. Consider a water tank where there is a huge mass stage. Yeah. Is supported by stretching right, and then lateral excitation is coming, but at the same time, vertical excitation is also coming. What will the behavior of this be? Vertical excitation is taken as two-thirds of the horizontal right. It can have a combination of recorded ground motion axes; all of you are doing earthquake engineering, right? So, you know that there can be three components: right, north, south, east to west, and then a vertical one, right? So, when we try to analyze this with a simple example, when we go bicycling, what kind of system is that? This is nothing but a noise-stabilized system. A baby cannot walk, but a human can. Right? Do you see that? So, all of these, you know, I mean there are other examples of noise stabilization. It is very important in physics to distinguish between very different kinds of systems, such as classical systems and others; you know there are systems in physics that use noise stabilization to effectively utilize the noise stabilization. So, it has, you know, other than practical application, it has physical, you know, I mean, this physics is very interesting. Okay, I'm assuming that this is excited by $A\cos(\omega t)$, right? This length is L, and I'm considering that it is basically much smaller than l, right? Okay. Well, then — if you write down the equation of motion, the best approach is to use the Lagrangian or Hamiltonian formalism. If we define this to be the origin, I can take this as x and this as y; then how will we define the x and y coordinates, right? Capital X and capital Y are to be what? $L\sin\theta$. This is L; $L\sin\theta$ is X, and Y is how much? $L\sin\theta + Y\cos\theta$. Whatever, right? Y is the vertical deflection that you are going to give. We have taken Y as equal to $A\cos(\omega t)$, right? So, then what is $\dot{x}$? $\dot{x}$ is $L\cos\theta\dot{\theta}$, and $\dot{Y} = -l\sin\theta\dot{\theta} + \dot{Y}$. Right? So, we calculate velocity $v^2 = \dot{x}^2 + \dot{y}^2$. You have to calculate velocity because when you apply the principle of Lagrangian. Lagrangian is kinetic energy minus potential energy $L = T - \pi$, okay. So, $v^2 = \dot{x}^2 + \dot{y}^2$.

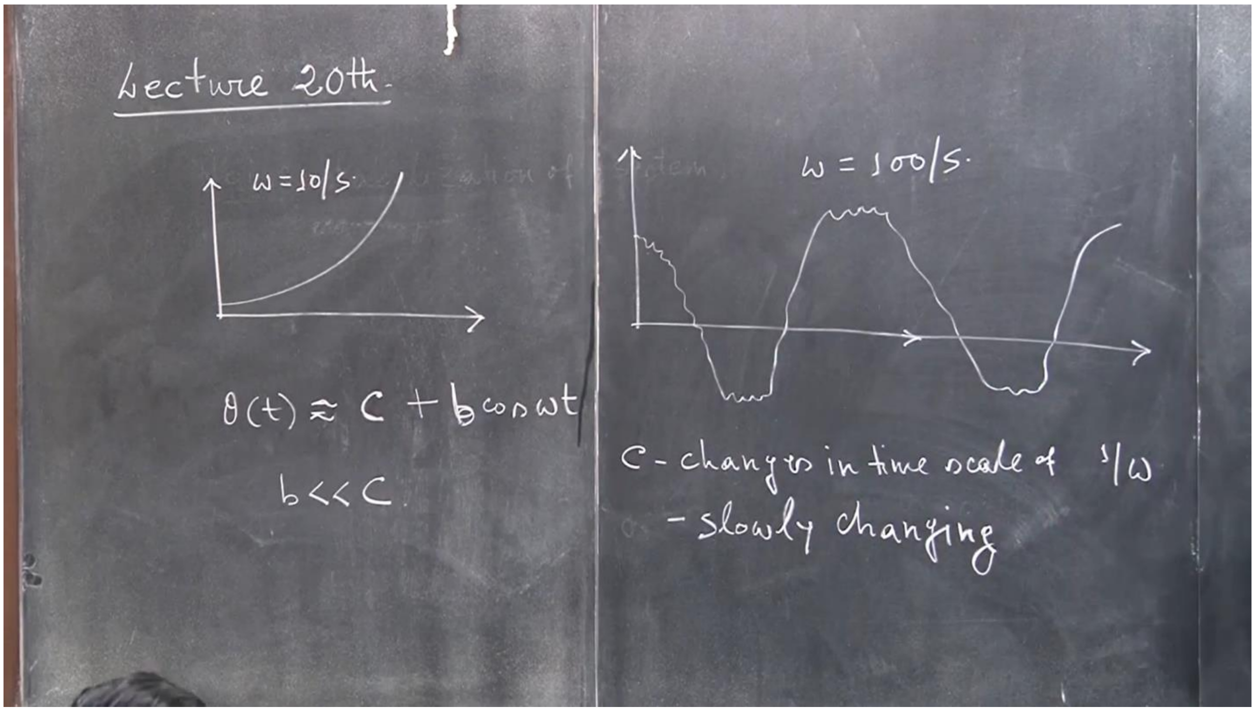


Then here you will get, if you simplify $(l^2\dot{\theta}^2 + \dot{Y}^2 - 2l\dot{Y}\dot{\theta}\sin\theta)$, when I'm writing the Lagrangian. So, $\frac{1}{2}mv^2$, where it means this one, $(L^2\dot{\theta}^2 + \dot{Y}^2 - 2L\dot{Y}\dot{\theta}\sin\theta) - mg(Y + l\cos\theta)$. You see that potential energy is nothing, mg, and then vertical displacement $mg(Y + l\cos\theta)$, right? So, this is $\frac{1}{2}mv^2$, you know, kinetic energy minus the potential energy. Okay, then you write down the equation of motion. What is the Lagrangian equation? The equation of motion is nothing but $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$. Right, we are not considering external excitations, damping, or any non-conservative forces. We are focusing only on the ideal conservative system. Okay, there can be damping and other factors, but that is not what we are concentrating on. So, if you just do it, then you will see that the way you will get this is from here: you will get the equation of motion to be $l\ddot{\theta} - \ddot{Y}\sin\theta = g\sin\theta$. Okay. And then, if you substitute $L\ddot{\theta} - \ddot{Y}\sin\theta = g\sin\theta$, now you substitute y equals what? y is nothing but $A\cos(\omega t)$, right? So, you know $\ddot{Y} = -A\omega^2\cos(\omega t)$. And that if you substitute, you'll ultimately get, by simplifying, $\ddot{\theta} + \sin(\theta)\left(\frac{A\omega^2\cos(\omega t)}{l} - \frac{g}{L}\right) = 0$, okay? So, I will just remove the top portion now, and I'll finally see the equation of motion, you know, and then we'll see what is going on. So ultimately, the equation of motion $\ddot{\theta} + \sin\theta\left(-\frac{g}{l} + \frac{A\omega^2\cos(\omega t)}{l}\right) = 0$.

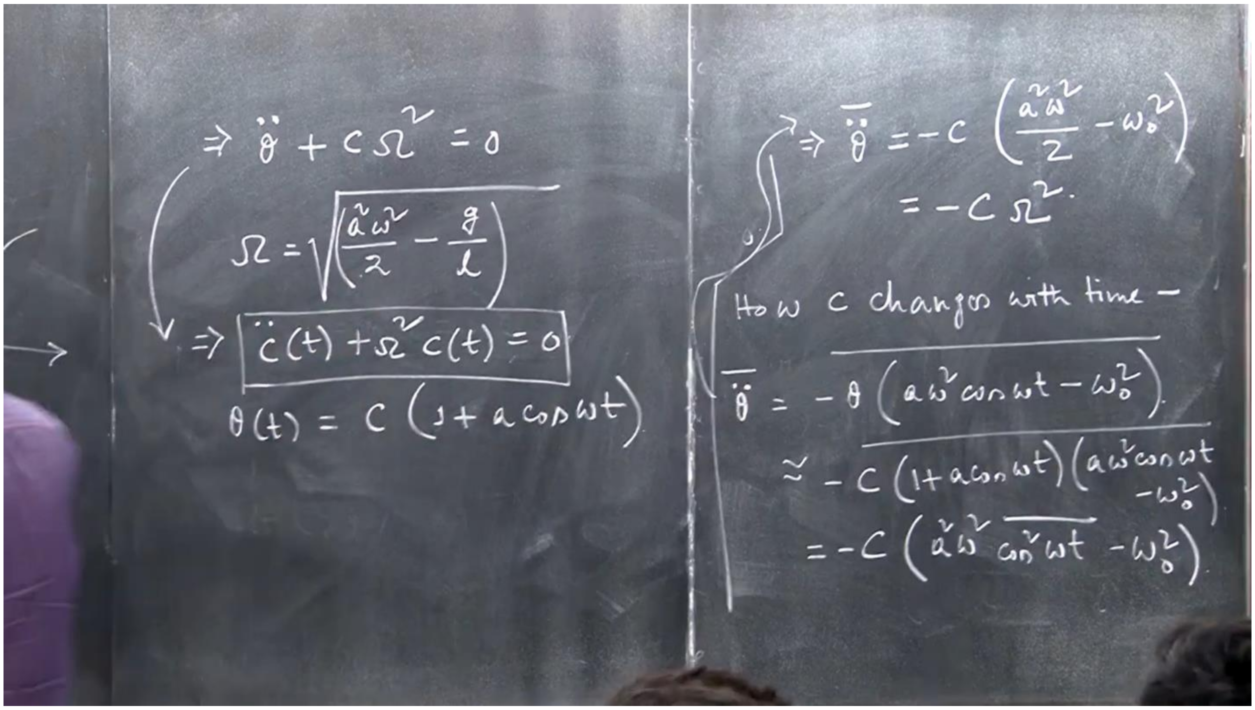


So, if you consider the pendulum, you can recall that in a simple pendulum, g divided by L is basically the natural frequency squared, correct? Effectively, what happens is that when we apply a vertical excitation to the base with amplitude $A\cos(\omega t)$, the effective vertical acceleration experienced by the pendulum is $-A\omega^2\cos(\omega t)$. This acceleration is counteracting the gravitational acceleration, so the effective acceleration that is coming is like this and is a function of nothing but this excitation frequency ωt, right? Fine. So, it's a reduced acceleration if you can recall $\omega_0 = \sqrt{g/l}$, right? If you know the frequency of the pendulum is how much $\sqrt{g/l}$ times the period. So ω_0 is $\sqrt{g/l}$, right? ω_0^2, right? Here I will introduce some dimensionless quantities. I'll introduce ω_0 as the square root of g divided by l, and I will write small a as equal to capital A divided by L. So, these are dimensionless quantities for simplifying the analysis. So, then, if we do, this equation ultimately simplifies to $\ddot{\theta} + \theta(a\omega^2\cos(\omega t) - \omega_0^2) = 0$. So, this is the ultimate equation. So here we will see the interplay between ω_0 and ω. So, we know that if ω is large enough, the stick will not fall, right? That means ω is much greater than ω_0. So, you know ω_0 looks like this, and ω dot will look like, you know, like this. What I'm trying to say is that the slow cycle corresponds to ω_0, while the fast oscillation is ω. This fast oscillation is what is termed "noise," whereas ω_0 is the signal. So, we can solve the equation by substituting, we can adapt various values of g, L, and different values of ω. I will show you the behavior over time with two different values of ω. Okay.

This is for ω, which is equal to, you know, 10 per second, and this one is for the other case. This is for ω equal to 100 per second. You see that in one case it is diverging, but in another case, it is trying to go, and whenever it tries to diverge, the noise stabilizes it. And it is compatible with the fact that if ω is very large, then the stick will not fall, correct? So, we can do a little analysis, and I won't go into very much detail, but the basic thing I will show is this: let us take $\theta(t)$; we want to find a solution for $\theta(t) = C + b\cos(\omega t)$.

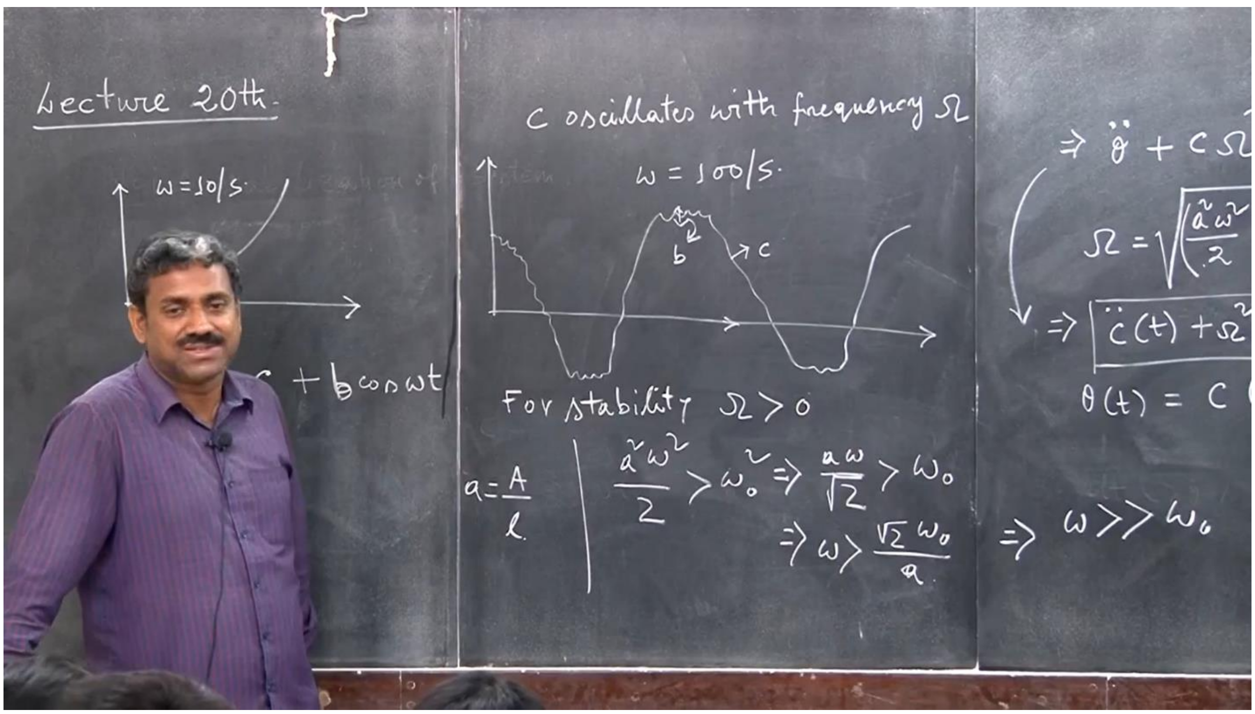


You can see $\theta(t) = c + B\cos(\omega t)$, where B is the amplitude of this small capital ω, and c is basically a function of t. We can assume this kind of solution. Okay. C is the small, slow cycle, and then ω is the one that is basically modulating it. Okay. We can assume that B is much smaller than C. This makes sense because C represents the slow oscillation, while B is the small, fast oscillation riding on top of it. So, that's why B is much smaller than C, which is justified. And another thing, C basically changes on a timescale of $1/\omega$. Okay. Since ω is very high, $1/\omega$ is very small, so that's why C(t) changes slowly. That means it is changing slowly. So then, that is the kind of solution that we see. Of course, this is a function of t. Please note that, huh? Now, if you substitute into the equation, we will get $-B\omega^2\cos(\omega t) + ca\omega^2\cos(\omega t) = 0$. If we find the leading order term from there, what we obtain is small $b =$ small a times capital C, okay? Small a is a small number, right?



So, from there we can write that $\theta(t) = c(1 + a\cos\omega t)$. Okay, we'll see how C changes with time. So, we'll try to find out the average acceleration. When I put a bar over it, that means it is average. You understand that, right? In statistics, we put a bar to find the mean, right? So, we average over a cycle. So $\overline{\ddot{\theta}(t)} = -\overline{\theta(a\omega^2\cos\omega t - \omega_0^2)}$

And this is equal to $-\overline{c(1 + a\cos\omega t)(a\omega^2\cos\omega t - \omega_0^2)}$, and there is also a bar, okay, and then it will be nothing but $-\overline{c(a^2\omega^2\cos^2\omega t - \omega_0^2)}$. this is $-\overline{C}$. Now $\cos^2\omega t$ if you just sum it up. Okay the average of $\cos^2\omega t$, taking us to be one. Okay if you integrate it over a cycle right, integration of $\cos^2\omega t$. You understand what I'm trying to do, right? Bar means I am integrating this over a complete cycle, okay? Then you will get $\left(\frac{a^2\omega^2}{2} - \omega_0^2\right)$, and that means it's nothing, nothing $-\overline{C\Omega^2}$. So, from here, what we see is that the motion will be $\ddot{C} + c\omega^2 = 0$. Where capital Ω is nothing but the square root of $\sqrt{\left(\frac{a^2\omega^2}{2} - \frac{g}{l}\right)}$. From here, we write that $\ddot{C}(t) + \omega^2 c(t) = 0$, which is basically the equation for the slowly varying component. So, this is the equation for the slowly varying component, okay? The high frequencies are written, okay? So, the slowly varying component oscillates at the frequency of capital Ω. So, the slowly varying component, C, oscillates with frequency ω, right? That is nothing but how ω is defined: $\sqrt{\frac{a^2\omega^2}{2} - \omega_0^2}$. Okay, from here you can clearly get the condition. See when ω will have some value if $\frac{a^2\omega^2}{2} > \omega_0^2$, right? So, for stability, ω must be greater than zero, and that is only possible when $\frac{\omega^2}{2} > \omega_0^2$. That means $\frac{a\omega}{\sqrt{2}} > \omega_0$ or $\omega > \sqrt{2}\omega_0/a$. Now, what is a? a is nothing but capital A divided by L; that is, $a = A/L$. Since L is much greater than A, a is a very small quantity. So, what is the excitation frequency that you should apply from below that should be much greater than whatever the slowly varying frequency is? By rotating, this means these conditions must be satisfied; only then will the system be stabilized.



So, this represents the condition for stability. From this, we can say that essentially, ω_0 corresponds to the frequency of the slowly varying component. The system will remain stable if the excitation or noise frequency applied to it is much higher, while CA remains a small quantity. So, that is much, much greater than ω_0. See, ω_0 divided by small a; small a is a small quantity. So, you can see that as ω increases further, this effect becomes more pronounced. If we remove the term involving a, then when ω is greater than ω_0, it is clear that ω must be much greater than ω_0. You see that because a is a small quantity, right? Clear. So that is the condition. So, I mean qualitative; if you want to say that, it means your vertical excitation should be large enough, and the frequency should be large enough to basically prevent it from falling laterally. Okay. And because it is so high, it means that what you see is a slowly varying component, which basically means that B is much smaller than C. So, the term $B\cos(\omega t)$ represents a small component. Here, ω is much greater than zero, and its amplitude is B. This means that the amplitude B is relatively small, corresponding to a high-frequency component with frequency ω. The capital C represents the overall amplitude of the system. So, what is happening when it tries to destabilize you is that this high frequency is basically stabilizing it by overriding the slowly varying component. Similarly, if the response diverges in this manner, the system becomes unstable. However, at higher frequencies, the response is stabilized, as the motion is brought back to a steady level. In this way, the high-frequency component helps the system maintain stability. So, you now have an understanding of noise stabilization. The explanation I have given is by no means comprehensive, but it provides a simple and intuitive example. Nevertheless, you can see why this phenomenon is referred to as noise stabilization. We are calling it noise because this is a very high frequency, comparing anything that varies slowly with a very high frequency, as low amplitude is termed noise when compared to the main signal. So, the main response is that there is a slowly varying component on top of which this noise is riding, high frequency and small amplitude. Okay, this is noise stabilization. Okay, so, uh, thank you very much for today's class. In the next class, we will cover a bit more about dynamics and stability. I will introduce the concept of Lyapunov stability. However, there is a lot to cover—an entire class could be devoted to dynamic instability. There is so much material in there, not only in civil engineering systems but also in fluid dynamics and more. We don't want to bore you with that, so in the next class, we'll cover a little bit, and then we'll go to the last chapter, which is on the buckling of cells. Thank you very much for today's session.