Stability of structure

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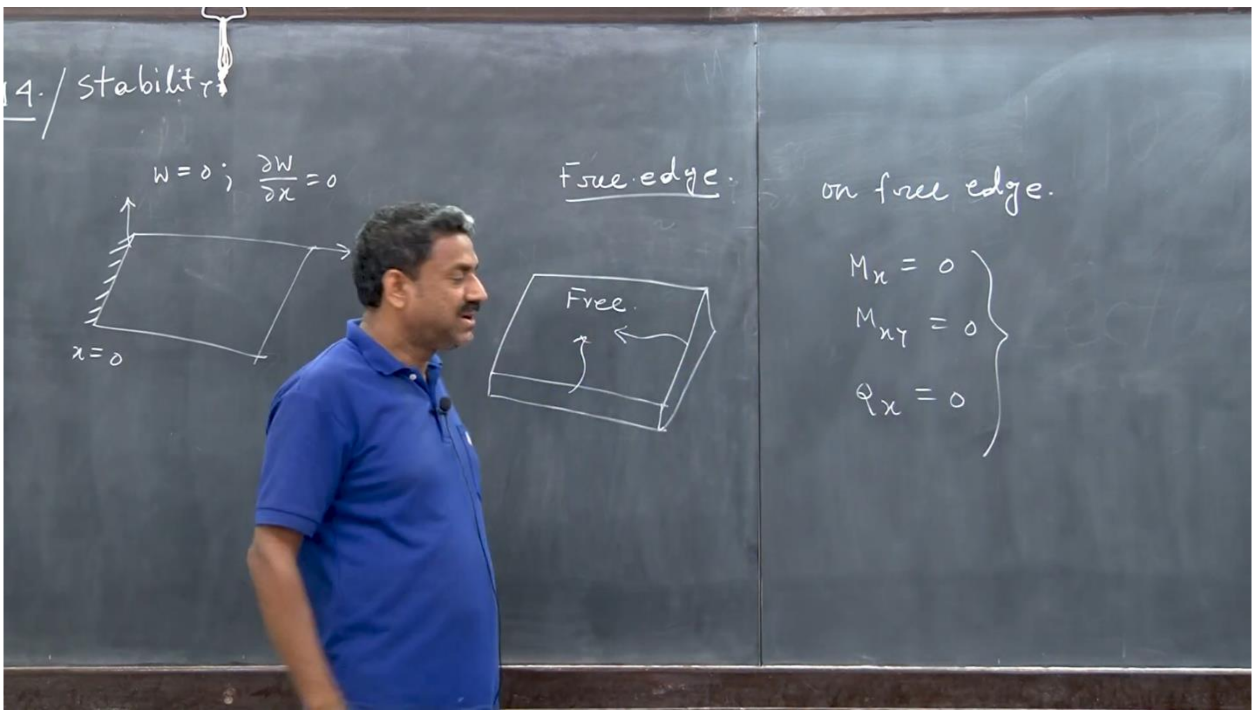
WEEK-07

Lecture 14: Shear Buckling of Plate

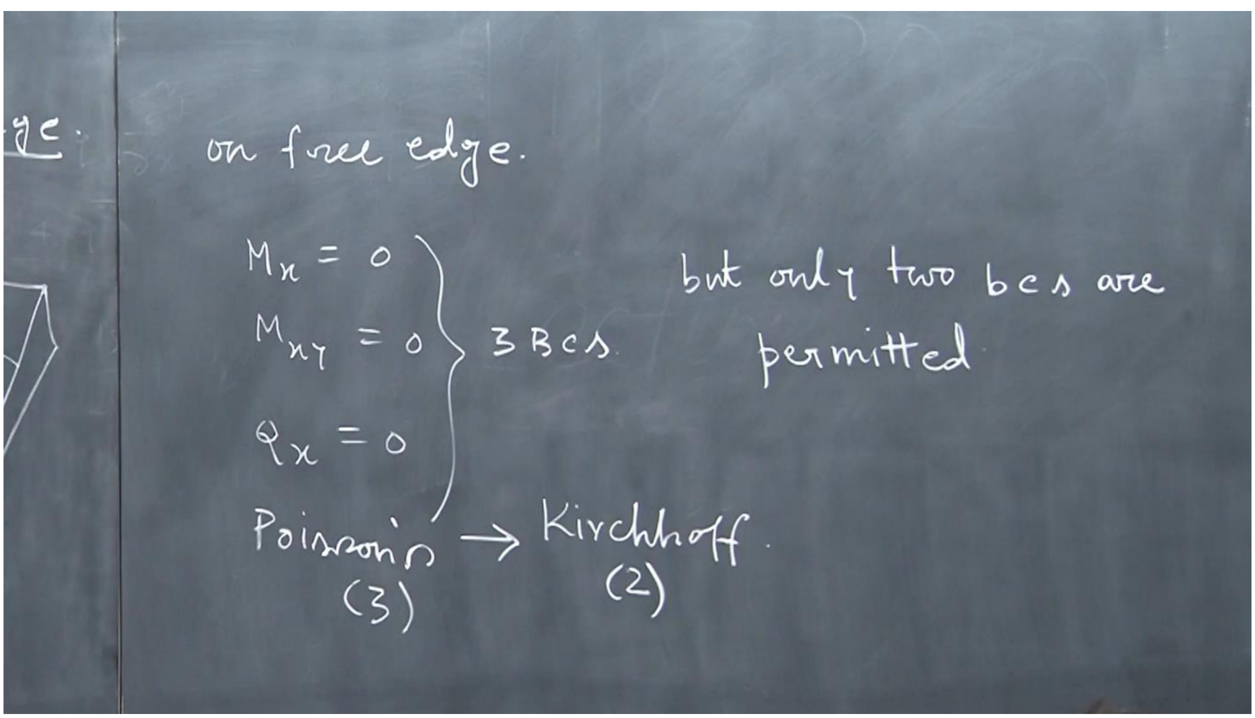
So welcome to lecture 14 on the stability of structures. So let us briefly recapitulate what we are doing. So we are discussing the buckling of a plate, and in the previous class, we have shown that you know buckling, while considering buckling of plate, basically you have to consider the its equilibrium in deformed configuration, perturb configuration But what we have seen is that out of plane means the equilibrium in the direction we considered is the same as whatever we use in the bending of a plate, right? So that means whatever little deviations may occur because of the deformed configuration, okay. On the vertical equilibrium equation, the force equilibrium equation along the vertical Z direction. That we don't basically consider; however, the contribution of the in-plane forces, because of the different contribution, has been taken into account. And that's what we derived, you know, the governing equation:

$$D\nabla^2\nabla^2 W(x,y) = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} + N_y \frac{\partial^2 w}{\partial y^2}$$

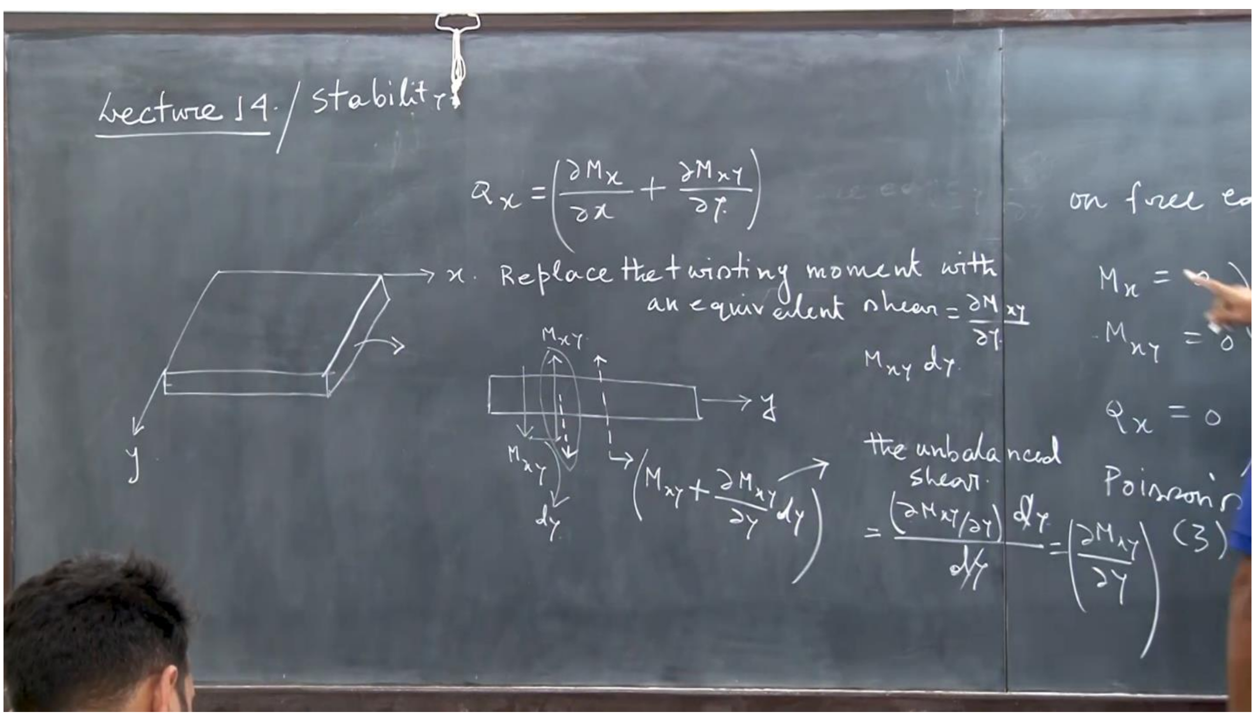
and then from there. So, these are the additional terms that are coming by considering the deformed configuration of the plate, whether this is the same as the equation from the beam ending right. And then, as far as buckling is concerned, N_x, N_y, and all these processes are assumed to be constant over the plate, right? They don't vary; that is strictly valid for the pre-buckling regime, okay. Now, then we have considered a simple example. To solve it, we have considered a simply supported plate, okay. With the governing equations, which are subjected to uniaxial forces and uniaxial compressive forces, we know the stresses, and then we solve it. We have seen that this results in an eigenvalue problem as before. We have solved the trigonometric eigenvalue problem. We assume some double Fourier series, you know, involving sine functions and by satisfying the boundary condition. And the boundary condition for the simply supported plate we have derived, we have seen that at each end, like $x = 0$ and $x = a$, $w = 0$; that is, the vertical deflection is zero, and then the bending moment is zero. That ultimately condensed out to be $\frac{\partial^2 w}{\partial x^2} = 0$, and then the other $\frac{\partial^2 w}{\partial y^2} = 0$. Solve this here. So, on each face, there are two sets of boundary conditions, right? And we have solved this thing. So, this is one approach to solve, you know, and then we have also seen that physically the behavior. So, we have assumed for the simplicity, we have assumed that well, there is only one single wave, sine wave prevail I mean, you know this only single sine wave is there, in the y direction and the other direction we have allowed multiple waves okay So then, we are restricting this is one kind of simplification right. So, we have seen that depending on the situation, we have minimized it. So, N_x is the critical value of the axial compression. So, then we minimize, and we have seen that, depending on the aspect ratio a/b, it shifts from one mode into a different mode, right? Okay. And then the minimum critical load is, if you can recall, $N_{cr,x}$ was $\pi^2 D/b^2$; it was four, I think. Okay.



So, from one mode to another, as we are changing the a/b ratio, when the a/b ratio appears as an integer, that means it can form a perfect square; then the critical load attains its minimum. Otherwise, there is a slight increase on both sides, okay? Whether a/b is less than an integer or a is greater than some particular integer, okay? And with increasing aspect ratio, mode shifting occurs, from the first mode to the second mode, the second mode to the third mode, the third mode to the fourth mode, and then while the shifting is occurring. So, if you see the Curve with a changing aspect ratio, the increase in the critical load that follows. So, the boundary follows a tongue-like structure, which is referred to as the Arnold tongue. And which is a very general, you know, kind of structure that you observe not only in buckling up plates but also in many other scenarios, especially in dynamic systems and other nonlinear dynamics. So, all these things we have done. Okay. We have also seen that we can study the bi-axial bending and buckling of a plate under bi-axial compression. Okay, where it is subjected to both N_x and N_y. We can simplify this equation for solving by treating N_x and N_y as appearing in a constant ratio. So, that means $r = N_x/N_y$. We don't consider all these things. But now, what we are going to do is focus on two things. See you all, whatever I am now going to discuss, you have perhaps learned in your plate, the theory of plate tectonics, but once again I would like to emphasize it. See, we know the boundary condition for the floor plate; you know, bending or buckling here is either simply supported, which is what we have learned: $w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0$. Now it can be fixed. If it is fixed, for example, if I have this end fixed, then over this end, $x = 0$. So, what will the boundary condition be? The boundary condition will be $W = 0$, and then the slope is zero. So, $\frac{\partial w}{\partial x} = 0$. Right? This is the boundary condition.

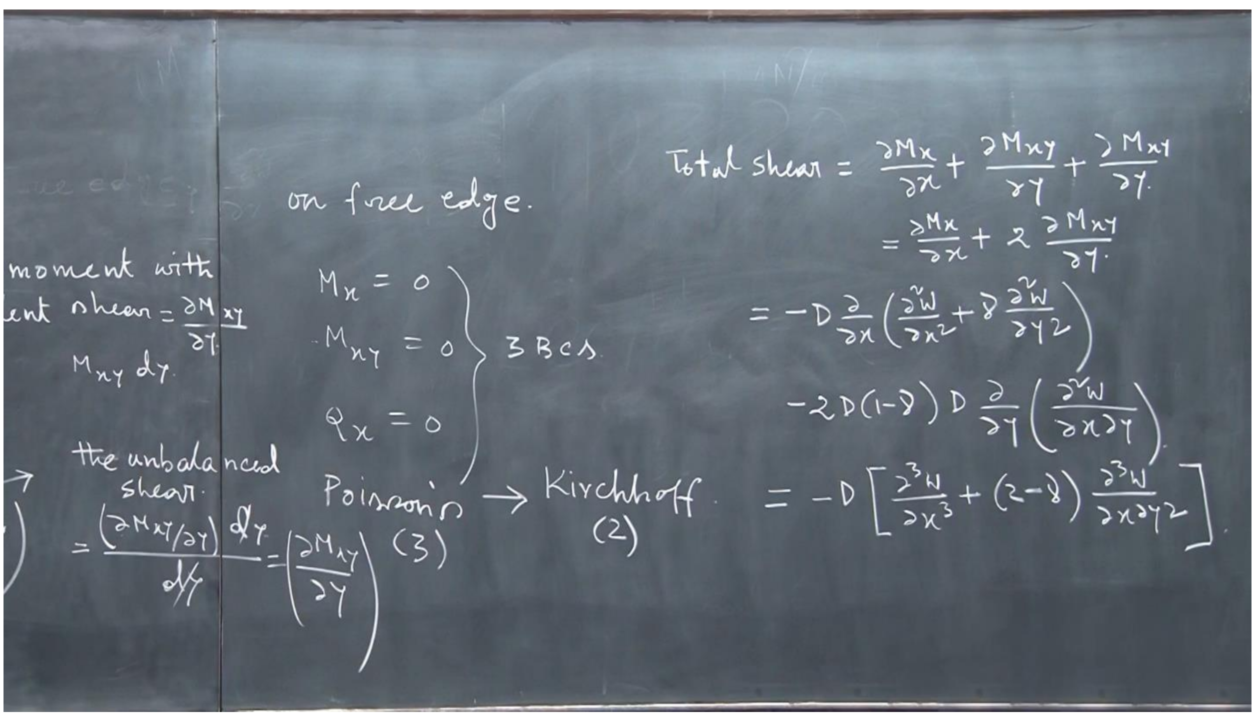


So, as far as pre-boundary conditions are concerned, simply supported is fixed. This is a fixed boundary. But what about the free edge? So, for Free Edge, there is a little problem that I would like to quickly address. Many of you might be aware that I'm considering this, and I'm assuming that these edges are free edges. Okay. Free edge. On the free edge, what will happen on the free edge? On the free edge, basically, see that the bending moment is zero. The shear force is zero, and the twisting moment is also zero, right? So, the bending moment $M_x = 0$. The twisting moment is to be zero and Q_x is also to be zero. But how many boundary conditions are there on any edge? Only two boundaries. So, these are three boundary conditions, right? But only two boundary conditions are permitted. Because the order of the differential equations is a fourth-order differential equation, right? So, what will you do? You have to somehow condense these three boundary conditions into two. So, these three boundary conditions were identified by Poisson, and they were modified by Kirchhoff. From three to two. So how will that work? I am going to show you. So, if we see that, what is q_x? There is a relation: $Q_x = \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}\right)$ right? Now this one, M_x, that's what I am going to do. What I'm going to do, I'm assuming that this is good. So, this is X and this is Y. Okay. So, on this, you know I'm considering this, you know. So, when it is along Y, okay.

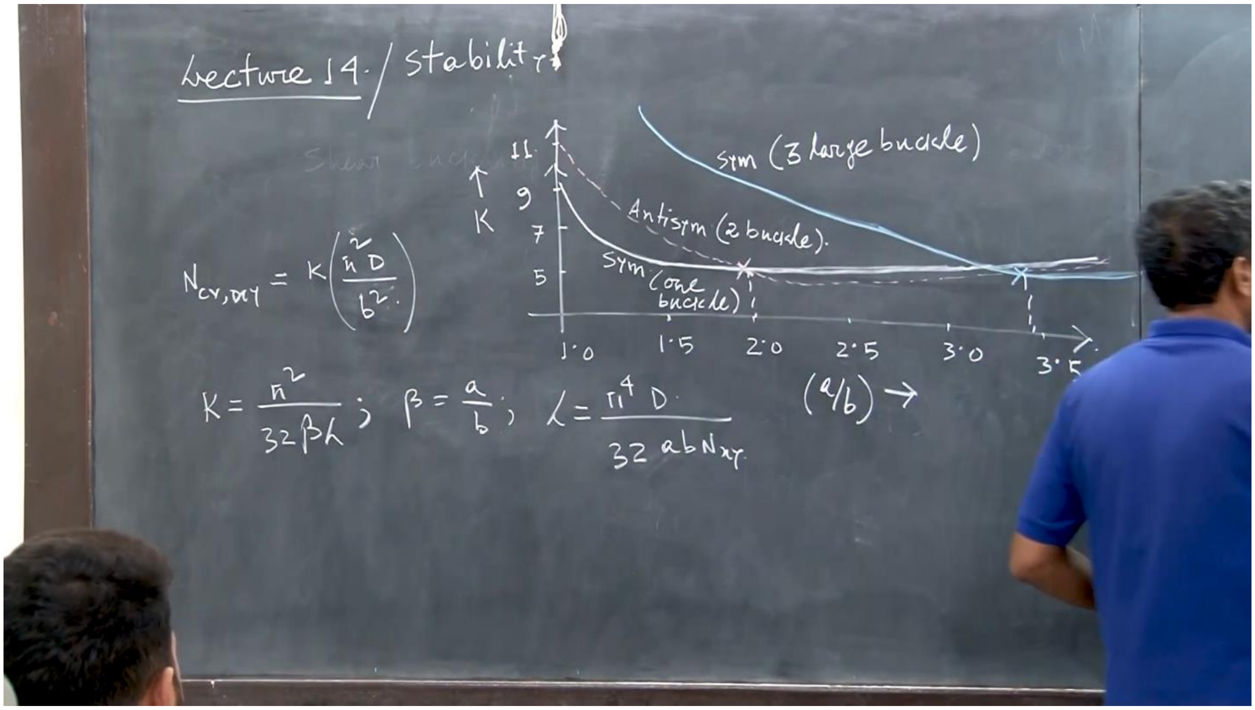


Now when it is subjected to M_{xy}, you know, maybe it is an M_{XY}, means, you know, I can consider that within a small stage M_{xy}. M_{xy} is the twisting moment per unit length. So, by the stretch of

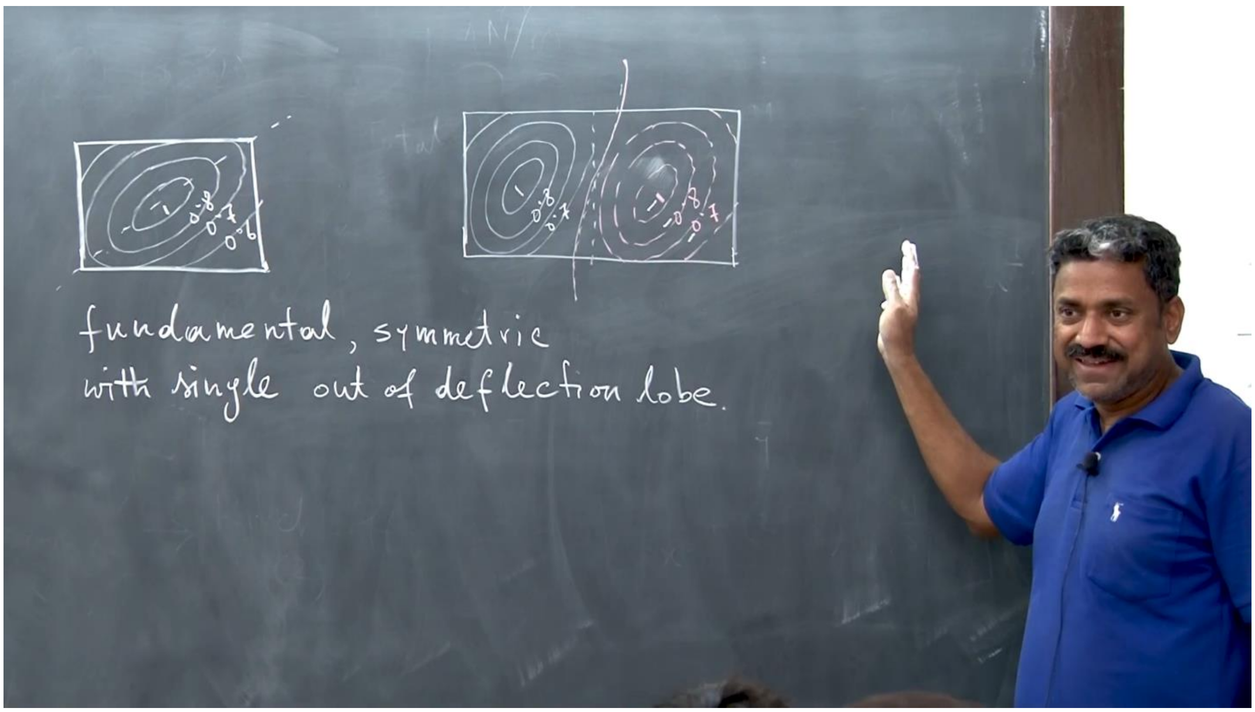
dx, how much is the twisting? M_{xy} into dy, isn't it? M_{xy} into dy, right? So, may I replace it with two forces that are a distance dy apart; can we do that? As equivalency d right, now over y, this is changing. so, once again now, we can also do it you know over next length, we also replace it with, this is $\left(M_x + \frac{\partial M_{xy}}{\partial y} dy\right)$. So, you know this is changing. Over a length of dy, this force is going to change; this will increase, right? Now you consider that over a segment dy, there is an unbalanced force. This force is an unbalanced force. So, just so you know, how much is the unbalanced shear? Shear is $\left(\frac{\partial M_{xy}}{\partial y}\right) dy$, and this is over a length of dy. So, per unit length, the unbalanced shear can be expressed as dx/dy. You understand that, over this length, M_{xy} is the twisting moment, so this is increasing in the positive x direction, right? So, over a length of dy, I can replace that twisting moment $M_{xy}dy$ with two shear forces distance dy apart. Now, the next segment I can also replace with two segments, this and this, right? Now, for one segment over this, take half of this. So, there is an unbalanced force, right? Because this is not balanced by this. Because, sure, you know the twisting moment is going to increase over the length, over this segment, right? Over the length, over the increasing y, right?



So, that unbalanced shear is how much? This is M_x and this is M_x plus. So, this one is the unbalanced one because upward, downward, right. I'm not considering that much; I mean, I'm not strict about the sign, etc., but you understand there's unbalanced shear, right? So, may I replace this twisting moment with an equivalent shear? So, what is equivalent shear? So, replace the twisting moment with an equivalent shear. And that is what magnitude ∂y is, right? So earlier it was Q_x and now this is additional. So, total Shear now, so both. Now $\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + \frac{\partial M_{xy}}{\partial y}$. So $\frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y}$, this is the total shear. So, now, do you understand? So, now we can combine these two. So, from three, the boundary condition is M_x; this is an independent boundary condition, and these two are combining into one. So, this shear is called Kirchhoff's shear. Okay. And then we can simplify by substituting. Okay. So, you just substitute. So, it is $-D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$. So, you will see that if you simplify, you will see that $\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x\,\partial y^2}\right]$. Ultimately, we have to express it in terms of the displacement. Because we have formulated everything in terms of displacement, right? So, in displacement order condition, the total shear is this, and at the free edge, make this one zero $\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x\,\partial y^2}\right] = 0$ right, bending moment you take separately. Okay. So, do you see, how these two have combined into one? So, this kind of simplification has been proposed by Kirchhoff's-Poisson, identifying the boundary condition and then the anomaly over there. Well, there is some mismatch. Okay. In order to bring consistency, you have to condense these two boundary conditions into one, and that was solved by Kirchhoff. Okay. This is called Kirchhoff's shear. Clear? And then M_x is the way you will hear; in here bending moment of course you want to consider to convert into displacement. Of course, this will be another equation: $\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} = 0$, right? But yes, this boundary is a little complicated. So, you cannot just intuitively assume some kind of specific form, okay? So, that's what, nevertheless, it can be solved using, of course, you require a digital computer; you require a computer, and you can use some, you know, more terms in the Fourier series. You know, and then you minimize both the differential, the error in the equation, as well as the error in the boundary. Okay. So, it can be done; that's not a problem. Okay. Similarly, the way you have physics, let me explain it first. So, the way you have it, if you solve it, then you will get the critical load to be, you know, the expression. Let me write it down. So, $N_{critical,xy}$, this is $k(\pi^2 D/a^2)$. Well, upon b^2, this k is nothing but $\pi^2/(32\beta\lambda)$. When β is with respect to (a/b) and λ is Anyway, what I'm trying to say is that it can be expressed as something into $(\pi^2 D/b^2)$. Okay, no k value. So, I'm just going to plot it over here. So, when I'm plotting it, I’m assuming this is 1.0, 1.5, 2.0, 2.5, 3.0. So, see how the mode shifting occurs when we want to demonstrate a similar structure as previously. Okay. So, this here is nothing but an a/b ratio; an a/b ratio is the aspect ratio. So, you see, when the a/b is less than 2 or 2.05, then.



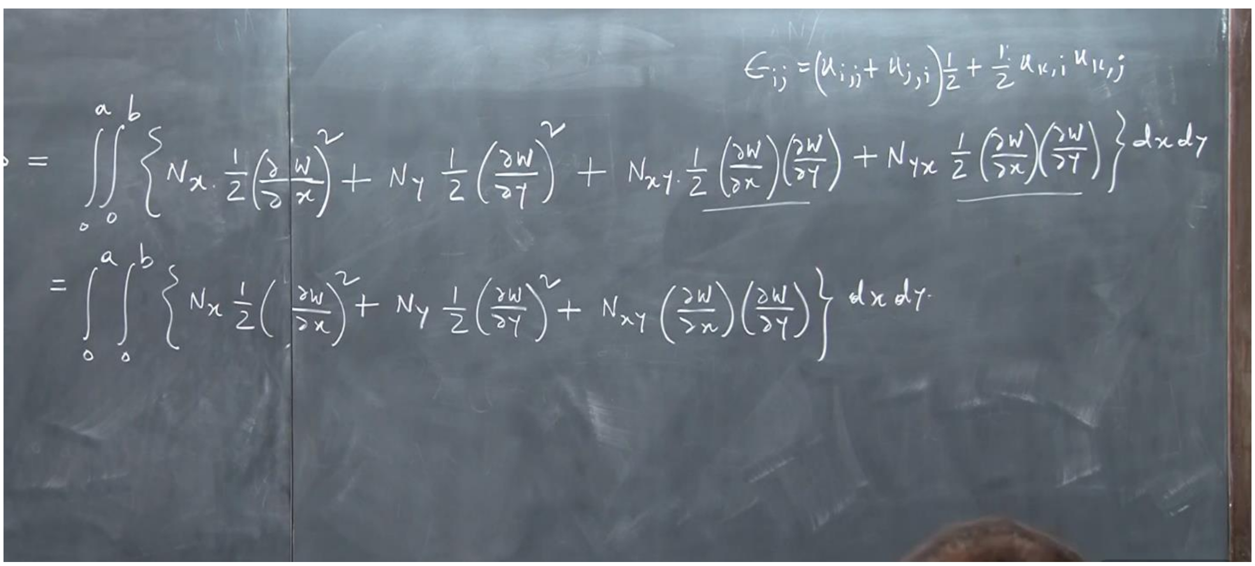
So, for all these modes, you are basically plotting how this K, which is nothing but K, varies for different aspects A/B. So, K versus this. This is for the one mode; this is the first fundamental buckling mode. Okay, this one. So, the first buckling mode is what you can see, which will consist of only one buckling lobe. Okay. And this is, of course, kind of symmetric buckling. One loop means one you know to be symmetric. Then the second mode is anti-symmetric because if there are two loops, this must be anti-symmetric. One will come out; another will go down. Right? Then there are two large buckles, and then there are three large buckles that are symmetric. So, what we see is how this buckle will look. Let me explain it to you. Okay. So, if I draw a contour, then it will look like this. That means, you know, maybe this is the value of one, then this is 0.8, 0.7, 0.6, something like upward. So, you know it will come; it is coming out, you see that. It is coming out, you know, but then the contour looks like this: it is being flattened outside. So here it is maximum; all this, you know, other places are going down. Do you understand the surface? Now, why is it a little oblate? Why is it, you know, shrinking in this direction? That is because you know, under shear, if it is subjected to pure shear, one diagonal will be in tension and another diagonal will be in compression. Right? So, this side is subject to tension. Some tension stiffening is occurring; that's what it is basically reducing the extent of this, uh, you know, outward deflection. You understand what I'm trying to say? On the other hand, in the other direction, it is compression. So, it is more susceptible to buckling. Sure, that's basically when it buckles under shear; what happens is that one diagonal is under tension while the other is under compression, and they are mutually perpendicular to each other, right? So, it will be shrinking along one direction, whether it will have little extension in the other direction. You see, that's essential. So, this is symmetric buckling because it is symmetric, right? So, this is the fundamental or symmetric mode. The fundamental or symmetric mode with a single out of deflection lobe is okay.



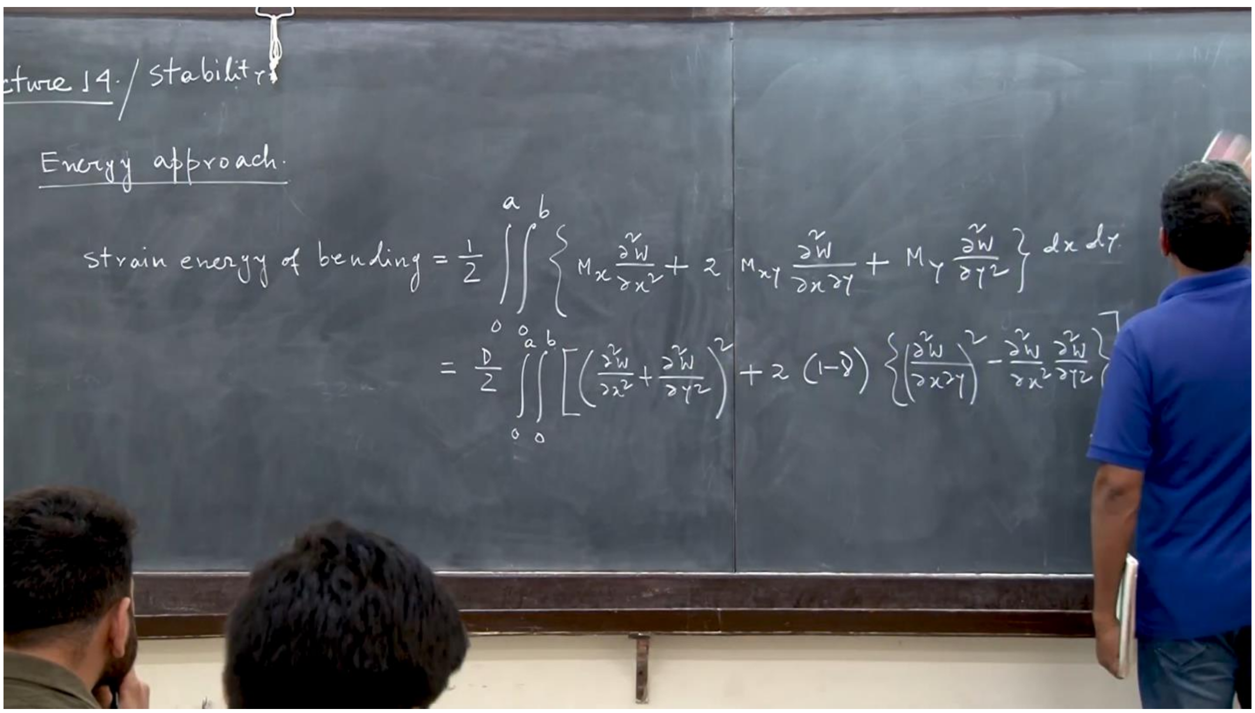
And essentially that's what is happening, where now this one, this one you see, and fundamentally it will give the lowest load; of course, K is starting from 9 and it is going down. With increasing aspect, this is reducing that; see here, this is a little different. The difference you must understand is from uniaxial buckling to shear buckling; in uniaxial buckling, it was diminishing. But then it will go up; it will go up depending on the A/B ratio. When the A/B ratio is an integer, that means it can perfectly accommodate the square shape. Then it will attain a lower value; otherwise, in both increasing or decreasing a/b aspect ratio, from an integer it will increase on both sides. That's what ratio will form this kind of Arnold tongue kind of structure. Here it is not, of course, shear;

also, Arnold Tongue will be there, because from here you see that from here then here. Yeah. These are also—this is one Arnold Tongue. This is another Arnold Tongue. Do you understand? But it is a little different; you know it is not monotonic. In this case, it is kind of decreasing mostly. You see that? And then these are the points of transition mode transition. Nevertheless, mode transition is akin to the same behavior as we see in the uniaxial compression. But the trend is different. As soon as it crosses 2.05, there is more shifting, and it is going to the anti-symmetric mode, which is energetically favorable. So, it will try to buckle in antisymmetric. So, there is a mode shifting occurring from symmetric mode to antisymmetric mode. Right? Then you further increase it by around 3.5 or so, and it will go to another symmetric mode, which consists of three buckling lobes. Let me draw it how it looks like. You see, so this is maybe one, this is 0.8, this is 0.7, or something like that. And maybe this is one, this is 0.8, 0.7, something like that. So, you see that these are all negative, okay? So, there is a ridge, which distinguishes between positive out-of-plane deflection and negative out-of-plane deflection. So, this fellow is coming out, and this fellow is going up. You have learned this kind of thing while studying, you know, warping torsion, right? Know that when you solve the Poisson equation, you'll get the warping functions, and then from there you'll plot that. So, here, the aspect ratio is around two. So, this pillow is going up. This fellow is going down. Once again, this little oblate shape is due to the tensile stiffening along the other diagonal direction. You see similarly. now, so this consist of this is second mode and this is anti-symmetric mode. This is positive. This is negative. Right? And it is having two buckling lobes. Right? Now you can go to the third; one fellow will go down, and the other two will come up, okay? when the aspect ratio is little larger than three. so, then it will you know, this will be, so sifting is indicated one these are the Arnold's tongue, okay. So, you understand the qualitative similarity between uniaxial buckling and shear buckling, right? There are qualitative matches, you know, but yes, the behaviors are different. Now we'll see how we'll do the energy analysis. So, for energy analysis, that is most convenient, right? That's what we have done in all the previous analyses we conducted. So, we'll try to find out, you know, the energy expression for the plate bending. So, you have done the energy expression for the plate bending theory. So, the energy approach. So, strain energy of bending. What you have learned is that the beam moment and the energy conjugate to moment curvature will give you the strain energy. $EI/2\int \left(\frac{d^2w}{dx^2}\right)^2 dx$ right or $\int \frac{M^2dx}{2EI}$, what is that, from where it is coming, bending moment into curvature multiply d^2/dx^2

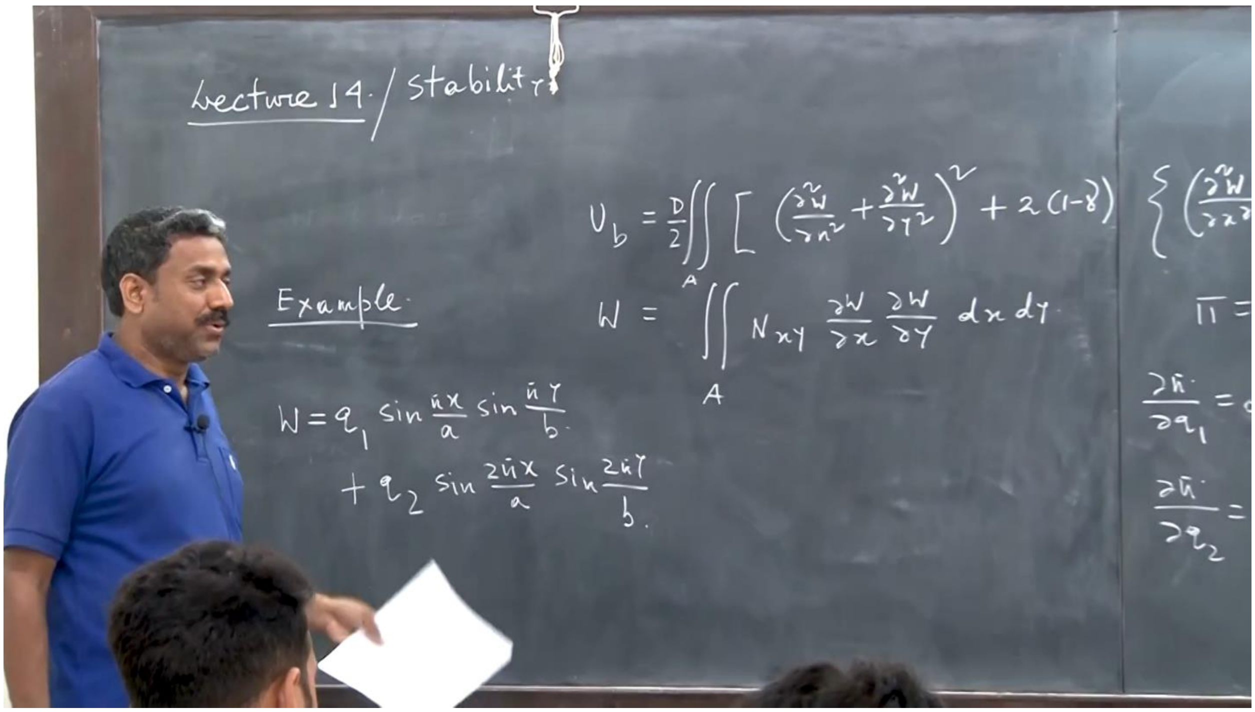
right. So here we have two bending moments and one torsional moment, right? So, for the respective curvature term, we should multiply, okay? And don't care about the signs. Moment multiplies a curvature. Okay. Now you substitute the equation. So, if you substitute, then M_x, you know all this $M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$ is. Take and substitute, and then you simplify. If you simplify, then you will get the final expression. I am not doing the simplification. You take everything we express in terms of displacement, because it is intuitive to assume the displacement field using some kind of approximation function. Okay, that's the way you do it for finite, you know, Galerkin methods and other things like the "Rayleigh-Ritz method." Okay, that's why. So, $D/2\iint$ you know, that is the expression for the strain energy of plate bending. Huh? So, we have this expression ready, and now we must find out the expression for the work done, okay. So, now work is done by the in-plane forces, right? So, work done by the in-plane forces due to out-of-plane deformation, right?



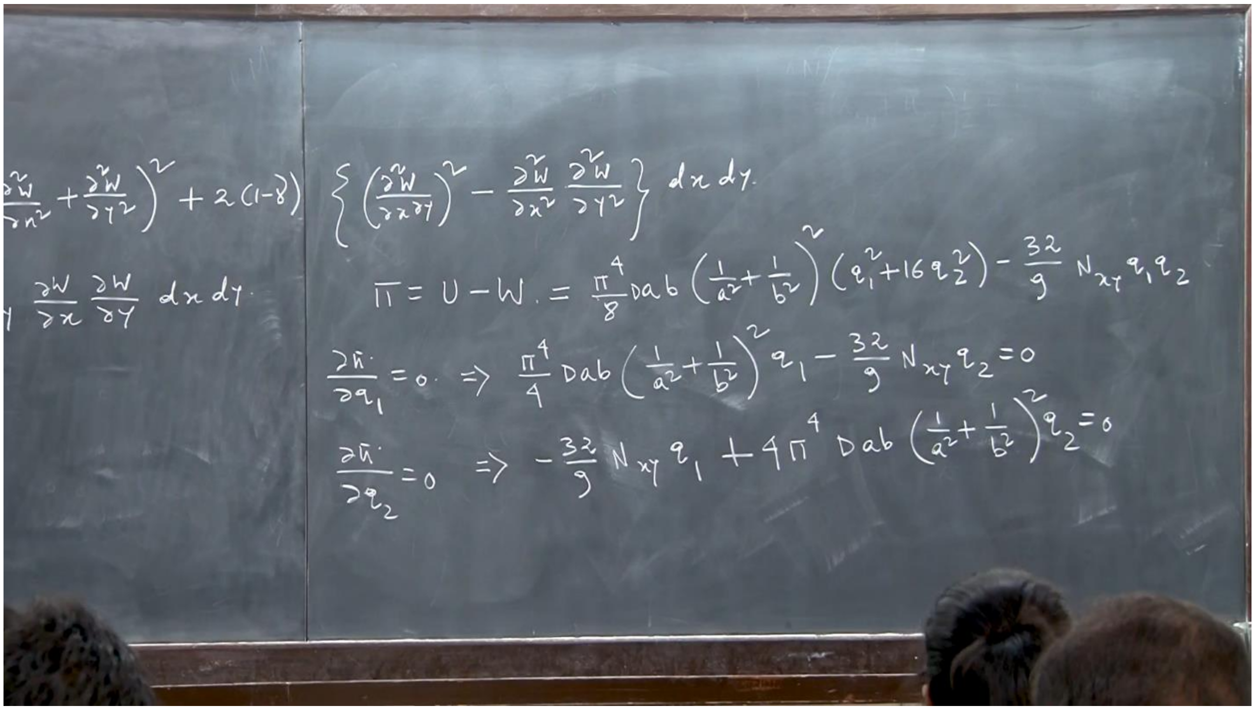
So, N_x, what is the respective you know in plane deformation? Because of out-of-plane deflection, you have to integrate the von Karman nonlinear term. ε_{xx} is what? $\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$, right? So $\frac{1}{2}\left(\frac{dw}{dx}\right)^2$, of course, when you are integrating, you will get the axial deformation, right? So, this one, huh? Please note that the in-plane strain is due to the out-of-plane deformation only, right? We're not considering that there is no in-plane deformation, no u or v, only w; similarly, you will have N_y half. This is for $N_y \frac{1}{2}\left(\frac{dw}{dy}\right)^2$, right? And then we can have N_{xy}. So, for the N_{xy} you see that it will be $\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w}{\partial x}\right)$, and for N_{yx} it will also have $\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)$. So, please note that there is, you know, this is half and half, and then both of half, if we sum it up, then you know this is from where this is coming, you can consider this. ε_{ij} is what? $\frac{1}{2}(u_{i,j}+u_{j,i})$ right plus then, of course $\frac{1}{2}U_{k,i}U_{k,j}$. Right. So, this is k comma i, k comma j and this is W Okay. So, if you want, I mean, I can solve this very small, you know. So now, for shear buckling, let us solve an illustrative example. An example for shear buckling, right? So, we are given a plate, you know, subjected to pure shear. So, when you do shear buckling, how will the expression look? So please note it down, and then I will write it down. Okay. So, you know strain energy is related to bending. The expression we have already you know derived



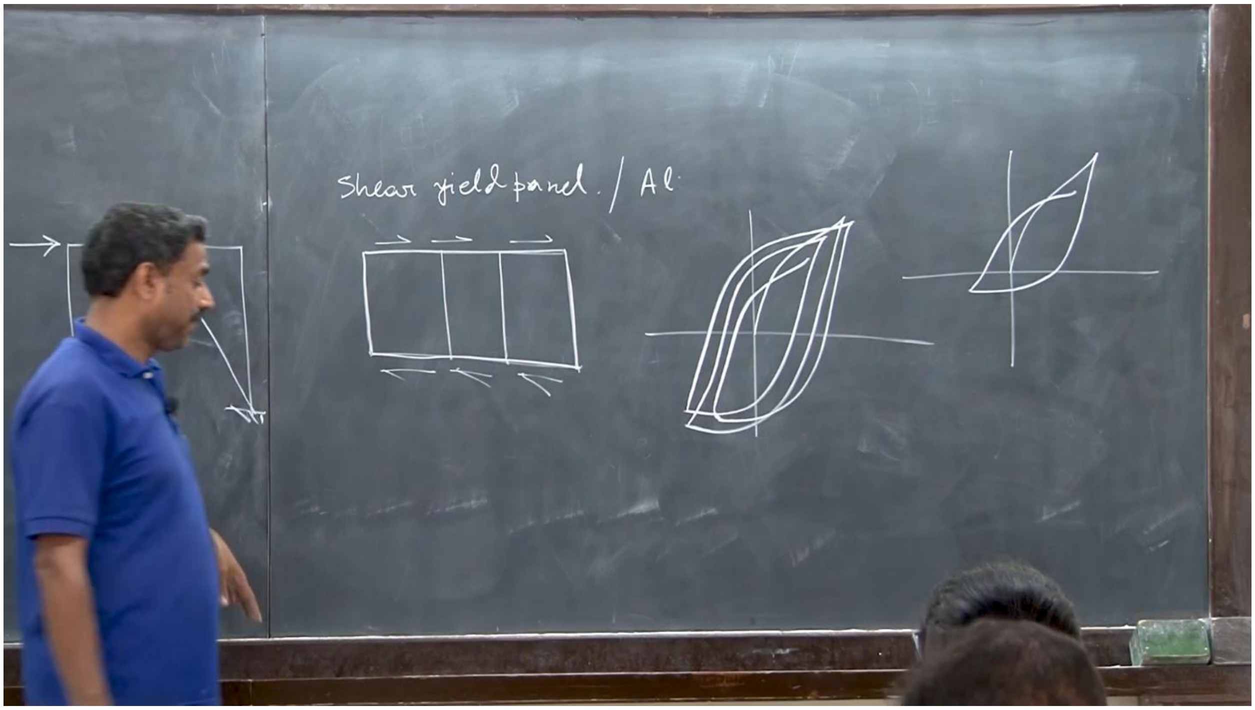
$D/2\int_0^a\int_0^b\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2(1-\nu)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]dxdy$ and then for the work done, only, and of course integrating the area. Only Shear N_{xy} is $\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)$. Now, the way we do it, you know, is that the potential energy is strain energy minus work done, right? We assume some w for the shear deflection, so we assume w to be; see if we want to include more terms, it's fine, but then if we include too many terms, we cannot do the h calculation. It will be, you know, too complicated; of course, if you are using a computer, then you can include as many terms as you want, right? And nowadays, with symbolic computation in, you know, Maple or Mathematica, you can do very simple integrations and others, right? So here I am writing $Q_1 \sin(\pi x/a) \sin(\pi y/b)$, two-term approximation I'm doing okay. $\sin(2\pi x/a) \sin(2\pi y/b)$. Please note that I should have also included $Q_3 \sin(\pi x/a) \sin(2\pi y/b)$ and $Q_4 \sin(2\pi x/a) \sin(\pi y/b)$, because you see this is $\pi x/a$ $\pi y/a$ single wave and this is a double wave, but the crossover between single and double should have also been included, right?



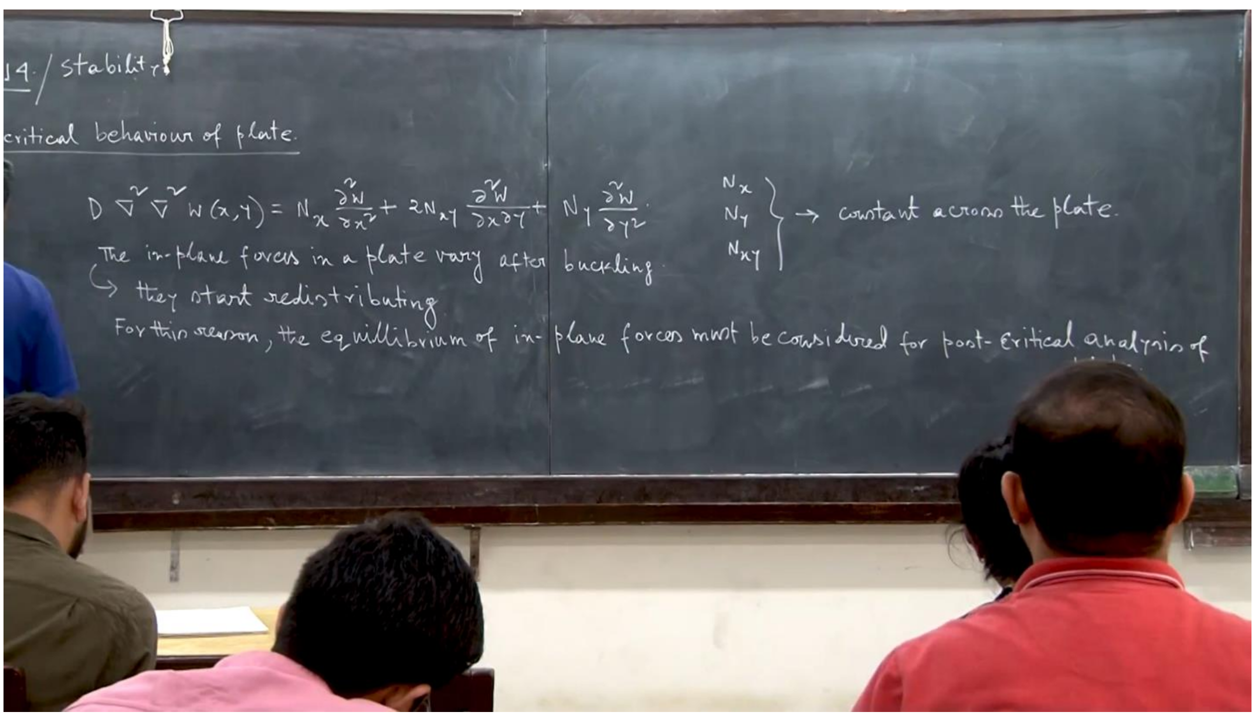
But even with this thing, you will get a reasonably good approximation. So, if you substitute here. And if I write down, you know, the express, carry out the integration over the domain x, y, okay? and then you will see that expression looks like you know $\pi^4/8Dab(1/a^2 + 1/b^2)(q_1^2 + 16q_2^2) + 32/9N_{xy}q_1q_2$. Huh? This way, with the expression on integrating. Okay, you see that everything is in terms of the generalized coordinates Q_1 and Q_2, right? Now you minimize the energy: $\partial\Pi/\partial q_1 = 0, \partial\Pi/\partial q_2 = 0$. and then the first equation you will get $\pi^4/4Dab(1/a^2 + 1/b^2)q_1 - 32/9N_{xy}q_2 = 0$ And $-32/9N_{xy} + 4a^4Dab\left(\frac{1}{a^2} + \frac{1}{b^2}\right)q_2 = 0$. You see that both the homogeneous system equations basically give rise to an eigenvalue problem, right? So, when we are approximating using this trigonometric function, you know, this is some kind of numerical technique, you know. I mean essentially doing error minimization while fitting. So, a trigonometric value problem is being converted into an algebraic eigenvalue problem.



So, you see there is Q_1 and Q_2. So, make the determinant zero, and you know that in this eigenvalue problem, the determinant must be zero. So, solving this characteristic equation, you will get $N_{xy}^{critical} = \frac{9}{32}\pi^4{}_{D}ab(1/a^2 + 1/b^2)$. So, that is critical. You see that one estimate of the critical load you are getting by doing so. and the respective mode you have already pre-determined right. So, for a single lobe and then for both, you have to find the values of Q_1 and Q_2, substitute them, and then plot the eigenvectors. And then you plot it, and you will get a surface. So, you will first mode, second mode of course here also you will see that you'll get it, you know, the first mode will be the symmetric and the second mode, you know, substitute and see what you get. Because from the algebraic eigenvalue problem, the eigenvector will give a relative value Q_1 Q_2. So, if the first vector is positive, both will be positive, and the second one will be positive and the negative, okay? So, then some asymmetry will be there; in the second mode, maybe you check it. Okay, I don't. So, this is, of course, the simplest approach to solve, given the fact that, you know the integral of the standard integral in terms of this signal function; you can directly substitute it. But yes, of course, if you want to include more terms, then you have to, you know, take help from a computer program, okay? But nevertheless, this, you know, expression is not a very bad approximation. It's a reasonably good, you know, approximation of the actual or exact load. Okay. So, where do you see shear buckling? Could you please give me some examples of where you see shear buckling? Of course, one example is that shear buckling is very common, and we all come across it while designing our plate guard, right? Plate girders have very long webs, so they are susceptible to shear buckling. And if you can recall, in steel design there is a provision that states if you know V/V_D, the shear force that is coming and the design strength you know is less than or equal to 0.6, but that is for a different reason. That two cases for the design of a beam, steel beam: one case is if V/V_D is less than or equal to 0.6; another is if it is greater than 0.6. That is something different. That is because of the influence of shear deformation. If the shear is a little too high, then it will affect the behavior; there will be significant shear deformation. And you have to take into account that one, but there are other provisions that we need to satisfy for the. Um, for the design of plate buckling, you know, plate design of, you know, plate girders, that you have to check for shear buckling. Okay. So that is one example, but there you will see a more common example; shear buckling is also exploited, you know, as a sacrificial device for energy dissipation in seismic-resistant structures. Uh, those of you who are working with Professor Durgesh Roy's group, you know, you'll see that he has developed the shear-yield panel. You know, still that shear-yield panel. So, you will see that, and then that is aluminum. So aluminum was the material that was used. But in not of the common aluminum, we have aluminum they have taken and then they do anneal, annealing on aluminum, do annealing that it reduces, there are some crystal re-orientation microsctructural things that happen. It reduces yield strength, okay. So, when it reduces the yield strength, you will see that there are stiffeners. If you go to the structural meaning lab, you will see those shear-yield panels, okay. So, when you know, then you install it, you know, in building. Okay. You will see that, the way it was installed, you know, consider that there is a bay. So, you know, maybe this is consenting; the braces will be there. Okay, and there the shear panel is. Okay, So, if it is subjected to lateral loading and cyclic loading, then what will happen? This fellow will be subjected to shear loading, right? Isn't it? Huh? And under shear loading, if it has low yield strength, then it will yield, and if it yields, then it will dissipate energy. Of course, it is not the monotonous loading; it is cyclic loading, and because it is cyclic loading, you'll see that energy dissipates. Something like that. So, the area under the loop will give. This is the hysteresis loop; that's what you will see there. Okay. So, the area under the loop, you know, in each cycle, it is dissipating energy. So that's basically helping in dissipating the input energy coming from seismic excitation. So, it's a control device, a passive control device. Okay. Now, the interesting thing is that you have to prevent it in order to be doing the experimental method correctly, right? Did you do that experiment? Inelastic buckling of a brace. Inelastic buckling of the brace. Huh? You have not done it yet.



So, you'll see that for the inelastic buckling of a brace, if you just take a simple brace and then allow it to buckle inelastically. You know, the tension will definitely be okay. But on compression, it will come and then it will go once again. Why is it not able to come? Because it is buckling prematurely, it is buckling prematurely. So, if it buckles prematurely, it's a geometric instability. It does not allow it to mobilize its full strength through wielding. So that's what you have to design in order to use it optimally. You have to provide, you know, a stiffener set so that it doesn't buckle prematurely or cause the plate to buckle. So aluminium plate buckling, of course, involves inelastic buckling, and yielding will, you know, precede the buckling. So, it is inelastic buckling, and there are formulas given by Gerald and others that they use. But this is one example of shear buckling; why shear buckling is utilized, you know. Not buckling is utilized; rather, yielding is utilized, and then you have to make sure that it doesn't buckle prematurely under shear. So, now we are going to consider post-critical behavior. So why have we considered? So, you see the way you are progressing over the course, you know. We have considered a very simple example, a toy example, and four different systems you know which represent four classes of instability behavior, right? We have considered, you know, critical load and then post-critical behavior, whether it is stable, unstable, or imperfection-sensitive, or things like that, right? Modal interactions and things like that, right? Now, we have demonstrated those simple behaviors, as illustrated by each of the toy systems in one practical example. We have considered the buckling of a column. And we have considered the post-buckling of the column. We have seen that the column doesn't have any significant post buckling strength. Even if the end rotation is 30°, it's only a 3 to 4% increase in post-critical capacity. So essentially, for the column, as soon as it buckles, it depletes all its load-carrying capacity. That is not the case with plate. So, plate has some interesting behavior. Plate has significant post-critical strength. Even if the plate buckles, it doesn't lose its capacity to carry load, and that is utilized, that is exploited even in code, you know, in design, and then code stipulations are made accordingly to exploit that. So that's what we will see: what is the mechanism, and what is the science behind that. Okay. So post-critical behavior of the plate. Now we have seen the governing equation for the plate buckling, that is D. Another 5 minutes, right? Hmm.



So, we have seen this equation that we have utilized for plate buckling, right? And here we have assumed that N_x, N_y, or N_{xy} are constant, okay, constant across the plate, huh? They remain identical, so there is no variation of N_x, N_y, and N_{xy}, and that's what we have found, the critical value of N_x, N_y, and N_{xy}, right? But as soon as a plate buckles, what will happen? This N_x, N_y, and N_{xy} will no longer remain constant over the plate. They will redistribute. And why redistribute? Because in the column, there is no provision for redistribution, as it is one-dimensional. But in a plate, it is two-dimensional. So, there are provisions for redistribution. However, two dimensions don't mean that it will always redistribute in a shell; there is nothing like this. But in the plate, it can redistribute, which means N_x, N_y, and N_{xy} can vary over the plate. You see that domain, so that is the first thing. So N_x, N_y, and N_{xy} are no longer constant. These fellows are no more constant; they vary, so the inflamed forces in a plate vary beyond after buckling. Once it buckled, it started redistributing. Okay. Reinforce when their plate varies. Okay. They start redistributing. So, if they start redistribution, then of course you have to enforce that the equilibrium is maintained for this reason. The equilibrium of in-plane forces must be considered for post-critical behavior. Post-critical analysis. Post critical analysis of the plate, you understand what I mean, right? We need to consider the out-of-plane equilibrium. Essentially, this is nothing but the equilibrium equation along the out-of-plane direction, which means along the Z-axis vertical direction, right? In-plane forces, the equilibrium of in-plane forces never arises because if N_x, N_y, and N_{xy} are constant, right? They're always in equilibrium, right? In both the reaction, they are under compression, right? But even though... When you buckle the plate at the end, it is always N_X; in a global sense, N_X and N_y are always maintained. These two need to be the same, right? PP, right? They are compressed, but inside the plate, away from the plate boundary, the in-plane forces get redistributed. So, a point that is away from the boundary, in the plane, must take into account the equilibrium of the forces along the in-plane direction. So that is the primary difference and then the main difference, between the buckling analysis or critical analysis and the postcritical analysis. and here it will show portray significantly different interesting behavior, that we going to explore. Okay. Okay. Next class, we will do it. Huh. Thank you.