Stability of Structures

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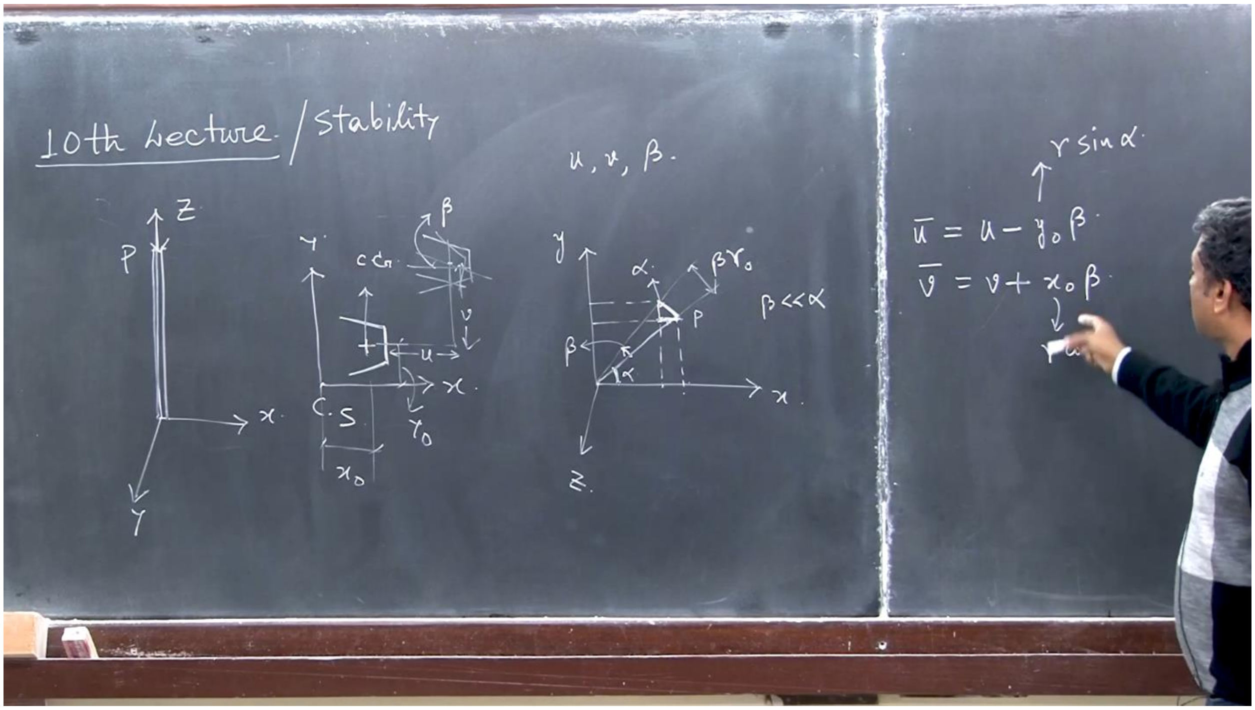
WEEK-05

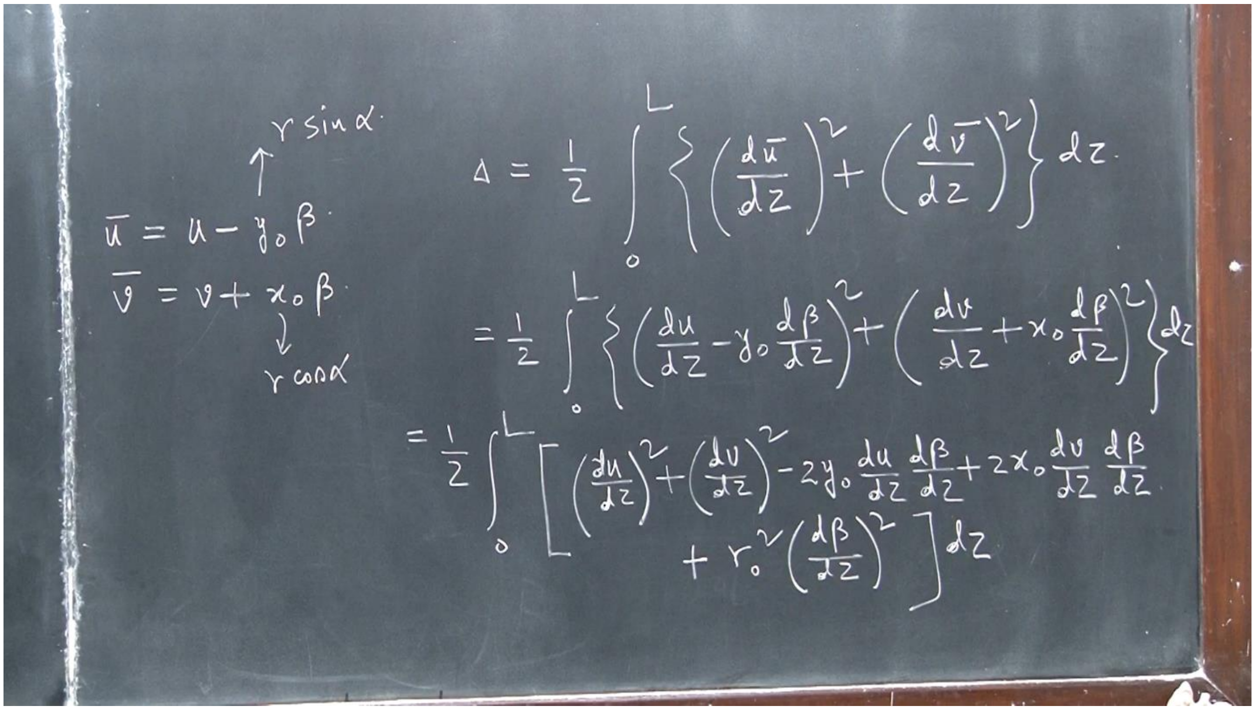
Lecture 10: Lateral Buckling of Beam

Welcome to the 10th lecture. So, what we are discussing, let us briefly recapitulate. So, we started with the torsional flexural buckling of columns, and we have emphasized the role of warping of cross sections, you know, open sections, especially in the world of open sections, okay. the design, which is many times governed by this torsional flexural buckling. So, we have derived the equation for the strain energy. So, strain energy, if you can recall, was contributed by the two translations u and v, as well as the twist, right? And then the twisting degree of freedom, which is the β twisting angle, was basically considered while deriving both components of torsion: one is Saint-Venant torsion and the other is warping torsion, right? And then the next step, since we are following the energy approach, is to derive the potential energy function. So, the strain energy component we have already derived, okay. I will write down the expression shortly. Then we are working with the expression for the work done. So, the expression for the work done, if you can recall that, was the choice of the coordinate system x, y, and z, right? And here was this column, and it was subjected to this load, right? And then, if you can recall the cross-section, the way we derive this x and y, we assume that, well, we have this kind of cross-section, right? I mean some arbitrary cross section, an open cross section, and then we have chosen the origin of the reference coordinate system to be the shear center, and then the CG is here. So, the CG has coordinates x_0 and y_0, and along with that, there will be two components of displacement: one is u along x, and the other is z along y, right? And here, there was this section that went there, and then after that, there was this rotation, right? β, okay. So, from this rotation, the angle is β; this angle is up to β. So, that was probably deformation, right? So, u, v, and β are the three, where u is the deflection due to bending, v is the deflection, and then β, right? So, then what we have emphasized is that because of this β, u and v will be modified, right? Because when we are considering a section, sorry, you consider this as x and this as y, and if you consider point B, P here, this is point P. You know, by twisting, if this angle is β and is taken as β, then this square k has been approximated by the line, and this length is nothing but β into r, or β into r is 0, rather. And then I am considering, and then we have seen that we have dropped a perpendicular here, and we have also dropped a perpendicular on the y-axis, right? So, if this angle is α, then this angle was α; this angle is also α.



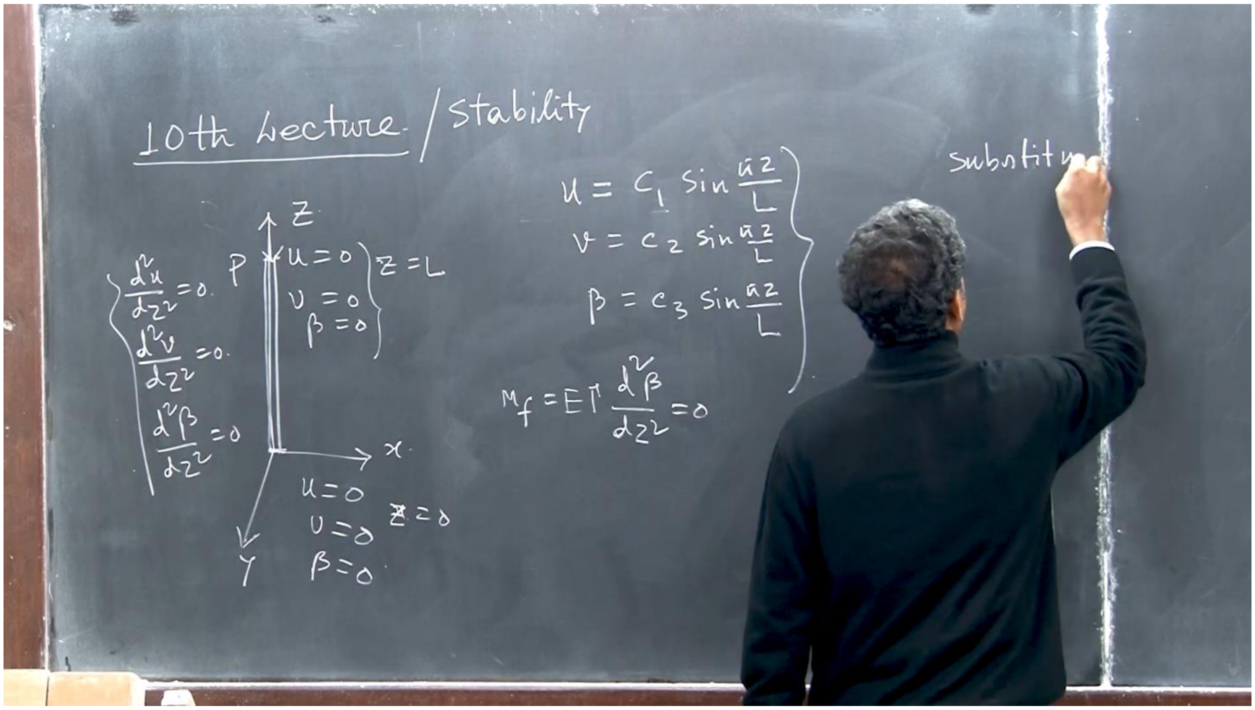
And then we have seen that u and v are the pure translational parts, right? So, because of β, the twist β, you know, u and v will be modified slightly. So, U will be modified as $U - y_0\beta$, and V bar is $v + x_0\beta$. How is it happening? Because you see that β, if we turn anti-clockwise to the right, then the positive x direction is here, okay? So then, this is causing a negative component of deformation at point B in the x direction; that is why the deformation in x basically reduces, right? Whether for V, it is increasing; that is why $V + x_0\beta$ and how x_0 y_0 is coming? Because you know if you take this, see this one is nothing but $\beta 0$, $\sin\beta$. So, $r\sin\beta$ is nothing but xy_0, right? $r\cos\beta$ is nothing but y_0, right? And $r\sin\beta$ is nothing but x_0, right? So, that way we have derived it, haven't we? Now this y_0 is coming, the condition that y_0 is $r\sin\alpha$ and x_0 is $r\cos\alpha$, if you can recall correctly. This angle is α; this angle is α; this angle is α. So, from here to here, I mean β is very, very less than α, right? For any finite α, right? So, that is what the component of $r\sin\alpha$ is basically; you know this is $r\sin\alpha$, this one, ok? This is nothing but y_0 $r\cos\theta$, fine. So now why are we doing this? Because the vertical deflection, the deflection along the z direction, can be obtained like this, you know. that is $\frac{d\bar{u}}{dz}, \frac{d\bar{V}}{dz}$ right, so, $\frac{1}{2}\int_0^l \left[\left(\frac{du}{dz} - y_0\frac{d\beta}{dz}\right)^2 + \left(\frac{dv}{dz} + x_0\frac{d\beta}{dz}\right)^2\right] dz$ right.

So, Here, $\frac{1}{2}[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2 - 2y_0\frac{du}{dz}\frac{d\beta}{dz} + 2x_0\frac{dv}{dz}\frac{d\beta}{dz} + r_0^2\left(\frac{d\beta}{dz}\right)^2]\, dz$. Right, it is fine, okay.

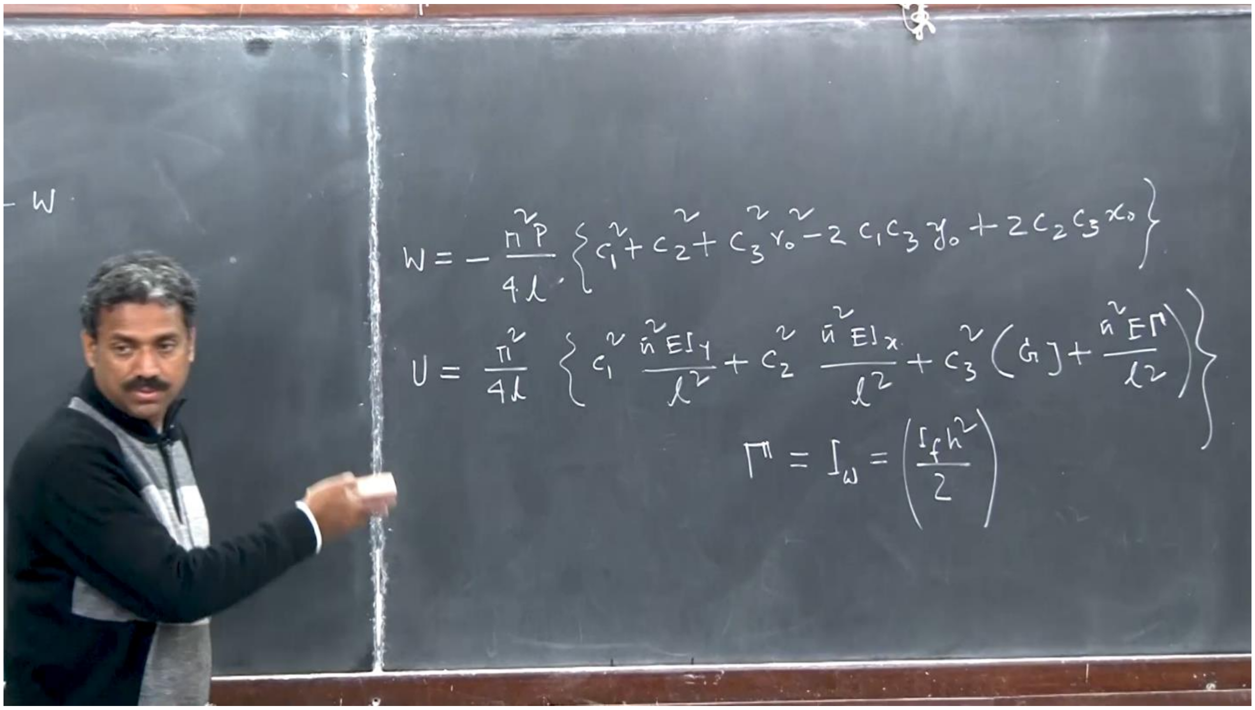


So now we will see that let us substitute. And see. So, what I will do first is find δ. We have assumed d is equal to, as you have seen, what the boundary condition was, and we have assumed that these are kind of a restraint against translation. So, $u(0) = 0$, $v(0) = 0$, $u(0)$ and $v(0)$ at $x = 0$, $x = x$, $z = l$, $z =$ right, and then β is also 0, right? β is zero, and then we have also assumed that it is free against warping at the end. So, $\frac{d^2\beta}{dz^2} = 0$, right? So, if you can recall that this was the boundary condition, $\frac{d^2u}{dz^2} = 0$, $\frac{d^2v}{dz^2} = 0$, $\frac{d^2\beta}{dz^2} = 0$, why? Because these are coming, it is rest, and the bending moment should be 0, right? Then it is not rest and against warping. So, not rest and against warping means, you know, what was the rest warping moment? $M_f = E\,\Gamma\frac{d^2\beta}{dz^2}$. So that must be 0, right? You see that was the reason why $\frac{d^2\beta}{dz^2}$. So, by this, you have assumed c_1 to be $C_1\sin\left(\frac{\pi z}{L}\right)$, V is equal to $C_2\sin\left(\frac{\pi z}{L}\right)$,, and $\beta = C_3\sin\left(\frac{\pi z}{L}\right)$,. This you see that, ok? So now you substitute that, okay? Substitute it there, and then what you are going to obtain is that final expression I am writing.

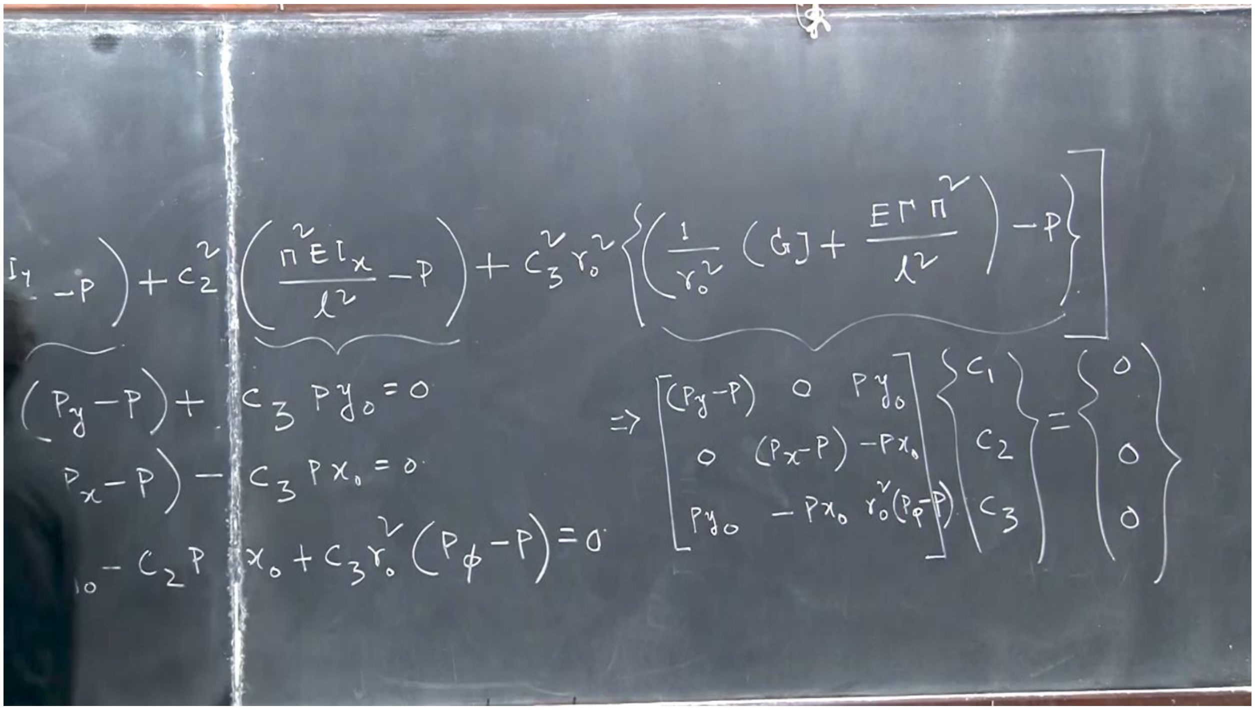
The standard integral is coming, and then you will see that the work done will be, and then you have to multiply by p, okay?



So, then $-\frac{\pi^2 P}{4L}[C_1^2 + C_2^2 + C_3^2 r_0^2 - 2C_1C_3Y_0 + 2C_2C_3X_0]$, you see that X_0, Y_0, and R_0, you just integrate it and you got these expressions fine. What are X_0, Y_0, and R_0? X_0 is nothing but the coordinate of the CG, and R_0 is the what? Radius is here, so that polar distance, right? Okay, that is the r_0^2. In fact, if you integrate over this area, you know, so x_0, y_0 are the coordinates and r_0 is for the present time; we are taking the square of x_0 plus the square root of y_0, okay. So that is r_0^2, and this is also nothing but the polar radius of gyration, right? Anyway, once we have done that, then what we will do is let us see. Now, this is work done and strain energy. If you can recall, we have already derived the strain energy expression. $\frac{y^2}{4L}$, this one we have derived $C_1^2 \frac{\pi^2 EI_Y}{L^2} + C_2^2 \frac{\pi^2 EI_x}{l^2} + C_3^2 \left(GJ + \frac{\pi^2 E\Gamma}{l^2}\right)$. X_0, y_0, and r_0 are okay; r^2 is nothing but $0^2 + y_0^2$. Please recall this; later I will come there, okay? One second. Please note that capital Γ, you know, is nothing but what? This is the warping constant, right? So, the warping constant is sometimes defined in literature as I_w, and for the I section, it was what? $\left(\frac{I_f H^2}{2}\right)$. If you can recall, I_f is nothing but the moment of area of the flange, right? And H is basically the depth of the wave, right? So that was okay. So now we obtain the total potential energy; so π is nothing but what? $u - w$, right? So, you can write down whatever the full expression of π is.

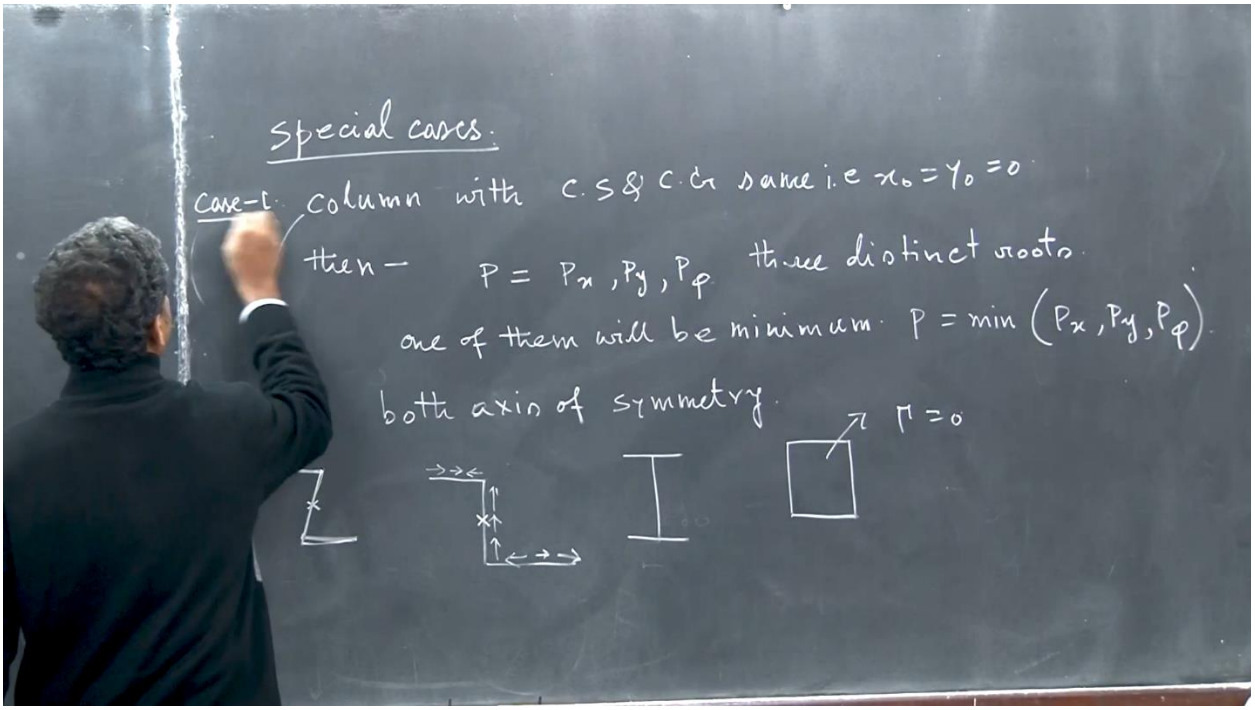


So, I am going to write it, and through some simplification and combining the terms, that combination I am just explaining to you, okay? I am removing these two parts, right? All of you have noted, right? So why am I writing in this combination? Because You see what I am doing with the coefficients C_1, C_2, and C_3, which appear in the work done as well as in the strain energy expression. So, I am just, you know, combining the term "ok," and then does it remind you of something? It will remind you of something, ok. So now I will solve it. We will do energy minimization. So $\frac{\partial \pi}{\partial c_1} = 0$ will give you one equation, you know sorry $\frac{\partial \pi}{\partial c_1} = 0$, $\frac{\partial \pi}{\partial c_2} = 0$, $\frac{\partial \pi}{\partial c_3} = 0$. So, if you do, when you differentiate, we get three expressions, okay. You see how we are getting the eigenvalue problem. So, what we have started with gives the same eigenvalue problem for a non-trivial solution. So, we are looking for an alternative equilibrium solution. The way we followed the simple system is the same approach we are following; the only difference is that here it is a real system. We started with the flexural buckling; now we will start with the torsional flexural buckling of the column. So, this leads to a homogeneous system, which is an eigenvalue problem, right? The determinant must be 0 for a non-trivial solution, and that is where we give the critical load, okay? So, if the determinant vanishes, then you see that it gives and that will yield the characteristic equation and things. Now, what we will say is that we are going to see if there is any general case for x_0, y_0, and non x_0 for any arbitrary cross section, right? x_0, y_0, r_0, not equal to zero, right, and then we can get the critical load P.

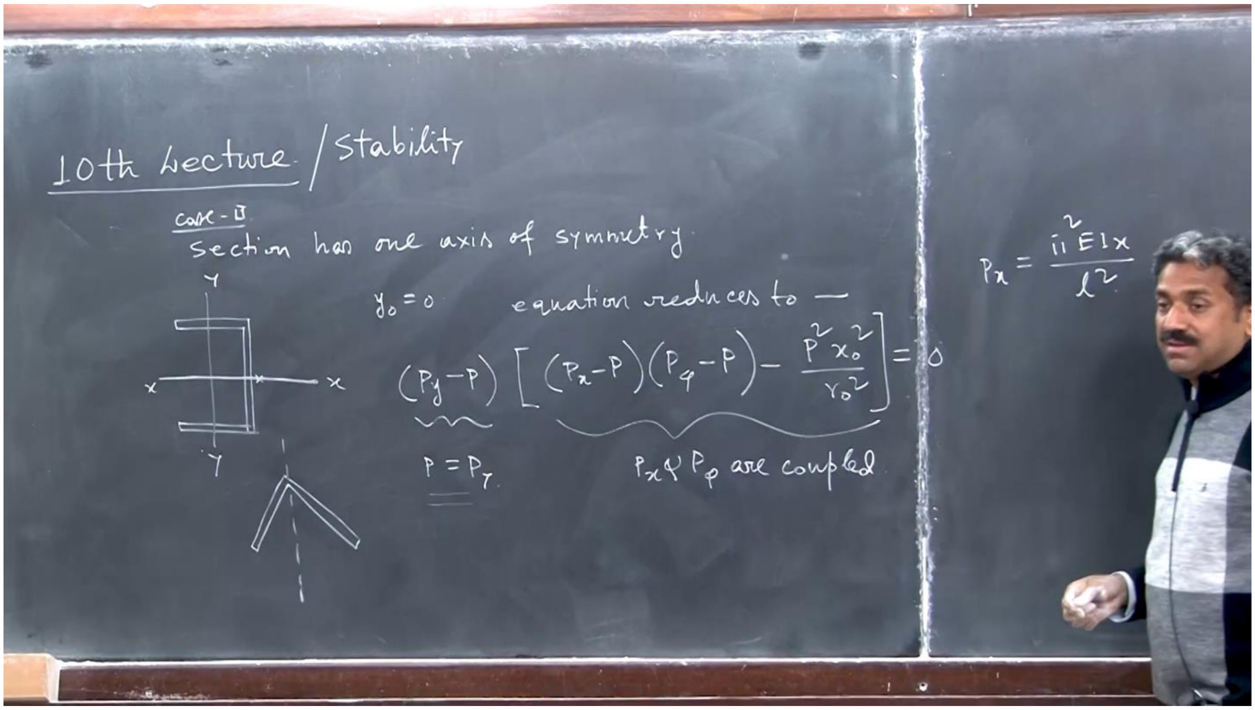


We can get critical P, solve for P correctly, and then that P can be expressed in terms of P_x, P_y, and P_ϕ. So, when I am writing here, what are P_x, P_y, and P_ϕ? So, these roots, you can see what P_x, P_y, and P_ϕ are, I am writing. Here, whatever we have written is $P_y = \left(\frac{\pi^2 EI_x}{L^2}\right)$, $P_\phi = \frac{1}{r_0^2}(GJ + \frac{\pi^2 E\Gamma}{L^2})$, $P_x = \left[\left(\frac{\pi^2 EI_y}{L^2}\right)\right]$, So, you see, these are the pure flexural buckling with respect to the x-axis, with respect to the y-axis, and this is pure torsional with respect to the x-axis. Here, pure flexural with respect to y. But please note that P cannot be alone; P_x, P_y, and P_ϕ must be included. It will be a combination for any arbitrary section where x_0, y_0, r_0 is not present; then P can be expressed as a combination of P_x and P_y. This P_x, $\frac{\pi^2 EI}{l^2}$, these are all familiar expressions to me; this refers to the uncoupled mode, basically, you know, the flexure mode. This is also the flexure mode; there is the y-axis, and this is the torsional mode. Please note that. See for the saint venant one, because it is equivalent to axial; that is why $\frac{GJ}{R^2}$ is coming okay, and here you know $\frac{\pi^2 E\lambda}{L^2}$, R_0^2

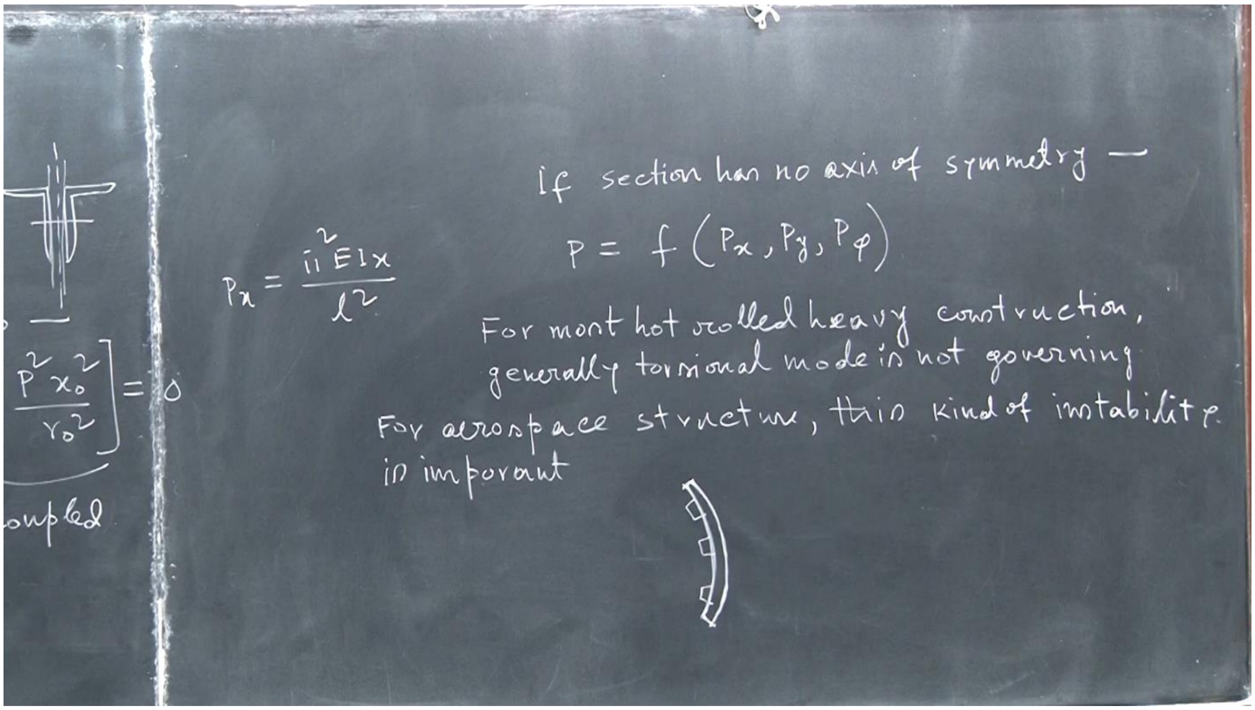
is that gyration okay. And $\pi^2 E$ capital ω, that is a similar expression to that flexure because of the warping moment, right?



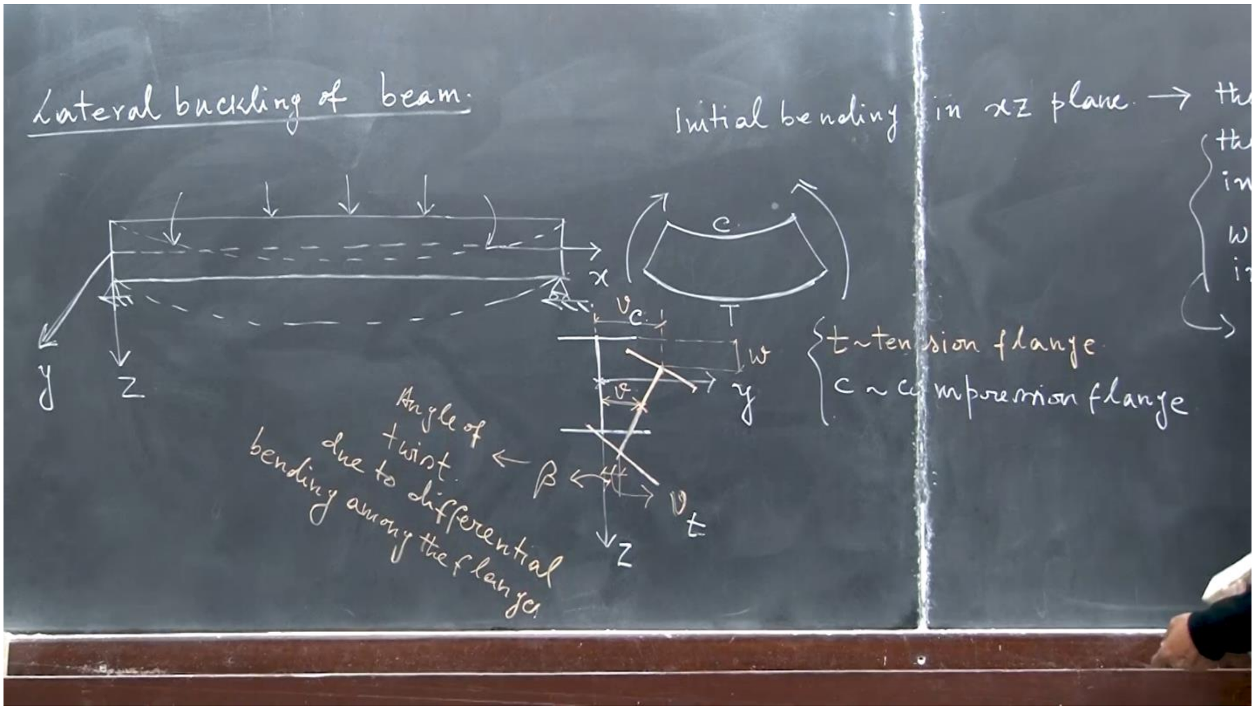
And that is why it is similar, you see, to that expression. Now we will consider special cases to see what really happens, okay. So special cases when we are considering. Column with shear center and CG. the same means $X_0 = Y_0 = 0$. x_0, $y_0 = 0$ and $r_0 = 0$. So, what happens when P is, so there will still be 3 rules, but P will be the minimum of these 3, okay? P will be, you know, So, then you will get 3 distinct roots for P. P will have P_x, P_y, and P_ϕ. So, three distinct roots, and it will buckle here; one of them will be a minimum. So, that means, it will buckle under which the critical load is minimum right, you see that under the value for which it will be minimum, so P will essentially be the minimum of anyone P_x, P_y, and P_ϕ; the minimum of this will be T, okay. But most of the high sections, hot rolled high sections, you know, especially the wide flange high sections, and it always happens that flexural is the minimum, okay, and it buckles in flexural mode, okay. You see, because it has both, if they are the same, that means it has both axes of symmetry; it has this kind of column with both axes of symmetry. You see that they have both axes of symmetry, x_0 and y_0, okay. So, it will buckle a minimum of any of the three, and in most cases for hot, basically wide flange hot rod sections, you know the flexural one is the minimum; that is what it buckles, okay. There are other examples also, you know. So, for that section, you can also see that for the Z section, Z is the shear center and the CG are the same. So, what is the shear flow diagram for the Z section? The shear flow diagram you will see for the Z section, if you consider, you know, from here whatever is coming, will be different, okay, and then from here.



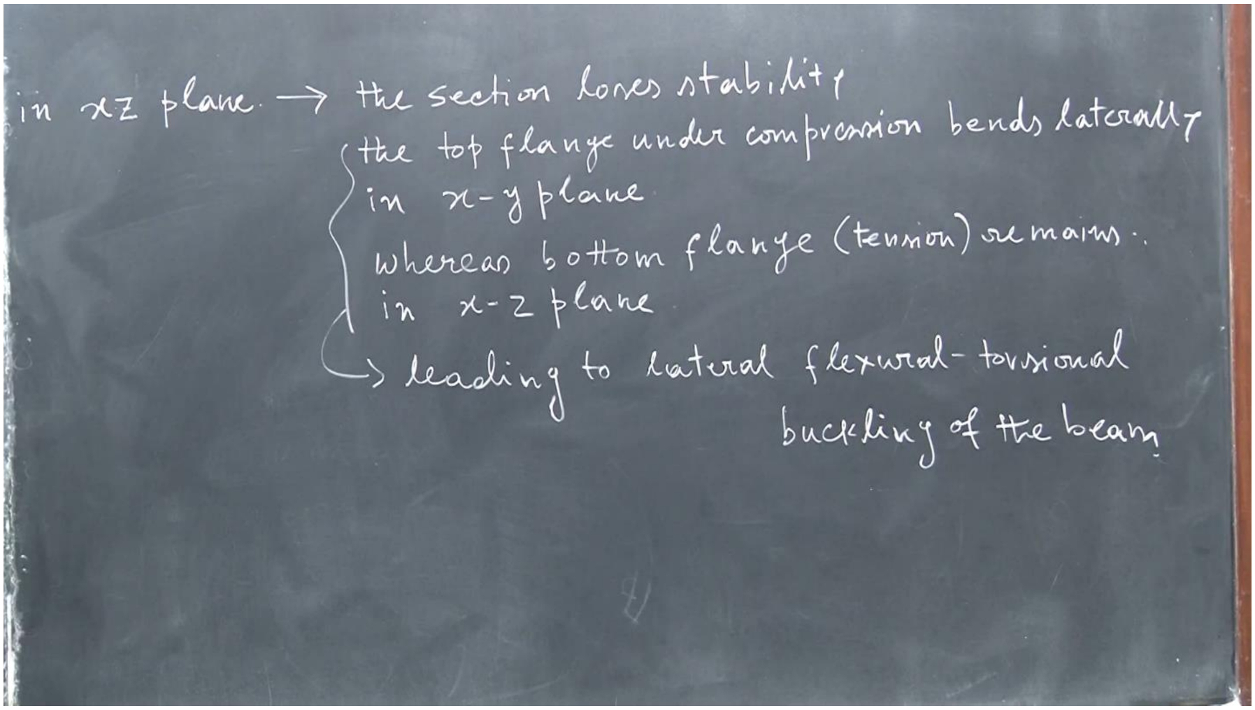
So, CG and this thing are the same, and then it will have that torsion, okay, and then there will be an I section; the I sections are both the axis of symmetry, right? of course, if you want to consider the rectangular section, what happen is that open section rectangular a closed section then warping is very negligible. So, you know the warping constant, capital ω becomes 0 for this. For this kind of capital ω, it becomes 0, and then this term will not be there, okay? This is very low working. Now, another case is where you know case 2 section. So, case 2, and this is case 1; case 2 is a special case 1. Case 2 is where the section has one axis of symmetry. So, it has one axis of symmetry, which means either x_0 or y_0 is zero. So, if we consider this, then it will be y_0. Consider that for a channel section, it has one axis of symmetry. If we consider this to be x and this to be y, the shear center will lie on this axis of symmetry, you see that. So, for here y_0 will be 0. If y_0 is 0, then what will happen? So, the equation reduces to the characteristic equation you know. The equation reduces this, so here you see that one root is, of course, $P = P_y$, and the other is a combination of P_x and P_ϕ, right? P_x and P_ϕ are coupled. Therefore, because there is a coupling between them, it cannot independently buckle with respect to the x-axis. What is P_x? If you can equal P_x, what was it? P_x was. $\frac{\pi^2 EI_x}{L^2}$. So, with respect to the x-axis, okay, with respect to, you know, sorry, so it will undergo flexural because it has one axis with respect to the axis of symmetry; it can independently buckle. It does not have torsional coupling, but as soon as it tries to buckle with respect to y, then it will be combined. So, it is torsional coupled in one way. The flexural buckling with respect to the y-axis and torsional buckling are coupled, right? So, it can either buckle independently with respect to this axis or a combination, right?



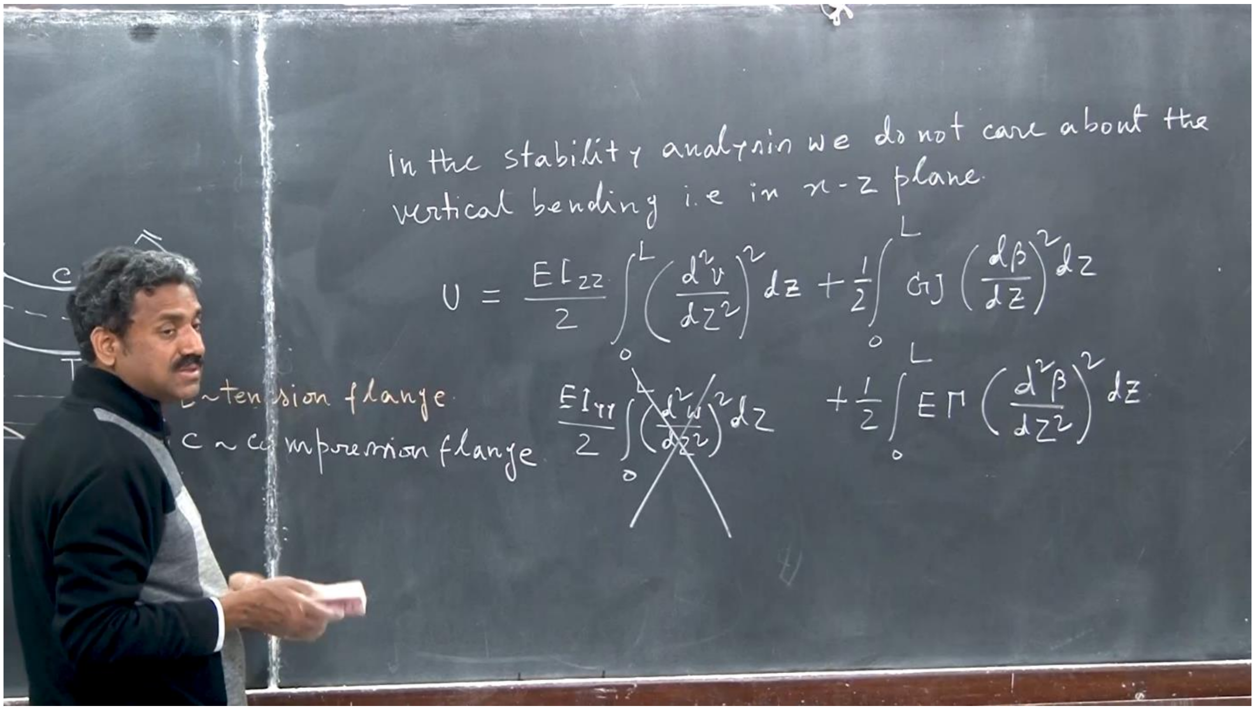
So, this is one example for the channel section, and then there are other examples. For example, if you want to consider a toy you know in a hat kind of section, right? Then you know this is also one axis of symmetry, right? Okay, so there are other examples. If the section has no axis of symmetry, then of course it has to be solved in terms of P_x, P_y, and P_ϕ. It has to be solved; you know P has to be solved in terms of P_x, P_y, and P_ϕ, okay? Because then all torsional and both flexural modes, as well as the torsional mode, are coupled. At a minimum, you will get whatever the load is by solving this characteristic equation, okay? So, you would say that for most of the hot-rolled sections, in heavy construction and structural construction, the torsional mode is not that important, but this is really true for aerospace structures. So, for most hot-rolled heavy construction, that is what we encounter in civil rights. Generally, the torsional mode is not governing. Here, the flexural modes are governing. However, for aerospace structures, this kind of instability and static instability is important. So, these are very common. So, for example, you know what happens with aerospace vehicles; if you consider the fuselage, then there is, you know, this fuselage made of a thin kind of shell. Of course, these are all stiffened shells. Because you can recall that these are all stiffeners, both the vertical stiffener and the longitudinal stiffener will be placed. But here you see this is, of course, here we consider that way; only this section will have one axis of symmetry. So there, for this kind of thing, torsional flexural buckling is common, okay? and that become the predominant design consideration for this. Anyway, so you understand that, for the torsional flexural buckling, we consider, if you see in civil design, only in one case that you will find this kind of. I do not know whether you can recall that design of the compression member if you consider only the Indian standard to be good. Design against torsional flexible buckling if only one angle section is connected to one side of the gusset plate. Then you have to check for the torsional, because there will be torsional flexural buckling, and there are some formulas given in the Indian Standard code. Unfortunately, the problem is that that formula is adapted from a conference paper published by IIT Madras. You know, fellow, this is not even validated or verified through experiments done at the lab; this is, you know, I was just sharing when one of the experts was saying this is for cold form sections. They have checked that and fitted that model. But that, they have put in, you know, in the hot roll section still designed for, I mean that there is inconsistency; you will not see that kind of formulas in other code, okay? Other code they do not do it that way. But you know, nevertheless, they keep a provision for checking against torsion; other than that, you will see that in most cases, the way we connect the I section around this gusset plate. We do not check for the torsional flexural one, okay? Because they are symmetric, they have either one plane of symmetry or both planes of symmetry, things like that, you, see? So, if you say two I section, which are connected back-to-back, on two sides of the gusset plate, that will have one axis of symmetry of the cross section, right? You see, what I am trying to tell you is that this kind of arrangement is like this. So, you will have this gusset plate; you know, in between you have this gusset plate here, mostly sectionally. So, it has one axis of symmetry; you see that flexural, and that is the load that will govern your flexural buckling, okay? You do not take it, but for aerospace vehicles and to see aerospace structures, these are very common. So anyway, now we have completed torsional and flexural buckling, okay. And we have seen that. So, this approach is similar; I do not know whether you are all doing earthquake engineering, and we have done structural dynamics, right? So, torsionally coupled structures are important in earthquake engineering, and there are total provisions for that in IS 1893. When you are in a building that is torsionally asymmetric, it means your center of mass is not coincident with the center of rigidity or the center of stiffness; therefore, the flexural mode and torsional mode are coupled. So, For example you know, So the same thing, so there you will see that you consider a building, and then assume that you have a shear, heavy shear wall in this two side, And then you do not have much thinner things, you know, these shear walls are maybe, then you will say whatever is the center of mass and your center of stiffness is towards that. So, you will have this eccentricity, right? So, this is the center of stiffness. The way here, the center of shear is whatever role is played by the shear center and CG; there is the CG, and here is the center of stiffness or center of rigidity. So, then this building becomes torsionally coupled, and when you need to get torsionally coupled, then of course you need to, you know, increase the demand by some percentage; from the other side, you see that, okay. And then, in seismic design, you always have to consider some minimal eccentricity, okay. 5 percent or whatever right there are formula in the code right 1893. So, because of the take, there are two take care of the accidental eccentricity ok. So, things are similar; torsional flexural coupling in buildings, in earthquake engineering, and here also torsional flexural buckling, okay. But the special cases are symmetric; one axis is symmetric, two axes are symmetric, and things like that, okay. Now what we are going to consider next is the lateral buckling of the beam. Happens, so we have just concluded the torsional flexural buckling of the curve. Lateral buckling of the beam, when it happens, is observed when you consider a beam that is subjected to a transverse load, right? It is bending in the vertical plane. Now what happened is that the top flange is under compression and the bottom flange is under tension. So, tension does not induce any instability, but under compression, the top flange tries to buckle. Therefore, the top flange will try to buckle, which means it will undergo out-of-plane deflection, while the bottom flange will remain in its plane. That means there is a difference; there will be a differential kind of displacement between the two ends. That will lead to, so that must be accompanied by twist. That is why you should also consider this in the design, specifically in steel design, when dealing with laterally unrestrained beams. You can recall that, at that time, you had to see that allowable bending stresses reduce significantly depending on the classes of imperfections you consider and then the slenderness ratio; those are the two prime factors, right? So that thing we are going to is a very complicated formula, especially for the case where there is a gradient in the bending moment.



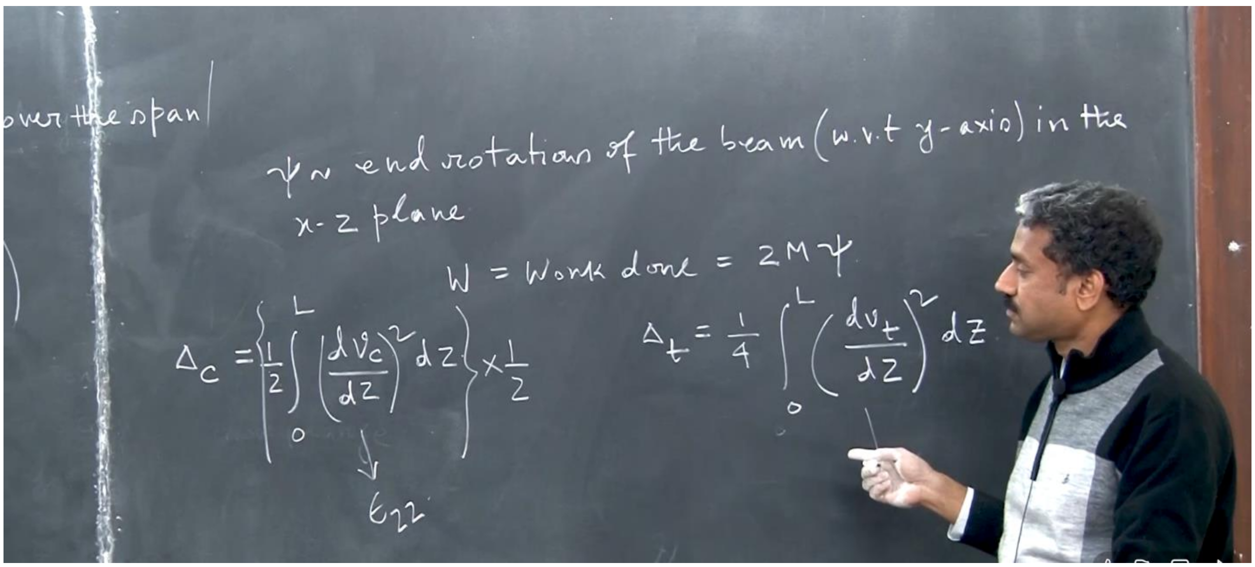
So, the bending moment is not uniform; it is a pure bending moment. You know uniform bending moment, then pure bending, and this, you know, the formula was simple. But if it was, you know there are shear forces and then there is a gradient in the bending moment diagram, then a complicated formula was there, right? We will try to derive the formula, which is still design code okay for the simple one, and we will see how it is coming along. So, what is really happening here is what you are considering. So, here is a, you know, beam. Okay, beam section, you see that? Maybe here is the solid simply supported. Okay. So, what happened then under transverse load, or if there is some bending? Okay, then what is happening? That this fellow will remain in the top portion is under compression, you see that. So, these are in compression, so the top portion is in compression and the bottom portion is in tension, right? So, under tension, the bottom plane is going to destabilize, but under compression, the top plane is going to destabilize. So, what is going to happen then? This fellow will remain in its place, but the top one will come out of plane deflection. You see that? This one, and then there will be lateral. So, what is happening? I am going to consider this one. Z is maybe and this axis is Y, and then along this, this is the X axis.



So, what I am going to consider is that the vertical plane is the xz plane, okay, yes, right hand in the quadrillion system. So, you see what is happening, initially the beam was remaining in equilibrium under vertical load, right? The bending was essentially happening in the XZ plane. The initial bending, so the initial bending in the XZ plane, was it happening, right? Then, beyond a critical bending, we are going to consider that it is subjected to pure bending, which means the uniform bending moment, because that simplifies our case. Then what is happening? This fellow is going to remain in its place, this flange, but then the top flange is coming; there is lateral bending in the xy plane, okay. So, what is happening? You see that the section loses stability. How? You know the top flange under compression bends laterally; that means in the XY plane, right? Whereas the bottom flange, under tension, remains in the XZ plane. So, this leads to lateral flexural torsional buckling of the beam. So essentially what I am going to draw is, you know, in the yz plane what is essentially happening. So, this is y and this is z. So, the top flange is going to deflect; this fellow, see, of course there is initial deflection, you know, in Z, but that we are not considering. So, this fellow is going here. So, this is the vertical plane W, and this is the top flange V, and this is along z; sorry, this is along y, and this is along z. Here, this is the top flange V, and this is compression. And then whatever this is, from here to here, this is what we are, you may be considering as, you know. So, you understand why this fellow is, why this twist is happening.

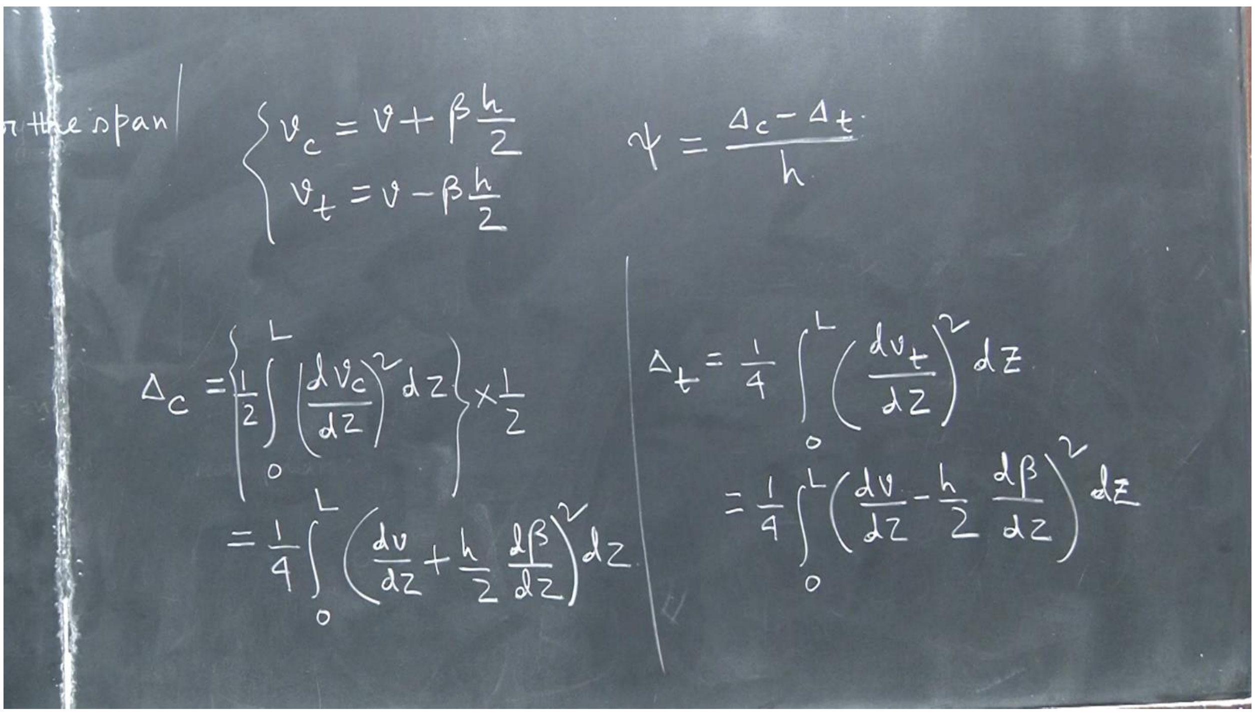


This angle is defined as β. β is the angle of twist, and this angle of twist results from differential bending in the flange, right? T is for tension, C is for compression, the top T is the tension flange, and C is for the compression flange. So, in the analysis, what will we do? In the analysis, we do not care about the vertical bending. Okay, why? Because in the stability analysis, we do not care about the vertical building that in xz plane. we do not care about the vertical building in the x-z plane, why? Because this is so, what do you mean when I say caring? We do not include the potential; we do not include the contribution of strain. And your vertical building in analysis, because of instability as far as vertical bending is concerned, other than inducing compression and tension in the top and bottom plane, it does not have any other role. The process of instability does not have any other role. Because in the vertical bending, the beam was under equilibrium with the transverse load, you know, prior to instability buckling, right? So we are not going to consider its contribution to the strain energy, okay? We are only going to consider the contribution of strain energy for the lateral buckling. So, this is lateral buckling, you see that. Some bending is happening in the what? xy plane, xy plane. This is bending in the xy plane, right? So, in the xy plane when it is happening, you know, let us see what will be the expression for the strain energy. So, the expression for the strain energy, you know, the strain energy expression we are going to consider is, of course, the you know EI because of lateral bending, that is the bending with respect to which axis, bending with respect to z axis, lateral bending is happening. So, $\frac{EI_{zz}}{2}\int_0^L \left(\frac{d^2v}{dz^2}\right)^2 dz$, right, is not it? V is basically the component for the lateral displacement, right? And that is what is essentially happening with V, you see that. So, this one is the, and then, of course, you will have this component, which is $\int_0^L \frac{GJ}{2}\left(\frac{d\beta}{dz}\right)^2 dz$. And then, in addition to that, what we are going to consider is $\frac{E\omega}{2}\int_0^L \left(\frac{d^2\beta}{dz^2}\right)^2 dz$, you see that? This is the flexural; you know this is the warping contribution by warping torsion because it is, anyway, when you know open section and then I section, right? So, this is because of warping torsion; this is Saint Venant torsion. You know this is the flexural component, right? So, all of you are wondering why I am not considering the contribution for the vertical building. Why I am not considering $\frac{EI_{YY}}{2}\int_0^l \left(\frac{d^2v}{dz^2}\right)^2 dz$ is because it has no contribution as far as losing stability is concerned. The vertical bending does not have; the bending in the vertical plane does not contribute to the instability process. Of course, it is contributing to the instability by causing that kind of, you know, stress condition which is conducive to instability, okay. This point you must understand clearly before proceeding; do you understand that, okay? So, when you say that I am not considering $\frac{EI_{yy}}{2}\int_0^l \left(\frac{d^2w}{dz^2}\right)^2 dz$, we are not considering this; you see that we do not need to consider it, okay. Now, so this, of course, then V β, and you know all these things we can assume some expression this fine. So, that is not a problem; we can solve it, okay. But let us now discuss the work done and how it is going to be okay. Because in estimating the potential energy function, we have the strain energy component as well as the work done. So, we will see how the work is coming. So, what is the external moment here? The work done is going to external forces. So, what I am considering is that this beam is subject to simplification; this beam is subjected to pure bending. What does it mean? Pure bending means that this is subjected to an end moment, which is M, and here is M. Pure bending means M over the span, the same moment, a uniform moment over the span. You see that. Why are we taking that? Because then it will be simplified; otherwise, it is possible, but it will be a little complicated, so okay. And here we are considering the royal the same; we are not considering the, you know, I mean, because torsional flexural buckling we have separately considered. You know, we have considered that this has both the axis of symmetry. Now we consider the situation of why this was such a complicated formula in your code when you were considering the laterally unrestrained beam. Because at that time both lateral buckling and torsional flexural buckling were present, you had to do a lot of calculations for the shear center, etc. We are having a separate treatment for the lateral buckling of beams, and we have a separate treatment for the lateral torsional buckling of columns and for the torsional flexural buckling of columns. Understand that? Good. So, then we are considering that it is a uniformly subjective moment. We can consider the distributed load, and so that is not a very big deal, but just for simplification, okay. Now if it is subjected to these two, your pure moment M, right? I am considering the end; these are applied, pure moment is applied, okay? Pure moments are applied at the end, you know.

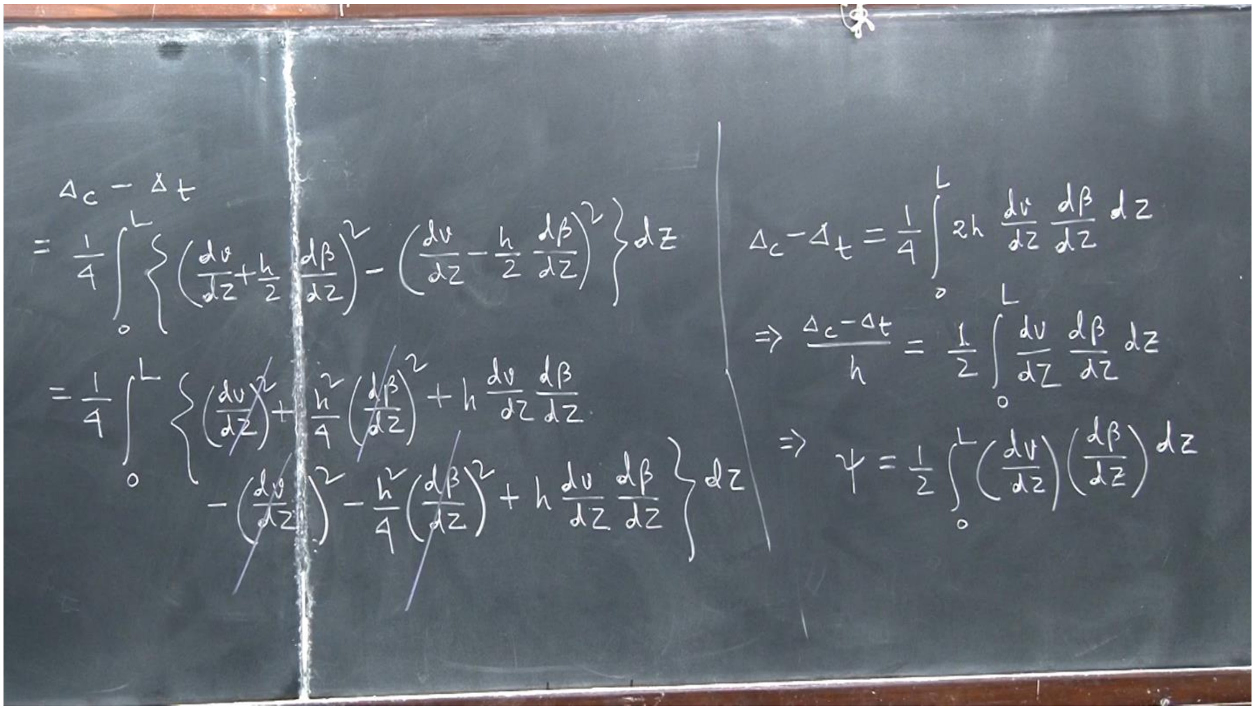


So, you know, this is also M and this is M, okay? There is no shear force, no shear deformation, okay? So now, how is M going to work? There must be some rotation. Yeah, rotation is happening. Why is rotation happening? Yeah, you see that. See. So, it was remaining vertical, and then it is rotating. Do you see that? This rotation, right? It remained vertical and then, you know, it started rotating. Do you see that? So, this, this angle. So M, and then I am defining this rotation of this, you know, this section, this one, this one; I am defining this angle to be ψ, and I am defining this angle to be ψ. So, what is that ψ rotation? ψ rotation is the rotation of the beam section. So, it is a rotation with respect to the y-axis, and then this rotation is happening in the xz plane, right? We have to find out how to determine ψ. ψ is the rotation at the end of the beam, and this rotation occurs with respect to the y-axis and in the xz plane, right? So, let us keep that expression okay, and we will come back to it later, and I am just removing this. So, let us get the work done. So, this Ψ is the end rotation of the beam. That means with respect to which axis? With respect to the

y axis, Y-axis in the xz plane Ψ. Okay, so work done is work $2M\Psi$. Why $2M$? Because both the end mm, so m into ψ, m into ψ both, so that is why this 2 is coming, $2m\psi$. Now we have to find out Ψ. How to find out Ψ? Do you see why this ψ is happening? This Ψ. See, this ψ is happening because you see that. Look, V, I am defining this deflection of this one, you know, and this deflection of this C, right? This one. So, ψ is what? See, ψ is nothing but this fellow; this fellow is V_c, this fellow is V_d, V_t, $V_c - V_t$ divided by h; h is the length, this length; that is the ψ, right? That is the angle. This is β, this is not ψ, this is β twist. ψ is little; you have to visualize it a little differently, ψ, okay? See what is happening; you see that?



See, this beam is subject to going out of plane deflection, right? So, what is in-plane displacement because of out-of-plane deflection? When it is laterally deflecting in the X-Z plane, what is the deformation along x? What is the in-plane deflection due to out-of-plane deflection? In-plane strain, you integrate von Karman strain, right? If you integrated, you know this is nothing but $\int_0^L \left(\frac{dV_c}{dz}\right)^2 dz$. If I mean. What is this? $\frac{1}{2}\int_0^L \left(\frac{\partial V_c}{\partial z}\right)^2 dz$. I am considering the top fiber, which is subjected to V_c, right? What is that V_c? This is what? V_c V_g $d\phi$, this is what? Von Karman strain of the, you know, this is out-of-plane deflection, right? Deflection along Y, right? Or the top fiber. So, this is nothing but ϵ, out-of-plane deflection, but the strain is axial strain; that means along Z, right? This is nothing but ϵ_{ZZ}, and this is nothing but ϵ_{zz} and half of that. So, if I want to consider only one plate, then it is not multiplied by half because this is contracting at both ends, right?



So, this is half of that; is that fine? Similarly, in tension, I am also right: $\frac{1}{2}\int_0^l \left(\frac{dv_t}{dz}\right)^2 dz$. Do you understand what I am doing? I am integrating the von Karman non-linear strain, which is in-plane strain due to out-of-plane deflection. The beam is undergoing out-of-plane deflection in the lateral direction along the y-axis, that is V_T. I am now considering the top fiber and the bottom fiber. And then for the top and bottom fiber, this is for compression fiber, and this is tensile fiber. So, I am integrating Von Karman strain over half of the length, which is $\frac{1}{4}$, because both ends are included. At one end, you know, this is the change in length of the compression fiber; this is the change in length of the tension fiber. So essentially, these are nothing but, you know, this one and this one, okay. But I mean just, okay. So now, what is V_c? V_c is nothing but $(v + \beta \frac{h}{2})$, $(v - \beta \frac{)h}{2}$. This you understand correctly. Now, Ψ is nothing but $\frac{\Delta_C - \Delta_T}{H}$. And now I am going to remove everything, this one, okay? I am going to $\Delta_C - \Delta_T$ is nothing but. So, work done, you know work done is $2M\Psi$ and then 22 will cancel out. $M\int_0^L \left(\frac{dv}{dz}\right)\left(\frac{d\beta}{dz}\right) dz$ is the work done. So, you find the expression for the strain energy and the expression for the work done. Okay. So, then we will be able to find out potential energy, and then we will subsequently discuss. Huh? So please be careful; you know, try to understand each and every point: how this work done expression is coming, why this Ψ end rotation is, how end rotation I'm relating with out-of-plane deflection V, as well as this twisting β, okay? This is a little tricky, not tricky, but please try to conceptualize it, okay? Thank you very much for today's class. Okay.