Introduction to Accounting and Finance for Civil Engineers
Professor. Sudhir Misra
Department of Civil Engineering, IIT Kanpur
Professor. Kumar Neeraj Jha
Department of Civil Engineering, IIT Delhi

Module No. #01
Lecture No. #04
Time Value of Money

Namaskar, and welcome to this lecture in this course on, Accounting and Finance for Civil Engineers.

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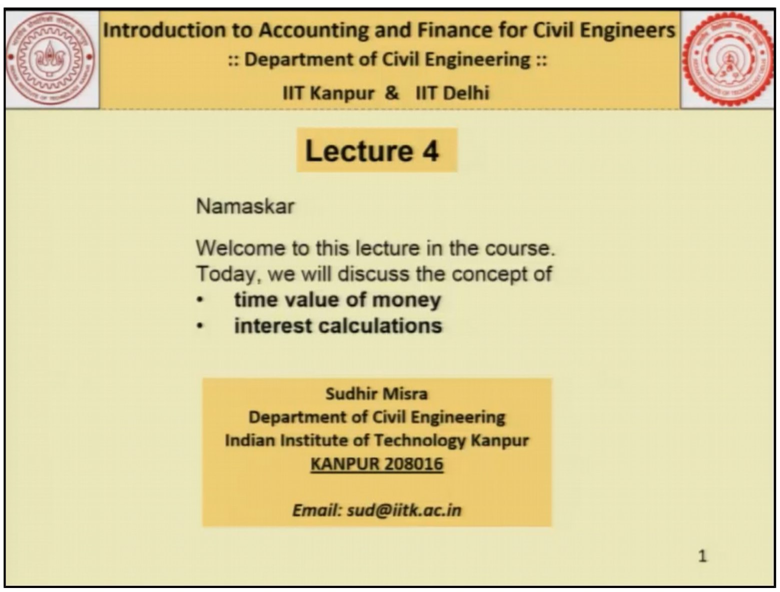

So today, we will get started, with the concept of time value of money, and do some simple interest calculations, to illustrate the point. Now, getting started, the concept is derived from, the cash flow diagram.

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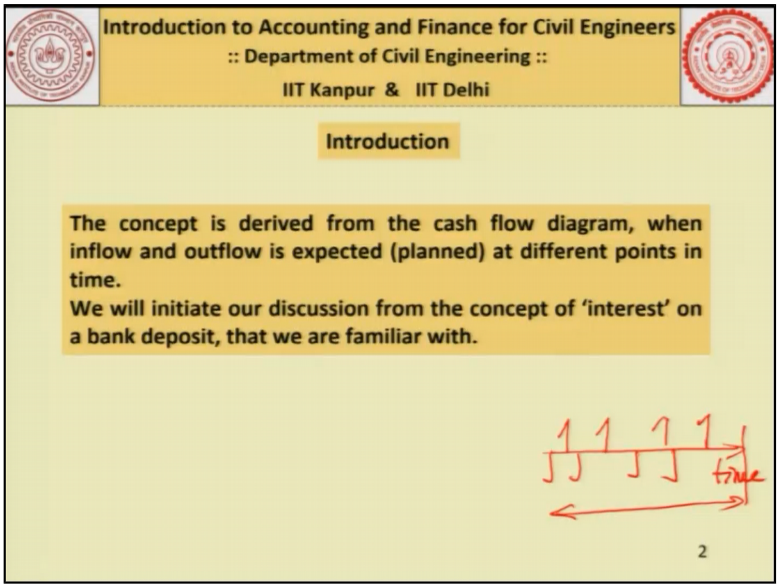

You will recall that, we constructed a cash flow diagram, last time. When the inflow and outflow of funds is expected or planned, at different points in time. So, what we are trying to say is that, if on the time axis, we expect outflows at different points, we expect certain inflows at different times. And, if this axis is long enough, that is, the time is long enough, that warrants the study on interest calculations, how the value of money changes over a period of time, and so on, then we need to use this concept of time value of money.

We will initiate our discussion, from the concept of interest on a bank deposit, that we are familiar with. Now, what is concept of interest in a bank deposit. We know that, if we deposit a 100 Rupees in the bank, at the end of 1 year, the bank may give us 105 or 107 or 109, depending on the rate of interest that is applicable, and the ways in which it is calculated. So, that is something, which we were familiar with.

Another concept, that we are familiar with, and we will use that point today, is the fact that, we borrow from the banks. We take a house loan, we may take a car loan, and so on. And, those loans have to be repaid. And obviously, when we take a 100 Rupees from the bank, what we pay back to the bank is not 100 Rupees. It is, a little more than 100 Rupees. And, that is something, which we will need to understand a little bit, before we get started with our discussion, as far as financing is concerned.

These concepts are very important to understand, when we are talking of, trying to organise finances for a project. That is what we are talked about, in the first few lectures, when we said, that one of the reasons, why civil engineers should be aware, or should be well versed with these methods is, because they need to plan, and organise, make sure, that funds, the right amount of funds are available to them, at the right point in time.

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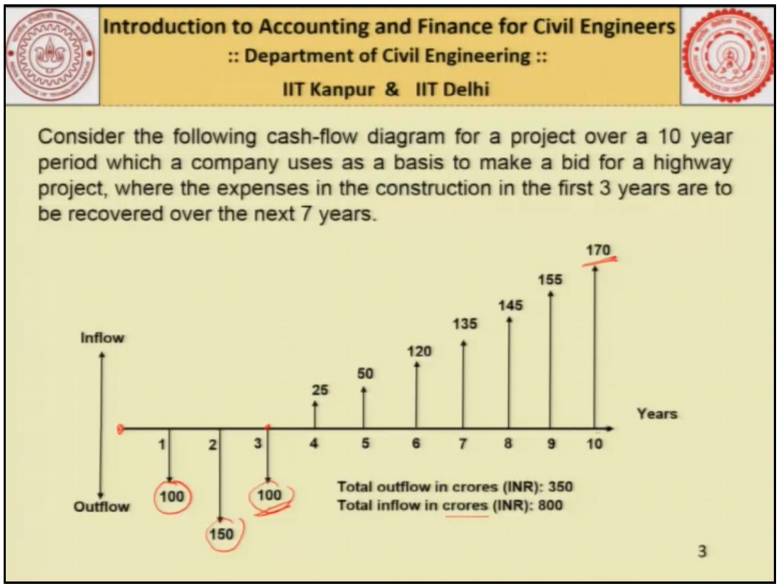

So, moving forward, this is the example, that we gave last time. That, here is a construction project, which involves, an expenditure of 100, 150 and 100 Crores. That is, a total expense, or an investment of 350, and we expect certain revenues. Now, what we have to see, and that is what we had mentioned last time also, how do we calculate, the present value of all these revenues, or all the investments, or the future value.

Because, at the end of it, if you want to compare, this 100 with this 170, we have either to bring this 170 to this point, or take this 100 to this point, then only we can make a rational comparison. These are the kind of things, which will become handy, when we try to compare options. And, that is what we will see, in the next class. So, moving forward, let us take another look at the cash flow diagrams.

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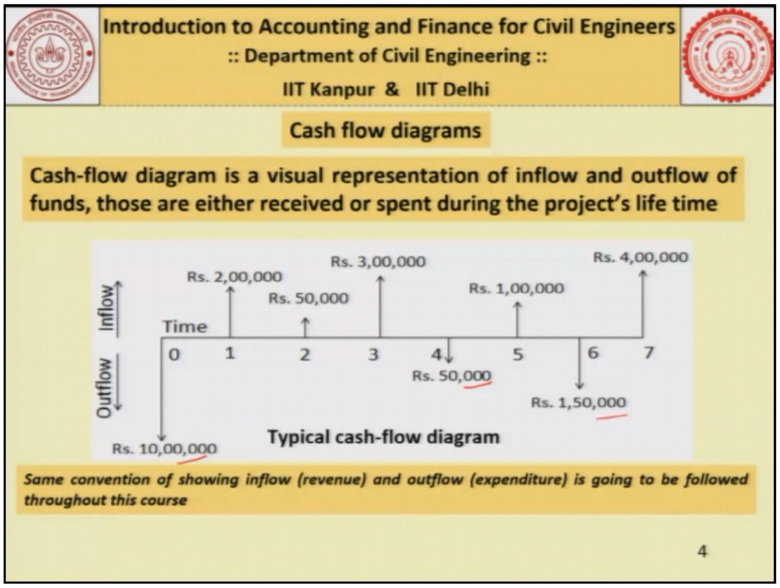

Cash flow diagrams, I have already talked to you, is a visual representation of inflow and outflow of funds, which are either received or spent, during the projects lifetime. And, this is another example, where we expect 150,000 being spent at different points in time, and certain amounts of revenue.

So, what we are trying to understand is, how do we make a comparison between, this cash flow, and another cash flow, which may have different inflow and outflow, even if the total inflow and outflow is the same. So, of course, we followed the same convention, as far as inflow and outflow is concerned, being plotted on the top half and the bottom half of the time axis.

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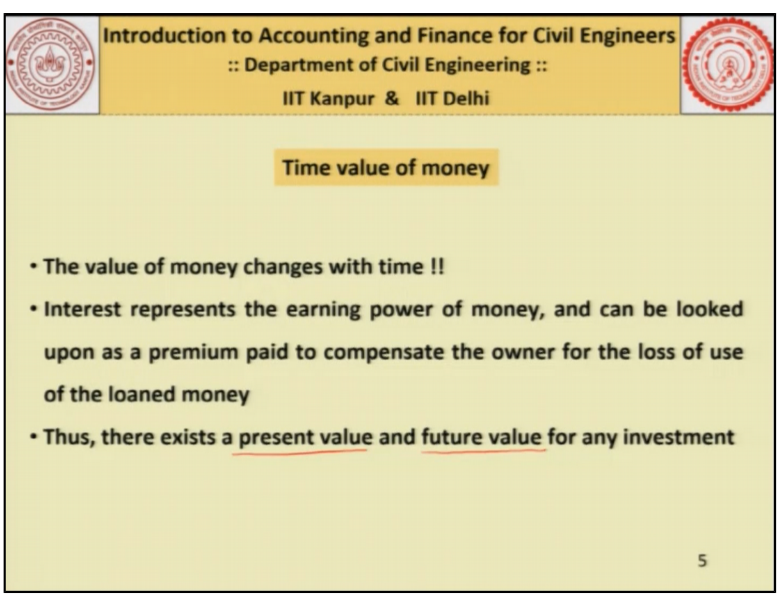

So, now coming to the formal definition of time value of money, the value of money changes with time. And, interest represents the earning power of money, and can be looked upon, as the premium paid, to compensate the owner, for the loss of use of the loaned money. So, our deposits in the bank can be looked upon as, loans to the bank. And because, we do not have the freedom to use those funds, the bank pays us the interest.

With those funds, the bank loans to other people. May be, some people take house loan, some industries take loans to set up industry, and so on. And, the bank charges them interest. Part of which, it repays to us, and part of it goes to, sustain the banks expenses, and the bank's profits. So, that is how, the money goes around. And thus, there exists, the concept of a present value, and a future value, for any investment. So, that is something, which we are going to look at, formally.

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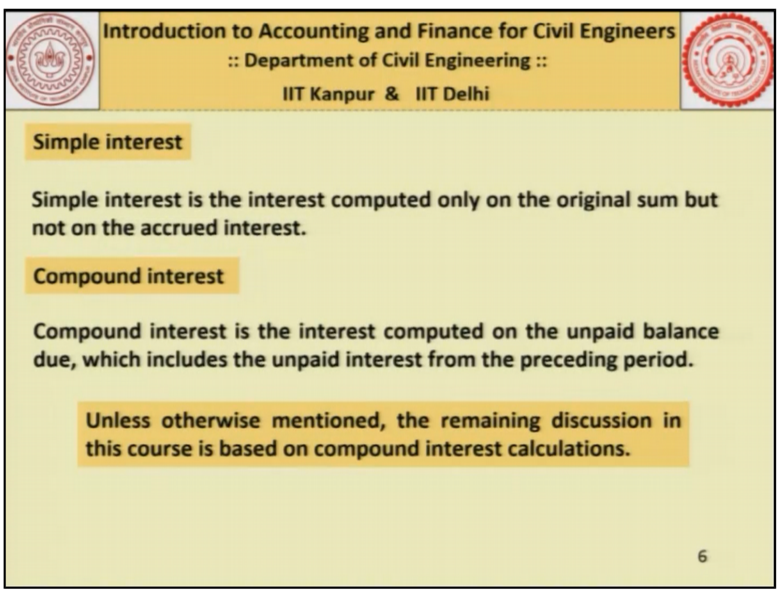

Now, we are already familiar, with the concept of simple interest. And, that is, the interest computed only on the original sum, but not on the accrued interest. That is what we know, from high school. And, when it comes to compound interest, it is the interest computed on the unpaid balance due, which includes the unpaid interest from the preceding period. Now, unless otherwise mentioned, the remaining discussion in this course is based on, compound interest calculations.

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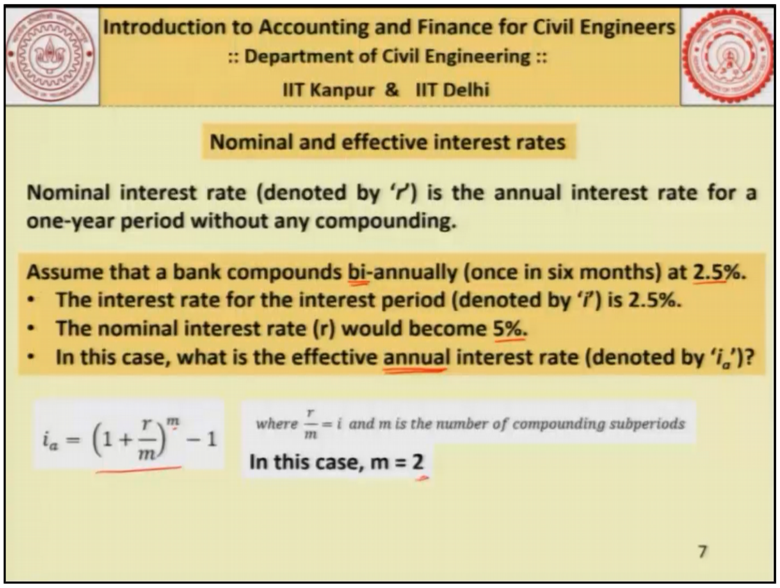

Now, let us try to define, the concept of nominal and effective interest rates. The nominal rate denoted by R, is the annual interest rate, for a 1-year period, without any compounding. Now assume, that a bank compounds an interest, bi-annually. Now, if the compounding is such that, it is at the rate of 2.5%. So, what it means is, it is a matter of terminology. What it means is, that the interest rate for the interest period denoted by I is, 2.5%. But, the nominal rate is 5%, because it is being compounded bi-annually.

So, it is done twice a year. Therefore, this 2.5 becomes 5. So, it is 2.5 times to, coming from the fact that, it is being compounded bi-annually. So, now in this case, what is the effective annual interest. We know this formula. And, we understand that, the effective annual interest rate in this case is, 1+R divided by M to the power of M, which in this case will be taken as 2. Because, we are trying to calculate, the effective annual interest rate, and we are talking of bi-annual system, and this is how, we will calculate it.

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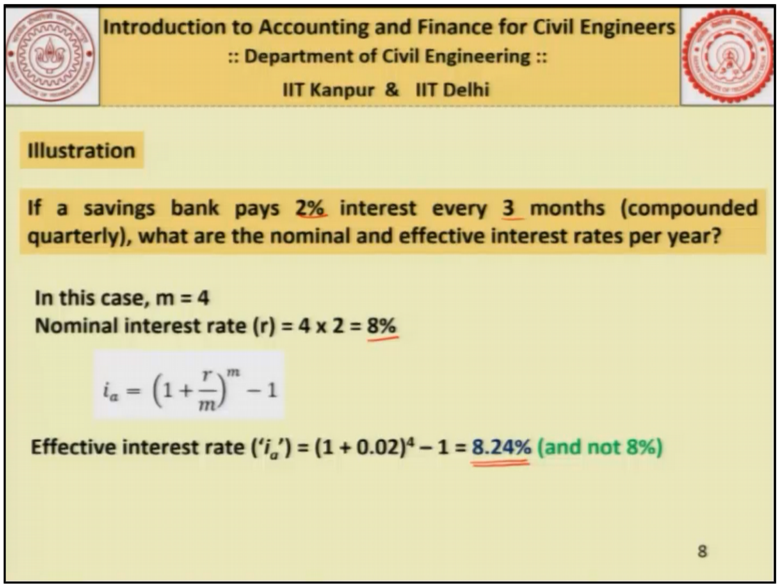

So, if you look at an example, or an illustration, suppose, a savings bank pays an interest of 2% every 3 months. Now, we are compounding it quarterly, what are the nominal, and effective rates of interest, per year. So, for that purpose, M becomes 4, because we are doing it quarterly. The nominal interest rate is 4 into 2, because 2 is the interest being paid, every 3 months.

So, the nominal interest rate is 8. And, the effective interest rate turns out to be, 8.24, and not 8. I would leave it to you as an exercise, to calculate the effective interest rate, in the previous case, where we had a 2.5% bi-annual interest. So, that is something, which I am leaving out to you, to do on your own. And, we will move forward.

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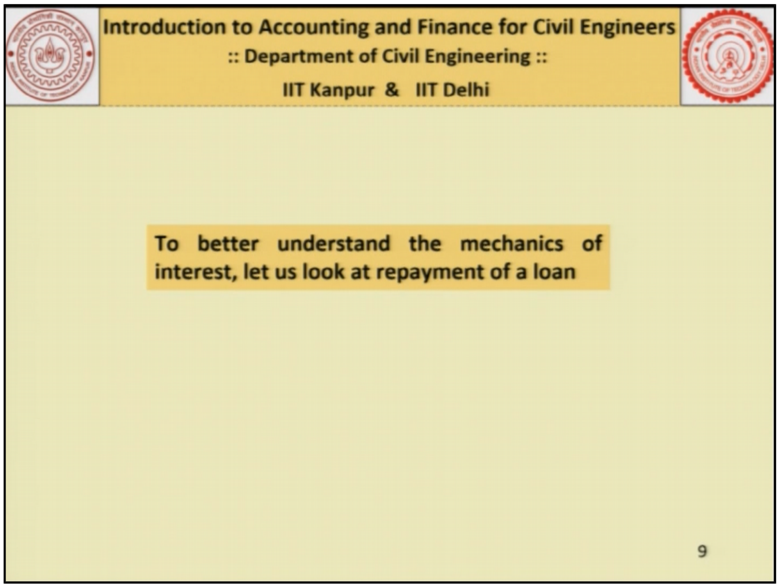

So, let us look at this whole concept once again, from the point of view of repayment of a loan. Now, these loans are taken, and have to be repaid over a period of time, which often times is substantially long. So, it depends on, how much we are repaying? What we have taken from the loan? What is the interest payable? And, the biggest thing is, the payment that we are making, what is it being used for? Let me clarify this.

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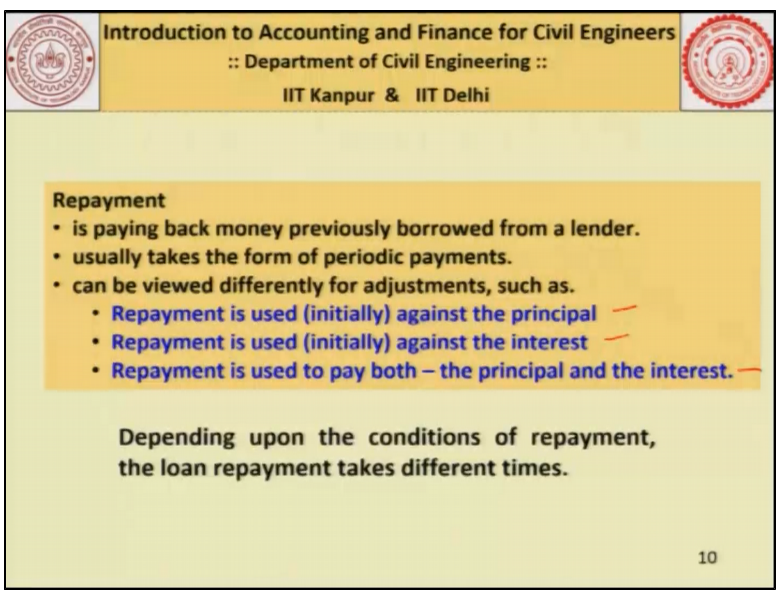

Repayment basically means, paying back the money, previously borrowed from a lender. Usually, this is done in the form of periodic payments. And, can be viewed differently for adjustments such as, repayment is used initially against the principal, repayment is used initially against the interest, and repayment is used to pay both, the principal and the interest.

This is something, which I will try to explain to you, through an illustrative calculation. Now, depending on the conditions of repayment, whether it is this, or it is this, or it is this one, the time that it takes to repay that loan, is quite different. And, let us try to see this, through an example.

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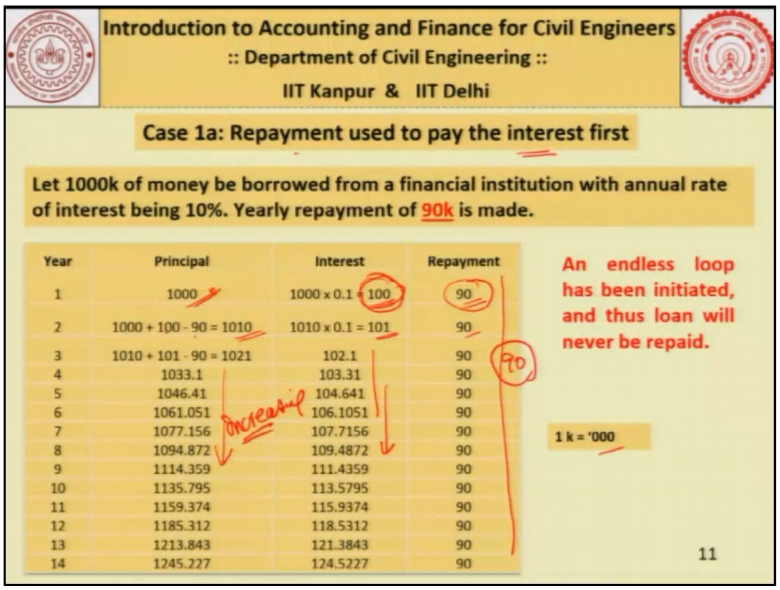

Let us take an illustrative example, on the condition that, the repayment will be used, to pay the interest first. If we take 10 Lakhs of money from a financial institution, with an annual rate of interest being 10%, and we make an annual repayment of 90,000. The condition being, that this 90,000 will first be used, to service the interest on that loan. What happens, here is a table, which tells us, that at the end of year-1, the principal on which the interest is going to be accounted, is 1,000. We are always talking in terms of 1000's.

So, the interest, that accrues on this amount is a 100, and the repayment is only a 90. So now, this 90, being less than this 100, obviously means, that the principal at the end of the 2nd year becomes, this 1,000 + this interest which accrues - the repayment which has been made, and 1,010. And, this 1,010, will attract an interest of a 101, in the next year.

And, since we are making an equal payment of 90 all the time, we find that, the principal is always increasing. And therefore, the interest accrued, is also increasing. And therefore, we are in a situation that, the loan can simply never be repaid. Even though, we are repaying a certain amount of money every time, the condition being that, the repayment is being used for the interest servicing first, simply means that, unless we at least clear this interest, there is no way that the principal is going to go down. So, that is one condition, that we have. So, this was one case.

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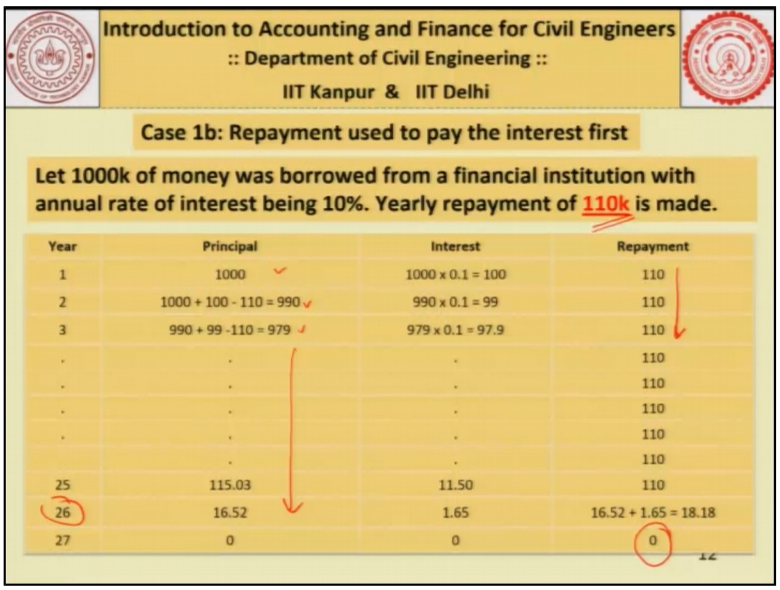

Now, we come to our second case, where still we are paying the interest first, but the payment is a 110,000. So, in that case, if you look at the same table, now what we are doing is, we are paying a 110, so we find that, the principal is gradually reducing. And, at the end of the, say 25, 26 years, we have been able to service this loan. So, I am leaving out the spreadsheet kind of calculation to you, to do as an exercise. You can change these values, and try to come up with your own numbers.

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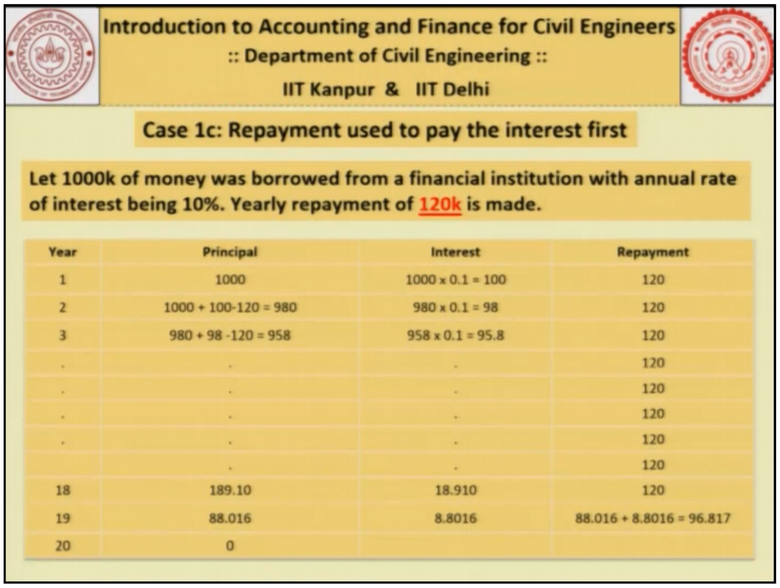

And, let me repeat this exercise, with a different amount of repayment annually. Let us try to make it a, 120K. And then, we find that, we are able to service the loan in about, 19, 20 years. So, this really shows that, if the condition was, that the repayment will be used to service the interest first, the obvious thing is that, the repayment amount should be at least equal to the interest, that the loan attracts in a given year.

And, after that, so long as that is met, if we keep on increasing the amount of repayment being made, the time it takes to repay the loan, could be reduced. Now, instead of the repayment being used to repay the interest, if the condition was, that it will be used to pay the principal first, then let us try to see, what happens to the numbers.

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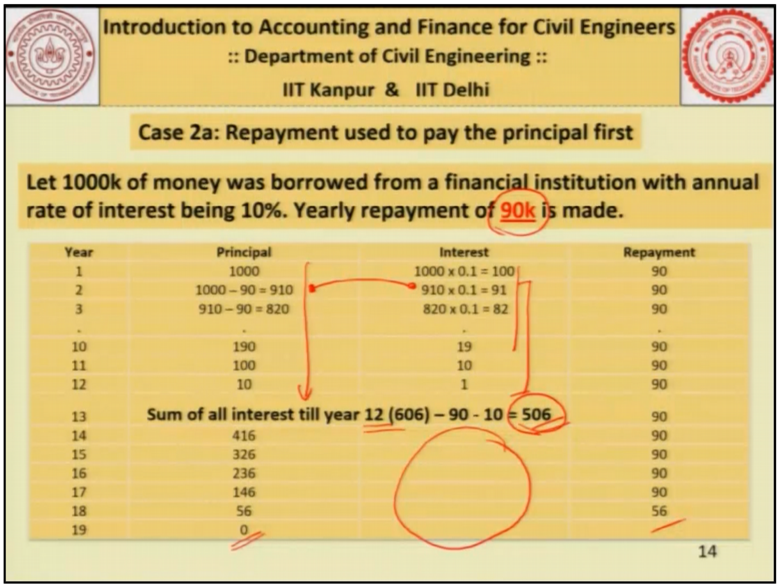

So, if we have the same conditions, 10 Lakhs of money borrowed, 10% of interest applicable, and the yearly payment is made for 90,000 as before, we find that, we are able to service the loan in about 19, 20 years. But, this is again, a very simplistic example. Because, what we are saying is, that this 90 is being used, to reduce the principal all the time. And, the interest is calculated only on, the principal of that particular year. And, this interest, does not attract any interest.

So, if these conditions are what are operating, then yes, we can at this point say that well, the sum of all the interest up to this point is, 506. And, in order to service this 506, without any interest being accumulated in this period, we are able to clear the loan. So, these are some simple assumptions, that we are making. Of course, they are not realistic, at times. The moment, we introduce more complications, you need to go back to the spreadsheet, and try to do your own calculations.

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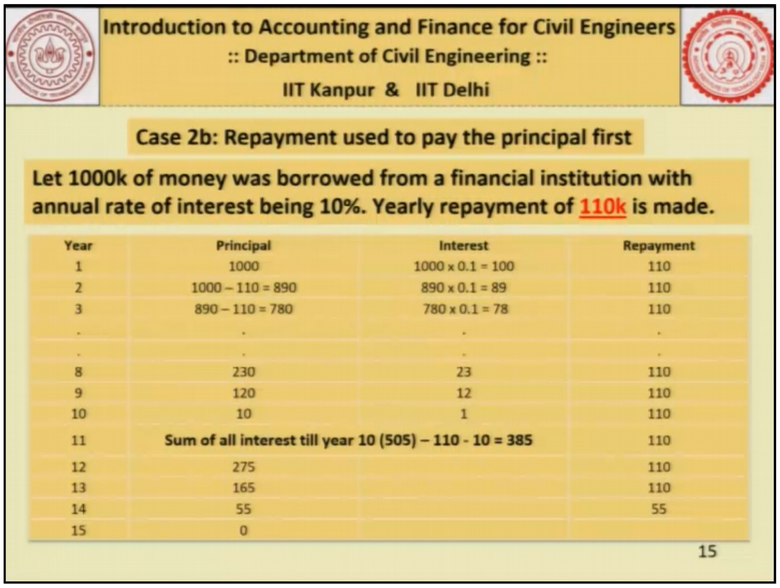

Year	Principal	Interest	Repayment
1	1000	1000 x 0.1 = 100	110
2	1000 – 110 = 890	890 x 0.1 = 89	110
3	890 – 110 = 780	780 x 0.1 = 78	110
.	.	.	.
.	.	.	.
8	230	23	110
9	120	12	110
10	10	1	110
11	Sum of all interest till year 10 (505) – 110 - 10 = 385		110
12	275		110
13	165		110
14	55		55
15	0		

Now, instead of 90,000, if we were to use the example of 110, which we also did last time, we find that, we are able to complete the payment in 15 years.

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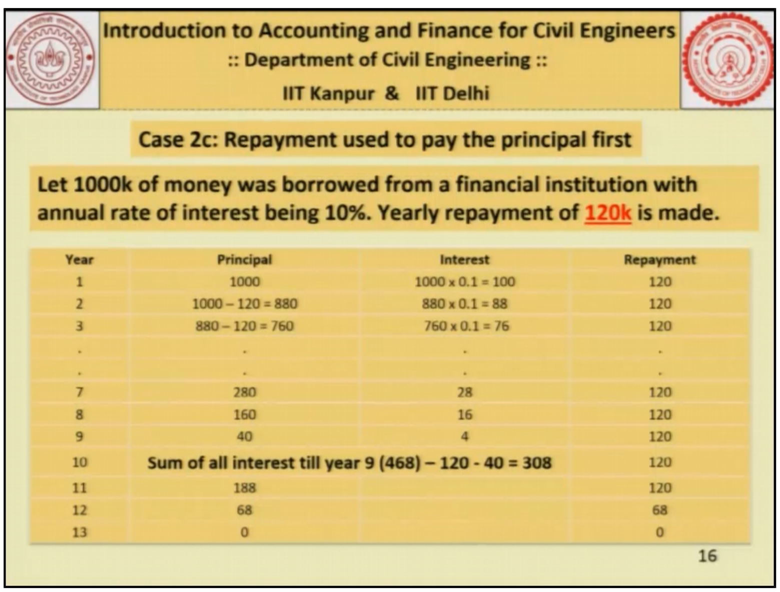

Year	Principal	Interest	Repayment
1	1000	1000 x 0.1 = 100	120
2	1000 – 120 = 880	880 x 0.1 = 88	120
3	880 – 120 = 760	760 x 0.1 = 76	120
.	.	.	.
.	.	.	.
7	280	28	120
8	160	16	120
9	40	4	120
10	Sum of all interest till year 9 (468) – 120 - 40 = 308		120
11	188		120
12	68		68
13	0		0

And, moving forward, if we were to pay 120, then the loan can be serviced in 13 years.

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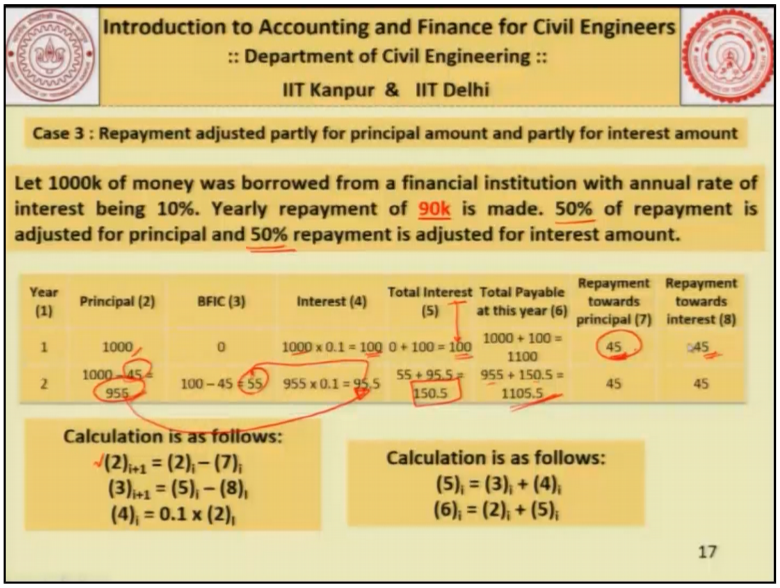

Now, let us look at a situation, where the repayment is adjusted, partly for the principal, and partly for the interest. What we have done so far is, that whether it was 90,000, whether it was a 110,000, or 120,000, that money was used to, either pay the interest that was accruing, or to reduce the principal. It was a 0-1. Now, let us try to obviously combine the two, and try to see that, if partly it was used for the principal, and partly for the interest, then how much time does it take.

So, for that, the first example is, we take 50-50. That is, 50% of the repayment is adjusted for the principal, and 50% for the interest. So, this is what is happening in this case. We have a principal of a 1,000. We pay back 90, out of which, 45 is counted towards the principal, and 45 towards the interest. So, the interest that has accrued here is, 1000 into 0.1, which is 10% of a 1,000, which is a 100. So, the total interest is, here.

So, this 45 here, goes to reduce my principal here, to 955. Since, I have a 100 as the interest, I have repaid 45 on that, so my interest remaining to be paid is 55. This 955, attracts an interest of 95.5. This 55 + 95.5, is this interest here, which is the total interest, that needs to be paid. And therefore, the total payable here becomes, 955 + 150.5, which is 1105.5.

So, this is how, we will continue to do the calculations. And, in a generic sense, what we have, is this. These are the columns, that we are talking about, 2, 3, 4, 5, 6, 7 and 8. And, we can calculate, how to make the entries, in the columns 2, 3, 4, 5, 6, and so on. And, of course, 7 and 8 are given to us, because we are just paying 90 every time.

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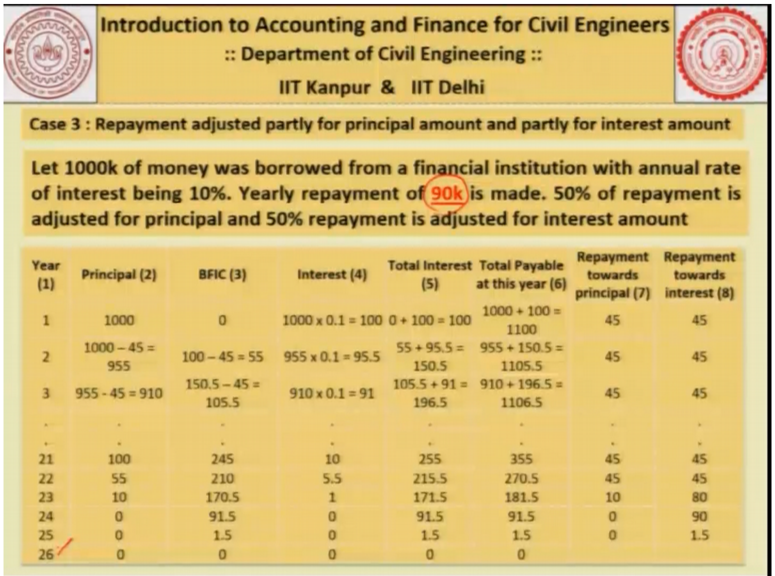

Year (1)	Principal (2)	BFIC (3)	Interest (4)	Total Interest (5)	Total Payable at this year (6)	Repayment towards principal (7)	Repayment towards interest (8)
1	1000	0	1000 x 0.1 = 100	0 + 100 = 100	1000 + 100 = 1100	45	45
2	1000 – 45 = 955	100 – 45 = 55	955 x 0.1 = 95.5	55 + 95.5 = 150.5	955 + 150.5 = 1105.5	45	45
3	955 - 45 = 910	150.5 – 45 = 105.5	910 x 0.1 = 91	105.5 + 91 = 196.5	910 + 196.5 = 1106.5	45	45
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
21	100	245	10	255	355	45	45
22	55	210	5.5	215.5	270.5	45	45
23	10	170.5	1	171.5	181.5	10	80
24	0	91.5	0	91.5	91.5	0	90
25	0	1.5	0	1.5	1.5	0	1.5
26	0	0	0	0	0		

So, this calculation, if we were to repeat on the spreadsheet, that we have been following, we find that, we are actually able to clear the loan in about, say 26 years. Now, you see that, this 90 was not sufficient to repay the loan, when it was being used, only to service the interest. But, so long as it is being used to service part of the principal, it still becomes possible, with the assumption that, the interest does not attract interest. So, you would notice in the last slide, that we did not have compounding interest on the interest. So, with that simplified assumption, it is possible to complete the payment on this loan, in say 26 years.

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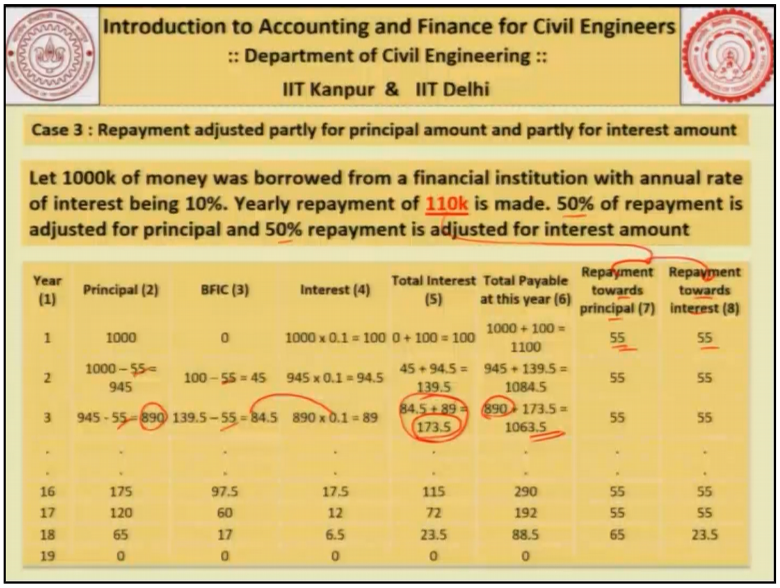

Year (1)	Principal (2)	BFIC (3)	Interest (4)	Total Interest (5)	Total Payable at this year (6)	Repayment towards principal (7)	Repayment towards interest (8)
1	1000	0	1000 x 0.1 = 100	0 + 100 = 100	1000 + 100 = 1100	55	55
2	1000 – 55 = 945	100 – 55 = 45	945 x 0.1 = 94.5	45 + 94.5 = 139.5	945 + 139.5 = 1084.5	55	55
3	945 - 55 = 890	139.5 – 55 = 84.5	890 x 0.1 = 89	84.5 + 89 = 173.5	890 + 173.5 = 1063.5	55	55
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
16	175	97.5	17.5	115	290	55	55
17	120	60	12	72	192	55	55
18	65	17	6.5	23.5	88.5	65	23.5
19	0	0	0	0	0		

Now, moving forward, if the amount again, was a 110,000, which is what we have used in the previous examples. And, we still stick to the, principal of 50-50, when we find that, we are able to pay the loan in, 19 years. Using the same arithmetic, that is, this 110 gets divided into two parts, paid towards the interest, paid towards the principal.

This component, we keep reducing all the time. This component, we keep reducing all the time. And, we find that, the total interest here, is a sum of this, plus this. And, the total payable is, this number here, plus this 890. So, if we keeping doing this arithmetic with 110,000 being paid every year, we are able to clear the loan in 19 years.

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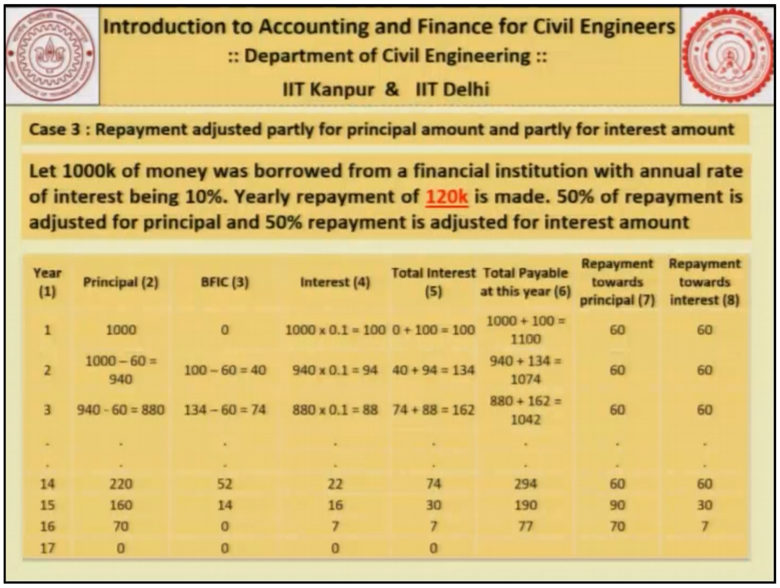

Similarly, if we were to repeat this exercise with 120, you will find that, we are able to pay it in 17 years.

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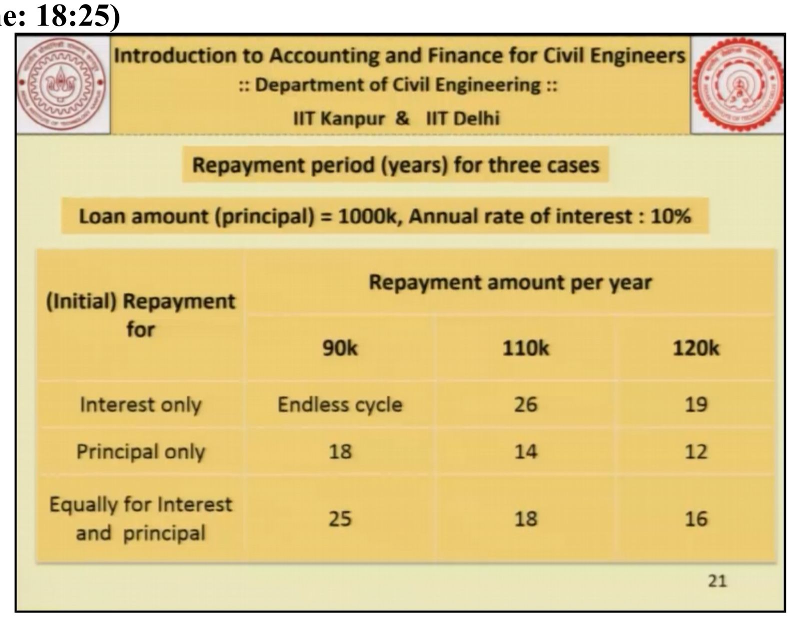

So, now this slide summarises, what happens, if we have 90, 110 and 120 being used, to service a loan amount of 10 Lakhs, at an annual interest of 10%, with this repayment being used, for the interest only, for the principal only, and equally for the interest and principal, with the assumption that, the interest does not attract interest. So, this table, is only a summary of the amount of time, that it takes to repay the amount. What I have not done here, is try to calculate that, how much money have we actually paid.

What I mean is that, if there was a calculation, that 110,000 is being paid for 20 years, then what you are paying totally is, 22 Lakhs. We were talking, in terms of 1,000's. And therefore, two, two, zero, zero, and three zeroes' here means, 22 Lakhs. So, what you should realise is that, in taking a loan of 10 Lakhs, what you landed up paying, was 22 Lakhs. Accept the fact that, it was to spread over 20 years, and you are making a payment of 1.1 Lakhs, every year.

So, this is something, which we have to understand. We can always try to translate that, into a cash flow diagram. And, say that, if we take a loan of 10 Lakhs, keep repaying 1.1, 1.1 every time, it will take 20 years, provided, the conditions that we talked about, were being operated, whether it is going to service the interest first, whether it is servicing the principal first, and so on and so forth.

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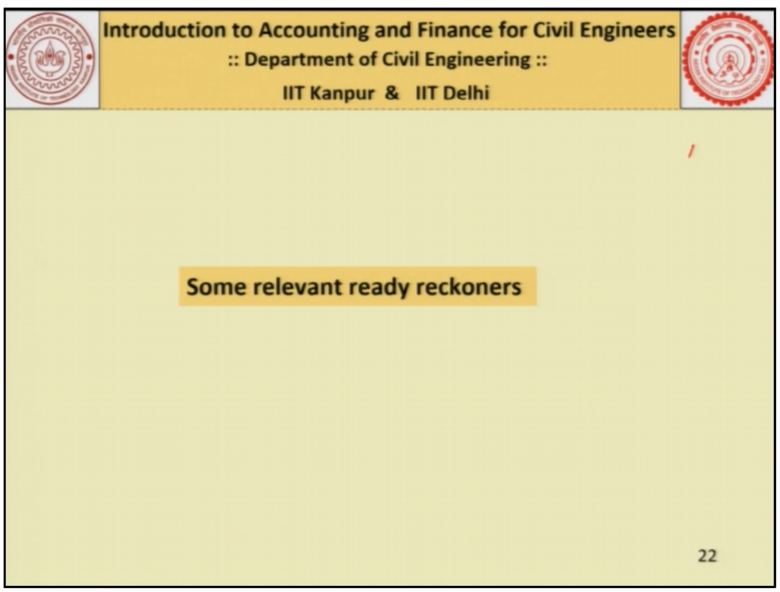

Now, let us try to look at some ready reckoners. Ready reckoners are easy to use tools, which help us do these calculations without, for example, the spreadsheet.

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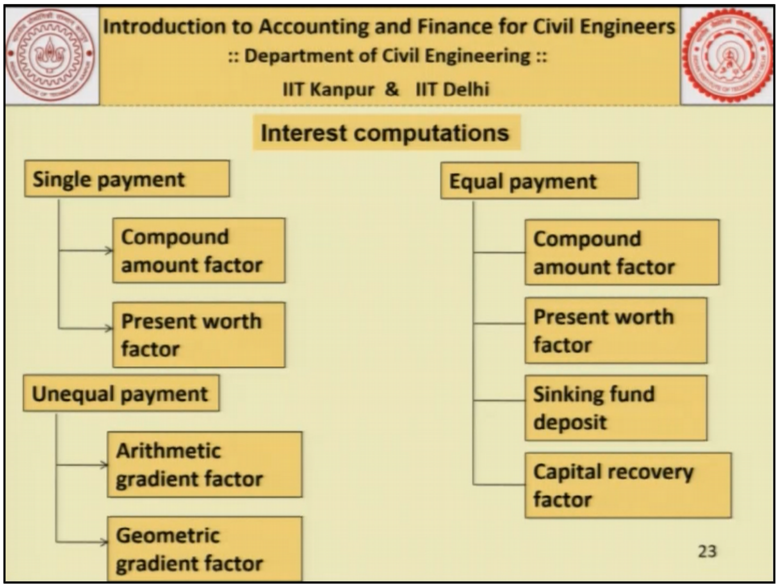

Now, what is the kind of interest computations, that are of interest to us. As far as single payment is concerned, what we would like to know, is the compound amount factor, and the present worth factor. If it is an equal payment, we would be interested to know, what is called the compound amount factor, the present worth factor, the sinking fund deposit, and the capital recovery factor.

And, if it was unequal payments, we are interested in, an arithmetic gradient factor, and a geometric gradient factor. So, these terminologies, is a matter of words. We do not really want to insist that, you should remember them, right away. But, the principal of it, is the following.

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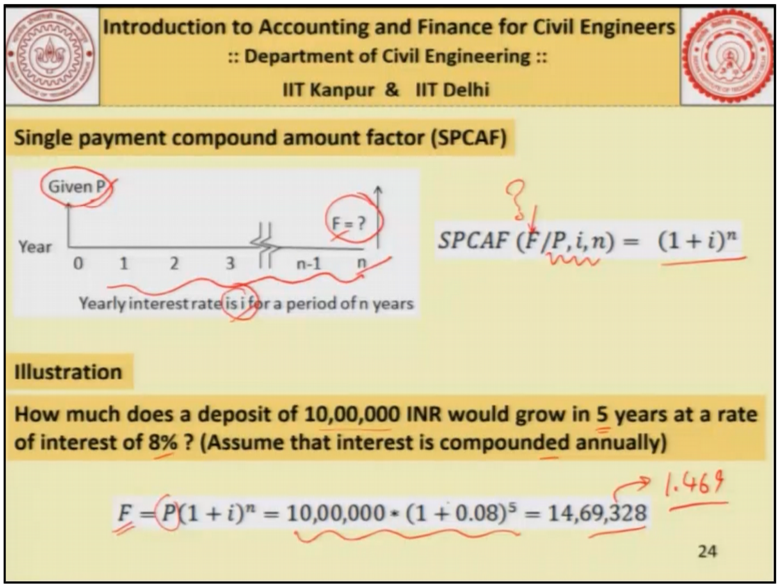

At the end of it, there is a principal involved, there is a time of repayment or a time which is involved, and there is a rate of interest involved, and there is a final value F, which is involved. Now, there can be different combinations when, out of these four variables, the P, the I, the N, and the F. Three are given, and one is unknown. In the case of the single payment compound amount factor, which is called the SPCAF, what we talk about is, what is this F, given P, I, and N. For that, the formula is this. This is basically, just the compound interest kind of a formula.

And, the illustrative example is, how much does a deposit of 10 Lakhs grow into, in 5 years, at a rate of 8% compounded annually. We are not getting in to, complications of bi-annual or quarterly compounded. But, if you make an illustrative example here, the F is 14.69328 Lakhs. Which means that, this here, is the formula. And, the factor that we are talking about is, 1.469. That is, if we have the P, we just need to know this value, and we will be able to calculate the F, provided, we also know the I, and the N.

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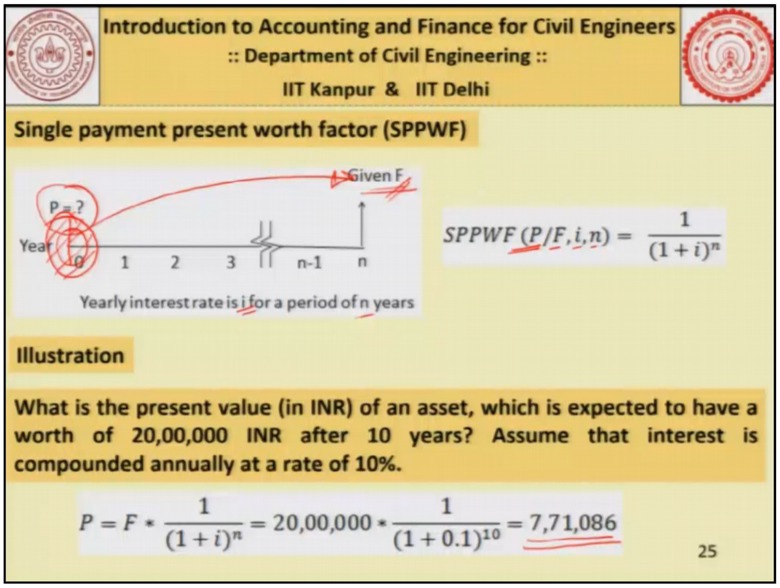

As far as, the single payment present worth factor is concerned, what we are talking about is, what is the present value or present worth of something, which will have a value of F, at the end of N years, with yearly interest rate being I, and the time period being N. In that case, it is just the other way around. It is the reciprocal of the previous discussion. And, what we are looking for is, what is the P, if the F, I, and N, are known. The illustrative example is given here. What is the present value in INR, of an asset, which is expected to have a worth of 20 Lakhs of Indian Rupees, in 10 years?

Assume that, the rate of interest compounded annually is 10%. And, we are talking of the answer to be, 7.71 Lakhs. What it means is that, if we have 7.71 Lakhs here, it will grow to a value of 20 Lakhs. That is, what is important to understand. From a construction perspective, or from the construction industries perspective, as a contractor, if you know for example that, you will need to buy equipment, which is going to cost 20 Lakhs in 5 years or 7 years, what is the kind of money that you need to set aside today.

If you are setting aside, a single instalment here, so that is the single deposit that you are creating here, with the intention, that this will grow into a certain amount of money here, which will enable you to buy a crane, or a bulldozer, or whatever it is. So, that is the kind of application, that we are talking about here, as far as a single payment present worth factor is concerned.

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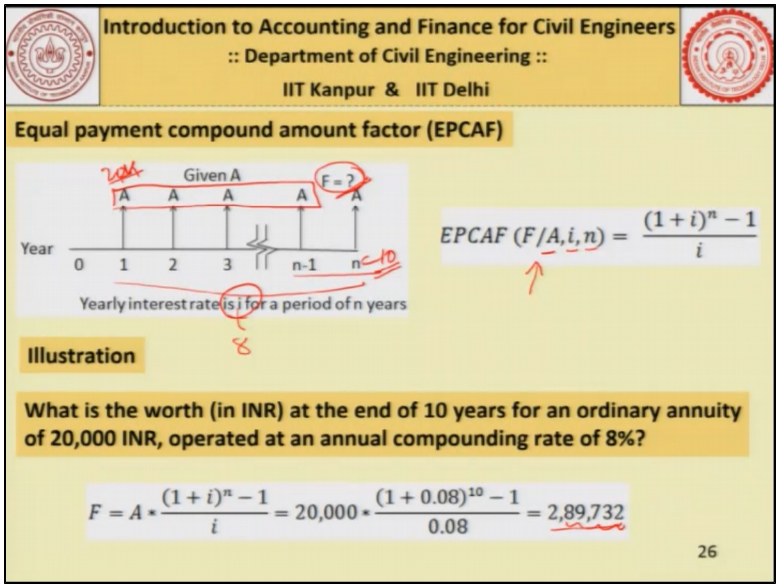

Now, let us try to look at, an equal payment compound amount factor. So, in that case, what we want to know is, what is the F, if we keep A, I, and N, as knowns. That is, if we set aside an amount of A annually, at an interest of I, over a period of N years, what will be the final value, that it will grow to. And, here is the ready reckoner formula for that. And, the illustrative example is, that what is the worth, at the end of 10 years, for an ordinary annuity of 20,000, operated at an annual compounding rate of 8%.

So, if we do that, what we are doing is, every year, we are setting aside 20,000. Why you want to know, what would it grow to, if this I was 8%, and we are talking in terms of M is equal to 10. So, if that is what we want to do, we find that, the F turns out to be 2,89,732

Which means that, the first 20,000 will attract an interest for 9 years, the next 20,000 will attract for 8 years and so on, if you keep doing that, we find that finally, the amount that we get is 2,89,000 against, what we have paid is, 2 Lakhs. So, this 89,732, is what is the cumulative interest, on all these deposits that, we have made every year.

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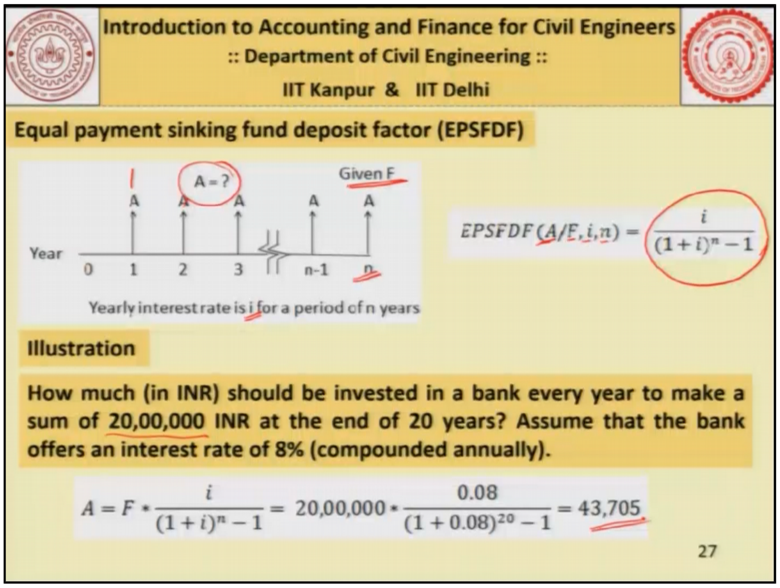

So, coming to the equal payment sinking fund deposit factor, we are talking of, if we want a certain F, we know the I, we know the N, how much should be set aside, every year. So, this factor really talks in terms of, what is the A, if F, I, and N, are known. And, this is the formula. For those of you, who are more academically inclined, I would think that, you should try to derive these formulae. It is not so difficult. We do not want to do this, in this course, because we do not have the time.

But, you should be able to derive these formulae. It is not so difficult. You just need to use the concept of compound interest, how much time does this attract interest, how many times does this attract interest, and so on, and try to do a summation, and so on and so forth. And, you should be able to come up with, these formulae on their own. Now, these formula have been tabulated. And, tables are available in order to help you, get the numbers.

And, that is why, I am calling them, ready reckoners. Now, as an illustrative example, how much INR should be invested in a bank every year, to make a sum of 20 Lakhs, at the end of 20 years, assume that the bank offers an interest of 8% compounded annually. So, if that happens, then you find that, an amount of 43,705 would suffice. So, you deposit 43,705 every year, for 20 years, and what you will get at the end is, 20 Lakhs.

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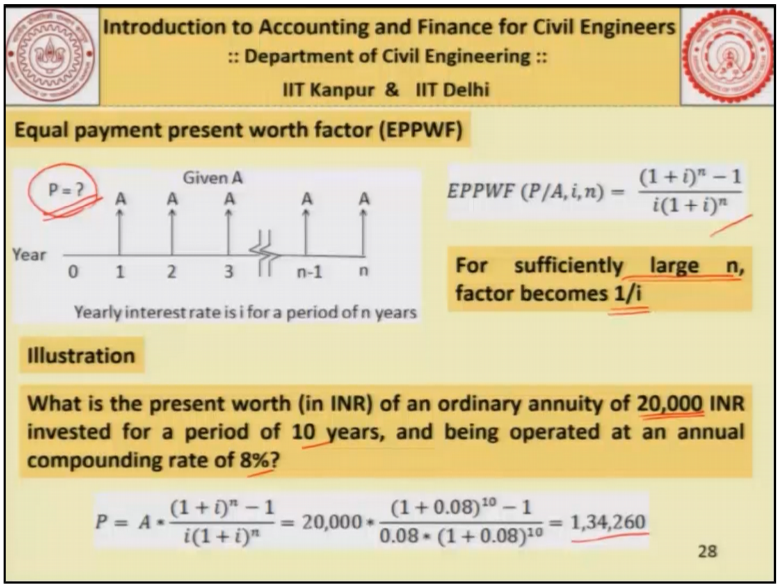

Moving forward, let us try to look at, the equal payment present worth factor. Which means that, what is the P, given A, I, and N. So, in that case, what is the illustration that we want to make. What is the present worth, of an ordinary annuity of 20,000, invested over a period of 10 years, and being operated at an annual compounding rate of 8%?

So, in that case, what we are looking at, is this. That, if we invest 20,000 for 10 years, at the rate of 8%, the present worth of this investment is 1,34,260. And, as a corollary to this formula, for a sufficiently large N, the factor becomes pretty close to, 1 upon I. You can try to check it out, on your own.

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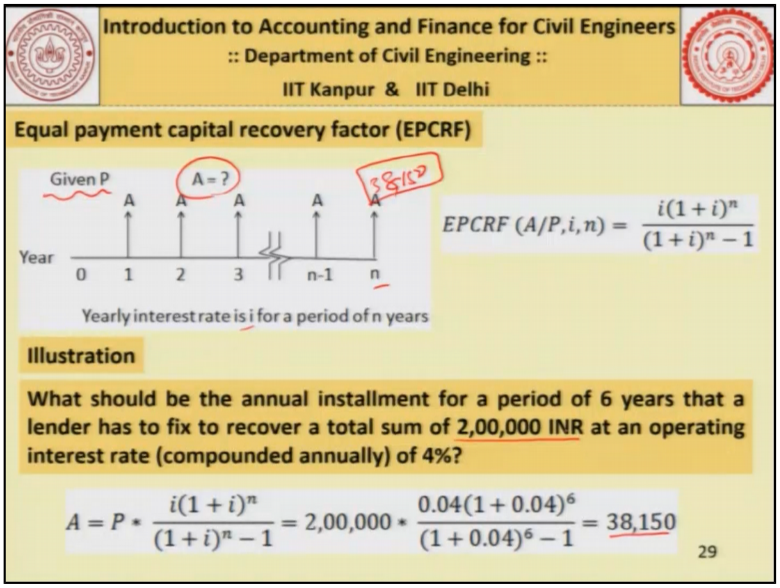

We move forward, and try to talk in terms of the, equal payment capital recovery factor. And there, what we are talking about is, given a P, what should be the A, if the N and I are known. This is the factor, that we use for this particular calculation. And, what it says is that, what should be the annual instalment for a period of 6 years, that a lender has to fix, to recover a total sum of 2 Lakhs, at an operating interest of 4%.

So, if that is what is the condition, then the A is 38.15. So, this 38.15 thousand, will pay a loan of 2 Lakhs, in 6 years, at the rate of 4%. Now, having completed all these factors very quickly, one slide after the other, what I would request you to do, is to look at the tables, try to do some calculations on your own, and also try to compare the numbers.

For example, we said that, if we have a certain annuity, over a period of time, and what was the F, try to see, what is the P involved with that, for the same conditions of I and N. Once you do that, you will get more insight into, what really is being talked about. Now, we are coming into a situation, where the cash flow series, may not consist only of constant amounts.

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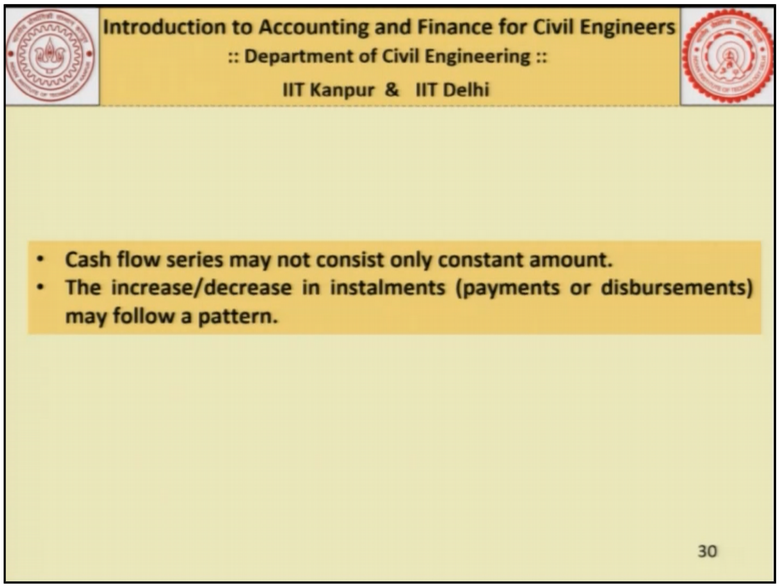

The increase and decrease in instalments, which is payments or disbursements, may follow a pattern. Again, these patterns are also, illustrative in nature. We are going to talk of arithmetic and geometric progressions. But of course, life is not as simple as that. Sometimes, there will be more distributions, there may be more complications, and then you will have to find out, the answers of your own.

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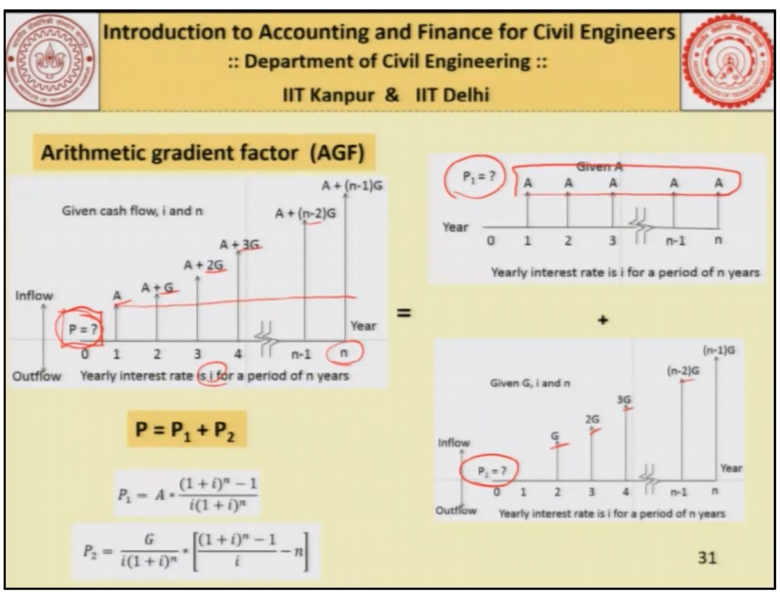

Now, coming to the arithmetic gradient factor, what we are talking about is, what is the present worth P, if this A keeps increasing every year, for a period of N years. And, the rate is I. It is like saying that well, today I have the paying capacity of a 100. But, I know that, I will be able to start paying more and more. I can give a 110 next year, 120 next year, and so on and so forth. Then, if that is the kind of series that we are able to follow, what is the present worth of that investment.

Now, in order to do that, this sheet is broken up into two parts. This A remains constant. That is, we take the A part of it, out from here. We just draw the line. And this, then becomes, G, 2G, 3G, and -2G, and -1G, and so on. So, if you do that, then the present worth P, is also broken up into two parts, or can be looked upon as having two components, one is P1 arising from here, and P2 which is arising from here. If you look at those two numbers, P1 we already know, and P2 was given by this formula.

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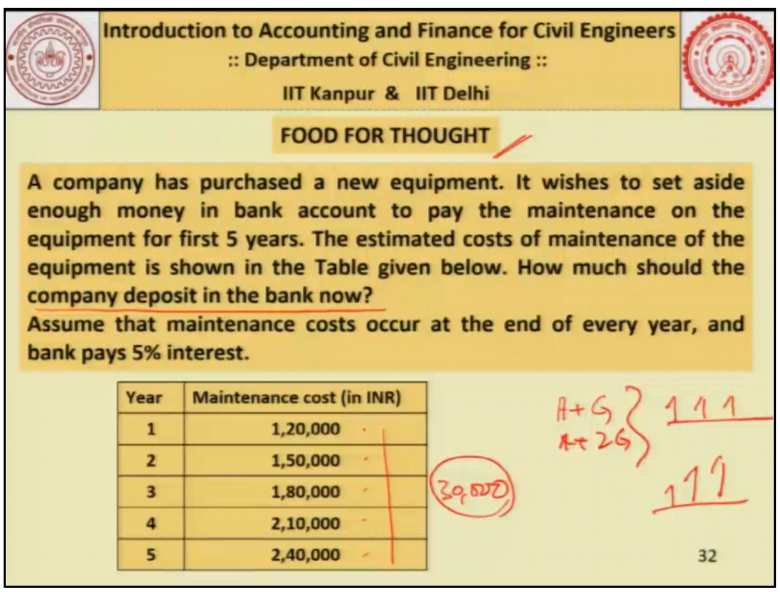

And, where do we use this kind of concept. A company has purchased A new equipment. It wishes to set aside enough money in a bank account, to pay the maintenance on the equipment, for the first 5 years. And, the estimated cost of maintenance of equipment, is shown below. So, we see that, it is a 120, 150, 180, 210, 240. So, we see that, this is increasing every year, at the rate of 30,000 Rupees.

And, that is a fair assumption. We can assume that, or it sounds logical, that as the equipment becomes older, the maintenance cost of that equipment increases. So, if we model it like this, then what should be the kind of company deposit, in the bank now. So, what should be the funds, that are set aside now. Since, this question is coming under, the food for thought scheme, we are not creating the answer for this, in this slide.

We probably do it in an assignment, at some point in time, but not now. What you need to do, is to go back to the previous slide, and try to break it up into A+G, A+2G, and so on. Try to get the A is at one place, as we did. Try to get, the G is increasing like this. And, use the formula, which was given, and try to come up with, the P1, and P2. Add them, and compare notes. Add them, and determine your answer.

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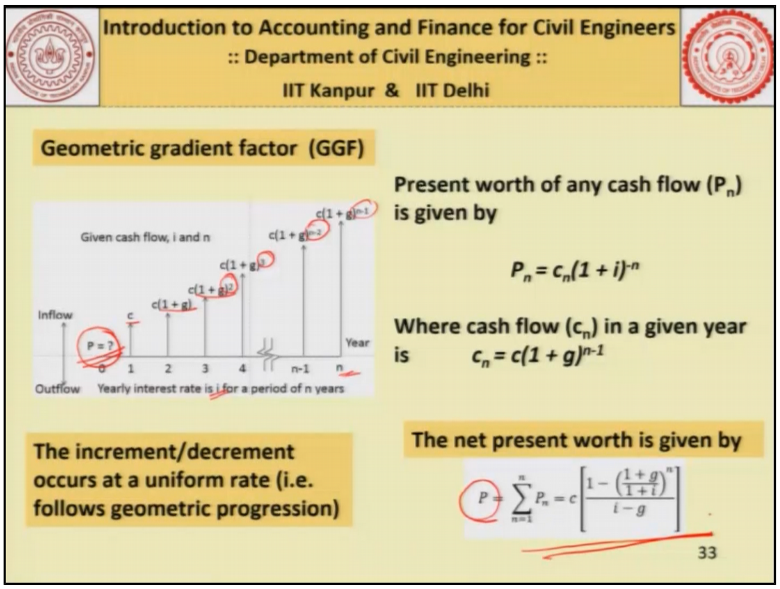

Now, as far as the geometric gradient factor is concerned, here, it is not arithmetic, it is geometric. Which means that, we are talking of, what is the P for C, C times 1+G, C times 1+G square, cubed, and so on, till it reaches, N years. And again, we were talking of, a rate of interest being I. So, for that, the increment or decrement occurs, at a uniform rate.

And, that is exactly, what is geometric progression, that we talk about. Or, the present worth is given by, PN is equal to CN times 1+ I to the power of –N, where cash flow CN in a given year is C times 1+ G to the power of N-1. And then, the net present worth, which is what we are looking for, that is, this P here, can be determined, using this formula.

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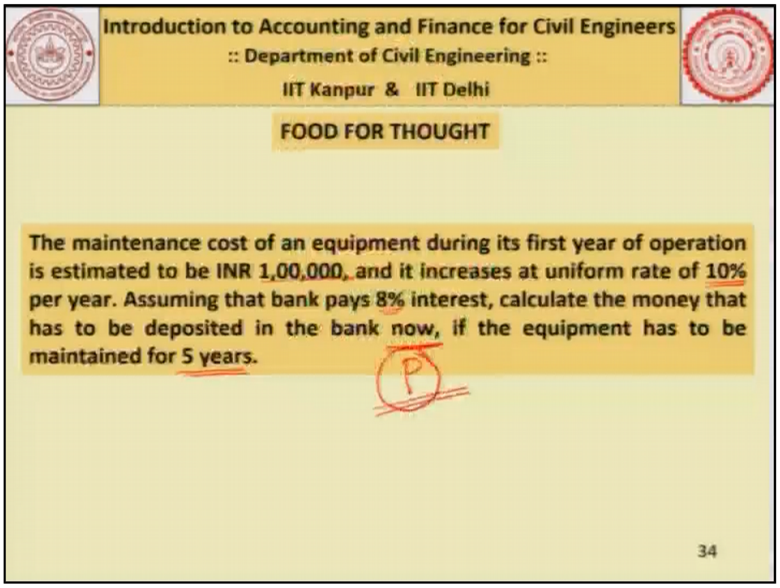

So, here again, it is a food for thought. We are not trying to solve this problem for you. What the statement says is, that the maintenance cost of an equipment, during the first year of operation is estimated to be, 1 Lakh. And, it increases uniformly, at the rate of 10% per year, assuming that the bank pays 8% interest. Calculate the amount of money, that has to be deposited in the bank, now. And, that is where, the P comes in.

So, what should be deposited now, if we expect the expenditure to start at 1 Lakh, increase at 10%, when the bank is giving us only 8%, and the equipment has to be maintained for 5 years. So now, you have to formulate this problem in the manner, which was given in the previous slide, and try to get the answer for. So, with this, we come more or less to the close of our discussion today. It has been a long day. And, we started with the principal of interest calculations.

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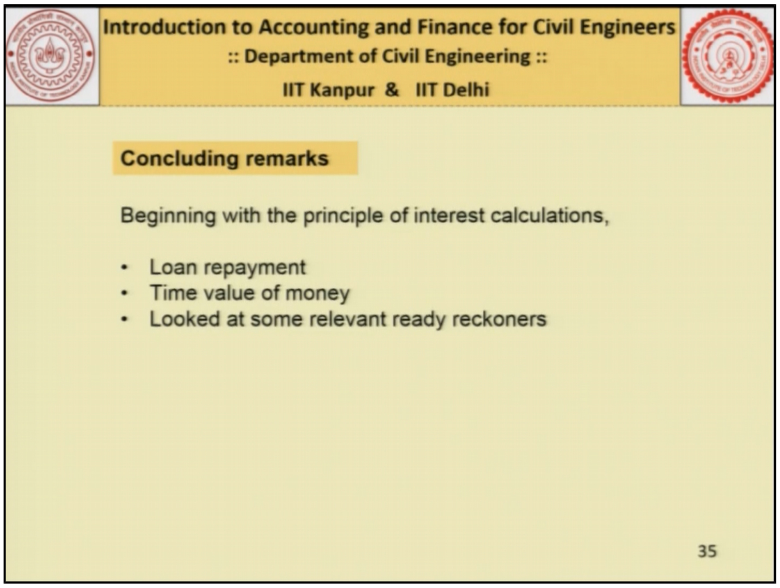

We talked a little bit about, loan repayment. We talked about, the idea of time value of money. And, we talked about, some relevant ready reckoners, which are available in the form of tables, which will enable you to determine the answers pretty quickly, rather than having to go to a spreadsheet all the time. Except that, you should remember that, those ready reckoners, or the kind of spreadsheet calculations that we did, make certain assumptions.

And, those assumptions are, what you should be very careful about, whether something is uniform, or it is not uniform, and so on, and that is what is going to make the calculations, more torturous for you. And, with this, we come to an end of the discussion today.

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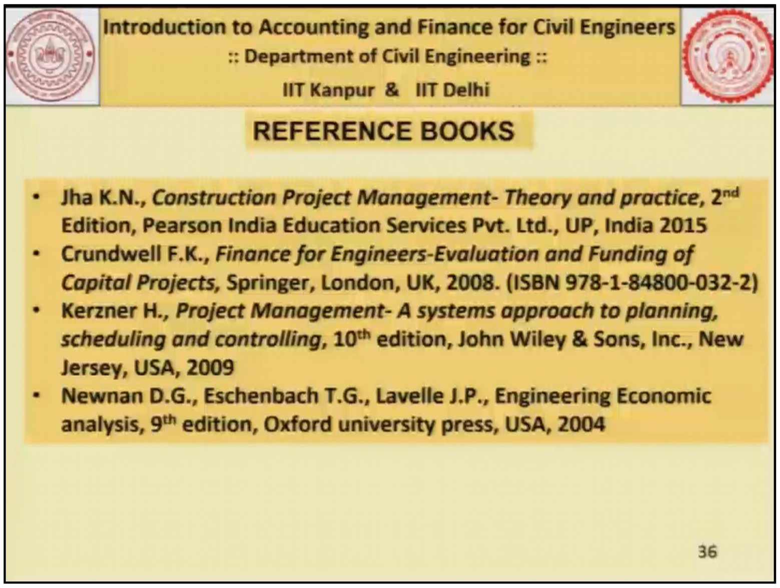

But, before that, here is a list of references, which you may like to use some time, to enhance your understanding. Thank you.