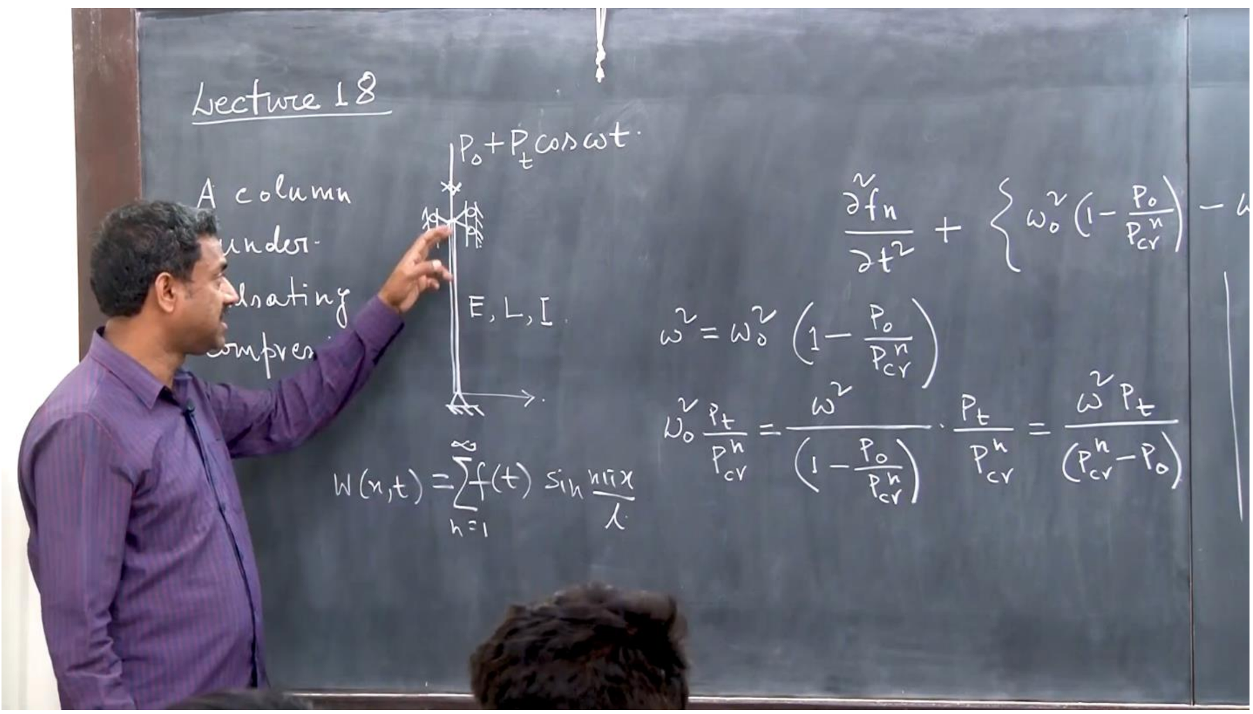
Stability of structure

Prof: Sudib Kumar Mishra
Department of Civil Engineering
IIT KANPUR

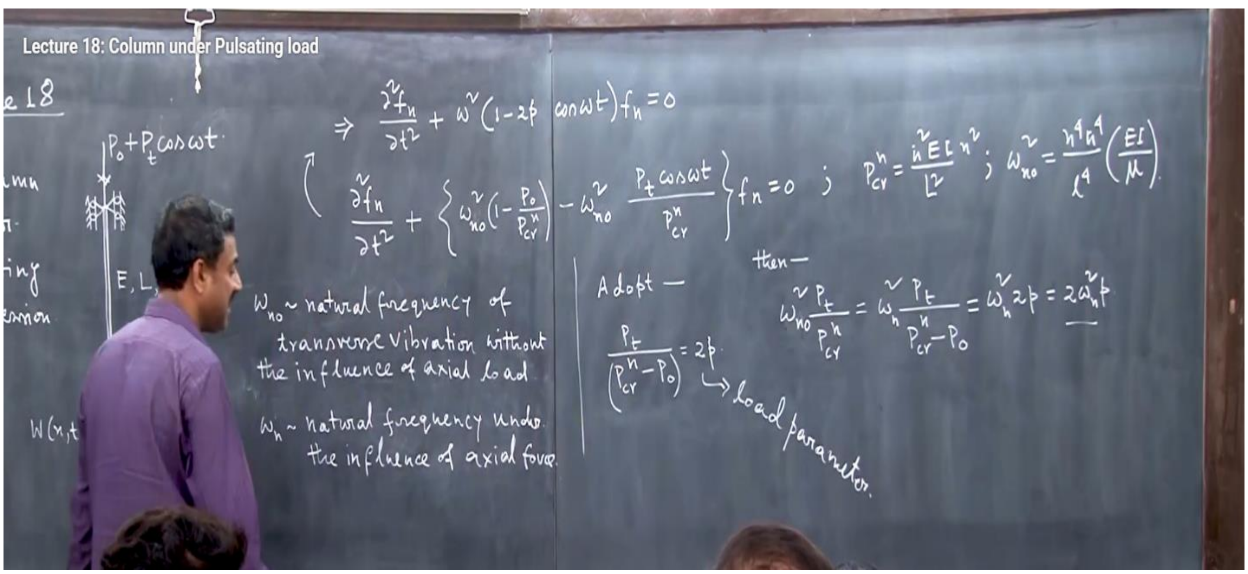
WEEK-09

Lecture 18: Column under Pulsating load

Welcome to lecture 18. So, what are we discussing? We are discussing a column under pulsating load and compressive load. So, consider a column; we assume, for simplification, that it is simply supported and allowed to deform in the axial direction.



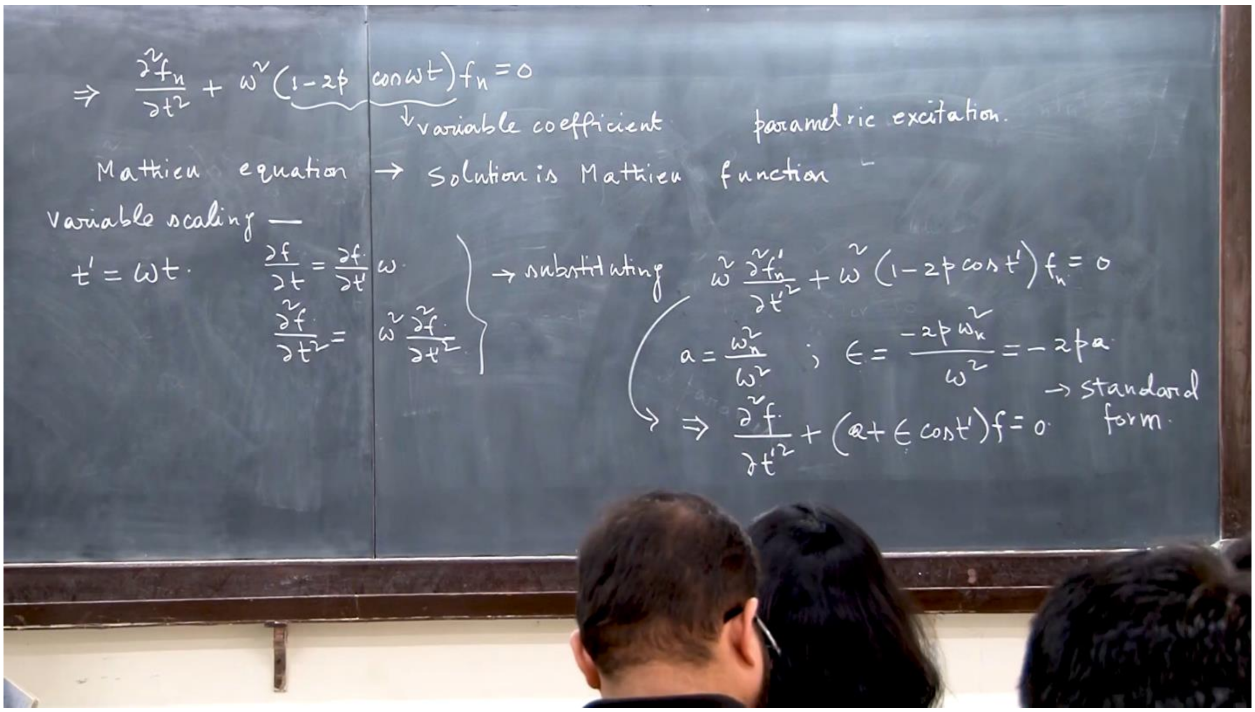
So, it is a static component P_0 and a dynamic component, that is P_t, right? $\cos(\omega t)$, ω is the loading frequency. If you write down the equation of motion, of course, that we have done in the previous class. We have to consider inertia and then the stiffness, elastic stiffness that is given by $EI\frac{d^4W}{dx^4}$, and then because of the axial force, there will be, you know, $P_0 + P_t\cos(\omega t)$ multiplied by $\frac{d^2W}{dx^2}$. And with the boundary conditions in the hinged L, w is equal to 0, w is the transverse deflection, and $\frac{d^2w}{dx^2}$ is equal to 0, right? in both the L. With this thing we assume that the solution of that equation to be in the form, so $W(x,t)$ is assumed to be in the form $\sum_{n=1}^{\infty} f(t)\sin(\frac{n\pi x}{L})$, and then you know in this form, and then the summation has a number of modes we can consider. When we assume sinusoidal functions, it satisfies all the boundary conditions, right? And then we substitute it. So essentially there is a separation of variables that is automatically occurring. You know this is the time; the spatial part we have assumed the solution, so we do not have anything to do with that. We are right now interested in the, you know, temporal part; that is what we can see, okay. What we have seen is that, let us define the several parameters, which you know we define as the natural frequency; you can recall that we have defined it from there. When we arrive at that equation you know, please note that I will put N_0, natural frequency. So, please distinguish between the loading frequency ω, which is the frequency of loading, whereas when I assign a subscript n, this is the natural frequency. So, here I will, I am going to put that n_0; please note that. Okay, so here I am going to put n, and I am going to put n_0. Okay.



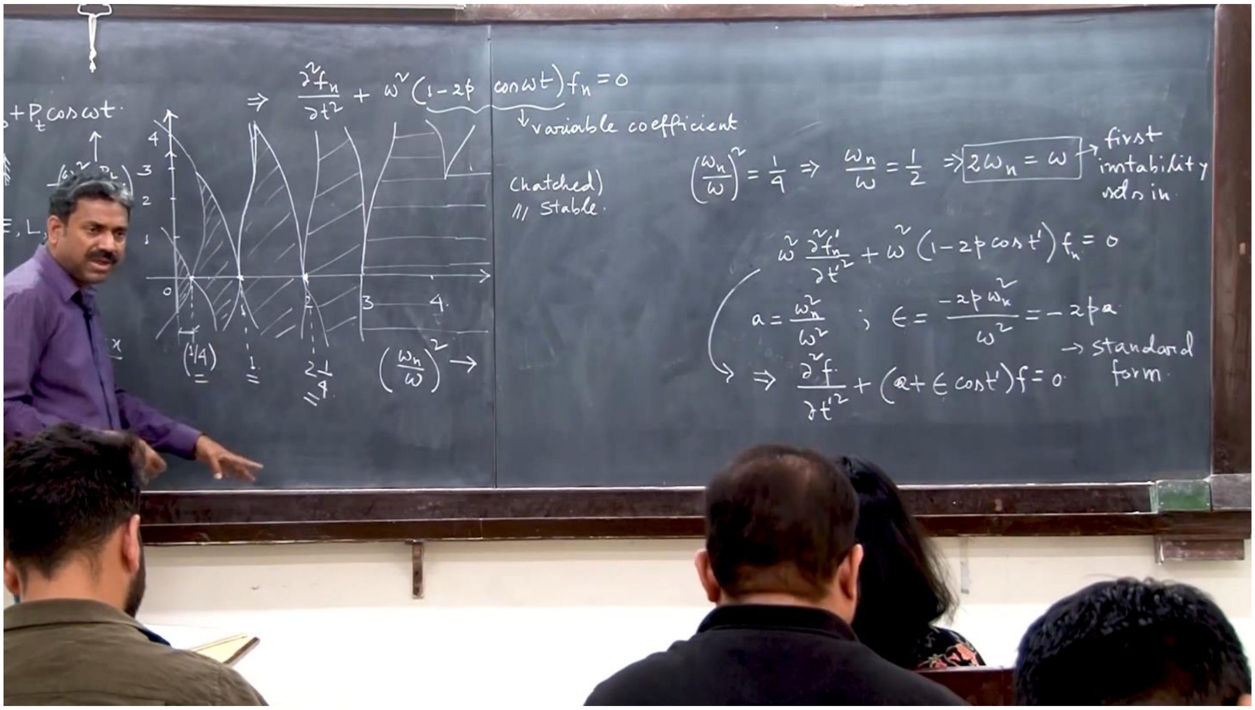
So in n_0, please see that, So we have, please recall that in the previous class we noted it down, so $P_{\text{critical } N}$ was nothing but $\frac{\pi^2 EI}{L^2}$, π^2 of course N^2 was also there, N^2, right, and that was critical loading for the n-th mode and then frequency, you know, n_0, you know, n_0^2 was written $\frac{n^4\pi^4}{L^4,}$ $\left(\frac{EI}{\mu}\right)$, μ is the mass per unit length of the column μ right, You can understand that EI/μ, so that is the natural frequency, okay, the lateral frequency of vibration. So, when I understand them, then we have defined all these quantities, please note that. When I define ω_{n0}, that is the natural frequency when not considering the effect of axial force. Please note that when there is axial force, there will be erosion in the lateral stiffness, as we have all learned. So, when I'm defining N_0 natural frequency without zero means, it is without considering the effect of axial force, right? That's what it is. However, when we are considering N, that means we have already considered the erosion P_0/P_{CR}, and do you see that? The natural frequency is under the influence of axial force. Axial force is only the static component. Clear? Then I have also normalized the other ω_{n0} p_t by P_L, something like this. And here we are defining this ratio for this is nothing but the ratio of the axial magnitude of the dynamic force. And divided by critical load minus $P_{cr} - P_0$, as is, to that of course this will be a small parameter; that is why it is too small. So, then all this simplification we have made, okay, clear? So now with all this simplification, how will this equation be further simplified? I am writing that this equation will be simplified as something like this.

$$\frac{\partial^2 f_n}{\partial t^2} + \omega^2(1 - 2p\cos\omega t)f_n = 0.$$

So that is the governing equation we have ultimately obtained. Ω is the free vibration frequency under actual static load. Ω_0^2 is the free vibration frequency without static load, and small p is the excitation parameter. I can write it down. Okay, so Ω, can we remove this thing? Remove these things. Okay. So, please write that ω_{n0} is the natural frequency of transverse vibration without the influence of axial load and ω_N is the natural frequency. Natural frequency under the influence of axial force and small p is a load parameter; small p is also a load parameter. It's a ratio of the course load parameter, but it's a critical parameter. Please note that what you can understand about what it signifies is the extent of the amplitude of the dynamic loading. Of course, it is normalized with respect to this, right? Clear? Now, what I will do is show you this equation; if you see, this equation is an ordinary differential equation. But you know it is homogeneous with variable coefficients, please note that; and also, what we have seen is that the loading itself is a parameter over there, okay? The right load magnitude itself is a system parameter; do you see that? So, the load magnitude of loading, you know, this P_t in the form of a normalized small p becomes a parameter of the system; that is why this kind of equation is called a parametric equation. Do you understand that? And excitation is called parametric excitation. So, this is parametric excitation. So, this equation can be simplified into standard form, using some modifications that I can show you. The modification is that this equation can be converted, I mean, into standard form; this is called the Matthew equation, okay, standard equation. And the solution of this equation can be expressed as the Matthew function, instead of Matthew's function. It is Matthew's equation, and it is Matthew's function, okay? Now, of course, the standard form of Matthew's equation is a little different, and we can definitely do that, you know, if you just scale the variable, okay? You can do very well in standard variable scaling, scaling of this time variable. So, the way you can do it is to define a variable τ t dot equal to tell ωt. So, the time variable multiplied by the excited, you know, ωt. This is the change in the scaling of the time variable, okay. Then with this, you can write $\partial f/\,\partial t =$ you know you can follow the chain rule of differentiation.

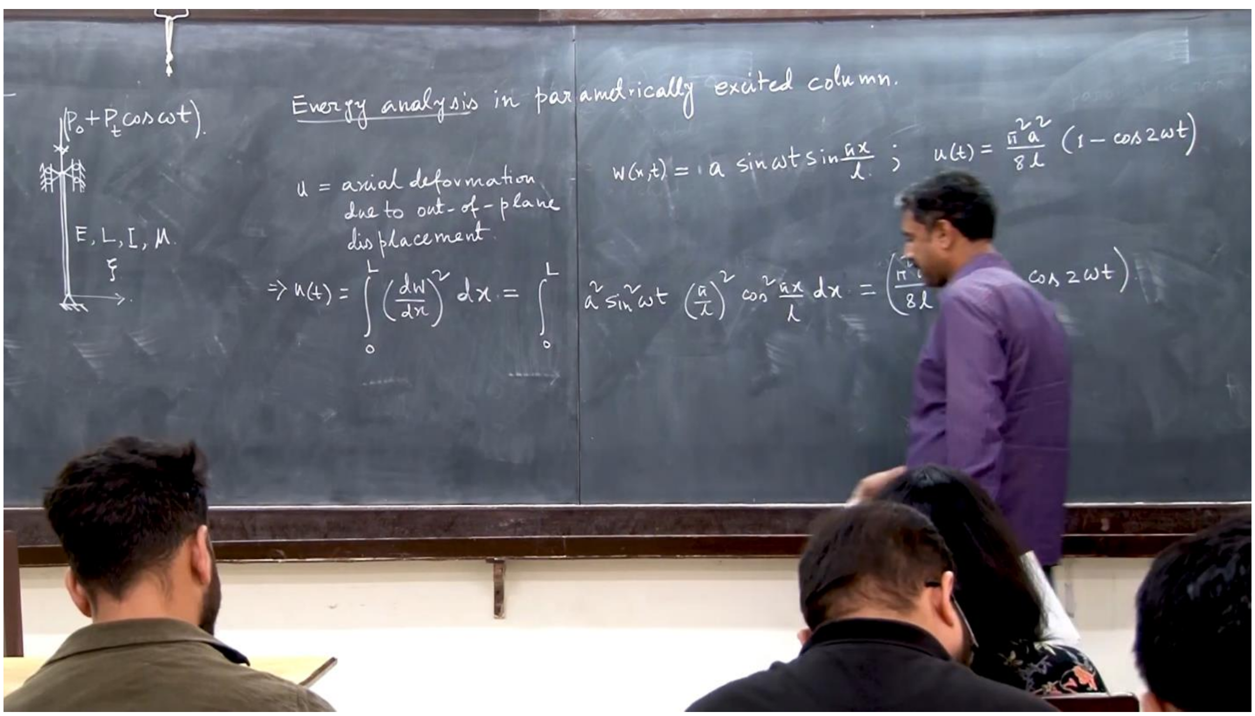


So, $\partial f/\,\partial t = \omega$, and you will see $\partial^2 f/\,\partial t^2$. I am not doing those things, but you can clearly see this is $\omega^2\,\partial^2 f/\,\partial t^2$, right? And then if you substitute this equation, we can write $\omega^2\,\partial^2 f_n/\,\partial t^2 + \omega^2(1 - 2p\cos(\omega t) f_n = 0$. I or F_N, whatever you would like to put, and then we can define some parameter A as $\omega_n^2/\omega.\,\omega_n^2$, you know, and ϵ to be $\epsilon = -2p\omega_n^2/\omega^2$; it is -2 small pa, and then this equation can be further simplified to be $\frac{\partial^2 f}{\partial t^2} + (a + \epsilon\cos\dot{t})f = 0$. So, this is a standard form of the Matthew equation if you go to MATLAB. Or any other standard mathematical software you can get the Mathieu function; they will plot it, but then the parameter will be small a, and this is the standard form of the Mathieu equation. The standard. You can clearly see that this is also Matthew's equation, just a variable scaling; the time variable has been scaled, you understand, so this is called the standard form of Matthew's equation, okay? So, when you use any mathematical standard form, fine. Now I can solve this equation; it can also be solved using the perturbation technique. Okay, so we can use point-curvilinear step perturbation. Here, as you see, since we have introduced it, we can assume F_n; we can assume asymptotic expansion, so we can understand ϵ to be a very small parameter because p is a small parameter. Then ω_n^2/ω, so treating ϵ as a small parameter and A as a constant, right? So, we can have an asymptotic expansion for both F as well as ω, you understand. Then if we do that. You know well that this one is here, so then use that asymptotic expansion and substitute it into the equation. And then equate various orders of the ϵ parameter: ϵ^0, ϵ^1. Please note that here the expansion with respect to the small parameter ϵ is already a variable in the system, and that controls the degree of non-linearity. So, then you can get the linearized equation of various orders: ϵ^0, ϵ^1, ϵ^2, and then sum everything up to get an asymptotic solution. That is possible to do, but here we are avoiding it; we are not going to do that. We know that Matthew's equation. Matthew function will give us the solution. So let us see how Matthew's function looks and from there how we can get the equation, you know, the stability condition, okay. So, for the standard Matthew function, I am going to draw it. How does it look? It is a little peculiar. Okay, so I'm going to draw it. So, you see that the hatched boundary is a stable boundary, okay, the hatched one, okay. and the hatched one is basically stable. otherwise, unstable. So, you can clearly see that, so my first instability will occur when it is, let me draw this function, that is the transition from stable to unstable, right? Then it will occur around 1, then it will be 2.14. So here, stable and unstable basically happen in $2\frac{1}{4}$ force. So here they need to occur at 1, and here 1 for 1, $\frac{1}{4}$ to 1, $2\frac{1}{4}$ force is basically the shifting from the stable to unstable. So, this diagram is, we are plotting the Matthew functions, right? This diagram has a name; it is called the Stuve diagram, okay? So higher instability lets us not care; we should concentrate on the first instability. So here, what we are plotting is basically $\left(\frac{\omega_n}{\omega}\right)^2$, okay. That's what, and on the right here, we are plotting essentially $-\left(\frac{\omega_{n0}^2}{\omega^2}\frac{P_t}{P_{crn}}\right)$. So, essentially, we are plotting this; we are plotting this. Now, whatever we are plotting does not matter; this is important. What we can see is that when the first instability sets in, when ω_n, you know, $\left(\frac{\omega_n}{\omega}\right)^2$ is essentially $1/4$. So, what does it mean that ω_n/ω is half, and then ω_n is essentially equal to $2\omega_n$ What does it mean? That means you know, this is the first instability that sets in, right? So, what is the kind of instability? The instability means that, of course, the second one happens when $\omega_n/\omega = 1$ and then $\omega/\omega_n^{1/4}$, right, a 2 then $1/4$, okay, the fraction. Now the first one is most important because that's where it is losing stability.

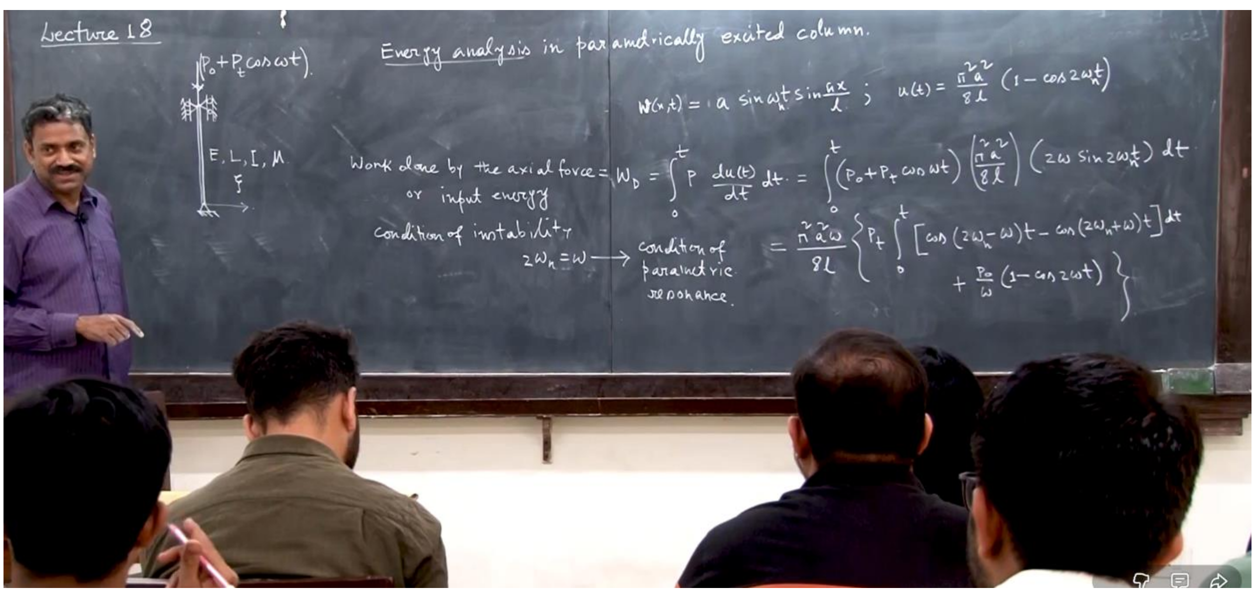


So, what is that kind of instability? Instability means the amplitude will increase. What? Unbounded, the amplitude of the vibration will be unbounded; that's the instability here. Dynamic instability means when you are compressing the column with axial forces, right? Then the vigorous vibration and lateral vibration will be way too high, right? So, under that condition, I can see that the frequency of the pulsating loading is twice the natural frequency of lateral vibration. And these natural frequencies are under the influence of the axial force, static axial force, right? What we can see then is that, well, there is a little difference, you see, from what we have seen? That ω, the loading frequency, has to be 2 times the frequency of the system, right? And this, it's still multiple but two times, what happens when $\omega = \omega_n$? We have learned that this is nothing but $\omega = \omega_n$ is the resonance, right? But this is not just any kind of resonance; this resonance occurs in a parametric system. Why is it a parametric system? Because the loading itself becomes a parameter that will govern the stiffness of the system, right? Because axial force is changing the properties of the system, this is called parametric resonance. But what is important to note here is that, under this condition, you know. Because we have assumed the axial force to be $P_0 + P_t$, you know

$\cos(\omega t)$, right? so even under that what do see, that this can take you know if you go back to here, in your, when in the expression of $\omega\omega_n$, When this vibration is occurring, the magnitude P_0 is then greater than P_{cr_n}. Meaning a column can sustain a load beyond the critical load. Okay, parametric resonance occurs when the system can because it's even, P_0 can be greater than the critical load. But in addition to that, you are getting some component dynamic load, so it can be higher than the critical load. Parametric resonance, when it occurs, may happen at loads beyond the critical load. So, the column can sustain the loading amplitude that is beyond critical. You understand that? Of course, part of it is static and part of it is dynamic, okay. So, what we see is that, this kind of instability occurs, known as parametric resonance, when the axial frequency is two times that of the natural frequency of lateral vibration; that condition you must recall, okay. We will try to establish the same thing using a different approach: energy analysis. So, you understand the Matthew equation, and then what is the Matthew function, what parameters you need, and how the stable and unstable boundaries are basically distinguished. And we have identified the point at which the transition from a stable to an unstable boundary occurs, right? We will try to establish the same thing using a different consideration and simple energy analysis. And that will be more intuitive to understand what is really happening, okay.

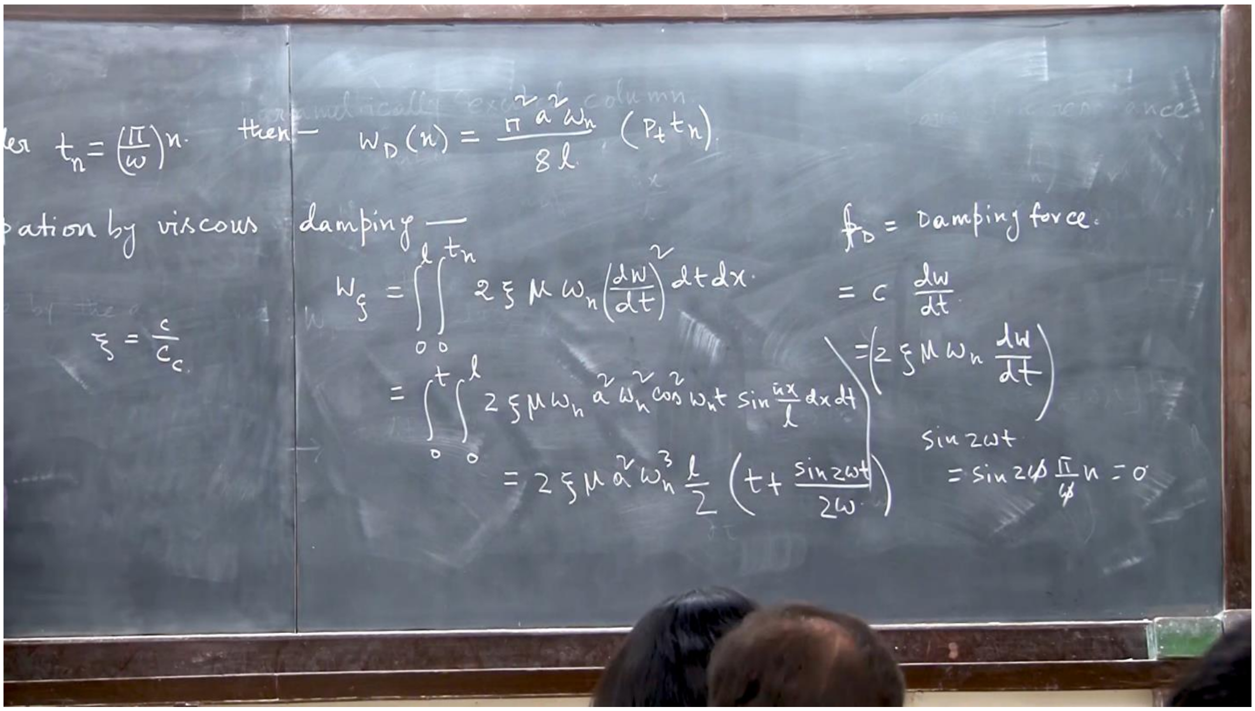


Here, please note that in this, we did not consider the effect of damping, but the effect of damping can be very easily included if we consider the energy analysis. So, I am considering inertia, of course, over there; now I am considering damping (ξ), right? we will consider sub-critical damping. So, this is we are going to do energy analysis. So, you see that here, what is the input energy to the system? So, this load P, $(P_0 + P_t\cos(\omega t))$ is working with what is the deformation U? Axial deformation is caused by out-of-plane displacement. This is what integration, Of course, $\int_0^L \left(\frac{dw}{dx}\right)^2 dx$ is a function of x and t; please note that. So, what we have assumed is that $w = f(t)\sin(n\pi x/L)$, right? Well, let us assume that $W(t)$ is basically some amplitude, maybe small $W = a\sin(\omega t) + \sin(\frac{\pi x}{l})$; I am assuming this is sin, okay, a little different from the previous. This is the expression for w, a function of x as well as t, right? So, then integrate it first to get u; of course, what you can see here is that it will be a function of t. So, then if you integrate from 0 to L, you know, $a^2\sin^2(\omega t)$ and $(\pi/l)^2\cos^2(\pi/l)dx$ right and then here it is, $\pi^2/8l$ and $1 - \cos$. Please note that here, just for simplification, we consider that, well, you know. So, this is P, you know, so $1 - \cos(2\omega t)$. So, what is the work done? Please have this expression. Okay. So, we got $u(t)$, and then $u(t)$ is, as you know, $\left(\frac{\pi^2 a^2}{8l}\right)(1 - \cos(2\omega t))$. Is the axial displacement along x? So now with this, what is the work done? What is the rate of work done?

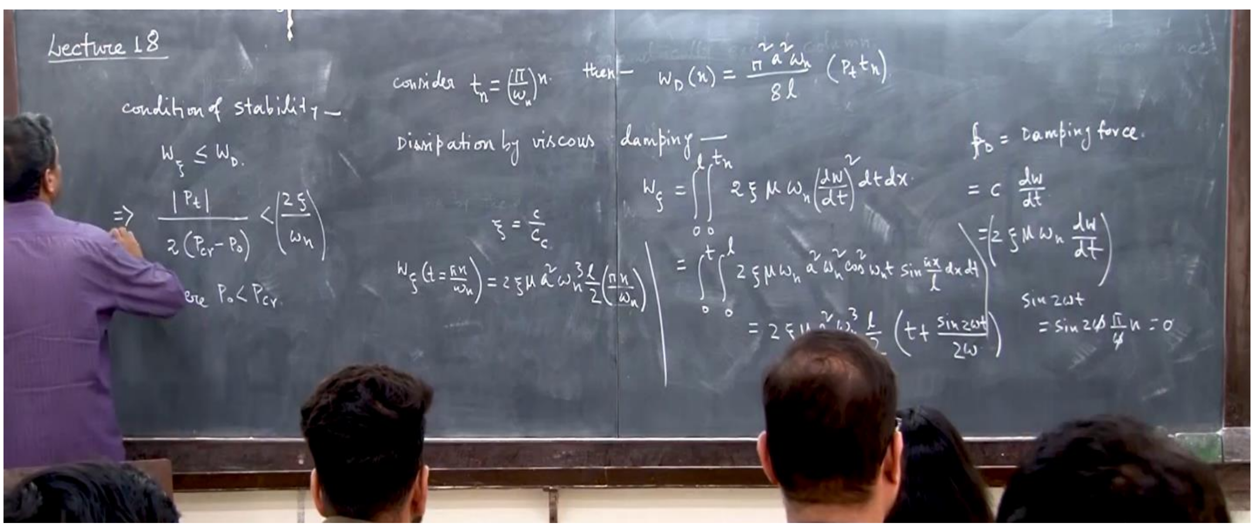


Work done over some cycle by these axial forces. What is integrating it? Of course you know, W as a function of two works done, W as a function, of course here you see that this is small w; you can take a small w, capital W okay. For the work done, I'm taking integration P, and what is the power? Force into velocity, right? Velocity means $\int_0^t \frac{du(t)}{dt} dt$, right? Then you integrate the energy, and you will get the power, right? You will get the total energy input, so the work done by the axial force or input energy over a cycle, over time, is something. So, if you integrate, then $\int_0^t (P_0 + P_t \cos(\omega t)), \frac{d}{dt}$ is $[\pi^2 a^2/8L(2\sin(2\omega t))]dt$. So, if you integrate it and things, then you'll see that π^2 I'm simplifying this expression; you integrated $\frac{\pi^2 a^2 \omega}{8L}$, you know, and then p_t integration from 0 to t $\cos(2)$. Yeah, please note that this is the excitation frequency, and I am assuming this should be a different frequency. So, I don't know what I should assume you know; I should, maybe I will put it ω_{cap}, I will put it ω_n. Okay. ω_n, ω_n please. Huh?

$P_T \int_0^t [\cos(2\omega_n - \omega)\mathrm{t} - \cos(2\omega_n + \omega)\mathrm{t}]\, dt + P_0/\omega(1 - \cos(2\omega t))$. So, what you see is that, this is under the integral. Now, please notice that, this integral is interesting. Please note that, seeing this, we will all recognize that the periodic solution $1 - \cos 2\omega t$ is nothing but $2\sin^2\omega t$, right? It's still a periodic function, right? $2\omega_n + \omega$ whatever the thing ultimately cos means, it is a periodic function, right? Now, consider a situation in which $2\omega_n = \omega$; then what will happen? Then it's $\cos 0$, which means 1, so 1 means the sum multiplied by dt. So, it's a function of t; if you integrate, and if beta becomes a function of t, do you understand that function of t means that with increasing time it will build up, build up, right? It will give the unbounded energy. So, from this and if the input energy, the work done here essentially means that you know, it is basically giving input to the system, right? That work is being stored as strain energy here. Or I mean, because it is happening dynamically, we are considering it as a kinetic energy input to the system. So, if a system cannot take unbounded kinetic energy. and it lead to it will lead to instability and that's what we can see because, so from this energy input. Do you understand how you know to, so the what is the condition of instability? The condition of instability is $2\omega_n = \omega$; that means once again, if the excitation frequency is two times the natural frequency. then it is leading to unbounded energy input to the system and instability and this is same as the condition of parametric resonance; it is identical to the condition of parametric resonance. This is a much simpler and more intuitive way to say it right. Now let us, because here, in earlier cases we didn't, find out some condition for how to dissipate this energy; of course, previously we didn't consider damping because there. We didn't have any mechanism to dissipate energy, but right now, if we consider viscous damping in the system or any form of damping in that matter, viscous is the simpler one. So, then, we will see how much energy is dissipated by viscous damping, and then we can arrive at a component condition of stability. It's not just that, okay? So let us do that. Keep in mind this W_D, this expression, okay? And now, what is the energy dissipation due to damping? Would you please tell me? Before that, we can do some simplification. We can assume that, as you can see, instead of making it in terms of T, we can consider a number of cycles, okay. And then we can count the input energy and make the integral definite instead of unbounded.

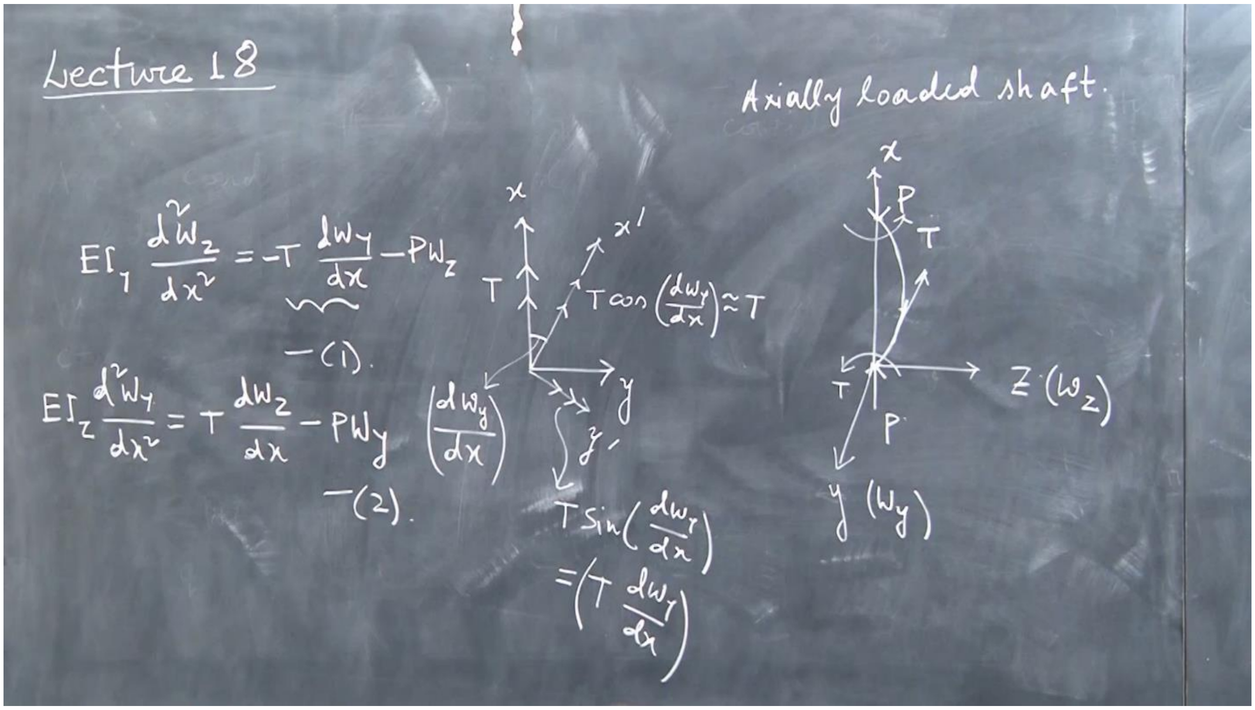


So, let us consider, you know, integrate it, consider t as π/ω. It is a half cycle into n. n is the number of half cycles; then you make the integral definite ω_D as a function of half cycle t. Then you will see that if I write down the expression, $\frac{\pi^2 a^2 \omega}{8L}(P_t T_n)$, T_n is nothing but the time duration of non-P_t. This is the input energy, okay now. So, why this is so is because we do not, I mean working with the integral oscillate will come out some. So, let us consider the number of cycles. It does not really matter; we can count how many cycles it will be stable before it becomes unstable, right? So, that dissipation by viscous damping will be what? Of course, here I will integrate what the dissipation is to ξ; ξ is the damping ratio C/C_c. The critical ratio of the damping coefficient was critical, so $2\xi\mu\omega$, this is the C, and then what? Dwell upon that, that's right; this is the expression, right? See damping force; damping force due to damping is what? C into dw/dt, and C is nothing but $2\xi\mu\omega_n dw/dt$, right? So, this is the damping force; this must be multiplied by the respective velocity. So, this will appear squared, right? And then if we integrate, you know, of course, over the $l\ dx$. So, substitute it: $\int_0^t \int_0^l 2\xi\mu\omega_n a^2\omega_n^2 \cos^2(\omega_n t)$ you know, and $\sin(\pi x/l) dx dt$. So, it is equal to $2\xi\mu a^2\omega_n^3 \frac{l}{2}(\mathrm{t} + \frac{\sin 2\omega t}{2\omega})$ And then you can substitute; once again, you can substitute t with t_n, and then you can find out, you know. So, if you substitute $t = t_n$ half sine, then t is equal to t_n. This is the $\sin(2\omega t)$. You know $\sin(2\omega t)$ means $\left(\frac{\pi}{\omega}\right)$n . ω will cancel out to $2\pi n$; $2\pi n$ will always be 0. So, $2\xi\mu\omega$ into T_n, you can write it right, so for each cycle, ok, then $\omega\xi$, $\omega\xi F(t) = \pi n/\omega_n$. This will be, you know, $2\xi\mu a^2\omega_n QL/2$, and t is $(\pi n/\omega_n)$. This is the other one, which is this. So, of course, these and these need to be balanced, right? This must be greater than that for stability, right? So, for the condition of stability, we want to derive the condition of stability. This is dissipation; this is input. So, the condition of stability will be— So, by "if you," I mean you can see that, you know, damping mark will be input energy input.

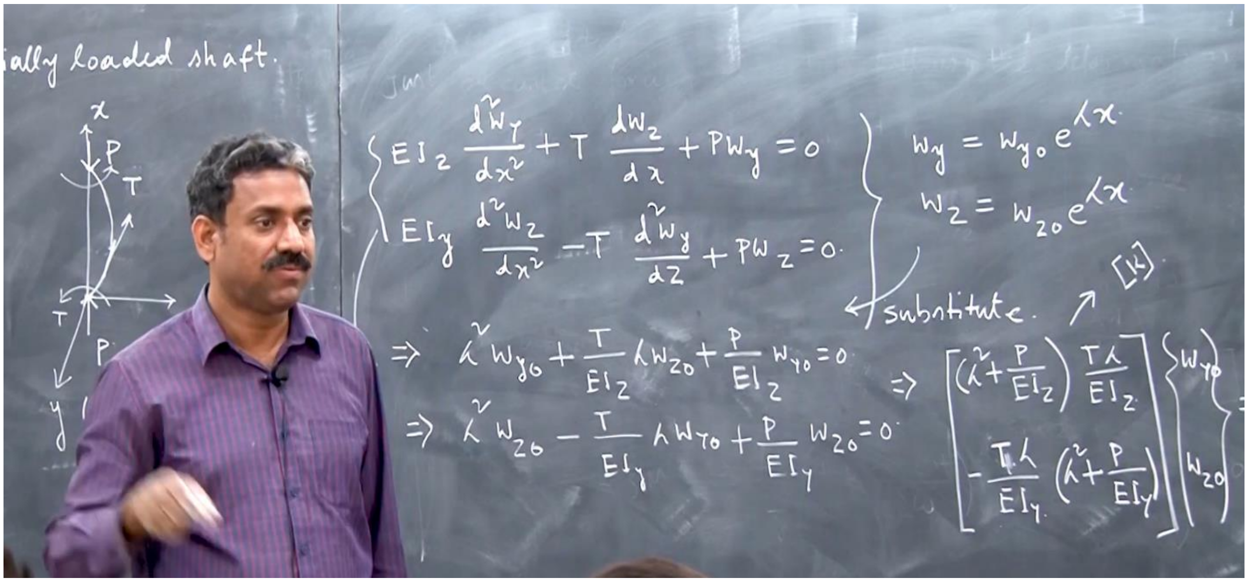


So here, if we simplify and substitute all the expressions I am writing, the final expression will be $P_t/(2P_{cr} - P_0) < (2\xi/\omega_n)$; that is the expression. P_r of course, $P < P_{cr}$. So, now this is for simple damping; you can see this is a ratio, right? P_t/P_{cr}. So, this is also a small quantity, and this is also a small quantity. So, this balance needs to be. This is for viscous damping, so you see that this expression is for viscous damping; you can exploit superelastic damping for stabilization. That's why you can put a super elastic wire and then, of course, this expression will be more complicated, okay? But you have to do that, okay? So, one thing to consider in the follower-proof ester is that there is a non-conservative force, but please note that just because the force is follower does not mean it is non-conservative, okay? For example, if you have a column here, and you consider this column, you know, this column, and maybe it is fixed with something, you know, something with another support here, okay. Support here and there. So, basically, it causes the force to always pass through this. But this is not an example of a follower force. So, what I mean is that a force which is directed in a particular direction, okay, doesn't mean that it will be non-conservative; this is not a case of a conservative force.

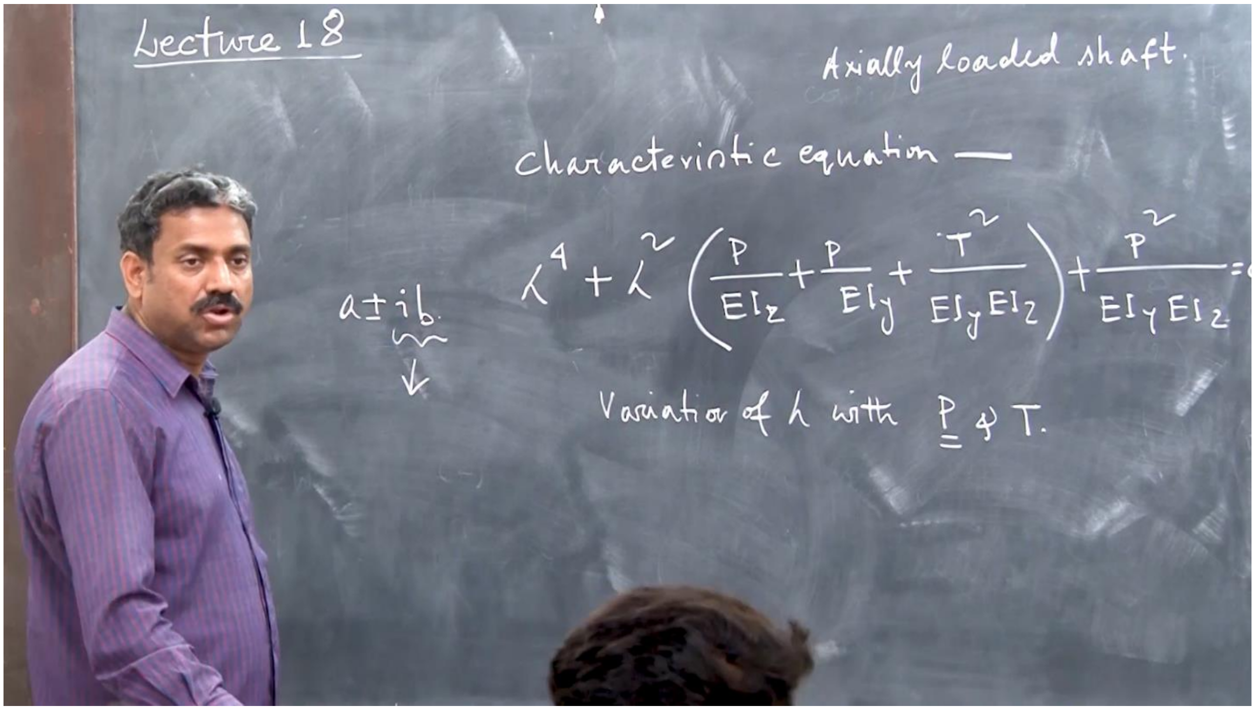
Follower force is a non-conservative force, but this force is not a non-conservative example. Just because the force is following the direction doesn't mean it is not; here, you also see if you encode and then deformation, you know, along with deformation, this fellow is following that it is non-conservative. Similarly, another example is if you consider a shell and then it is subjected to hydrostatic forces. So, hydrostatic force means it's always perpendicular to this. Right? So, whenever it is deforming under this force, whatever little deformation occurs in things, it will also be perpendicular to that. Right? So, you may think that, well, this is also a follower force, right? It is always perpendicular because it follows the deformation of the surface. Hydrostatic forces are always perpendicular to the surface, right? Isn't that so? But do you think it's a non-conservative force? No, it is not a non-conservative force because, I mean, this kind of force you can see, and it will not cause any instability in that matter. Because this system cannot pump infinite energy, this deformation will still be finite; as soon as you attain equilibrium, it will stay in its place. So, this is also not an example where instability will occur, okay fine. However, if we consider a spherical shell or any shell that matters, and it is subject to the aerodynamic force and the aerodynamic pressure is moving, you know the airflow is flowing, then it will cause instability. Why? Because that is a continuous source pumping energy, then at that time it becomes conservative, but not in a pure hydrostatic case. So, please distinguish this case from a case in which there is a, there is an, you know. And this is subjected to aerodynamic forces; some airflow under this airflow will cause instability, flutter, and other issues. That I will come later, so this is a non-conservative example; this is not. Similarly, I will also demonstrate to you some examples in which, just because force, magnitude, and direction are maintained, So these are the examples in which the forces are changing their direction on the deformation of the structure, following the deformation of the structure, but they are not non-conservative. Similarly, it should not be driven by the false notion that just because force is maintained, the direction of force remains constant; it does not ensure that it is a conservative system. I will show you this example. So, just because forcing direction follows the deformation of a system doesn't mean it is a non-conservative system, as given here. Conversely, just because the force maintains its direction does not mean that the is the force maintained, its reaction does not mean that the system is conservative. Give me an example of the second kind. So, this is a very interesting system and is often encountered not in civil engineering practice but in mechanical engineering. So, you have seen, you know the shaft, right? The shaft, which is, you know, of course, this will all, there will be bearing. And this is subjected to torque, you know; subjected to torque, right? It is under axial compression. So, what will happen in this example, which is a typical non-conservative example? Let me explain it to you. So, I am considering the situation. So, I consider a shaft like this to be okay; it will have something like this. This is the deformation subjected to axial force P here, axial force P here, right? And then it is also subjected to this kind of torque T while it is moving to the right, torque T. So, then what will happen? So, if you consider, you have to consider the section, and then you consider its, you know, of course, when you want to study stability, you have to consider the equilibrium equation in the deformed configuration. So, I am considering the equilibrium of this shaft in the deformed configuration, okay. So, what am I going to do? Now, axially loaded shaft.



And this is also called a gyroscopic system. Huh? So then, if you consider that projected in the xy plane. So, if you project, you can clearly see that if I draw a tangent here, the x-axis will change here. So, this is the X-axis, X dot, and this is the Y dot, right? And I am defining this as the torque; I am defining it with an arrow diagram, and what is this angle? What is this angle? Along the y direction, I am assuming the deflection along the y direction will be W_y, and the deflection along the z direction is expressed as W_z, right? So, if it is deflecting along Y, this means that the angle is nothing but dW_y/dx, okay? So, this angle is something like that. Now you can see, there is this; this is nothing but $T\cos(dW_y/dx)$. It is basically TdW_y/dx, right? Sorry, cos of this thing, how can it be? cos is small; it is equal to t, right? If it is very small, it is right, and then there is another component here, that component, so this component is the top, right? And this component will be bending, right? This component will be what? $T\sin(dW_y/dx)$, so, I can write dW_y/dx, right? This is the bending component, so now if I write down the equation for this, in this plane, of course this is called bending, so this is with respect to the y-axis, right? So, may I write this equation to be EI, so the deflection is happening in the y; you know deflection is going to occur if it is along y. Deflection will occur along the z direction, right? If the bending with respect to the y-axis will cause deflection, what happens? Along the z-axis, right?



So, that is what $EI_y d^2W_z/dx^2$ will be equal to. And this is y. Y will be equal to TdW_y/dx, and then along with that, because of the axial force, there will be an additional bending moment P into W_y, right? If the deflection is along z, it will be PW_y, right? So, this is the component of torque that is contributing to bending, and then this bending with respect to the z axis, so deflection will be along y, right? So, this will also be PW_z is the same, okay. This will be one equation, okay. The other equation, similarly, if you consider and project it over the z, okay, then you will get, similarly, I will project it over the xz plane instead of the xy plane; I will project everything in the xz plane. And in the xz plane, you will see the equation $EI_z d^2W_y/dx^2 = TdW_z/dx - PW_y$, equation 2. In one case, what will happen? In one case, you know about this torque, and this bending we are considering, right? So, in one case, if it is positive, you know, the bending moment, if it is along the positive Y-axis, then in another case, when you are projecting along XZ, that will be negative. So, see, M is equal to what? M is equal to EId^2w/dx^2 is equal to m. In one case, if the curvature and bending moment are along the same direction, otherwise it is negative. So, with one choice of coordinate, you will get $EId^2w/dx^2 = m$; in other words, it is minus, okay? So, this condition will cause a situation in which one of them will be positive; in both cases, one will be negative and another will be positive, okay. So now if I combine these two, at the system then what I will write is basically nothing but the choice of coordinate system, and in both the equations if I write it, it will look like $EI_z d^2W_y/dx^2 + TdW_z/dx + PW_y = 0$; in the other case, it is $EI_y d^2W_z/dx^2 - Td^2w/dz + PW_z = 0$. This kind of system of equations you will get. The interesting part here is that So, let me explain to you once again what is happening: you have your column and your shaft that are subjected to torque, and they are also subjected to axial force; we have considered their deformed configuration. And we are considering the deformed configuration; if I cut a section, I will take it in the deformed configuration. If I resolve it into components, whatever the torque is, I resolve it into two components, one considering the deformation to be small; in bending, the deformation along x is whatever the moment, which will basically be torque, right? But in respect to other axes, whatever moment is coming will be a bending moment. So, the components of those two bending moments, of course, we can clearly understand that there will be one bending moment component because of the different configuration along Z, and one will be along Y, right? So that we have considered, and then we have the right, and then there will be another component that is coming because of the axial force; because of this deflection, it will also contribute to moment P into W, right? So, in both cases, the total bending moment we are taking is nothing but the bending equation, $M = EId^2W/dx^2$, right? So, when it is bending with respect to the Z axis, then, of course, with respect to, you know, the Y axis, the deflection is along the Z direction, right? So EId^2w_z/dx^2 that is fine, and then P into Z, right? That is the contribution, and then this is the component of torque that is contributing to the moment. However, in one case, it is negative while one place is positive. Why is this happening? Because if you project in the x, y and x, z planes, in one case the bending moment and curvature will be in the same direction and in another case, it will be opposite, okay? That is why this negative sign will come, clear? So, all these details I am not going to, but I will write down the equation here, right? And when I am writing down the equation here, you can see this is the linear simultaneous equation, right? Two differential equations, simultaneous differential equations. The important part is that W_y, and if there is no time involved in this space, it is all static analysis; we can assume the solution to be ${W_y}_0 e^{\lambda x}$, and here ${W_z}_0 e^{\lambda x}$. And we assume something like that; then you substitute it here, you assume this solution and then substitute. If you substitute, then he will see that I have done it, and I will just write quickly to save time. This will lead to an equation of something like $\lambda^2 W_{y0} + \frac{T}{EI_z}\lambda W_{z0} + \frac{p}{EI_z}W_{y0} = 0$. In other cases, it will be $\lambda^2 W_{z0} - \frac{T}{EI_y}\lambda W_{y0} + \frac{P}{EI_y}W_{Z0} = 0$. So, from here, what we see is that $\begin{bmatrix} \left(\lambda^2 + \frac{P}{EI_z}\right) & \frac{T}{EI_z}\lambda \\ -\frac{T}{EI_y}\lambda & \left(\lambda^2 + \frac{P}{EI_y}\right) \end{bmatrix} \begin{Bmatrix} W_{y0} \\ W_{z0} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ Please note the equation; here there is a very interesting part: the interesting part is that you see this led to essentially what? An eigenvalue problem is homogeneous. But there is an interesting thing here: if we take this to be a stiffness matrix, of course, it can be a stiffness matrix, right? The way I mean, knowingly or unknowingly, we have considered what is part of the material stiffness matrix. Because there is E_{ij} and all this term is there, the geometric part is nothing but P, right? Do you see that the diagonal terms are positive? I mean, well, you know, λ^2, this whatever, but the off-diagonal terms, you see that here it is, you know, sorry. Off-diagonal terms, you know these are asymmetric. Do you see that? Have you seen the geometry of a stiffness matrix being asymmetric? Stiffness matrices are always symmetric, right? The point stiffness matrix can be asymmetric. The positive stiffness matrix, a conventional stiffness matrix, has several properties; one is, of course, positive definite, and another is that it needs to be symmetric. Why? Because of the matrix reciprocal theorem, if there is a positive energy flow and deformation, applying a load at the i-th point will result in the same deformation at the j-th point as applying a load at the j-th point. Why? Because there is a positive energy flow from i to j, then there is the same energy flow from j to i. When an asymmetric stiffness matrix occurs, several cases arise. The following forces, whatever I have solved, if you cast them into the finite element equation, I will tell you how to do it. Then you will see that there will be an asymmetric stiffness matrix. Here, it is also asymmetric. So, the asymmetric stiffness matrix is a manifestation of the fact that the system becomes non-conservative. This kind of situation also occurs when the system, as you know, is subjected to damage in a 3D continuum. In damage mechanics, you will sometimes see that because of damage, it loses this symmetry, and another case in an elastic system is the non-conjugability. So, this is an asymmetric stiffness matrix. I do not know in finite element analysis whether you have studied the problem where there is fluid flow, such as flow in a pipe or one-dimensional convection, including convection and species transport. So, there is a flow, and you are putting a drop of ink there. The ink will flow from this point to that point, in the direction of the flow. Will the ink ever come from that point here? No. You understand where the reversibility is broken. So, the direction of flow basically governs, and now that I don't know whether you have solved that equation, if you can recall, that will also give you a stiffness matrix which will be asymmetric, non-symmetric. Convective transport of species is also a mathematical issue; there is very elegant mathematics involved, and this problem is called a non-self-adjoint problem. All these problems which have, you know, symmetry are self-adjoint problems; you can have an adjoint defined for the system. But for the asymmetric, it will be non-self-adjoint, okay, non-adjoint, okay.



There is mathematics; please read a little bit about it. For those of you who are taking the optimization course, you might have found that you can see dynamic stability also for a system, and we also find out about self-adjoint systems. But now, when you solve this, how many eigenvalues for the characteristics and determinant will vanish? The determinant will vanish if you need it to vanish; then what does it give? This gives you the characteristic equation, and that equation will give us the eigenvalue upon solving. Now that you know the characteristic equation by determining the characteristics equation,

$$\lambda^4 + \lambda^2\left(\frac{P}{EI_z} + \frac{P}{EI_y} + \frac{T^2}{EI_y EI_z}\right) + \frac{P^2}{EI_y EI_z} = 0.$$

This will give us sets of eigenvalues and eigenvectors, right? Now, with P, you have to see the variation of λ with P, axial force, P, and of course torque. So, there are two control parameters P and T, and then you can define the stability boundary. You can clearly see that for a valid solution, if λ is, of course, because of asymmetry, it will be complex. It will appear in complex conjugate, right? And then the complex conjugate's real part should be negative, and then the imaginary part, okay? So, depending on the situation, in some places it will give you a stable solution, while in others it will give an unstable solution. We can, I will discuss this problem and how you have to play with the sets of eigenvalues. To decide whether it is stable or unstable, I will discuss that in the next class, but you understand that you will get eigenvalues from the characteristic equation. And those eigenvalues will appear in complex conjugates; if $a + ib$ is an eigenvalue, then $a - ib$ is also an eigenvalue. Now this will give you the sine and cosine time of the solution; that's fine, that's perfectly stable,'a' will play a role,'a' has to be negative for a bounded value. But if b is, you know, not positive or b vanishes, then it will not give a stable solution, right? All these things we will discuss, but what I want to point out for this system, at least, is that here you see the case in which the load is not changing with its deformable configuration structure, yet the system is non-conservative. So non-conservative loading can be identified for physical consideration of whether there is potential to pump up, you know, unbounded energy, or secondly, a very good example mathematically to check is the asymmetry of the system matrix, clear. Thank you for today.