Stability of structure

Prof: Sudib Kumar Mishra

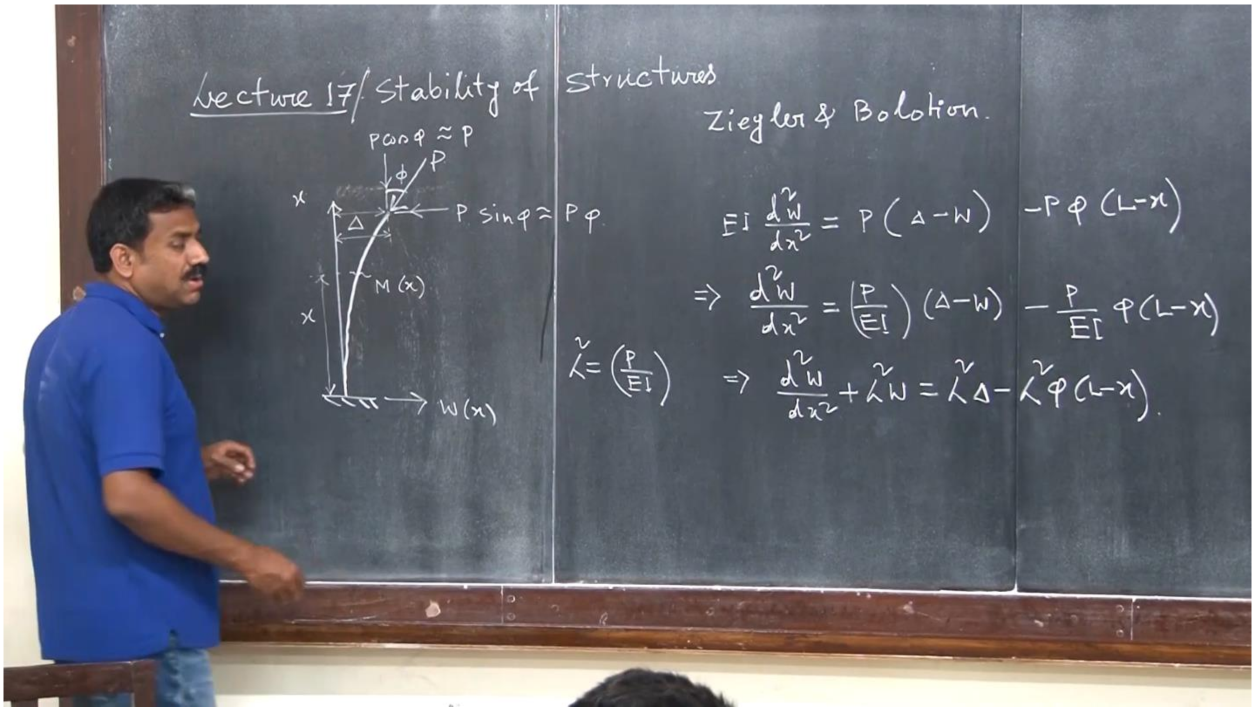
Department of Civil Engineering

IIT KANPUR

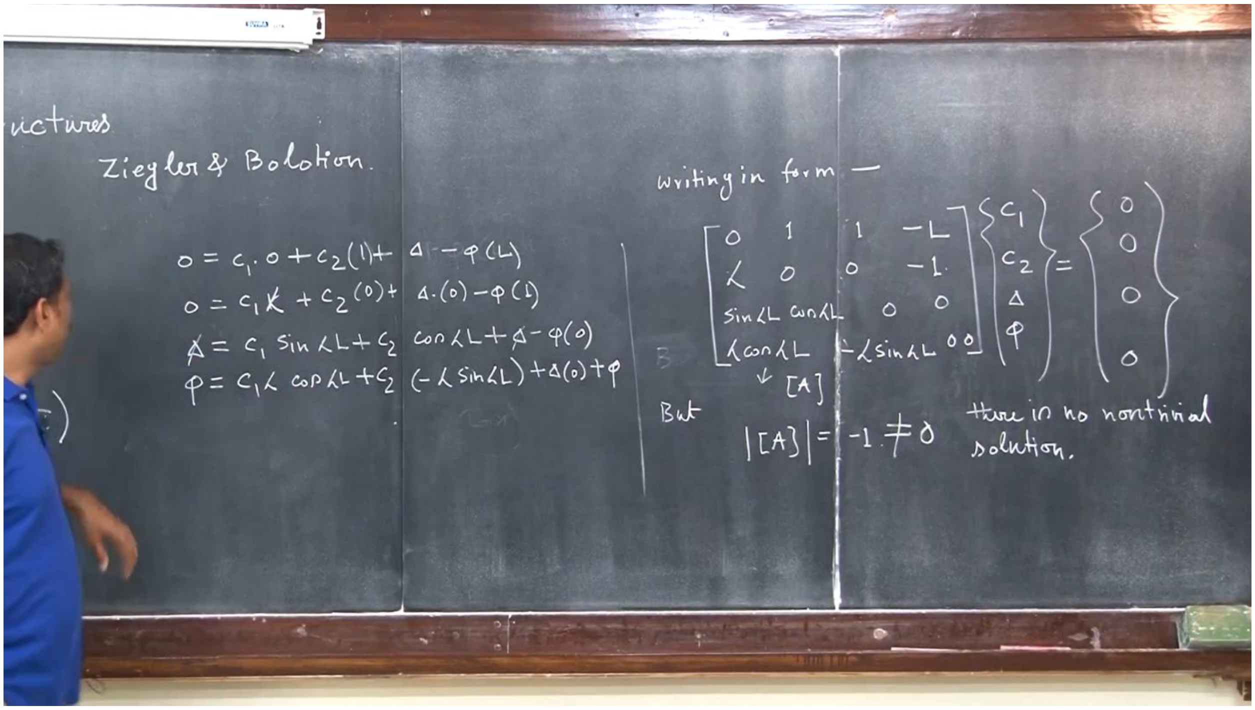
WEEK-09

Lecture 17: Column under Follower Force

Okay, welcome to Lecture 17. So, what we discussed in the previous class was the stability of structures, and then we started by introducing the concept of dynamic stability. So, the reason to study dynamic stability is that sometimes we need to assess, of course, the obvious, and the first reason is that we have to study the stability of the system under dynamic excitation, right? In the number of cases where you know the loading will be time-dependent, we cannot apply the static formalism that we have derived, either energy-based or equilibrium-based. You know, based on the equilibrium equation, we need to consider the inertia and the kinetics of the problem, right? And even before that, to start with, we can demonstrate a case. If you want to study the stability of the system using the static method, you will see that you will fail. The static method itself cannot solve the problem. So, that's what we wanted to study. And these particular classes of problems are those in which the force is non-conservative. Till now whatever we have considered all the forces you know those were conservative forces, which were not you know causing any dissipation. Okay, you see, there is a problem that we are currently looking into here. This force is non-conservative; I will explain to you why the force is non-conservative. So, here we are considering this problem; basically, many people tried to solve it, but because they were using all static methods, they were unable to solve it. Notably, the contribution to dynamic instability was made by Ziegler and Bolotin. Okay, so Bolotin has books on the dynamic stability of structures. There is a very thin book, but it is a good book. Good treatment of the subject. So, you are considering a problem. The problem we are considering is a little different from whatever we have considered previously. In all cases, we are considering stability, you know, static stability, namely the buckling of a column in which the force remained vertical all the time. But here the forces are following the tangent right. So, that's why it was a follower force, and then with this kind of choice of coordinate system, we have seen. First, let us try to solve it using the static method. We have started solving this in the previous class. So, you know if it is following this; we are defining this angle ϕ with the tangent, and then we are writing down the bending moment. At any section, at the distance X, you know that the bending moment $EI\frac{d^2w}{dx^2}$ is something like this; capital Δ is nothing but the deflection at the end, and then we basically obtain this equation. which is a linear differential equation with inhomogeneous right-hand side term. So, it will have a complementary function and a particular integral complementary function, of course, because it's a $d^2 + \lambda^2\ d$ operator plus minus $i\lambda$ complex root, right.

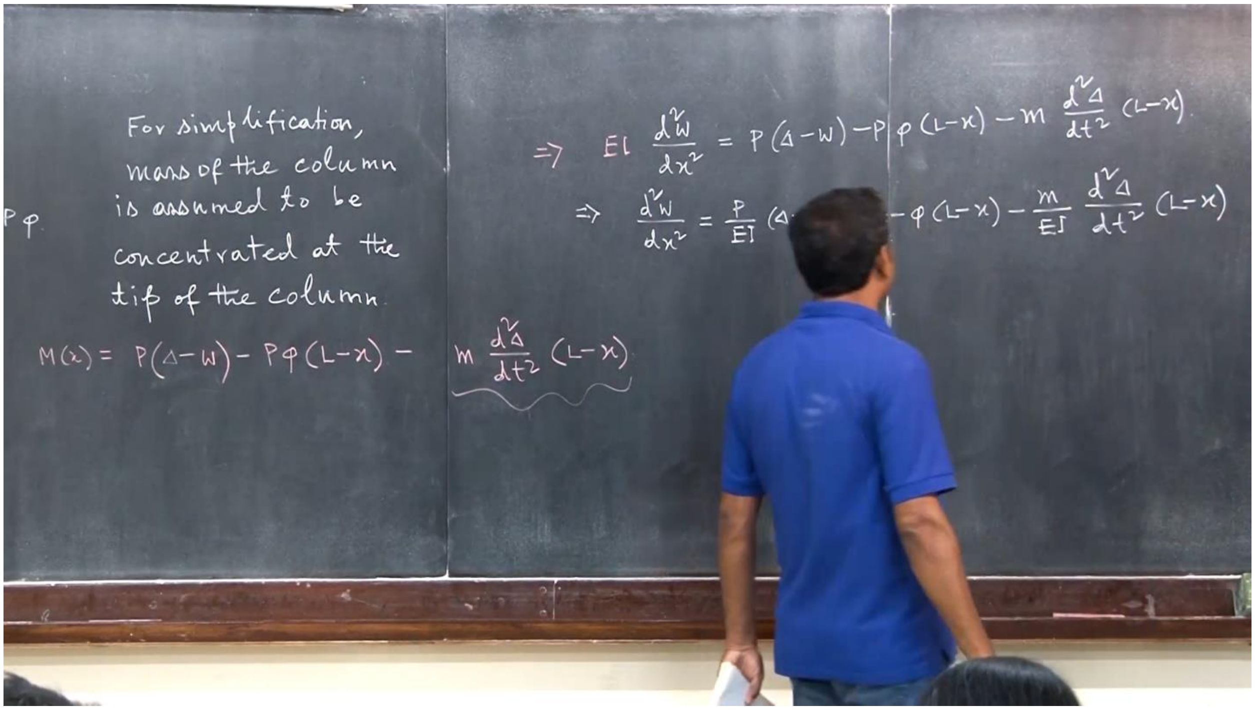


So, it will consist of sine, cosine, the complementary function, and the particular integral we have found using the differential operator, and we obtain something like this, right? And then we input the boundary condition; there will be four boundary conditions at $x = 0$: no deflection and slope is zero at $x = l$ at the end, the deflection is well, and the deflection is δ. And then the slope is, you know, ϕ, okay. So, now if we enforce all of them, of course you will see there are two c_1 and c_2 δ right. So, let us enforce this and let us obtain four equations; let me write down the equations, okay. So, as you see, first condition: if we enforce $x = 0$, I'm writing down the equation, okay? $0 = c_1 \cdot 0 + c_2 \cdot (1) + \Delta - \phi(L)$ then $0 = c_1\lambda + c_2(0) + \Delta \cdot (0) - \Phi$ this is one. Then $\delta = C_1\sin(\lambda L) + C_2\cos(\lambda L) + \Delta - \phi \cdot (0)$, and then $\phi = C_1\lambda\cos(\lambda L) + C_2$, you differentiate it, reinforcing this one. $C_2(-\lambda\sin(\lambda L)) + \Delta \cdot (0) + \phi$. So, four conditions; you know, four equations, right? Fine. You can see this δ; δ will cancel out, you know. So, δ multiplied by zero essentially gives you four simultaneous equations, and all are homogeneous systems of equations. Let me write it down in matrix form. If you write it in matrix form, you will see writing in matrix form $\begin{bmatrix} 0 & 1 & 1 & -L \\ \boldsymbol{\lambda} & 0 & 0 & -1 \\ sin\boldsymbol{\lambda}L & cos\boldsymbol{\lambda}L & 0 & 0 \\ \boldsymbol{\lambda}cos\boldsymbol{\lambda}L & -\boldsymbol{\lambda}sin\boldsymbol{\lambda}L & 0 & 0 \end{bmatrix}$,, so then we'll get to know. Okay. Although we have assumed that this is, you know, δ and this is ϕwe don't know these values; right, those are also unknown.

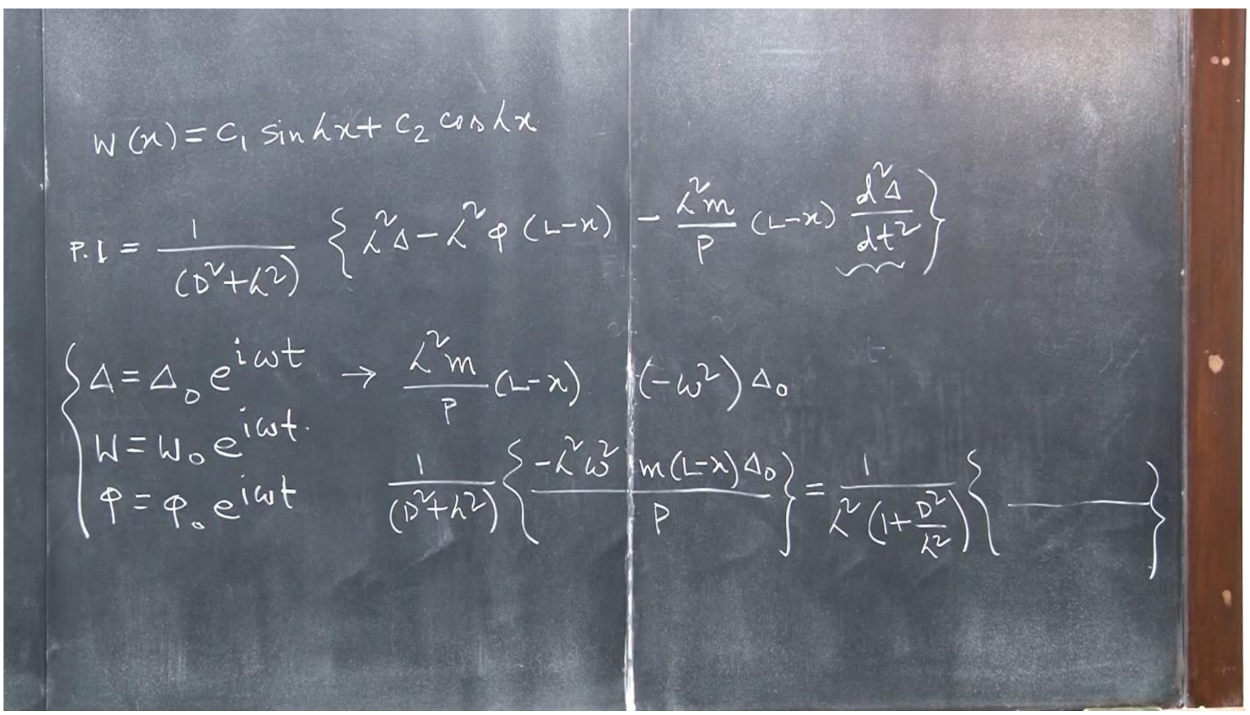


So then, of course, you know that for a non-trivial solution, the trivial solution will be that all things are zero, and in that case, that will give you the fundamental path. But because then you know the system does remain in its equilibrium position, right? Let us see if it has any non-trivial solution; for a non-trivial solution, the determinant needs to be zero. But its determinant, if I take this matrix to be A, is the determinant of this matrix A. You will see this is -1. It is never zero. For a non-trivial solution, the determinant must vanish. It is -1. What does it mean? That means there cannot be any non-trivial solution. Right? There is no non-trivial solution. What does it imply? That implies that the system cannot lose its stability. The system cannot lose it stability. Right? Because if there is no alternate configuration, if this system doesn't provide any you know non-trivial solution in perturbed configuration under follower force, it cannot lose its stability. Right? Ultimately, any stability problem gives rise to an eigenvalue problem. Right? Here, we can also see, but what are we doing? We are following, once again, the equilibrium approach; we have considered the equilibrium in the deformed configuration, right? But we see that, with this kind of approach, people were actually very confused. Because they thought that, well, I mean, then what is the problem? That means under this kind of follower force, the column will never lose stability. So, that gives a false notion you I'm writing it down. So, the fact that static method does not admit the existence of an alternative configuration. It implies that a column under follower force would never lose stability. It will never be unstable. That is a wrong notion. This is a wrong notion as well. This is a wrong notion or conclusion. The wrong notion of conclusion is arrived at by a static method. Right? Static fact many people were confused and then scientists where you know it was misleading; the static approach was misleading that was giving it never loose stability that cannot happen okay. So, the way you know that basically perplexed the scientist you know for quite several decades you know to see that this cannot lose style which is a wrong notion. Then people started applying the kinetic method, and with the kinetic method, we will see that it really gives the correct solution, which admits that there indeed exist alternate configurations. And then it can still lose stability under the follower force. Let us do that. Let us solve the problem using a dynamic method by incorporating the inertia force. Okay. And by admitting, I mean by considering the kinetics of the problem as well. And I will tell you why this kinetic or dynamism in the problem is important to consider to reach the correct conclusion. So, for that, "For simplification in dynamic analysis, we have assumed distributed elasticity." We consider the elasticity distributed, but let us assume that the mass is concentrated at its end; for simplification, the mass of the column is assumed to be concentrated at the tip of the column. So, mass is m. I'm considering the mass to be m. Okay. So, this is the mass. Okay. So, then, with this, what will happen? There will be little change. So, this will also contribute to the moment. Right? So, if you consider once again, in addition to that, whatever the force is, what the forces were, you know well earlier that there is this P force, you know. And then its horizontal component and vertical component are contributing to the moment, right? The way we write down the moment expression, you know. So, M_X that we have written as $P(\Delta - w)$, right? This one then minus $P\phi(L - X)$, right? In addition to that, these two forces are contributing, but if you consider inertia, then what will this fellow also contribute? The inertia force, if the displacement is in this direction, is also going to act like this. Inert, what is the inertia force? I am assuming this mass. So, mass m and then $\ddot{u}$, right? Mass m into $\ddot{u}$, right?

And basically $\ddot{\delta}$, huh? That's what we assume, right? So, you please have this thing in the system. Here, $-m$ is the concentrated force; $m\ddot{\delta}$ means $\frac{d^2\Delta}{dt^2}$, right?

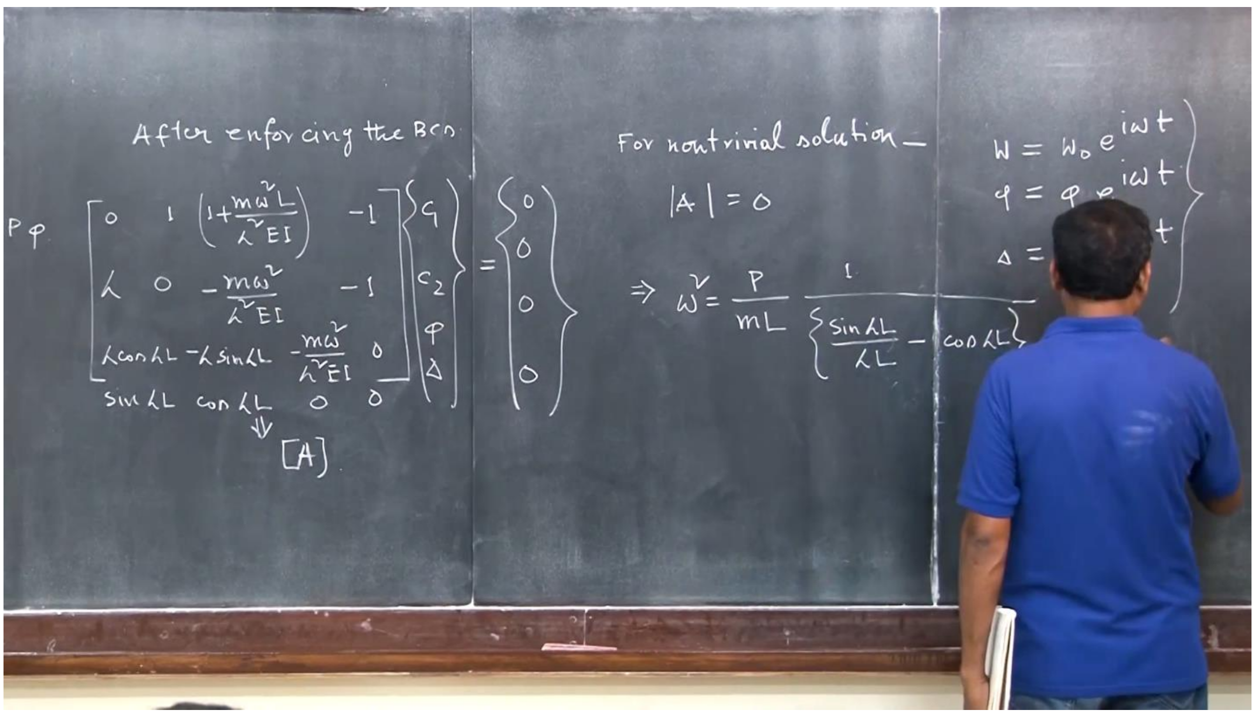


And then that is the force. So, that once again you take the moment, so then this moment will be $(l-x)$ right. So, what was the additional thing that I'm considering? I'm considering the inertia of the column, and then the inertia was due to concentrated mass that is assumed to be considered at the end. That is one kind of simplification. So, we have considered the mass to be concentrated at the tip, but the system of the column has distributed elasticity. Right? Okay. So, with this once again moment is nothing but $EI\frac{d^2w}{dx^2} = P(\Delta - w) - P\phi(l-x) - m\frac{d^2\delta}{dt^2}(l-x)$, right? Or you just write it like this: $\frac{d^2w}{dx^2} = \frac{P}{EI}(\Delta - w) - \frac{P}{EI} - \phi(l-x) - \frac{M}{EI}\frac{d^2\Delta}{dt^2}(L-x)$. Okay. like earlier the way, we have solved it, $\frac{P}{EI} = \lambda^2 w$, $\frac{d^2w}{dx^2} + \lambda^2 w = \lambda^2\Delta - \lambda^2\phi(l-x)$, this expression when you write, $-\frac{M}{EI}\frac{d^2\Delta}{dt^2}(L-x)$. So, that is the thing. Huh. Now, once again there will be one complimentary solution, and then there is a particular integral of course the boundary condition. Let us define it here. Boundary conditions will be $w(x=0) = 0$, $\frac{dw}{dx}(x=0) = 0$ you know, these are at $x = 0$ and then the other end $w(x=l) = \Delta$ and then $\frac{dw}{dx}(x=L) = \Phi$ right. Note that it's a second-order equation, so we can express the derivative until first we don't require other boundary conditions to be defined, right? You can simplify this a little bit; this expression you can simplify. You'll see that $\frac{m}{EI}$ can be written as $\frac{m}{P}$, and then $\frac{P}{EI}$, and $\frac{P}{EI}$ is nothing but λ^2, so $\lambda^2\frac{m}{p}$. So, we can write it like this. Huh? Of course, we can write down the solution now. The solution will be a complementary function, and particularly, once again, the complementary function will always be $\cos\theta$, $\sin\theta$, right? you know c_1 let me $c_1\sin(\lambda x) + c_2\cos(\lambda x)$ and then here. we can do one thing you know then what we can do you know for particular integral you know see The particular integral will be $\left(\frac{1}{D^2+\lambda^2}\right)$, and then this one is $\left\{\lambda^2\Delta - \lambda^2\phi(l-x) - \lambda^2\frac{m}{P}(L-x)\frac{d^2\Delta}{dt^2}\right\}$. So, this part is what we found out previously. So, you just take λ out and then you operate on it. Here for δ, what will we do? We can assume the solution to be $\delta = \delta_0 e^{i\omega t}$; that's what we can assume, okay? $e^{i\omega t}$, huh? Can that allow a solution? It will allow a periodic solution, right? So, similarly, if we do this, then of course from here, this term comes as $\lambda\frac{m}{p}(l-x)$, and this one, you know, makes a difference twice.

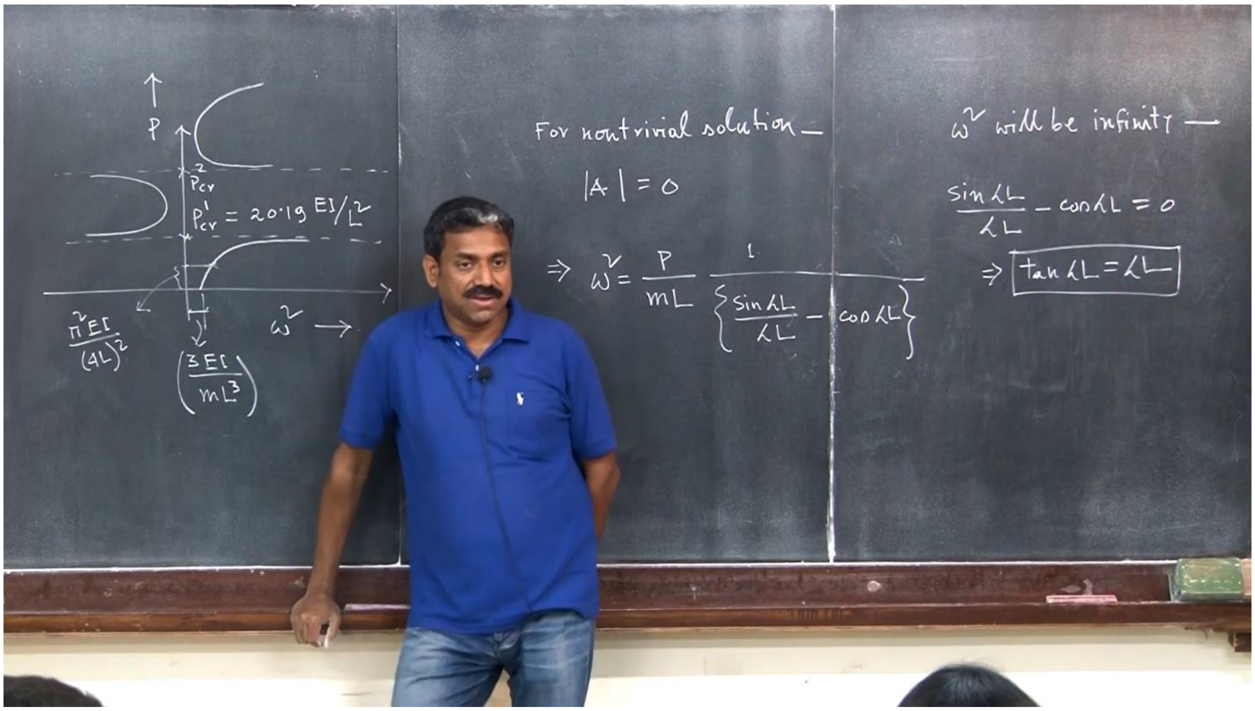


So, it will be $-\omega^2$ and Δ_0 and $e^{i\omega t}$. Okay. Fine, $e^{i\omega t}$, right? Now, see, you may wonder that, well, there is only $e^{i\omega t}$, but what you can do in the governing equation when you assume the solution is in terms of w. And there is ϕ right, so you can also assume that w also expresses $w_0e^{i\omega t}$ and $\phi\delta$.

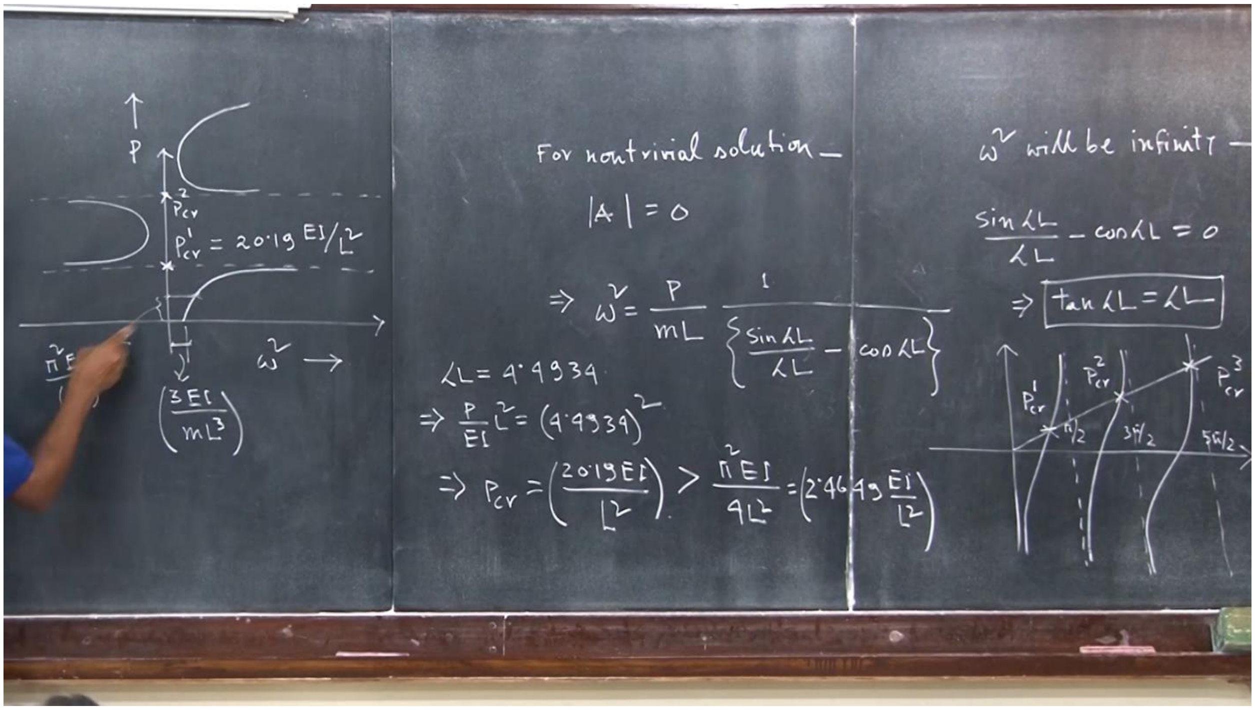
You know ϕ is also $\phi_0 e^{i\omega t}$. You see that? So, for dynamics, you know this solution can be assumed, and if you substitute $e^{i\omega t}$, the term will cancel out from both sides, right? Understand what I'm trying to say. So, what will remain here is basically this term, $\lambda^2 \frac{m}{p}(l-x)(-\omega^2)\Delta_0$, okay? And then if you operate on this, you know, $\left(\frac{1}{D^2+\lambda^2}\right)$, you know, you just operate on this, so it will be $\left\{-\lambda^2\omega^2 \frac{m(l-x)\Delta_0}{p}\right\}$, huh, we just take λ out. So, if you take λ out, $\frac{1}{\lambda^2\left(1+\frac{D^2}{\lambda^2}\right)}$ and things you know, this λ will cancel out. and then you basically take inverse of that and then you operate on this ultimately this term will remain you understand, how to find out the particular integral right.



So, I'm writing the final expression. The final expression, the solution that you are going to get, is expressed in terms. I'm removing this. So, are you okay? What, so $w(x)$ is nothing but c_1. So, after enforcing the boundary condition, you will see the equation something like this; I'm writing it in matrix form, okay? $\begin{Bmatrix} C_1 \\ P_2, \\ \phi \\ \delta \end{Bmatrix}$, and. this form right and all homogeneous boundary conditions right. So, this is equal to $\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$ and here let me write $\begin{bmatrix} 0 & 1 & (1+\frac{m\omega^2 l}{\lambda^2 EI}), & -1 \\ \lambda & 0 & -\frac{m\omega^2}{\lambda^2 EI} & -1 \\ \lambda\cos(\lambda l) & -\lambda\sin(\lambda l) & -\frac{m\omega^2}{\lambda^2 EI} & 0 \\ \sin(\lambda l) & \cos(\lambda l) & 0 & 0 \end{bmatrix}$, and this is okay. You see that this is a homogeneous system of equations. Okay. Now this is essentially what we see: the addition of this inertia force, okay, in the previous case, and then converting the problem into a dynamic problem. Okay. We have assumed that all this W ϕ, whatever the variable was, follows this, allows the periodic solution, right? You see that we have assumed this, and then we have substituted. because W is both a function of X and a function of T, right? So, then from there let us see whether it, one second, this also gives rise to an eigenvalue problem, right? So, the determinant of this thing…We need to vanish for a non-trivial solution; this determinant of A, I'm assuming this is matrix A, must be zero. Then, if you solve it, I'm just directly writing this, huh? If you solve this, then you'll see ω to be Ω^2; rather, $\omega^2 = \frac{P}{mL}\left\{\frac{1}{\frac{\sin(\lambda L)}{\lambda L} - \cos(\lambda L)}\right\}$. That is the value of ω^2. So that means, you know, well I mean although the column doesn't buckle, it loses its stability under static consideration. but dynamic consider it can lose stability for the given value of ω^2. Okay. Now So, ω depends on what? The axial force, which is the force, the frequency of vibration, and the square of the natural frequency of vibration will depend on the amount of the axial force. I mean the follower force magnitude P because λ is nothing but $\sqrt{\frac{P}{EI}}$. Now λ^2 is nothing but $\frac{P}{EI}$. So, let us see how ω^2 changes, right? So, when it does, see as soon as Ω is really right. Ω is real and then positive; you know it will give you, know it doesn't really matter, right? Ω, as soon as it is real, will give you what kind? The periodic solution and periodic solution mean it is stable, right? But if ω^2 is negative, that means ω becomes imaginary. Then what will happen? Then it will have what? It will have what? If it has this kind of value $e^{\text{something}}$ like you know if ω is imaginary $I\omega$ is $I\lambda$ then $I^2\lambda t$. $I^2\lambda$ means $-\lambda t$, right? You know $-\lambda t$ is still fine because λt means it will still decay. But it will not allow the periodic solution. But if it is $+\lambda t$, then what will happen? It will decay, right? You understand, so that will indicate some kind of divergence. I mean some kind of instability; let us see what happened to ω, okay? So, if you plot the value of ω and ω^2, then we'll see what it looks like. Okay? Let us. So, what we see is that ω^2 will be zero, and this is zero. ω^2 will not be zero. ω^2 will be infinity. When this is zero, the denominator can always be zero. Right here. you see that $\frac{\sin(\lambda L)}{\lambda L} - \cos(\lambda L) = 0$. Which does mean $\tan(\lambda L) = \lambda L$. You see that when $\tan(\lambda L) = \lambda L$, which is once again the outcome of this trigonometric value problem, right? So, what we see is that for this condition, ω^2 will be infinity. That's what you see; understand that this is increasing and going to infinity, and then suddenly there, and then from zero, see initially it is positive, then for a certain value, it will be zero. Then it will be negative.

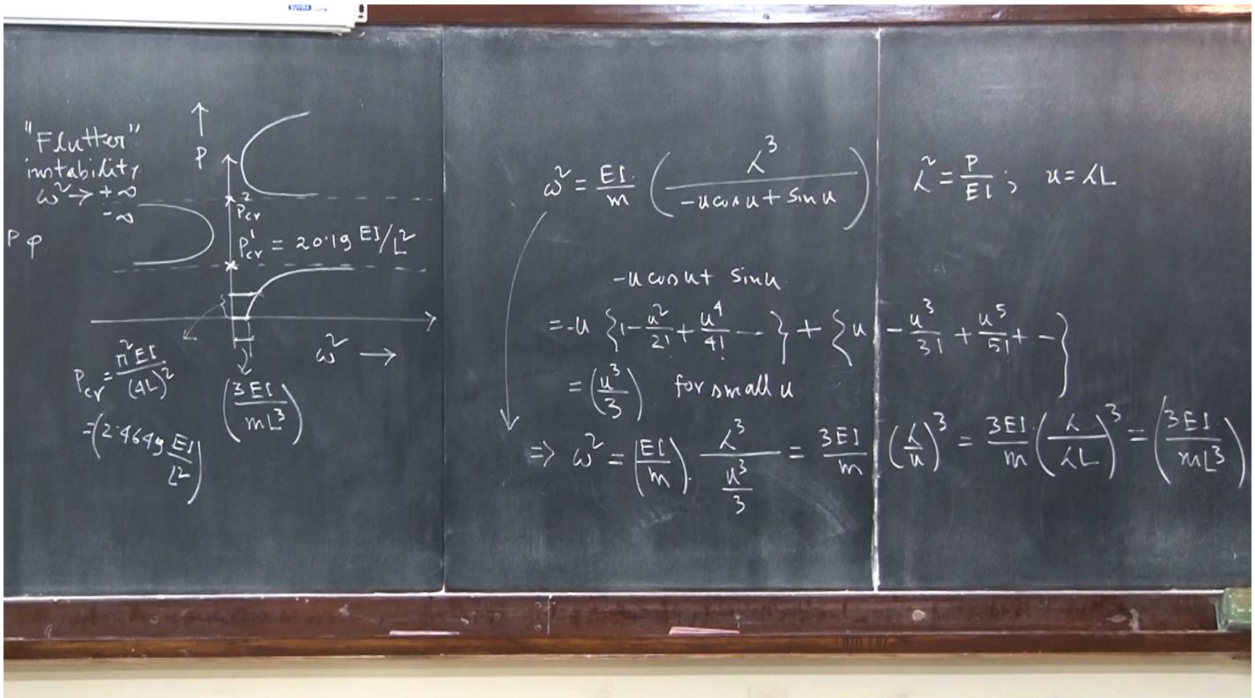


So, you see that the ω^2 is positive, then suddenly it becomes infinity. When it is infinity, $\tan(\lambda L) = \lambda L$; then it is changing like this. Then once again, it is being positive, minus infinity to it is between plus infinity, and then it is something. So, with varying P, ω^2 is changing its value; ω^2 can be positive or negative. Okay. Now, when it is going to infinity, that means it's, of course, instability, right? You understand, right? So, the first thing is, if you solve it $\tan(\lambda l) = \lambda L$, you will know how many roots there will be. So, if you can see how this tangent looks, you know the values of tangent. So, these are the tangents, right? Look, this is the kind of function tangents look like, right? isn't it? huh. So, $\tan(\frac{\pi}{2})$, $\tan(\frac{3\pi}{2})$ and $\tan(\frac{5\pi}{2})$ isn't it? That's what is the tangent this tangent function looks like this. Negative is also right. So, then λ_n, if you draw it, you know, this is the first root, this is the second root, and this is the third root that corresponds. This root will give you the first critical load $P_{\text{critical 1}}$. This will give the second critical load; this will give the third critical load, you see that, isn't it? So, the first critical load where it is infinite is from ω^2; this ω^2 is positive because this one is lesser than this one. So, it is positive; it is positive, so it is increasing; then it is being infinity, right? So, what? p is equal to 1, this value, and p_{cr1} is basically $20.\ 19\frac{EI}{l^2}$ because λL is equal to λ value, I think, 4.54. Okay. Yeah.



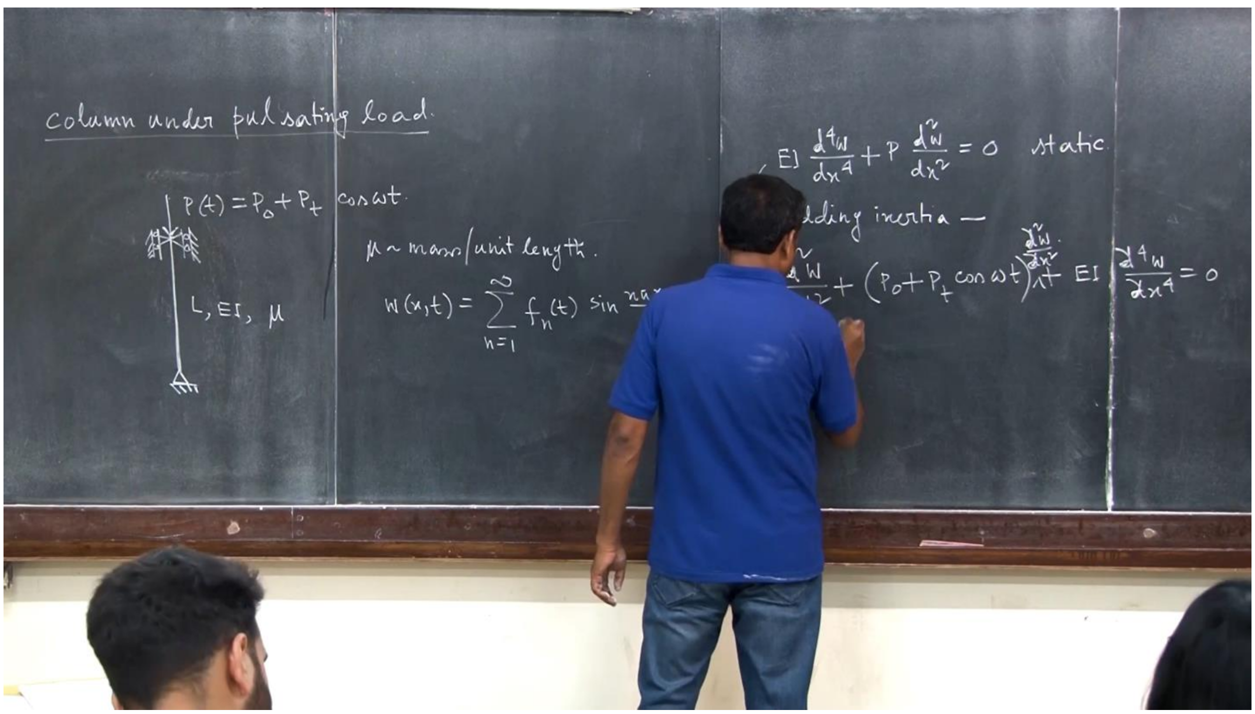
λL is basically 4.4934. Okay. And then λ^2 means $\frac{P}{EI}L^2$ is equal to whatever the square of 4.4934, and then P_{critical} is the same as $\left(20.19\frac{EI}{L^2}\right)$. That is the value; now what will happen when it is? So, this is similarly the second critical load you can also find out, okay. Now, don't you think this is much greater than whatever $\frac{\pi^2 EI}{(2L)^2}$ by $\frac{4}{L^2}$? So, mass if you know $\frac{\pi^2 E}{4L^2}$ is what? How much is that square? $\frac{E}{4L^2}$ is nothing but $\left(2.4649\frac{EI}{L^2}\right)$, right? So, it is almost like 10 times increase in the critical load. Look, you see, this is the first critical load static buckling load, $P_{\text{critical static}}$. So, this is nothing but$\left(2.4649\frac{EI}{L^2}\right)$, you see that. So, this is the dynamic load, the following load, which we get for the static case for a column that is subjected to axial compression. It's not a follower force; then the respective critical load is how much? $\frac{\pi^2 EI}{\text{effectively over } 2L \text{ to } L^2}$, so that's what is 2.4649, but this critical load is 20.1, almost 10 times increase when it is a follower force. Why is there so much increase? So, much increase is one second; let me tell you. This is because, you see, when it is follower force, then a component of it, basically, see when it is vertically, you know, compressing it, you know, then this is basically a destabilizing moment, right? But for a follower force component, this is stabilizing, you see that, right? And so is the inertia. So, inertia and the component of the follower force have a stabilizing effect on the column; that's what enhances the critical load for instability under the follower force. and that's what lead to a 10 time increase in the critical load. for the follower force, understand that secondly now you also think about the non-conerivity that what is happening That when this load is acting and this deflection is occurring, right? So, this load—see, this fellow is not working; work done is what? Force multiplied by displacement along this direction, right? Okay. If the force is a vector, then displacement is a vector. So, you take the cross product, right? If it is 90°, then $\cos 90°$ is zero. So, when it is vertically applied, no work is done. But when it is horizontal, this component is acting on this lateral deflection. So that basically is doing work, and that work is dissipating; that is non-conservative, and that's why the follower force is non-conservative. You understand why the follower force is non-conservative because a component of the follower force, which in this case is transverse to the direction, basically acts on the lateral deflection. To result in work done and thereby dissipation. That's what this follower force is a non-conservative force; that's why force is a non-conservative force, clear? Now, so of course when ω^2 is infinity, then it cannot vibrate, right? So, this kind of instability is called flutter. This kind of dynamic instability is a flutter instability. So, Flutter is a dynamic instability. Huh? ω tending to ω^2 as it approaches infinity plus ω plus infinity or minus infinity. Okay. Flutter instability and ω tending to zero are called divergence instability. That I will talk with you later. What is divergence? I see you have done it. So, you understand flutter instability and under follower force, and why it causes flutter instability. Right? Now, what is this? This is the, you know, frequency ω^2, when there is no axial force $p = 0$. And how will you get this? Well, you know, you just expand this if $\sin(\lambda)$, you know, let me explain it to you. Yeah. We are doing this kind of simplification, and then you basically do it now. You do $-u\cos u + \sin u$; expand it if you expand it, so $-u$, and then $\left\{1 - \frac{u^2}{2!} + \frac{u^4}{4!}\right\}$, you know things, and plus $\left\{u - \frac{u^3}{3!} + \frac{u^5}{5!} +\right\}$. So, from there, if you simplify, you will see that this is giving you u^3, okay? For very small u, when P tends to zero. Okay? For small u, huh? Small U means small P as well, right? So then if you just simplify it, this is $\omega^2 = \frac{P}{M} . \frac{\lambda^3}{u^3}$ over 3. So, it is $\frac{3EI}{m} \lambda$ over

u^3 is equal to $\frac{3EI\lambda}{u}$ u is equal to λ is nothing λl^3. So, $\left(\frac{3EI}{ML^3}\right)$. What does it mean? See $\frac{3EI}{ML^3}$. What is $\left(\frac{3EI}{ML^3}\right)$? If you take a cantilever and there is a tip load P, what is the deflection? $\frac{PL^3}{3EI}$. So, steepness is what? $\frac{3EI}{L^3}$. Right? And you multiply to divide mass.



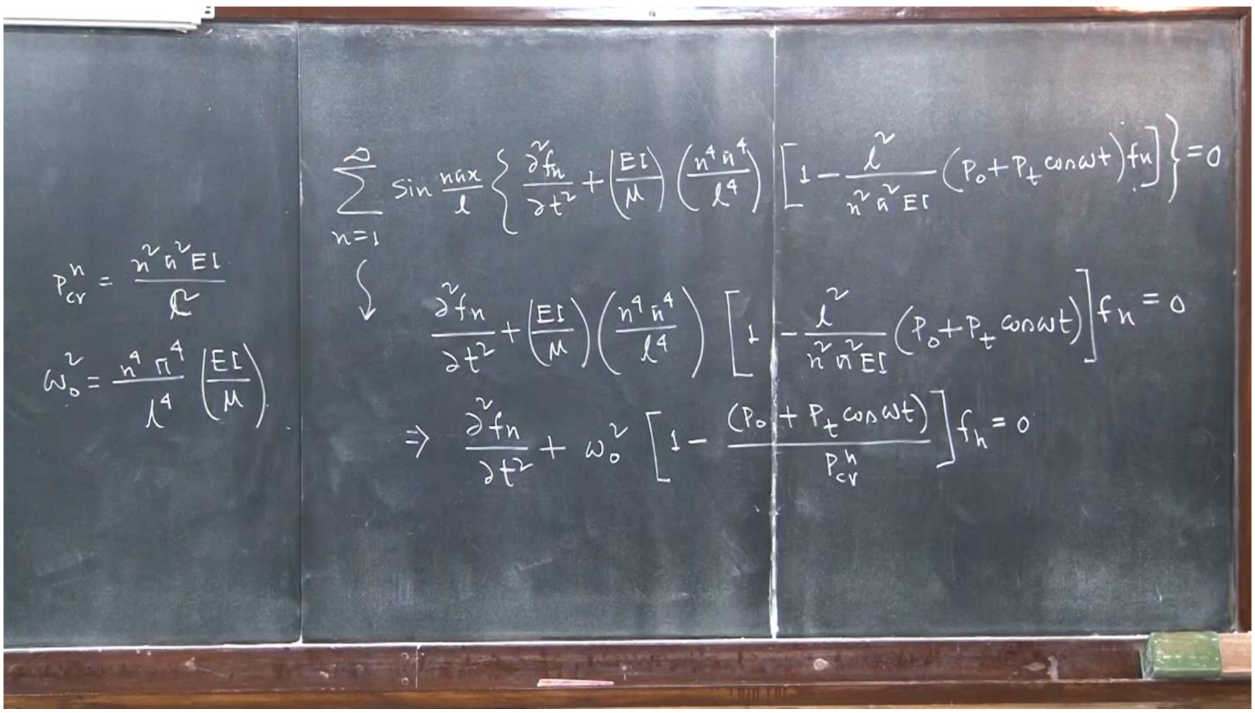
So that is ω is the square root of $\frac{k}{m}$. Do you see that this ω^2 and there is no axial follower force? This is nothing but the same as $\frac{3EI}{ml^3}$. Understand that, and I'm proving it by doing this expansion. Clear? So, flutter instability ω is tending to infinity. I will go into much detail, and later I will explain more generalized concepts about flutter. Right now, try to see that this is a flutter instability. At which the frequency is jumping between positive infinity and negative infinity. Hence, it ceases to have periodic solutions. Right? Now, another form of static instability, but manifested as a dynamic instability, is called divergence instability. So, flutter and divergence are the two most common forms of instability. But see, what is divergence? Divergence means you do not need to have even a follower force. We just consider the column that is under critical load. Okay, P_{critical}. Now go back to your beam, which is subjected to critical load. You know static; what is static deflection? Static deflection is, basically, out-of-plane deflection. If there is axial force, it reduces the softening of the system. Right? Go back to your stiffness matrix and geometric stiffness matrix; at critical load, $k - k_g$ determinant becomes what? That's when the determinant becomes zero, right? Isn't it? So, if there is no stiffness, what will the frequency be if there is no stiffness? If you consider a column that is subjected to axial compression, there is no follower force in axial compression. If the load is below the critical load or the critical load for buckling, then there will still be some stiffness, but with increasing axial compression, its stiffness erodes. That's what we have demonstrated both from, you know, the beam-column equation as well as geometrically, you know, by formulating it in terms of the stiffness matrix, I mean the finite element matrix method, right. At critical load, it virtually erodes all the stiffness, right? And if there is no stiffness frequency, you know stiffness, then what is frequency? Frequency becomes zero. And that frequency becoming zero means what? That if it vibrates, will it be allowed to vibrate? If there is no frequency, what will happen? It will deflect in one direction. Right. Zero frequency. Right. That is called divergence. So, divergence, although in a dynamic sense, is called divergence, but divergence is nothing but a dynamic equivalent of the static instability buckling, right? So, divergence is nothing but the manifestation of buckling up. So, is that clear in dynamic terms? Whereas Flutter is basically the case, the frequency is never going to be zero; rather, it is going to be plus infinity to minus infinity. There are other things. Okay. You know this is only for the column and is subject to follower force. But I will come, and I will explain to you how you can identify which will flutter and which will diverge. Okay. We have to see the change in frequency ω with the follower force or any other non-conservative forces. The follower force is just an example of a non-conservative force. But there are other forces that are non-conservative. For example, follower force is an idealized force. But a bridge under aerodynamic conditions, you know, flowing wind affects bridge aerodynamics. Okay. Under, you know, aerodynamic loading, that is also a non-conservative force. Hydrodynamic force is also a non-conservative force. So, you will see in one of those cases we will consider, instead of the magnitude of the follower force, we will consider the magnitude of the wind speed, okay? As the control parameter, you will see that we have to track the change in frequency with the increasing parameter, which is triggering instability. And there you will see that flutter will be indicated by coalitions of mode and the coalition and bifurcation of mode. So the real part of frequency will indicate something. it will also reflect something in the imaginary part. Okay. Please note that here, why it is going to infinity because then the system doesn't have any damping. Ω is here; Ω is when this is negative. Ω will only be imaginary. There is no damping. If there were damping, it would never go to infinity;

rather, it would go to a very large value. And if there were letting lamping you in ω, it will not be purely imaginary. It will appear in complex; it will have a complex value. So, ω^2 will be $\pm ib$ in this form if there is damping. So, you can see that the real part basically represents damping, while the imaginary part basically represents the frequency at which it will vibrate. Okay, we'll come there later. Huh? Fine. Okay. So, now do you understand why we require to study dynamic instability? Now, what will we do second? We will consider a case. Which is more familiar? You are more familiar with it because this is a direct extension of the instability from the column buckling. Because here, instead of a statically applied load, the axial force is being applied dynamically, which means it is a fluctuating axial force. So, the column is under axial pulsating load. So, the column is under pulsating load compression. Okay. Here we consider the column, and this column has length L, flexural rigidity EI, and mass per unit length μ. μ is the mass per unit length. So, this column is subjected to force.



A force-time dependent force; part of it is static and part of it is dynamic. $\cos\omega t$ you see that, so P_0 is the statically applied force and p_t is the amplitude of the dynamically applied load $p_t\cos\omega t$ okay So, essentially, why do you see this kind of force on a column that is subjected to this kind of excitation? Of course, you know a column; you consider the bridge pair, which is supporting the deck, and then maybe in the bridge, where this support is simply two different supports.

Nowadays, bridges are mostly subjected to continuous dynamic loads. So, vehicles are actually moving under that dynamic load, right? Sudden dynamic load, right? Of course, that will not be continuous. Similarly, a column that is supporting some kind of rotating machinery. Okay. So, we will also apply this kind of axially pulsating force, right? Now, with this, what will be the equation of motion for this? You see, we know the beam-column equation. What is the beam-column equation? Beam column equation is nothing but $EI\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = 0$, that is the beam column equation or maybe laterally. If a laterally applied load is present, then it will be something; this is the static beam-column equation, right?



Now we have to add what? This is the static equation, right? We all derived that, static. On the static beam column equation, right? Here, inertia will come into the picture. So, adding inertia to this equation, what will it be? So, $\mu\frac{d^2w}{dt^2}$ + here p will be $(P_0 + p_t\cos\omega t)$ and then $+EI\frac{d^4w}{dx^4} = 0$. That is the governing equation because this is inertia, and this is basically, well, here will be the $\frac{d^2w}{dx^2}$, huh, and because it will appear in partial because w is a function of x. So, let me write it partially. Okay, please. Huh? Partially. Now, we have to solve this equation. How will you solve this equation? I mean there are different ways to solve it. But One thing, let us assume that this column is basically a simply supported column. Okay, so that is for simplification. Okay, so you know earlier this is simply supported and there is also a hinge there. So, say of course it will; we must allow axial deformation, right? So then with this, we can assume $w(x, t)$, and we assume something like that summation, you know, maybe some term, you know. So, $f_n(t)$ and then $\sin(\frac{n\pi x}{l})$ something like that, okay, $\sum_{n=1}^{\infty} sin\frac{n\pi x}{l}$ means you know this will take the different you know harmonics in the sign of course when you assume $\sin(\frac{n\pi x}{l})$, it satisfy all the boundary condition right. so simply supported $W = 0$ and $\frac{d^2w}{dx^2} = 0$ right. Here, please note that it's a fourth-order derivative coming, so you have to consider shear force and bending moment, those things as well in the boundary condition, okay? So, simply supported, you know what ϕ means? $w = 0$, and the bending moment is zero. So, from there, by satisfying both, you can assume this. So, if you substitute it, you assume this kind of solution and then substitute. You substitute what you are going to get; let me write it down. You are going to get an equation something like this. So, $n = 1$ to ∞, find $\frac{n\pi x}{l}$ you know $\frac{\partial^2 f}{\partial t^2} + \left(\frac{EI}{\mu}\right)\left(\frac{n^4\pi^4}{L^4}\right)\left[1 - \frac{l^2}{n^2\pi^2 EI}(P_0 + p_t\cos\omega t)\right] f_n = 0$. Okay, you see that if you differentiate with respect to time twice, you know, uh, that's what you see in time. So, that's why the double derivative is coming, and second, if you differentiate twice and then four times with respect to the spatial coordinate, which is also sinusoidal, sine will remain sine. So, that's why I'm taking it out, and of course, these coefficients will come l^2 and $\pi^2\ EI$ coefficients. Okay, fine. You just substitute and then do. Huh? I'll simplify this. So, what we see is that we can take out, and from there, I will specially space the equation for the space we can take out separately, and for the time equation, we can take out separately, right? So, then from here we can write down $\frac{\partial^2 f_n}{\partial t^2}$, I mean I can use the you know total derivative also, we do not require to then $\frac{EI}{\mu}$ and $\frac{n^4\pi^4}{L^4}n^2$ Go ahead. This one, huh? So, understand that $\frac{\pi^2 EI}{L^2}$ is nothing but P_{cr}. Okay. Here also you can write P_{cr}. We'll simplify this subsequently. Okay. And now we will use a certain quantity to simplify. We'll define the critical load for the N-th mode as $N^2\frac{\pi^2 EI}{L^2}$. This is P_{cr}. Then we can also define ω_0^2. Ω_0 natural frequency $\frac{n^4\pi^4}{l^4}$, sorry, $l^2\ \frac{EI}{\mu}\ \omega_0^2$ is the natural frequency in the absence of axial force. And we also define. So, then if we define thatthen this equation can further be simplified as $\frac{\partial^2 f_n}{\partial t^2} + \omega_0^2$ you know ω_0^2, you know, $\left[1 - \frac{P_0 + p_t\cos\omega T}{P_{\text{critical } N}}\right]$, you know, $F_N = 0$. We will simplify it further, okay? The next class, but for the time being, we'll define this. So, this is the natural frequency in the absence of axial force, ω_0^2. You can see this is a ratio of the, you know, K by M, right? So, this is elasticity, and then this is the mass, right? So, π is coming, and critical load. If you simplify it something like this further, we'll simplify it further where we'll consider the natural frequency in the presence of axial force. So, $\omega_0^2(1 - \frac{p_0}{P_{cr}})$, we'll define as ω_n^2 and things like that. In the next class, we'll do that. Okay. Thank you very much for today's class.